Artificial Intelligence
ENCS 434

Fuzzy logic

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Fuzzy logic

- Introduction, or what is fuzzy thinking?
- Fuzzy sets
- Linguistic variables and hedges
- Operations of fuzzy sets
- Fuzzy rules
- Mamdani fuzzy inference
- Sugeno fuzzy inference
- Case study
- Summary

Introduction

- Experts rely on **common sense** when they solve problems.
- **How can we represent expert knowledge that uses vague and ambiguous terms in a computer?**
- Fuzzy logic is not logic that is fuzzy, but logic that is used to describe fuzziness. Fuzzy logic is the theory of fuzzy sets, sets that calibrate vagueness.
- Fuzzy logic is based on the idea that all things admit of degrees. Temperature, height, speed, distance, beauty – all come on a sliding scale. The motor is running *really hot*. Tom is a *very tall* guy.

Introduction, or what is fuzzy thinking?

- Boolean logic uses sharp distinctions. It forces us to draw lines between members of a class and non-members. For instance, we may say, Tom is tall because his height is 181 cm. If we drew a line at 180 cm, we would find that David, who is 179 cm, is small. Is David really a small man or we have just drawn an arbitrary line in the sand?

- Fuzzy logic reflects how people think. It attempts to model our sense of words, our decision making and our common sense. As a result, it is leading to new, more human, intelligent systems.

Introduction, or what is fuzzy thinking?

- **Fuzzy**, or multi-valued logic was introduced in the 1930s by **Jan Lukasiewicz**, a Polish philosopher. While classical logic operates with only two values 1 (true) and 0 (false), Lukasiewicz introduced logic that extended the range of truth values to all real numbers in the interval between 0 and 1.
- He used a number in this interval to represent the *possibility* that a given statement was true or false.
- For example, the possibility that a man 181 cm tall is really tall might be set to a value of 0.86. It is *likely* that the man is tall.
- This work led to an inexact reasoning technique often called **possibility theory**.

Introduction, or what is fuzzy thinking?

- Later, in 1937, **Max Black** published a paper called "Vagueness: an exercise in logical analysis". In this paper, he argued that a continuum implies degrees.
- Imagine, he said, a line of countless "chairs". At one end is a Chippendale. Next to it is a near-Chippendale, in fact indistinguishable from the first item. Succeeding "chairs" are less and less chair-like, until the line ends with a log.
- When does a *chair* become a *log*? Max Black stated that if a continuum is discrete, a number can be allocated to each element. He accepted **vagueness as a matter of probability**.

Introduction, or what is fuzzy thinking?

- In 1965 **Lotfi Zadeh**, published his famous paper “Fuzzy sets”. Zadeh extended the work on possibility theory into a formal system of mathematical logic, and introduced a new concept for applying natural language terms.

- This new logic for representing and manipulating fuzzy terms was called **fuzzy logic**, and Zadeh became the Master of *fuzzy logic*.

Introduction, or what is fuzzy thinking?

- ***Why fuzzy?***

  As Zadeh said, the term is concrete, immediate and descriptive; we all know what it means. However, many people in the West were repelled by the word *fuzzy*, because it is usually used in a negative sense.

- ***Why logic?***

  Fuzziness rests on fuzzy set theory, and fuzzy logic is just a small part of that theory.

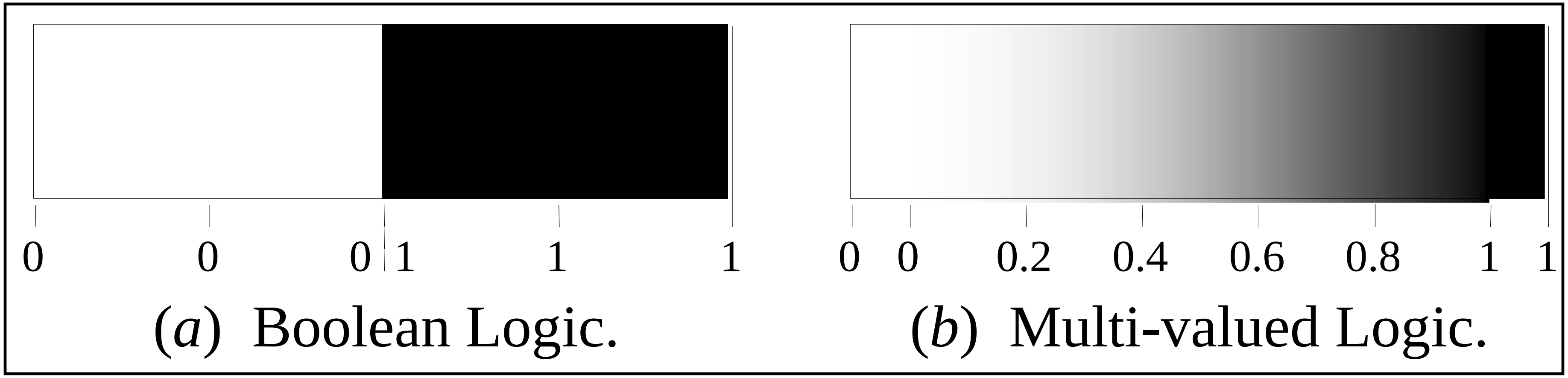
Fuzzy logic

- **Fuzzy logic is a set of mathematical principles for knowledge representation based on degrees of membership.**

- Unlike two-valued Boolean logic, fuzzy logic is **multi-valued**. It deals with **degrees of membership** and **degrees of truth**. Fuzzy logic uses the continuum of logical values between 0 (completely false) and 1 (completely true). Instead of just black and white, it employs the spectrum of colours, accepting that things can be partly true and partly false at the same time.

Fuzzy logic

Range of logical values in Boolean and fuzzy logic



Fuzzy sets

- The concept of a **set** is fundamental to mathematics.

- However, our own language is also the supreme expression of sets. For example, *car* indicates the *set of cars*. When we say *a car*, we mean one out of the set of cars.

Fuzzy sets

- Crisp set theory is governed by a logic that uses one of only two values: true or false. This logic cannot represent vague concepts, and therefore fails to give the answers on the paradoxes.

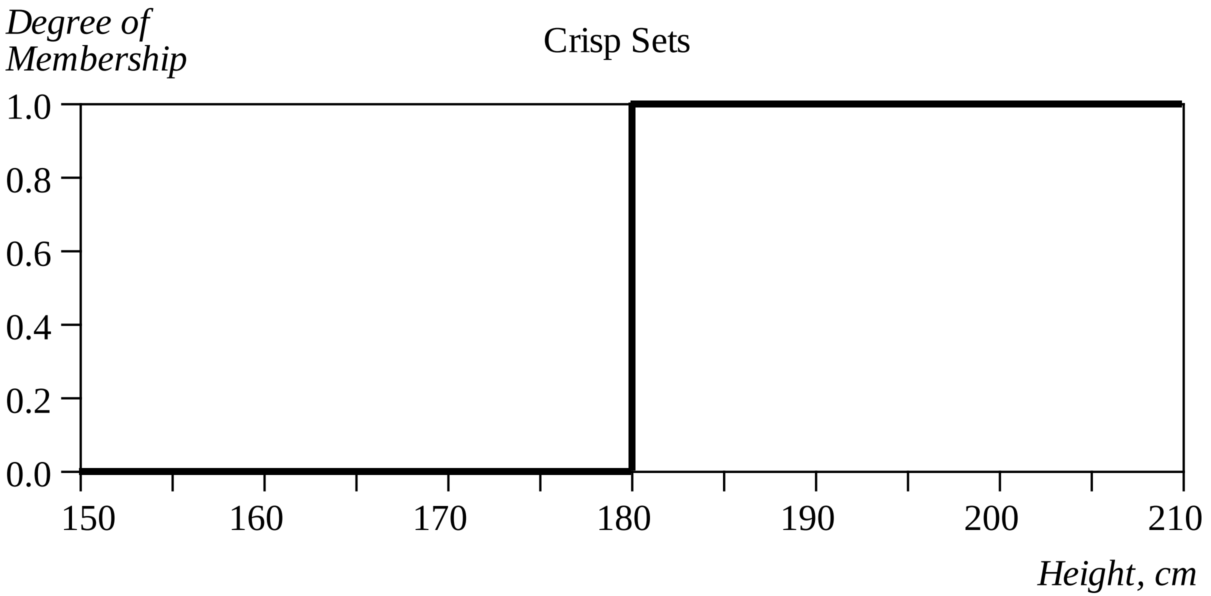
- The basic idea of the fuzzy set theory is that an element belongs to a fuzzy set with a certain degree of membership. Thus, a proposition is not either true or false, but may be partly true (or partly false) to any degree. This degree is usually taken as a real number in the interval [0,1].

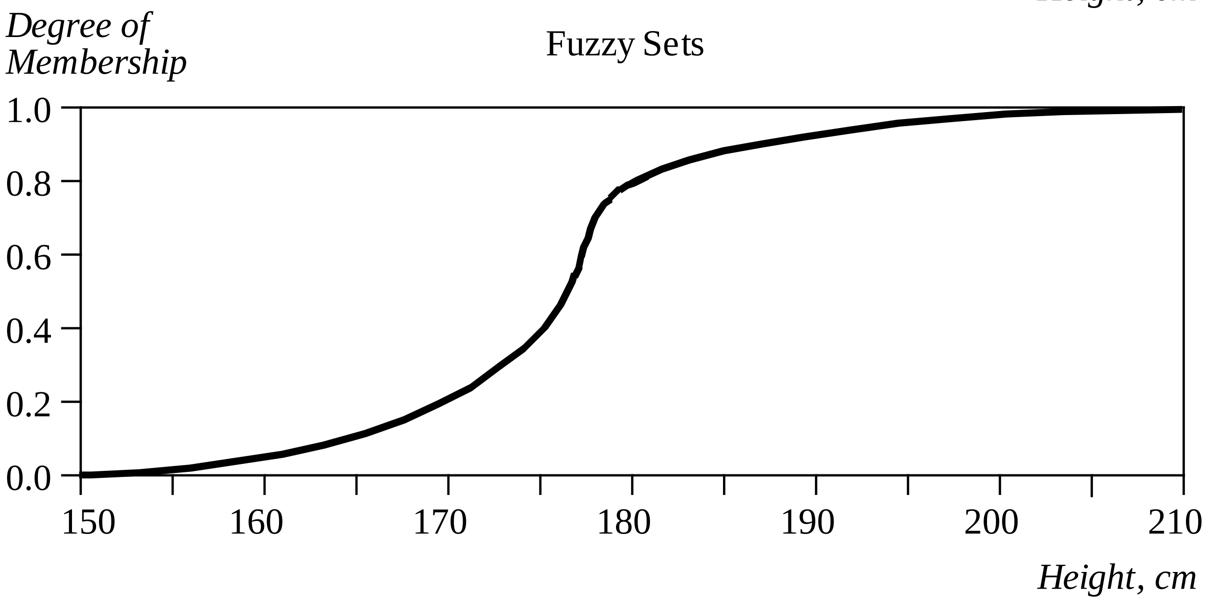
Fuzzy sets

- The classical example in fuzzy sets is *tall men*. The elements of the fuzzy set “tall men” are all men, but their degrees of membership depend on their height.

Name	Height, cm	Degree of Membership	
		Crisp	*Fuzzy*
Chris	208	1	1.00
Mark	205	1	1.00
John	198	1	0.98
Tom	181	1	0.82
David	179	0	0.78
Mike	172	0	0.24
Bob	167	0	0.15
Steven	158	0	0.06
Bill	155	0	0.01
Peter	152	0	0.00

Crisp and fuzzy sets of "*tall men*"





Fuzzy sets

- The x-axis represents the **universe of discourse** – the range of all possible values applicable to a chosen variable. In our case, the variable is the man height. According to this representation, the universe of men's heights consists of all tall men.

- The y-axis represents the **membership value of the fuzzy set**. In our case, the fuzzy set of "*tall men*" maps height values into corresponding membership values.

Fuzzy sets

- **A fuzzy set is a set with fuzzy boundaries.**
  - Let X be the universe of discourse and its elements be denoted as x. In the classical set theory, **crisp set A of X is defined as function $f_A(x)$ called the characteristic function of A**

$f_A(x)$: $X \to \{0, 1\}$, where

$$f_A(x) = \begin{cases} 1, \text{if } x \in A \\ 0, \text{if } x \notin A \end{cases}$$

This set maps universe X to a set of two elements. For any element x of universe X, characteristic function $f_A(x)$ is equal to 1 if x is an element of set A, and is equal to 0 if x is not an element of A.

Fuzzy sets

- In the fuzzy theory, fuzzy set A of universe X is defined by function $\mu_A(x)$ called the *membership function* of set A

$$\mu_A(x): X \rightarrow [0, 1], \text{ where } \begin{aligned} \mu_A(x) &= 1 \text{ if } x \text{ is totally in } A; \\ \mu_A(x) &= 0 \text{ if } x \text{ is not in } A; \\ 0 < \mu_A(x) &< 1 \text{ if } x \text{ is partly in } A. \end{aligned}$$

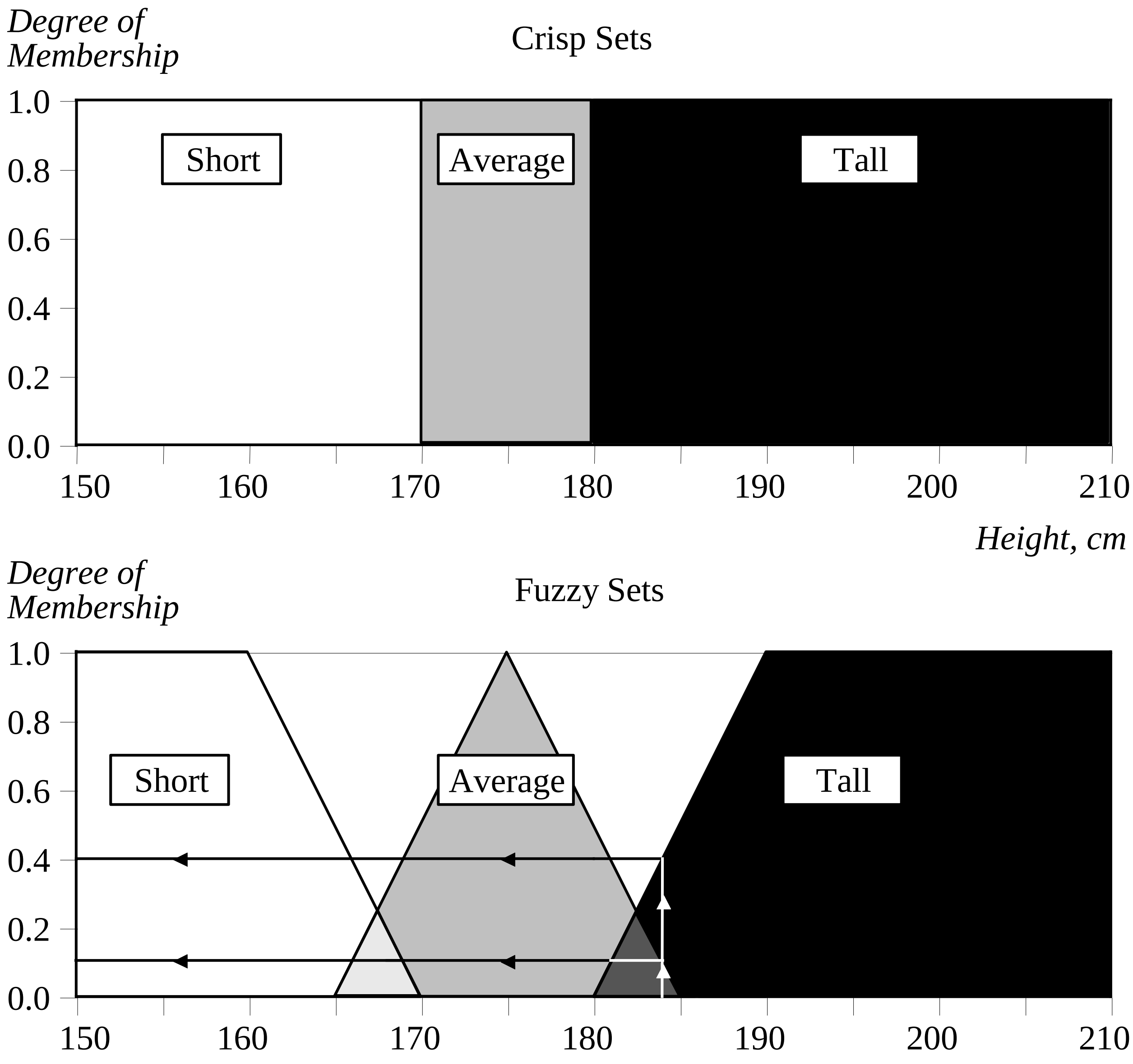
This set allows a continuum of possible choices. For any element x of universe X, membership function $\mu_A(x)$ equals the degree to which x is an element of set A. This degree, a value between 0 and 1, represents the **degree of membership**, also called **membership value**, of element x in set A.

How to represent a fuzzy set in a computer?

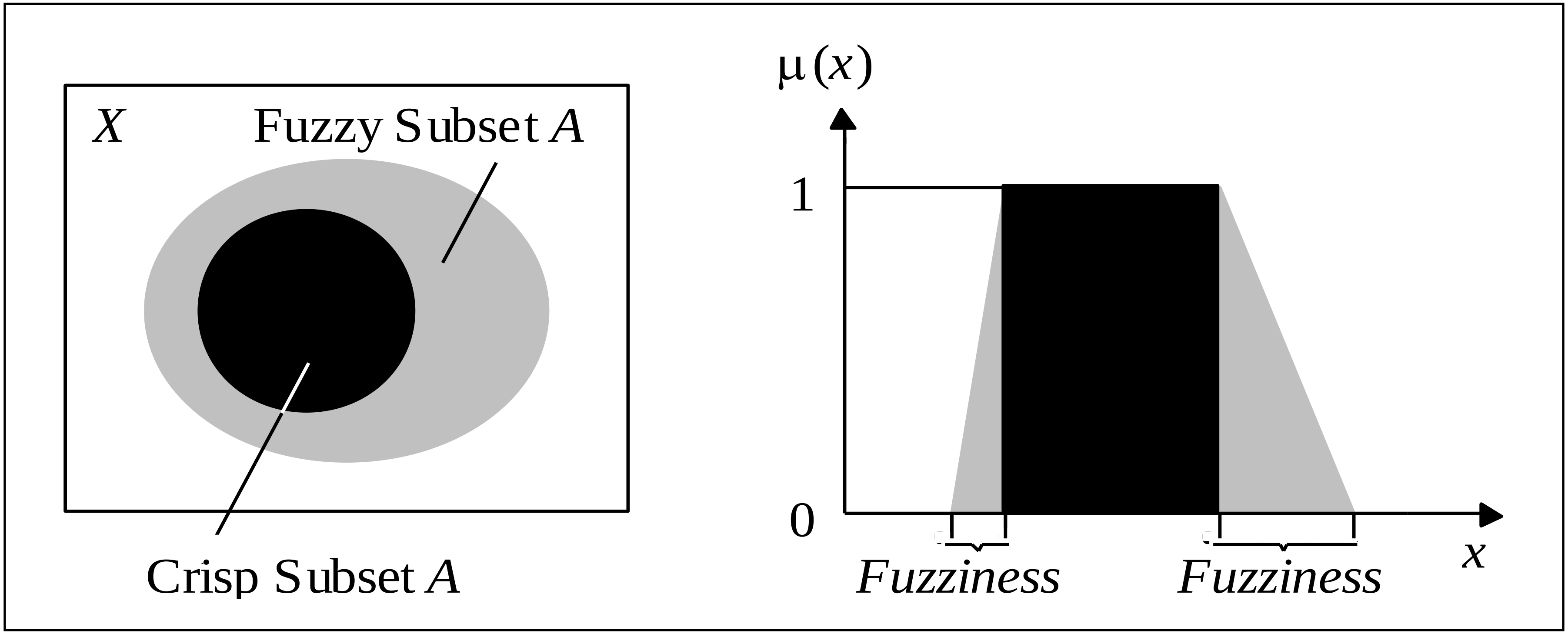
- First, we determine the membership functions. In our "*tall men*" example, we can obtain fuzzy sets of *tall*, *short* and *average* men.

- The universe of discourse – the men's heights – consists of three sets: *short*, *average* and *tall men*. As you will see, a man who is 184 cm tall is a member of the *average men* set with a degree of membership of 0.1, and at the same time, he is also a member of the *tall men* set with a degree of 0.4.

Crisp and fuzzy sets of short, average and tall men



Representation **of crisp and fuzzy subsets**



Typical functions that can be used to represent a fuzzy set are sigmoid, gaussian and pi. However, these functions increase the time of computation. Therefore, in practice, most applications use **linear fit functions**.

Linguistic variables and hedges

- At the root of fuzzy set theory lies the idea of linguistic variables.

- **A linguistic variable is a fuzzy variable.** For example, the statement "John is tall" implies that the linguistic variable *John* takes the linguistic value *tall*.

linguistic variables

In fuzzy expert systems, linguistic variables are used in fuzzy rules. For example:

IF wind is strong
THEN sailing is good

IF project_duration is long
THEN completion_risk is high

IF speed is slow
THEN stopping_distance is short

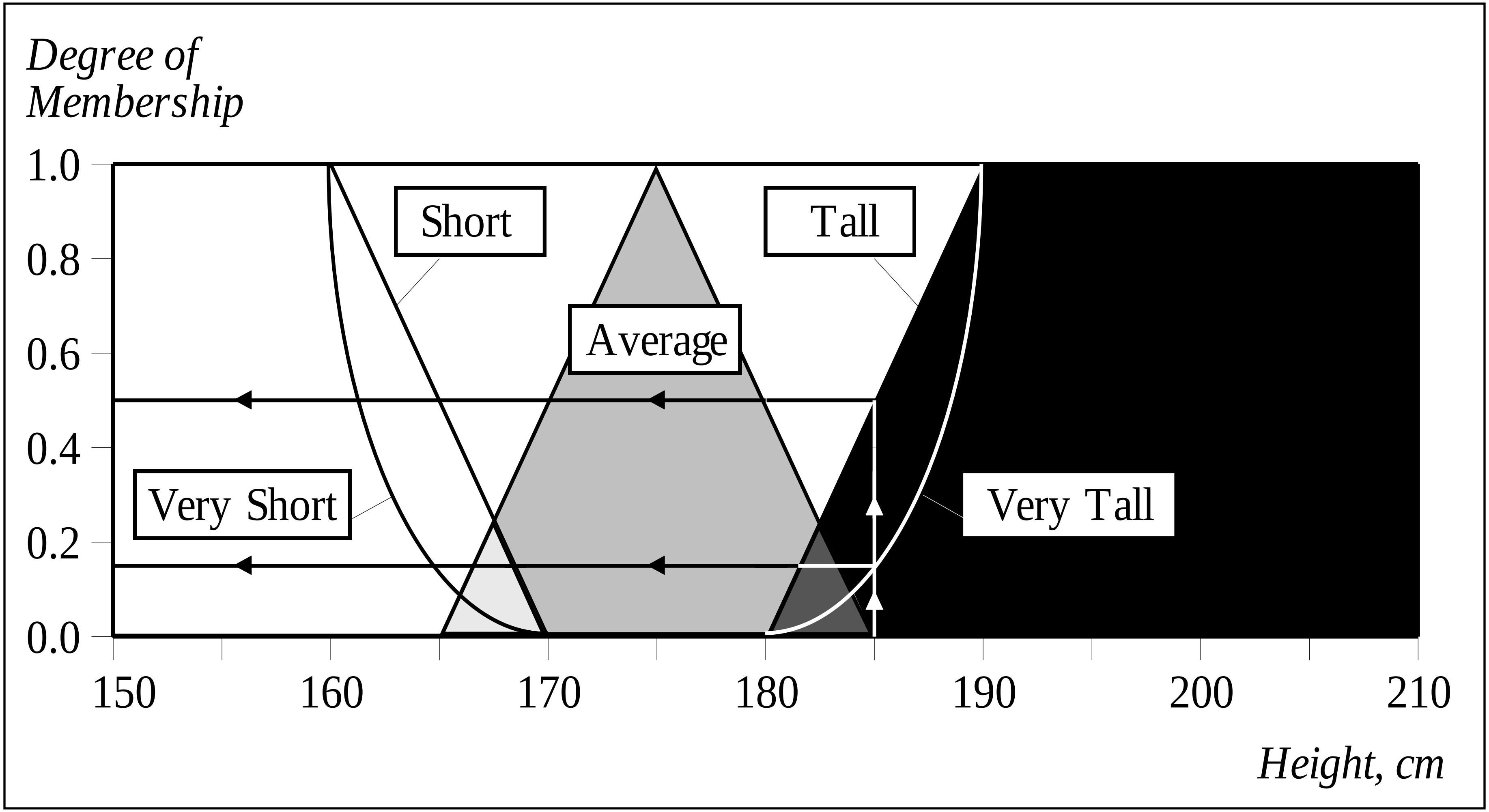
linguistic variables

- The range of possible values of a linguistic variable represents the universe of discourse of that variable. For example, the universe of discourse of the linguistic variable *speed* might have the range between 0 and 220 km/h and may include such fuzzy subsets as *very slow*, *slow*, *medium*, *fast*, and *very fast*.

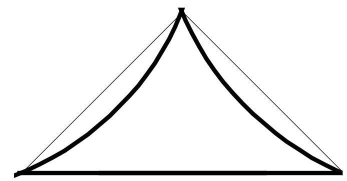
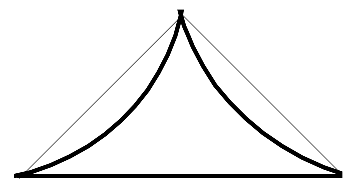
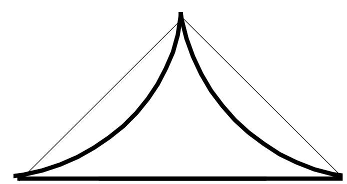
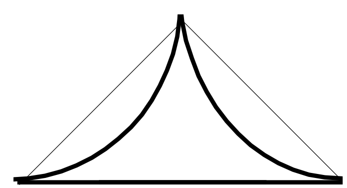
- A linguistic variable carries with it the concept of fuzzy set qualifiers, called *hedges*.

- **Hedges are terms that modify the shape of fuzzy sets. They include adverbs such as *very*, *somewhat*, *quite*, *more or less* and *slightly*.**

Fuzzy sets with the hedge *very*



Representation of hedges in fuzzy logic

Hedge	*Mathematical Expression*	*Graphical Representation*
A little	$[\mu_A(x)]^{1.3}$	
Slightly	$[\mu_A(x)]^{1.7}$	
Very	$[\mu_A(x)]^{2}$	
Extremely	$[\mu_A(x)]^{3}$	

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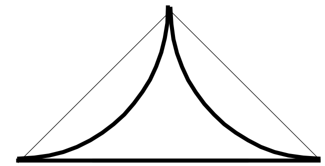
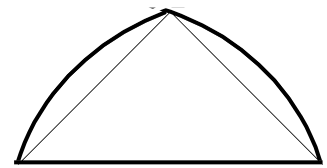
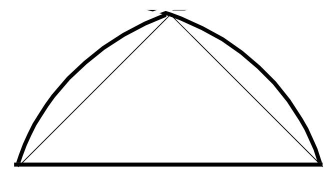
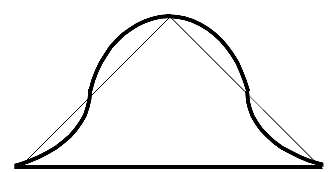
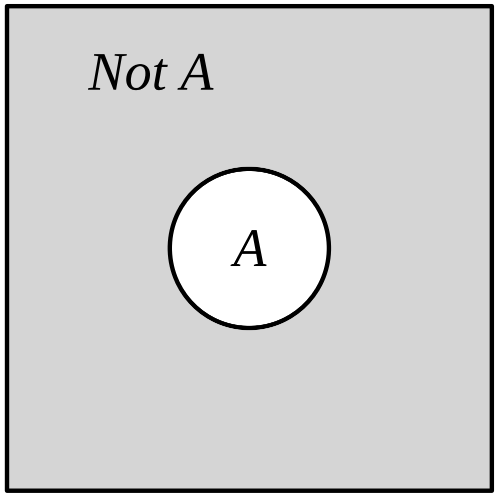
Representation of hedges in fuzzy logic (cont.)

Hedge	*Mathematical Expression*	*Graphical Representation*
Very very	$\mu_A(x)\ ^4$	
More or less	$\sqrt{\mu_A(x)}$	
Somewhat	$\sqrt{\mu_A(x)}$	
Indeed	$2\ \ \mu_A(x)\ ^2$ if $0 \le \mu_A \le 0.5$ $1 - 2\ \ 1 - \mu_A(x)\ ^2$ if $0.5 < \mu_A \le 1$	

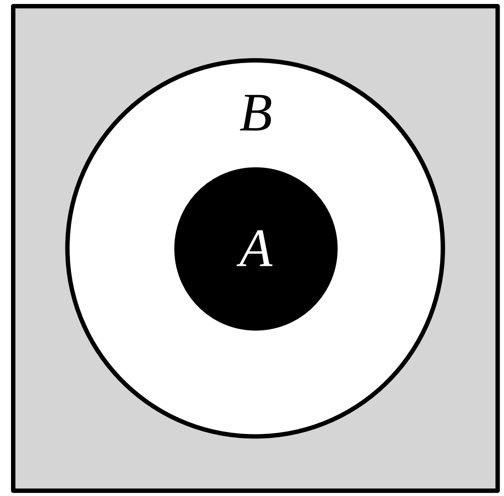
Operations of fuzzy sets

The classical set theory developed in the late 19th century by Georg Cantor describes how crisp sets can interact. These interactions are called **operations**.

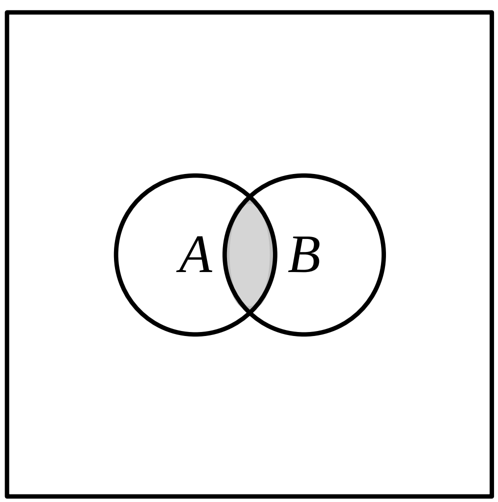
Cantor's sets



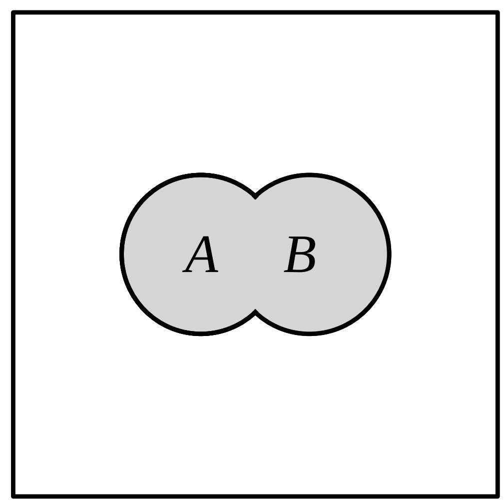
Complement



Containment



Intersection



Union

Operations of fuzzy sets

Complement

<u>*Crisp Sets*:</u> *Who does not belong to the set?*

<u>*Fuzzy Sets*:</u> *How much do elements not belong to the set?*

The complement of a set is an opposite of this set. For example, if we have the set of *tall men*, its complement is the set of *NOT tall men*. When we remove the tall men set from the universe of discourse, we obtain the complement. If A is the fuzzy set, its complement $\neg A$ can be found as follows:

$$\mu_{\neg A}(x) = 1 - \mu_A(x)$$

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Operations of fuzzy sets

■ Containment

Crisp Sets: *Which sets belong to which other sets?*

Fuzzy Sets: *Which sets belong to other sets?*

Similar to a Chinese box, a set can contain other sets. The smaller set is called the **subset**. For example, the set of *tall men* contains all tall men; *very tall men* is a subset of *tall men*. However, the *tall men* set is just a subset of the set of *men*. In crisp sets, all elements of a subset entirely belong to a larger set. In fuzzy sets, however, each element can belong less to the subset than to the larger set. Elements of the fuzzy subset have smaller memberships in it than in the larger set.

Operations of fuzzy sets

Intersection

Crisp Sets: Which element belongs to both sets?

Fuzzy Sets: How much of the element is in both sets?

In classical set theory, an intersection between two sets contains the elements shared by these sets. For example, the intersection of the set of *tall men* and the set of *fat men* is the area where these sets overlap. In fuzzy sets, an element may partly belong to both sets with different memberships. A fuzzy intersection is the lower membership in both sets of each element. The fuzzy intersection of two fuzzy sets *A* and *B* on universe of discourse *X*:

$$\mu_{A \cap B}(x) = \min\,[\mu_A(x), \mu_B(x)] = \mu_A(x) \cap \mu_B(x),$$

where $x \in X$

Operations of fuzzy sets

- **Union**

<u>Crisp Sets</u>: Which element belongs to either set?

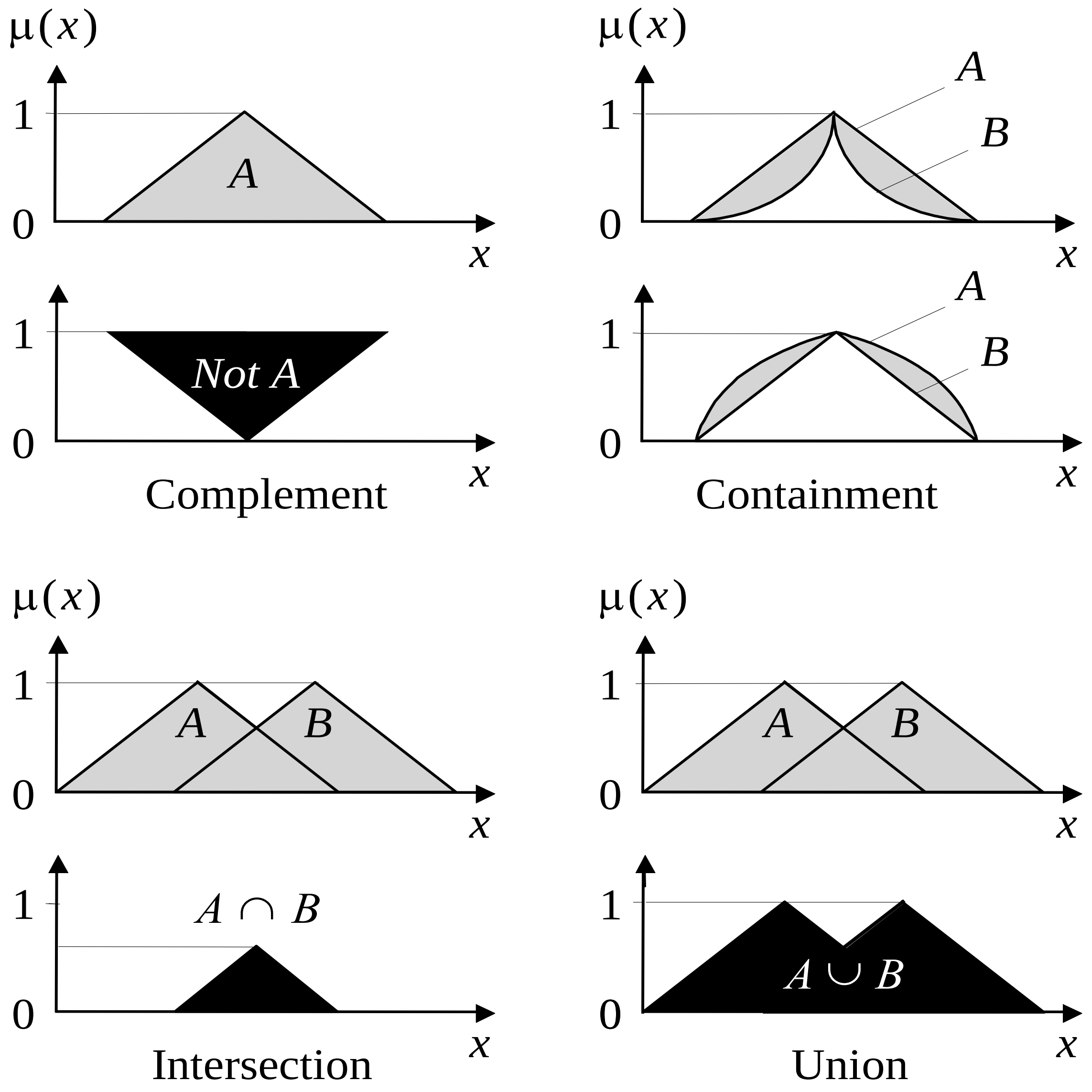
<u>Fuzzy Sets</u>: How much of the element is in either set?

The union of two crisp sets consists of every element that falls into either set. For example, the union of *tall men* and *fat men* contains all men who are tall **OR** fat. In fuzzy sets, the union is the reverse of the intersection. That is, the union is the largest membership value of the element in either set. The fuzzy operation for forming the union of two fuzzy sets *A* and *B* on universe *X* can be given as:

$$\mu_{A \cup B}(x) = max\,[\mu_A(x), \mu_B(x)] = \mu_A(x) \cup \mu_B(x),$$

where $x \in X$

Operations of fuzzy sets



Fuzzy rules

In 1973, **Lotfi Zadeh** published his second most influential paper. This paper outlined a new approach to analysis of complex systems, in which Zadeh suggested capturing human knowledge in fuzzy rules.

What is a fuzzy rule?

A fuzzy rule can be defined as a conditional statement in the form:

IF x **is** A
THEN y **is** B

where x and y are linguistic variables; and A and B are linguistic values determined by fuzzy sets on the universe of discourses X and Y, respectively.

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What is the difference between classical and fuzzy rules?

- A classical IF-THEN rule uses binary logic, for example,

Rule: 1
IF speed is > 100
THEN stopping_distance is long

Rule: 2
IF speed is < 40
THEN stopping_distance is short

- The variable *speed* can have any numerical value between 0 and 220 km/h, but the linguistic variable *stopping_distance* can take either value *long* or *short*. In other words, classical rules are expressed in the black-and-white language of Boolean logic.

We can also represent the stopping distance rules in a fuzzy form:

Rule: 1
IF speed is fast
THEN stopping_distance is long

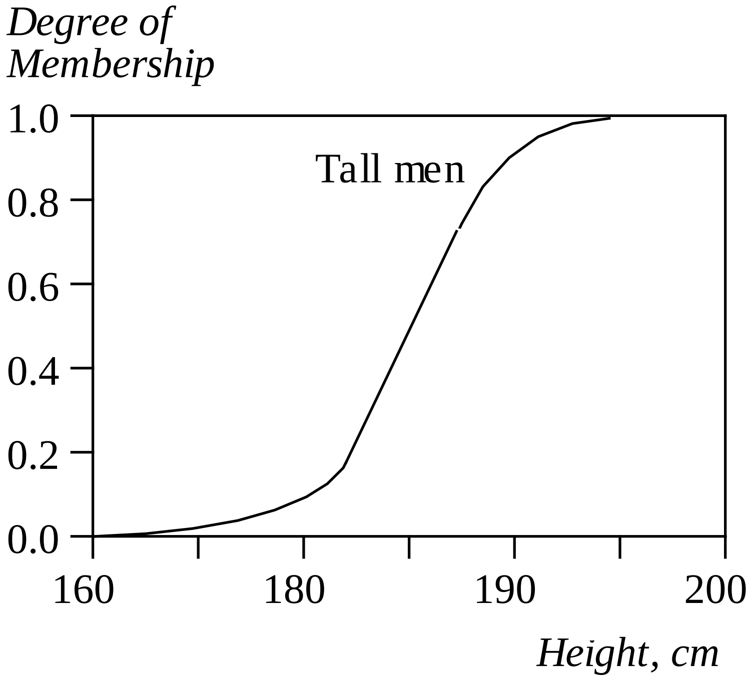
Rule: 2
IF speed is slow
THEN stopping_distance is short

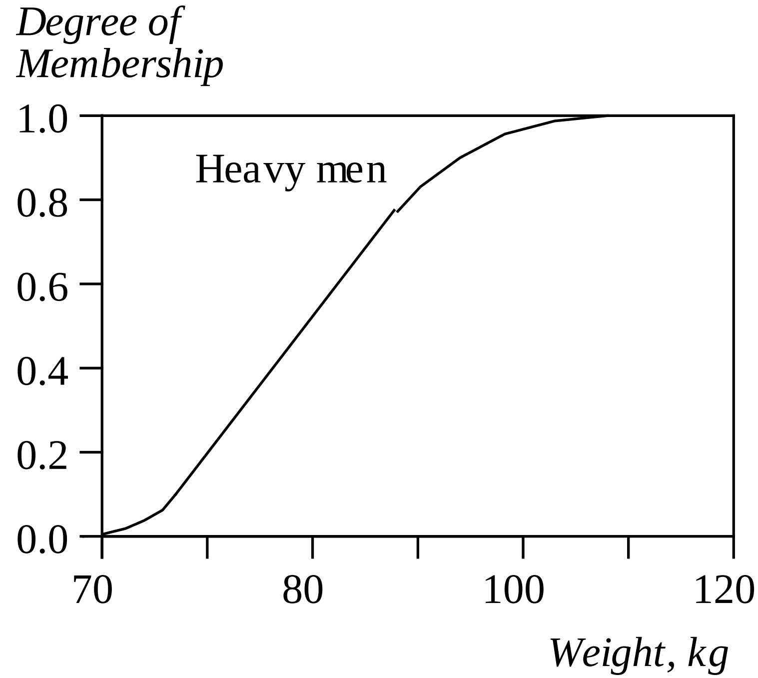
- In fuzzy rules, the linguistic variable *speed* also has the range (the universe of discourse) between 0 and 220 km/h, but this range includes fuzzy sets, such as *slow*, *medium* and *fast*. The universe of discourse of the linguistic variable *stopping_distance* can be between 0 and 300 m and may include such fuzzy sets as *short*, *medium* and *long*.

Fuzzy rules

- Fuzzy rules relate fuzzy sets.
- In a fuzzy system, all rules fire to some extent, or in other words they fire partially. If the antecedent is true to some degree of membership, then the consequent is also true to that same degree.

Fuzzy sets of *tall* and *heavy* men



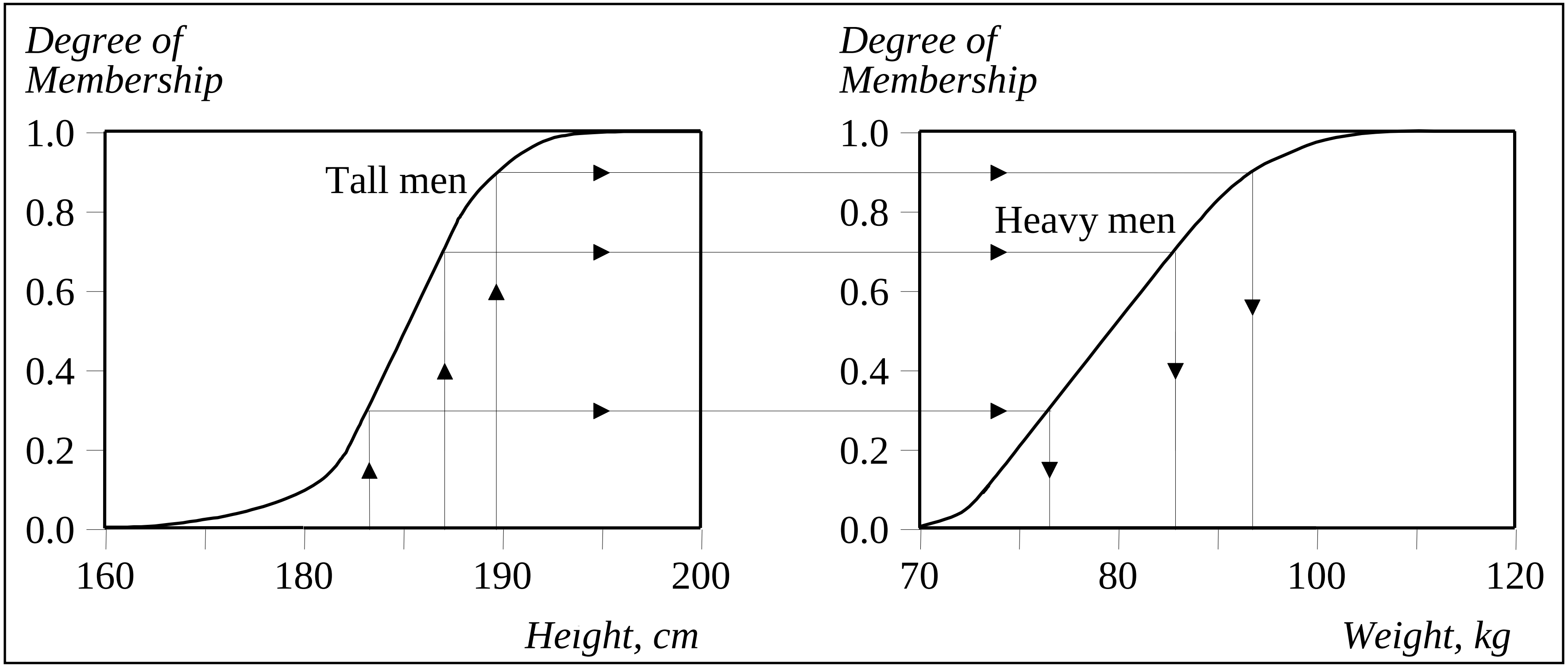


These fuzzy sets provide the basis for a weight estimation model. The model is based on a relationship between a man's height and his weight:

IF height is *tall*
THEN weight is *heavy*

Fuzzy rules

The value of the output or a truth membership grade of the rule consequent can be estimated directly from a corresponding truth membership grade in the antecedent. This form of fuzzy inference uses a method called **monotonic selection**.



Fuzzy rules

A fuzzy rule can have multiple antecedents, for example:

IF project_duration is long
AND project_staffing is large
AND project_funding is inadequate
THEN risk is high

IF service is excellent
OR food is delicious
THEN tip is generous

Fuzzy rules

The consequent of a fuzzy rule can also include multiple parts, for instance:

IF temperature is hot
THEN hot_water is reduced;
cold_water is increased

Fuzzy inference

The most commonly used fuzzy inference technique is the so-called Mamdani method. In 1975, Professor **Ebrahim Mamdani** of London University built one of the first fuzzy systems to control a steam engine and boiler combination. He applied a set of fuzzy rules supplied by experienced human operators.

Mamdani fuzzy inference

- The Mamdani-style fuzzy inference process is performed in four steps:
  - fuzzification of the input variables,
  - rule evaluation;
  - aggregation of the rule outputs, and finally
  - defuzzification.

Mamdani fuzzy inference

We examine a simple two-input one-output problem that includes three rules:

Rule: 1
IF *x* **is** *A*3
OR *y* **is** *B*1
THEN *z* **is** *C*1

Rule: 2
IF *x* **is** *A*2
AND *y* **is** *B*2
THEN *z* **is** *C*2

Rule: 3
IF *x* **is** *A*1
THEN *z* **is** *C*3

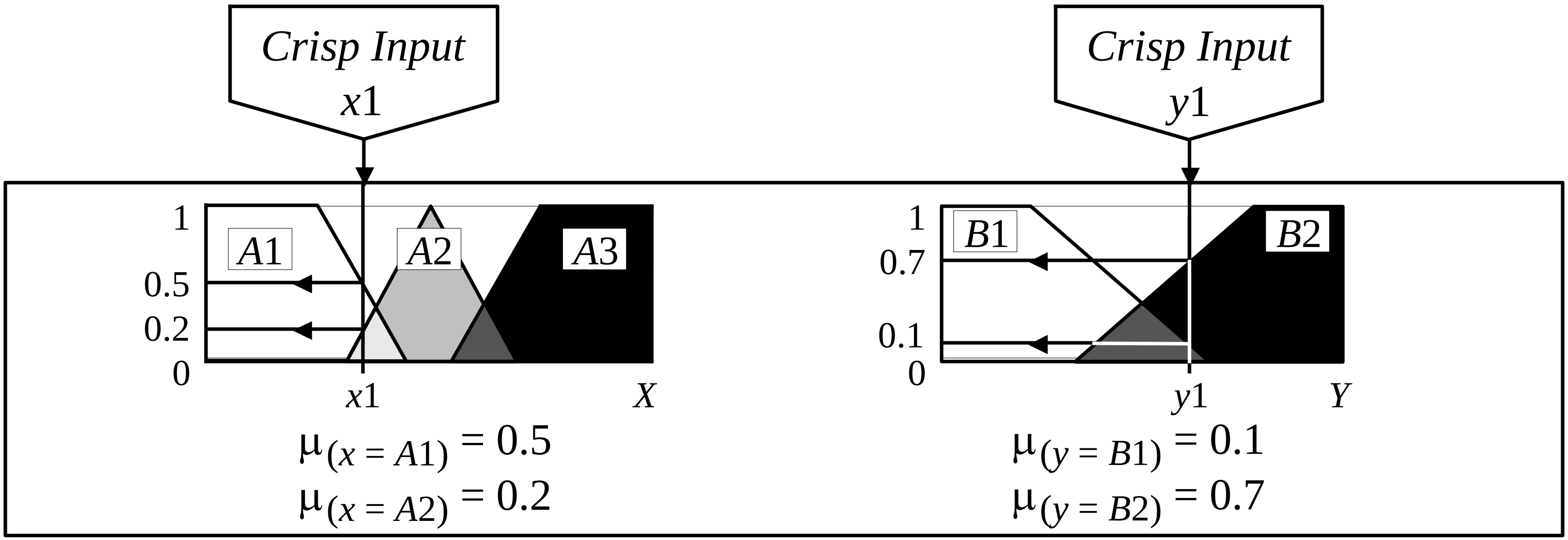
Rule: 1
IF ***project_funding*** **is** ***adequate***
OR ***project_staffing*** **is** ***small***
THEN ***risk*** **is** ***low***

Rule: 2
IF ***project_funding*** **is** ***marginal***
AND ***project_staffing*** **is** ***large***
THEN ***risk*** **is** ***normal***

Rule: 3
IF ***project_funding*** **is** ***inadequate***
THEN ***risk*** **is** ***high***

Step 1: Fuzzification

The first step is to take the crisp inputs, *x*1 and *y*1 (*project funding* and *project staffing*), and determine the degree to which these inputs belong to each of the appropriate fuzzy sets.



Step 2: Rule Evaluation

The second step is to take the fuzzified inputs, $\mu_{(x=A1)} = 0.5$, $\mu_{(x=A2)} = 0.2$, $\mu_{(y=B1)} = 0.1$ and $\mu_{(y=B2)} = 0.7$, and apply them to the antecedents of the fuzzy rules. If a given fuzzy rule has multiple antecedents, the fuzzy operator (AND or OR) is used to obtain a single number that represents the result of the antecedent evaluation. This number (the truth value) is then applied to the consequent membership function.

Step 2: Rule Evaluation

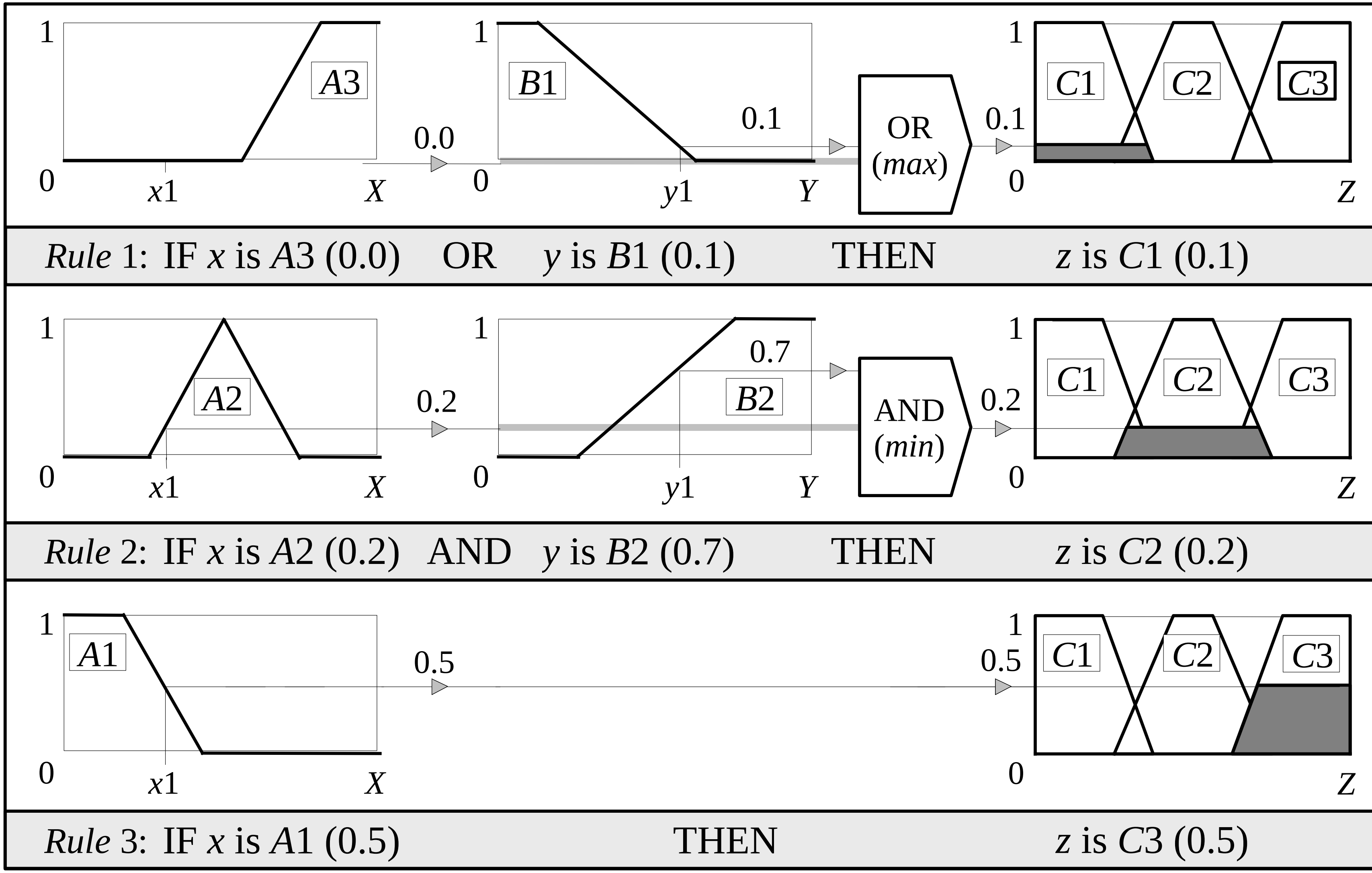
To evaluate the disjunction of the rule antecedents, we use the **OR fuzzy operation**. Typically, fuzzy expert systems make use of the classical fuzzy operation **union**:

$$\mu_{A \cup B}(x) = max\ [\mu_A(x),\ \mu_B(x)]$$

Similarly, in order to evaluate the conjunction of the rule antecedents, we apply the **AND fuzzy operation intersection**:

$$\mu_{A \cap B}(x) = min\ [\mu_A(x),\ \mu_B(x)]$$

Mamdani-style rule evaluation



Rule Correlating - clipping

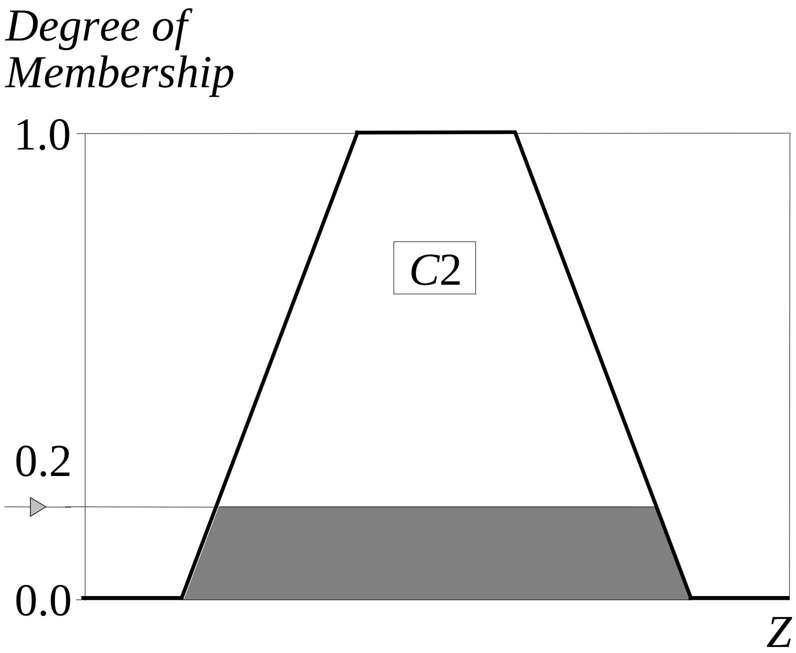
- Now the result of the antecedent evaluation can be applied to the membership function of the consequent.

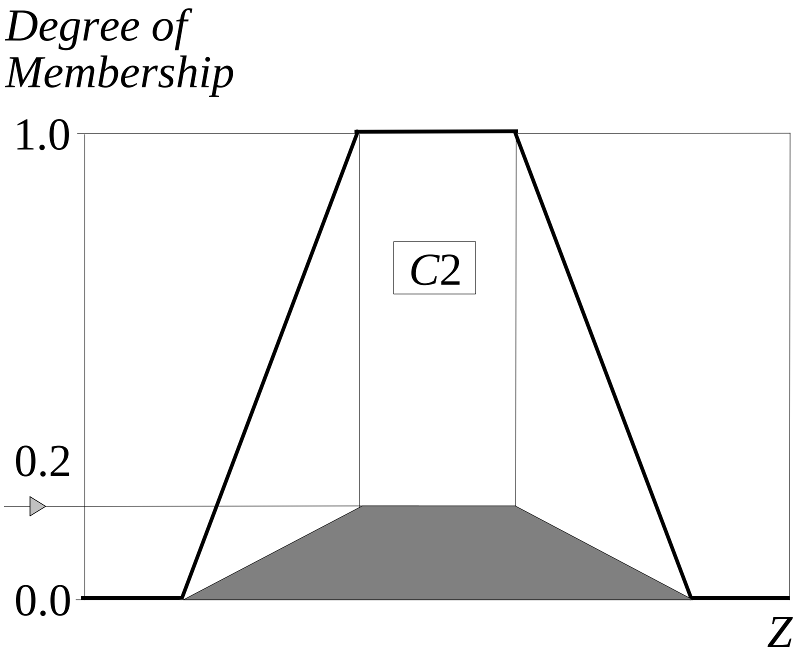
- The most common method of correlating the rule consequent with the truth value of the rule antecedent is to cut the consequent membership function at the level of the antecedent truth. This method is called **clipping**. Since the top of the membership function is sliced, the clipped fuzzy set loses some information. However, clipping is still often preferred because it involves less complex and faster mathematics, and generates an aggregated output surface that is easier to defuzzify.

Rule Correlating - Scaling

- While clipping is a frequently used method, **scaling** offers a better approach for preserving the original shape of the fuzzy set. The original membership function of the rule consequent is adjusted by multiplying all its membership degrees by the truth value of the rule antecedent. This method, which generally loses less information, can be very useful in fuzzy expert systems.

Clipped and scaled membership functions



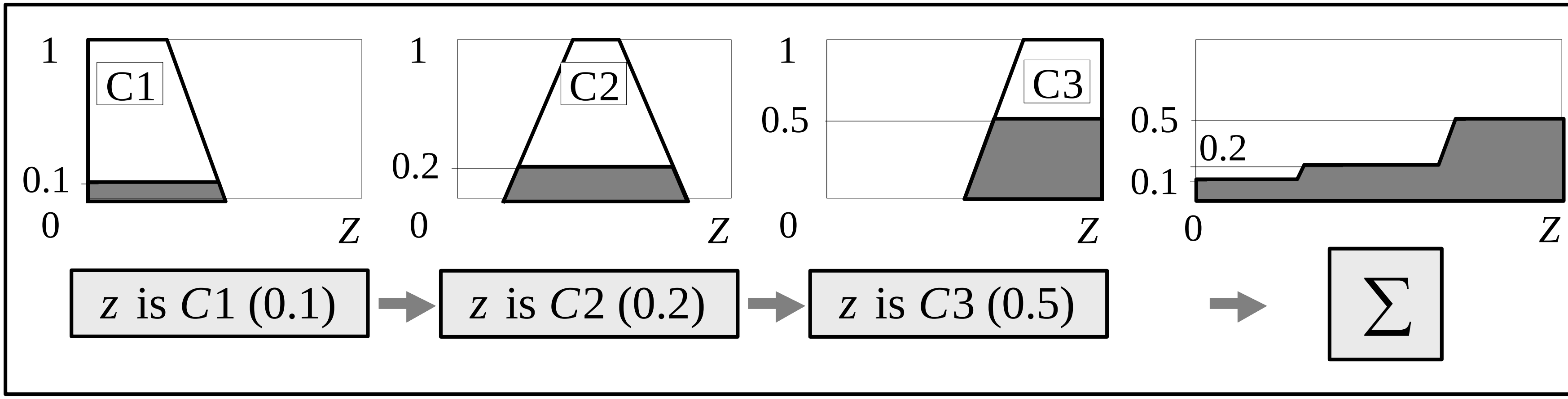


Step 3: Aggregation of the rule outputs

Aggregation is the process of unification of the outputs of all rules. We take the membership functions of all rule consequents previously clipped or scaled and combine them into a single fuzzy set.

The input of the aggregation process is the list of clipped or scaled consequent membership functions, and the output is one fuzzy set for each output variable.

Aggregation of the rule outputs



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Step 4: Defuzzification

The last step in the fuzzy inference process is defuzzification. Fuzziness helps us to evaluate the rules, but the final output of a fuzzy system has to be a crisp number. The input for the defuzzification process is the aggregate output fuzzy set and the output is a single number.

Step 4: Defuzzification

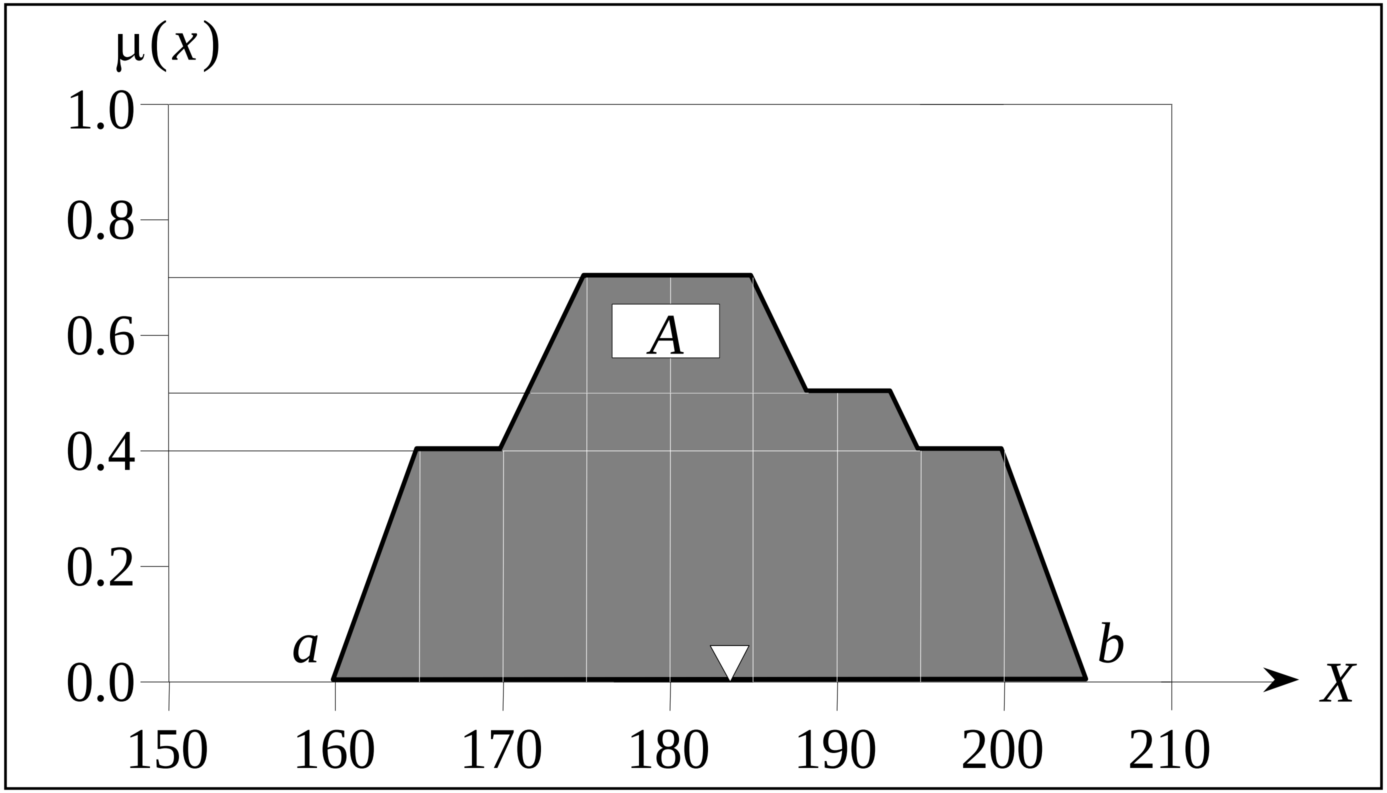
- There are several defuzzification methods, but probably the most popular one is the **centroid technique**. It finds the point where a vertical line would slice the aggregate set into two equal masses. Mathematically this **centre of gravity (COG)** can be expressed as:

$$COG = \frac{\int_a^b \mu_A(x)\, x\, dx}{\int_a^b \mu_A(x)\, dx}$$

$$COG = \frac{\sum_{x=a}^{b} \mu_A(x)x}{\sum_{x=a}^{b} \mu_A(x)}$$

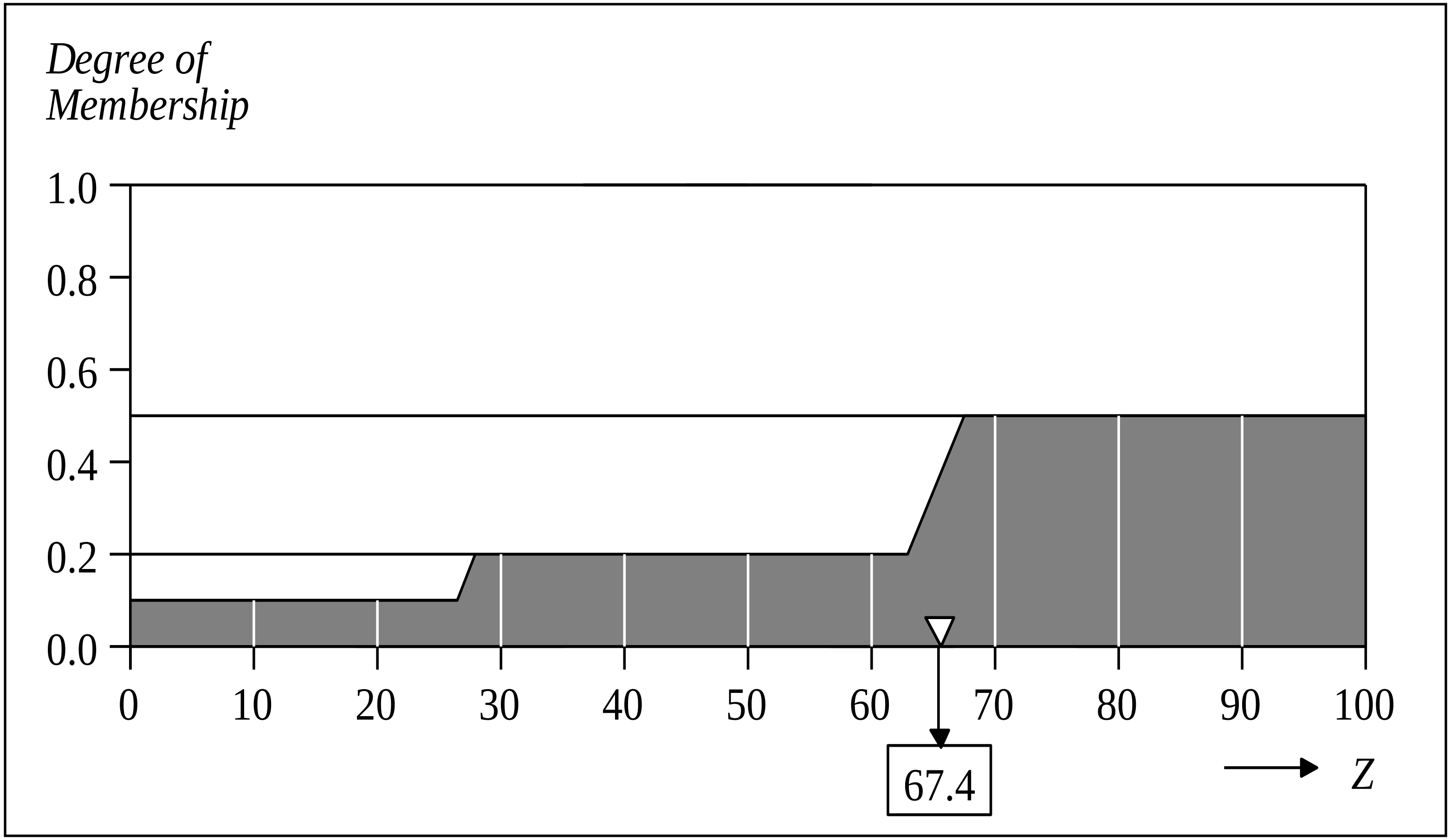
Step 4: Defuzzification

- Centroid defuzzification method finds a point representing the centre of gravity of the fuzzy set, *A*, on the interval, *ab*.
- A reasonable estimate can be obtained by calculating it over a sample of points.



Centre of gravity (COG):

$$COG = \frac{(0+10+20)\times 0.1 + (30+40+50+60)\times 0.2 + (70+80+90+100)\times 0.5}{0.1+0.1+0.1+0.2+0.2+0.2+0.2+0.5+0.5+0.5+0.5} = 67.4$$



Sugeno fuzzy inference

- Mamdani-style inference, as we have just seen, requires us to find the centroid of a two-dimensional shape by integrating across a continuously varying function. In general, this process is not computationally efficient.
- **Michio Sugeno** suggested to use a single spike, a *singleton*, as the membership function of the rule consequent. A **singleton**, or more precisely a **fuzzy singleton**, is a fuzzy set with a membership function that is unity at a single particular point on the universe of discourse and zero everywhere else.

Sugeno-style fuzzy inference is very similar to the Mamdani method. Sugeno changed only a rule consequent. Instead of a fuzzy set, he used a mathematical function of the input variable. The format of the **Sugeno-style fuzzy rule** is

IF x is A
AND y is B
THEN z is $f(x, y)$

where x, y and z are linguistic variables; A and B are fuzzy sets on universe of discourses X and Y, respectively; and $f(x, y)$ is a mathematical function.

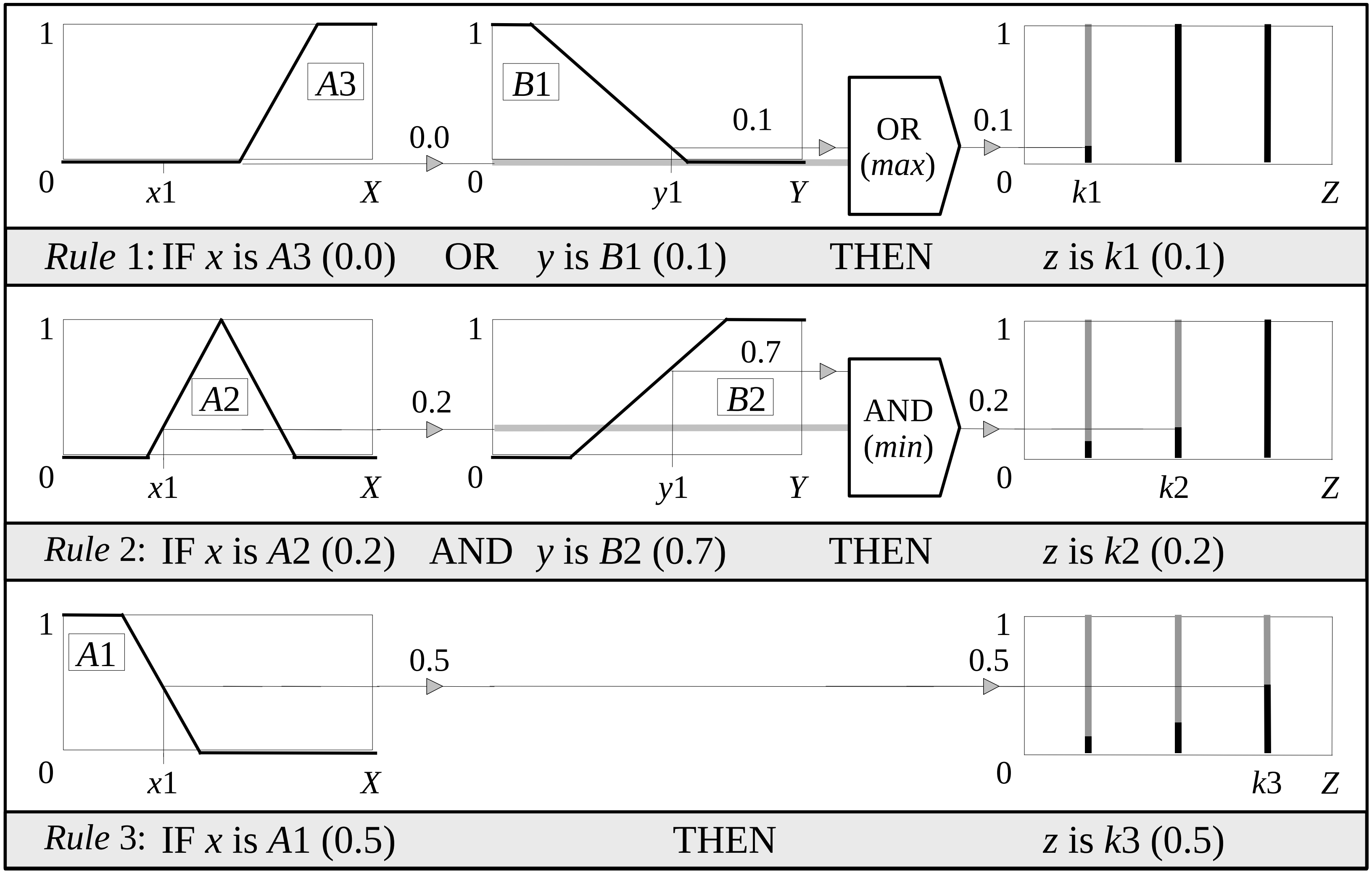
The most commonly used **zero-order Sugeno fuzzy model** applies fuzzy rules in the following form:

IF ***x*** **is** ***A***
AND ***y*** **is** ***B***
THEN ***z*** **is** ***k***

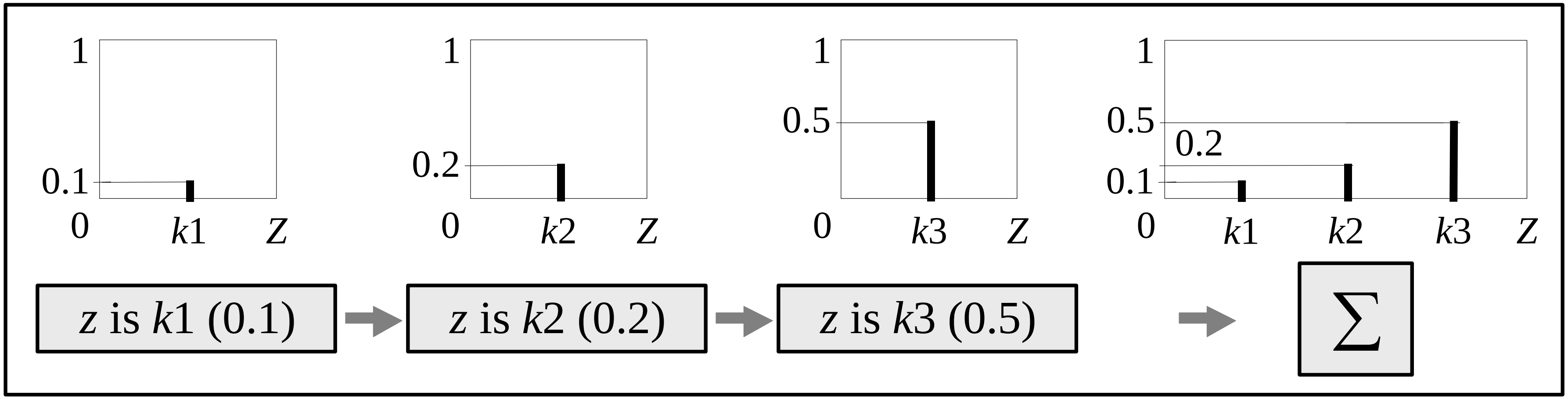
where *k* is a constant.

In this case, the output of each fuzzy rule is constant. All consequent membership functions are represented by singleton spikes.

Sugeno-style rule evaluation



Sugeno-style aggregation of the rule outputs



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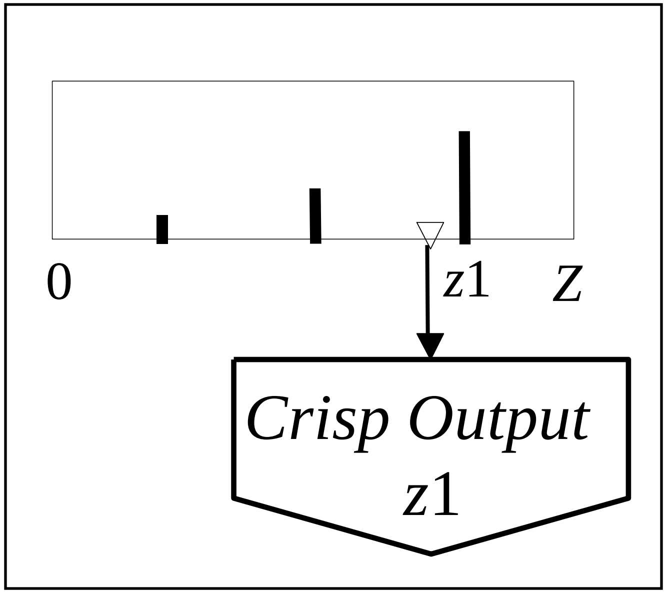
Weighted average (WA):

$$WA = \frac{\mu(k1)\times k1 + \mu(k2)\times k2 + \mu(k3)\times k3}{\mu(k1)+\mu(k2)+\mu(k3)} = \frac{0.1\times 20 + 0.2\times 50 + 0.5\times 80}{0.1+0.2+0.5} = 65$$

Sugeno-style defuzzification



How to make a decision on which method to apply – Mamdani or Sugeno?

- Mamdani method is widely accepted for capturing expert knowledge. It allows us to describe the expertise in more intuitive, more human-like manner. However, Mamdani-type fuzzy inference entails a substantial computational burden.
- On the other hand, Sugeno method is computationally effective and works well with optimisation and adaptive techniques, which makes it very attractive in control problems, particularly for dynamic nonlinear systems.

Building a fuzzy expert system: case study

- A service centre keeps spare parts and repairs failed ones.
- A customer brings a failed item and receives a spare of the same type.
- Failed parts are repaired, placed on the shelf, and thus become spares.
- The objective here is to advise a manager of the service centre on certain decision policies to keep the customers satisfied.

Process of developing a fuzzy expert system

1. Specify the problem and define linguistic variables.
2. Determine fuzzy sets.
3. Elicit and construct fuzzy rules.
4. Encode the fuzzy sets, fuzzy rules and procedures to perform fuzzy inference into the expert system.
5. Evaluate and tune the system.

***Step* 1**: **Specify the problem and define linguistic variables**

There are four main linguistic variables: average waiting time (mean delay) *m*, repair utilisation factor of the service centre ρ, number of servers *s*, and initial number of spare parts *n*.

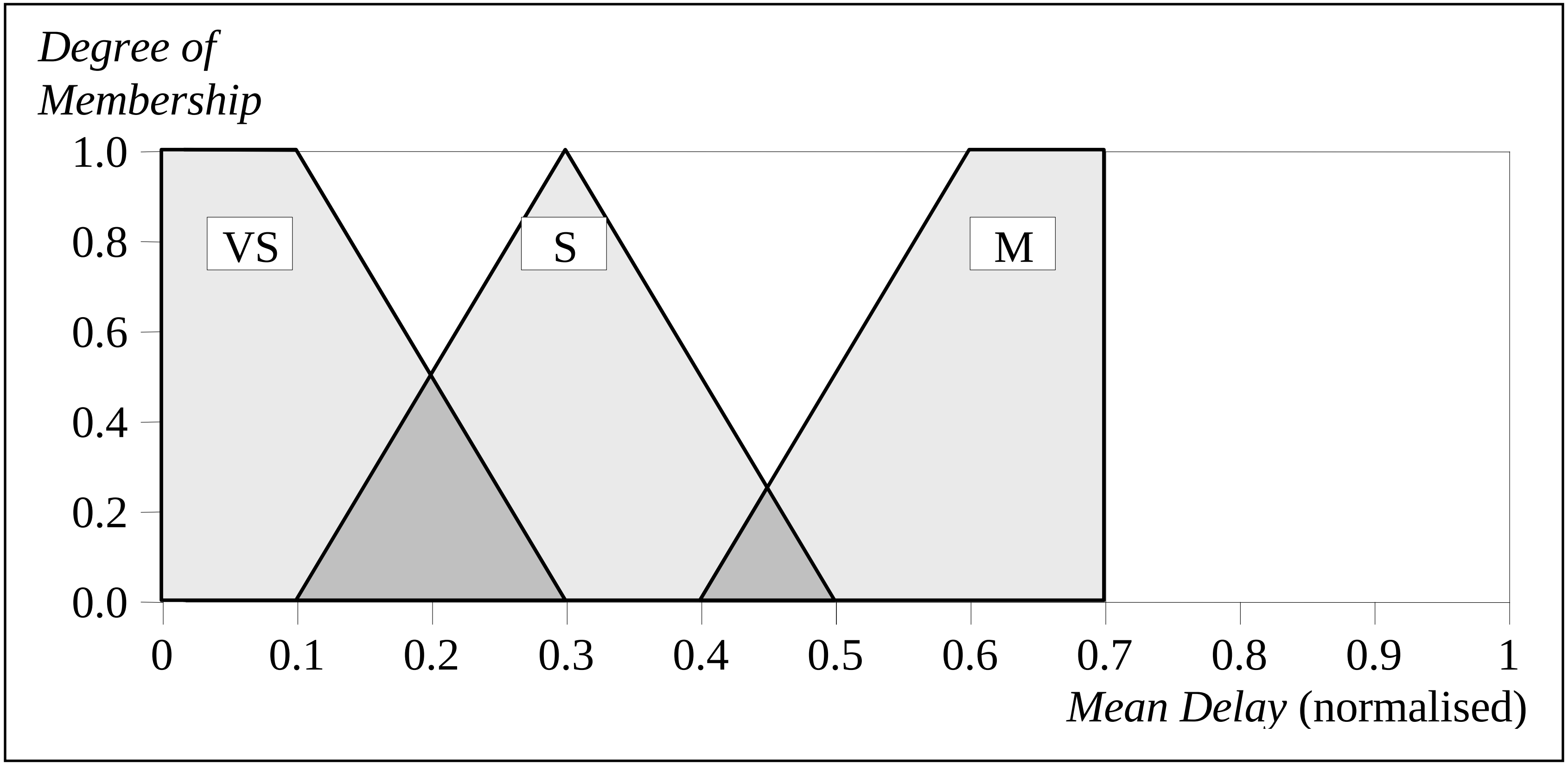
Linguistic variables and their ranges

Linguistic Variable: *Mean Delay, m*		
Linguistic Value	Notation	Numerical Range (normalised)
Very Short	VS	[0, 0.3]
Short	S	[0.1, 0.5]
Medium	M	[0.4, 0.7]
Linguistic Variable: *Number of Servers, s*		
Linguistic Value	Notation	Numerical Range (normalised)
Small	S	[0, 0.35]
Medium	M	[0.30, 0.70]
Large	L	[0.60, 1]
Linguistic Variable: *Repair Utilisation Factor*, ρ		
Linguistic Value	Notation	Numerical Range
Low	L	[0, 0.6]
Medium	M	[0.4, 0.8]
High	H	[0.6, 1]
Linguistic Variable: *Number of Spares, n*		
Linguistic Value	Notation	Numerical Range (normalised)
Very Small	VS	[0, 0.30]
Small	S	[0, 0.40]
Rather Small	RS	[0.25, 0.45]
Medium	M	[0.30, 0.70]
Rather Large	RL	[0.55, 0.75]
Large	L	[0.60, 1]
Very Large	VL	[0.70, 1]

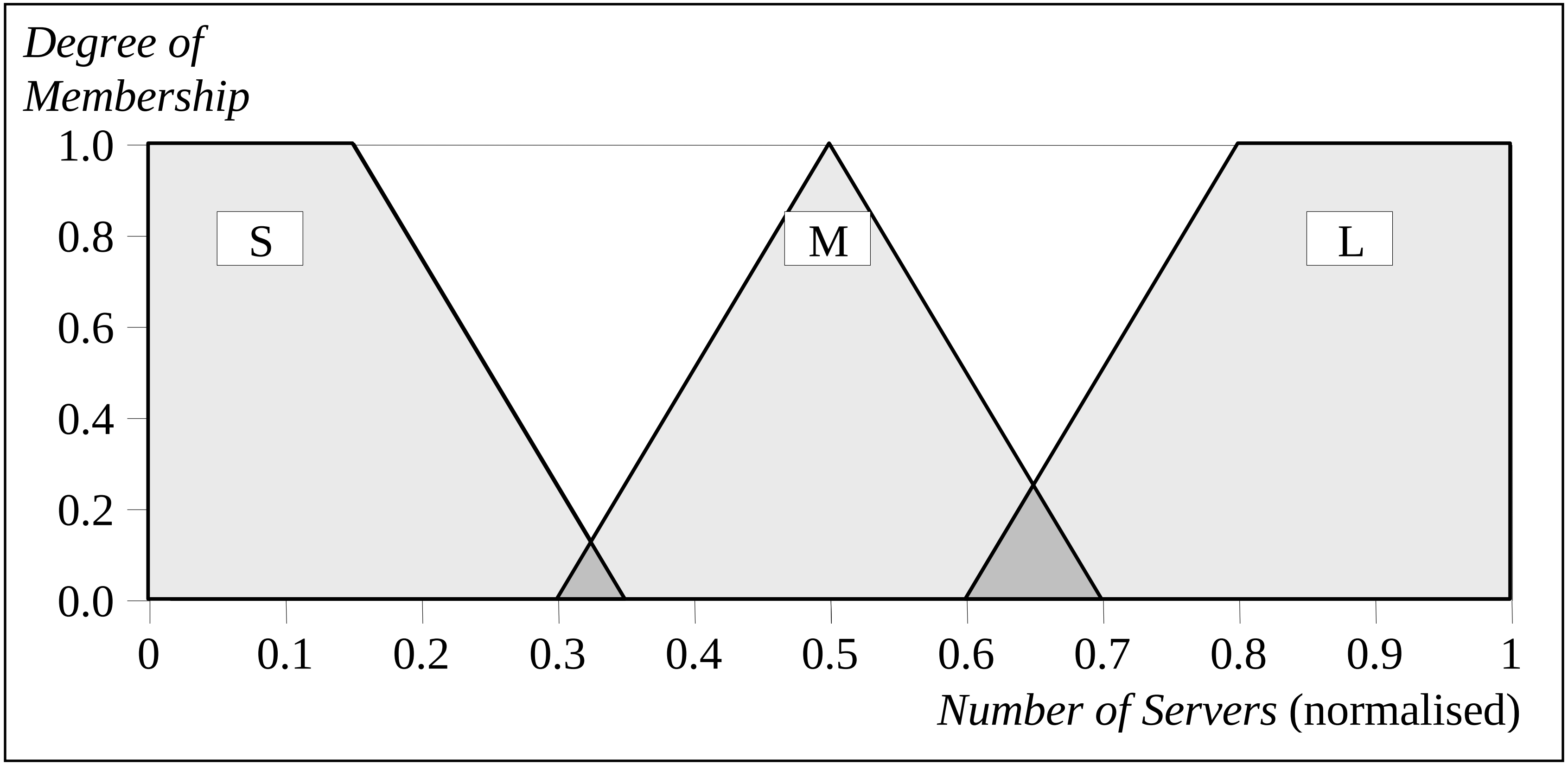
Step **2**: Determine fuzzy sets

Fuzzy sets can have a variety of shapes. However, a triangle or a trapezoid can often provide an adequate representation of the expert knowledge, and at the same time, significantly simplifies the process of computation.

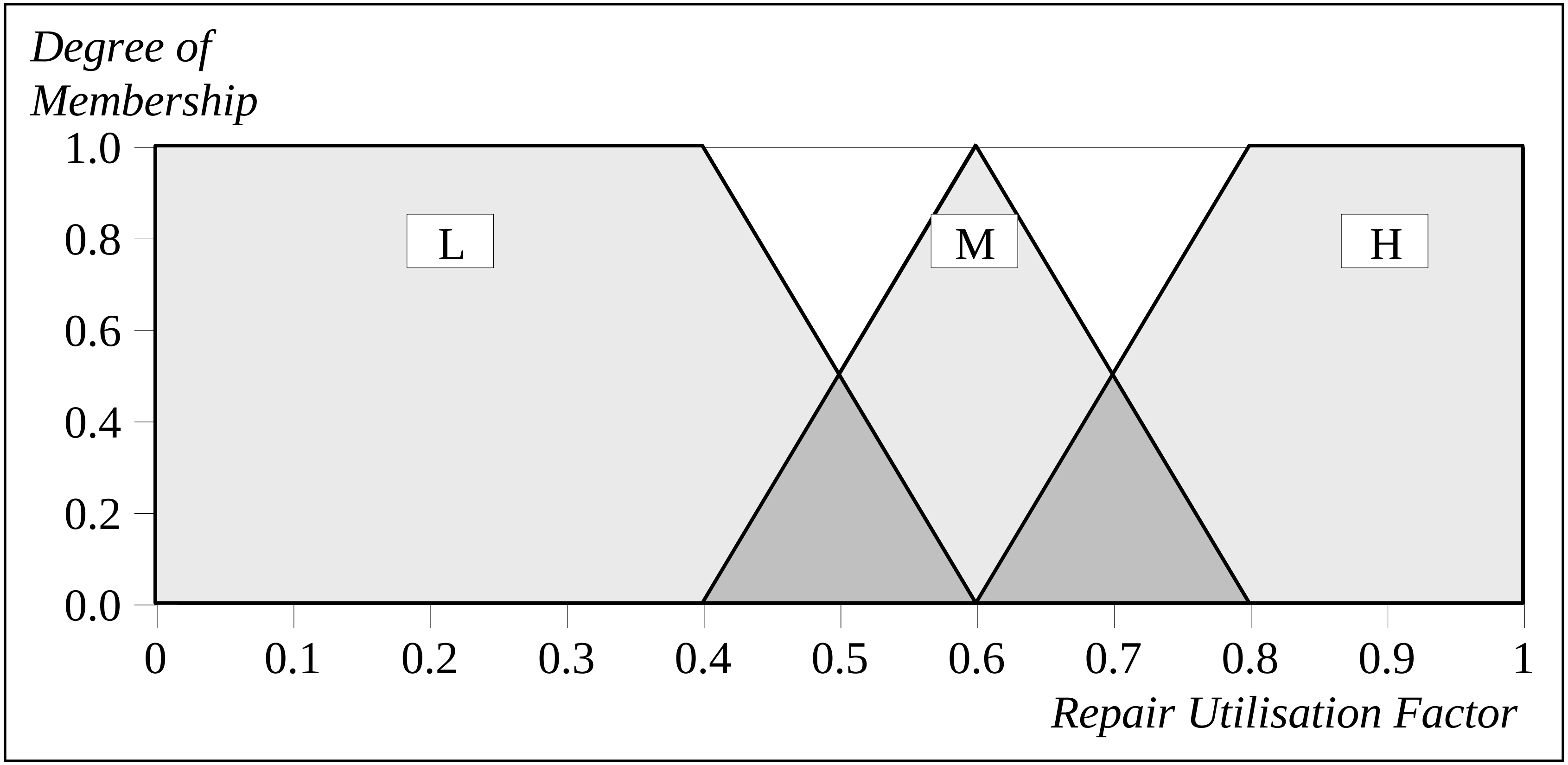
Fuzzy sets of *Mean Delay m*



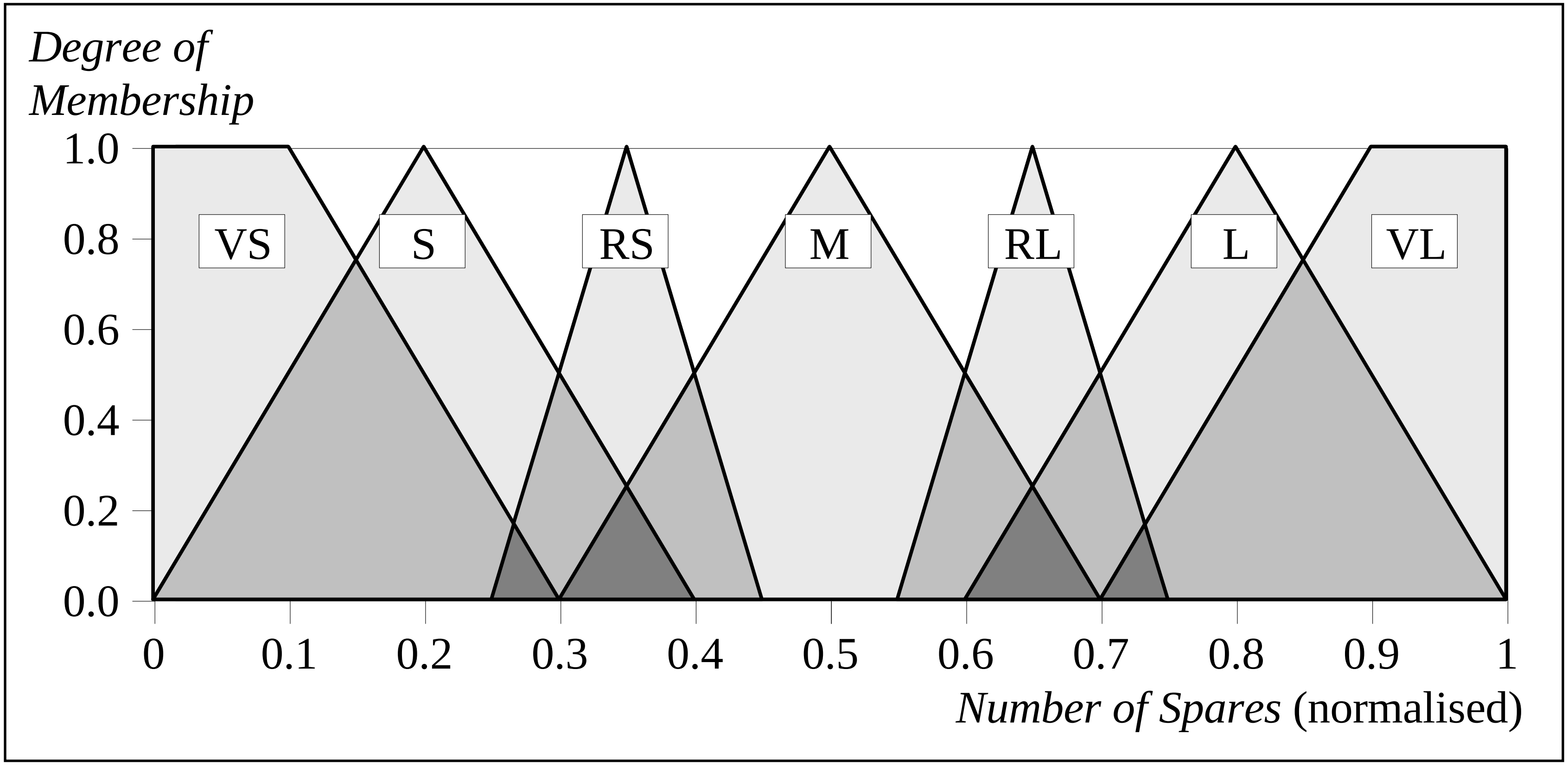
Fuzzy sets of *Number of Servers s*



Fuzzy sets of *Repair Utilisation Factor ρ*



Fuzzy sets of *Number of Spares n*

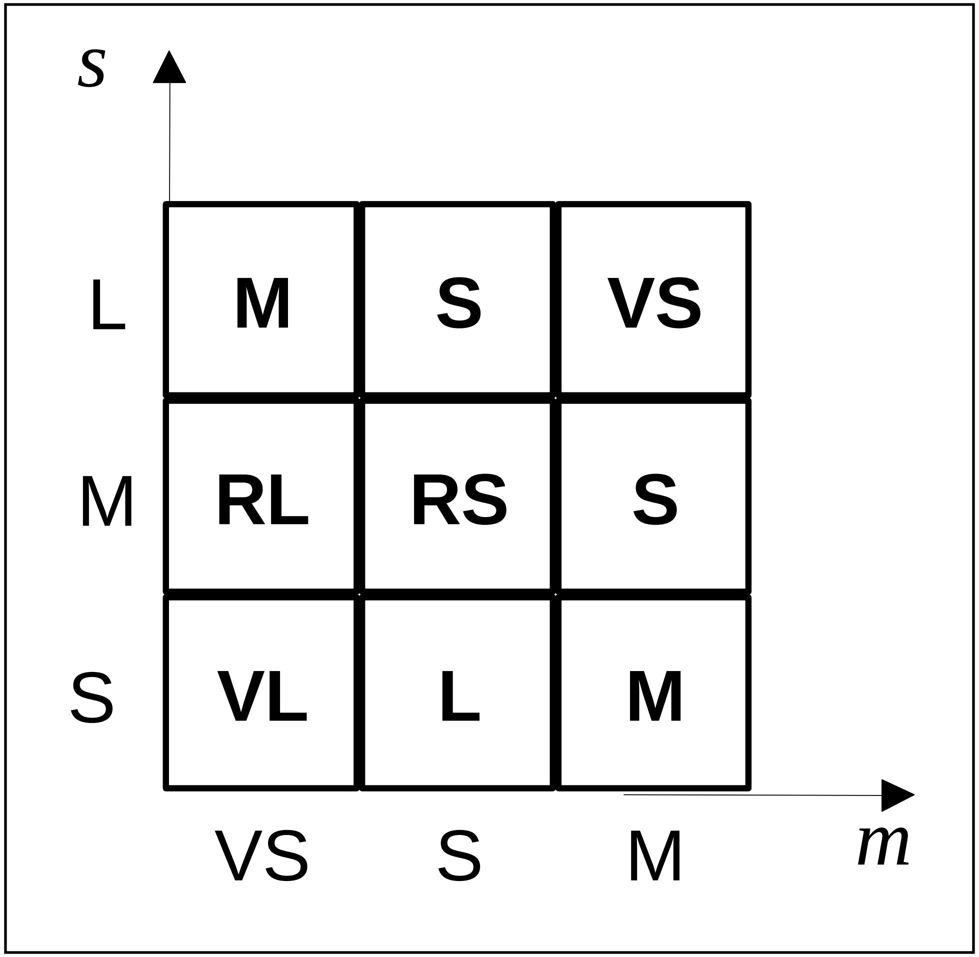


***Step* 3**: **Elicit and construct fuzzy rules**

To accomplish this task, we might ask the expert to describe how the problem can be solved using the fuzzy linguistic variables defined previously.

Required knowledge also can be collected from other sources such as books, computer databases, flow diagrams and observed human behaviour.

The square FAM representation



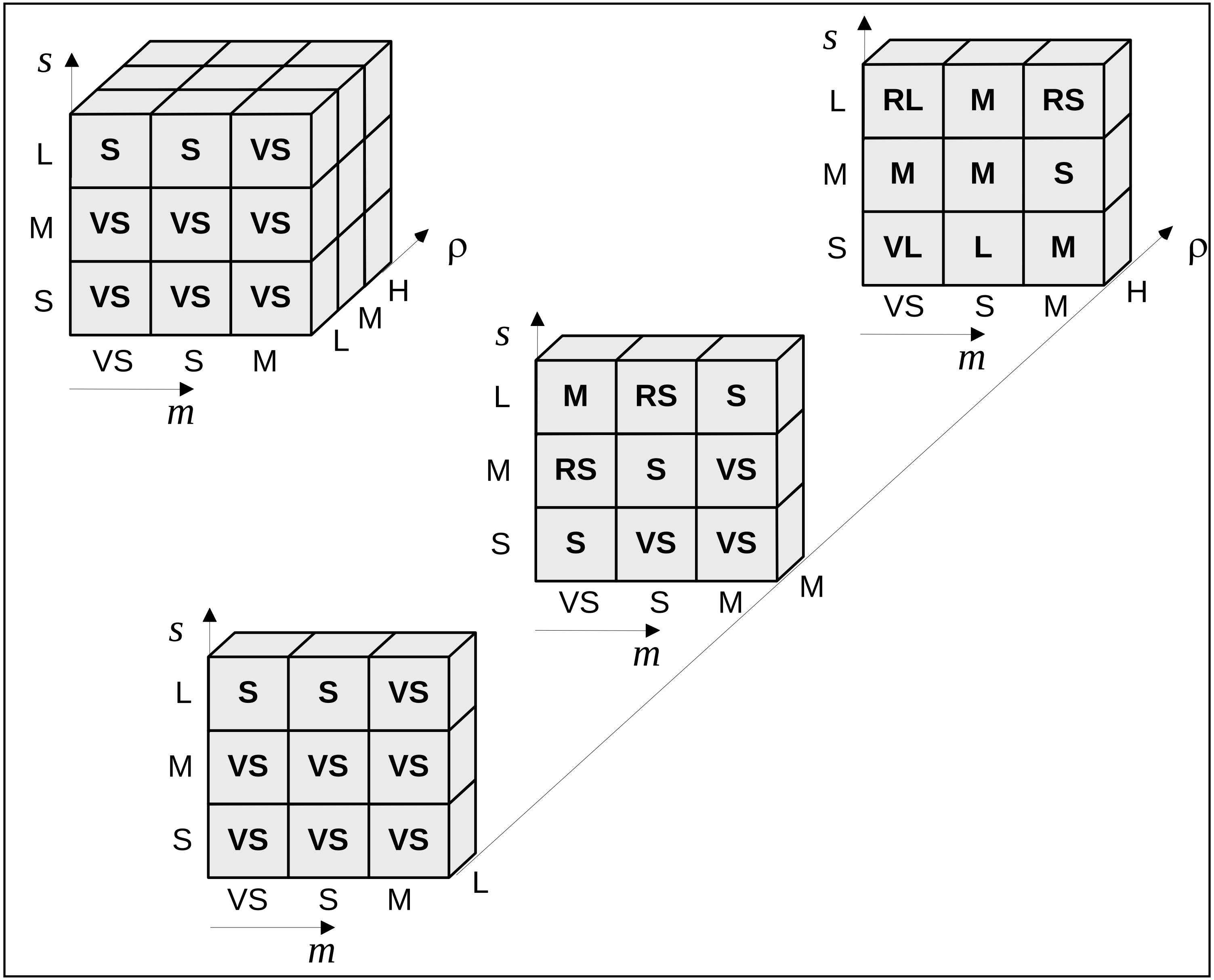
The rule table

Rule	*m*	*s*	ρ	*n*	Rule	*m*	*s*	ρ	*n*	Rule	*m*	*s*	ρ	*n*
1	VS	S	L	VS	10	VS	S	M	S	19	VS	S	H	VL
2	S	S	L	VS	11	S	S	M	VS	20	S	S	H	L
3	M	S	L	VS	12	M	S	M	VS	21	M	S	H	M
4	VS	M	L	VS	13	VS	M	M	RS	22	VS	M	H	M
5	S	M	L	VS	14	S	M	M	S	23	S	M	H	M
6	M	M	L	VS	15	M	M	M	VS	24	M	M	H	S
7	VS	L	L	S	16	VS	L	M	M	25	VS	L	H	RL
8	S	L	L	S	17	S	L	M	RS	26	S	L	H	M
9	M	L	L	VS	18	M	L	M	S	27	M	L	H	RS

Rule Base 1

1. If (utilisation_factor is L) then (number_of_spares is S)
2. If (utilisation_factor is M) then (number_of_spares is M)
3. If (utilisation_factor is H) then (number_of_spares is L)
4. If (mean_delay is VS) and (number_of_servers is S) then (number_of_spares is VL)
5. If (mean_delay is S) and (number_of_servers is S) then (number_of_spares is L)
6. If (mean_delay is M) and (number_of_servers is S) then (number_of_spares is M)
7. If (mean_delay is VS) and (number_of_servers is M) then (number_of_spares is RL)
8. If (mean_delay is S) and (number_of_servers is M) then (number_of_spares is RS)
9. If (mean_delay is M) and (number_of_servers is M) then (number_of_spares is S)
10. If (mean_delay is VS) and (number_of_servers is L) then (number_of_spares is M)
11. If (mean_delay is S) and (number_of_servers is L) then (number_of_spares is S)
12. If (mean_delay is M) and (number_of_servers is L) then (number_of_spares is VS)

Cube FAM of Rule Base 2



***<u>Step 4</u>*: Encode the fuzzy sets, fuzzy rules and procedures to perform fuzzy inference into the expert system**

To accomplish this task, we may choose one of two options: to build our system using a programming language such as C/C++ or Pascal, or to apply a fuzzy logic development tool such as MATLAB Fuzzy Logic Toolbox or Fuzzy Knowledge Builder.

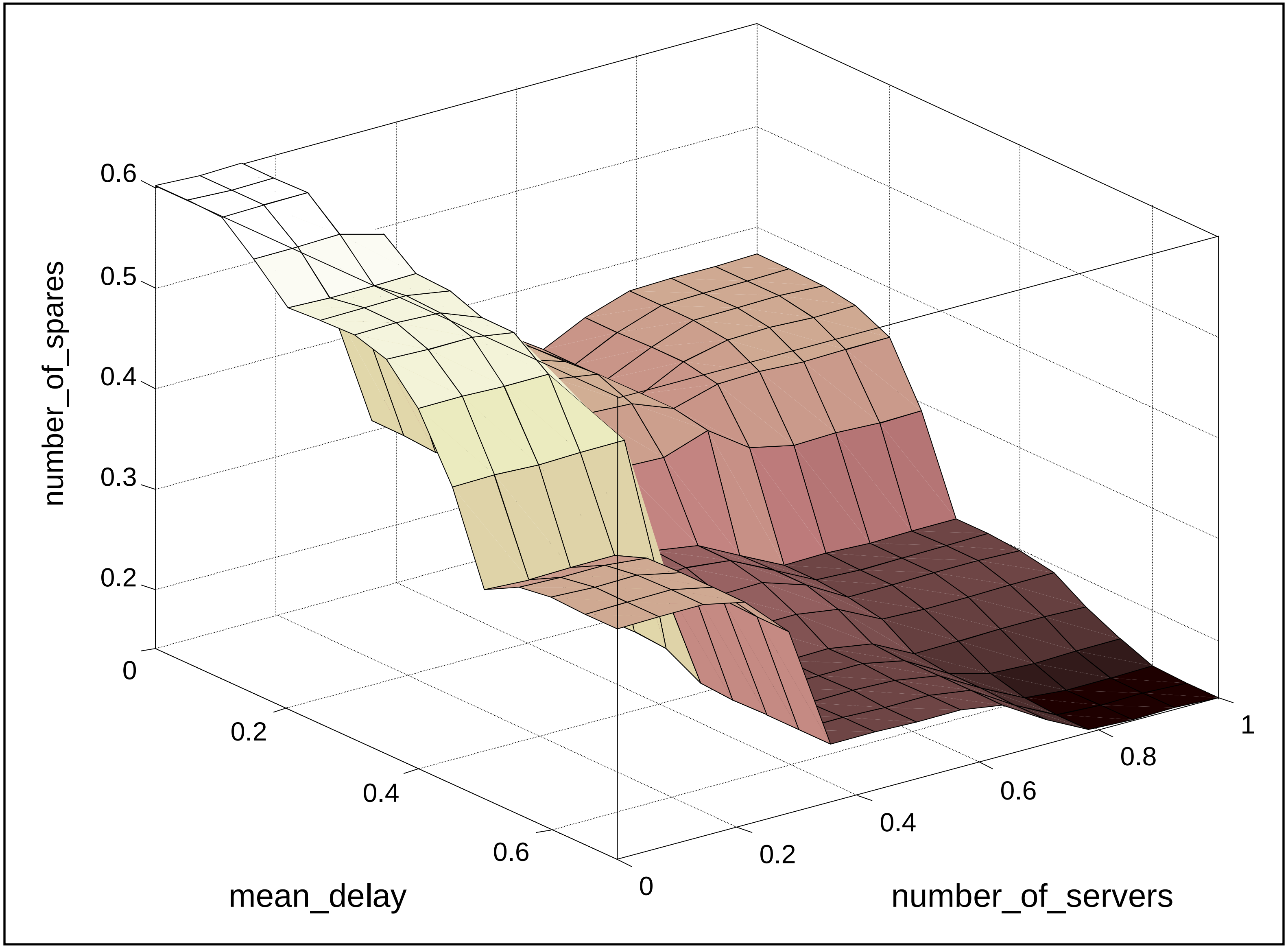
***Step* 5**: **Evaluate and tune the system**

The last, and the most laborious, task is to evaluate and tune the system. We want to see whether our fuzzy system meets the requirements specified at the beginning.

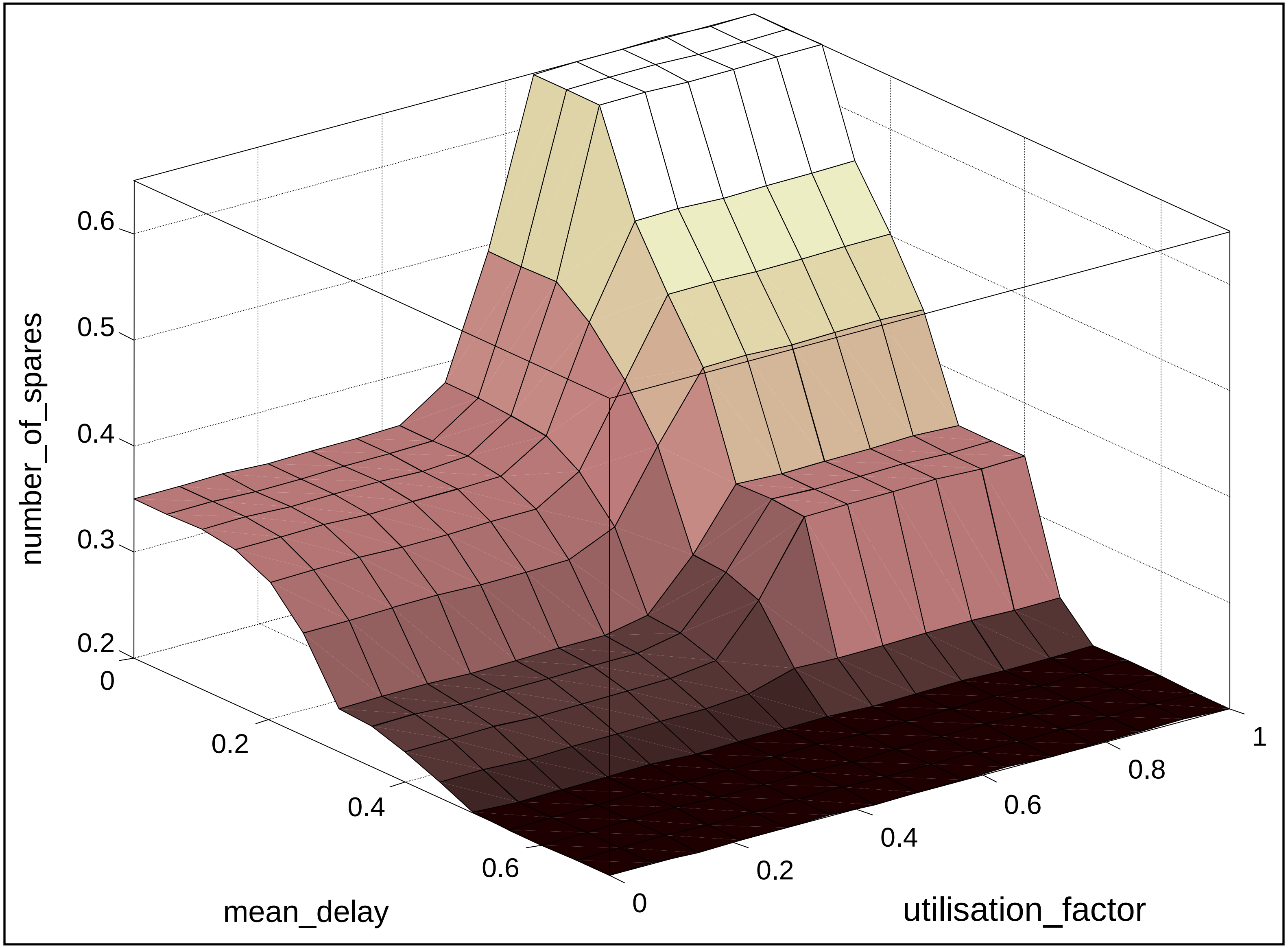
Several test situations depend on the mean delay, number of servers and repair utilisation factor.

The Fuzzy Logic Toolbox can generate surface to help us analyse the system's performance.

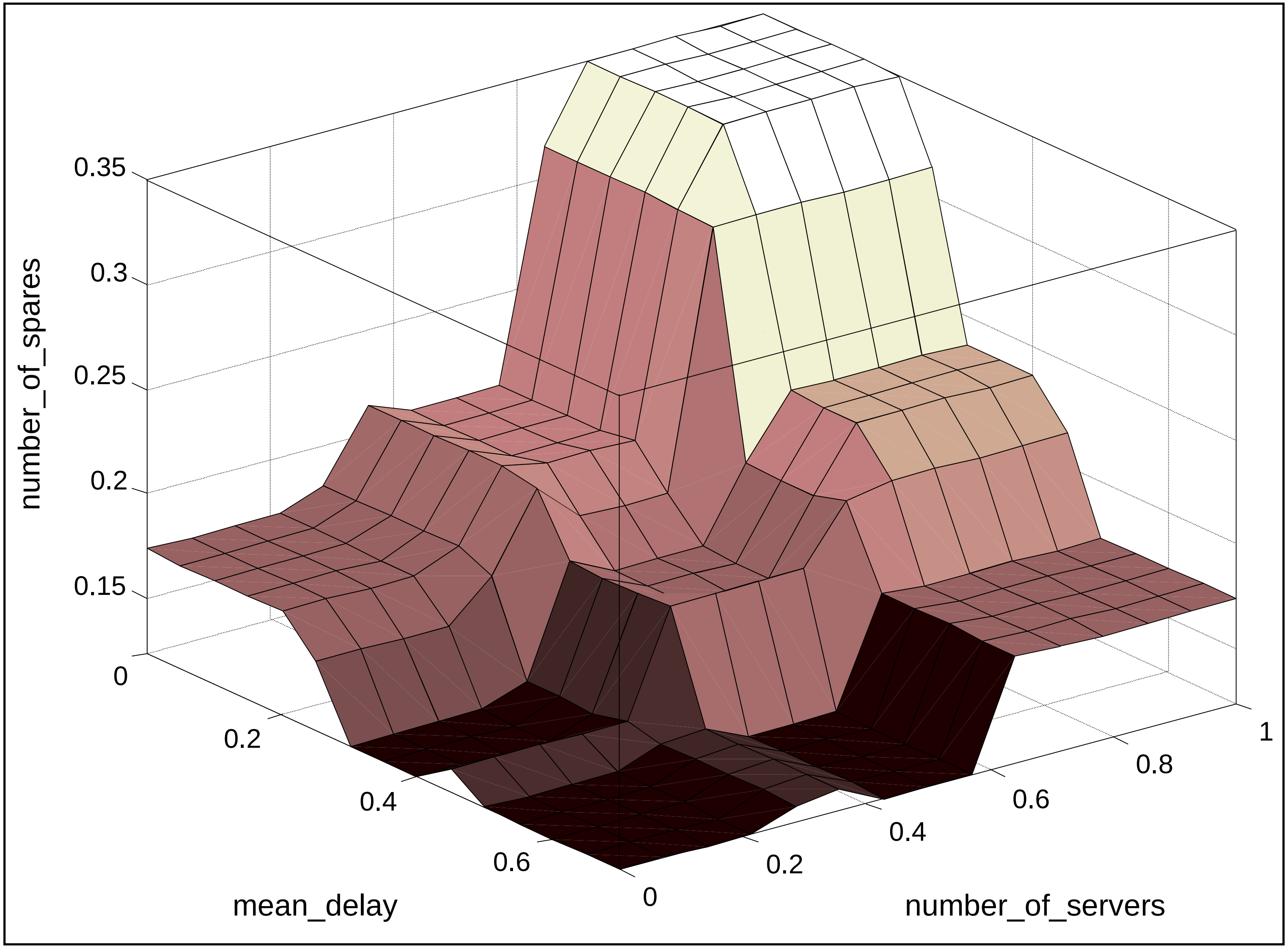
Three-dimensional plots for Rule Base 1



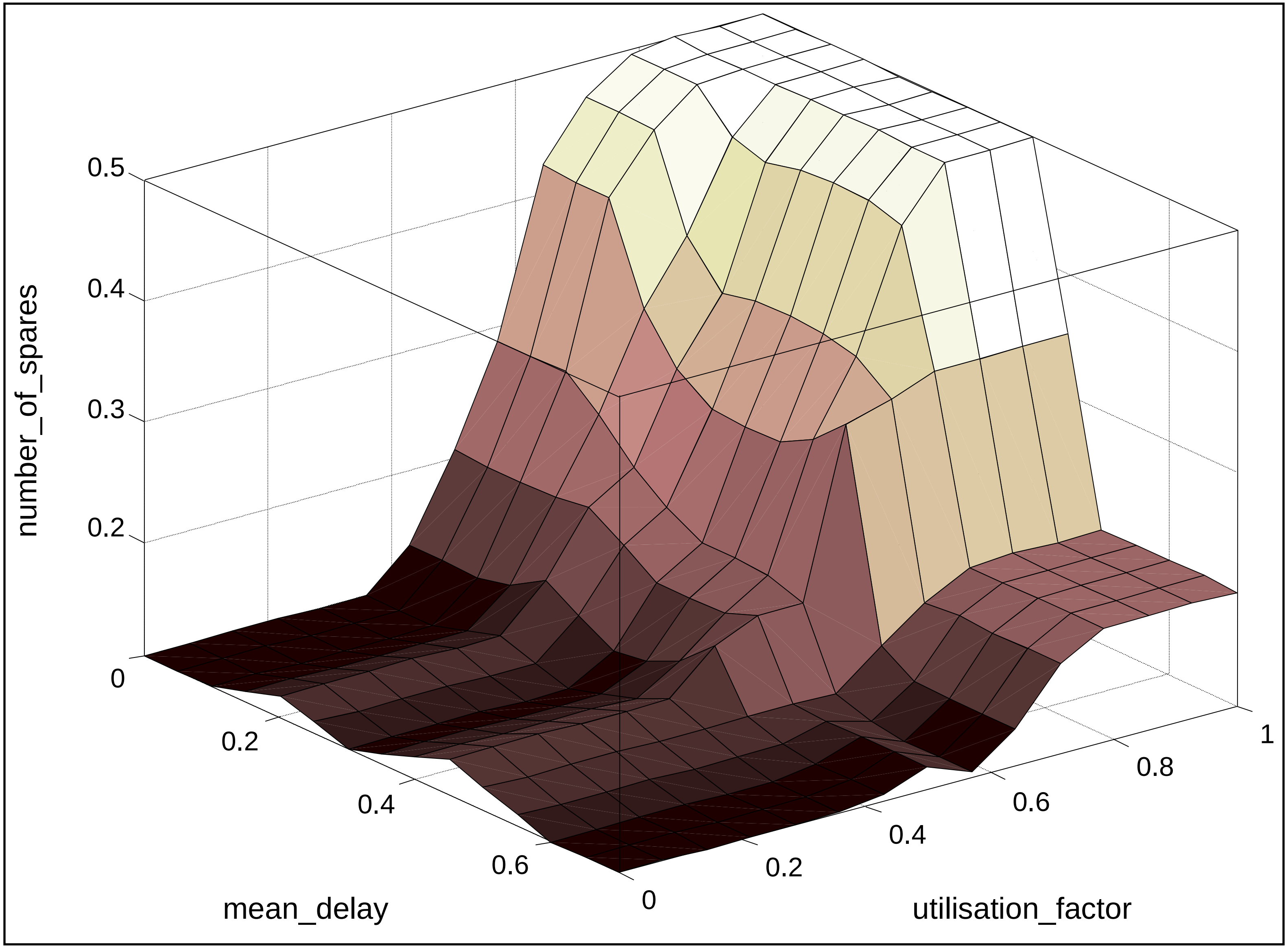
Three-dimensional plots for Rule Base 1



Three-dimensional plots for Rule Base 2



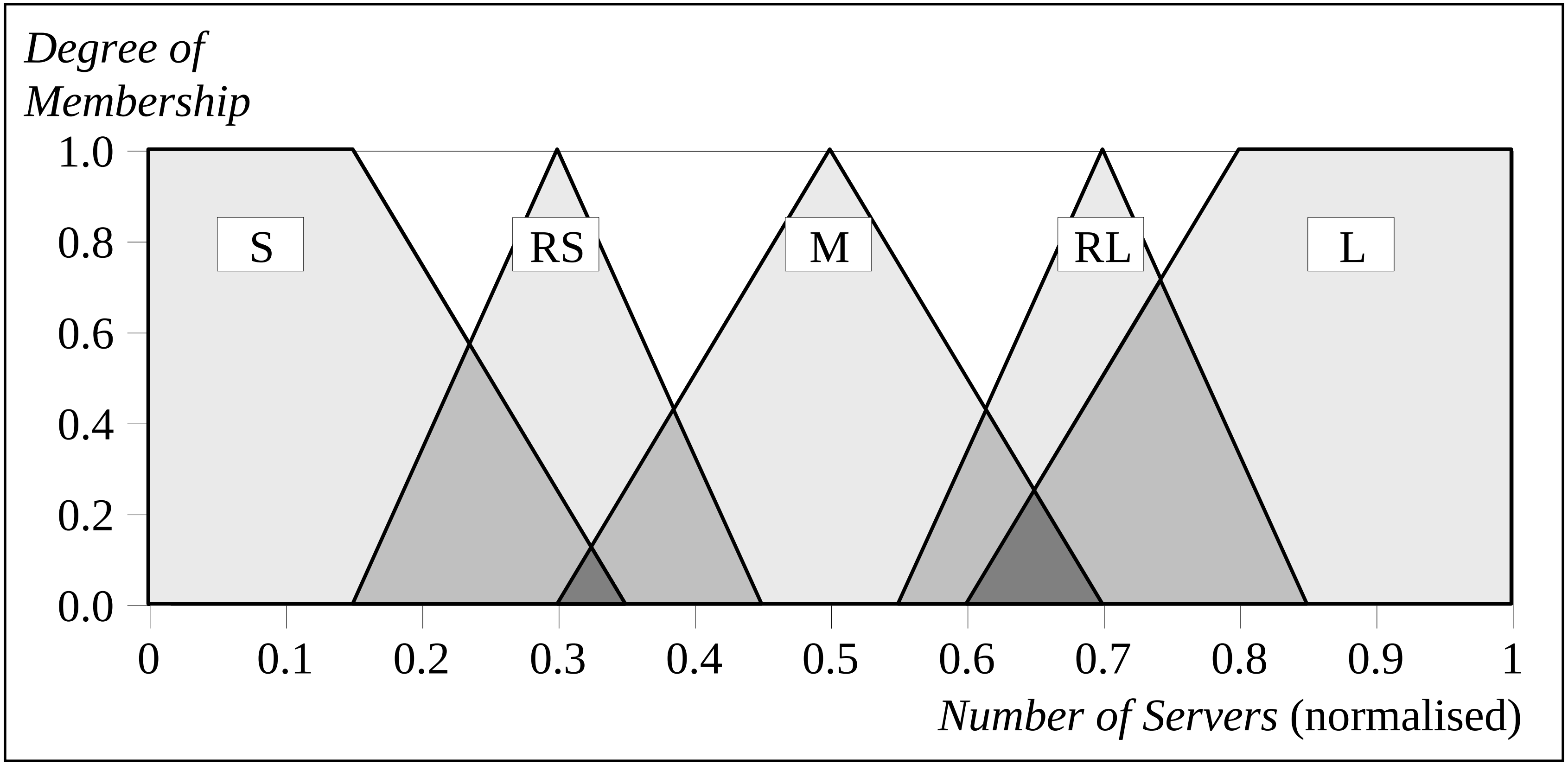
Three-dimensional plots for Rule Base 2



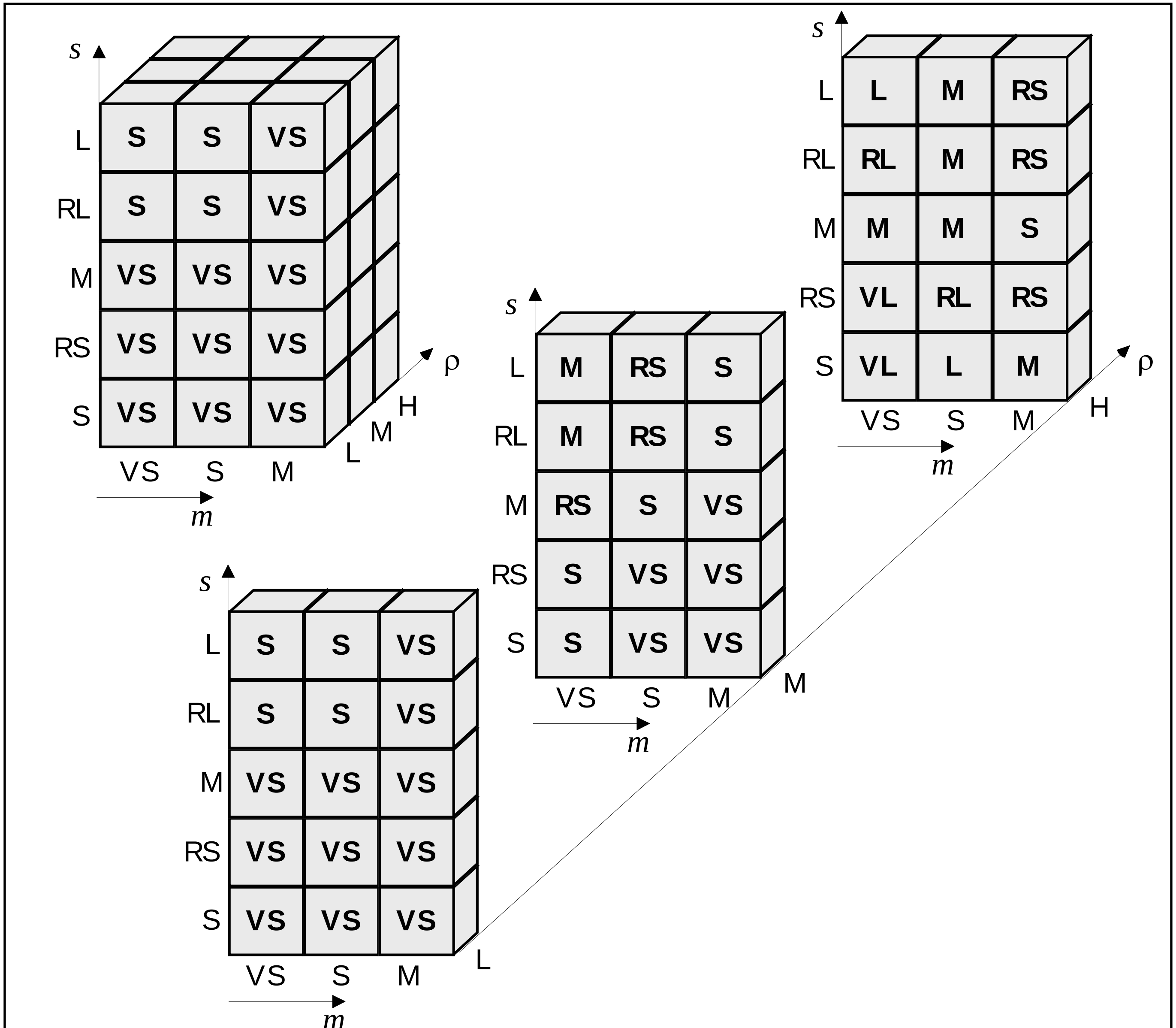
However, even now, the expert might not be satisfied with the system performance.

To improve the system performance, we may use additional sets – *Rather Small* and *Rather Large* – on the universe of discourse *Number of Servers*, and then extend the rule base.

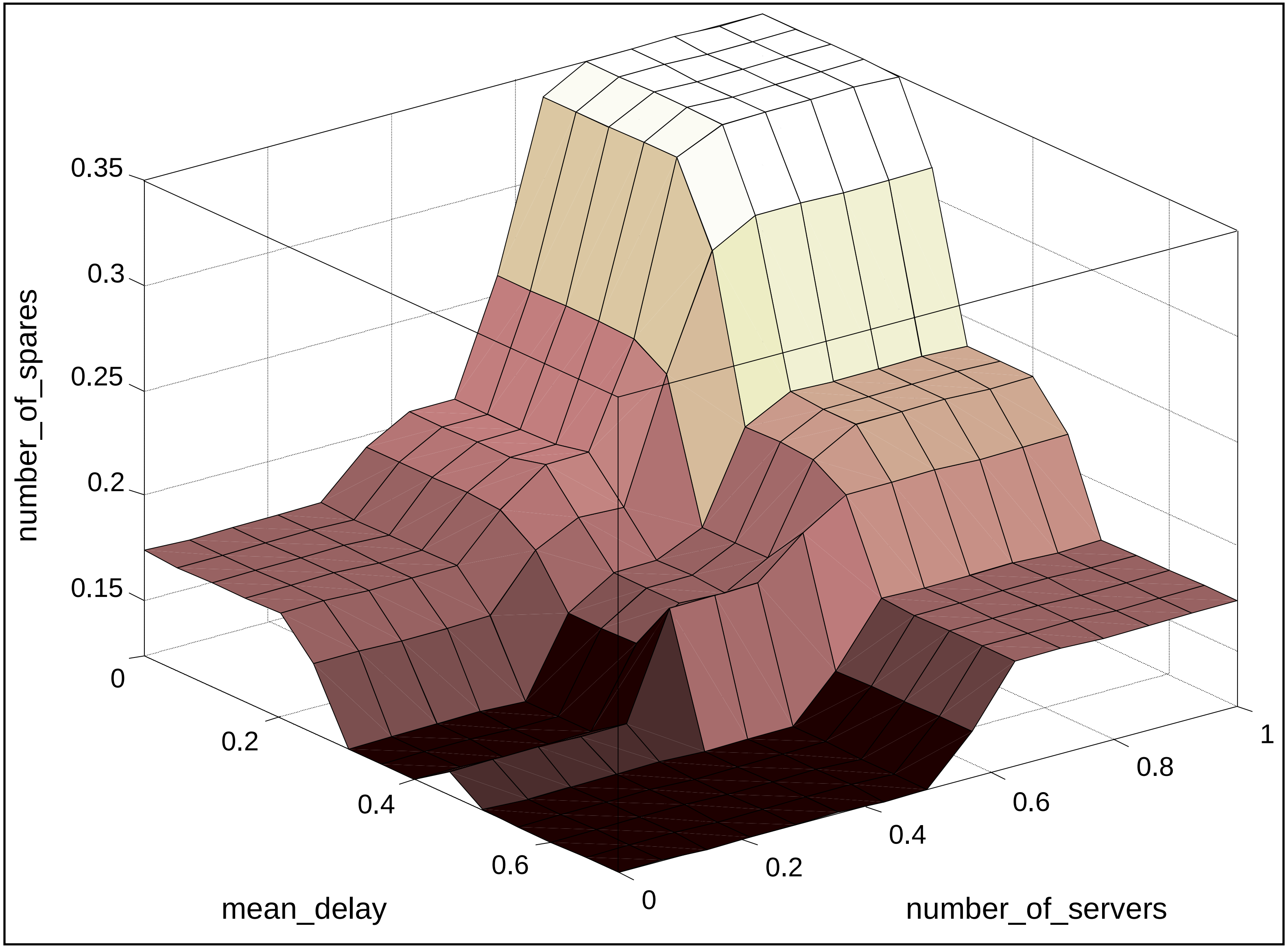
Modified fuzzy sets of *Number of Servers s*



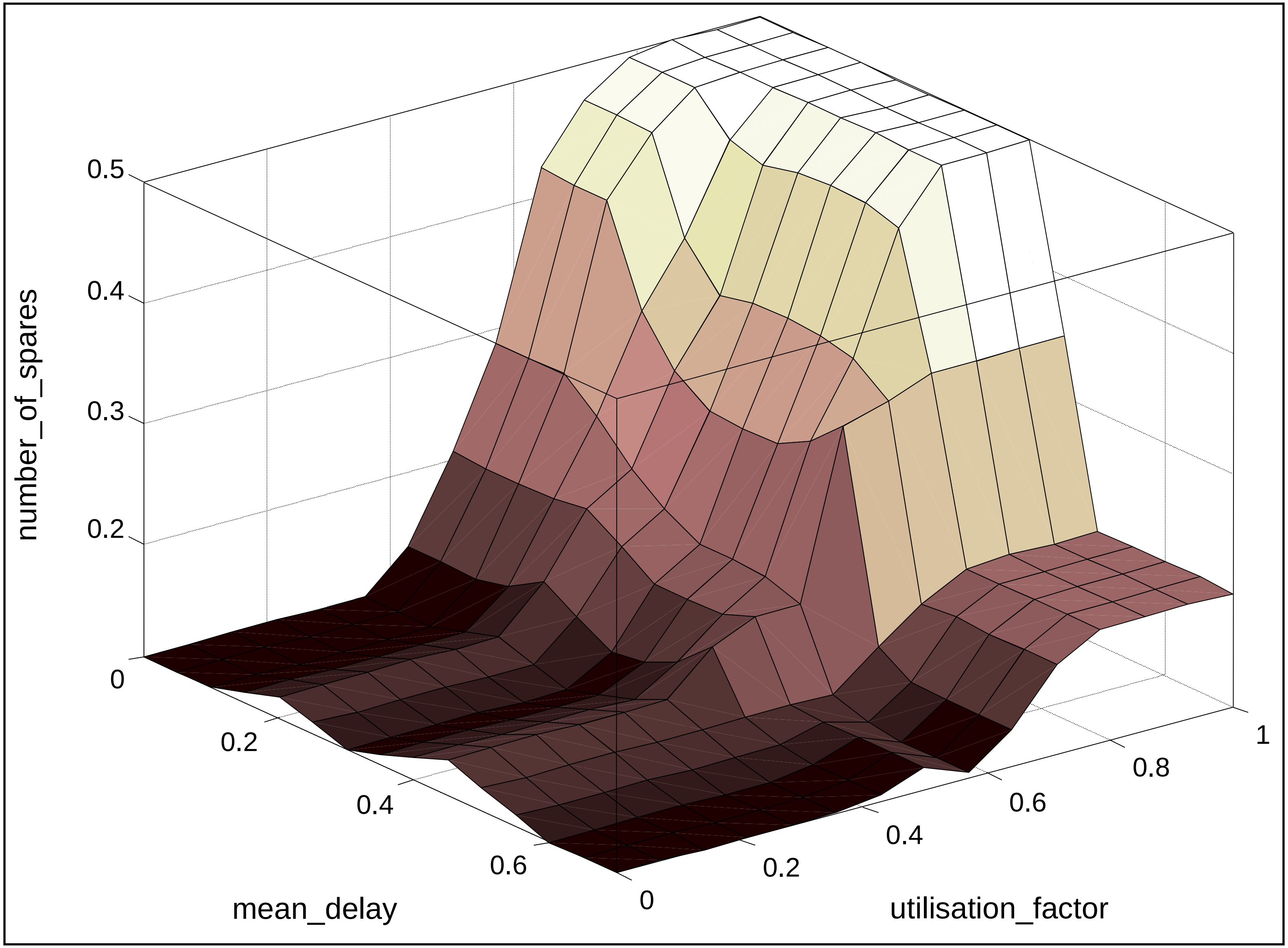
Cube FAM of Rule Base 3



Three-dimensional plots for Rule Base 3



Three-dimensional plots for Rule Base 3



Tuning fuzzy systems

1. Review model input and output variables, and if required redefine their ranges.
2. Review the fuzzy sets, and if required define additional sets on the universe of discourse. The use of wide fuzzy sets may cause the fuzzy system to perform roughly.
3. Provide sufficient overlap between neighbouring sets. It is suggested that triangle-to-triangle and trapezoid-to-triangle fuzzy sets should overlap between 25% to 50% of their bases.

4. Review the existing rules, and if required add new rules to the rule base.
5. Examine the rule base for opportunities to write hedge rules to capture the pathological behaviour of the system.
6. Adjust the rule execution weights. Most fuzzy logic tools allow control of the importance of rules by changing a weight multiplier.
7. Revise shapes of the fuzzy sets. In most cases, fuzzy systems are highly tolerant of a shape approximation.