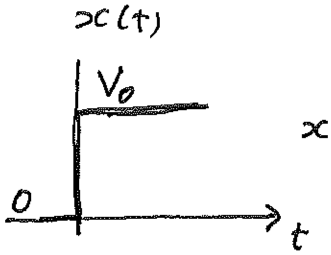
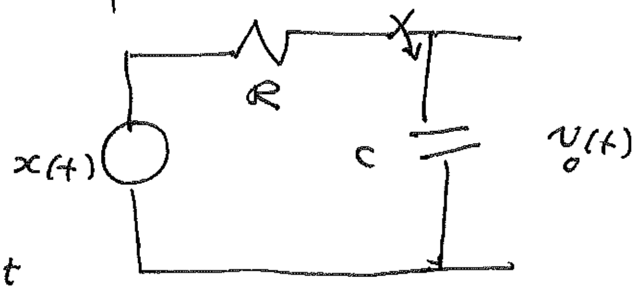
① Experiment 4

RC circuit





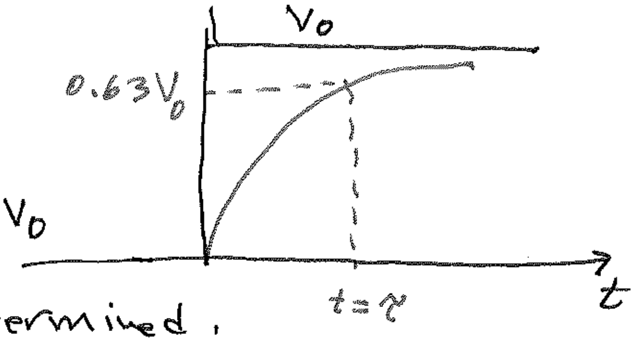
First, let

$x(t) = V_0\, u(t)$;

the output is

$$v(t) = V_0\left(1 - e^{-t/RC}\right) = V_0\left(1 - e^{-t/\tau}\right)$$

τ : time constant.



when $t = \tau$,

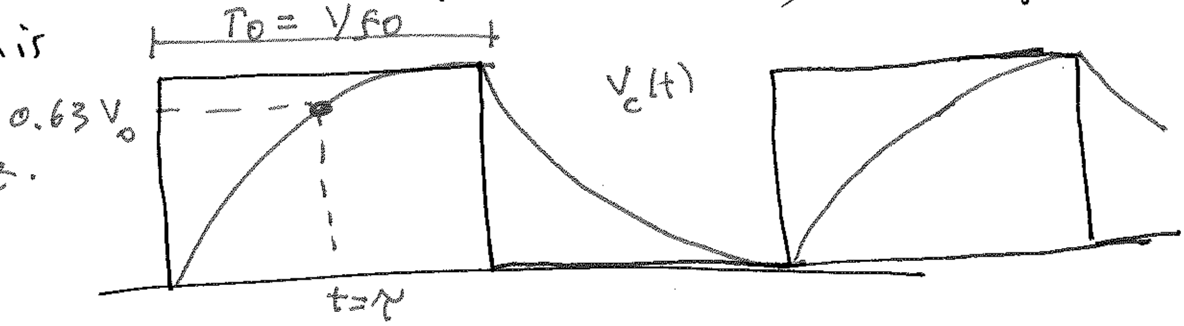
$$v(\tau) = V_0(1 - e^{-1}) = 0.63\, V_0$$

From this τ can be determined.

Limiting values ; when $t \to \infty$, $v_0 \Rightarrow V_0$

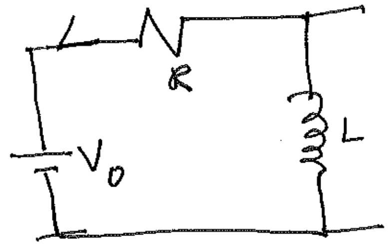
when the input is a square function, the output looks like this

$f_0 = 50\,Hz$.



RL circuit

The D.E. due to a step input is



$$V_0\, u(t) = Ri + L\frac{di}{dt}$$

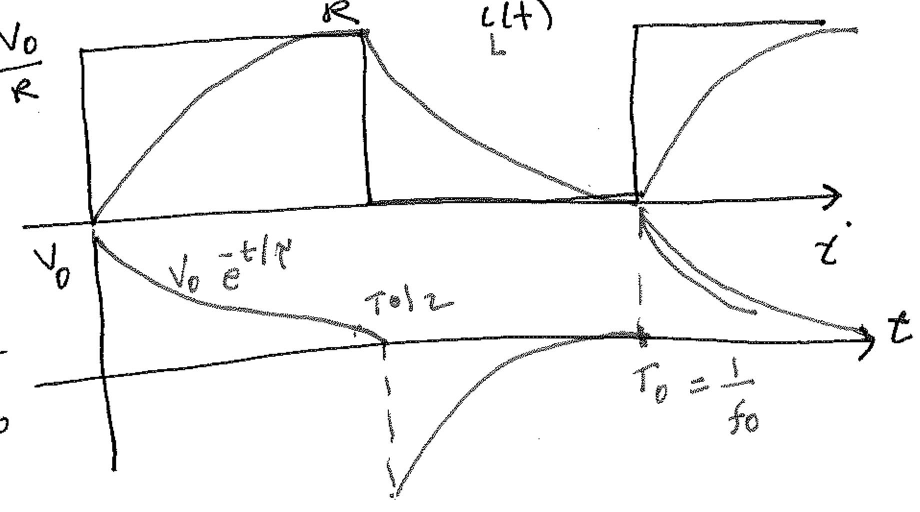
$$\Rightarrow i(t) = \frac{V_0}{R}\left(1 - e^{-t/\tau}\right) ; \tau = \frac{L}{R}$$

Also,

$$V_L = L\frac{di}{dt}$$

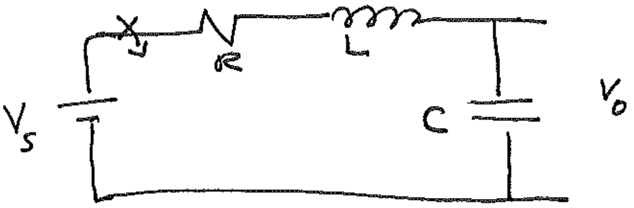
$$V_R = Ri = V_0\left(1 - e^{-t/\tau}\right)$$

$$V_L = V_0 - V_R = \begin{cases} V_0 e^{-t/\tau} & 0 < t < T_0/2 \\ -V_0 e^{-t/\tau} & \frac{T_0}{2} < t < T_0 \end{cases}$$



Experiment 4

RLC circuit



D.E

$$V_s = Ri + L\frac{di}{dt} + V_0$$

$$i = C\frac{dV_0}{dt}$$

$$\Rightarrow \boxed{\frac{d^2V_0}{dt^2} + \frac{R}{L}\frac{dV_0}{dt} + \frac{V_0}{LC} = \frac{V_s}{LC}}$$

- The solution is the sum of the final voltage $V_f = V_s$ and the natural response

$$V_0 = V_s + V_{natural}$$

$$V_0 = \begin{cases} V_s + A_1 e^{s_1 t} + A_2 e^{s_2 t} & \text{if } \alpha > \omega_0 \quad \text{overdamped} \\ V_s + e^{-\alpha t}\left(B_1 \cos \omega_d t + B_2 \sin \omega_d t\right) & \text{if } \alpha < \omega_0 \quad \text{underdamped} \\ V_s + e^{-\alpha t}\left(D_1 + D_2 t\right) & \text{if } \alpha = \omega_0 \quad \text{critically damped} \end{cases}$$

$$\boxed{s_1, s_2 = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2}}$$

$$\underset{\text{attenuation}}{\alpha = \frac{R}{2L}}, \quad \underset{\text{undamped freq.}}{\omega_0 = \frac{1}{\sqrt{LC}}}, \quad \underset{\text{damping freq.}}{\omega_d = \sqrt{\omega_0^2 - \alpha^2}}$$

- critical damping when $\alpha^2 = \omega_0^2$

$$\left(\frac{R}{2L}\right)^2 = \frac{1}{LC} \Rightarrow \boxed{R = \frac{4L}{C}}$$

$$s_1 = s_2 = -\alpha$$

- over damped / s_1, s_2 are real
- underdamped $s_1, s_2 = -\alpha \pm j\omega_d$ ← damped frequency

$$= -\alpha \pm j\sqrt{\omega_0^2 - \alpha^2}$$

The exponential decay

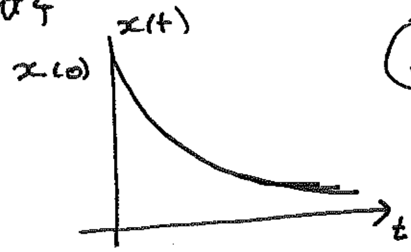
Experiments

3

Consider the function

$$\boxed{x(t) = x(0)\, e^{-\alpha t}\, u(t).} \quad (1)$$

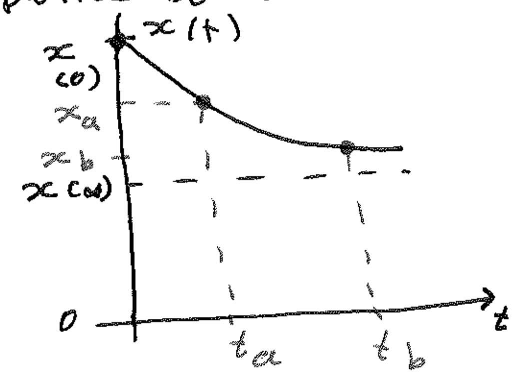
This function is plotted here



Note that as $t \to \infty$, $x(t) \to 0$

Now, consider the signal plotted below

The signal decays exponentially reaching the limiting value $x(\infty)$ as $t \to \infty$.



The mathematical formula for $x(t)$ is

$$\boxed{x(t) = x(\infty) + (x(0) - x(\infty))\, e^{-\alpha t}\, u(t)} \quad (2)$$

How to determine α in (2)

$$x_a = x(\infty) + (x(0) - x(\infty))\, e^{-\alpha t_a}$$

$$\Rightarrow e^{-\alpha t_a} = \frac{x_a - x(\infty)}{(x(0) - x(\infty))} \quad (a)$$

Also,

$$x_b = x(\infty) + (x(0) - x(\infty))\, e^{-\alpha t_b}$$

$$\Rightarrow e^{-\alpha t_b} = \frac{x_b - x(\infty)}{(x(0) - x(\infty))} \quad (b)$$

Dividing (a) by (b),

$$e^{-\alpha(t_a - t_b)} = \frac{x_a - x(\infty)}{x_b - x(\infty)} = e^{\alpha(t_b - t_a)}$$

$$\alpha(t_b - t_a) = \ln\left(\frac{x_a - x(\infty)}{x_b - x(\infty)}\right)$$

$$\Rightarrow \boxed{\alpha = \frac{1}{t_b - t_a} \ln\left(\frac{x_a - x(\infty)}{x_b - x(\infty)}\right)}$$

We shall take $\boxed{(t_b - t_a) = T_d}$; i.e., one period of the damping function