

## Complex Roots: Example

$$H(z) = \frac{z+3}{(z+5)(z^2+4z+5)}$$

Method ①: using complex (first order) roots

Method ②: using second order polynomials

$$H(z) = \frac{A}{z+5} + \frac{Bz+C}{z^2+4z+5}$$

$$A = \frac{(z+5)H(z)}{z+5} = -0.2.$$

$$Az^2 + 4Az + 5A + Bz^2 + 5Bz + Cz + 5C = z + 3$$
$$(A+B)z^2 + (4A+5B+C)z + (5A+5C) = z + 3$$

$$\Rightarrow \left. \begin{array}{l} A+B=0 \\ 4A+5B+C=1 \\ 5A+5C=3 \end{array} \right\} \Rightarrow \begin{array}{l} A=-0.2 \\ B=0.2 \\ C=0.8 \end{array}$$

$$\text{So, } H(z) = \frac{-0.2}{z+5} + \frac{0.2z+0.8}{z^2+4z+5}$$

\* An exponential (or other function) in the numerator

e.g.  $H(z) = \frac{(z+3)z^{-3}}{z^3+7z^2+10z} = \frac{z+3}{z(z+2)(z+5)} \cdot z^{-3}$

$$= \left( \frac{A_1}{z} + \frac{A_2}{z+2} + \frac{A_3}{z+5} \right) z^{-3}$$

$$= \frac{0.3}{z} z^{-3} - \frac{1}{6} \cdot \frac{1}{z+2} z^{-3} - \frac{2}{15} \cdot \frac{1}{z+5} z^{-3}$$

### Repeated Root Example:

e.g.  $H(z) = \frac{z+3}{z(z+2)^2(z+5)} = \frac{A_1}{z} + \frac{A_2}{z+2} + \frac{A_3}{(z+2)^2} + \frac{A_4}{z+5}$

$$A_1 = z \cdot H(z) \Big|_{z=0} = \frac{3}{20}$$

$$A_3 = (z+2)^2 H(z) \Big|_{z=-2} = \frac{1}{6}$$

$$A_4 = (z+5) H(z) \Big|_{z=-5} = \frac{2}{45}$$

①  $A_2 = \frac{d}{dz} \left[ (z+2)^2 H(z) \right] \Big|_{z=-2} = \frac{d}{dz} \left( \frac{z+3}{z(z+5)} \right) \Big|_{z=-2}$

$$= \frac{z(z+5) - (z+3)(z+5)}{z(z+5)^2} \Big|_{z=-2} = \frac{-7}{36}$$

### ② Cross-multiplication method

$$z(z+2)^2(z+5) \cdot \frac{z+3}{z(z+2)^2(z+5)} = z(z+2)^2(z+5) \left[ \frac{A_1}{z} + \frac{A_2}{z+2} + \frac{A_3}{(z+2)^2} + \frac{A_4}{z+5} \right]$$

$$z+3 = (z+2)(z+5)A_1 + z(z+2)(z+5)A_2 + z(z+5)A_3 + z(z+2)^2A_4$$

$$\Rightarrow \begin{cases} A_1 + A_2 + A_4 = 0 \\ 9A_1 + A_2 + A_3 + 4A_4 = 0 \\ 24A_1 + 10A_2 + 5A_3 + 4A_4 = 0 \\ 20A_1 = 3 \end{cases}$$

$$\Rightarrow \begin{bmatrix} 1 & 1 & 0 & 1 \\ 9 & 7 & 1 & 4 \\ 24 & 10 & 5 & 4 \\ 20 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} A_1 \\ A_2 \\ A_3 \\ A_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 3 \end{bmatrix}$$

$$\begin{aligned} A_1 &= 0.15 \\ A_2 &= -0.1944 \\ A_3 &= -0.1667 \\ A_4 &= 0.0444 \end{aligned}$$