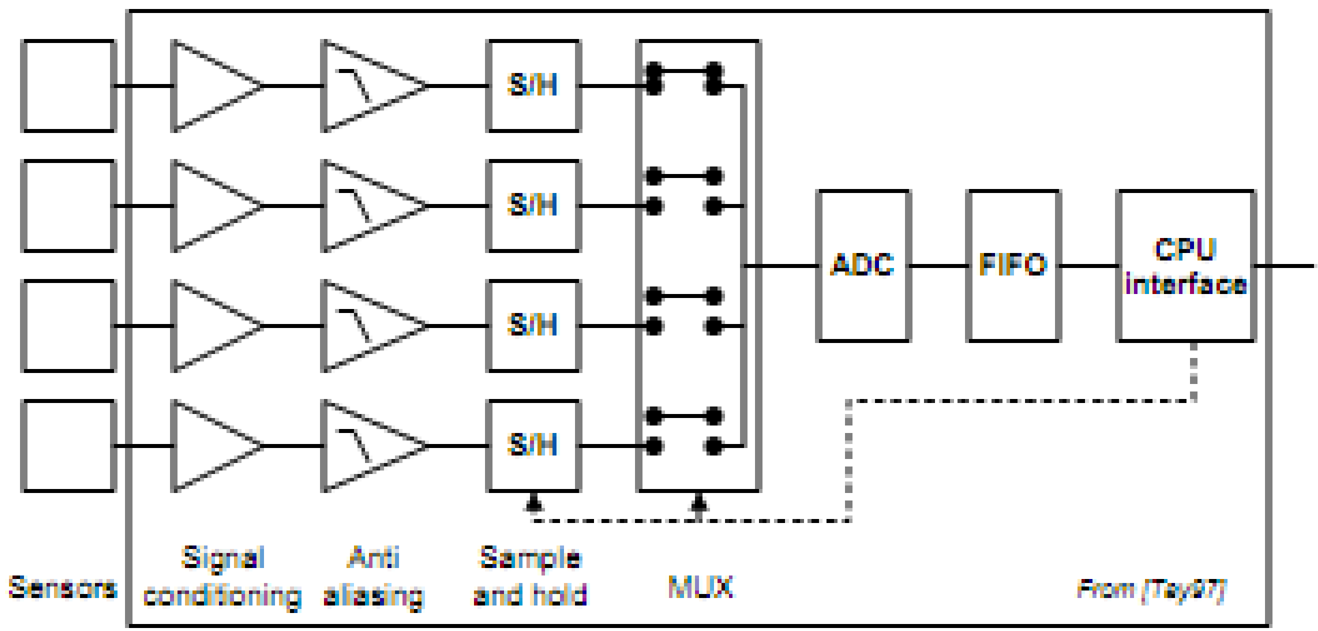
Data Acquisition I

- Architecture of DAQ systems
- Signal conditioning
- Aliasing

Architecture of data acquisition systems

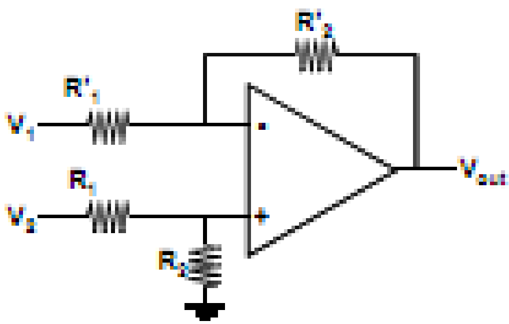


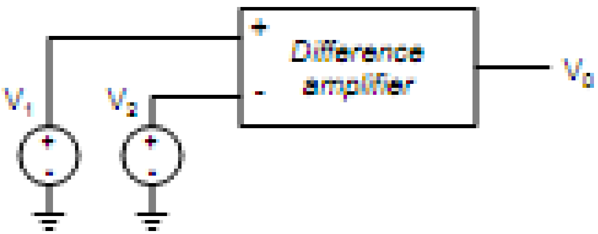
Signal conditioning

- Instrumentation amplifiers
- Filters
- Integrators/differentiators

Instrumentation amplifiers

- Consider the difference amplifier we saw in the previous lecture

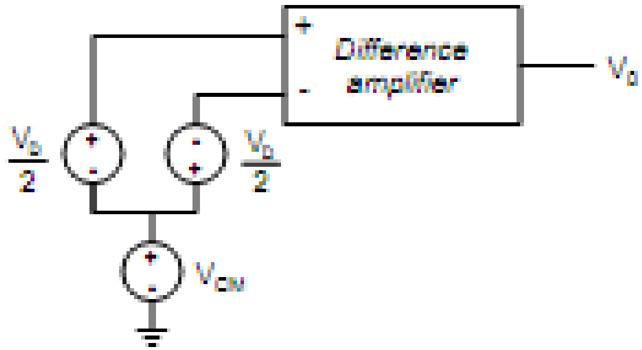




- We define COMMON-MODE and DIFFERENCE-MODE voltage as

$$V_{CM} = \frac{V_2 + V_1}{2}$$

$$V_D = V_2 - V_1$$



Instrumentation amplifiers

- **As a result of a mismatch in the resistors ($R'_k \neq R_k$), the differential inputs may not have the same gain**

$$V_0 = G(V_2 - V_1) \overset{R'_k \neq R_k}{=} G_2V_2 - G_1V_1 = G_2\left(-\frac{V_D}{2} + V_{CM}\right) - G_1\left(\frac{V_D}{2} + V_{CM}\right) =$$

$$= -V_D\left(\frac{G_2 + G_1}{2}\right) + V_{CM}(G_2 - G_1) = -V_DG_D + V_{Cm}G_{CM}$$

- **We define COMMON-MODE REJECTION RATIO as**

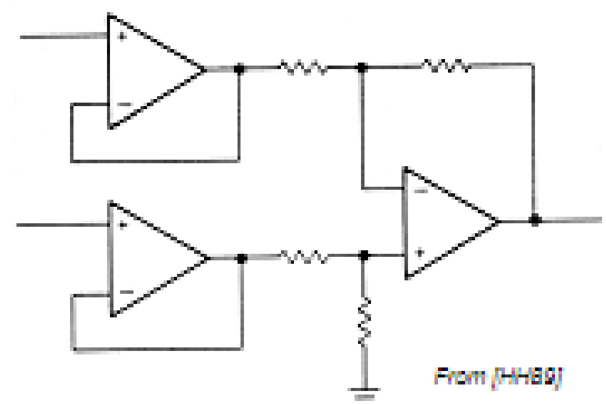
$$CMRR = 20\log_{10}\left(\frac{G_D}{G_{CM}}\right) = 20\log_{10}\left(\frac{G_2 + G_1}{2(G_2 - G_1)}\right)$$

  - CMRR is, in practice, a function of frequency, and its magnitude decreases with increasing frequency

- **An additional shortcoming of the difference amplifier is its LOW INPUT IMPEDANCE**

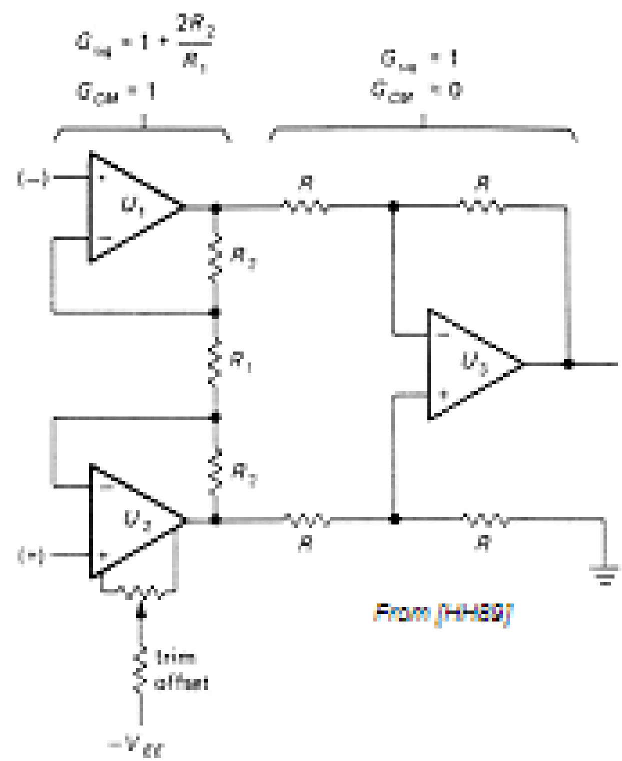
Instrumentation amplifiers

- The term INSTRUMENTATION AMPLIFIER is used to denote a difference amplifier with
  - High gain (INA2126)
  - Single-ended output
  - High input impedance
  - High CMRR
- High input impedance may be achieved by buffering the differential inputs
  - This solution, however, requires high CMR both in the followers and in the final op-amp
    - Otherwise, since the input buffers have unity gain, all the CM rejection must come in the output op-amp, requiring precise resistor matching

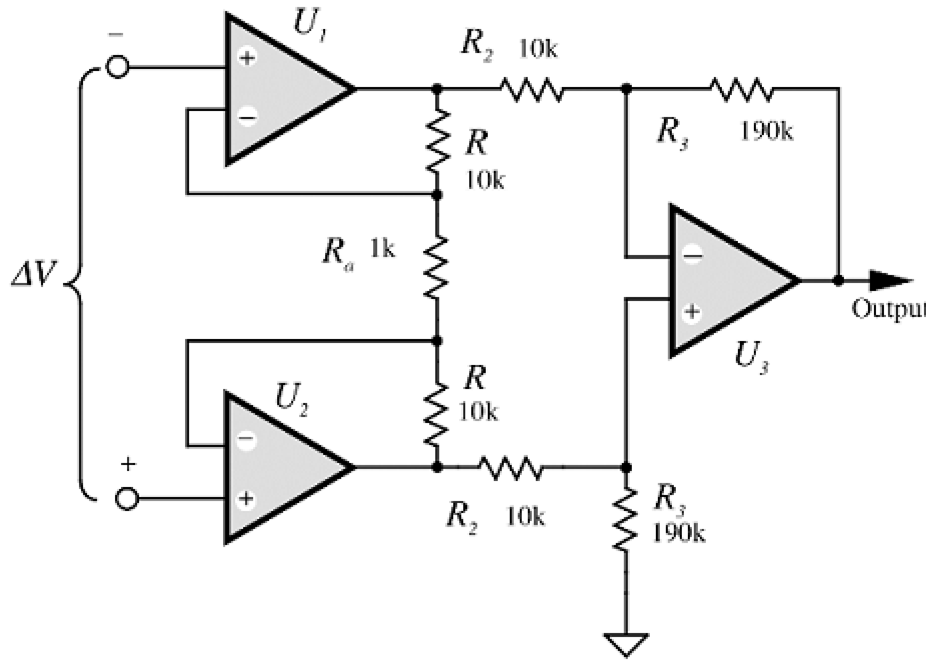


Common mode rejection ratio

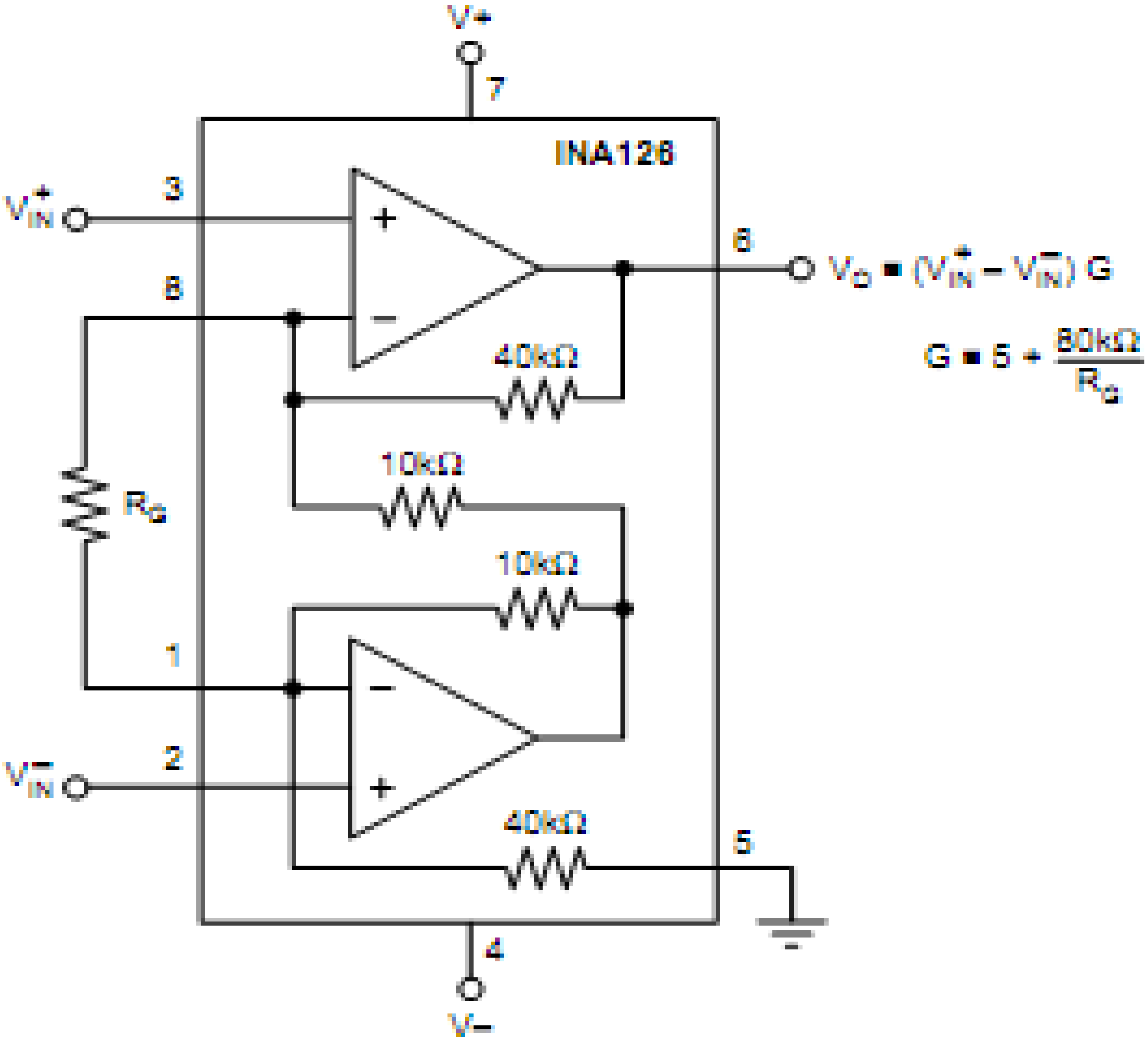
- A better solutions is the "standard" instrumentation amplifier shown below
  - Input stage provides high GD and unity GCM
    - Close resistor (R2) matching is NOT critical
  - As a result, the output op-amp (U3) does not require exceptional CMRR and resistor matching in U3is not critical
  - Offset trimming can be done at one of the input op-amps



INA…



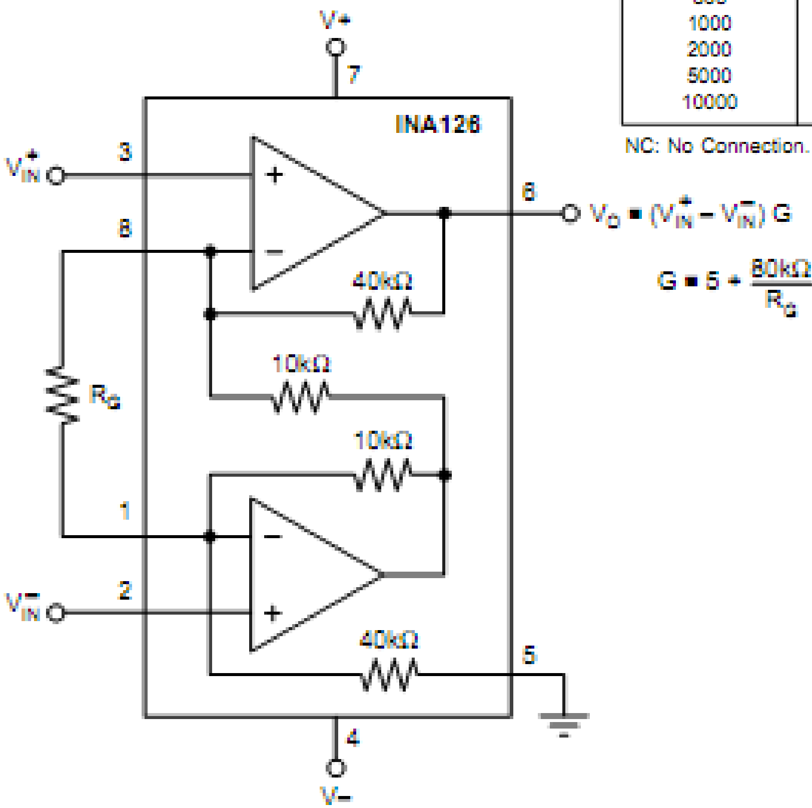
$$A = \left(1 + \frac{2R}{R_a}\right)\frac{R_3}{R_2}$$


V+
7
INA126
3
V_{IN}^{+}
6
$V_O = (V_{IN}^{+} - V_{IN}^{-})\,G$
$G = 5 + \frac{80k\Omega}{R_G}$
8
40kΩ
10kΩ
R_G
10kΩ
1
2
V_{IN}^{-}
40kΩ
5
4
V−

INA126.....

DESIRED GAIN (V/V)	R_G (Ω)	NEAREST 1% R_G VALUE
5	NC	NC
10	16k	15.8k
20	5333	5360
50	1779	1780
100	842	845
200	410	412
500	162	162
1000	80.4	80.6
2000	40.1	40.2
5000	16.0	15.8
10000	8.0	7.87

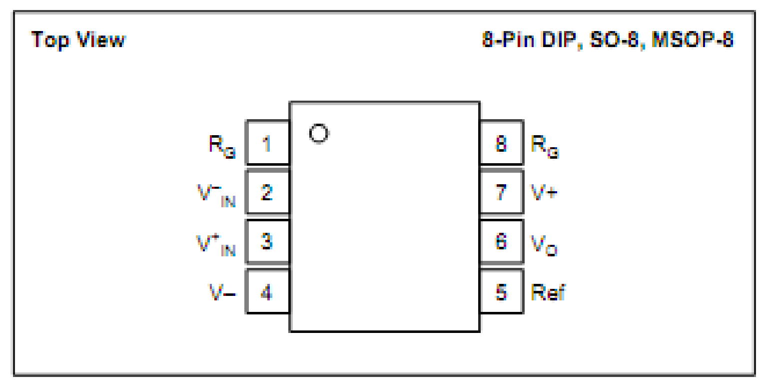
NC: No Connection.



APPLICATIONS

- INDUSTRIAL SENSOR AMPLIFIER: Bridge, RTD, Thermocouple
- PHYSIOLOGICAL AMPLIFIER: ECG, EEG, EMG
- MULTI-CHANNEL DATA ACQUISITION
- PORTABLE, BATTERY OPERATED SYSTEMS

PIN CONFIGURATION (Single)

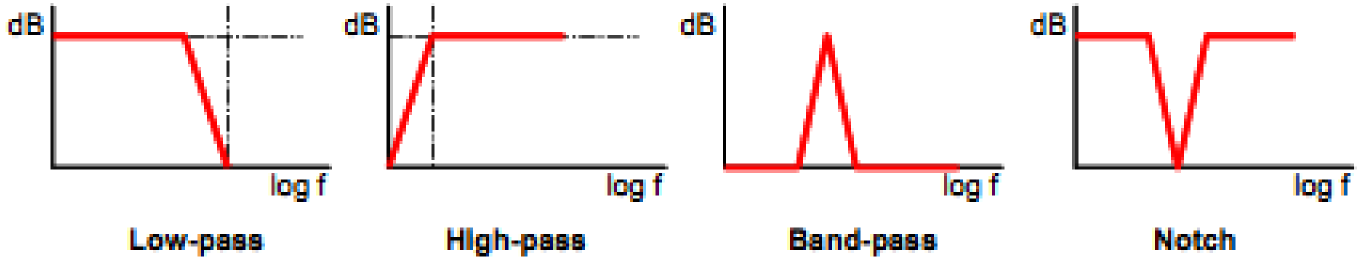


Filters

- Filters are used to remove unwanted bandwidths from a signal
- Filter classification according to implementation
  - Active filters include RC networks and op-amps
    - Suitable for low frequency, small signal
    - Active filters are preferred since avoid the bulk and non-linearity of inductors and can have gains greater than 0dB
    - However, active filters require a power supply
  - Passive filters consist of RCL networks
    - Simple, more suitable for frequencies above audio range, where active filters are limited by the op-map bandwidth
- Digital filters
  - DSP is beyond the scope of this course

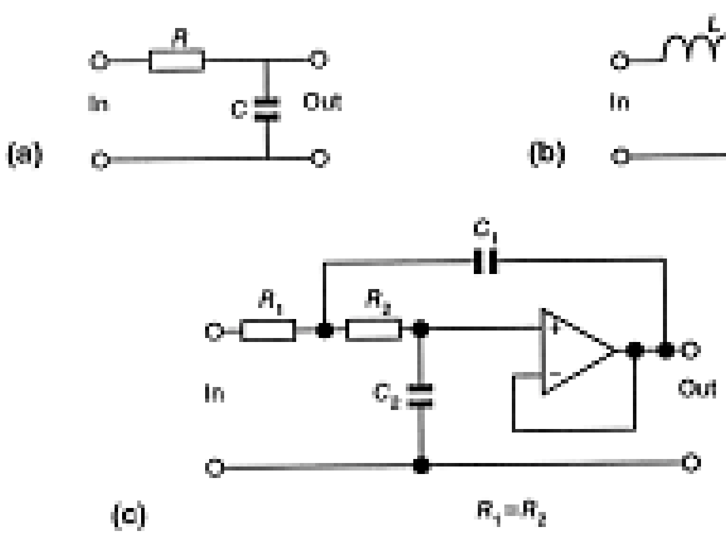
Filters

- Filter classification according to frequency response
  - Low-pass filter
  - n High-pass filter
  - Band-pass filter
  - Band-stop (Notch)

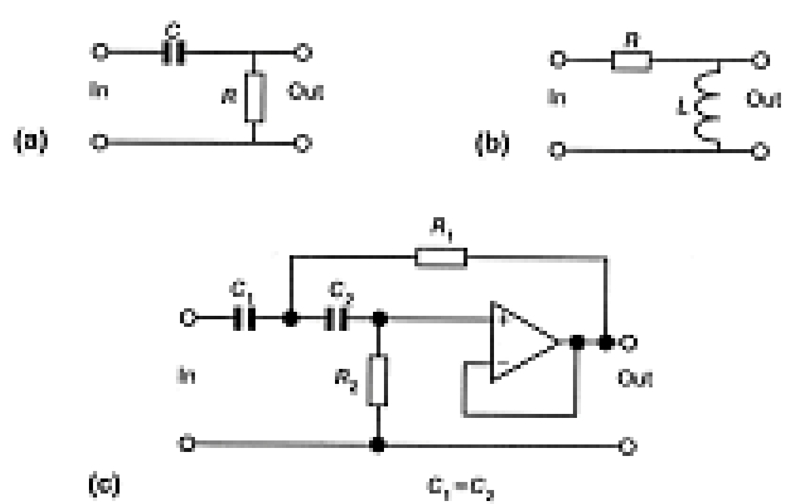


Low- and high-pass filters

- Low pass filters
- High pass filters



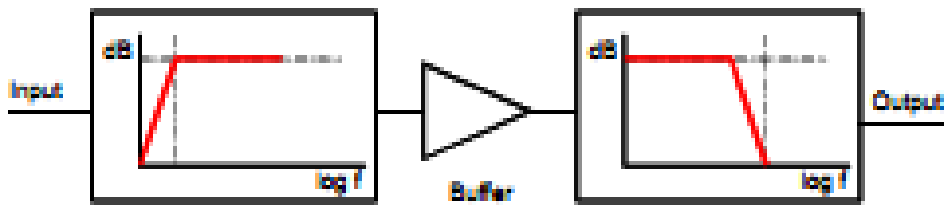
From [Ram98]



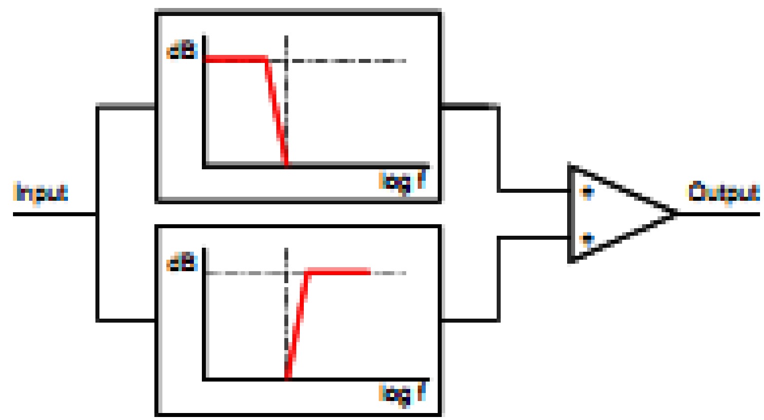
From [Ram98]

Band-pass and band-stop filters

- **Band-pass**
  - High-pass and low pass in series
    - High-pass should usually precede
      - Corner frequency of low-pass must then be higher
    - If these are passive filters they should be buffered in between

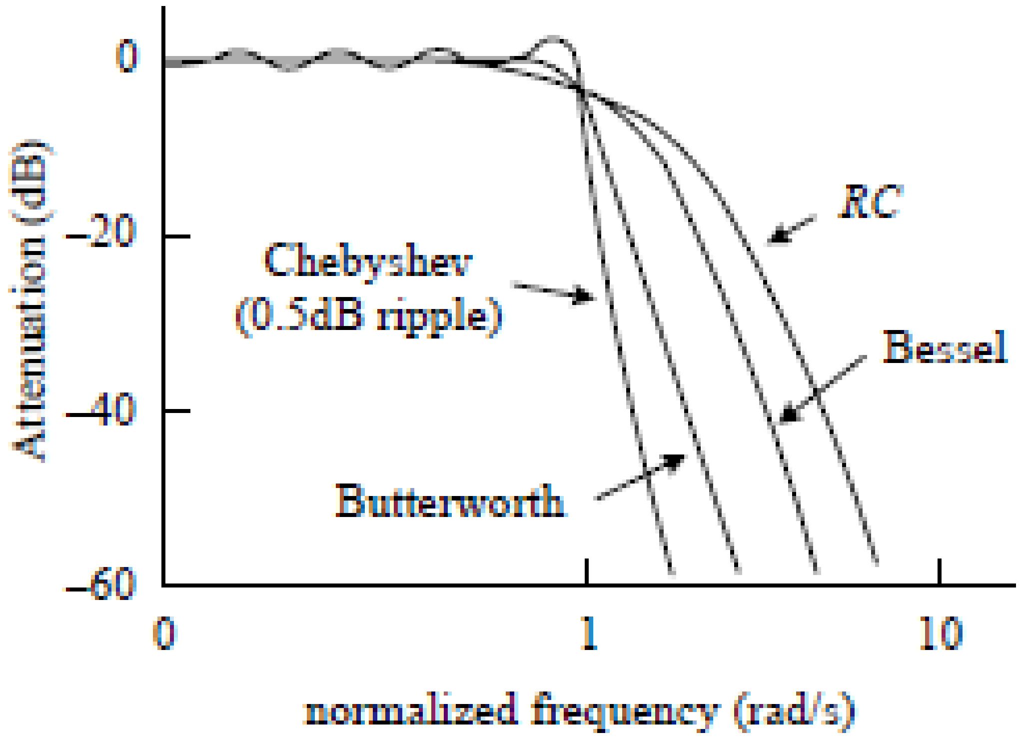


- **Band-stop**
  - High-pass and low-pass in parallel followed by a summer
    - Corner frequency of high-pass must be higher

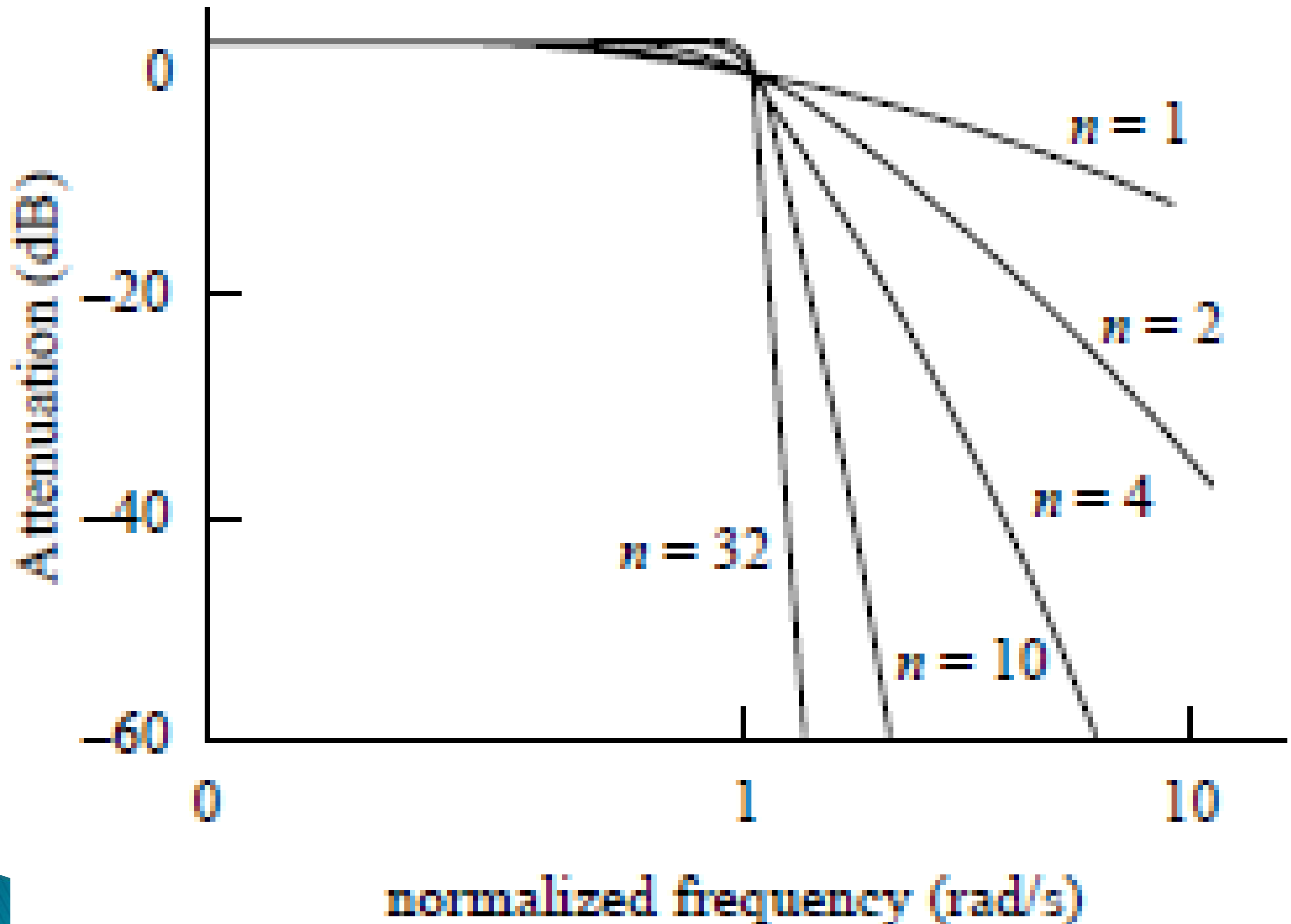


Types of Filters

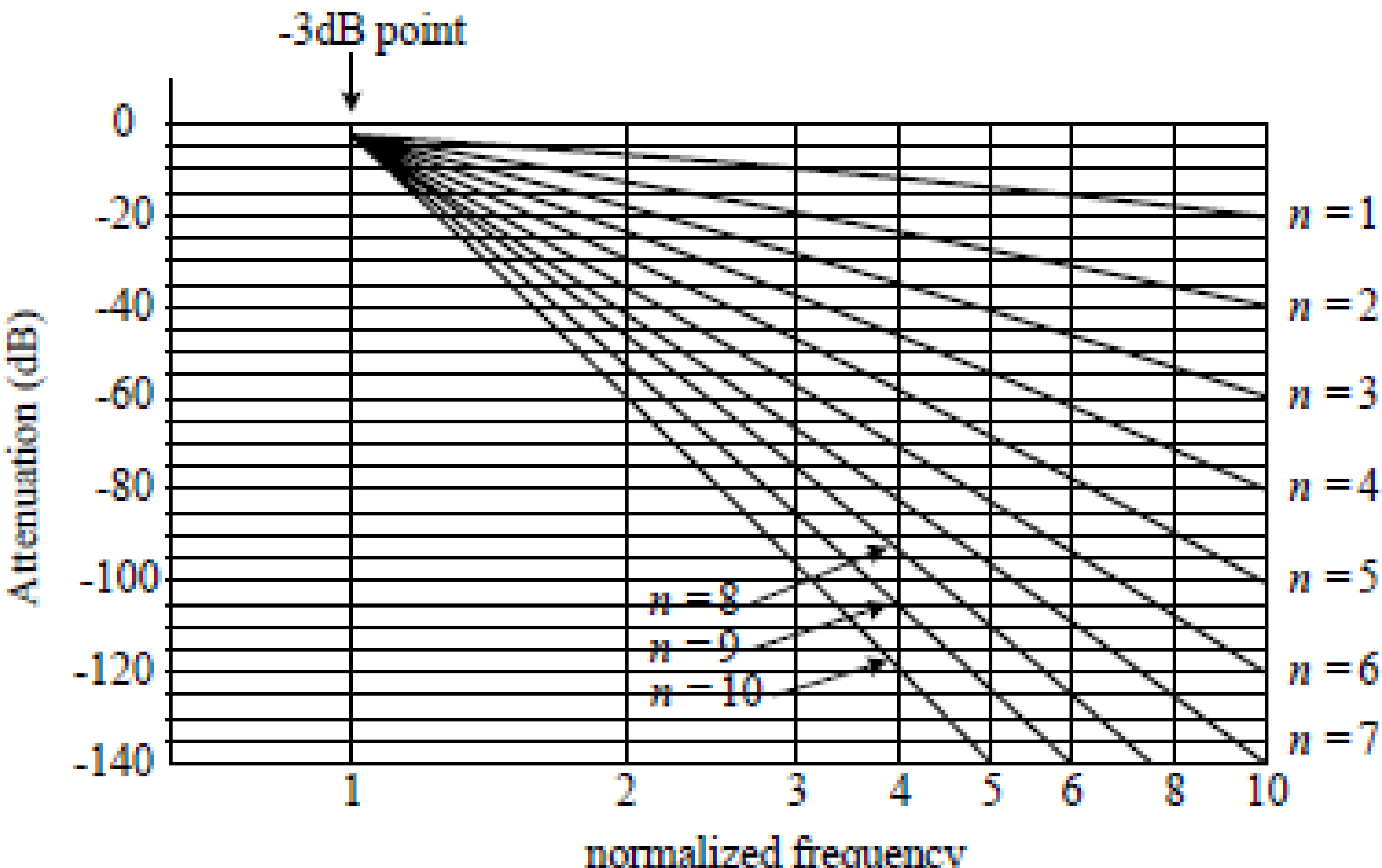
$$T(S) = \frac{V_{out}}{V_{in}} = \frac{N_m S^m + N_{m-1} S^{m-1} + \cdots + N_1 S + N_0}{D_n S^n + D_{n-1} S^{n-1} + \cdots + D_1 S + D_0}$$



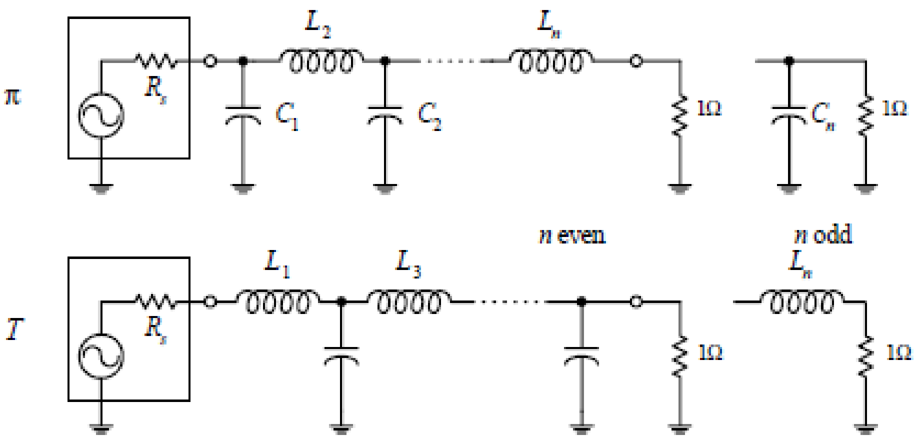
Normalized low-pass Butterworth filter reponse curves



Attenuation curves for Butterworth LPF



LC Low Pass Filter network



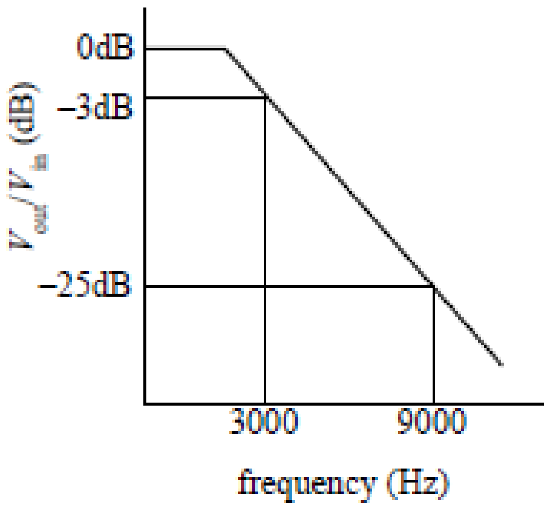
π {T} n	R_s {$1/R_s$}	C_1 {L_1}	L_2 {C_2}	C_3 {L_3}	L_4 {C_4}	C_5 {L_5}	L_6 {C_6}	C_7 {L_7}
2	1.000	1.4142	1.4142					
3	1.000	1.0000	2.0000	1.0000				
4	1.000	0.7654	1.8478	1.8478	0.7654			
5	1.000	0.6180	1.6180	2.0000	1.6180	0.6180		
6	1.000	0.5176	1.4142	1.9319	1.9319	1.4142	0.5176	
7	1.000	0.4450	1.2470	1.8019	2.0000	1.8019	1.2470	0.4450

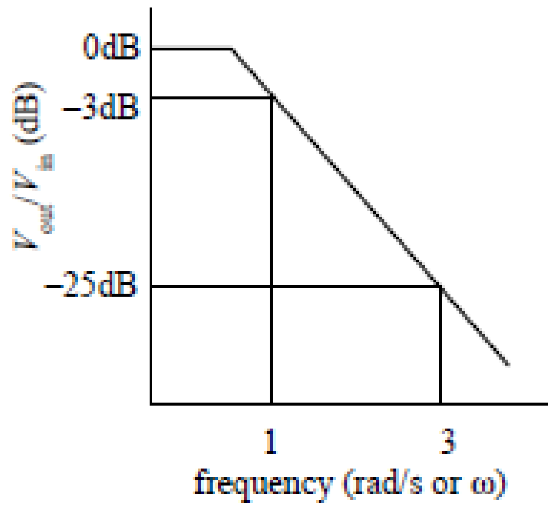
Note: Values of L_n and C_n are for a 1-Ω load and –3-dB frequency of 1 rad/s and have units of H and F. These values must be scaled down. See text.

Example

Suppose that you want to design a low-pass filter that has a f_{3dB} = 3000 Hz (attenuation is –3 dB at 3000 Hz) and an attenuation of –25 dB at a frequency of 9000 Hz—which will be called the *stop frequency* f_s. Also, let's assume that both the signal-source impedance R_s and the load impedance R_L are equal to 50 Ω. How do you design the filter?

Step 1: Normalization



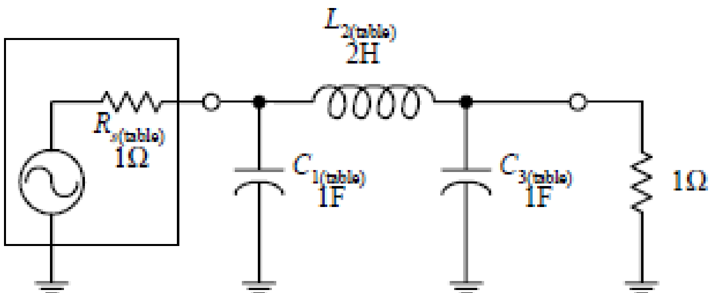


Step 2&3: Pick Response Curve and determine number of Poles

n=3 from the curve for Butterworth

Step 4: Create Normalized Filter

π

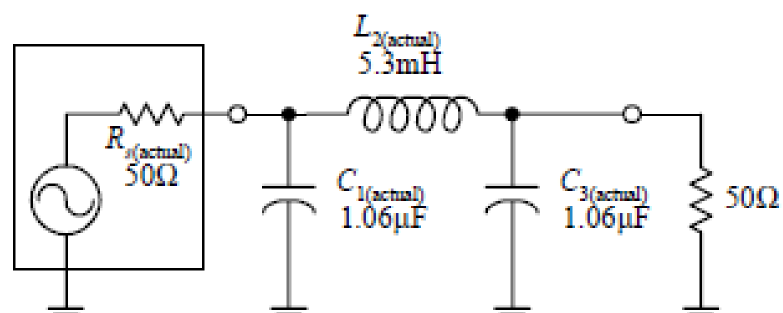


Step 5: Frequency and Impedance Scaling

$$L_{2(actual)} = \frac{R_L L_{2(table)}}{2\pi f_{3dB}} = \frac{(50\Omega)(2\text{ H})}{2\pi(3000\text{ Hz})} = 5.3\text{ mH}$$

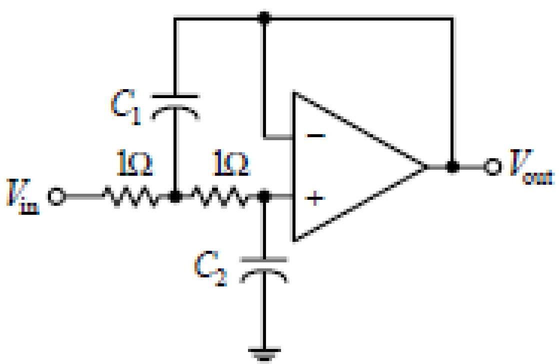
$$C_{1(actual)} = \frac{C_{1(table)}}{2\pi f_{3dB} R_L} = \frac{1\text{ F}}{2\pi(3000\text{ Hz})(50\ \Omega)} = 1.06\ \mu\text{F}$$

$$C_{3(actual)} = \frac{C_{3(table)}}{2\pi f_{3dB} R_L} = \frac{1\text{ F}}{2\pi(3000\text{ Hz})(50\ \Omega)} = 1.06\mu\text{F}$$

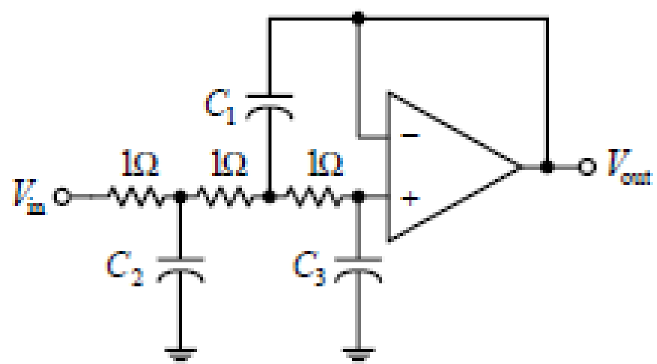


Active Filter Design

Basic two-pole section



Basic three-pole section



Normalized Curves

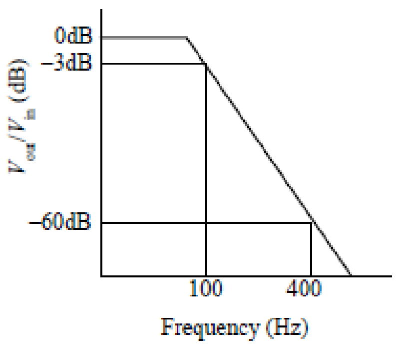
TABLE 8.2 Butterworth Normalized Active Low-Pass Filter Values

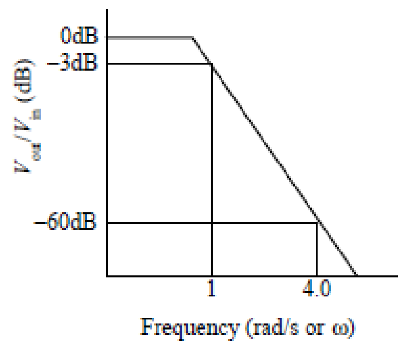
ORDER n	NUMBER OF SECTIONS	SECTIONS	C_1	C_2	C_3
2	1	2-pole	1.414	0.7071	
3	1	3-pole	3.546	1.392	0.2024
4	2	2-pole	1.082	0.9241	
		2-pole	2.613	0.3825	
5	2	3-pole	1.753	1.354	0.4214
		2-pole	3.235	0.3090	
6		2-pole	1.035	0.9660	
	3	2-pole	1.414	0.7071	
		2-pole	3.863	0.2588	
7		3-pole	1.531	1.336	0.4885
	3	2-pole	1.604	0.6235	
		2-pole	4.493	0.2225	
8		2-pole	1.020	0.9809	
	4	2-pole	1.202	0.8313	
		2-pole	2.000	0.5557	
		2-pole	5.758	0.1950	

Exqample

Suppose that you wish to design an active low-pass filter that has a 3-dB point at 100 Hz and at least 60 dB worth of attenuation at 400 Hz—which we'll call the *stop frequency* f_s.

Solution

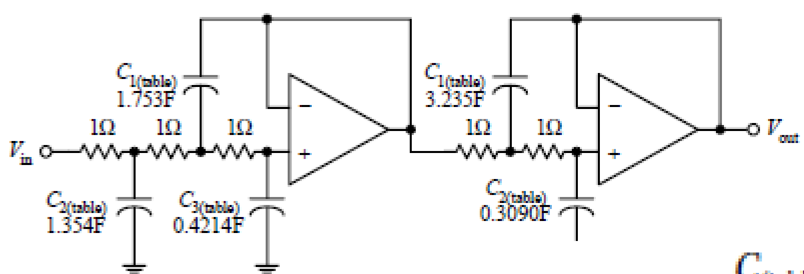




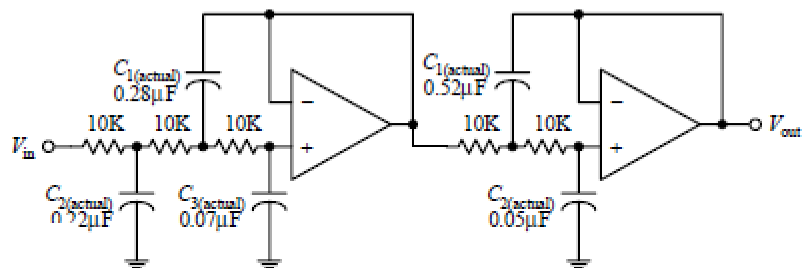
$$A_s = \frac{f_s}{f_{3dB}} = \frac{400 \text{ Hz}}{100 \text{ Hz}} = 4$$

n=5 from the curves

Normalized low-pass filter



Final low-pass filter



$$C_{(actual)} = \frac{C_{(table)}}{Z \cdot 2\pi f_{3dB}}$$

$$R_{(actual)} = ZR_{(table)}$$

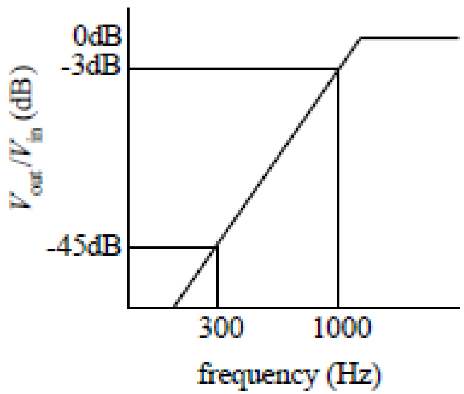
High Pass Filter -Passive-

Suppose that you want to design a high-pass filter that has an *f3dB = 1000 Hz and an*
attenuation of at least −45 dB at 300 Hz—which we call the *stop frequency fs. Assume*
that the filter is hooked up to a source and load that both have impedances of 50 Ω
and that a Butterworth response is desired

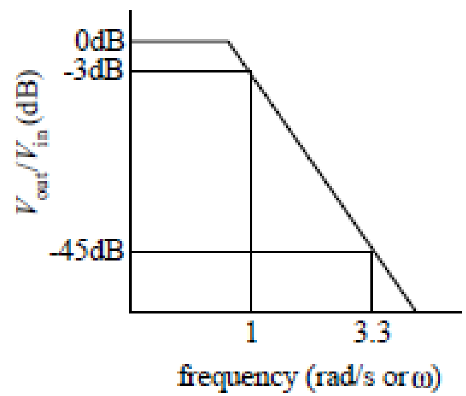
How do you design the filter?

Solution

Frequency Response Curve



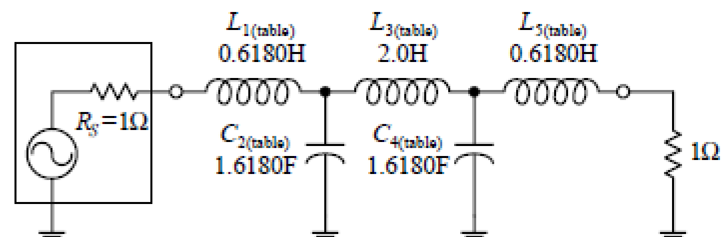
Normalized translation to low-pass filter



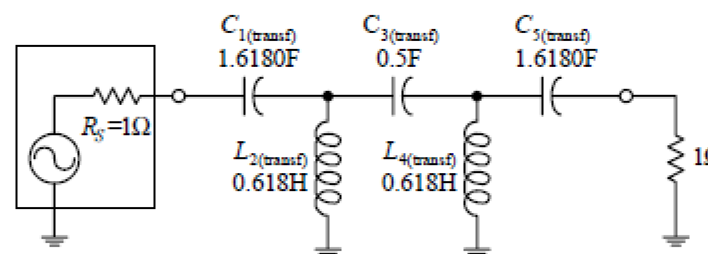
$$A_s = \frac{f_{3dB}}{f_s} = \frac{1000\text{ Hz}}{300\text{ Hz}} = 3.3$$

n=5 from the curves

Start with a "*T*" low-pass filter...



Transform low-pass filter into a high-pass filter...



To convert the low-pass into a high-pass filter, replace the inductors with capacitors that have value of $1/L$, and replace the capacitors with inductors that have values of $1/C$. In other words, do the following:

$$L_{2(transf)} = 1/C_{2(table)} = 1/1.6180 = 0.6180\text{ H}$$

$$L_{4(transf)} = 1/C_{4(table)} = 1/1.6180 = 0.6180\text{ H}$$

$$C_{1(transf)} = 1/L_{1(table)} = 1/0.6180 = 1.6180\text{ F}$$

$$C_{3(transf)} = 1/L_{3(table)} = 1/2.0 = 0.5\text{ F}$$

$$C_{5(transf)} = 1/L_{5(table)} = 1/0.6180 = 1.6180\text{ F}$$

Now Scaling

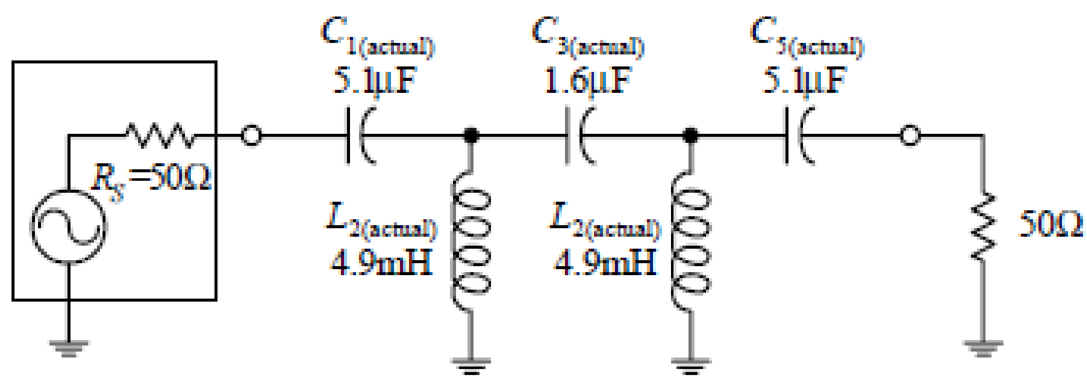
Next, frequency and impedance scale to get the actual component values:

$$C_{1(actual)} = \frac{C_{1(trans)}}{2\pi f_{3dB} R_L} = \frac{1.618 \text{ H}}{2\pi(1000 \text{ Hz})(50\ \Omega)} = 5.1\ \mu\text{F} \qquad L_{2(actual)} = \frac{L_{2(trans)} R_L}{2\pi f_{3dB}} = \frac{(0.6180 \text{ F})(50\ \Omega)}{2\pi(1000 \text{ Hz})} = 4.9 \text{ mH}$$

$$C_{3(actual)} = \frac{C_{3(trans)}}{2\pi f_{3dB} R_L} = \frac{0.5 \text{ H}}{2\pi(1000 \text{ Hz})(50\ \Omega)} = 1.6\ \mu\text{F} \qquad L_{4(actual)} = \frac{L_{4(trans)} R_L}{2\pi f_{3dB}} = \frac{(0.6180 \text{ F})(50\ \Omega)}{2\pi(1000 \text{ Hz})} = 4.9 \text{ mH}$$

$$C_{5(actual)} = \frac{C_{5(trans)}}{2\pi f_{3dB} R_L} = \frac{1.618 \text{ H}}{2\pi(1000 \text{ Hz})(50\ \Omega)} = 5.1\ \mu\text{F}$$

Impedance and frequency scale high-pass filter to get final circuit



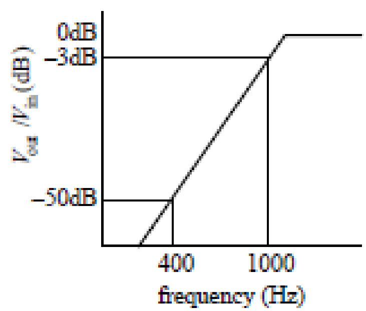
High Pass Filter–Active–

suppose

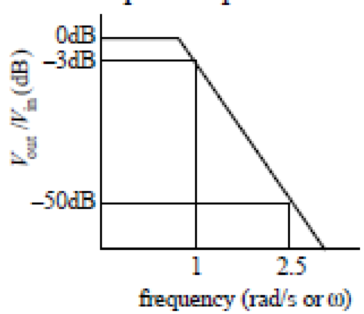
that you want to design a high-pass filter with a –3-dB frequency of 1000 Hz and 50 dB worth of attenuation at 300 Hz. What do you do?

n=5 from the curves

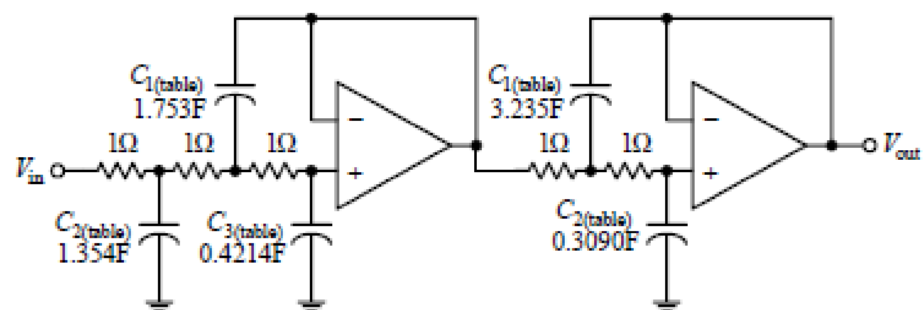
High-pass frequency response



Translation to normalized low-pass response



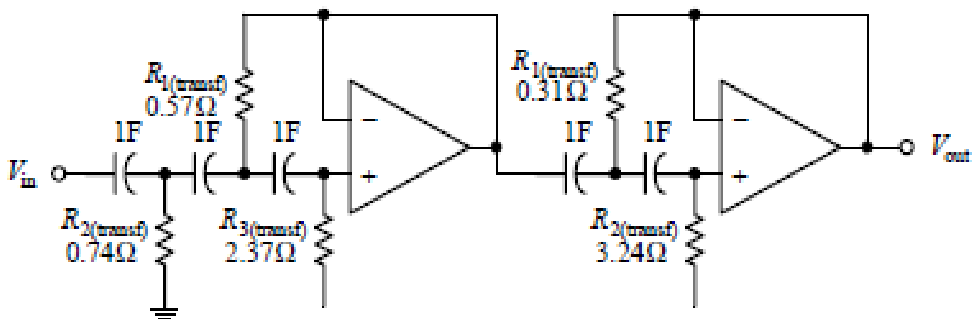
Normalized low-pass filter



Next, the normalized low-pass filter must be converted into a normalized high-pass filter. To make the conversion, exchange resistors for capacitors that have values of $1/R$ F, and exchange capacitors with resistors that have values of $1/C$ Ω.

Follow…..

Normalized high-pass filter (transformed low-pass filter)

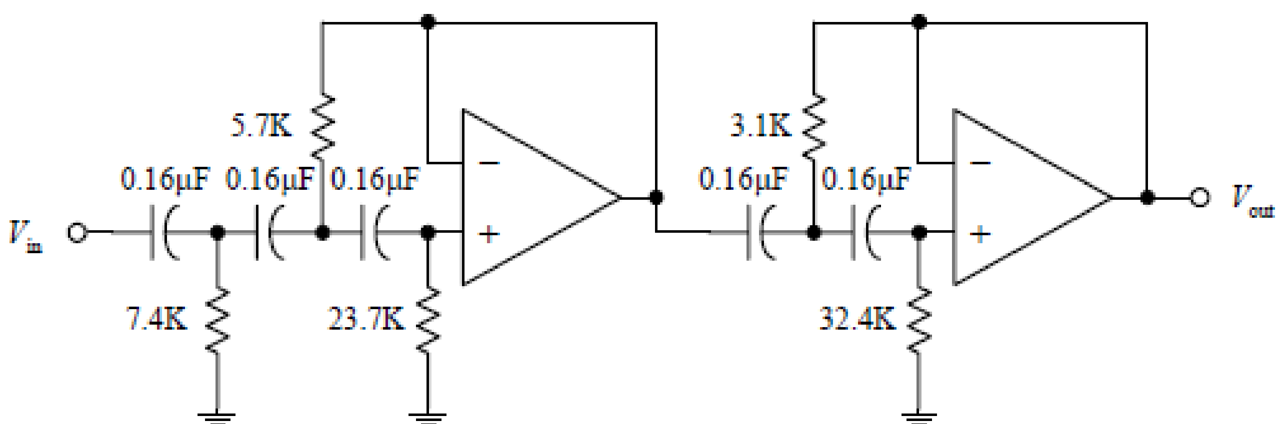


$$C_{(actual)} \frac{C_{(transf)}}{Z \cdot 2\pi f_{3dB}}$$

$$R_{(actual)} = ZR_{(transf)}$$

let $Z = 10{,}000\ \Omega$.

Final high-pass filter



Bandpass Filter Design-Wide Band-Passive-

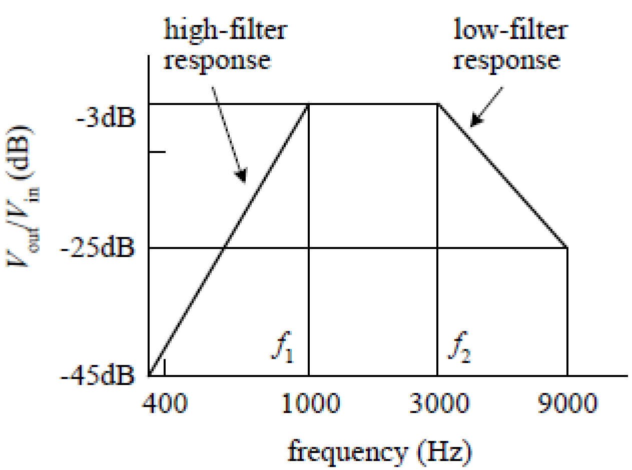
When is the filter wide band??

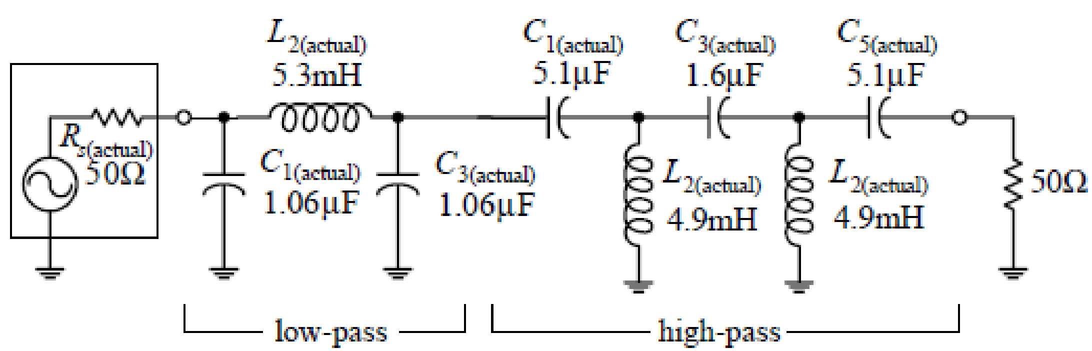
If f_2/f_1 is greater than 1.5,

Suppose that you want to design a bandpass filter that has –3-dB points at $f_1 = 1000$ Hz and $f_2 = 3000$ Hz and at least –45 dB at 300 Hz and more than –25 dB at 9000 Hz. Also, again assume that the source and load impedances are both 50 Ω and a Butterworth design is desired.

Low-pass	–3 dB at 3000 Hz
	–25 dB at 9000 Hz
High-pass	–3 dB at 1000 Hz
	–45 dB at 300 Hz

Bandpass Response Curve





Just Cascade

Bandpass Filter Design–Wide Band–Active–

Suppose that you want to design a bandpass filter that has –3-dB points at $f_1 = 1000$ Hz and $f_2 = 3000$ Hz and at least –30 dB at 300 and 10,000 Hz. What do you do?

$$\frac{f_2}{f_1} = \frac{3000\text{ Hz}}{1000\text{ Hz}} = 3$$

Low-pass: –3 dB at 3000 Hz

–30 dB at 10,000 Hz

High-pass: –3 dB at 1000 Hz

–30 dB at 300 Hz

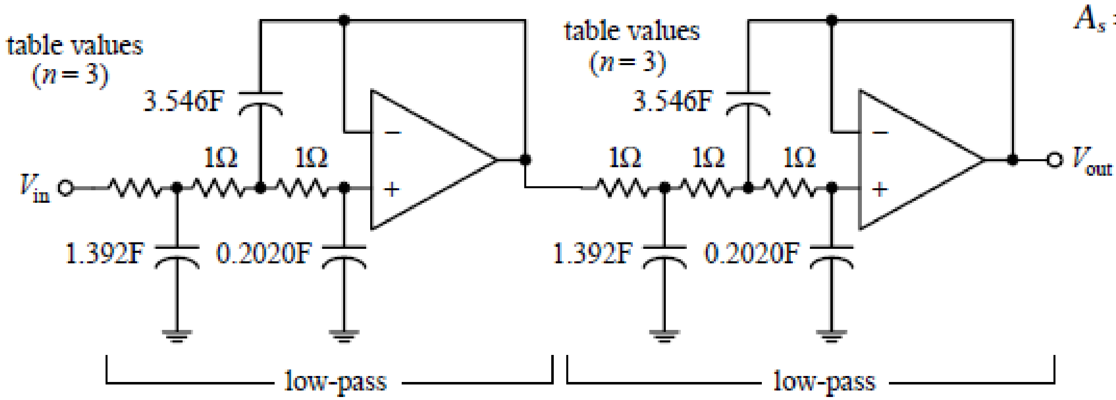
The steepness factor for the low-pass filter is

$$A_s = \frac{f_s}{f_{3dB}} = \frac{10{,}000\text{ Hz}}{3000\text{ Hz}} = 3.3$$

while the steepness factor for the high-pass filter is

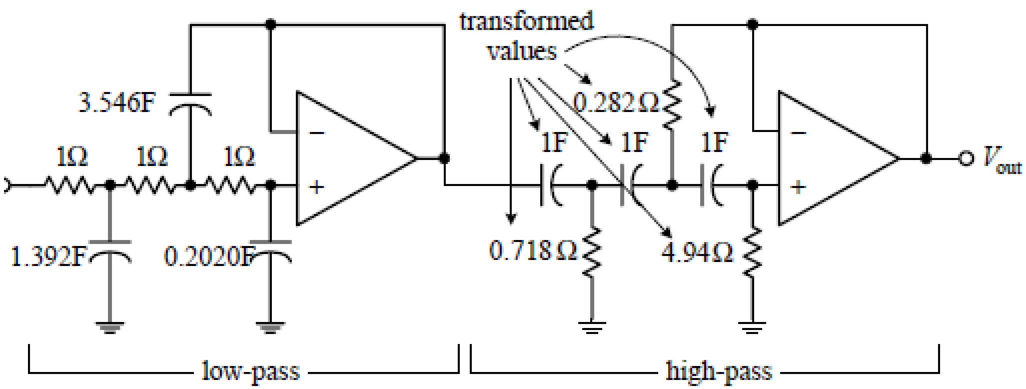
$$A_s = \frac{f_{3dB}}{f_s} = \frac{1000\text{ Hz}}{300\text{ Hz}} = 3.3$$

Normalized low-pass/low-pass inital setup

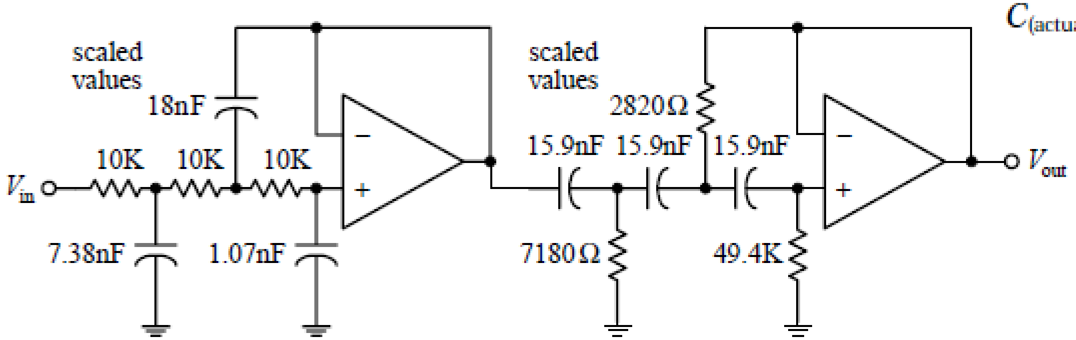


Follow…

Normalized and transformed bandbass filter



Final bandpass filter



Low-pass section:

$$C_{(actual)} = \frac{C_{table}}{Z \cdot 2\pi f_{3dB}} = \frac{C_{table}}{Z \cdot 2\pi(3000\ \text{Hz})}$$

High-pass section:

$$C_{(actual)} = \frac{C_{table}}{Z \cdot 2\pi f_{2dB}} = \frac{C_{table}}{Z \cdot 2\pi(1000\ \text{Hz})}$$

Narrow Bandwidth BPF–Passive (optional)

Suppose that you want to design a bandpass filter with –3-dB points at $f_1 = 900$ Hz and $f_2 = 1100$ Hz and at least –20 dB worth of attenuation at 800 and 1200 Hz. Assume that both the source and load impedances are 50 Ω and that a Butterworth design is desired.

Since $f_2/f_1 = 1.2$, which is less than 1.5, a narrow-band filter is needed.

geometric center frequency $$f_0 = \sqrt{f_1 f_2} = \sqrt{(900\text{ Hz})(1100)} = 995\text{ Hz}$$

Next, compute the two pair of geometrically related stop-band frequencies by using

$$f_a f_b = f_0^2$$

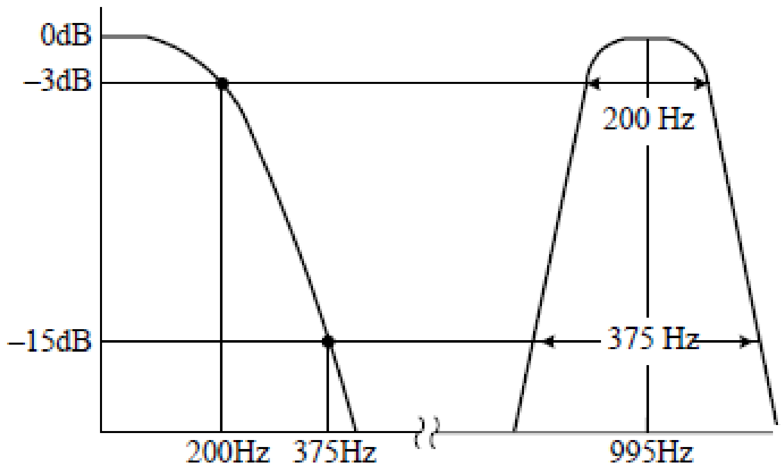
$$f_a = 800\text{ Hz} \qquad f_b = \frac{f_0^2}{f_a} = \frac{(995\text{ Hz})^2}{800\text{ Hz}} = 1237\text{ Hz} \qquad f_b - f_a = 437\text{ Hz}$$

$$f_b = 1200\text{ Hz} \qquad f_a = \frac{f_0^2}{f_b} = \frac{(995\text{ Hz})^2}{1200\text{ Hz}} = 825\text{ Hz} \qquad f_b - f_a = 375\text{ Hz}$$

Choose the pair having the least separation, which represents more severe requirements–375 Hz

Follow.....

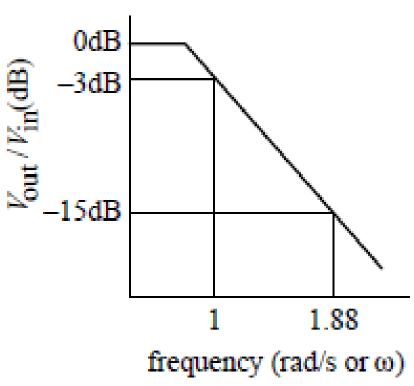
Low-pass bandpass relationship



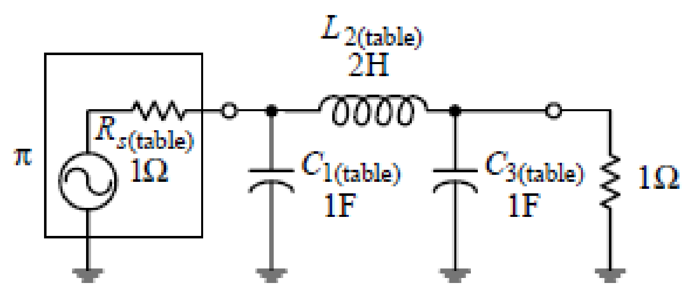
$$A_s = \frac{\text{stop-band bandwidth}}{\text{3-dB bandwidth}} = \frac{375\ \text{Hz}}{200\ \text{Hz}} = 1.88$$

From the curve n=3

Normalized low-pass response



Normalized low-pass filter



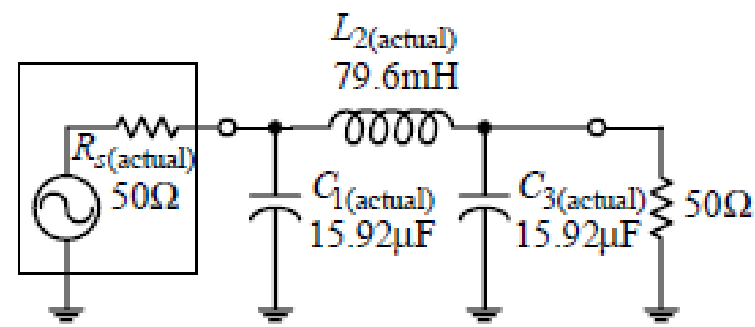
Follow……

Impedance and frequency scaled low-pass filter

$$C_{1(\text{actual})} = \frac{C_{1(\text{table})}}{2\pi(\Delta f_{BW})R_L} = \frac{1\ \text{F}}{2\pi(200\ \text{Hz})(50\ \Omega)} = 15.92\ \mu\text{F}$$

$$C_{3(\text{actual})} = \frac{C_{3(\text{table})}}{2\pi(\Delta f_{BW})R_L} = \frac{1\ \text{F}}{2\pi(200\ \text{Hz})(50\ \Omega)} = 15.92\ \mu\text{F}$$

$$L_{2(\text{actual})} = \frac{L_{2(\text{table})}R_L}{2\pi(\Delta f_{BW})} = \frac{(2\ \text{H})(50\ \Omega)}{2\pi(200\ \text{Hz})} = 79.6\ \text{mH}$$



The important part comes now. Each circuit branch of the low-pass filter must be resonated to f_0 by adding a series capacitor to each inductor and a parallel inductor to each capacitor. The *LC* resonant equation is used to determine the additional component values:

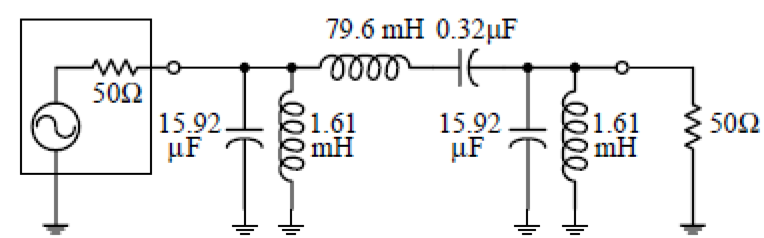
$$f_0 = \frac{1}{2\pi\sqrt{LC}}$$

$$L_{(\text{parallel with } C1)} = \frac{1}{(2\pi f_0)^2 C_{1(\text{actual})}} = \frac{1}{(2\pi \cdot 995\ \text{Hz})^2(15.92\ \mu\text{F})} = 1.61\ \text{mH}$$

$$L_{(\text{parallel with } C3)} = \frac{1}{(2\pi f_0)^2 C_{3(\text{actual})}} = \frac{1}{(2\pi \cdot 995\ \text{Hz})^2(15.92\ \mu\text{F})} = 1.61\ \text{mH}$$

$$C_{(\text{series with } L2)} = \frac{1}{(2\pi f_0)^2 L_{2(\text{actual})}} = \frac{1}{(2\pi \cdot 995\ \text{Hz})^2(79.6\ \text{mH})} = 0.32\ \mu\text{F}$$

Final bandpass filter



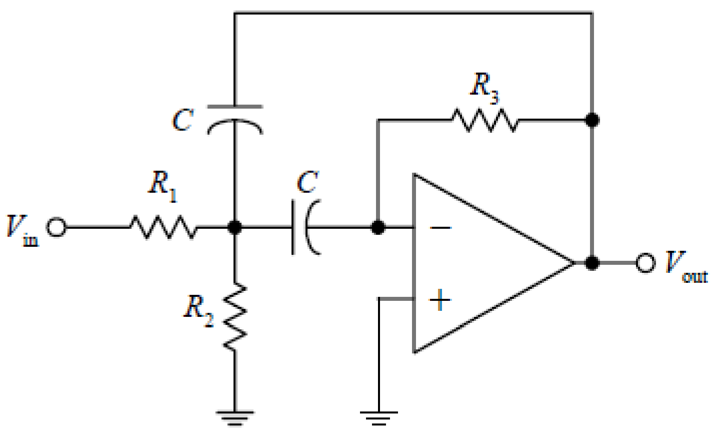
BPF–Narrow Band–Active Design

Suppose that you want to design a bandpass filter that has a center frequency f_0 = 2000 Hz and a –3-dB bandwidth $\Delta f_{BW} = f_2 - f_1 = 40$ Hz. How do you design the filter?

Since $f_2/f_1 = 2040\text{ Hz}/1960\text{ Hz} = 1.04$,

No Cascading

Narrow-band filter circuit



$$Q = \frac{f_0}{f_2 - f_1} = \frac{2000\text{ Hz}}{40\text{ Hz}} = 50$$

$$R_1 = \frac{Q}{2\pi f_0 C} \qquad R_2 = \frac{R_1}{2Q^2 - 1} \qquad R_3 = 2R_1$$

Choose convenient value of C, let it 0.01 µF

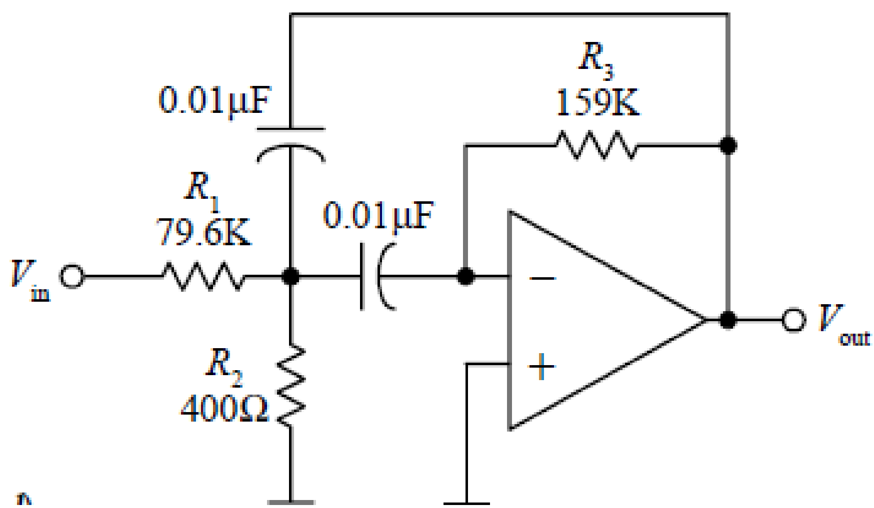
$$R_1 = \frac{50}{2\pi(2000\text{ Hz})(0.01\ \mu\text{F})} = 79.6\text{ k}\Omega$$

$$R_2 = \frac{79.6\text{ k}\Omega}{2(50)^2 - 1} = 400\ \Omega$$

$$R_3 = 2(79.6\text{ k}\Omega) = 159\text{ k}\Omega$$

Final Design …

Final filter circuit



Passive Notch Filter (optional)

EXAMPLE

Suppose that you want to design a notch filter with –3-dB points at $f_1 = 800$ Hz and $f_2 = 1200$ Hz and at least –20 dB at 900 and 1100 Hz. Let's assume that both the source and load impedances are 600 Ω and that a Butterworth design is desired.

First, you find the geometric center frequency:

$$f_0 = \sqrt{f_1 f_2} = \sqrt{(800\text{ Hz})(1200\text{ Hz})} = 980\text{ Hz}$$

Next, compute the two pairs of geometrically related stop-band frequencies:

$$f_a = 900\text{ Hz} \qquad f_b = \frac{f_0^2}{f_a} = \frac{(980\text{ Hz})^2}{900\text{ Hz}} = 1067\text{ Hz}$$

$$f_b - f_a = 1067\text{ Hz} - 900\text{ Hz} = 167\text{ Hz}$$

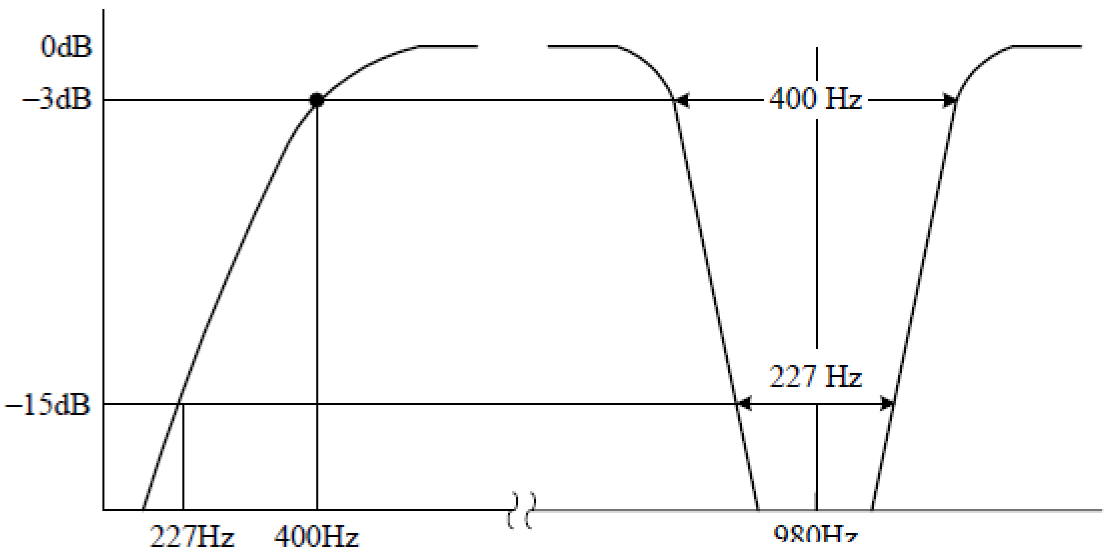
$$f_b = 1100\text{ Hz} \qquad f_a = \frac{f_0^2}{f_b} = \frac{(980\text{ Hz})^2}{1100\text{ Hz}} = 873\text{ Hz}$$

$$f_b - f_a = 1100\text{ Hz} - 873\text{ Hz} = 227\text{ Hz}$$

Choose the pair of frequencies that gives the more severe requirement—227 Hz.

Follow....

High-pass bandpass relationship

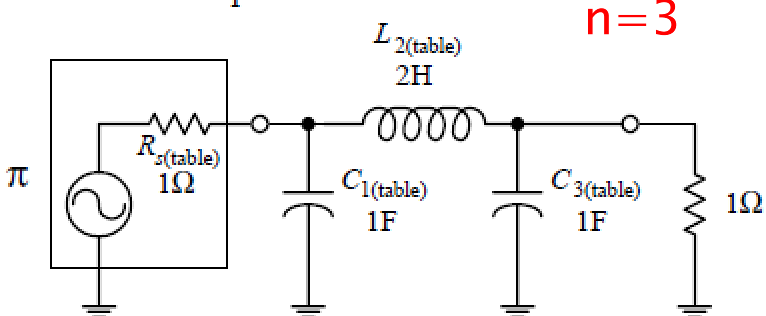


$$A_s = \frac{\text{3-dB bandwidth}}{\text{stop-band bandwidth}} = \frac{400 \text{ Hz}}{227 \text{ Hz}} = 1.7$$

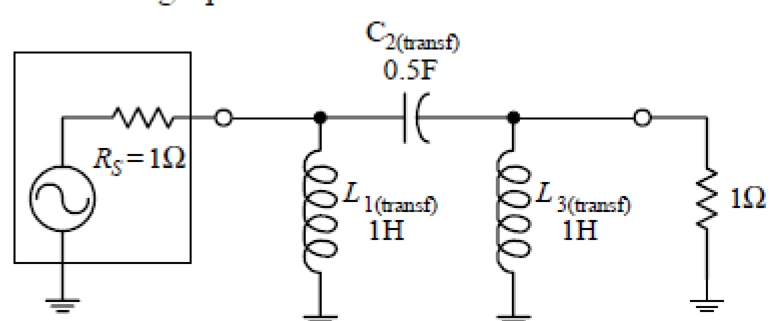
$$L_{1(\text{transf})} = 1/C_{1(\text{table})} = 1/1 = 1 \text{ H}$$
$$L_{3(\text{transf})} = 1/C_{3(\text{table})} = 1/1 = 1 \text{ H}$$
$$C_{2(\text{transf})} = 1/L_{2(\text{table})} = 1/2 = 0.5 \text{ F}$$

Normalized low-pass filter

n=3

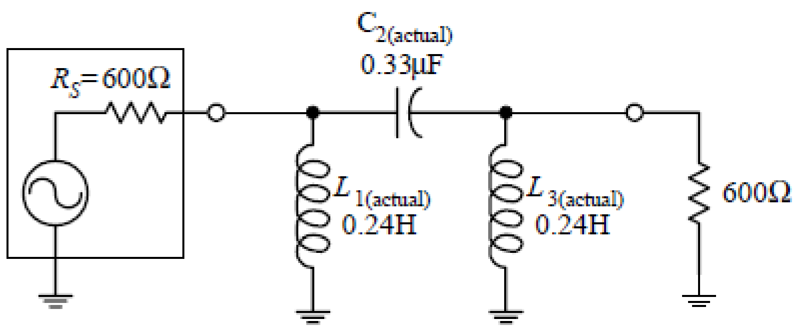


Normalized high-pass filter



Follow…

Actual high-pass filter

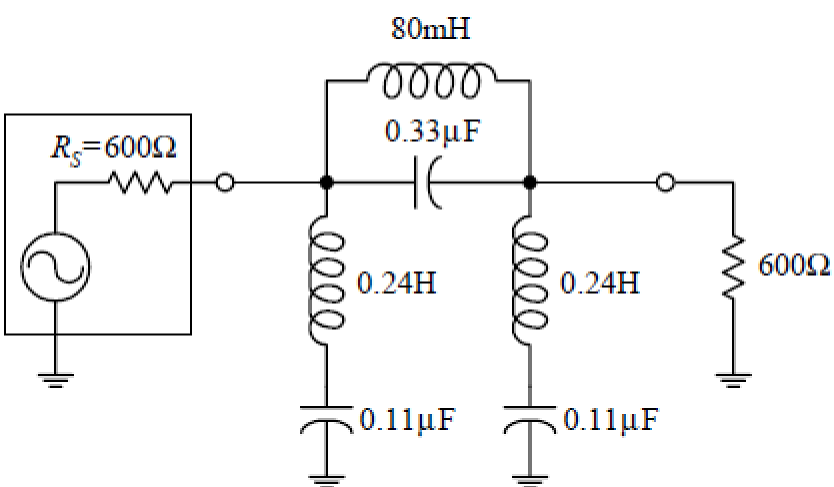


$$L_{1(\text{actual})} = \frac{R_L L_{1(\text{transf})}}{2\pi(\Delta f_{BW})} = \frac{(600\ \Omega)(1\ \text{H})}{2\pi(400\ \text{Hz})} = 0.24\ \text{H}$$

$$L_{3(\text{actual})} = \frac{R_L L_{3(\text{transf})}}{2\pi(\Delta f_{BW})} = \frac{(600\ \Omega)(1\ \text{H})}{2\pi(400\ \text{Hz})} = 0.24\ \text{H}$$

$$C_{2(\text{actual})} = \frac{C_{1(\text{transf})}}{2\pi(\Delta f_{BW})R_L} = \frac{(0.5\ \text{F})}{2\pi(400\ \text{Hz})(600\ \Omega)} = 0.33\ \mu\text{F}$$

Final bandpass filter



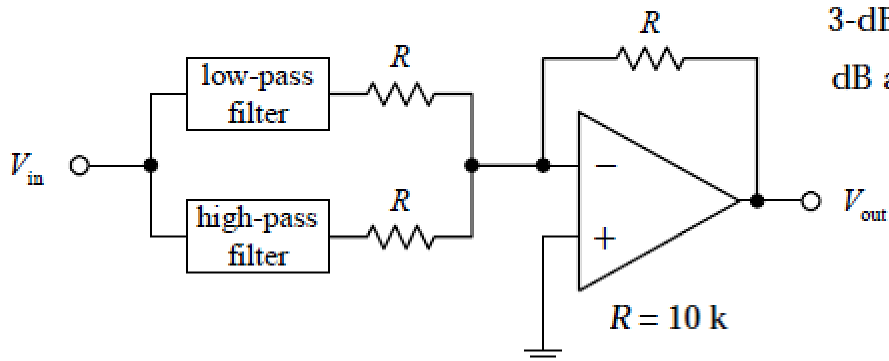
$$C_{(\text{series with } L1)} = \frac{1}{(2\pi f_0)^2 L_{1(\text{actual})}} = \frac{1}{(2\pi \cdot 400\ \text{Hz})^2(0.24\ \text{H})} = 0.11\ \mu\text{F}$$

$$C_{(\text{series with } L3)} = \frac{1}{(2\pi f_0)^2 L_{3(\text{actual})}} = \frac{1}{(2\pi \cdot 400\ \text{Hz})^2(0.24\ \text{H})} = 0.11\ \mu\text{F}$$

$$L_{(\text{parallel with } L1)} = \frac{1}{(2\pi f_0)^2 C_{2(\text{actual})}} = \frac{1}{(2\pi \cdot 400\ \text{Hz})^2(0.33\ \mu\text{F})} = 80\ \text{mH}$$

Active Notch Filter –Wide-Band

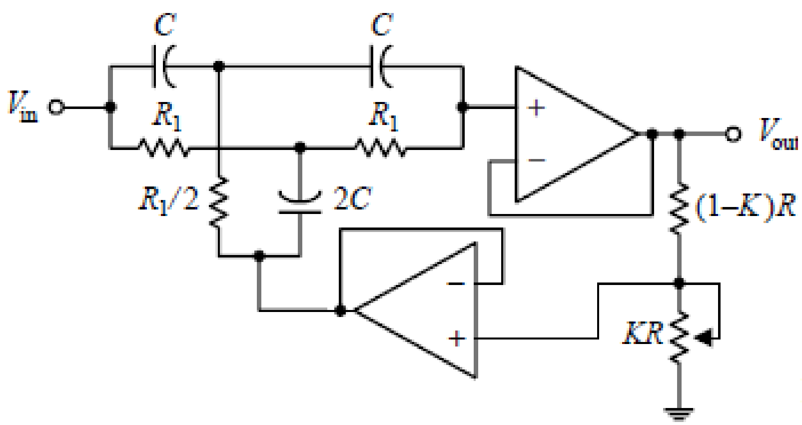
Basic wide-band notch filter



For example, if you need a notch filter to have –3-dB points at 500 and 5000 Hz and at least –15 dB at 1000 and 2500 Hz,

Narrow-Band Notch Filter

Improved notch filter



Suppose that you want to make a "notch" at f_0 = 2000 Hz and desire a –3-dB bandwidth of Δf_{BW} = 100 Hz. To get this desired response, do the following: First determine the Q:

$$Q = \frac{\text{"notch" frequency}}{-3\text{-dB bandwidth}} = \frac{f_0}{\Delta f_{BW}} = \frac{2000\ \text{Hz}}{100\ \text{Hz}} = 20$$

$$R_1 = \frac{1}{2\pi f_0 C} \quad \text{and} \quad K = \frac{4Q-1}{4Q}$$

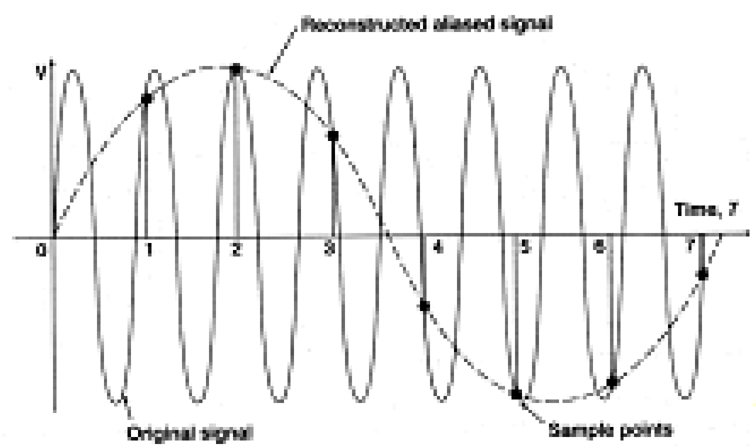
Now arbitrarily choose R and C; say, let $R = 10$ k and $C = 0.01$ µF. Next, solve for R_1 and K:

$$R_1 = \frac{1}{2\pi f_0 C} = \frac{1}{2\pi(2000\ \text{Hz})(0.01\ \mu\text{F})} = 7961\ \Omega$$

$$K = \frac{4Q-1}{4Q} = \frac{4(20)-1}{4(20)} = 0.9875$$

Anti-aliasing

- The sampling theorem
  - A continuous signal can be represented completely by, and reconstructed from, a set of instantaneous measurements or samples of its voltage which are made at equally-spaced times. The interval $T(=1/f_S)$ between such samples must be less than one-half the period of the highest-frequency component f_{MAX} in the signal
  - In other words: you must sample at least twice the rate of the maximum frequency in your signal to prevent aliasing ($F_S \geq 2F_{MAX}$)
  - The sampling rate $F_S = 2F_{MAX}$ is called the Nyquist rate



Anti-aliasing

- The effects of aliasing can also be observed on the frequency spectrum of the signal
- In the figures below
  - F_1 appears correctly since $F_1 \leq F_S/2$
  - F_2, F_3 and F_4 have aliases at 30, 40 and 10Hz, respectively
  - You can compute these aliased frequencies by <u>folding</u> the spectrum around $F_S/2$ or with the expression

$$\text{Alias frequency } \hat{F} = \min \left| kF_S - F \right|_{\forall k \text{ integer}}$$

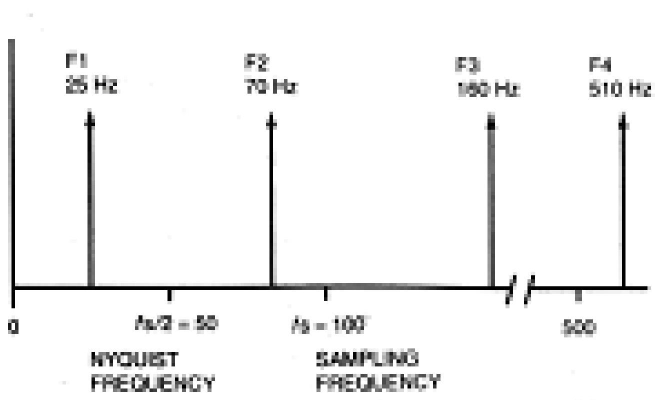


FIGURE 117.5 Spectral of signal with multiple frequencies.

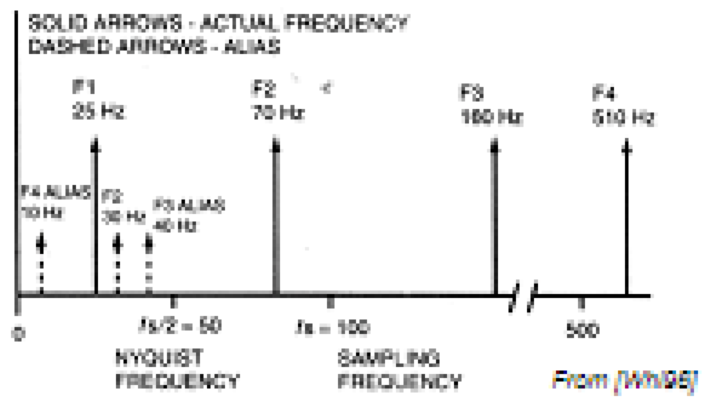
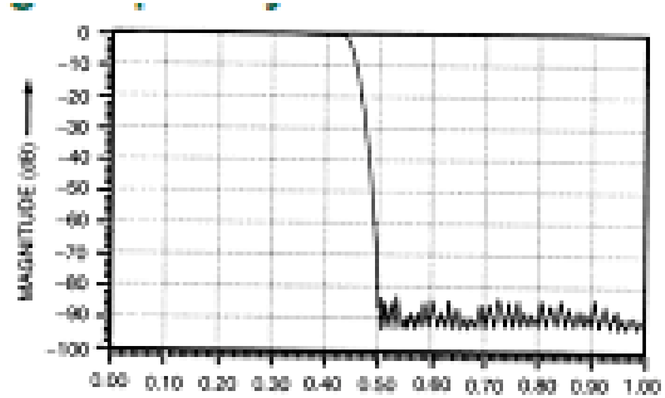


FIGURE 117.6 Spectral of signal with multiple frequencies after sampled at $fs = 100$ Hz.

Anti-aliasing filters

- An anti-aliasing filter is a low-pass filter designed to filter out frequencies higher than the sampling frequency
  - An anti-aliasing filter should have
    - Steep cut-off and
    - Flat response in the frequency band
- Typical filters are:
  - Butterworth: flattest response in the frequency band but phase shifts well below the break frequency
  - Bessel: phase shift proportional to frequency, so the signal is not distorted by the filter
    - Recommended for anti-aliasing if it is important to preserve the waveform
  - Chebyshev: steepest cut-off but it has ripples in the band-pass



From [Wh96]