Answers to HMM questions (1), (2) and (3) in question sheet 2.

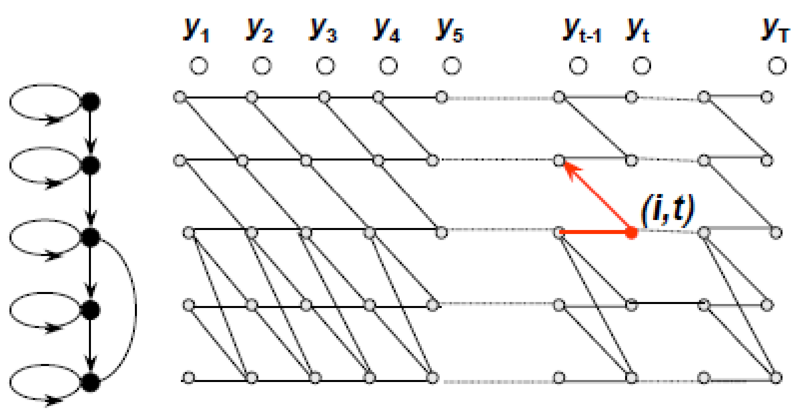
ASR: Decoding

- Let $X = \{x_1, \ldots, x_T\}$ be a state sequence of length T
- The joint probability of Y and X is given by:

$$p(Y, X) = b_{x_1}(y_1) \prod_{t=2}^{T} a_{x_{t-1}x_t} b_{x_t}(y_t)$$

- i.e. the product of the state-output and state transition probabilities along the state sequence
- $p(Y)$ is the sum of $P(Y,X)$ over all sequences X
- $P(Y, \hat{X})$ is the probability of an observation sequence Y and the optimum state sequence $\hat{X}$

Viterbi Decoding

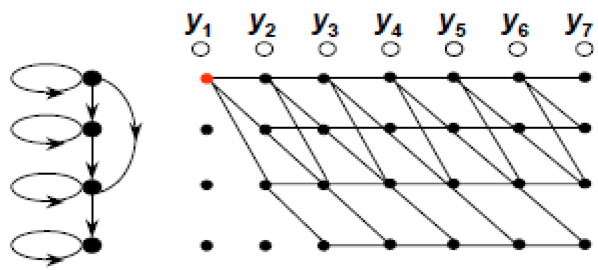


$p_t(i)$ = Prob($y_1, \ldots, y_t$, opt sequence to (i,t))

$p_t(i) = \max \{p_{t-1}(i-1)a_{i-1,i},\ p_{t-1}(i)a_{i,i}\} b_i(y_t)$

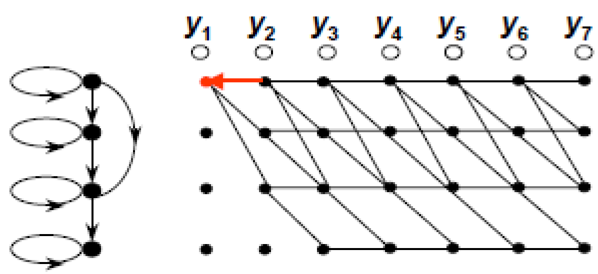
Viterbi Decoding

- State-time trellis



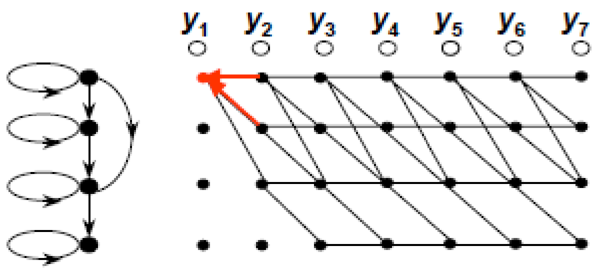
$$\alpha_1(1) = b_1(y_1) = 0.6$$

Viterbi Decoding



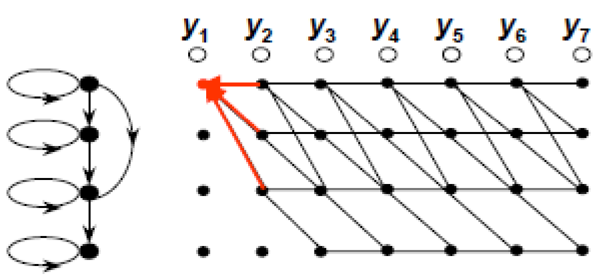
$$\alpha_2(1) = \alpha_1(1) a_{11} b_1(y_2) \quad = 0.6 * 0.5 * 0.2 = 0.06$$

Viterbi Decoding



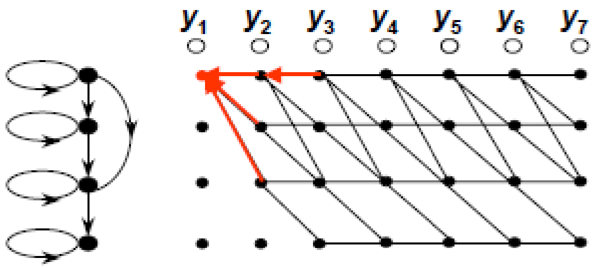
$$\alpha_2(2) = \alpha_1(1)a_{12}b_2(y_2) = 0.6 * 0.2 * 0.7 = 0.084$$

Viterbi Decoding



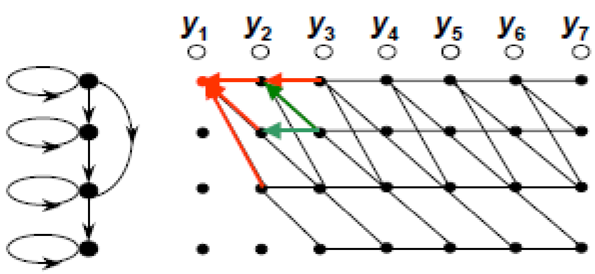
$$\alpha_2(3) = \alpha_1(1)a_{13}b_3(y_2) = 0.6 * 0.3 * 0.4 = 0.072$$

Viterbi Decoding



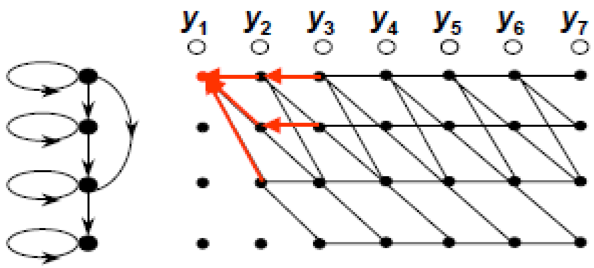
$$\alpha_3(1) = \alpha_2(1) a_{11} b_1(y_3) = 0.06 * 0.5 * 0.6 = 0.018$$

Viterbi Decoding



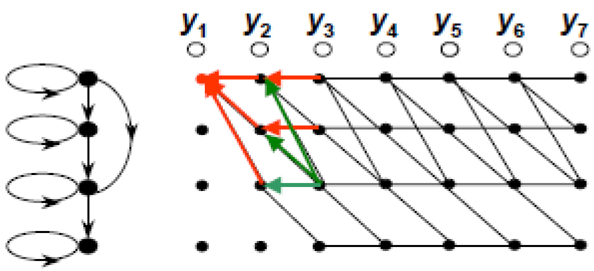
$$\alpha_3(2) = \max \begin{cases} \alpha_2(1) a_{12} b_2(y_3) = 0.06 * 0.2 * 0.2 = 2.4 * 10^{-3} \\ \alpha_2(2) a_{22} b_2(y_3) = 0.084 * 0.6 * 0.2 = 0.01008 \end{cases}$$

Viterbi Decoding



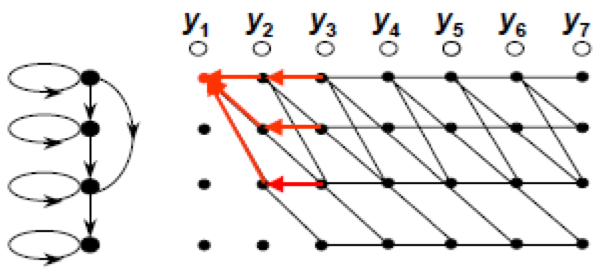
$$\alpha_3(2) = \max \begin{cases} \alpha_2(1) a_{12} b_2(y_3) & = 0.06 * 0.2 * 0.2 = 2.4 * 10^{-3} \\ \alpha_2(2) a_{22} b_2(y_3) & = 0.084 * 0.6 * 0.2 = 0.01008 \end{cases}$$

Viterbi Decoding



$$\alpha_3(3) = \max \begin{cases} \alpha_2(1) a_{13} b_3(y_3) & = 0.06 * 0.3 * 0.2 = 3.6 * 10^{-3} \\ \alpha_2(2) a_{23} b_3(y_3) & = 0.084 * 0.4 * 0.2 = 6.72 * 10^{-3} \\ \alpha_2(3) a_{33} b_3(y_3) & = 0.072 * 0.6 * 0.2 = 8.64 * 10^{-3} \end{cases}$$

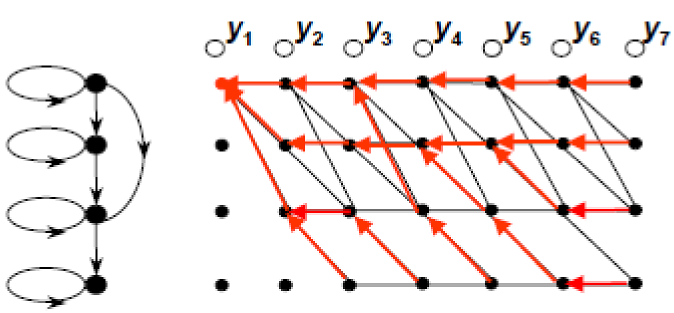
Viterbi Decoding

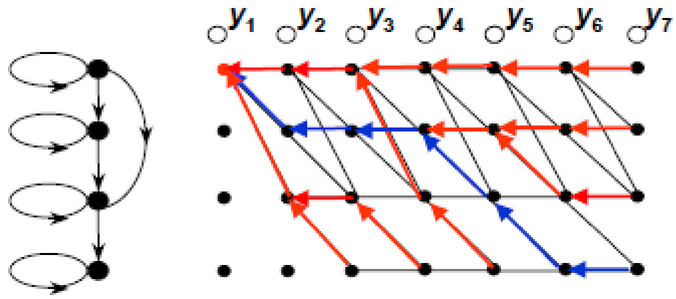


$$\alpha_3(3) = \max \begin{cases} \alpha_2(1) a_{13} b_3(y_3) & = 0.06 * 0.3 * 0.2 = 3.6 * 10^{-3} \\ \alpha_2(2) a_{23} b_3(y_3) & = 0.084 * 0.4 * 0.2 = 6.72 * 10^{-3} \\ \alpha_2(3) a_{33} b_3(y_3) & = 0.072 * 0.6 * 0.2 = 8.64 * 10^{-3} \end{cases}$$

Viterbi Decoding

- Continue in a similar manner
- Final overall probability $P(Y, \hat{X}) = \alpha_7(4) = 1.73 * 10^{-4}$




y_1 y_2 y_3 y_4 y_5 y_6 y_7