Faculty of Engineering and Technology

Electrical and Computer Engineering Department

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Report for Experiment #5

First and second order circuits

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1. <u>Abstract:-</u>

The aims of experiment:

- examine the behavior of first-order and second-order circuit in response to a step input
- . RC and RL series circuits were connected with a pulse voltage source.
- examine the characteristic of the response in RC &RL (charging and discharging) and measure the value of τ As long as changing the connection of the circuit.
- examine the characteristic of the response in parallel and series RLC circuit (over damped, critical damped, under damped) and takes some measurement (V_a,V_b,t_a,t_b,$V(\infty)$)

Contents

2.Theory :-

First-Order circuits with DC sources (Step Response):

In mathematics there are certain kinds of functions defined. One of these, is the unit step function.

Unit step function is defined as:

$$\begin{Bmatrix} u(t) = 0 & for\ t \leq 0^- \\ u(t) = 1 & for\ t\ \geq 0^+ \end{Bmatrix}$$

Defining $t' = t - t_0$ then the function would be $u(t') = u(t - t_0)$

And thus:

$$\begin{Bmatrix} u(t - t_0) = 0 & for\ t \leq {t_0}^- \\ u(t - t_0) = 1 & for\ t \geq {t_0}^+ \end{Bmatrix}$$

The graph of this function is described in the following figures:

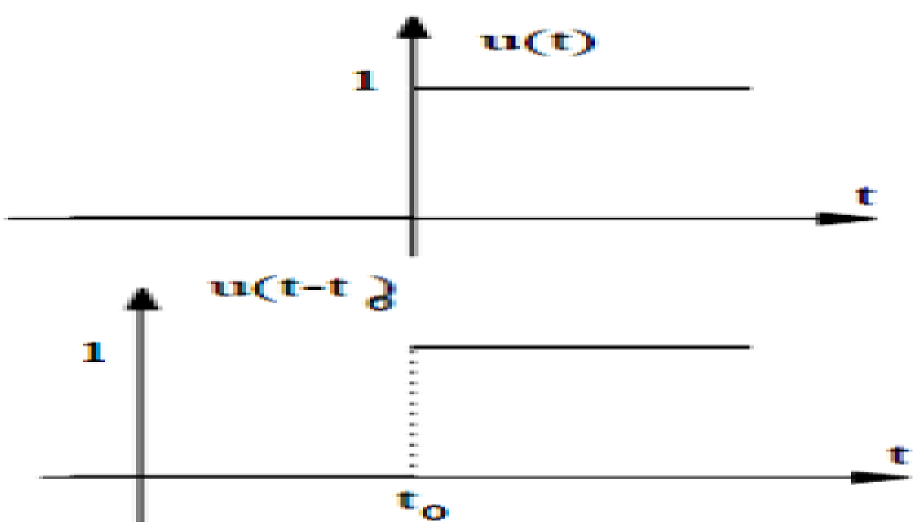


Figure 1

However, these step functions are useful and are one way top describe switches in electrical circuits as the following.

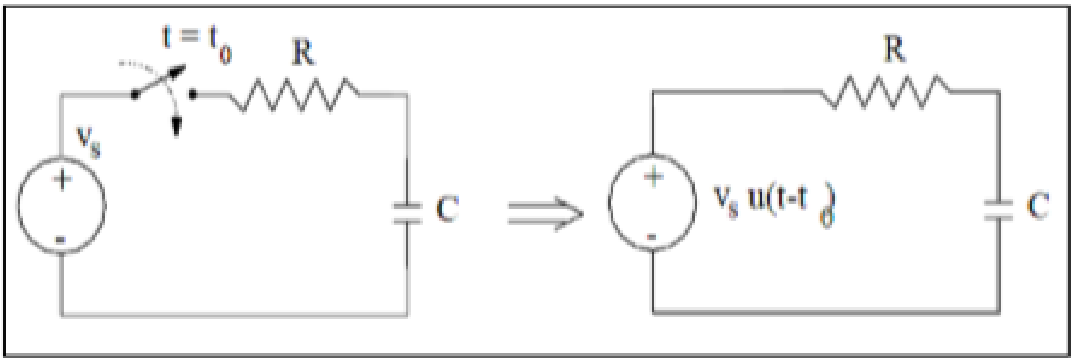


Figure 5.1

Step Response of an RC circuit:

Take a look at the previous circuit, as you notice the switch will close at time t=0, and the capacitor will begin to charge. Anyhow, the capacitor will have an initial voltage of V_0. Now if we took KVL on this circuit, it will result in $R_{ic} + V_c = V_s$ where this can be also written as:

$RC\frac{dV_c}{dt} + V_c = V_s$ and for t>0, $V_c(0^+)=V_c(0^-)=V_0$ and through these values we can say that we found the natural solution to the RC circuit.

$$V_{c,n} = Ke^{-\frac{t}{\tau}}\, and\ \tau = RC$$

And now to find the forced response, take Vc,f=A and when we substitute in the differential equation we will get.

$RC\frac{dA}{dt} + A = V_s$ ➔ $v_{c,f} = A = V_s$ and thus after all the solution will have both parts (Natural and forced) $V_C = V_{C,n} + V_{C,f} = Ke^{-\frac{t}{\tau}} + v_s$ also the constant K can be found from the initial conditions $V_c(0^+)=V_c(0^-)=V_0$

Thus $V_C(0^+) = K + v_s$ and so $K = V_C(0^+) - v_s$. Adding to this, $V_C(\infty) = V_S$, where this leads us finally to, $V_C(t) = v_s + [V_C(0^+) - v_s]e^{-\frac{t}{\tau}}$ ➔ $V_C(t) = V_C(\infty) + [V_C(0^+) - V_C(\infty)]e^{-\frac{t}{\tau}}$

Where $V_C(0^+) = initial\ voltage$

$V_C(\infty) = final\ voltage$

Finally at T➔∞ or at t=5τ the circuit will reaches its steady state. Where the current in the capacitor will be zero and the voltage will reach V_s. as seen in the expression of V_c seen in the figure.

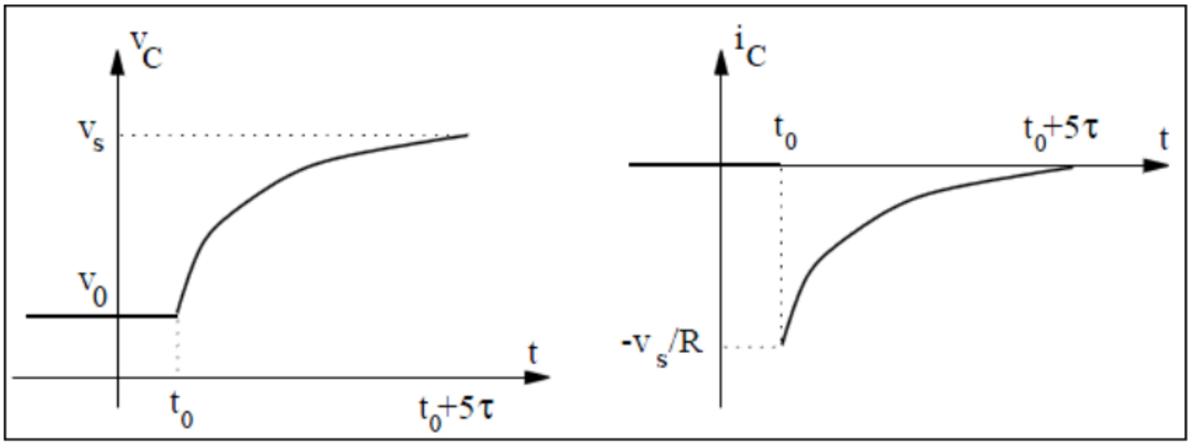


Figure 5.2

However, if the switch was closed at time t=t_0 then the voltage on the capacitor would be as the following:

$$V_C(t) = V_C(\infty) + [V_C(0^+) - V_C(\infty)]e^{-\frac{(t-t0)}{\tau}}$$

Step response of an RL circuit:

If we take the RL circuit in the following figure:

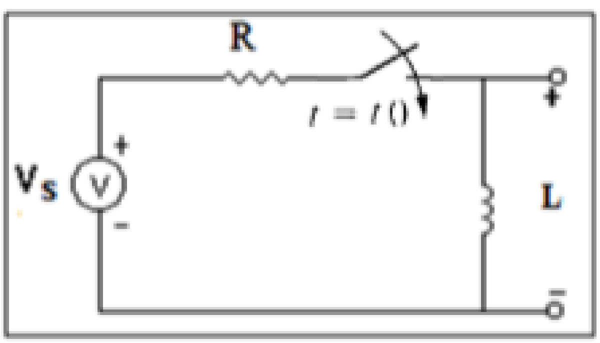


Figure 5.3

When we close the switch at time t=t0 the inductor will have an initial current of $I_L(t_0^-)$. Now, what left is finding the waveform of the inductor current, which will have the same procedure as the one in the capacitor circuit. Anyhow we can use the above equation as a general response equation and substitute to find the response of interest which is I_L , but the difference is that the time constant is $\tau = \frac{L}{R}$. Finally, when we leave the circuit for a long time or mathematically until t➔∞ the circuit will have reached a steady state condition. The inductor will be replaced with a short and $I_L(\infty) = i_s = V_s/R$.

Where this will also lead to,

$$I_L(t) = I_L(\infty) + [I_L(0^+) - I_L(\infty)]e^{-\frac{(t-t0)}{\tau}} \quad t \geq t0$$

The Natural and Step Response of a Series RLC Circuit:-

Finding the Natural and step response of series RLC circuits follows the same path used to find the Natural and Step responses of parallel RLC circuits. This is due to the similarity in the differential equations, as they both have, the same form.

If we took an example, such as the one in fig 5.4:

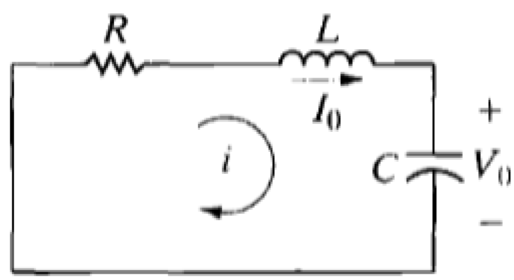


Fig5.4

The circuit in figure 5.4 used to illustrate the natural response of series RLC circuit.

We can take a loop around the circuit. And we would arrive at the differential equation:

$$Ri + L\frac{di}{dt} + \frac{1}{C}\int_0^t i d\tau + V_0 = 0$$

Now, to start working with this equation we differentiate it:

$$R\frac{di}{dt} + L\frac{d^2i}{dt^2} + \frac{i}{C} = 0$$

Rearranging

$$\frac{d^2i}{dt^2} + \frac{R}{L}\frac{di}{dt} + \frac{i}{LC} = 0$$

From this eq. we find that the Characteristic equation is:

$s^2 + \frac{R}{L}s + \frac{1}{LC} = 0$, and by solving the roots $s_{1,2} = -\frac{R}{2L} \pm \sqrt{(\frac{R}{2L})^2 - \frac{1}{LC}}$

Describing the neper frequency of the series RLC as $\alpha = \frac{R}{2L}\ rad/s$ and by describing the resonant radian frequency as $\omega_0 = \frac{1}{\sqrt{LC}}$ rad/s we can say that $s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2}$

Adding to this, we note the difference between the neper frequency of the parallel and series RLC circuits. However, they share the same damped and resonant radian frequencies.

The response of current will have 3 forms:

1) Overdamped ($w_0^2<a^2$)
2) Underdamped ($w_0^2>a^2$)
3) Critically Damped ($w_0^2=a^2$)

Thus the solution will be :

$i(t) = A_1e^{s_1t} + A_2e^{s_2t}$ (Overdamped)

$i(t) = B_1e^{-\alpha t}\cos(\omega_d t) + B_2e^{-\alpha t}\sin(\omega_d t)$ (Underdamped)

$i(t) = D_1te^{-\alpha t} + D_2e^{-\alpha t}$ (Critically Damped)

When we obtain the the natural current response we can find the natural voltage response across any circuit element.

To simplify the analysis, we assume that the initial energy in the circuit is zero.

If we apply Kirchhoff's Voltage law to the circuit in the figure 5.5 we get:

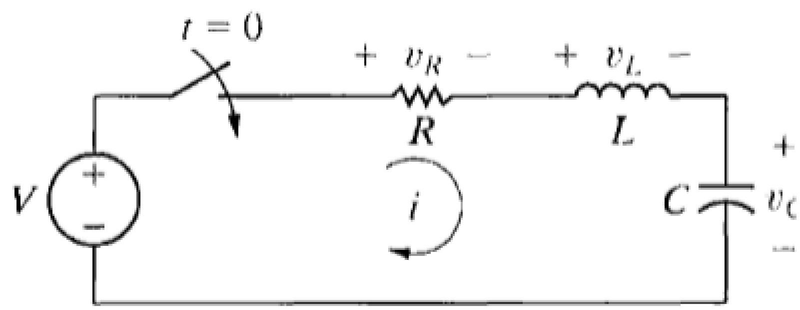


Figure 5.5

The circuit in figure 5.5 used to illustrate the step response of series RLC circuit.

$$V = Ri + L\frac{di}{dt} + V_C$$

And we know that the relation between the current and the voltage on the capacitor is:

$i = C\frac{dV_C}{dt}$, where $\frac{di}{dt} = C\frac{d^2V_C}{dt^2}$, and so substituting in the previous equation and simplifying :

$$\frac{d^2V_C}{dt^2} + \frac{R}{L}\frac{dV_C}{dt} + \frac{V_C}{LC} = \frac{V}{LC}$$

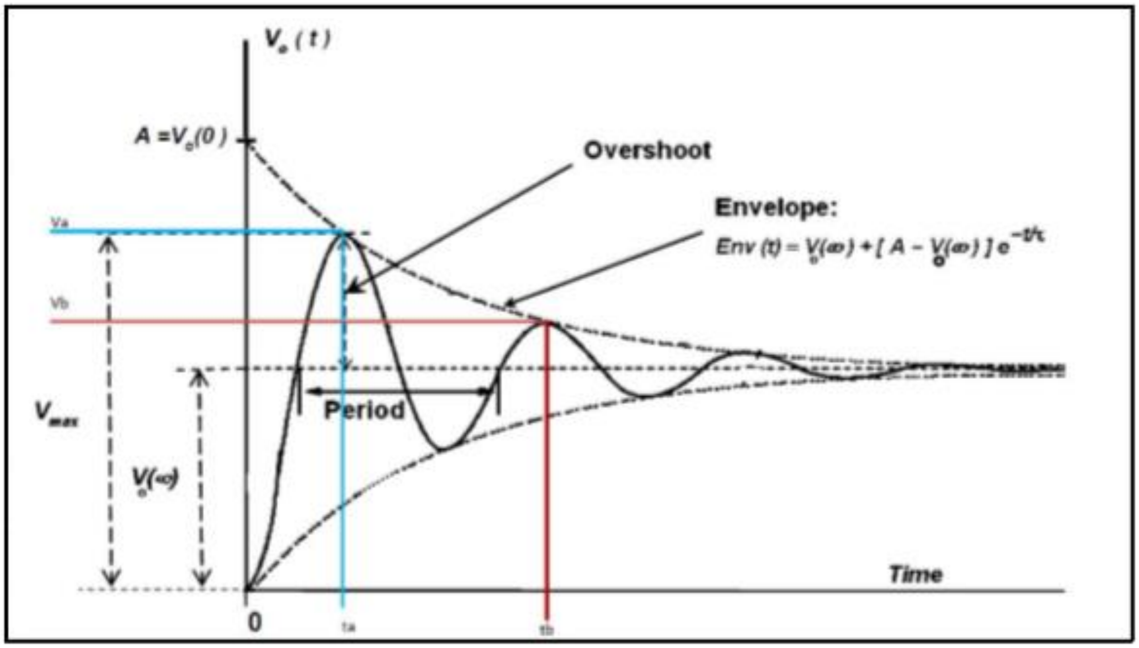
Thus, the three possible solutions for V_C will be:

$V_C(t) = V_f + A_1'e^{s_1t} + A_2'e^{s_2t}$ (Overdamped)

$V_C(t) = V_f + B_1'e^{-\alpha t}\cos(\omega_d t) + B_2'e^{-\alpha t}\sin(\omega_d t)$ (Underdamped)

$V_C(t) = V_f + D_1'te^{-\alpha t} + D_2'e^{-\alpha t}$ (Critically Damped)

Where V_f is the final value of V_C, and therefore we notice that the final value is the DC Voltage source "V".



Decay time constant $$\tau = \frac{t_b - t_a}{ln\left(\frac{V_a - V_{o(\infty)}}{V_b - V_{o(\infty)}}\right)}$$

Damping Coefficient

Damped radian frequency $$\omega_d = \frac{2\pi}{t_b - t_a}$$

2. Procedure:-

step response for first-order RC circuit.

1.the circuit in Figure 5.6 was connected

2. the function generator was set to give square wave with 50 Hz and $6V_{p\text{-}p}$.

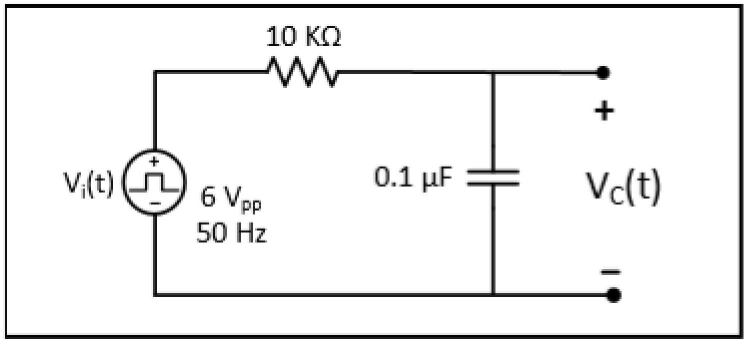


Figure 5.6

.The oscilloscope was set to display the wave form $V_c(t)$ produced in the circuit above(Figure 5.6).

. The wave form of $V_c(t)$ was displayed using oscilloscope ,the 63% of peak value of input voltage was calculated and by using the cursor the point of intersection was determined and hence the time constant(τ_c) was measured .

The circuit in the figure 5.7 was connected as same as figure 5.6 but, the location of the capacitor and the resistor was changed.

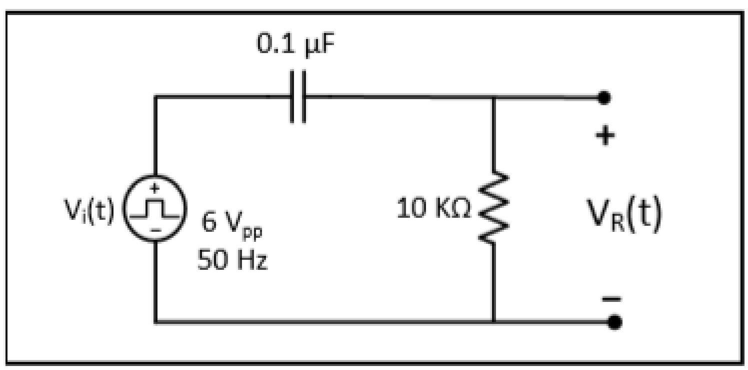


Figure 5.7

.The oscilloscope was set to display the wave form $V_R(t)$ produced in the circuit above(figure 5.7).

. The wave form of $V_R(t)$ was displayed using oscilloscope ,the 37% of peak value of output voltage was calculated and by using the cursor the point of intersection was determined and hence the time constant(τ_R) was measured.

step response for first -order RL circuit :-

1.the circuit in Figure 5.8 was connected

2. the function generator was set to give square wave with 50 Hz and $6V_{p-p}$.

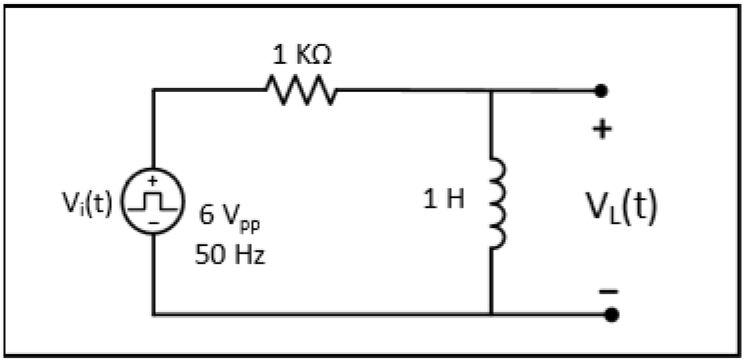


Figure 5.8

.The oscilloscope was set to display the wave form $V_L(t)$ produced in the circuit above(figure 5.8).

. The wave form of $V_L(t)$ was displayed using oscilloscope ,the 37% of peak value of output voltage was calculated and by using the cursor the point of intersection was determined and hence the time constant(τ_L) was measured .

The circuit in the figure 5.9 was connected as same as figure 5.8 but, the location of the inductor and the resistor was changed.

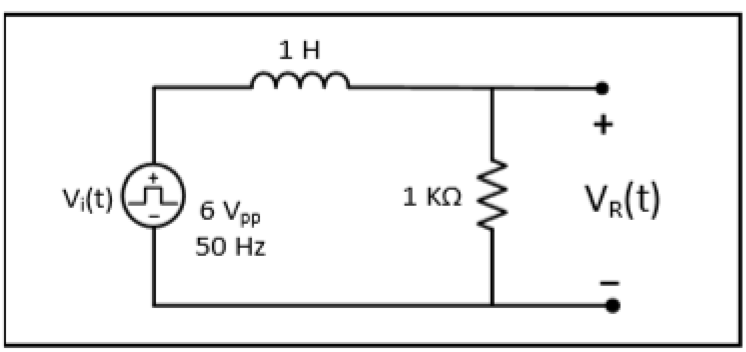


Figure 5.9

.The oscilloscope was set to display the wave form $V_R(t)$ produced in the circuit above(figure 5.9).

. The wave form of $V_R(t)$ was displayed using oscilloscope ,the 63% of peak value of output voltage was calculated and by using the cursor the point of intersection was determined and hence the time constant(τ_R) was measured .

step response of second-order series RLC circuit :

1.the circuit in Figure 5.9 was connected

2. the function generator was set to give square wave with 100 Hz and $3V_{p\text{-}p}$.

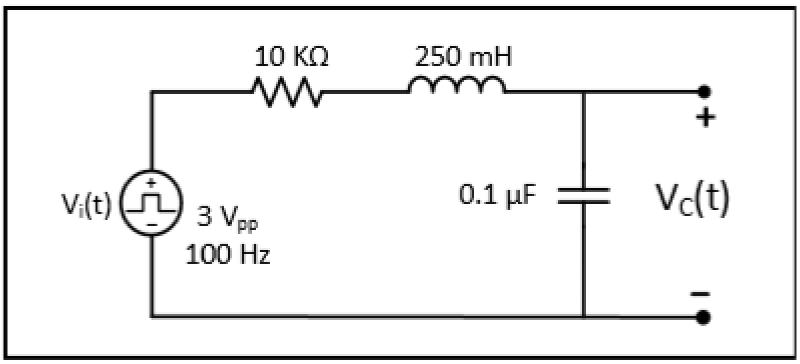


Figure 5.10

.The oscilloscope was set to display the wave form $V_c(t)$ produced in the circuit above(figure 5.10).

. The parasitic resistance of the inductor was measured using DVM and recorded in table 5.2

.the value of variable resistor was changed causing changing in the characteristic of the $V_c(t)$ in three cases :

Case A: The variable resistance was set to its maximum (10K) and its value was measured with noticing that the total resistance of the circuit is the sum of the value of the variable resistor, the output resistance of the function generator and the parasitic resistance of the inductor, this case called (over damped)

Case B : The variable resistor was set at normal value to equal the critical value of(Rc) that was measured in the prelab ,this case called (critical damped)

Case c : The variable resistor was set at low value ,and in this case the value of (Va,Vb,Ta,Tb,V(inf)) was measured using amplitude cursor and time cursor and recorded in table 5.3,this called (under damped)

4- Step response of second-order parallel RLC circuit:

1.the circuit in Figure 5.11 was connected

2. the function generator was set to give square wave with 100 Hz and $3V_{p\text{-}p}$.

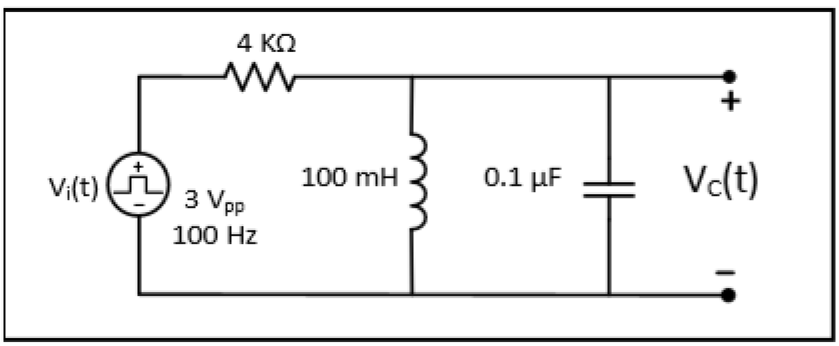


Figure 5.11

.The oscilloscope was set to display the wave form $V_c(t)$ produced in the circuit above(figure 5.11).

. The parasitic resistance of the inductor was measured using DVM and recorded in table 5.2

.the value of variable resistor was changed causing changing in the characteristic of the $V_c(t)$ in three cases :

Case A: The variable resistance was set to its maximum (4K) and its value was measured with noticing that the total resistance of the circuit is the sum of the value of the variable resistor, the output resistance of the function generator and the parasitic resistance of the inductor, the values of(Va,Vb,ta,tb,V(inf)) was measured using amplitude cursor and time cursor and recorded in table5.4 , this case called (under damped).

Case B: : The variable resistor was set at normal value to equal the critical value of(Rc) that was measured in the prelab ,this case called (critical damped).

Case c : The variable resistor was set at low value(approximated to short circuit),this case called (over damped).

3. *Data:-*

Step response for first-order RC circuit.

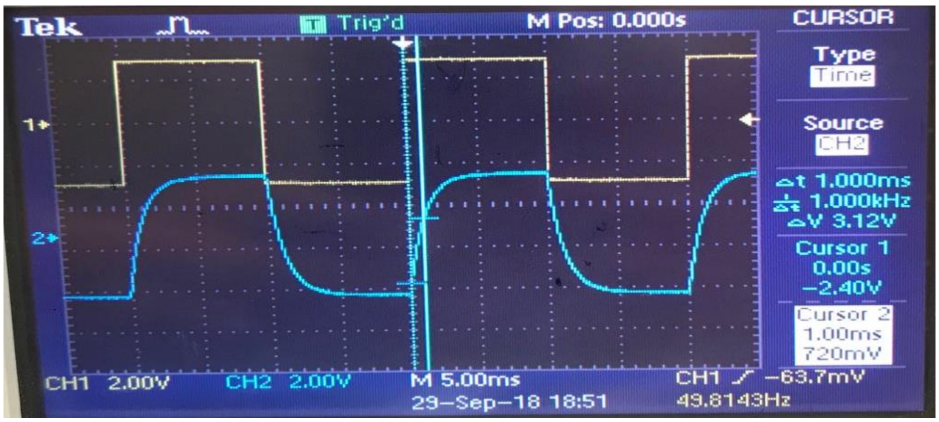


Figure 5.12

From the figure above, the existence of the capacitor makes the response goes to the final value with time (i.e. not instantaneously). Which means that the response takes time to reach final value (3V), the time that the response took was 20ms, which means that τ = $\frac{5ms}{5}$ = 1ms, and that was represented in the theory (τ= RC = 1ms).

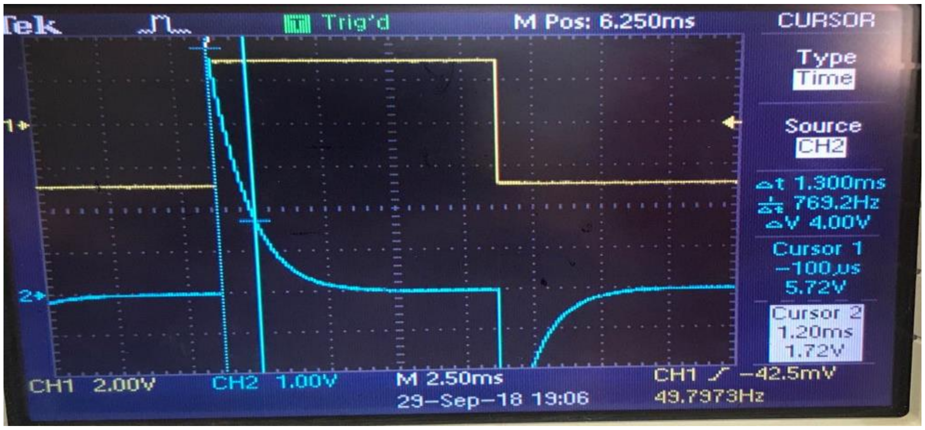


Figure 5.13

From the figure above, the existence of the capacitor makes the response

Of the resistor discharging (i.e. not instantaneously). from the maximum value(3V) to the steady state (0V) which takes time approximated to 1ms

Assume that the time of charging voltage on the capacitor equals the time of discharging the voltage on the resistor .

Noticed that there is some error between the theoretical and experimental value of τ ,$\tau_{\text{theoretical}}$ =1ms, $\tau_{\text{experimental}}$ =1.3ms

$E = \frac{1.3-1}{1.3} = 23\%$

Step response for first-order RL circuit.

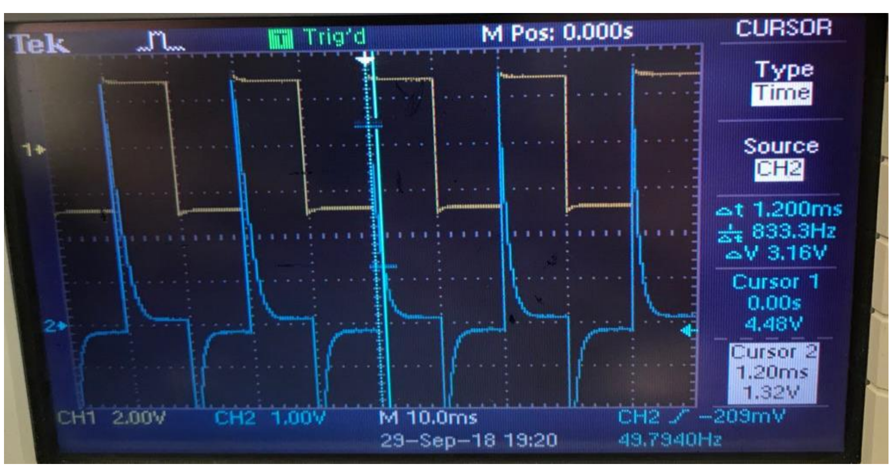


Figure 5.14

From the figure above, the existence of the inductor makes the response goes to the final value with time (i.e. not instantaneously). Which means that the response takes time to discharging from (3V) to the final value (steady state) (0V), the time that the response took was 20ms, which means that $\tau = \frac{5ms}{5}$ = 1ms, and that was represented in the theory ($\tau = L/R$ = 1ms).

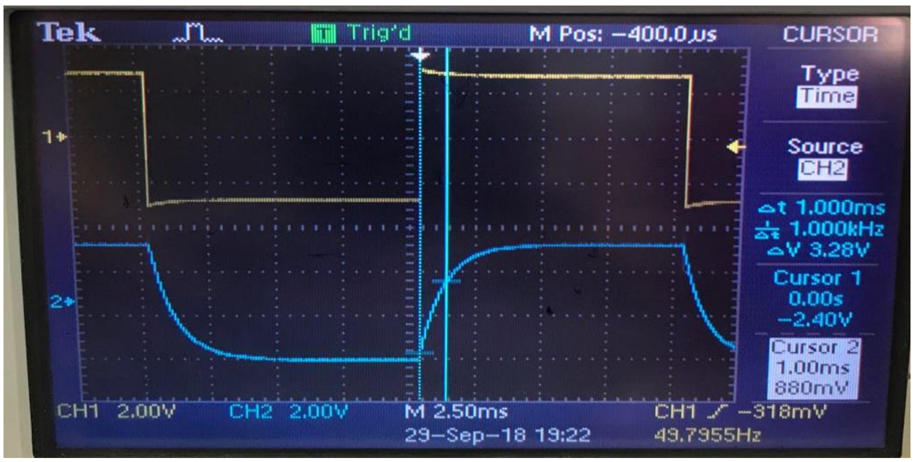


Figure 5.15

From the figure above, the existence of the makes the response goes to the final value with time (i.e. not instantaneously). Which means that the response takes time to reach final value (3V)which takes time approximated to 1ms

Assume that the time of discharging voltage on the inductor equals the time of charging the voltage on the resistor .

Noticed that there is some error between the theoretical and experimental value of τ ,$\tau_{theoretical}$ =1ms, $\tau_{experimental}$ =1.2ms

$E = \frac{1.2-1}{1.2}$ = 16%

Step response for second- order series RLC circuit

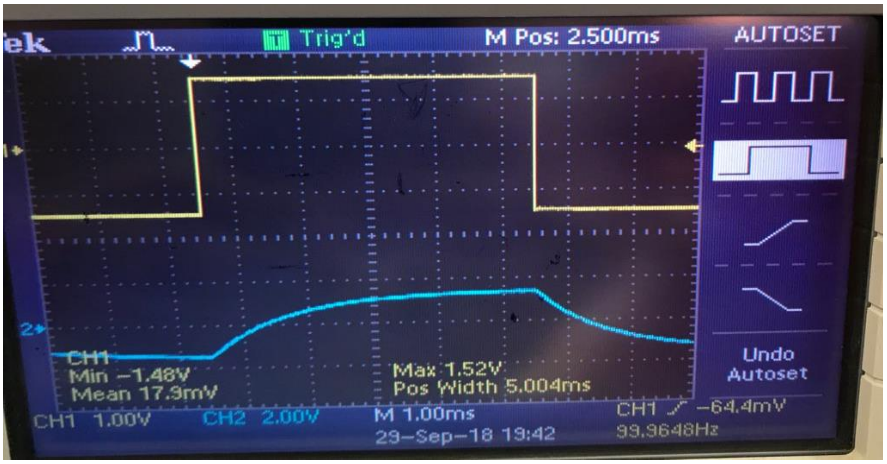


Figure 5.16

From the figure above, the existence of series connection between the inductor and the capacitor and the high value of resistor makes the response charging rapidly and reach the steady state (3V) in time depends on the roots of characteristic equation that calculated in prelab ,this response called (over damped response).

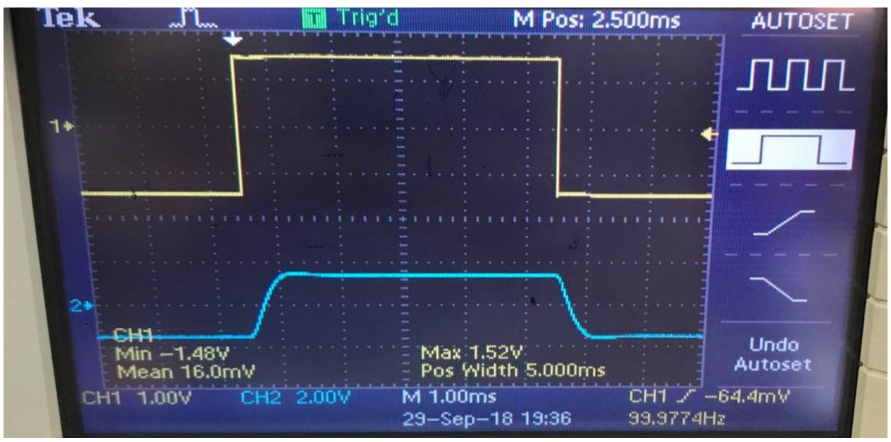


Figure 5.17

From the figure above, noticed that the response reach the steady state value (3V)more rapidly than Figure 5.16 (takes less time) as result of changing the value of variable resistor to the critical value(R_c) that calculated in prelab .this response called (critical damped)

The time needed to reach the steady state depends on the value of(α) that measured in the prelab .

Note: The circuit connection is same as the Figure 15.6

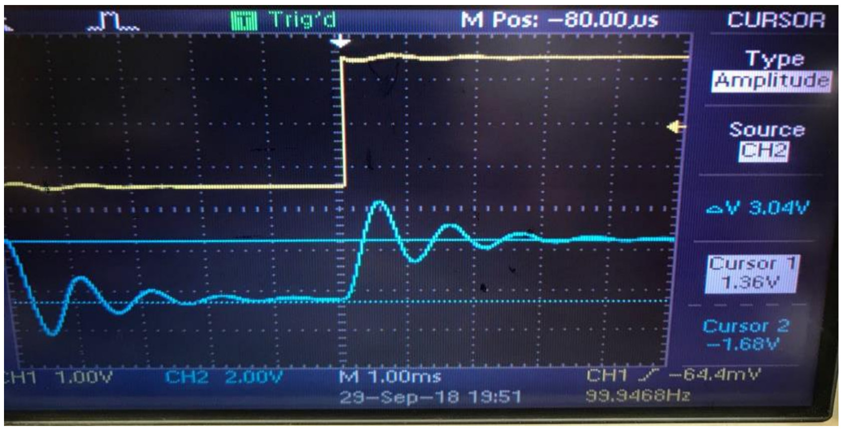


Figure 5.18

From the figure above, noticed that the response oscillated around the final value(steady state) before reach it ,this case happened as result of decreasing the value of variable resistor to small value ,also, the value of V_a Measured in the first maximum peak of the response(over shoot) and value of t_a measured in that moment, , the value of V_b Measured in the second maximum peak of the response and value of t_b measured in that moment,

The value of V(∞) is same as the final value of the response(steady state) and equal (3V).

In the two cases above(5.17,5.16) the value of response didn't exceed the final value ,but in this case the value of response exceed the final value (over shoot) before reach it ,so we have to wary of damage the load ,this type of response called (under damped)

.

Note: The circuit connection is same as the Figure 5.16

Step response for second- order parallel RLC circuit

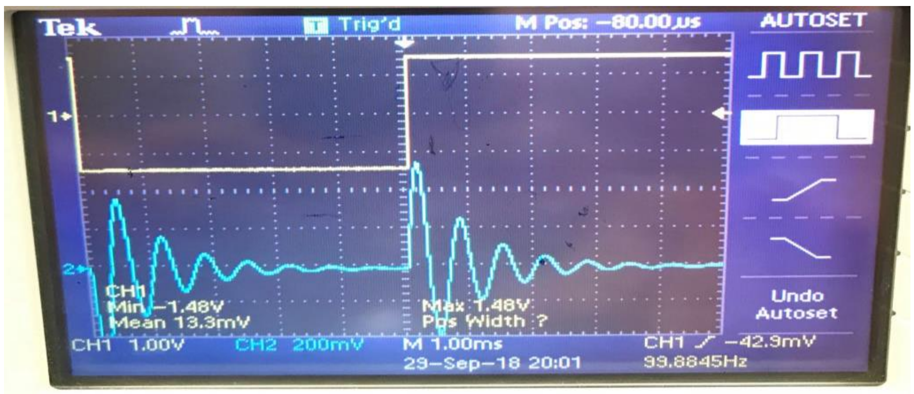


Figure 5.19

From the figure above, the existence of parallel connection between the inductor and the capacitor and the high value of resistor makes the response oscillation around the final value(steady state) before reach it.

The value of V_a Measured in the first maximum peak of the response(over shoot) and value of t_a measured in that moment, , the value of V_b Measured in the second maximum peak of the response and value of t_b measured in that moment,

The value of V(∞) is same as the final value of the response(steady state) and equal (0V).

In this case the value of response exceed the final value (over shoot) before reach it ,so we have to wary of damage the load ,this type of response called (under damped)

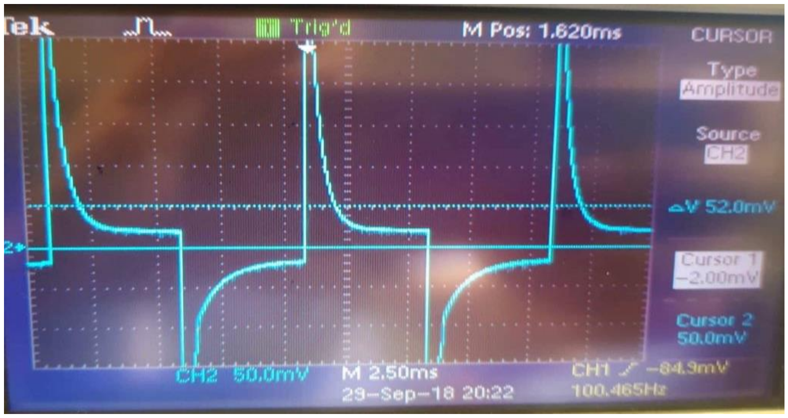


Figure 5.20

From the figure above, noticed that the response reach the steady state value (0V)more rapidly than Figure 5.19(takes less time) as result of changing the value of variable resistor to the critical value(R_c) that calculated in prelab .this response called (critical damped)

The time needed to reach the steady state depends on the value of(α) that measured in the prelab .

Note: The circuit connection is same as the Figure 5.19

Figure 5.21

From the figure above, the response discharging rapidly and reach the steady state (0V) in time depends on the roots of characteristic equation that calculated in prelab ,this response called (over damped response).

Noticed that the response in this case reach the steady state slower than the previous case(figure 5.20)but it more rapid than the case in figure 5.19

This three cases(figure 5.19,5.20,5.21) differed in each other only in the value of variable resistor (150Ω - 4kΩ)

4. *Tables*

Step response of first-order RC circuit

Table 5.1

R actual	10 kΩ

Step response of first-order RL circuit

Table 5.2

R actual	**R inductor**
10 kΩ	**53Ω**

Step response of second-order Series RLC circuit

under damped response

Table 5.3

V_a	**t_a**	**V_b**	**t_b**	**$V(\infty)$**
5V	**0.48ms**	**3.92V**	**1.5 ms**	**3V**

Step response of second-order parallel RLC circuit

under damped response

Table 5.4

V_a	t_a	V_b	t_b	$V(\infty)$
0.53V	120 µs	0.148 V	0.48 ms	0V

5. *Conclusion*

Four types of circuit were connected in this experiment ,first-order(RL&RC)circuit and second-order (series &parallel)RLC circuit

Some measurement were talked in the (RC & RL)circuit like τ and noticed that the theoretical value was differed from the experimental value and the error was measured above.

the RLC circuit (series and parallel) were connected and the three cases of response(over damped , critical damped, under damped) were explained and some measurement were talked (V_a ,t_a ,V_b ,t_b , $V(\infty)$).

6. *References*

- Circuits Lab manual.
- Nillson & Riedel electric circuit 9th edition <chapter 7&8>.