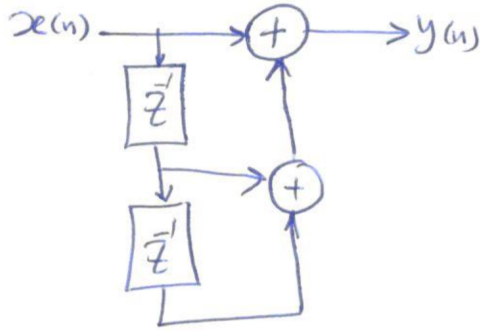
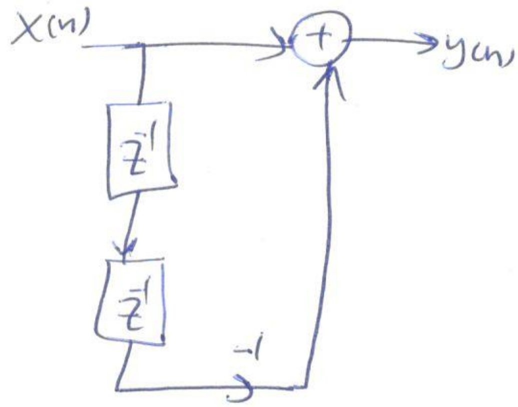
Examples:

Two filter structures are shown below. Show that both filters have linear phase?





$$y(n) = x(n) + x(n-1) + x(n-2)$$

$$H(z) = 1 + z^{-1} + z^{-2}$$

$$H(e^{jw}) = 1 + e^{-jw} + e^{-j2w}$$

$$= e^{-jw}\left(e^{jw} + e^{-jw} + 1\right)$$

$$= e^{-jw}\left(1 + 2\cos w\right)$$

Phase $= -w$

$\therefore$ linear phase

$$y(n) = x(n) - x(n-2)$$

$$H(z) = 1 - z^{-2}$$

$$H(e^{jw}) = 1 - e^{-j2w}$$

$$= 2j\,e^{-jw}\left(\frac{e^{jw} - e^{-jw}}{2j}\right)$$

$$= 2\,e^{j\frac{\pi}{2}}\,e^{-jw}\sin w$$

$$= e^{j\left(\frac{\pi}{2} - w\right)}\left(2\sin w\right)$$

$\therefore$ Phase $= \frac{\pi}{2} - w \Rightarrow$ linear phase.

Example: Consider a filter described by

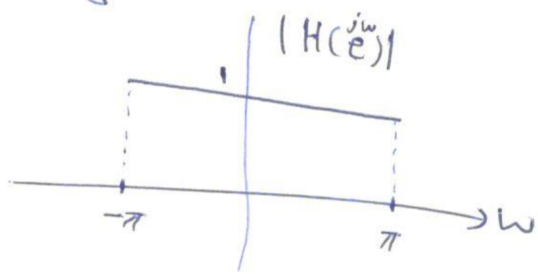
$$H(z) = \frac{a + c z^2}{c + a z^2}, \text{ where } a \text{ and } c \text{ are constants.}$$

Show that the magnitude response $|H(e^{j\omega})|$ is unity for all ω.

$$|H(e^{j\omega})|^2 = H(e^{j\omega}) \cdot H^*(e^{j\omega}) = \frac{c + a e^{-j2\omega}}{a + c e^{-j2\omega}} \cdot \frac{c + a e^{j2\omega}}{a + c e^{j2\omega}}$$

$$= \frac{c^2 + a^2 + ac\left[e^{j2\omega} + e^{-j2\omega}\right]}{a^2 + c^2 + ac\left[e^{j2\omega} + e^{-j2\omega}\right]} = 1$$

This is an All-pass filter



Example: Find and Draw parallel realization of a third order system $H(z)$

$$H(z) = \frac{23 + 40z^{-1} + 36z^{-2} + 19z^{-3}}{(2 + z^{-1})(5 + 2z^{-1} + 3z^{-2})}$$

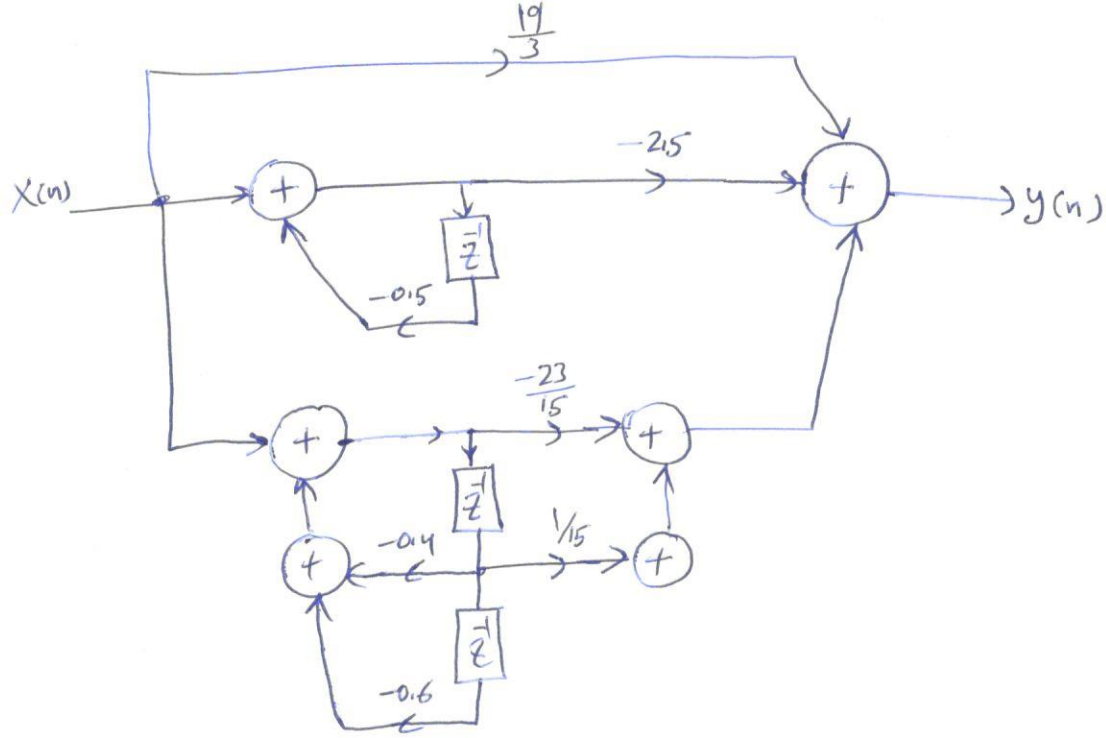
by long-division, by cover-up, by cross multiplication

$$= \underbrace{A}_{H_0(z)} + \underbrace{\frac{B}{2 + z^{-1}}}_{H_1(z)} + \underbrace{\frac{C - Dz^{-1}}{5 + 2z^{-1} + 3z^{-2}}}_{H_2(z)}$$

$$= \frac{19}{3} - \frac{5}{2 + z^{-1}} - \frac{1}{3}\frac{(23 - z^{-1})}{5 + 2z^{-1} + 3z^{-2}}$$

$$= \frac{19}{3} - \frac{5}{2}\frac{1}{1 + 0.5z^{-1}} - \frac{1}{3}\frac{(23 - z^{-1})}{5(1 + 0.4z^{-1} + 0.6z^{-2})}$$

$$= \frac{19}{3} + \frac{-2.5}{1 + \frac{1}{2}z^{-1}} + \frac{-\frac{23}{15} + \frac{1}{15}z^{-1}}{1 + 0.4z^{-1} + 0.6z^{-2}}$$

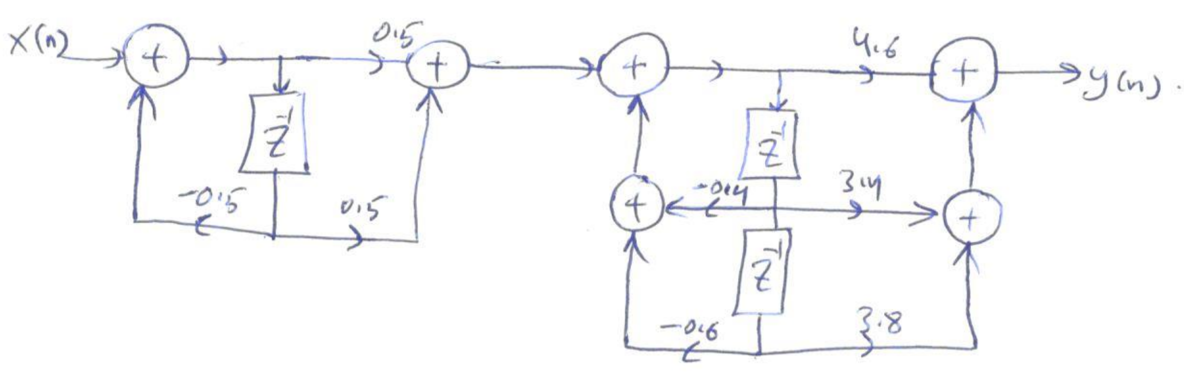


A cascad realization of this third-order system is given by

$$H(z) = \frac{(1+z^{-1})}{(2+z^{-1})} \cdot \frac{23+17z^{-1}+19z^{-2}}{5+2z^{-1}+3z^{-2}}$$

$$= \frac{1+z^{-1}}{2(1+\frac{1}{2}z^{-1})} \cdot \frac{23+17z^{-1}+19z^{-2}}{5(1+0.4z^{-1}+0.6z^{-2})}$$

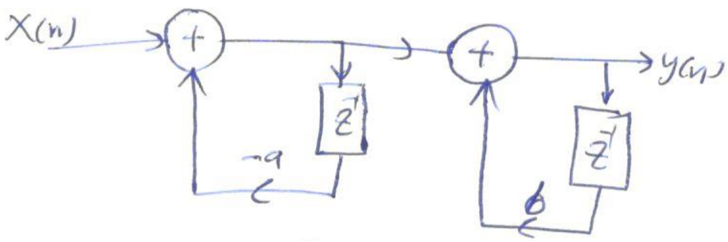
$$= \left(\frac{0.5+0.5z^{-1}}{1+\frac{1}{2}z^{-1}}\right) \cdot \left(\frac{4.6+3.4z^{-1}+3.8z^{-2}}{1+0.4z^{-1}+0.6z^{-2}}\right)$$



Example:

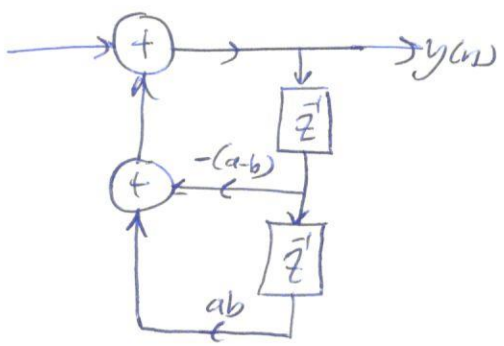
Implement the following system in the Cascade, direct form II and Parallel structures. All coefficients are real!

(a) $$H(z) = \frac{1}{(1+az^{-1})(1-bz^{-1})}$$



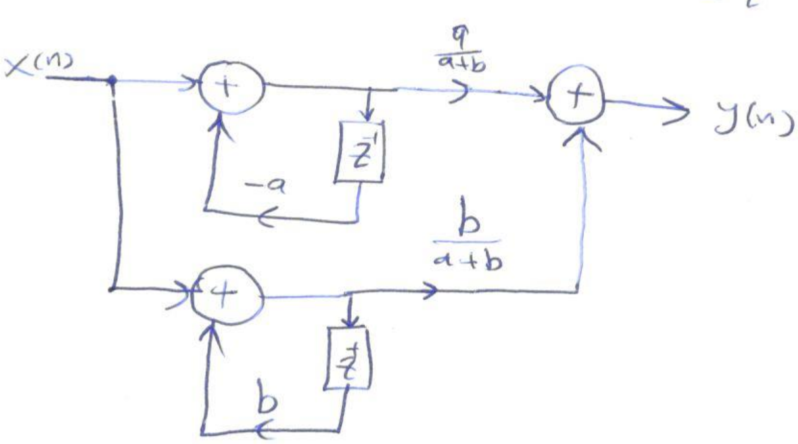
Cascade.

$$H(z) = \frac{1}{1+(a-b)z^{-1} - abz^{-2}}$$

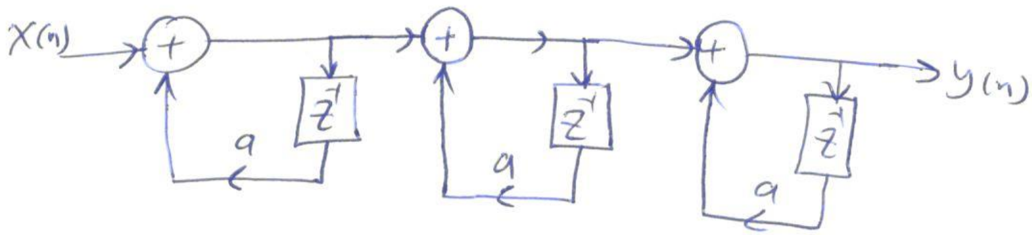


Direct form II

$$H(z) = \frac{A}{1+az^{-1}} + \frac{B}{1-bz^{-1}} = \frac{\frac{a}{a+b}}{1+az^{-1}} + \frac{\frac{b}{a+b}}{1-bz^{-1}}$$



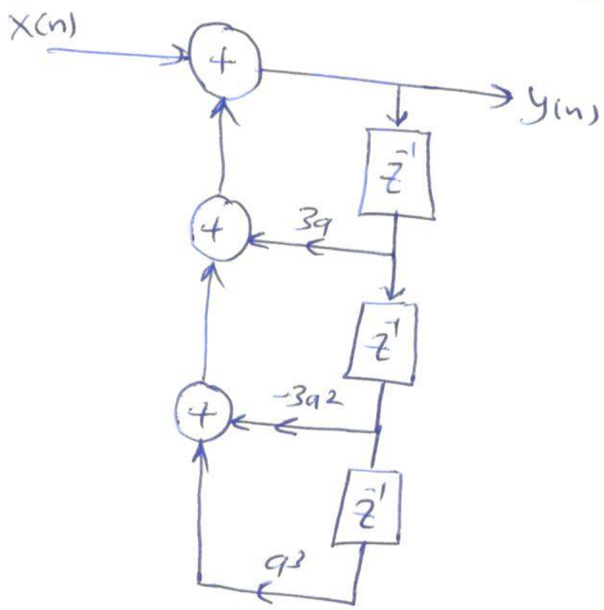
(b) $H(z) = \dfrac{1}{(1 - az^{-1})^3}$



Cascade

* No parallel structure exists because partial fraction expansion cannot be performed.

$H(z) = \dfrac{1}{1 - 3az^{-1} + 3a^2z^{-2} - a^3z^{-3}}$

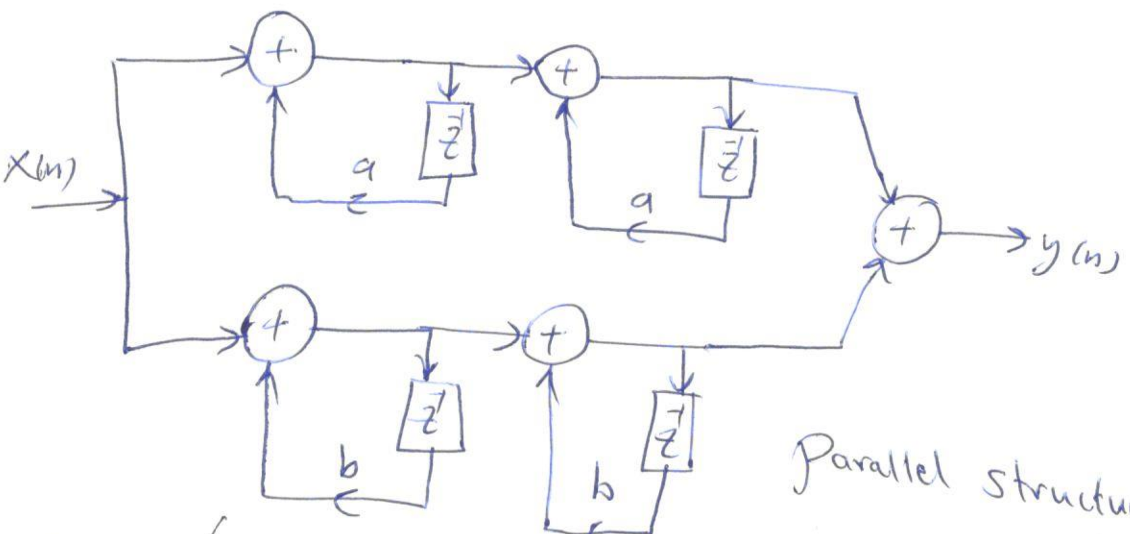


Direct form II

(c) $H(z) = \dfrac{1}{(1-az^{-1})^2} + \dfrac{1}{(1-bz^{-1})^2}$ ← parallel.

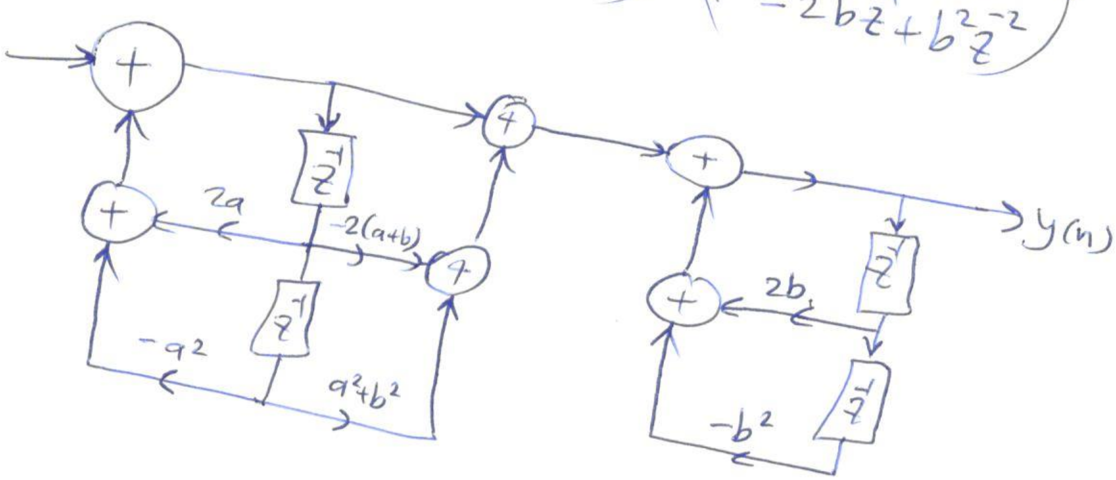
$$= \frac{1-2(a+b)z^{-1}+(a^2+b^2)z^{-2}}{(1-2az^{-1}+a^2z^{-2})(1-2bz^{-1}+b^2z^{-2})}$$ ← Cascade

$$H(z) = \frac{1}{(1-za^{-1})(1-za^{-1})} + \frac{1}{(1-bz^{-1})(1-bz^{-1})}$$



Parallel structure.

$$H(z) = \left(\frac{1-2(a+b)z^{-1}+(a^2+b^2)z^{-2}}{(1-2az^{-1}+a^2z^{-2})}\right)\left(\frac{1}{1-2bz^{-1}+b^2z^{-2}}\right)$$



Cascade structure.