## Decimation

$$x(n) \longrightarrow \boxed{\downarrow M} \longrightarrow x_d(n) = x(nM)$$

In CT-sampling:

$$X(e^{j\omega}) = \frac{1}{T}\sum_{k=-\infty}^{\infty} X_c\left(j\left(\frac{\omega}{T} - \frac{2\pi k}{T}\right)\right)$$

similarly, $x_d(n) = x(nM) = x_c(nT')$ with $T' = MT$

$$X_d(e^{j\omega}) = \frac{1}{T'}\sum_{r=-\infty}^{\infty} X_c\left(j\left(\frac{\omega}{T'} - \frac{2\pi r}{T'}\right)\right)$$

$$= \frac{1}{MT}\sum_{r=-\infty}^{\infty} X_c\left(j\left(\frac{\omega}{MT} - \frac{2\pi r}{MT}\right)\right)$$

$$\Rightarrow \boxed{X_d(e^{j\omega}) = \frac{1}{M}\sum_{i=0}^{M-1} X\left(e^{j\left(\frac{\omega}{M} - \frac{2\pi i}{M}\right)}\right)}$$

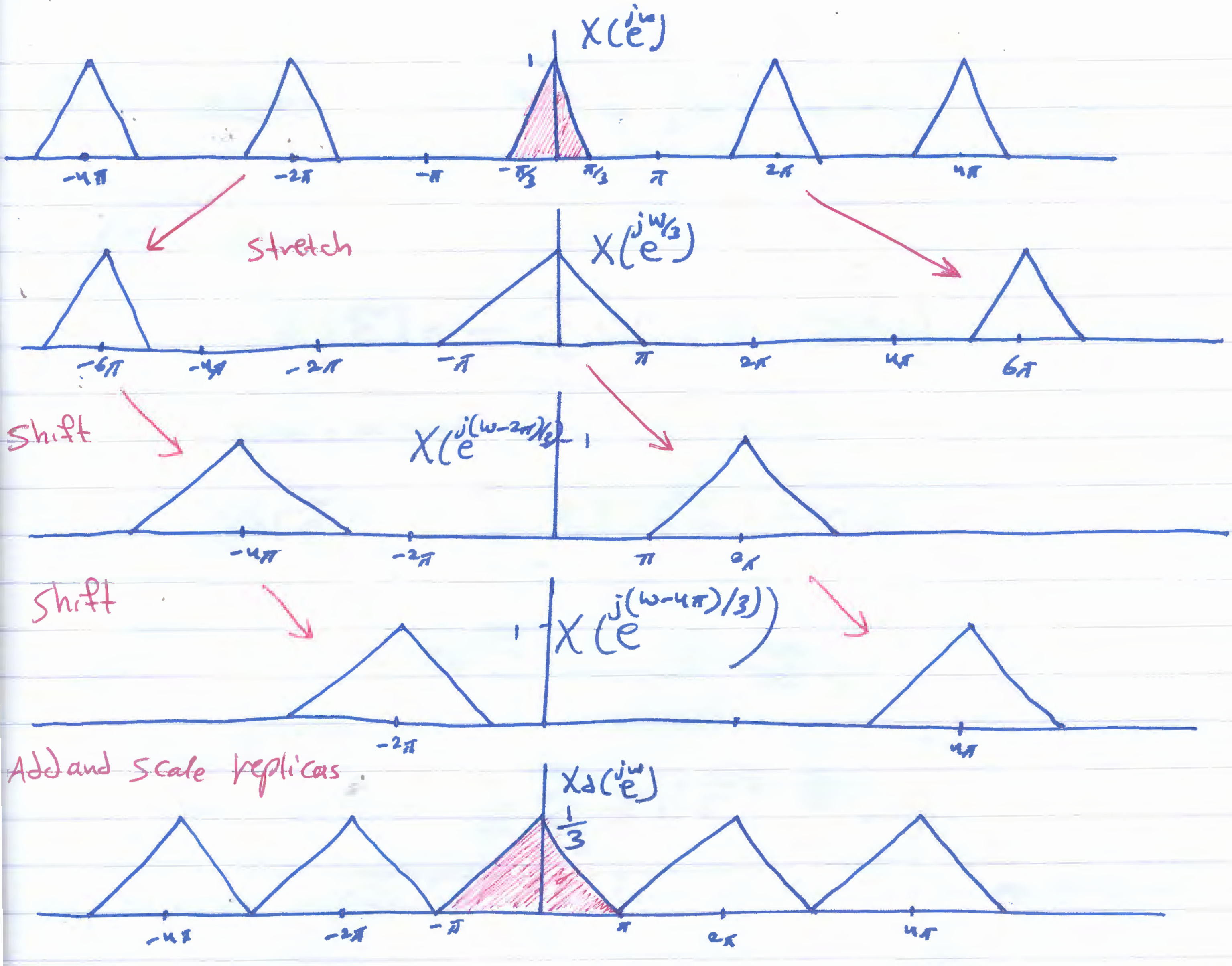
So, To skitch $X_d(e^{j\omega})$ directly from $X(e^{j\omega})$ follow 3 steps:

1. Stretch $X(e^{j\omega})$ by a factor $M$ to obtain $X(e^{j\frac{\omega}{M}})$. note that the highest freq. of $X(e^{j\omega})$, $\omega_H$, is "repositioned" to freq. $\omega = \omega_H \cdot D$.

2. Create and put $M$ copies of $X(e^{j\frac{\omega}{M}})$ at freq. $\omega = 2\pi i$ (integer multiple of $2\pi$) for $i = 0, 1, 2, \ldots, M-1$.

3. Add the $M$ streched and shifted replicas and then divide by $M$ to obtain the spectrum $X_d(e^{j\omega})$ of the downsampled sequence $x_d(n) = x(nM)$.

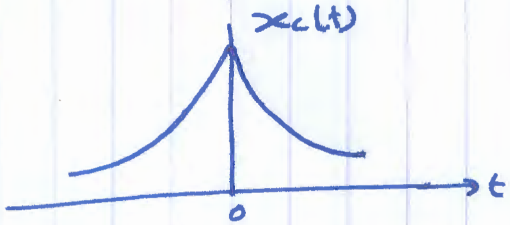
Example. Let $M=3$ and Bandwidth of signal $x(n)$ is $\pi/3$.



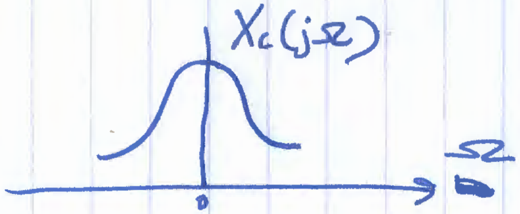

**Interpolation:**

$$x(n) \rightarrow \boxed{\uparrow L} \rightarrow x_L(n) = x(n/L)$$

$$X_L(e^{j\omega}) = X(e^{j\omega L})$$




$\xleftrightarrow{\text{CTFT}}$

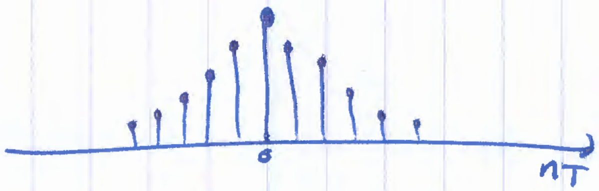



↓ Sampling

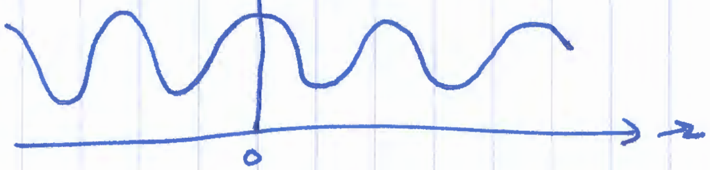
$x(n) = x_c(nT)$

↓ periodization

$\tilde{X}(e^{j\Omega T}) = \frac{1}{T}\sum_{k=-\infty}^{\infty} X_c\left(j\Omega - j\frac{2\pi k}{T}\right)$




$\xleftrightarrow{\text{DTFT}}$

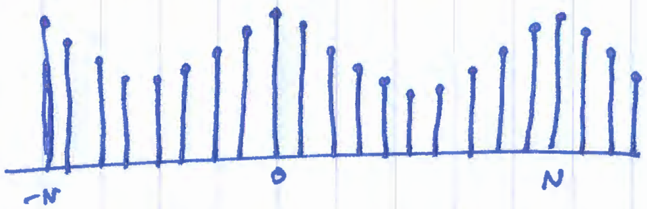



↓ periodization

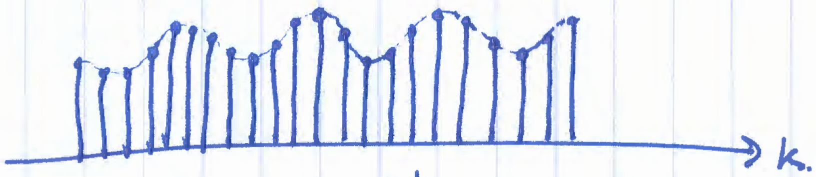
$\tilde{x}(n) = \sum_{l=-\infty}^{\infty} x_c(nT - lNT)$

↓ Sampling

$X(k) = \frac{1}{T}\sum X_c\left(j\frac{2\pi k}{NT} - j\frac{2\pi k}{T}\right)$




$\underset{N}{\xleftrightarrow{\text{DFT}}}$




$X(k) = \sum_{n=0}^{N-1} x(n) W_N^{kn}$

relationship between CTFT, DTFT and DFT.