COMP5331: Knowledge Discovery and Data Mining

Acknowledgement: Slides modified based on the slides provided by Lawrence Page, Sergey Brin, Rajeev Motwani and Terry Winograd, Jon M. Kleinberg

PageRank & HITS: Bring Order to the Web

- Background and Introduction
- Approach – PageRank
- Approach – Authorities & Hubs

Motivation and Introduction

- Why is Page Importance Rating important?
  - New challenges for information retrieval on the World Wide Web.
    - Huge number of web pages: 150 million by1998
    - 1000 billion by 2008
    - Diversity of web pages: different topics, different quality, etc.
- Hard to imagine no ranking algorithms in search engine.

Motivation and Introduction

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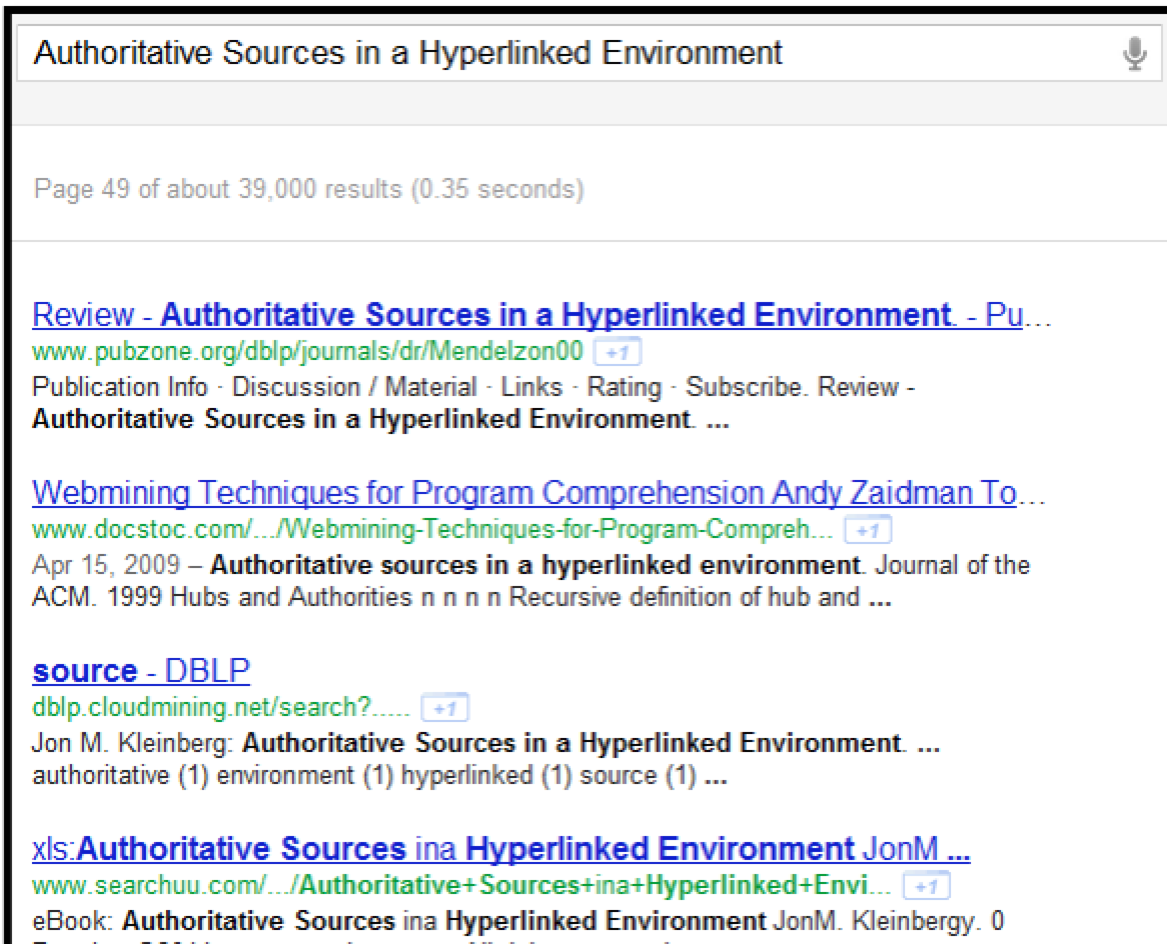
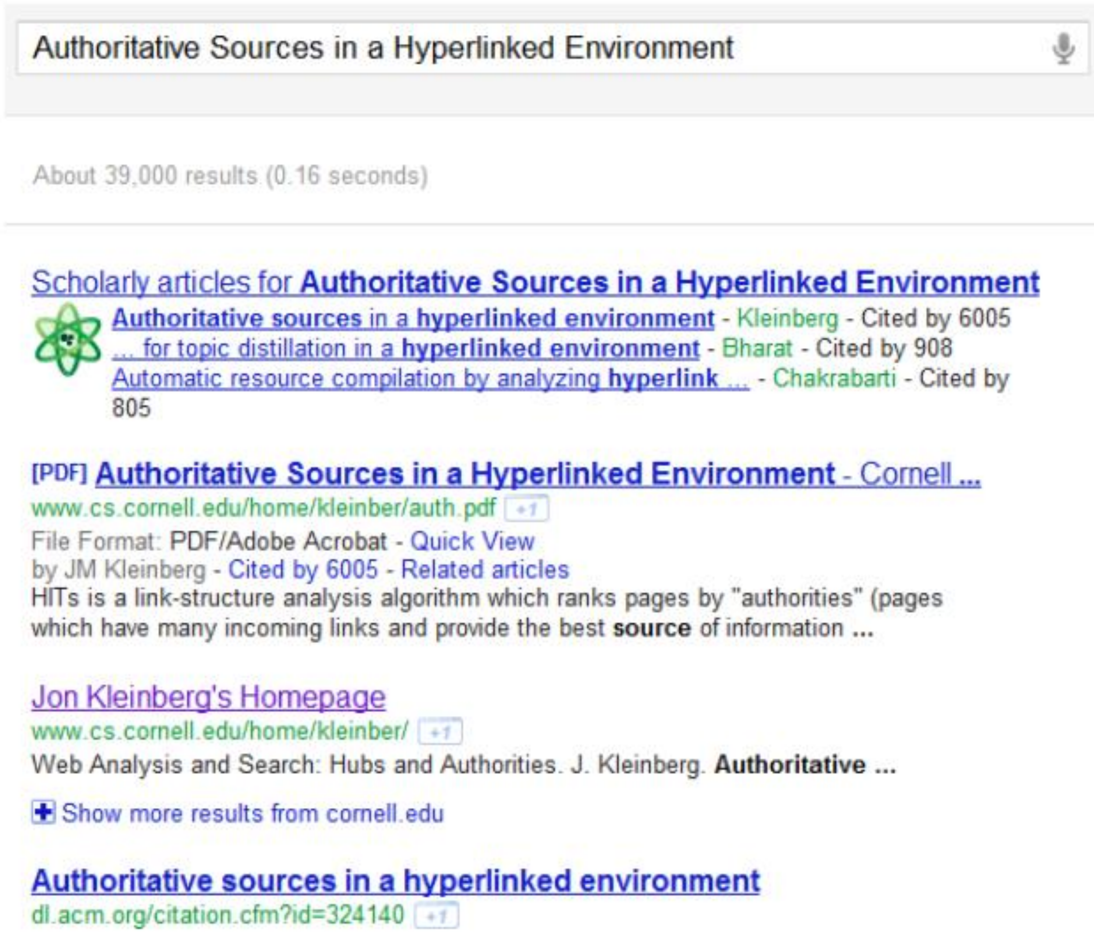


Motivation and Introduction

- Modern search engines may return millions of pages for a single query. This amount is prohibitive to preview for human users.
- Ranking algorithms will process the search results and only show the most useful information to the search engine user.

Motivation and Introduction



PageRank: History

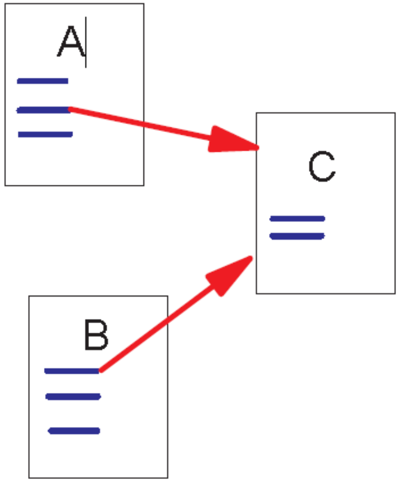
- PageRank was developed by Larry Page (hence the name *Page*-Rank) and Sergey Brin.

- It is first as part of a research project about a new kind of search engine. That project started in 1995 and led to a functional prototype in 1998.

- Shortly after, Page and Brin founded Google.

Link Structure of the Web

- 150 million web pages → 1.7 billion links



Backlinks and Forward links:
➢A and B are C's backlinks
➢C is A and B's forward link

Intuitively, a webpage is important if it has a lot of backlinks.

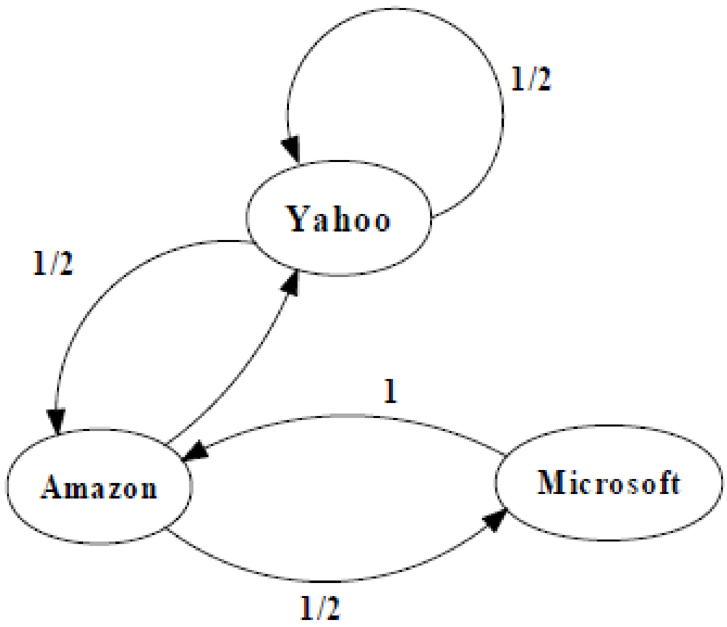
What if a webpage has only one link off www.yahoo.com?

PageRank: A Simplified Version

$$R(u) = c \sum_{v \in B_u} \frac{R(v)}{N_v}$$

- u: a web page
- B_u: the set of u's backlinks
- N_v: the number of forward links of page v
- c: the normalization factor to make $||R||_{L1} = 1$ ($||R||_{L1} = |R_1 + \ldots + R_n|$)

An example of Simplified PageRank



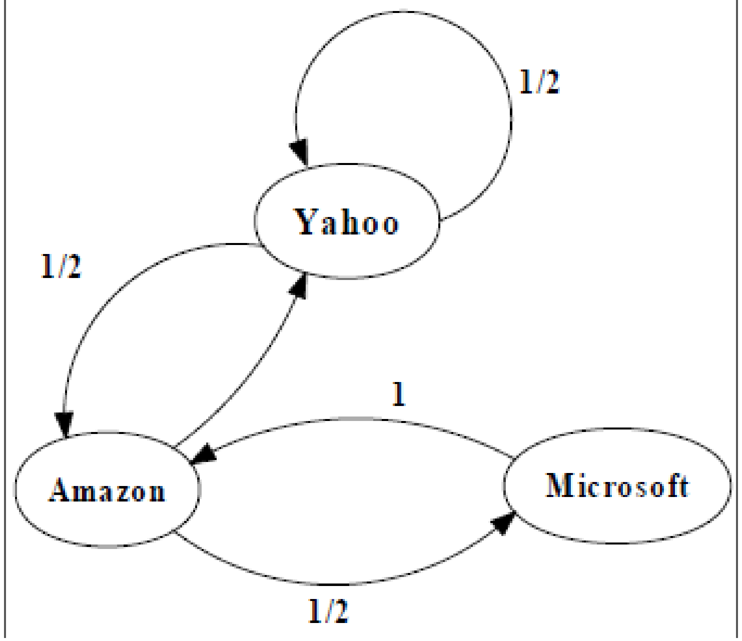
$$M = \begin{bmatrix} 1/2 & 1/2 & 0 \\ 1/2 & 0 & 1 \\ 0 & 1/2 & 0 \end{bmatrix}$$

$$\begin{bmatrix} \text{yahoo} \\ \text{Amazon} \\ \text{Microsoft} \end{bmatrix} = \begin{bmatrix} 1/3 \\ 1/3 \\ 1/3 \end{bmatrix}$$

$$\begin{bmatrix} 1/3 \\ 1/2 \\ 1/6 \end{bmatrix} = \begin{bmatrix} 1/2 & 1/2 & 0 \\ 1/2 & 0 & 1 \\ 0 & 1/2 & 0 \end{bmatrix} \begin{bmatrix} 1/3 \\ 1/3 \\ 1/3 \end{bmatrix}$$

PageRank Calculation: first iteration

An example of Simplified PageRank



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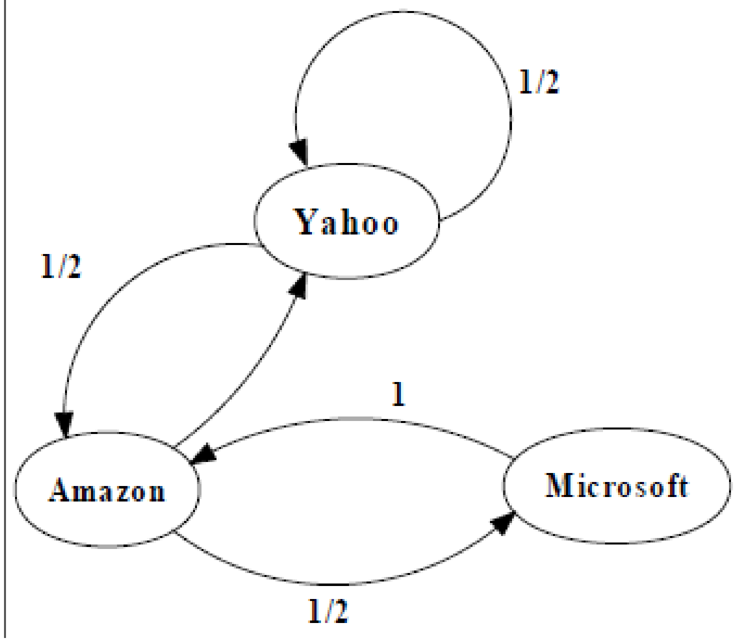
$$\begin{bmatrix} 5/12 \\ 1/3 \\ 1/4 \end{bmatrix} = \begin{bmatrix} 1/2 & 1/2 & 0 \\ 1/2 & 0 & 1 \\ 0 & 1/2 & 0 \end{bmatrix} \begin{bmatrix} 1/3 \\ 1/2 \\ 1/6 \end{bmatrix}$$

PageRank Calculation: second iteration

An example of Simplified PageRank



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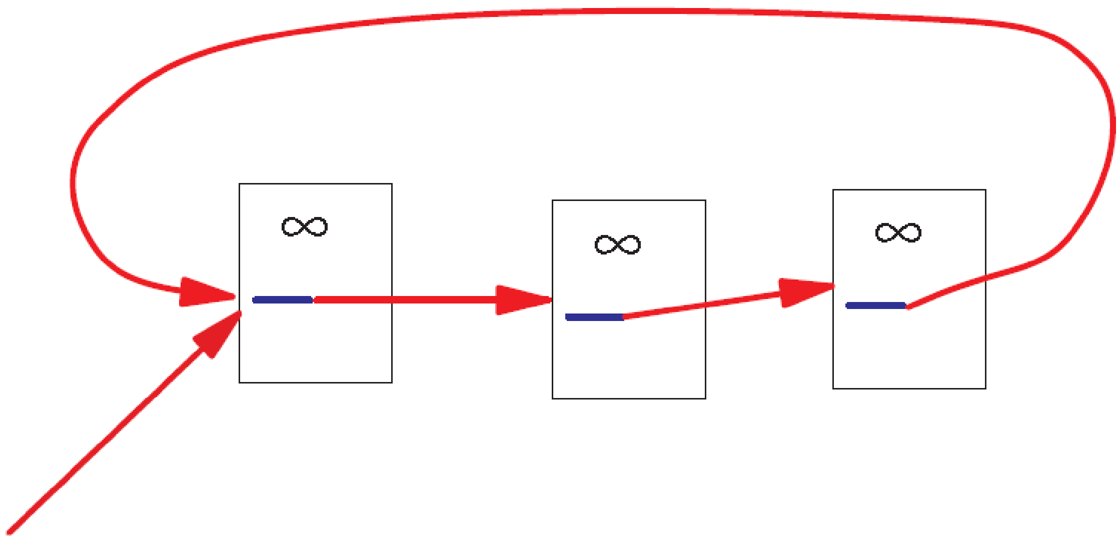
$$\begin{bmatrix} \text{yahoo} \\ \text{Amazon} \\ \text{Microsoft} \end{bmatrix} = \begin{bmatrix} 1/3 \\ 1/3 \\ 1/3 \end{bmatrix}$$

$$\begin{bmatrix} 3/8 \\ 11/24 \\ 1/6 \end{bmatrix} \begin{bmatrix} 5/12 \\ 17/48 \\ 11/48 \end{bmatrix} \dots \begin{bmatrix} 2/5 \\ 2/5 \\ 1/5 \end{bmatrix}$$

Convergence after some iterations

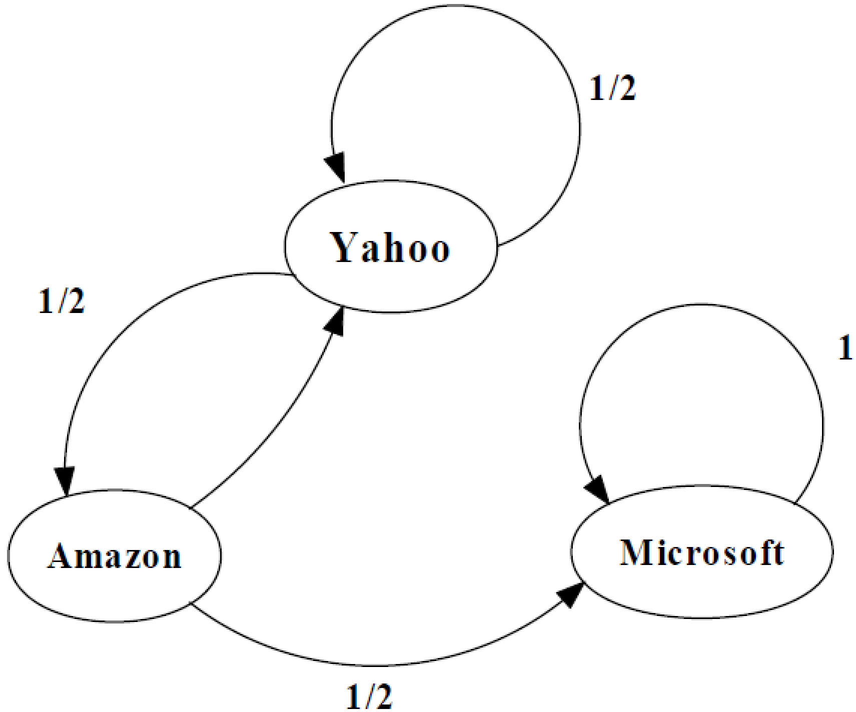
A Problem with Simplified PageRank

A loop:



During each iteration, the loop accumulates rank but never distributes rank to other pages!

An example of the Problem



$$M = \begin{bmatrix} 1/2 & 1/2 & 0 \\ 1/2 & 0 & 0 \\ 0 & 1/2 & 1 \end{bmatrix}$$

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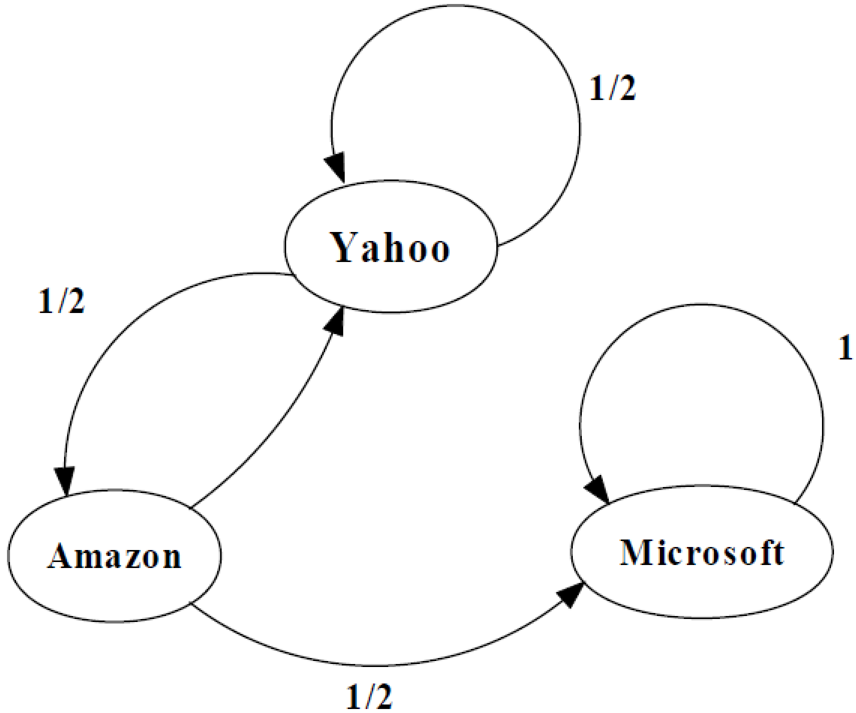
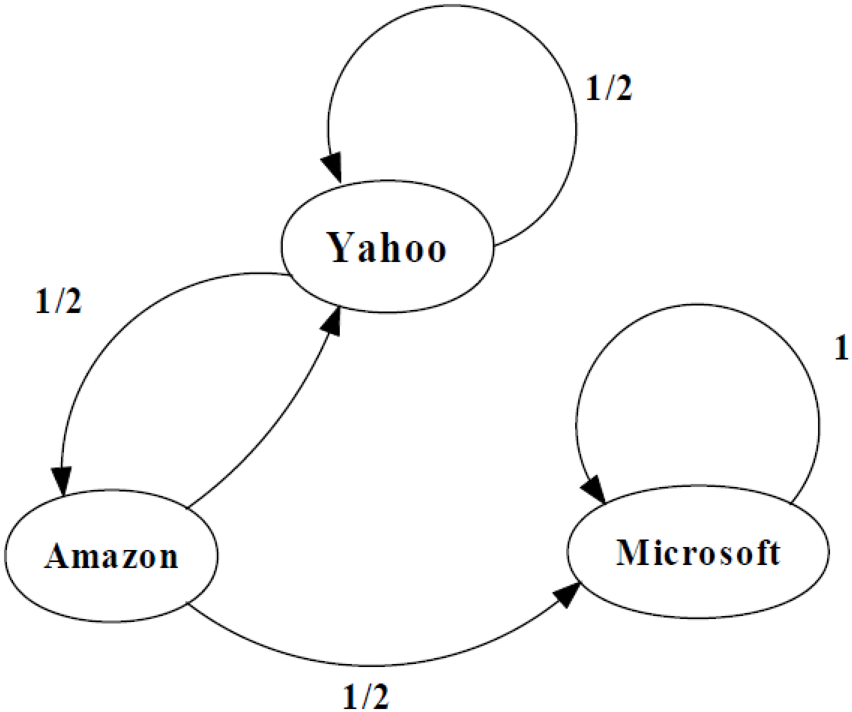


$$M = \begin{bmatrix} 1/2 & 1/2 & 0 \\ 1/2 & 0 & 0 \\ 0 & 1/2 & 1 \end{bmatrix}$$

$$\begin{bmatrix} \text{yahoo} \\ \text{Amazon} \\ \text{Microsoft} \end{bmatrix} = \begin{bmatrix} 1/3 \\ 1/3 \\ 1/3 \end{bmatrix}$$

$$\begin{bmatrix} 1/4 \\ 1/6 \\ 7/12 \end{bmatrix} = \begin{bmatrix} 1/2 & 1/2 & 0 \\ 1/2 & 0 & 0 \\ 0 & 1/2 & 1 \end{bmatrix} \begin{bmatrix} 1/3 \\ 1/6 \\ 1/2 \end{bmatrix}$$

An example of the Problem



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$$\begin{bmatrix} 5/24 \\ 1/8 \\ 2/3 \end{bmatrix} \begin{bmatrix} 1/6 \\ 5/48 \\ 35/48 \end{bmatrix} \;\ldots.\; \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

Random Walks in Graphs

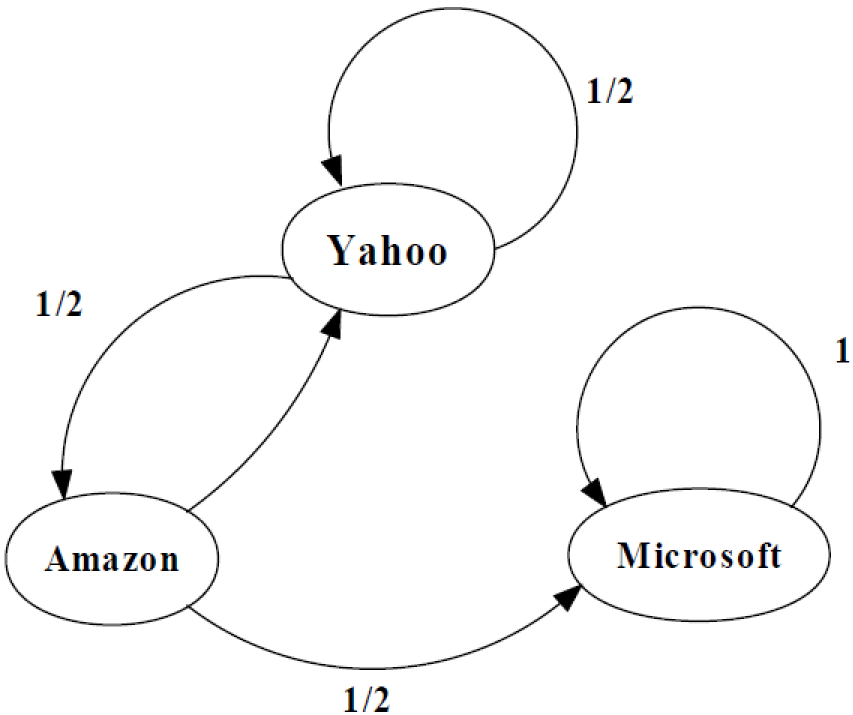
- The Random Surfer Model
  - The simplified model: the standing probability distribution of a random walk on the graph of the web. simply keeps clicking successive links at random
- The Modified Model
  - The modified model: the "random surfer" simply keeps clicking successive links at random, but periodically "gets bored" and jumps to a random page based on the distribution of E

Modified Version of PageRank

$$R'(u) = c_1 \sum_{v \in B_u} \frac{R'(v)}{N_v} + c_2 E(u)$$

E(u): a distribution of ranks of web pages that “users” jump to when they “gets bored” after successive links at random.

An example of Modified PageRank



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$C_1 = 0.8 \qquad C_2 = 0.2$

$$\begin{bmatrix} 0.333 \\ 0.333 \\ 0.333 \end{bmatrix} \begin{bmatrix} 0.333 \\ 0.200 \\ 0.467 \end{bmatrix} \begin{bmatrix} 0.280 \\ 0.200 \\ 0.520 \end{bmatrix} \begin{bmatrix} 0.259 \\ 0.179 \\ 0.563 \end{bmatrix} \ldots \begin{bmatrix} 7/33 \\ 5/33 \\ 21/33 \end{bmatrix}$$

Dangling Links

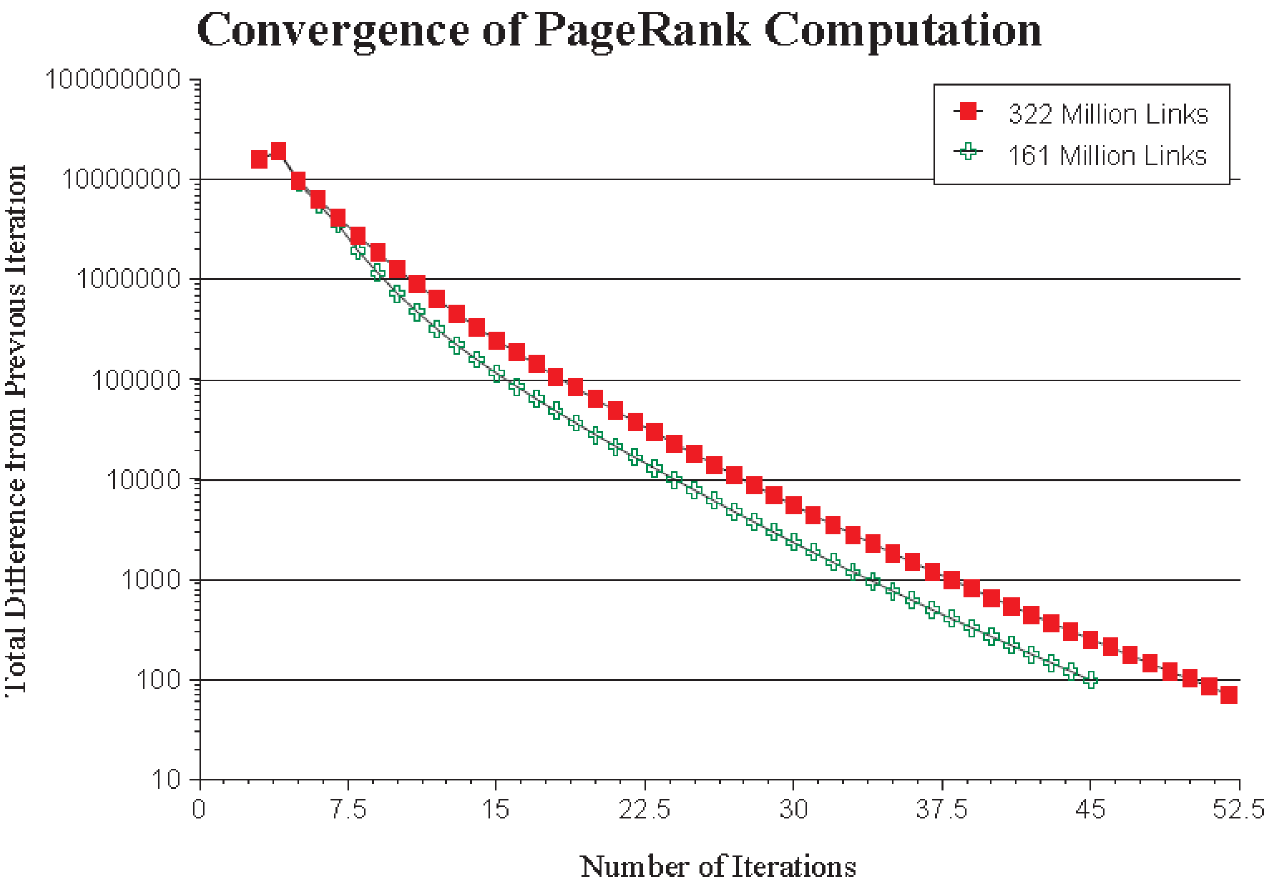
- Links that point to any page with no outgoing links
- Most are pages that have not been downloaded yet
- Affect the model since it is not clear where their weight should be distributed
- Do not affect the ranking of any other page directly
- Can be simply removed before pagerank calculation and added back afterwards

PageRank Implementation

- Convert each URL into a unique integer and store each hyperlink in a database using the integer IDs to identify pages
- Sort the link structure by ID
- Remove all the dangling links from the database
- Make an initial assignment of ranks and start iteration
  - Choosing a good initial assignment can speed up the pagerank
- Adding the dangling links back.

Convergence Property

- PR (322 Million Links): 52 iterations
- PR (161 Million Links): 45 iterations
- Scaling factor is roughly linear in *logn*



Convergence Property

- The Web is an expander-like graph
  - Theory of random walk: a random walk on a graph is said to be rapidly-mixing if it quickly converges to a limiting distribution on the set of nodes in the graph. A random walk is rapidly-mixing on a graph if and only if the graph is an expander graph.
  - Expander graph: every subset of nodes S has a neighborhood (set of vertices accessible via outedges emanating from nodes in S) that is larger than some factor α times of $|S|$. A graph has a good expansion factor if and only if the largest eigenvalue is sufficiently larger than the second-largest eigenvalue.

PageRank vs. Web Traffic

- Some highly accessed web pages have low page rank possibly because
  - People do not want to link to these pages from their own web pages (the example in their paper is pornographic sites…)
  - Some important backlinks are omitted

use usage data as a start vector for PageRank.

Hypertext-Induced Topic Search(HITS)

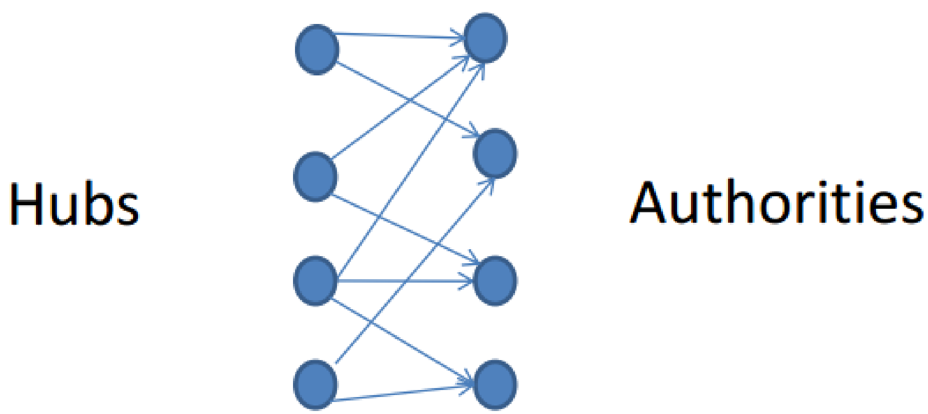
- To find a small set of most “authoritative” pages relevant to the query.
- Authority – Most useful/relevant/helpful results of a query.
  - “java” – java.com
  - “harvard” – harvard.edu
  - “search engine” – powerful search engines.

Hypertext-Induced Topic Search(HITS)

- Or Authorities & Hubs, developed by Jon Kleinberg, while visiting IBM Almaden
- IBM expanded HITS into Clever.
- Authorities – pages that are relevant and are linked to by many other pages
- Hubs – pages that link to many related authorities

Authorities & Hubs

- Intuitive Idea to find authoritative results using link analysis:
  - Not all hyperlinks related to the conferral of authority.
  - Find the pattern authoritative pages have:
    - Authoritative Pages share considerable overlap in the sets of pages that point to them.



Authorities & Hubs

- First Step:
  - Constructing a focused subgraph of the WWW based on query
- Second Step
  - Iteratively calculate authority weight and hub weight for each page in the subgraph

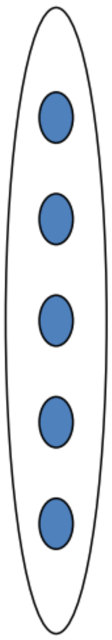
Constructing a focused subgraph

- Why not find authorities on the entire WWW?
  - The algorithm is non-trivial.
  - not necessary when there is a query.
- Objective: S_σ
  - S_σ is relatively small.
  - S_σ is rich in relevant pages.
  - S_σ contains most (or many) of the strongest authorities
- Solution:
  - Generate a Root Set Q_σ from text-based search engine
  - Expand the root set

Constructing a focused subgraph

Subgraph (σ, Ɛ t,d)
σ : a query string
Ɛ : a text-based search engine.
t, d: natural numbers.
Let *R denote the top t results of* Ɛ *on* σ

Set *S := R*
For each page *p ε R*
  Let $\Gamma^+(p)$ *denote the set of all pages p points to.*
  Let $\Gamma^-(p)$ *denote the set of all pages pointing to p.*
  Add all pages in $\Gamma^+(p)$ *to S.*
  If $(\Gamma^-(p)) < d$ *then*
    Add all pages in Γ *(p) to S.*
  Else
    Add an arbitrary set of *d pages from* $\Gamma^-(p)$ *to S*
End



Root Set

Constructing a focused subgraph

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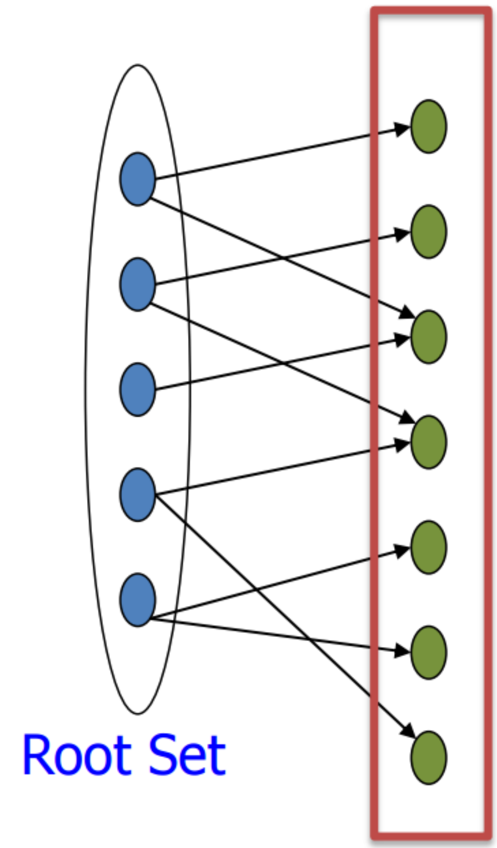

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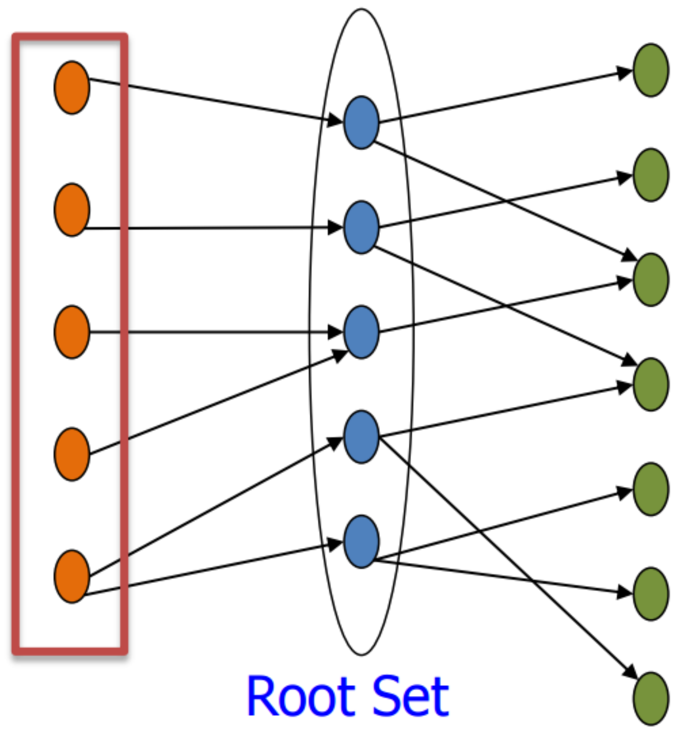
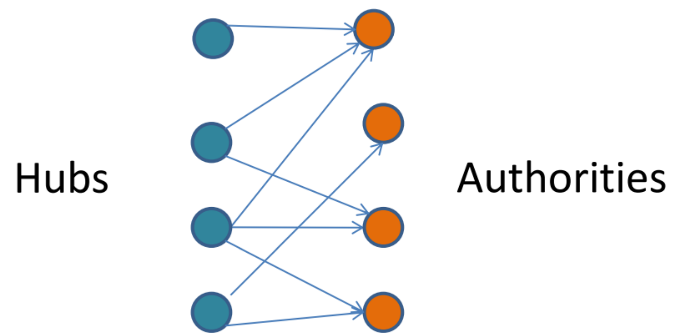


Computing Hubs and Authorities

- Rules:
  - A good hub points to many good authorities.
  - A good authority is pointed to by many good hubs.
  - Authorities and hubs have a mutual reinforcement relationship.



Computing Hubs and Authorities

- Let authority score of the page i be $x(i)$, and the hub score of page i be $y(i)$.
- mutual reinforcing relationship:
- I step:

$$x(i) = \sum_{(j,i)\in E} y(j)$$

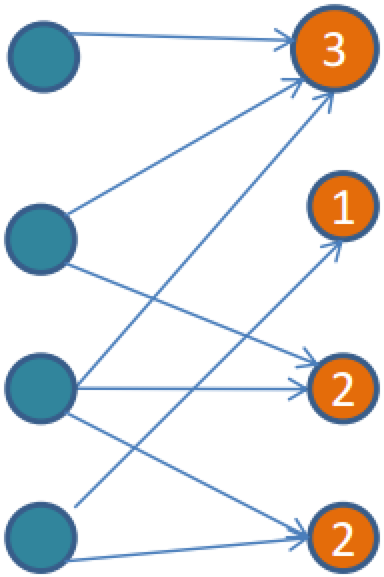
- O step:

$$y(i) = \sum_{(i,j)\in E} x(j)$$

Example (no normalization)

1st Iteration

I Step



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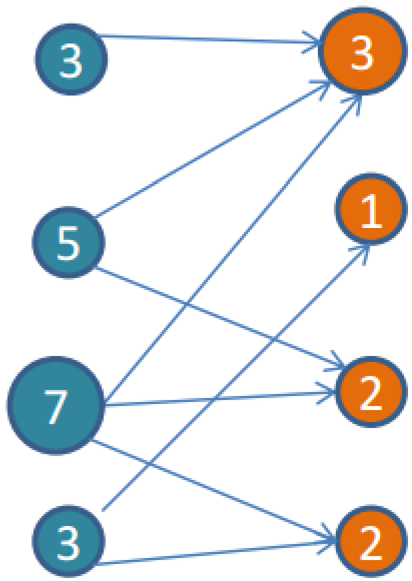
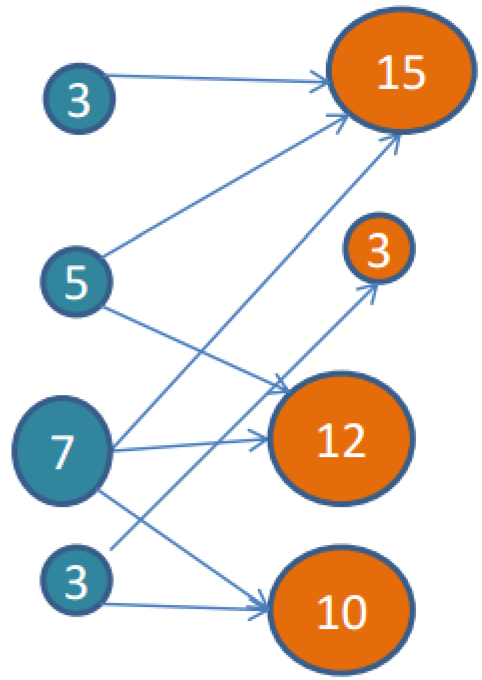


Example (no normalization)

2nd Iteration

I Step



Example (no normalization)

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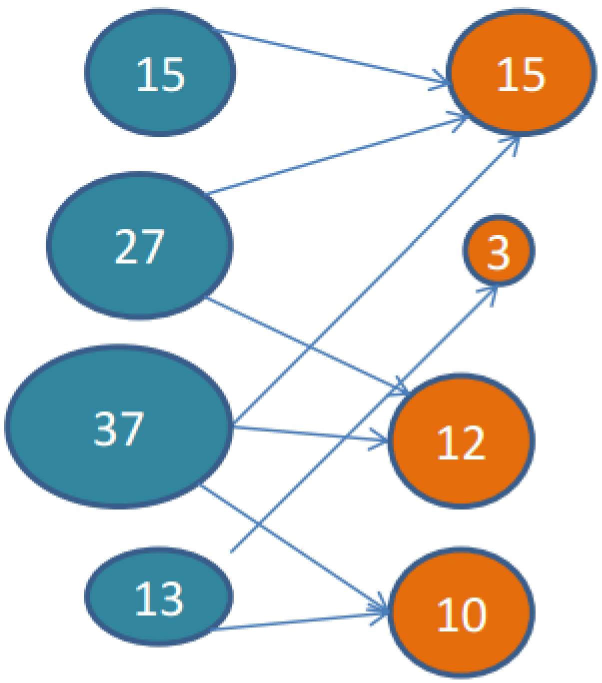
O Step



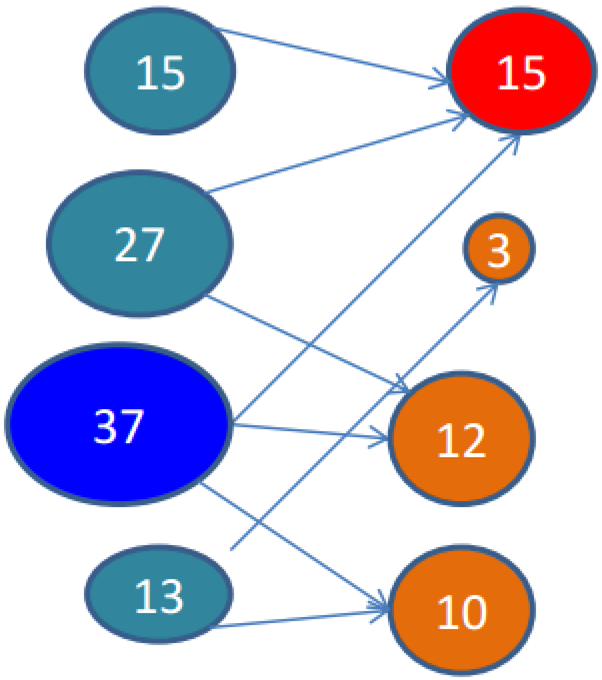
Example (no normalization)

- 2nd Iteration
- I Step
- O Step

...

...

...



The Iterative Algorithm

Iterate(G,k)

G: a collection of n linked pages

k: a natural number

Let z denote the vector $(1, 1, 1, \ldots, 1) \in \mathbf{R}^n$.

Set $x_0 := z$.

Set $y_0 := z$.

Initialization

For $i = 1, 2, \ldots, k$

Apply the $\mathcal{I}$ operation to (x_{i-1}, y_{i-1}), obtaining new x-weights x'_i.

Apply the $\mathcal{O}$ operation to (x'_i, y_{i-1}), obtaining new y-weights y'_i.

Normalize x'_i, obtaining x_i.

Normalize y'_i, obtaining y_i.

End

Return (x_k, y_k).

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Normalize y'_i, obtaining y_i.

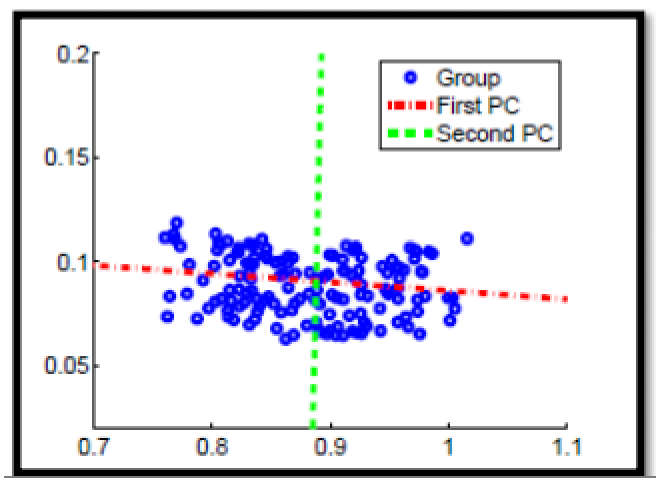
Normalization

End

Return (x_k, y_k).

A Statistical View of HITS

- 1^{st} Eigenvalue of AA^T = singular value of A
- 1^{st} Eigenvector of AA^T = transform vector to the 1^{st} principal component.
- Principal Component:
  - Matrix A a set of vectors.
  - The dimension where vectors significantly distributed



A Statistical View of HITS

- The weight of authority equals the contribution of transforming the dataset to first principal component.
  - Importance of this vector for the distribution of whole dataset.
- From the statistical view:
  - HITS can be implemented by PCA
  - HITS is different from clustering using dimensionality reduction.
  - The number of samples of PCA is limited.

PageRank v.s. HITS

- PageRank
  - Computed for all web pages stored prior to the query
  - Computes authorities only
  - Fast to compute
- HITS
  - Performed on the subset generated by each query.
  - Computes authorities and hubs
  - Easy to compute, real-time execution is hard.

Which one is more suitable for large scale data set??

Summary

- PageRank is a global ranking of all web pages based on their locations in the web graph structure
- PageRank uses information which is external to the web pages – backlinks
- Backlinks from important pages are more significant than backlinks from average pages
- The structure of the web graph is very useful for information retrieval tasks.

- HITS – Find authoritative pages; Construct subgraph; Mutually reinforcing relationship; Iterative algorithm