When the input to a causal LTI system is

$$x[n] = -\frac{1}{3}\left(\frac{1}{2}\right)^n u[n] - \frac{4}{3} 2^n u[-n-1]$$

The z-transform of the output is

$$Y(z) = \frac{1+z^{-1}}{(1-z^{-1})(1+\frac{1}{2}z^{-1})(1-2z^{-1})}$$

(a) Find z-transform of x[n]?

(b) What is the region of convergence (ROC) of Y(z)?

(c) Find the system function of this system H(z)? Plot its zero-pole diagram and indicate ROC?

(d) Is the system stable?

The signal y[n] is the output of an LTI system with impulse response h[n] for a given input x[n]. Assuming signal y[n] is stable and its z-transform Y(z) has a pole-zero diagram in the left figure, and that signal x[n] is also stable and its pole-zero diagram shown in the right figure.

z-plane

z-plan
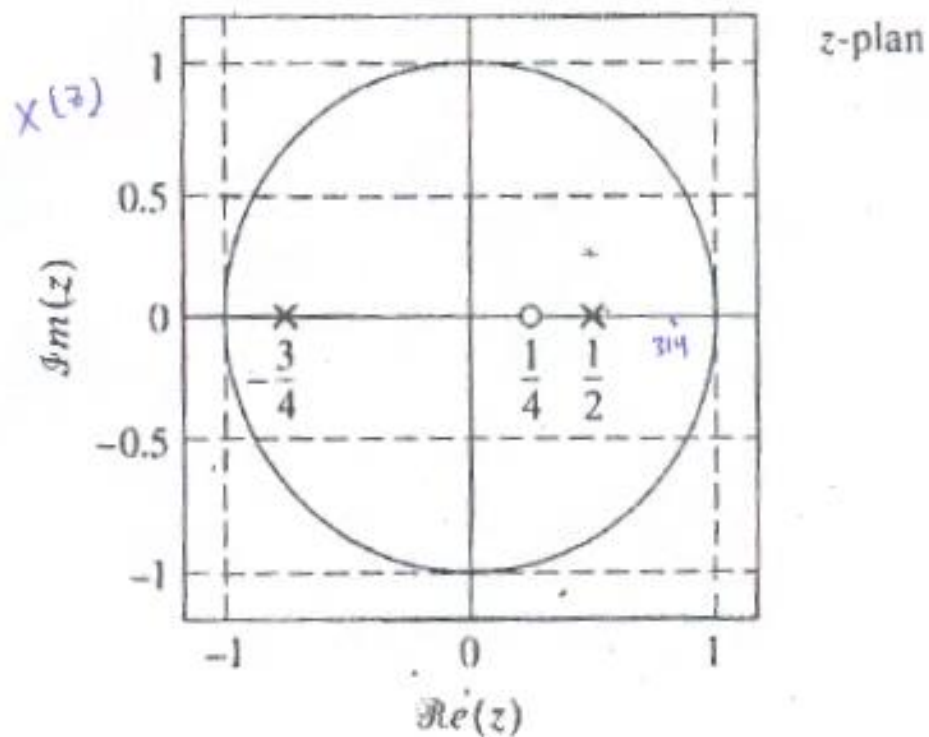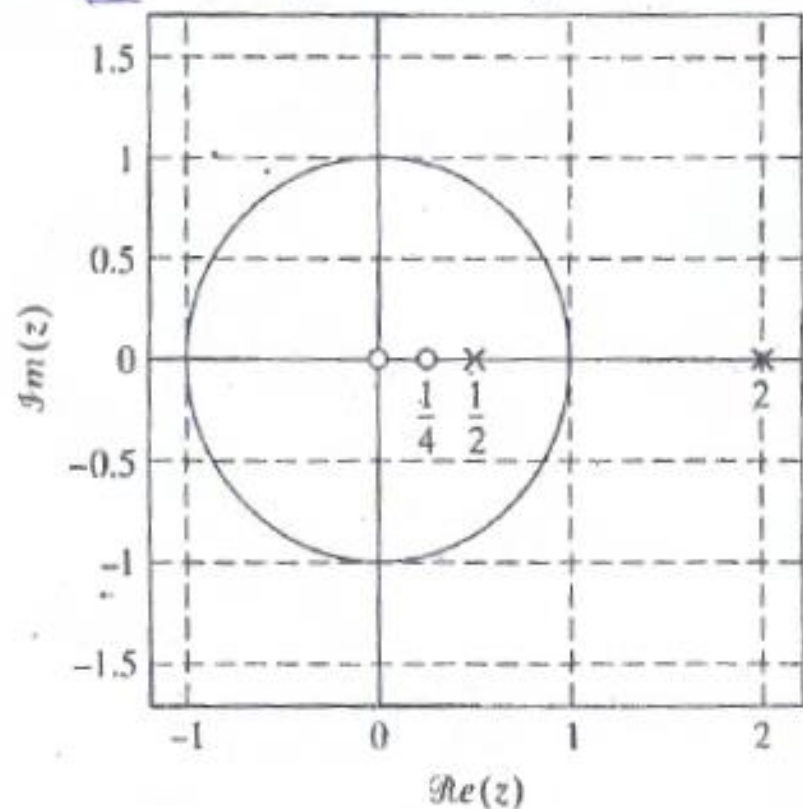
(a) What is the region of convergence (ROC) of Y(z)?
(b) Is y[n] left sided, right sided or two sided?
(c) What is the ROC of X(z)?
(d) Is x[n] a causal sequence? That is, does x[n]= 0 for n<0?
(e) Draw the pole-zero plot of H(z) and specify its ROC.
(f) Is h[n] anticausal? That is, does h[n]=0 for n>0?

The system function H(z) of a causal LTI system has pole-zero plot shown in the following figure. It is also know that $H(z) = \dfrac{3}{4}$ when z=1.
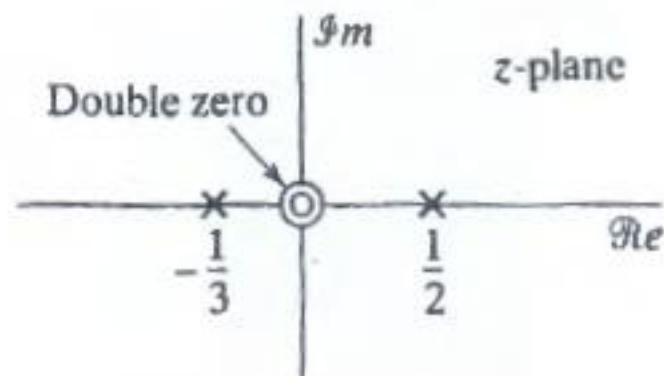
(a) Determine H(z).

(b) Determine impulse response h(n) of the system.

(c) Determine the response of the system to the following input signal:

$$x(n) = u(n) - \frac{1}{2}u(n-1)$$