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17-8-2021

# ENEE2360 Analog Electronics ✓

## T11: Operational Amplifiers

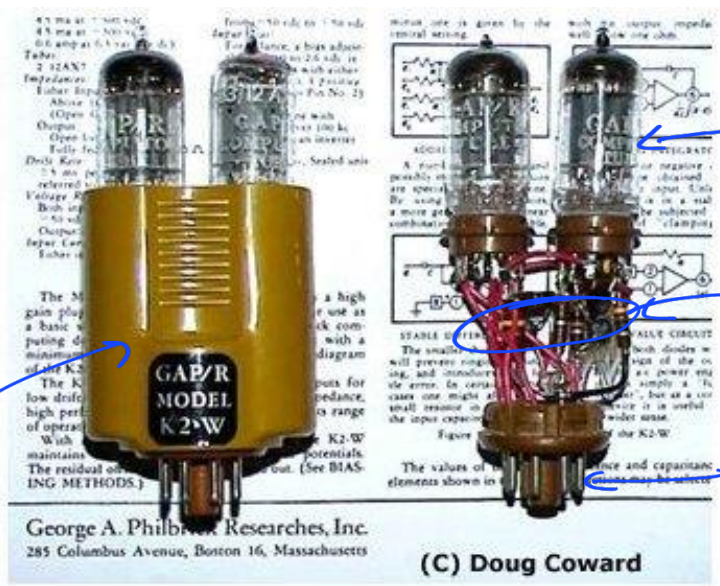
Instructor : Nasser Ismail

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## Operational Amplifiers

- Early Operational Amplifiers were constructed with vacuum tubes and were used in analog computers to perform mathematical operations.
- Even as late as 1965, vacuum tube operational amplifiers were still in use and cost in the range of \$75.
- These days, they are linear Integrated circuits (IC) that use low voltage dc supplies, they are reliable and inexpensive
- The operational amplifier has become so cheap in price (often less than \$1.00 per unit) and it can be used in so many applications

# Early Vacum Tube Operational Amplifiers



## The Philbrick Operational Amplifier (1952)

From "Operational Amplifier", by Tony van Roon: <http://www.uoguelph.ca/~antoon/gadgets/741/741.html>

## Operational Amplifiers

Operational was used as a descriptor early-on because this form of amplifier can perform operations of :

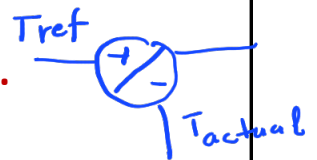
- Adding signals
- Subtracting signals
- Integrating signals,  $\int x(t)dt$
- Differentiation of signals,

The applications of operational amplifiers ( shortened to op amp ) have grown beyond those listed above.



## What can you do with Op amps?

- You can make music louder when they are used in stereo equipment.
- You can amplify the heartbeat by using them in medical cardiographs.
- You can use them as comparators in heating systems <sup>& cooling</sup>.
- You can use them for Math operations
- And many other applications in all fields of engineering

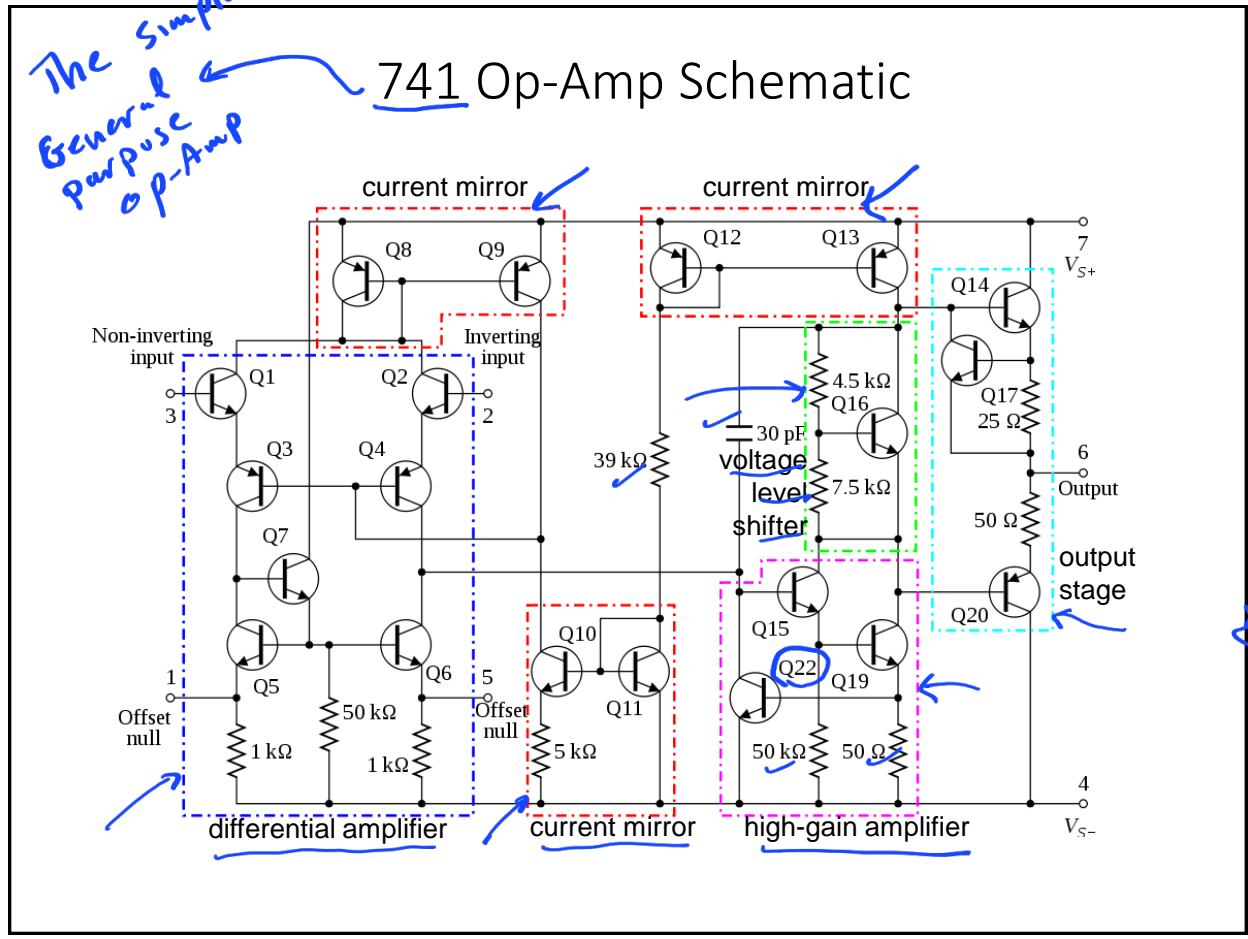


## Operational Amplifiers → ?

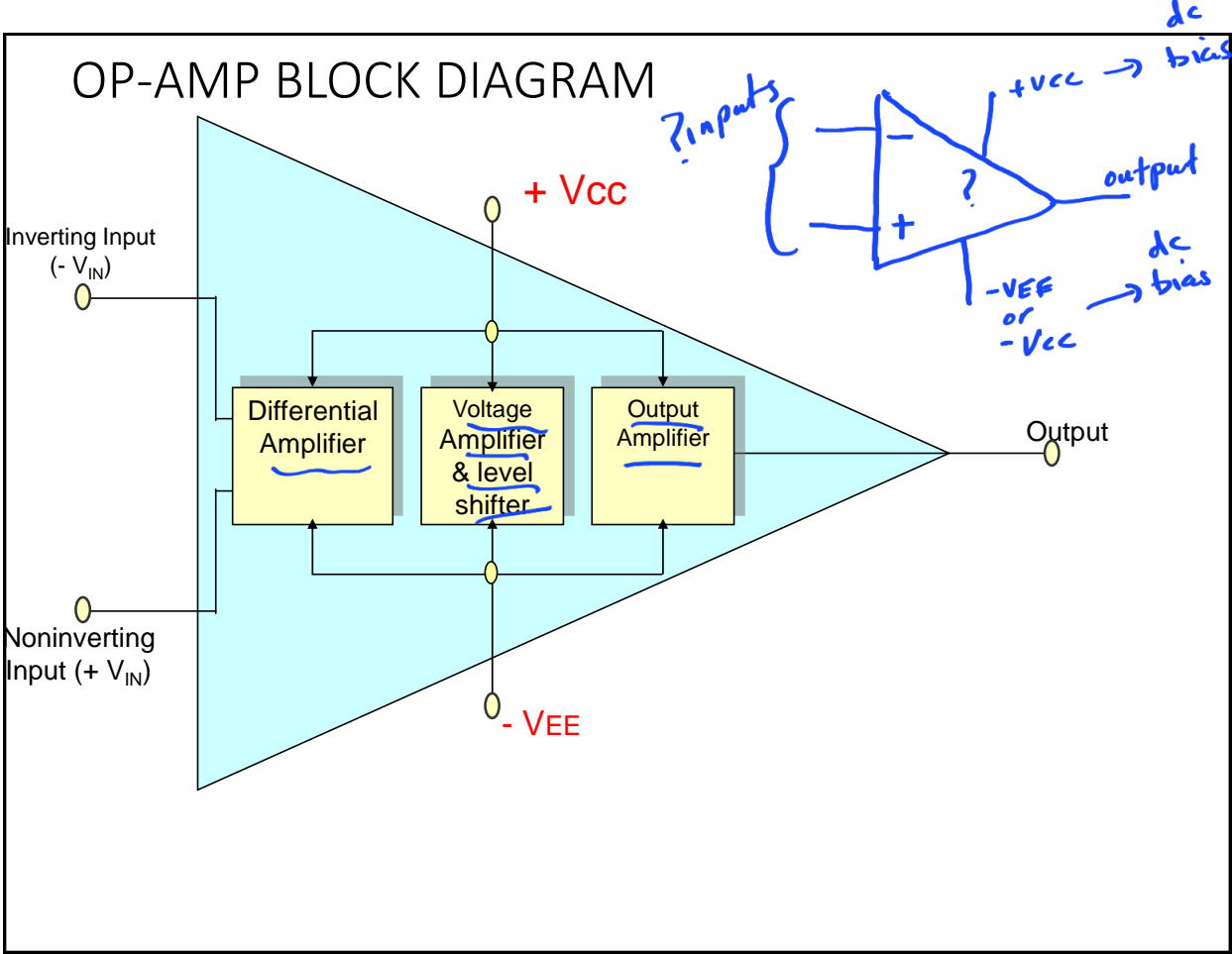
- In this course we will be concerned with *how to use the op amp as a device.*
- The internal configuration (design) is beyond the scope of our study and can be covered in an advanced electronics course.
- The complexity is illustrated in the following block diagram and detailed circuit.

*The simplest  
General  
purpose  
Op-Amp*

### 741 Op-Amp Schematic



*low  
bias*



## OP-AMP CHARACTERISTICS

1. Very high input impedance (in mega ohms)
2. Very high gain (> 100,000)
3. Very low output impedance (in ohms)

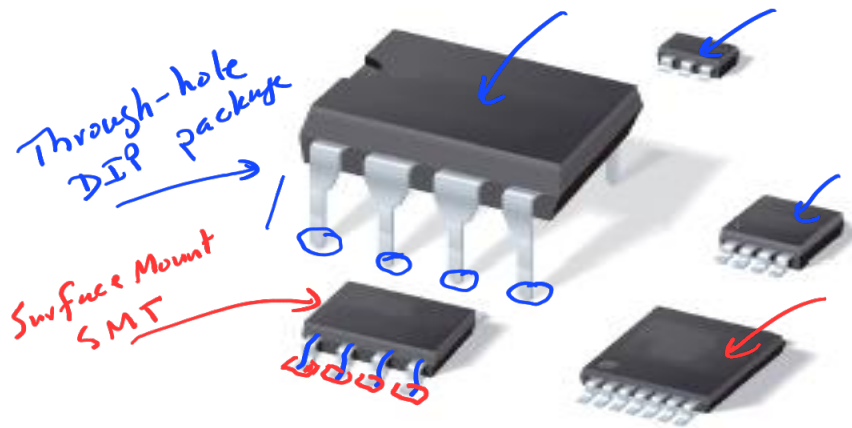
\* OP-AMP is a differential, voltage amplifier with high gain.

## Operational Amplifiers

Fortunately, we do not have to assemble a circuit with so many transistors and resistors in order to get and use the op amp

The circuit in the previous slide is usually encapsulated into a dual in-line pack (DIP)

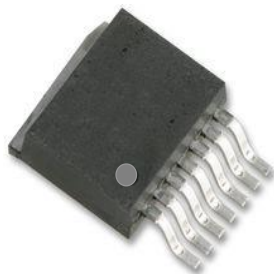
For a single LM741, the pin connections for the chip are shown below.



### Packaging Types



(a) Op Amp 741  
8-pins DIP package

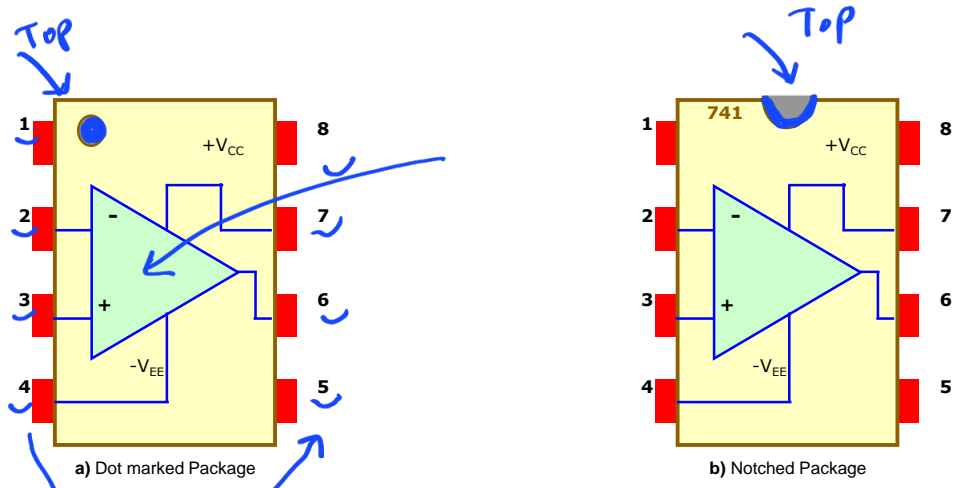


(b) OPA547FKTWT  
DIP SMT package  
Op Amp packages



(c) TO-5 metal can  
8-Leads package

# OP-AMP pins identification



Op Amp pins Identification

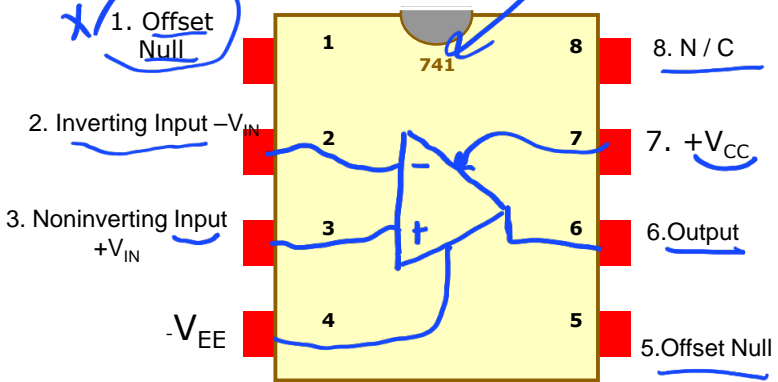


What are these pins?

*Data sheet*  
*X* 1. Offset Null

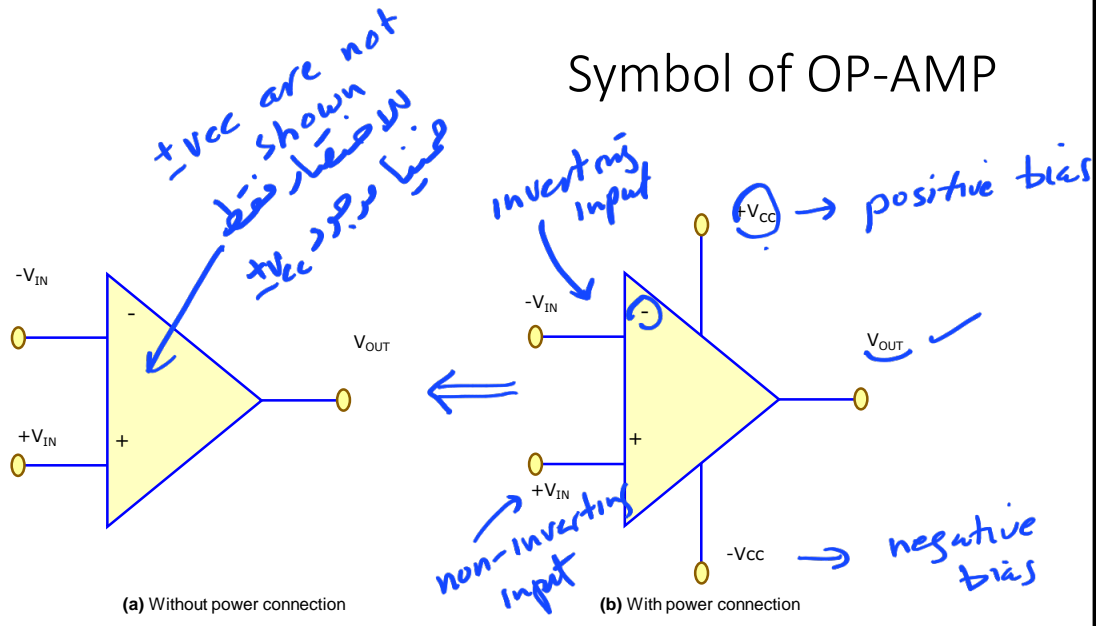
*Standard 8-pin DIP pack*

*not connected (not used)*



Op Amp pins Description

### Symbol of OP-AMP

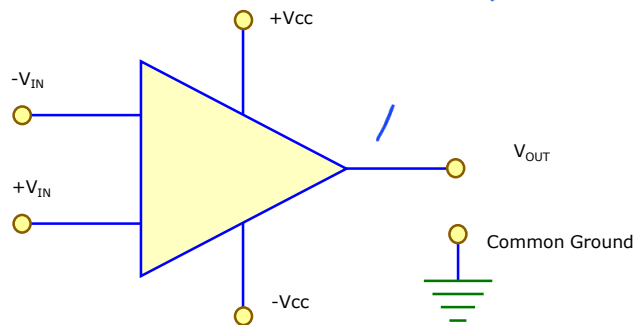


Op Amp Schematic Symbols

## Most Op Amps require dual power supply with common ground

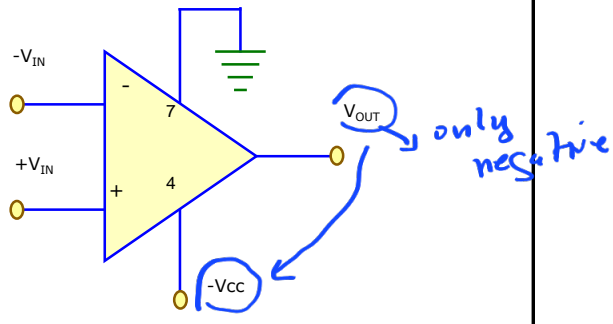
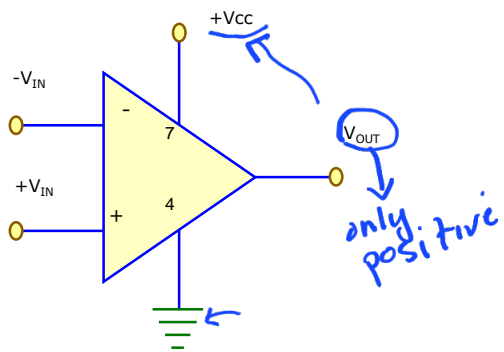
Positive Supply +Vcc to pin7 in 741 opamp

Negative Supply -Vcc to pin4 in 741 opamp



Dual Supply Voltages connection

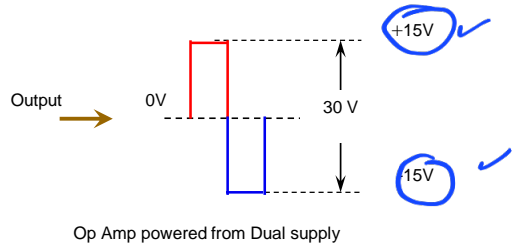
Some Op Amps work on single supply also (with some restrictions)



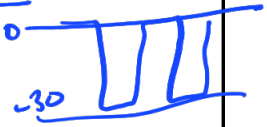
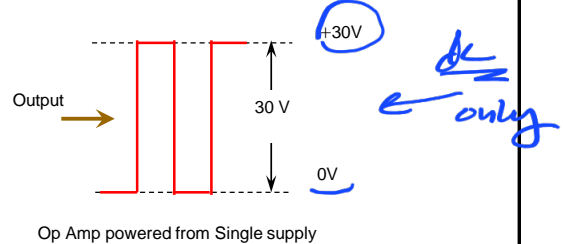
Single Supply Voltages connection

### Advantage of dual power supply

Using dual power supply will let the op amp to output true AC voltage.



*can be  
ac or dc*



# What is dual power supply?

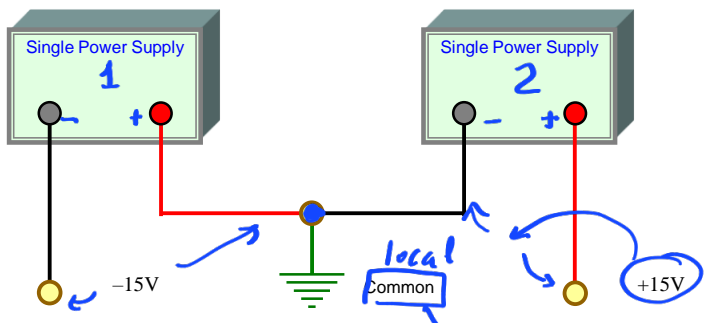


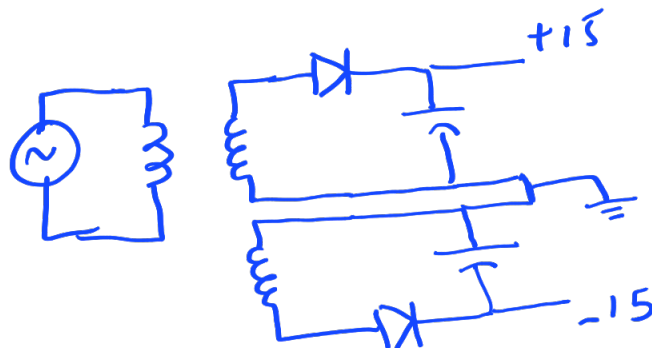
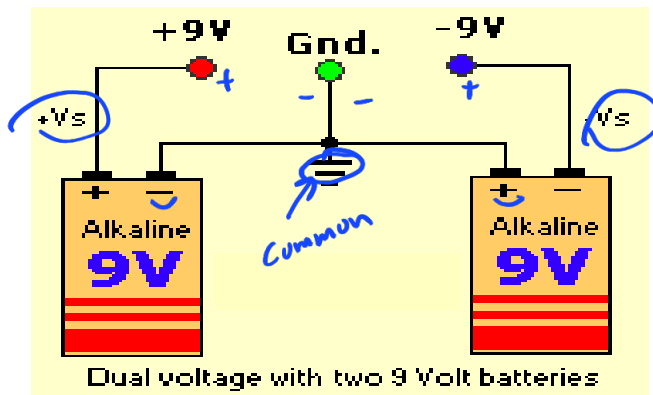
Figure 18 Dual Power Supply

Do not connect to earth GND

Earth GN

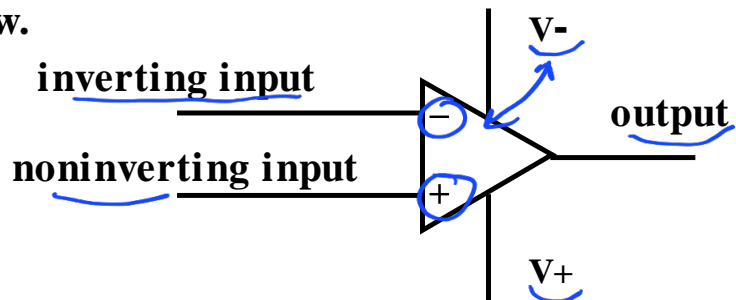
# How can you make a dual power supply using two 9V batteries?

What is the voltage between + of first battery and - of second battery?



## Operational Amplifiers

The basic op amp with supply voltage included is shown in the diagram below.



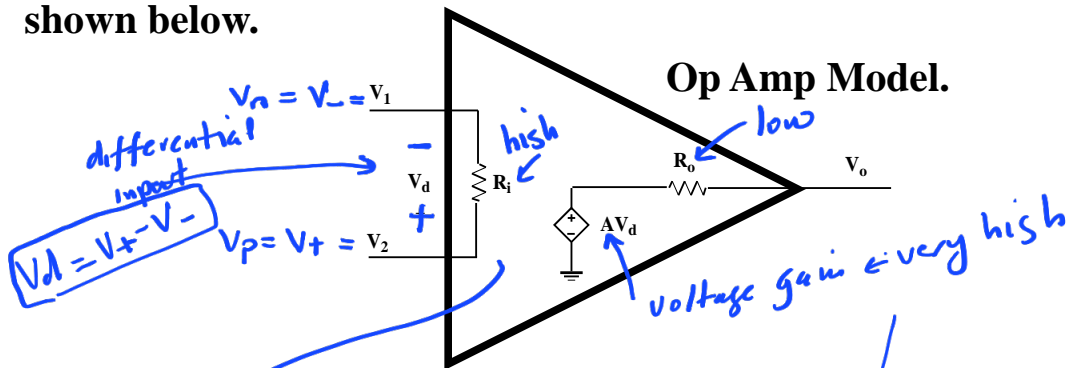
In most cases only the two inputs and the output are shown for the op amp.

However, one should keep in mind that supply voltage is required, and a ground.

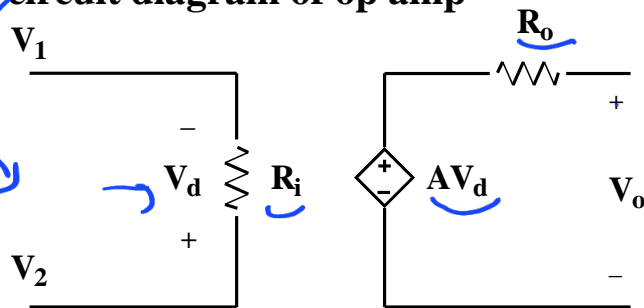


## Operational Amplifiers Model

A model of the op amp, with respect to the symbol, is shown below.



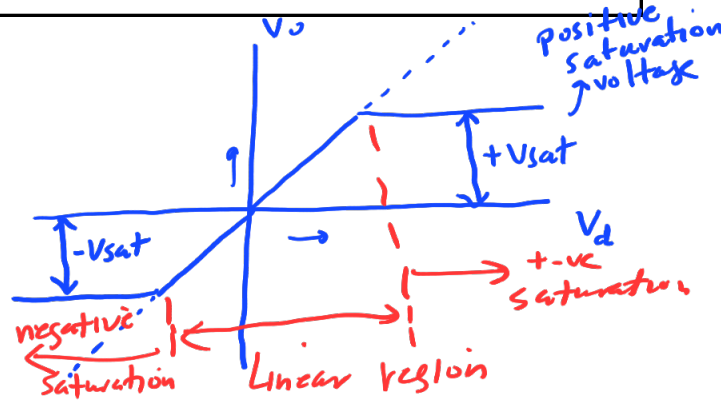
Working circuit diagram of op amp



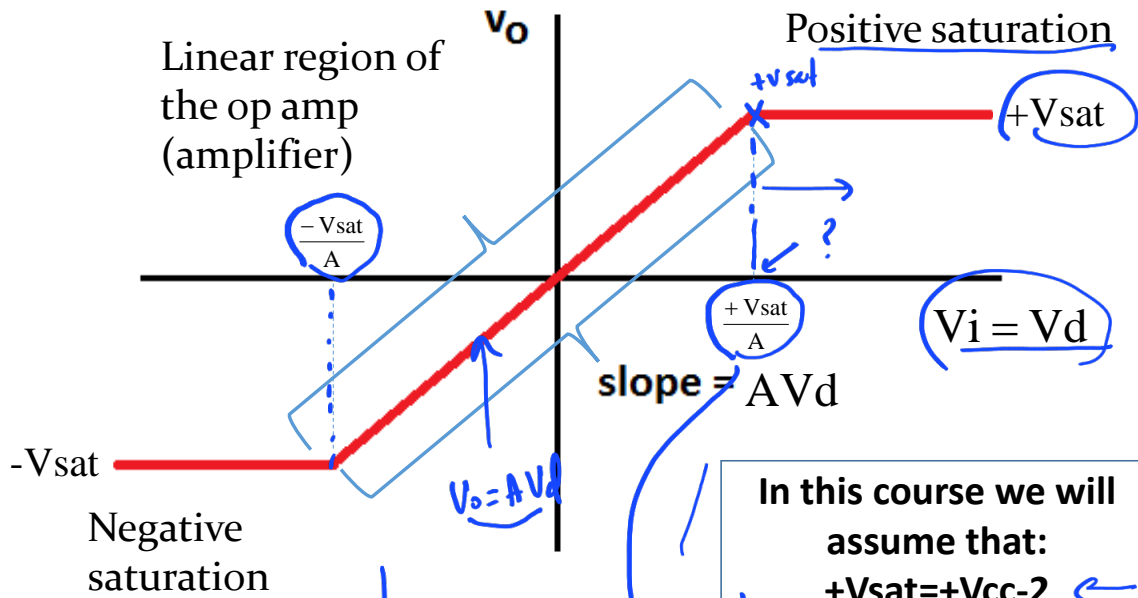
$$+V_{sat} \cong +V_{CC} - 2$$

$$-V_{sat} \cong -V_{CC} + 2$$

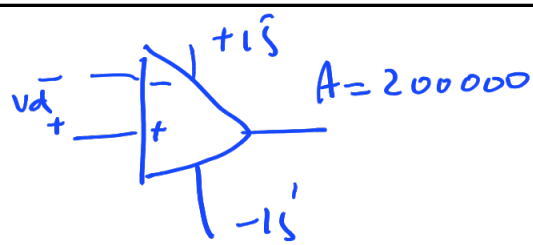
*to be used in this course*



### Voltage Transfer Characteristic



In this course we will assume that:  
 $+V_{sat} = +V_{cc} - 2$   
 $-V_{sat} = -V_{cc} + 2$   
 In reality value must be taken from data sheets



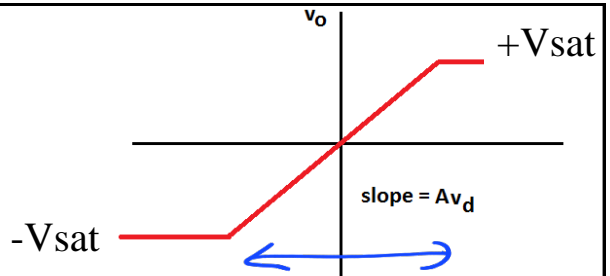
$\pm V_{sat} = \pm 13$

$\frac{+V_{sat}}{A} = \frac{13}{200000} = 65 \mu V$

$\frac{-V_{sat}}{A} = -65 \mu V$

# Output Voltage

- Real Op Amp



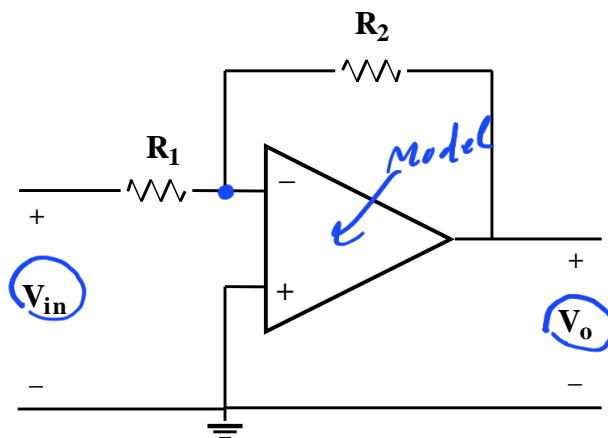
Mode of operation	Voltage Range	Output Voltage
Positive Saturation	$A V_d > +V_{sat}$	$v_o = +V_{sat} \approx +V_{cc} - 2$
Linear Region	$-V_{sat} < A V_d < +V_{sat}$	$v_o = A V_d$
Negative Saturation	$A V_d < -V_{sat}$	$v_o = -V_{sat} \approx -V_{cc} + 2$

The voltage produced by the dependent voltage source inside the op amp is limited by the voltage applied to the positive and negative rails and  $\pm V_{sat}$  level which is approximated by the formulas above

## Operational Amplifiers Analysis (Exact)

As an application of the previous model, consider the following configuration.

Find  $V_o$  as a function of  $V_{in}$  and the resistors  $R_1$  and  $R_2$ .



Op amp functional circuit.

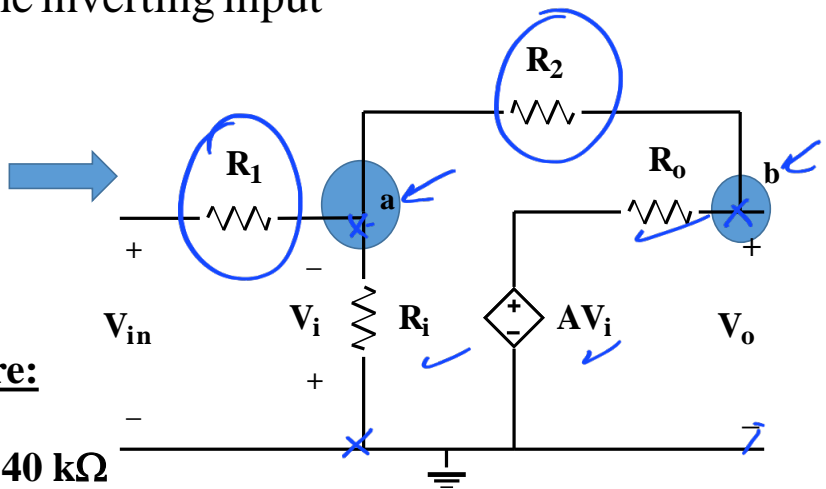
## Operational Amplifiers

$V_i = V_d = V_p - V_n = V_+ - V_-$  differential input voltage

$V_p = V_+$  voltage at the non - inverting input

$V_n = V_-$  voltage at the inverting input

### Equivalent circuit



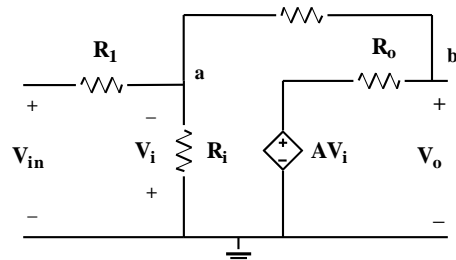
### Component values are:

- $R_1 = 10 \text{ k}\Omega$        $R_2 = 40 \text{ k}\Omega$   
 $R_o = 50 \text{ }\Omega$   
 $A = 100,000$        $R_i = 1 \text{ meg }\Omega$

## Operational Amplifiers

Exact solution

We can write the following equations for nodes a and b.



$$\begin{aligned}
 R_1 &= 10 \text{ k}\Omega & R_2 &= 40 \text{ k}\Omega \\
 R_o &= 50 \Omega \\
 A &= 100,000 & R_i &= 1 \text{ meg } \Omega
 \end{aligned}$$

KCL at A ✓

$$\frac{V_{in} + V_i}{R_1} = \frac{-V_i}{R_i} - \frac{V_i + V_o}{R_2}$$

(1)

KCL at B ✓

$$V_o = R_o \left[ \frac{-(V_i + V_o)}{R_2} \right] + AV_i \quad (2)$$

## Operational Amplifiers

Equation 1 simplifies to;

$$\frac{V_{in} + V_i}{10k} = \frac{-V_i}{1000k} - \frac{V_i + V_o}{40k}$$

$$-25V_o - 126V_i = 100V_{in} \quad (3) \leftarrow$$

Equation 2 simplifies to;

$$V_o = 50 \left[ \frac{-(V_i + V_o)}{40k} \right] + 100,000V_i$$

$$(4.005 \cdot 10^5)V_o - (4 \cdot 10^9)V_i = 0 \quad (4) \leftarrow$$

## Operational Amplifiers

From Equations (3 ) and (4) we find;

$$V_o = -3.99V_{in} \quad (5)$$


**This is an expected answer.**

**Fortunately, we are not required to do elaborate circuit analysis, as above, to find the relationship between the output and input of an op amp. Simplifying the analysis is our next consideration.**



## Operational Amplifiers Models

For most operational amplifiers,

$R_i$  is 1 Meg  $\Omega$  or larger and

$R_o$  is around 50  $\Omega$  or less.

The open-loop gain,  $A$ , is greater than 100,000.

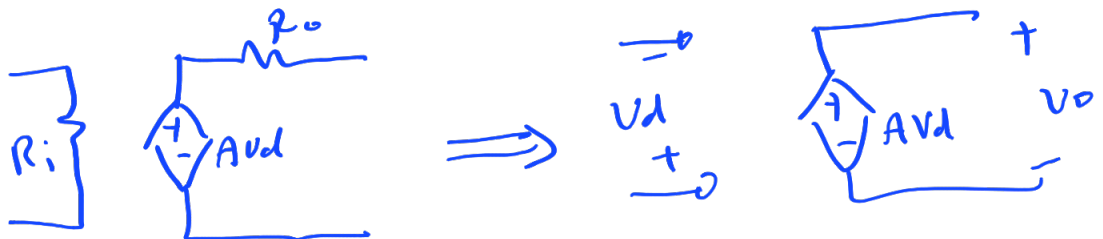
### Ideal Op Amp Model:

The following assumptions are made for the ideal op amp.

1. Infinite open-loop gain;  $\Rightarrow A \cong \infty$

2. Zero output ohms;  $\Rightarrow R_o = 0$

3. Infinite input ohms;  $\Rightarrow R_i = \infty$

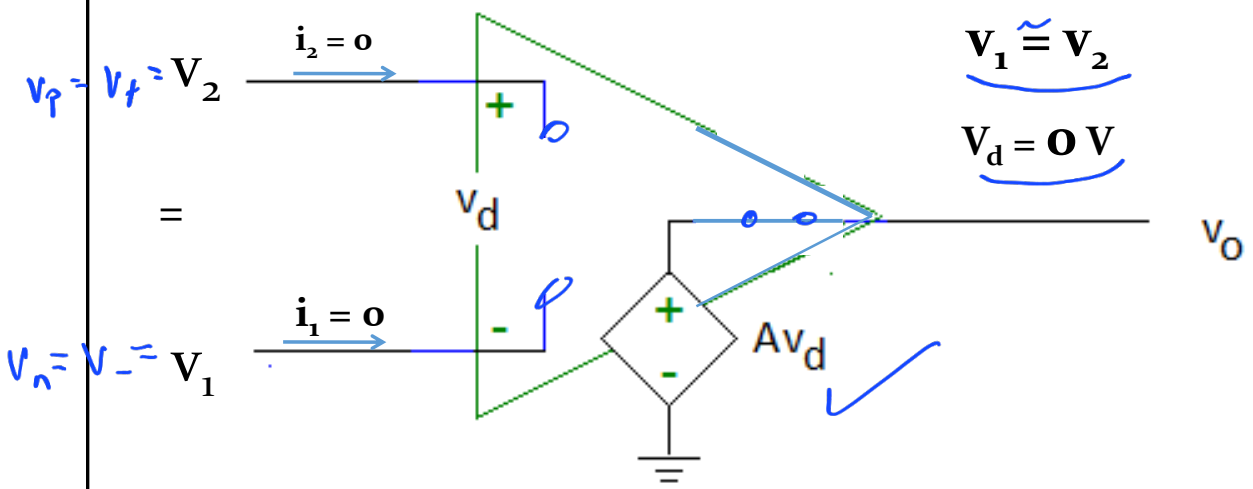


op-amp  
Ideal-Model

\*\* To be used in this  
course

# Ideal Op Amp Model

Because  $R_i$  is equal to  $\infty\Omega$ ,  
the voltage across  $R_i$  is **0 V**.

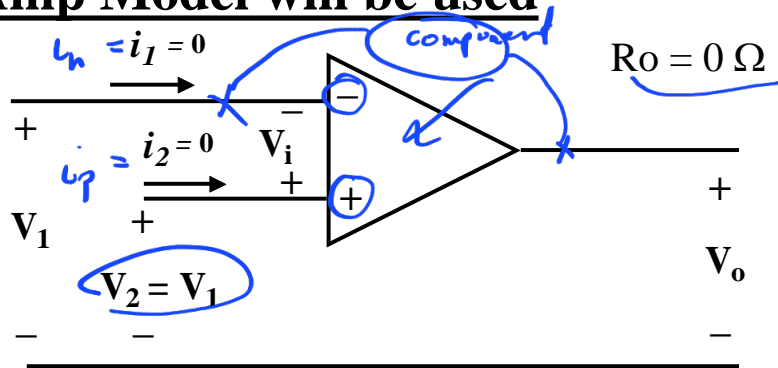


**Important Note:**

**Ideal Op Amp**

**Only Ideal Op Amp Model will be used from now on:**

$R_i = \infty \Omega$



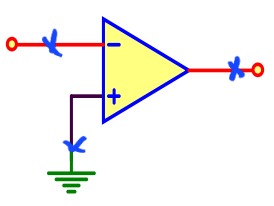
(a)  $i_1 = i_2 = 0$ : Due to infinite input resistance.

(b)  $V_i$  is negligibly small;  $V_1 \cong V_2$ .

The op amp forces the voltage at the inverting input terminal to be equal to the voltage at the noninverting input terminal if there is some component connecting the output terminal to the inverting input terminal.

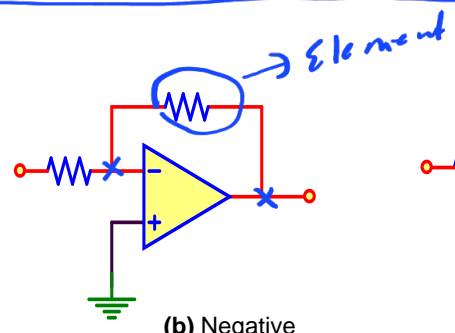
*applications*

# OP-AMP CONFIGURATIONS



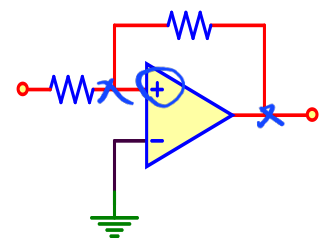
(a) No Feedback  
(open loop  
comparator circuit)

*$+V_{sat}$  or  $-V_{sat}$*



(b) Negative  
Feedback

*1)*



(c) Positive Feedback

- No feedback : Open loop (used in comparators)
- Negative feedback : Feedback to the inverting input (Used in amplifiers)
- Positive feedback : Feedback to the non inverting input (Used in oscillators) and Schmitt triggers ( comparators with hysteresis)

*\*\**

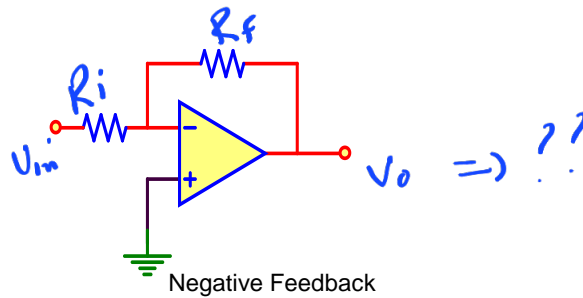
*✓*

## OP-AMPS WITH NEGATIVE FEEDBACK

The two basic amplifier circuits with negative feedback are:

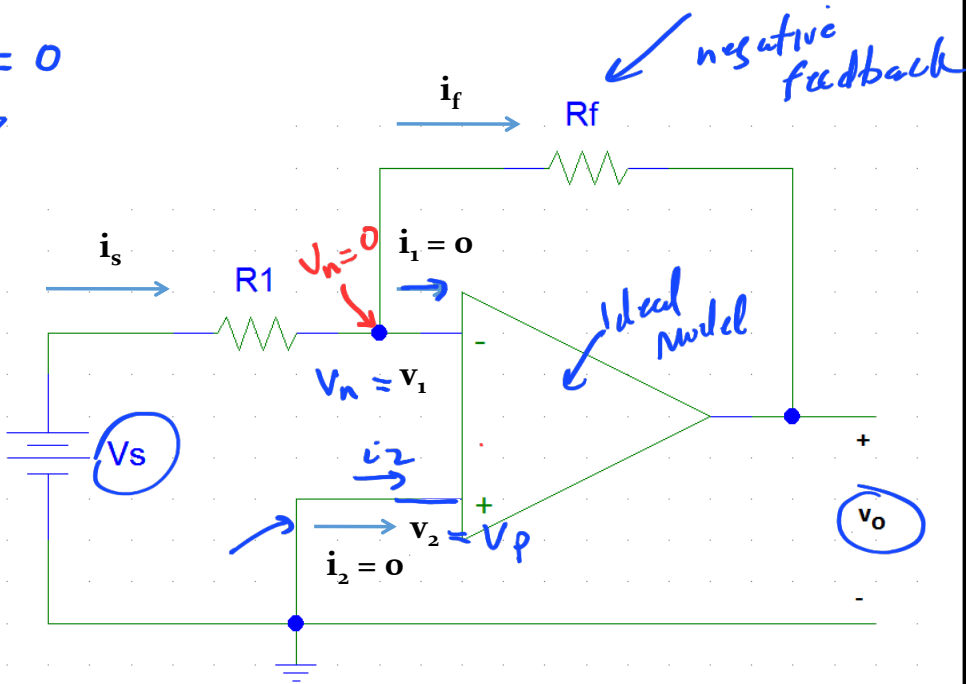
- The non-inverting Amplifier. ←
- The inverting Amplifier ←

(Note: Negative feedback is used to limit the gain)

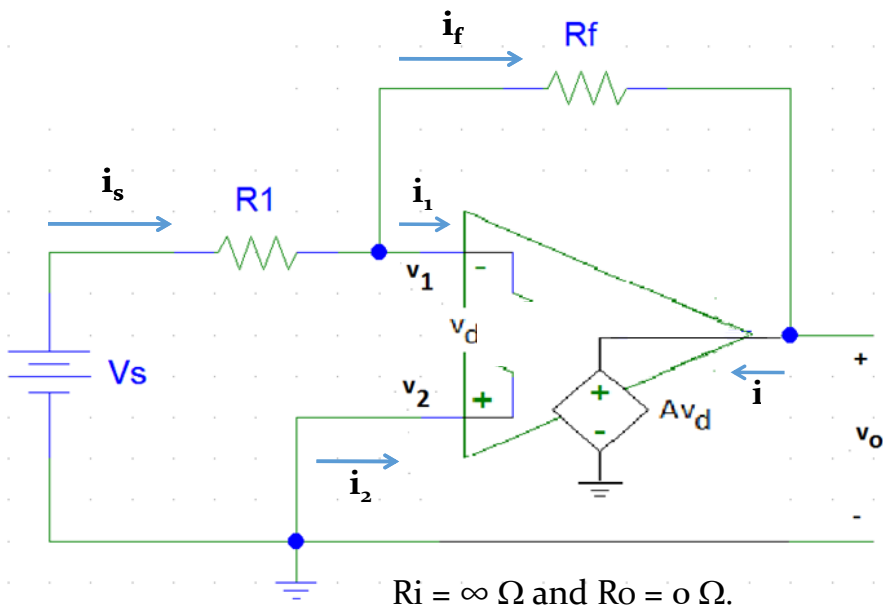


Example #2: Closed Loop Gain  $A_v = \frac{V_o}{V_s}$   
(inverting amplifier)

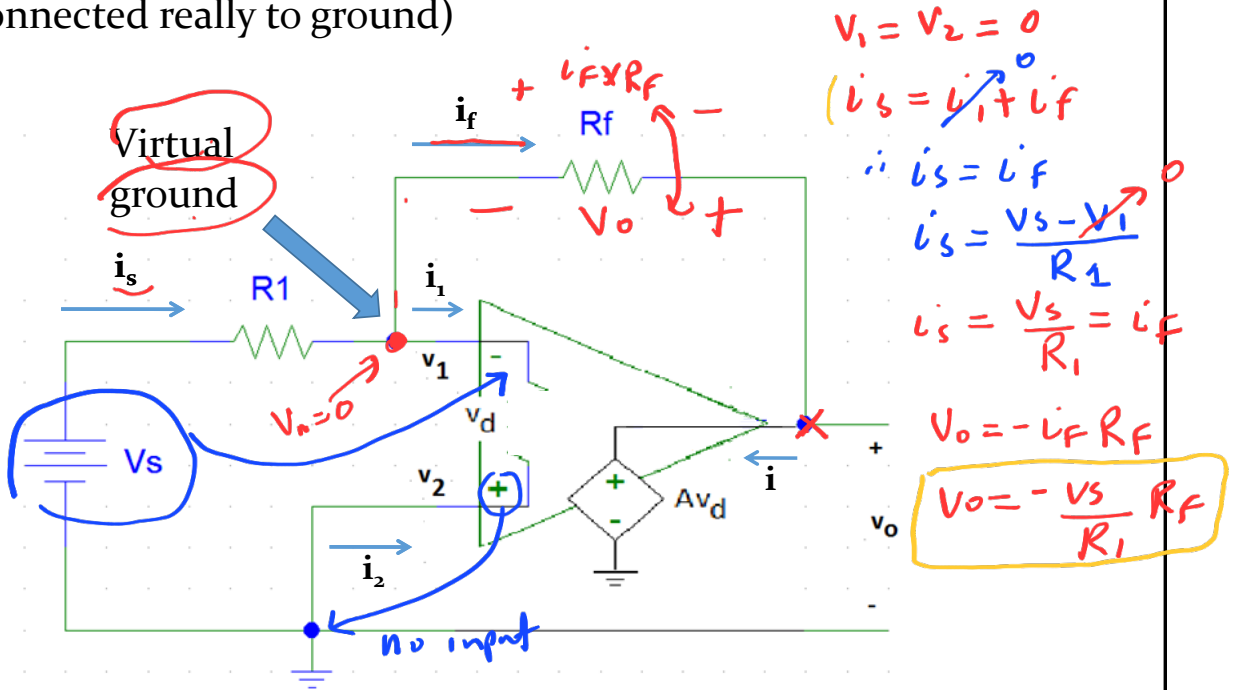
$\begin{cases} i_1 = i_2 = 0 \\ V_n = V_p \end{cases}$   
but  
 $V_p = 0$   
 $\therefore V_n = 0$



Example #2 (con't)



The op amp outputs a voltage  $V_o$  such that  $V_1 = V_2$ .  
 $V_2 = V_p = 0 \rightarrow V_1 = V_n = 0$   
 (virtual ground - potential equal 0 even though it is not connected really to ground)

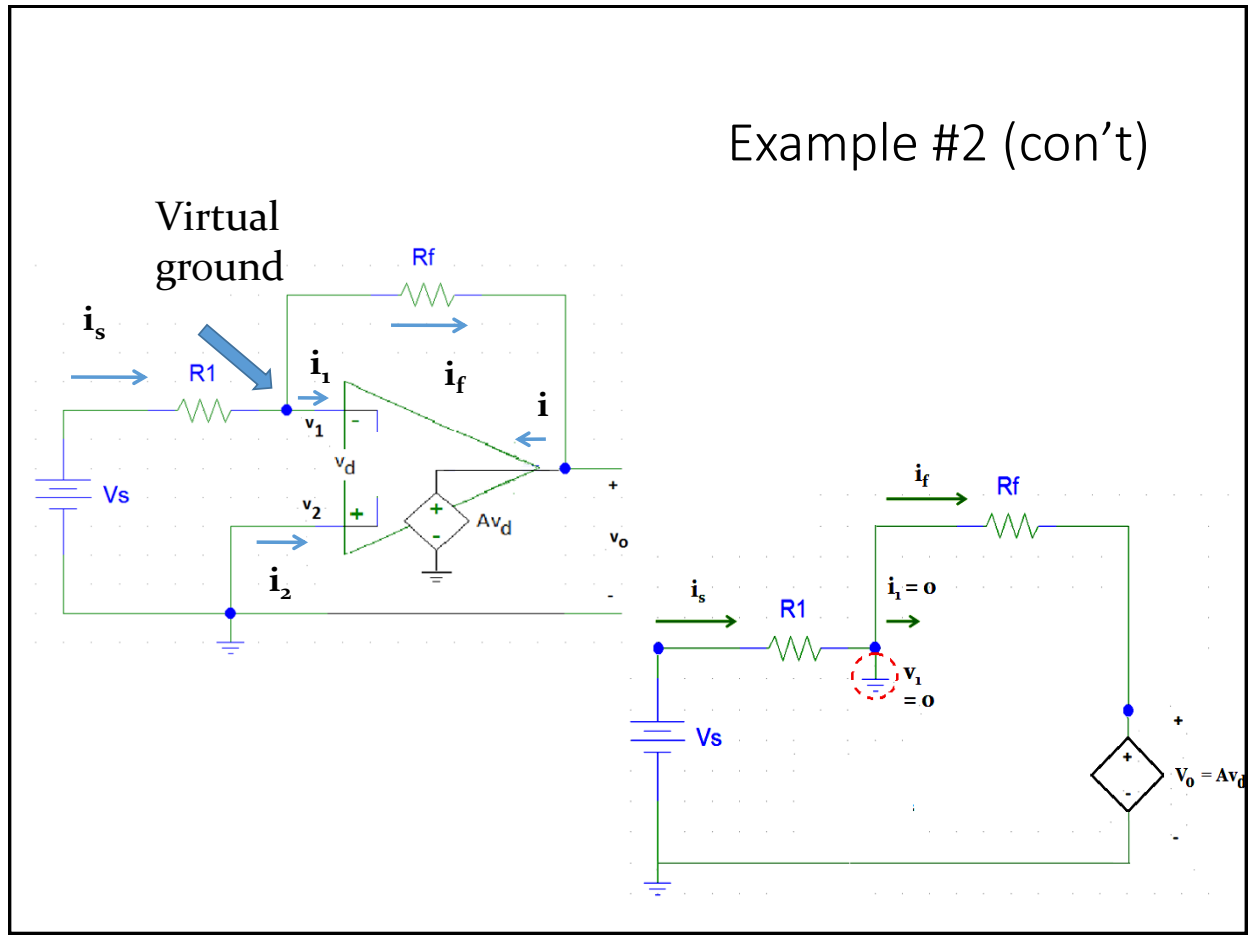


$$V_o = -\frac{R_f}{R_1} \cdot V_s \Rightarrow A_v = \frac{V_o}{V_s} = -\frac{R_f}{R_1}$$

↑  
Inverting Amplifier  
Gain



Example #2 (con't)



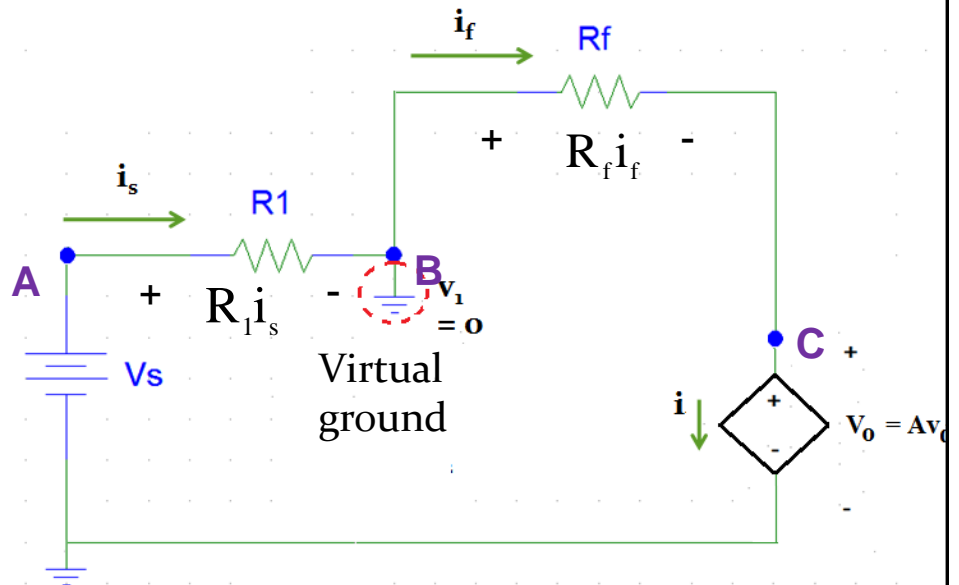
$$V_2 = 0V \Rightarrow V_1 = 0V$$

$$V_s = R_1 i_s$$

$$v_o = -R_f i_f$$

$$i_s = i_f = i$$

Example #2: Closed Loop Gain

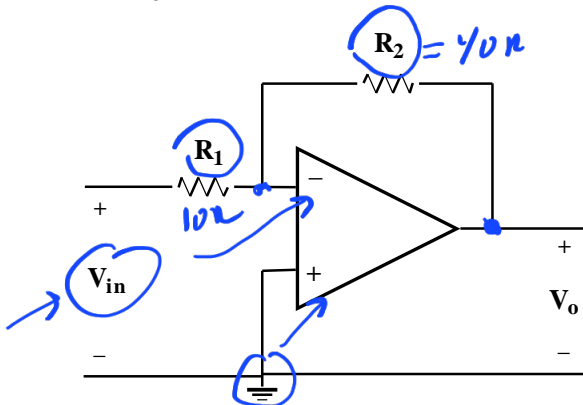


$$A_v = v_o / v_s = \frac{-R_f i_f}{R_1 i_s}$$

$A_v = -R_f / R_1$  → This circuit is known as an inverting amplifier.

## Inverting Amplifier (previous example of slide 30)

Find  $V_o$  in terms of  $V_{in}$  for the following configuration.



*inverting Amplifier*

$$V_o = -\frac{R_2}{R_1} V_{in}$$

With  $R_2 = 40 \text{ k}\Omega$  and  $R_1 = 10 \text{ k}\Omega$ , we have

$$V_o = -4V_{in}$$

Earlier we got

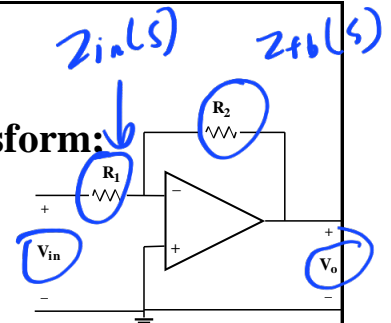
$$V_o = -3.99V_{in}$$

$$\frac{0.01}{4} \times 100\% = 0.25\% \text{ error}$$

## Inverting Op Amp:

When  $V_i = 0$  in and we apply the Laplace Transform:

$$\frac{V_0(s)}{V_{in}(s)} = \frac{-R_2}{R_1}$$



In fact, we can replace  $R_2$  with  $Z_{fb}(s)$  and  $R_1$  with  $Z_1(s)$  and we have the important expression;

$$\frac{V_0(s)}{V_{in}(s)} = \frac{-Z_{fb}(s)}{Z_{in}(s)}$$

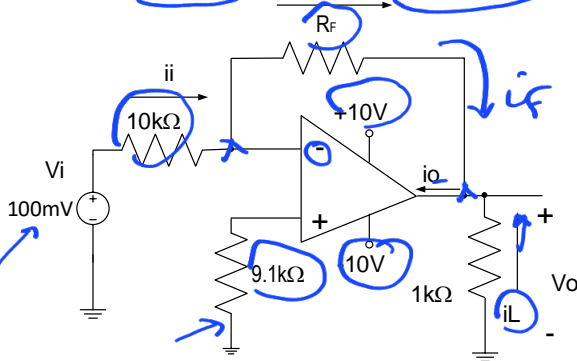
At this point in circuits we are not able to appreciate the use of this equation. We will revisit this at a later point in circuits but for now we point out that judicious selections of  $Z_{fb}(s)$  and  $Z_{in}(s)$  leads to important applications in

- Analog Compensators in Control Systems
- Analog Filters
- Application in Communications

**Example**

Find the value of  $V_o$  and  $I_o$  and verify if the opamp is in linear or saturation mode for two values of feedback resistor; assume  $I_o(\max) = 20 \text{ mA}$ :

- 1)  $R_F = 100 \text{ k}\Omega$  2) if  $R_F = 2 \text{ M}\Omega$



**Important**

$I_o(\max)$  is few mA for most opamps which limits the values of resistors to be used to kohm range

$$V_p = V(+)=0V$$

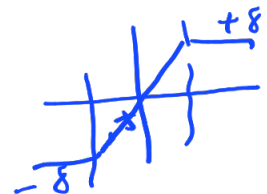
$$i_1 = \frac{V_1}{R_1}$$

$$i_i = i_f = \frac{V_i}{10 \text{ k}} = \frac{100 \text{ mV}}{10 \text{ k}\Omega} = 10 \mu\text{A}$$

$$1) V_o = -\frac{R_F}{10 \text{ k}\Omega} V_i = -10 V_i = -1V$$

$$V_o > -V_{\text{sat}}$$

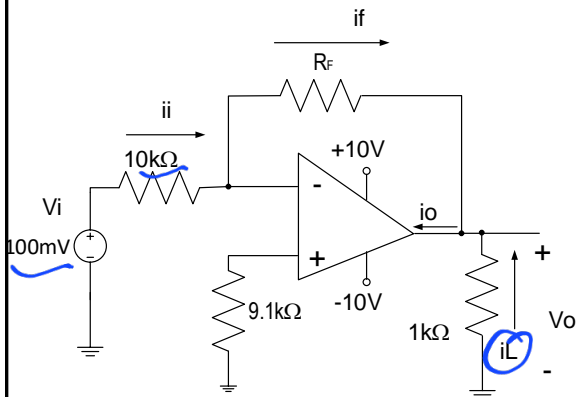
Linear mode



**Example**

**Find the value of  $V_o$  and  $I_o$  and verify if the opamp is in linear or saturation mode for two values of feedback resistor:**

- 1)  $R_F=100k\Omega$  2)  $R_F=2M\Omega$**



$$i_o = i_f + i_L$$

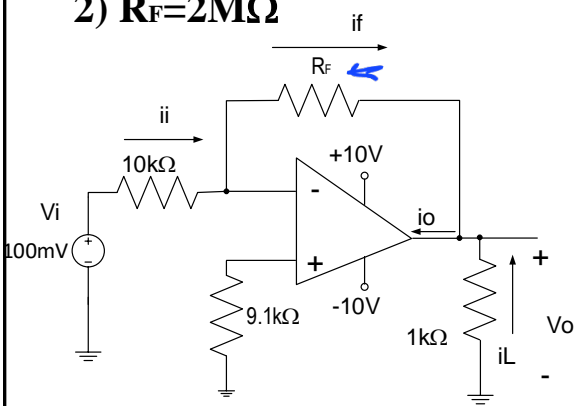
$$i_L = \frac{1V}{1k} = 1mA$$

$$i_o = 10\mu A + 1mA = 1.01mA < I_o(max)$$

20mA

**Example continued**

2)  $R_F = 2M\Omega$



$$i_i = i_f = \frac{V_i}{10k} = \frac{100mV}{10k} = 10 \mu A$$

$$2) V_o = -\frac{R_F}{10k} V_i = -200 V_i = -20V$$

$$V_o < -V_{sat}$$

$$-V_{sat} = -8V$$

Saturation mode  $\Rightarrow V_o$  is limited to  $-V_{sat}$

$$\therefore V_o = -V_{sat} = -8V$$

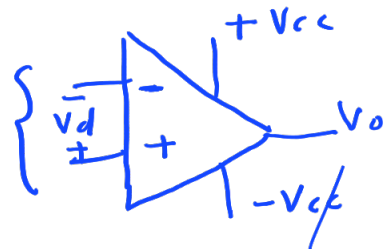
$$i_L = \frac{8V}{1k} = 8mA$$

$$i_o = 10 \mu A + 8mA = 8.01mA < I_o(max)$$

20mA

End of L22

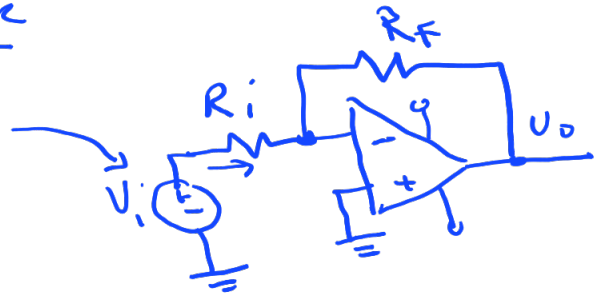
op-amp



Negative Feedback

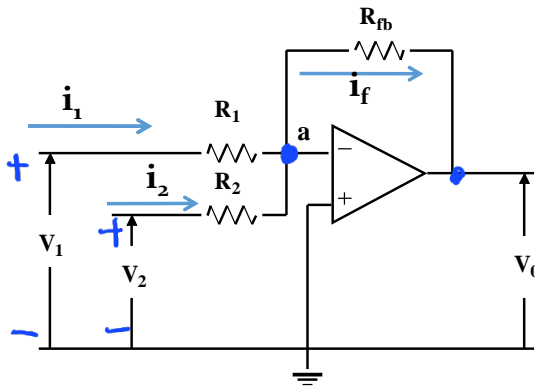
1. Inverting Amplifier

$$V_o = -\frac{R_f}{R_i} V_i$$



Inverting Adder or Summing Amplifier

Summing Amplifier: This is an application of inverting amplifier



$$V_p = V_{(+)} = 0V$$

$$i_1 = \frac{V_1}{R_1} \quad i_2 = \frac{V_2}{R_2}$$

$$V_o = -R_f i_f$$

$$i_f = i_1 + i_2$$

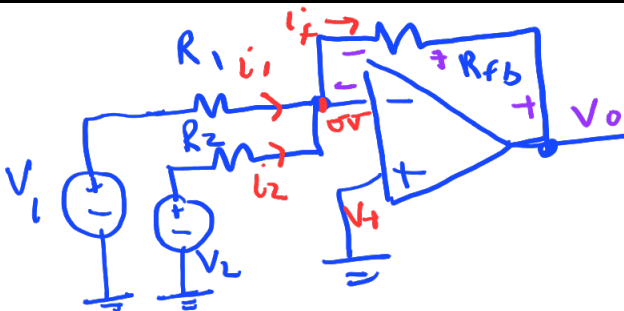
$$V_o = -\left[ \left( \frac{R_{fb}}{R_1} \right) V_1 + \left( \frac{R_{fb}}{R_2} \right) V_2 \right]$$

$$V_o = -R_f [i_1 + i_2]$$

If  $R_1 = R_2 = R_{fb}$  then,

$$V_o = -[V_1 + V_2]$$

Therefore, we can add signals with an op amp



$$V_+ = 0, V_- = V_+ = 0$$

$$i_1 = \frac{V_1}{R_1}; i_2 = \frac{V_2}{R_2};$$

$$i_f = i_1 + i_2$$

$$V_o = -i_f R_{fb} = -(i_1 + i_2) R_{fb}$$

$$V_o = -\left( \frac{V_1}{R_1} + \frac{V_2}{R_2} \right) R_{fb}$$

$$\text{if } R_1 = R_2 = R_{fb} \Rightarrow V_o = -(V_1 + V_2) *$$



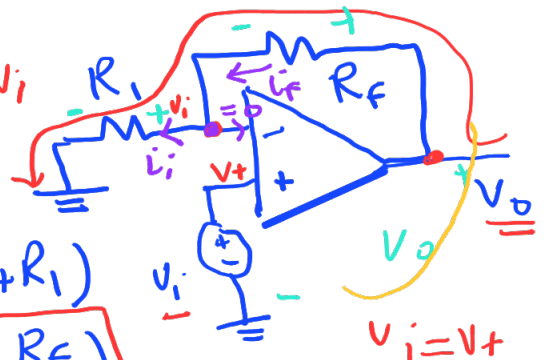
## 2) Non-Inverting

$V_+ = V_-$  ; but  $V_+ = V_i$  ;  $\therefore V_- = V_i$

$i_i = \frac{V_i}{R_i} = i_f$

$V_o = i_f R_f + i_i R_i = i_f (R_f + R_i)$

$V_o = \frac{V_i}{R_i} (R_f + R_i) = V_i \left( 1 + \frac{R_f}{R_i} \right)$



### The non-inverting op amp.

The non-inverting op amp has the input voltage connected to its (+) terminal while no voltage at the negative terminal

$V_n = V_1 = V_p = V_s$

$i_+ = i_- = 0$

$V_o = V_{R1} + V_{R2}$

$V_{R1} = V_1 = i_1 R_1$

$V_{R2} = i_2 R_2$

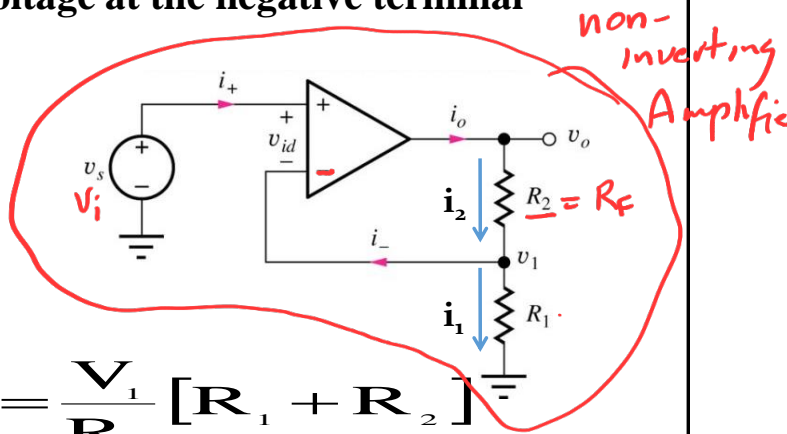
but

$i_1 = i_2 = \frac{V_1}{R_1}$

$V_o = \frac{V_1}{R_1} [R_1 + R_2]$

which gives,

$V_o = \left( 1 + \frac{R_2}{R_1} \right) V_s$



## Example: Non-inverting Amplifiers

Example: Find  $V_o$  for the following op amp configuration.

$$V_x = \frac{6k}{6k + 2k} 4V$$

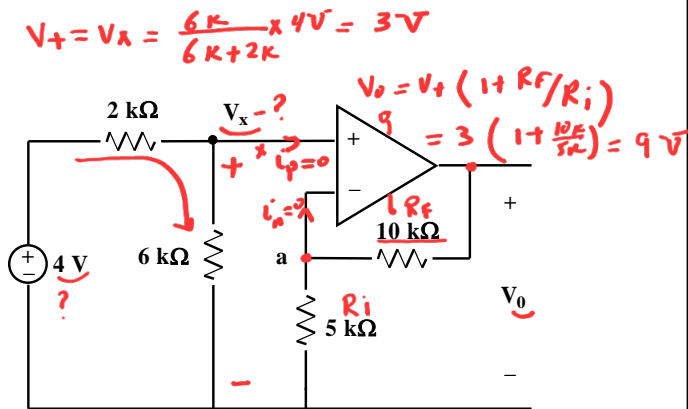
$$V_x = 3V$$

$$V_o = \left(1 + \frac{R_F}{R_i}\right) V_x$$

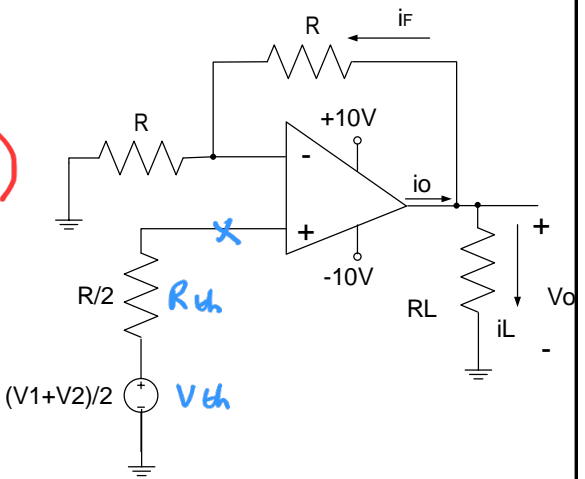
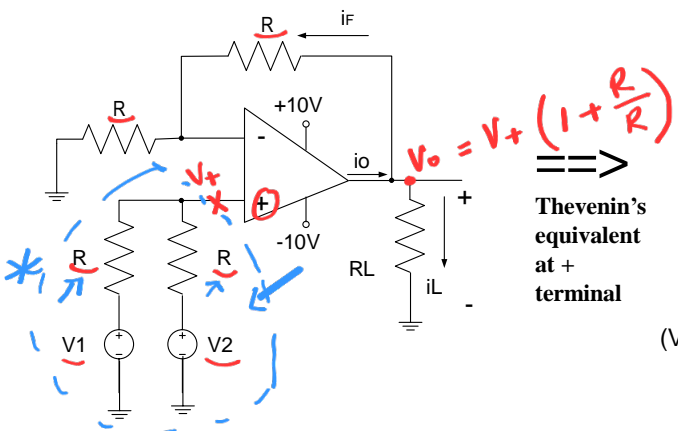
$$V_o = \left(1 + \frac{10k}{5k}\right) 3V$$

$$V_o = 9V$$

Make sure that:  $-V_{sat} < V_o < +V_{sat}$



# Non-Inverting Adder



$$V_{TH} = \left(\frac{R}{R+R}\right)V_1 + \left(\frac{R}{R+R}\right)V_2$$

$$= \left[\frac{V_1 + V_2}{2}\right]$$

$$V_o = \left(1 + \frac{R}{R}\right)V_+$$

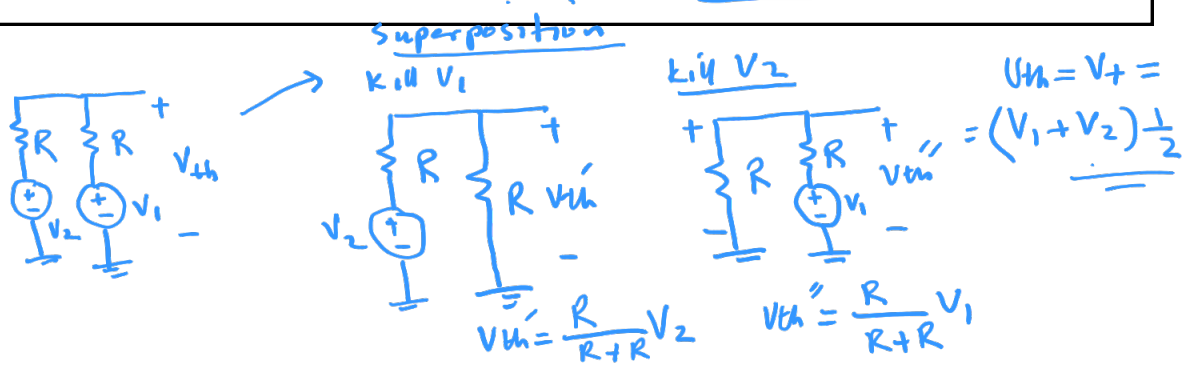
$$V_o = \left(1 + \frac{R}{R}\right) \left[\frac{V_1 + V_2}{2}\right]$$

$$V_+ = V_{TH} \quad \leftarrow \text{(since } i_+ = 0)$$

adder

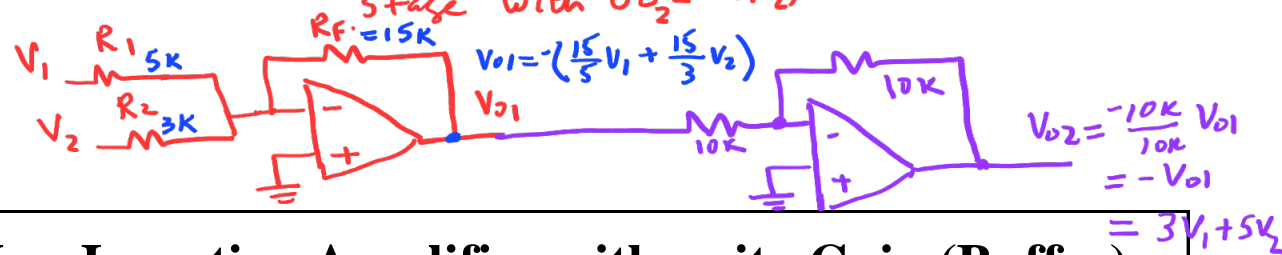
$$V_o = 2 \left[\frac{V_1 + V_2}{2}\right]$$

$$V_o = [V_1 + V_2]$$

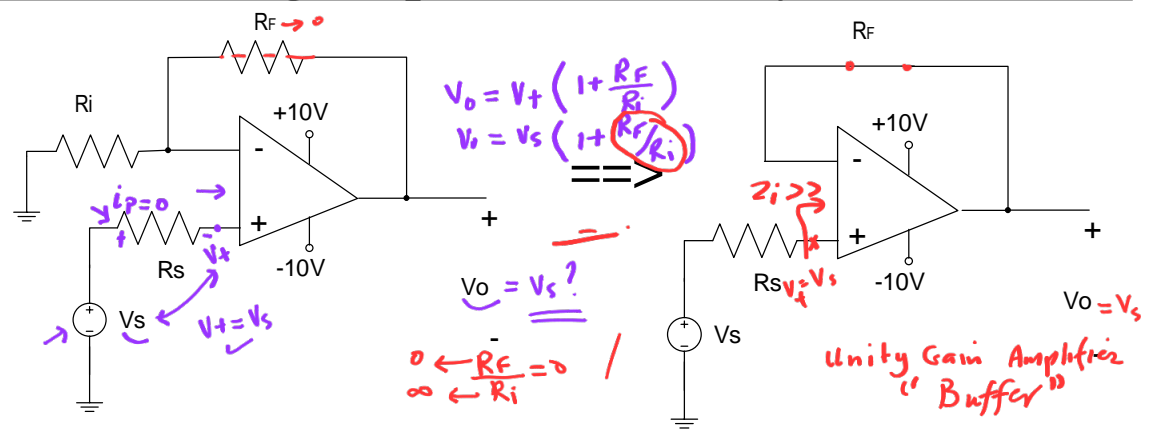


⇒ Design An Amplifier that have two input voltages  $V_1$  &  $V_2$  such that the output  $V_o = 3V_1 + 5V_2$  (use inverting Amplifiers)

Solution → with inverting Amplifier  $V_{o1} = -(3V_1 + 5V_2)$   
 → we can have a second stage with  $V_{o2} = -V_{in2}$ ,  $V_{o1} = V_{in2}$



**Non-Inverting Amplifier with unity Gain (Buffer)**



$$V_o = V(-)$$

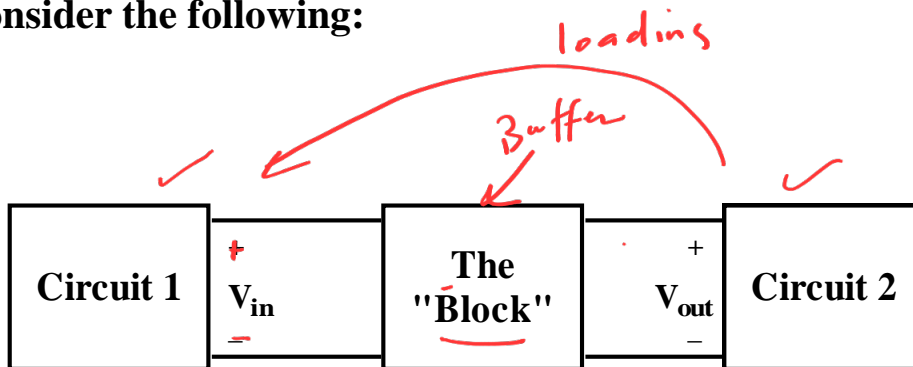
$$V(-) = V(+) = V_s$$

$$V_o = V_s$$

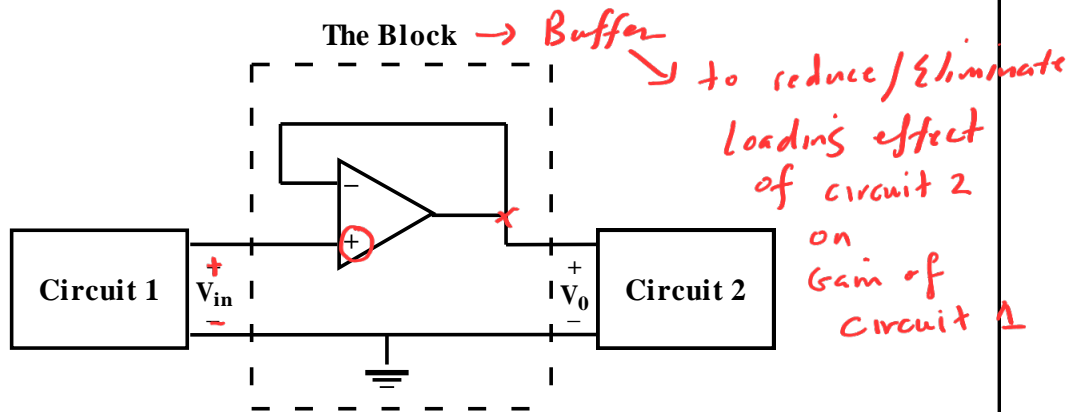
$$\frac{V_o}{V_s} = 1$$

## Buffer or Isolation Amplifier or Voltage Follower

- Applications arise in which we wish to connect one circuit to another without the first circuit loading the second.
- This requires that we connect to a “block” that has infinite input impedance and zero output impedance.
- An operational amplifier does a good job of approximating this.
- Consider the following:

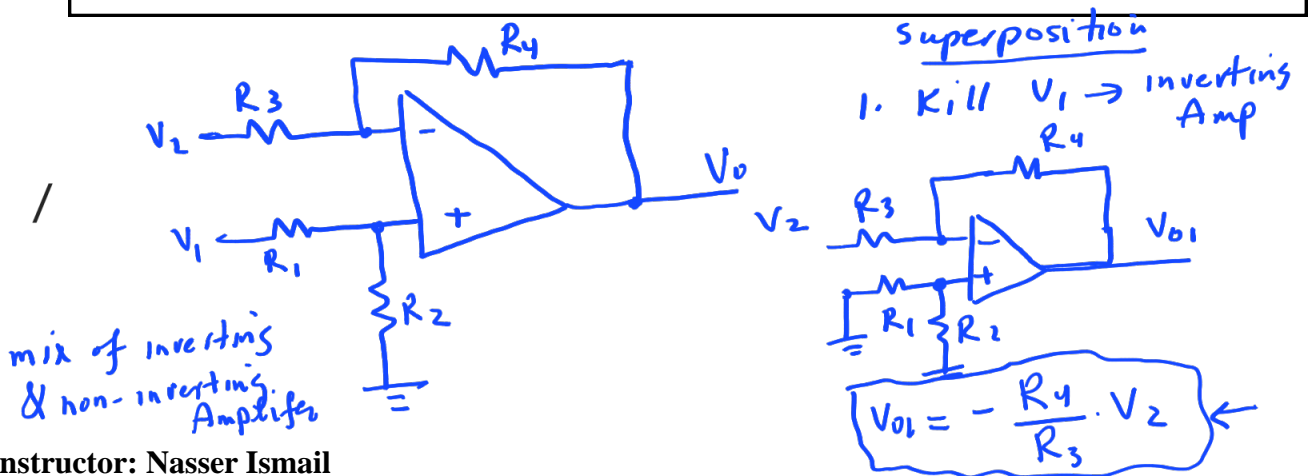


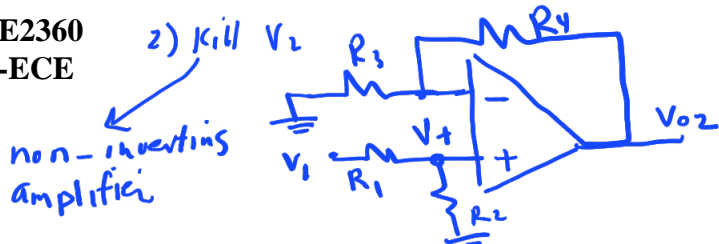
## Buffer or Isolation Amplifier or Voltage Follower



Circuit isolation with an op amp.

It is easy to see that:  $V_0 = V_{in}$





$$V_{o2} = \left(1 + \frac{R_4}{R_3}\right) \left(\frac{R_2}{R_1 + R_2}\right) V_1$$

$$V_o = V_{o1} + V_{o2}$$

$$V_o = \left(\frac{R_3 + R_4}{R_3}\right) \left(\frac{R_2}{R_1 + R_2}\right) V_1 - \frac{R_4}{R_3} V_2$$

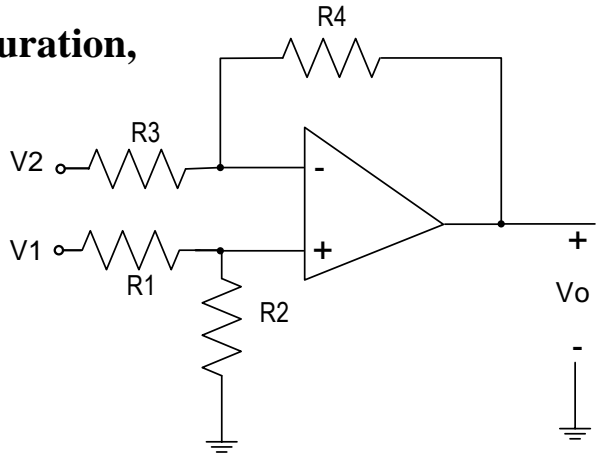
$$V_o = a V_1 - b V_2$$

$$a = \left(\frac{R_3 + R_4}{R_3}\right) \left(\frac{R_2}{R_1 + R_2}\right)$$

$$b = \frac{R_4}{R_3}$$

## Difference Amplifier (Subtractor)

For the following op amp configuration, in order to find  $V_o$  we can use Superposition:



- 1) Short  $V_2$  and find contribution of  $V_1$  to  $V_o$  ==> non-inverting amp

$$V_o = V_{o1}$$

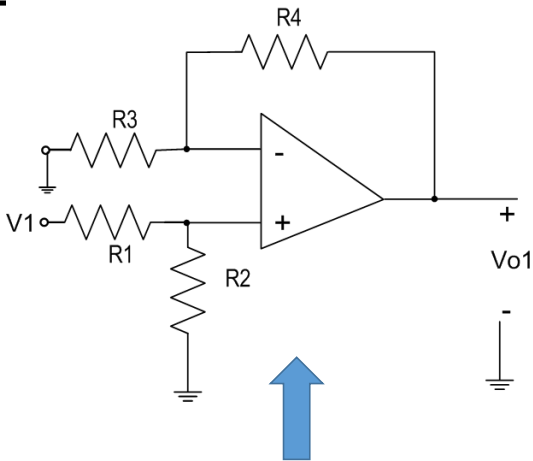
- 2) Short  $V_1$  and find contribution of  $V_2$  to  $V_o$  ==> inverting amp

$$V_o = V_{o2}$$

- 3) The total output is found by summing the two results above

$$V_o = V_{o1} + V_{o2}$$

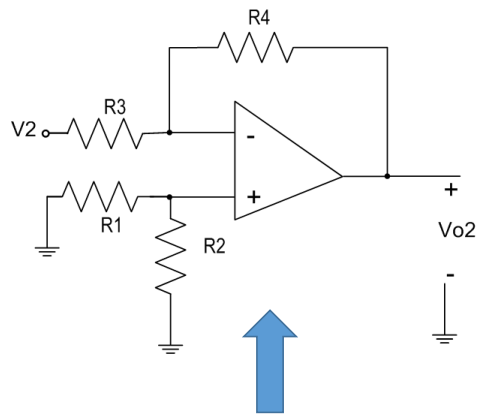
## Difference Amplifier (Subtractor)



**Non-Inverting Amplifier**

$$V_{o1} = \left(1 + \frac{R_4}{R_3}\right) V_+$$

$$V_{o1} = \left(1 + \frac{R_4}{R_3}\right) \left(\frac{R_2}{R_1 + R_2}\right) V_1$$

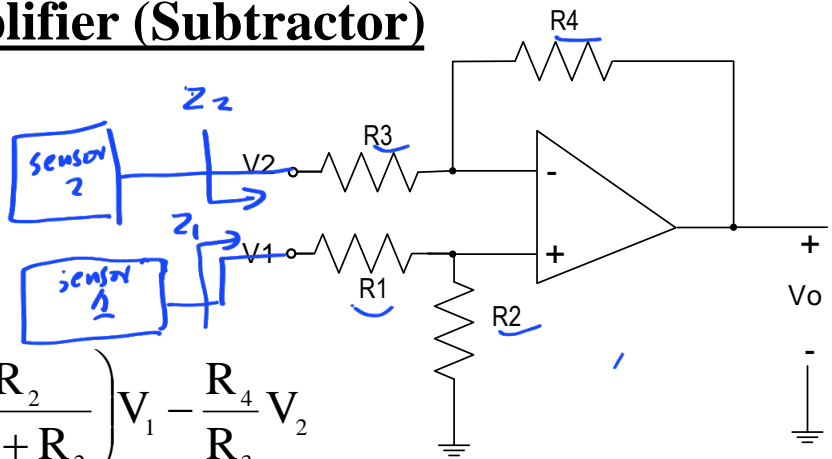


**Inverting Amplifier**

$$V_{o2} = -\frac{R_4}{R_3} V_2$$



## Difference Amplifier (Subtractor)



$$V_o = \left(1 + \frac{R_4}{R_3}\right) \left(\frac{R_2}{R_1 + R_2}\right) V_1 - \frac{R_4}{R_3} V_2$$

$$V_o = aV_1 - bV_2$$

$$a = \left(1 + \frac{R_4}{R_3}\right) \left(\frac{R_2}{R_1 + R_2}\right) = \frac{R_2}{R_3} \left(\frac{R_1 + R_2}{R_1 + R_2}\right) = \frac{R_2}{R_3} = \frac{mR}{mR} = \frac{R}{m}$$

$$b = \frac{R_4}{R_3} = \frac{mR}{R} = m$$

let  $R_1 = R_3 = R$   
 $R_2 = R_4 = mR$   
 $\therefore V_o = m(V_1 - V_2)$

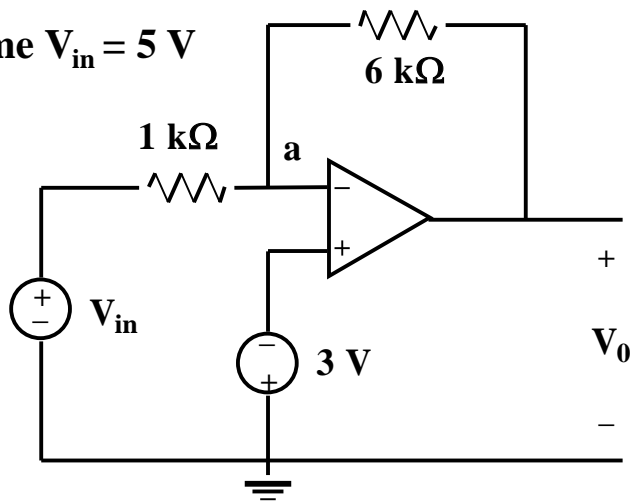
under this condition this is a difference amplifier \*\*

$V_1, V_2$  } signals from sensors which is low in value  
 $mV$ 's  $\Rightarrow$  Volts

$V_o = m(V_1 - V_2)$   
 amplification factor « gain »

**Example** Consider the op amp configuration below.

Assume  $V_{in} = 5\text{ V}$

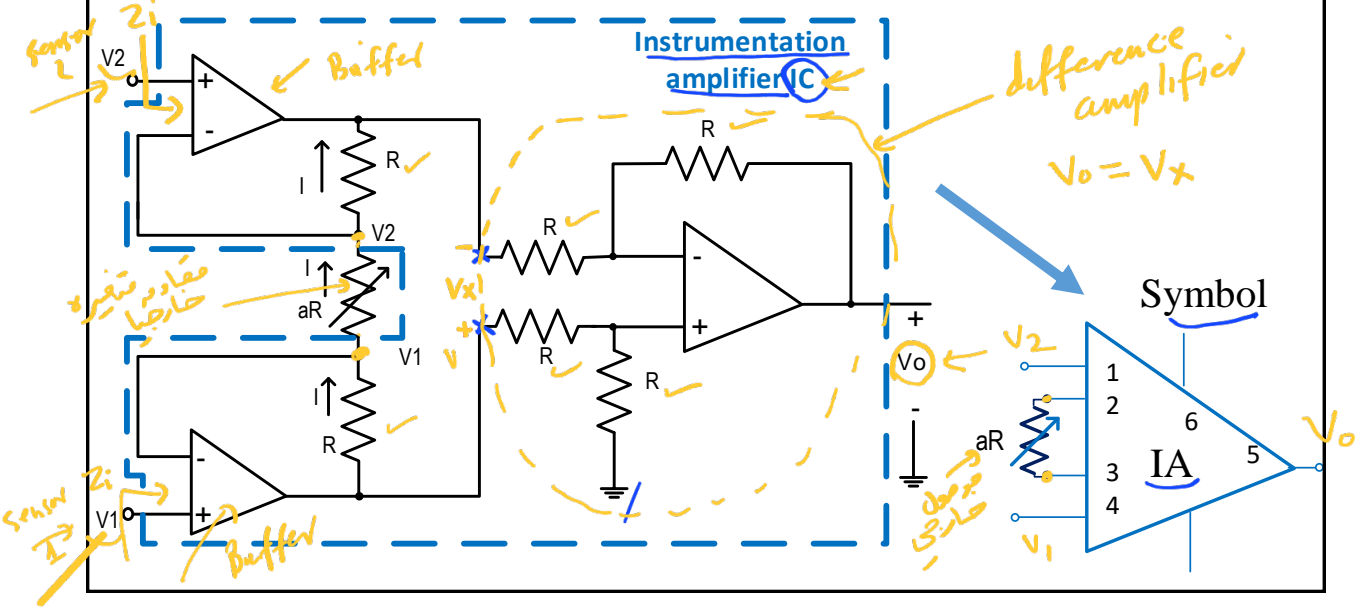


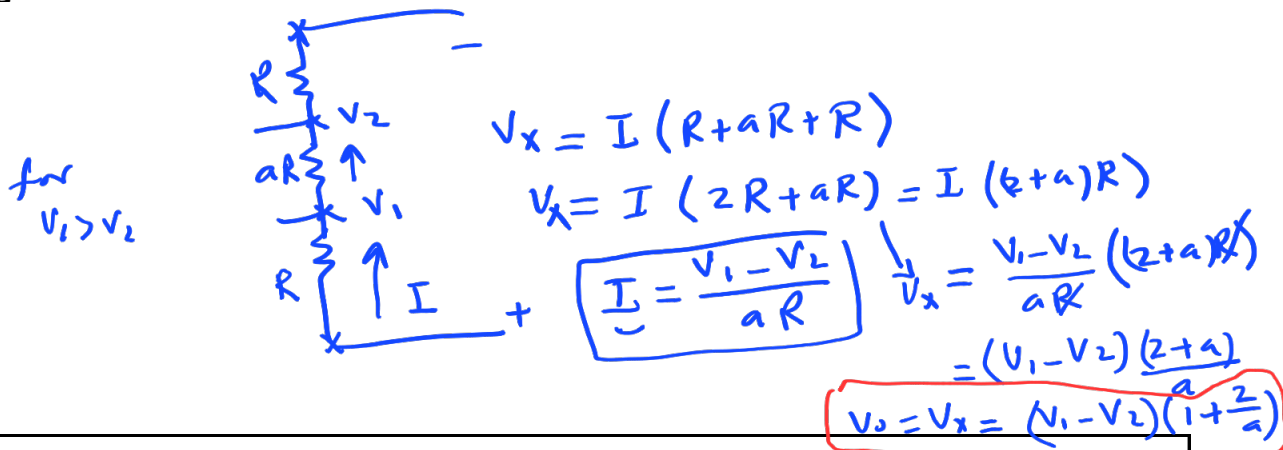
$$V_o = \left(1 + \frac{6k}{1k}\right)(-3) - \left(\frac{6k}{1k}\right)(5)$$
$$= -21 - 30 = -51\text{ V}$$

**Since  $V_o = -51\text{ V}$**  (op amp will saturate and  $V_o$  will be limited to  $-V_{sat}$ )

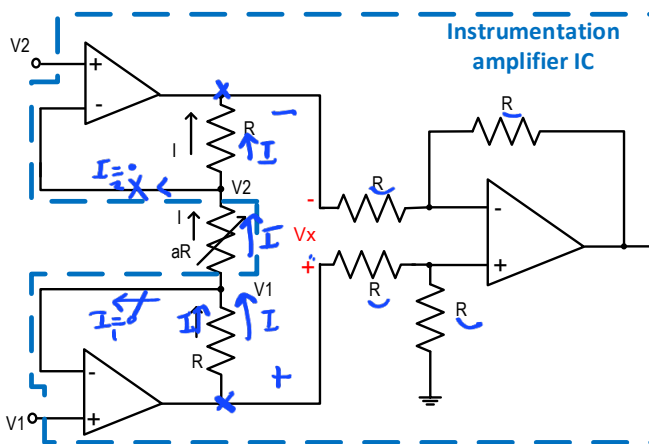
# Instrumentation Amplifier

- The previous difference amplifier has low input impedance and it is difficult to vary the gain “m”
- The instrumentation amplifier solves this problem by adding a buffer stage and a difference amplifier stage to solve the disadvantages of difference amplifier





### Instrumentation Amplifier



assume  $V_1 > V_2$

$$I = \frac{V_1 - V_2}{aR}$$

$$V_x = (R + R + aR)I$$

$$V_x = (R + R + aR) \frac{V_1 - V_2}{aR}$$

$$V_x = \left( \frac{(2 + a)R}{aR} \right) (V_1 - V_2)$$

$$V_x = \left( 1 + \frac{2}{a} \right) (V_1 - V_2)$$

$$V_o = mV_x$$

for  $m = 1$

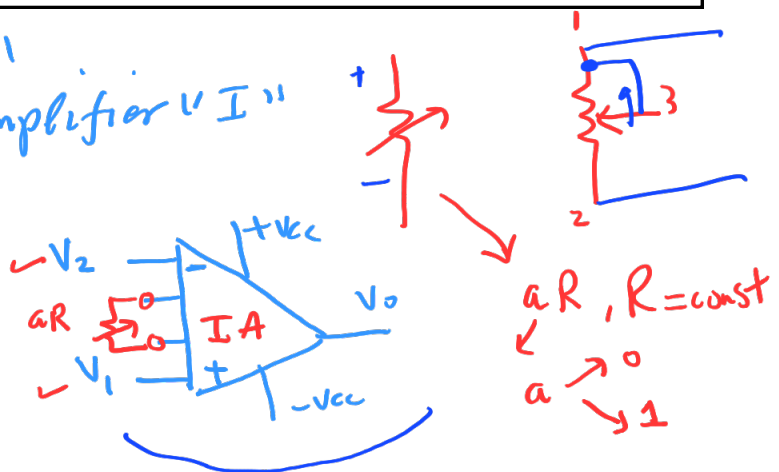
$$V_o = V_x = \left( 1 + \frac{2}{a} \right) (V_1 - V_2)$$

- Only by varying the value of potentiometer aR, the output can be adjusted
- R is an internal resistance given in data sheet of "IA" IC

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Instrumentation Amplifier "IA"

$$V_o = \left( 1 + \frac{2}{a} \right) (V_1 - V_2)$$



Wheatstone Bridge

**Example: Instrumentation Amplifier (self study)**

$$V_o = V_x = \left(1 + \frac{2}{a}\right)(E_1 - E_2)$$

$$E_1 = \frac{R_1}{R_1 + R_1} E = \frac{1}{2} E$$

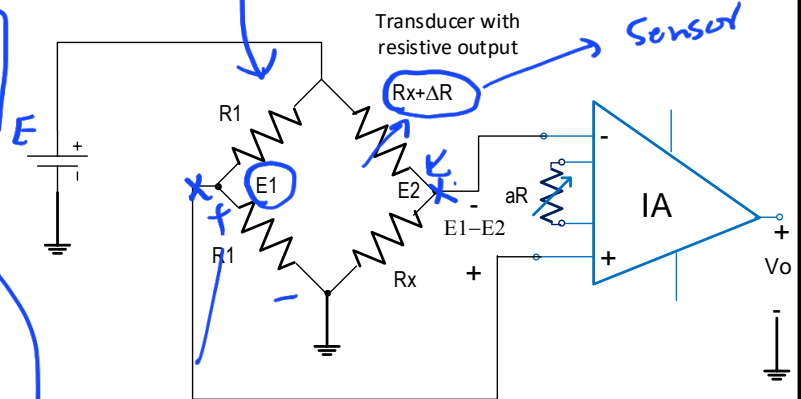
$$E_2 = \frac{R_x}{R_x + R_x + \Delta R_x} E$$

$$E_1 - E_2 = \frac{1}{2} E - \frac{R_x}{2R_x + \Delta R_x} E$$

$$= \frac{2R_x + \Delta R_x}{2(2R_x + \Delta R_x)} E - \frac{2R_x}{2(2R_x + \Delta R_x)} E$$

$$E_1 - E_2 = \frac{E}{2} \left( \frac{2R_x + \Delta R_x - 2R_x}{2R_x + \Delta R_x} \right)$$

$$= \frac{E}{2} \left( \frac{\Delta R_x}{2R_x + \Delta R_x} \right)$$



## Instrumentation Amplifier

assume  $\Delta R_x \ll R_x$

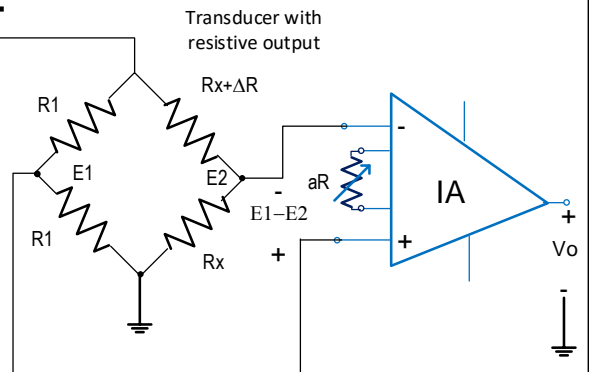
$$V_o = E_1 - E_2 = \left(1 + \frac{2}{a}\right) (E_1 - E_2)$$

$$V_o \propto \Delta R_x = \left(1 + \frac{2}{a}\right) \frac{E}{4R_x} (\Delta R_x)$$

let  $R_1 = 10k$ ;  $E = 10V$ ;  $\Delta R_x = 10$ ;

$aR = 50.12 \text{ ohm}$

(potentiometer is set to 50.12 ohms)



$$a = \left(\frac{aR}{R}\right) = \left(\frac{50.12}{10000}\right) = 0.005013$$

$$\left(1 + \frac{2}{a}\right) = 400$$

$$V_o = (400) \left(\frac{10}{4}\right) * \frac{10}{10000} = 1V$$

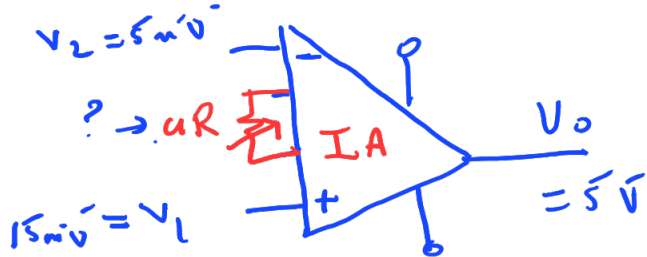
Example : Use an instrumentation amplifier in order to get an output  $V_o = 5V$  if  $V_1 = 15mV$  (sensor 1),  $V_2 = 5mV$  (sensor 2) & given that internal IA resistance =  $20k\Omega$

$$V_o = \left(1 + \frac{2}{a}\right)(V_1 - V_2)$$

$$5 = \left(1 + \frac{2}{a}\right)(15m - 5m)$$

$$5 = \left(1 + \frac{2}{a}\right)(10mV)$$

$$1 + \frac{2}{a} = \frac{5V}{10mV} = \frac{5000mV}{10mV} = 500$$



$$1 + \frac{2}{a} = 500$$

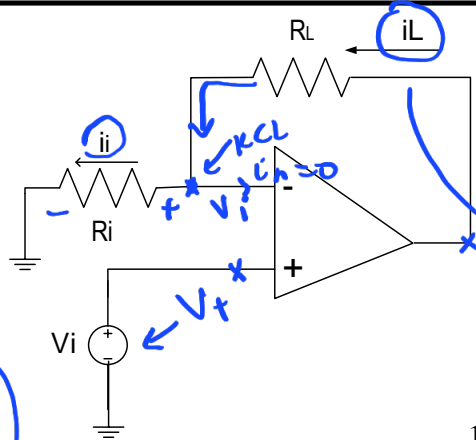
$$\frac{2}{a} = 499$$

$$a = \frac{2}{499}$$

$$aR = \frac{2 \times 20k}{499}$$

$$= 80.16 \Omega$$

## Voltage to Current converter



$$V_+ = V_-$$

$$i_p = i_n = 0$$

replace  $R_L$  by  
current measuring  
Device

$$i_i = \frac{V_i}{R_i}$$

$$i_i = i_L$$

$$\text{let } V_i = 1V; R_i = 1k$$

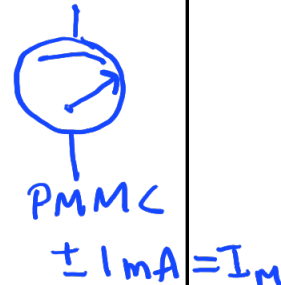
$$i_L = \frac{1V}{1k} = +1mA$$

$$i_L = \frac{1V}{1k} = +1mA$$

$$\text{let } V_i = -1V; R_i = 1k$$

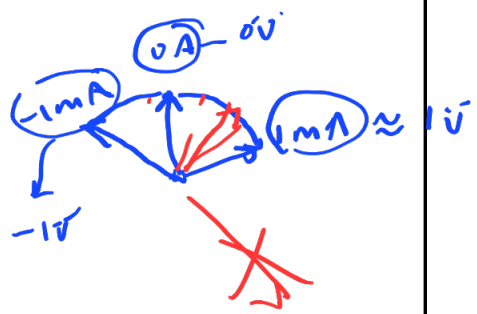
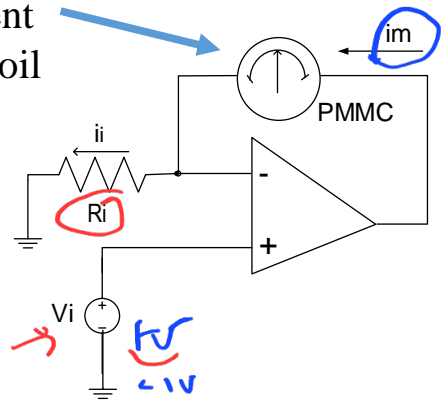
$$i_L = \frac{-1V}{1k} = -1mA$$

Here we converted  $\pm 1V$  to  $\pm 1mA$



## Voltage to Current converter

PMMC: Permanent magnet moving coil



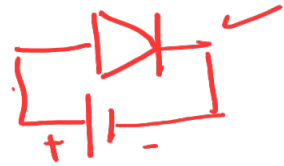
$$i_i = \frac{V_i}{R_i}$$

$$i_i = i_m$$

let  $V_i = \pm 1V$ ;  $R_i = 1k$

$$i_m = \frac{1V}{1k} = \pm 1mA$$





## Current to Voltage converter

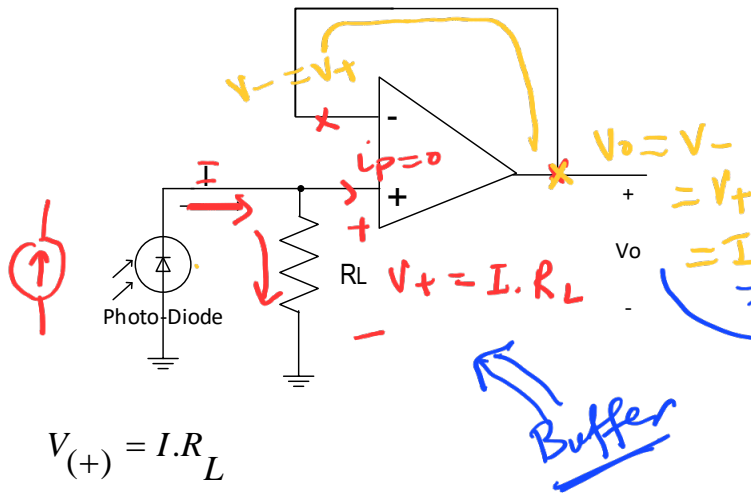


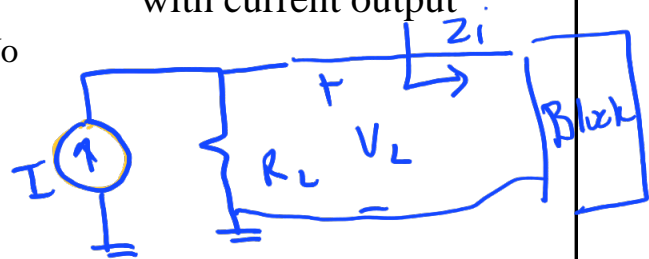
Photo-diode  
Is a diode which is biased by certain type of light

$$V_{(+)} = I \cdot R_L$$

$$V_o = V_{(-)} = V_{(+)} = I \cdot R_L$$

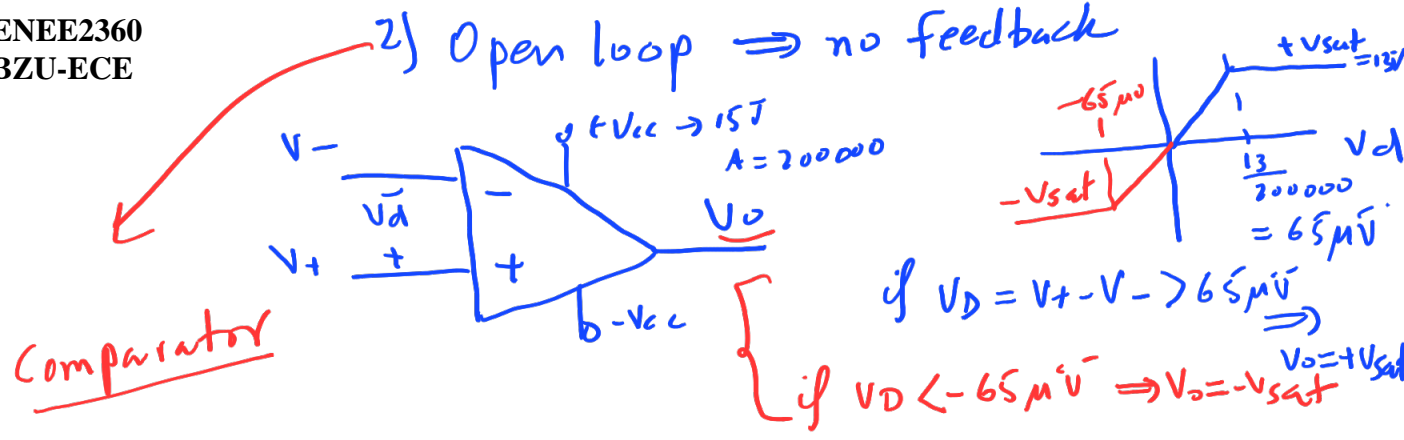
Here we converted current I to voltage  $V_o$

I - can be any current source, sensor or device with current output

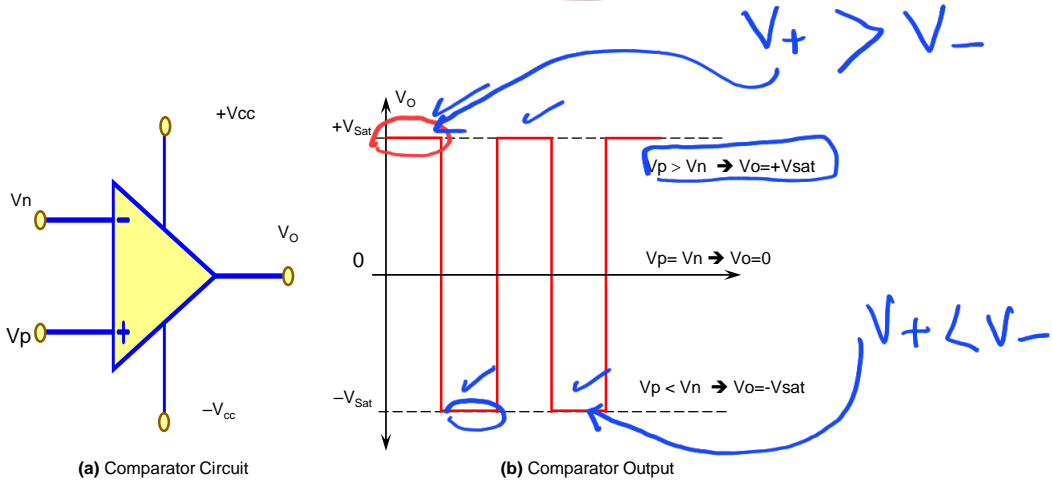


So far ; circuits with Negative feedback

- inverting
  - non-inverting
  - Difference
  - Instrumentation
  - Voltage to current
  - current to voltage
- } Amplifiers
- } converters



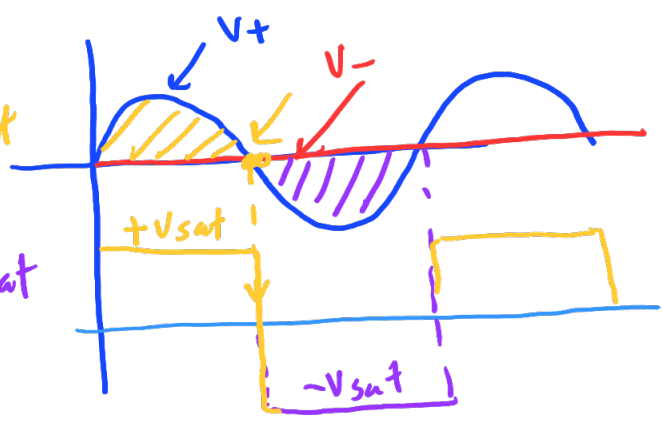
**OP AMP as a Comparator (compares 2 voltages and produces a signal to indicate which is greater)**



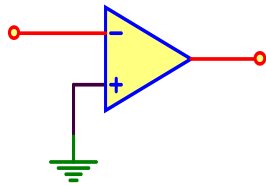
Applications of Comparators

- Analog to digital converters (ADC)
- Counters (e.g. count pulses that exceed a certain voltage level).
- Cross Over Detectors

Comparator  
 $V_+ > V_- \Rightarrow V_o = +V_{sat}$   
 $V_+ < V_- \Rightarrow V_o = -V_{sat}$



## Comparator : Zero -Level detector



Exact analysis:

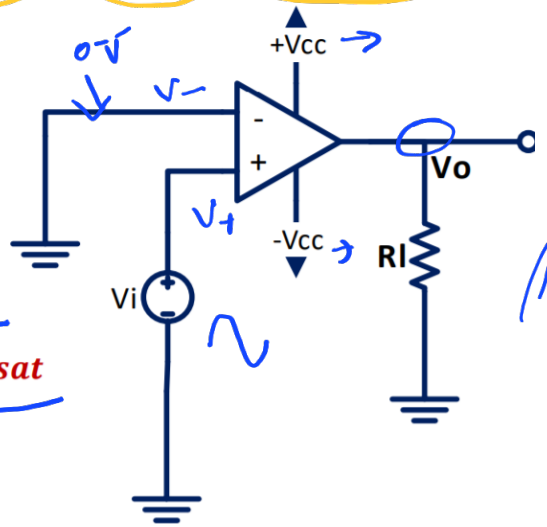
When  $v_d > 65\mu V$  ;  $V_o = +V_{sat}$

When  $v_d < -65\mu V$  ;  $V_o = -V_{sat}$

Approximate analysis

When  $v_d > 0V$  ;  $V_o = +V_{sat}$

When  $v_d < 0V$  ;  $V_o = -V_{sat}$

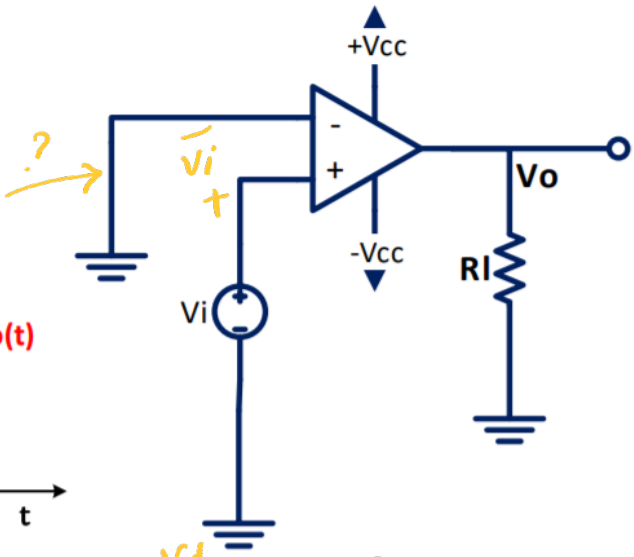
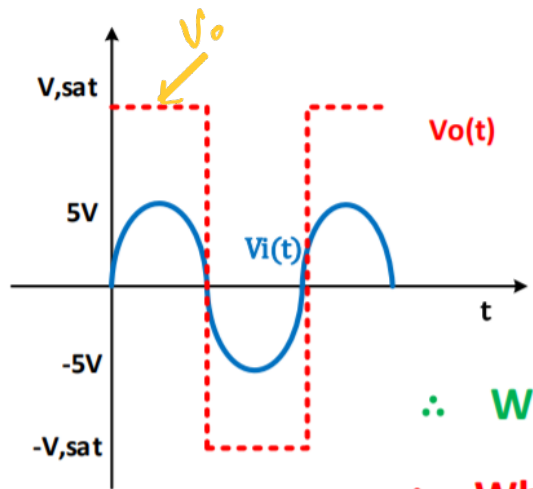


$$v_d = v_+ - v_- \Rightarrow v_+ > v_-$$

$$\Rightarrow v_+ < v_-$$

### Comparator : Zero -Level detector

$V_i(t) = 5 \sin \omega t \text{ v}$   
 $\pm V_{sat} = \pm 13 \text{ v}$



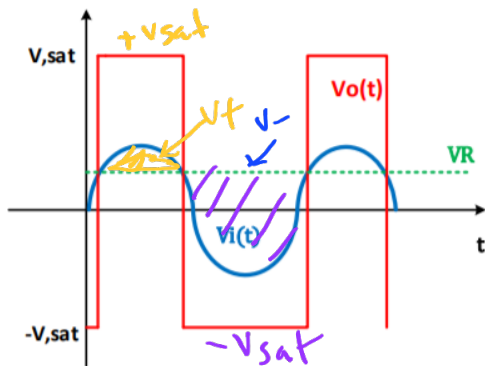
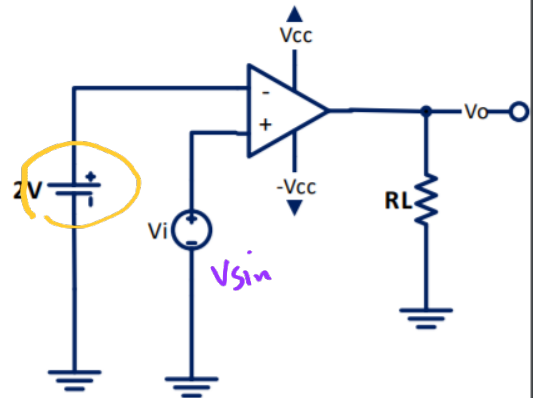
$\therefore$  When  $V_i > 0V$  ;  $V_o = +V_{sat}$   
 $\therefore$  When  $V_i < 0V$  ;  $V_o = -V_{sat}$

### Non Zero -Level detector

$V_i(t) = 5 \sin \omega t \text{ v}$   
 $\pm V_{sat} = \pm 13 \text{ v}$

When  $V_i > 2\text{V}$  ;  $V_O = +V_{sat}$

When  $V_i < 2\text{V}$  ;  $V_O = -V_{sat}$



$V_{sin} - 2 > 0 \rightarrow V_{sin} > 2$

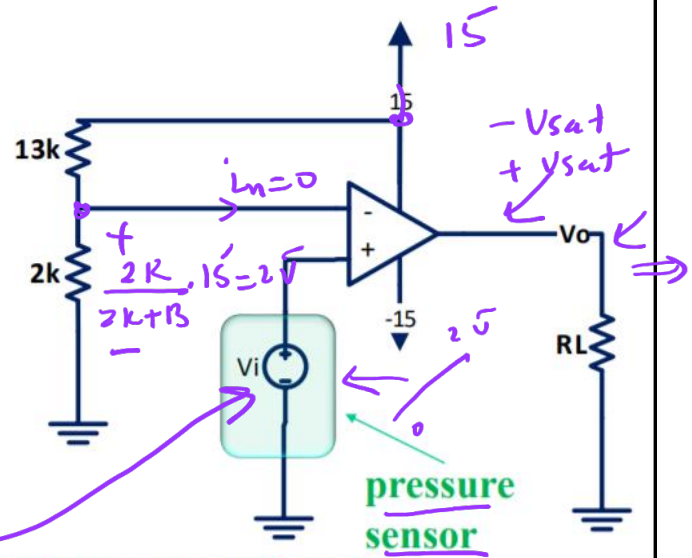
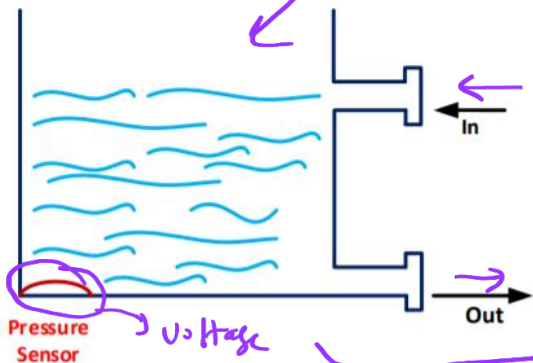
When  $v_d > 0\text{V}$  ;  $V_O = +V_{sat}$

When  $v_d < 0\text{V}$  ;  $V_O = -V_{sat}$

$V_{sin} - 2 < 0 \rightarrow V_{sin} < 2$

### Practical Non Zero –Level detector

Application



The pressure sensor generates a voltage proportional to the water level in the tank

When water level reaches the maximum allowable level

$V_i = 2V$

$V(-) = 2V$

When  $V_i > 2V$  ;  $V_O = +V_{sat}$

When  $V_i < 2V$  ;  $V_O = -V_{sat}$

### Voltage-Level detector with LEDs:

When  $V_i > 2V$  ;  $V_o = +V_{sat}$

∴ Red LED is ON

∴ green LED is OFF

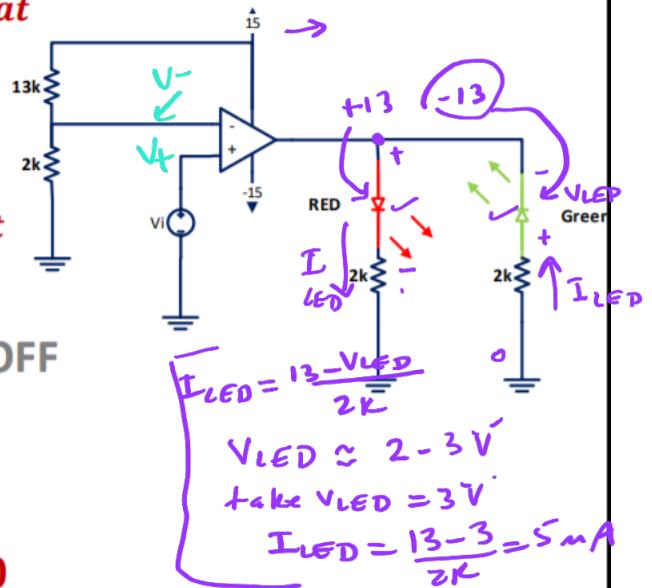
When  $V_i < 2V$  ;  $V_o = -V_{sat}$

∴ green LED is ON

∴ Red LED is OFF

When  $V_i = 2V$  ;  $V_o = 0$

∴ green LED and the Red LED are OFF



## Example

- Given how an op amp functions, what do you expect  $V_o$  to be if  $v_2 = 5V$  when:

1.  $V_s = 0V$ ?

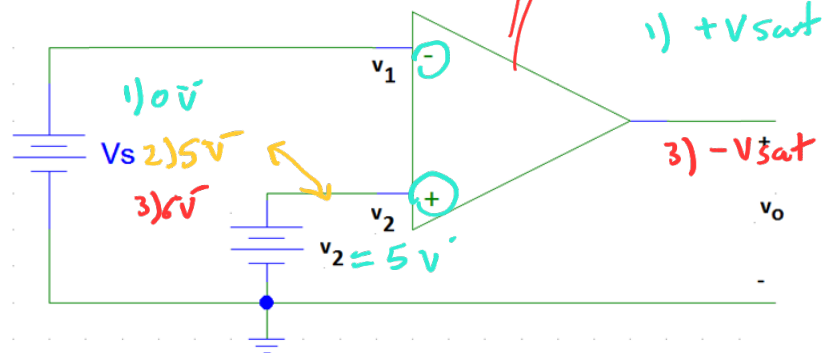
Answer  $V_o = +V_{sat}$

2.  $V_s = 5V$ ?

Answer  $V_o = 0$  (practically impossible to have both  $V_1 = V_2$ )

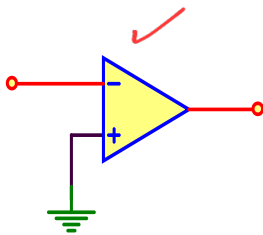
3.  $V_s = 6V$ ?

Answer  $V_o = -V_{sat}$

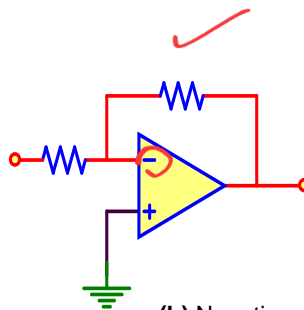




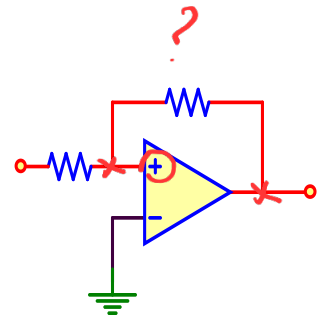
## OP-AMP CONFIGURATIONS



(a) No Feedback  
(open loop  
comparator circuit)



(b) Negative  
Feedback



(c) Positive Feedback

- No feedback : Open loop (used in comparators)
- Negative feedback : Feedback to the inverting input (Used in amplifiers)
- Positive feedback : Feedback to the non inverting input (Used in oscillators) and Schmitt triggers ( comparators with hysteresis)

a **Schmitt trigger** is a comparator circuit with hysteresis.

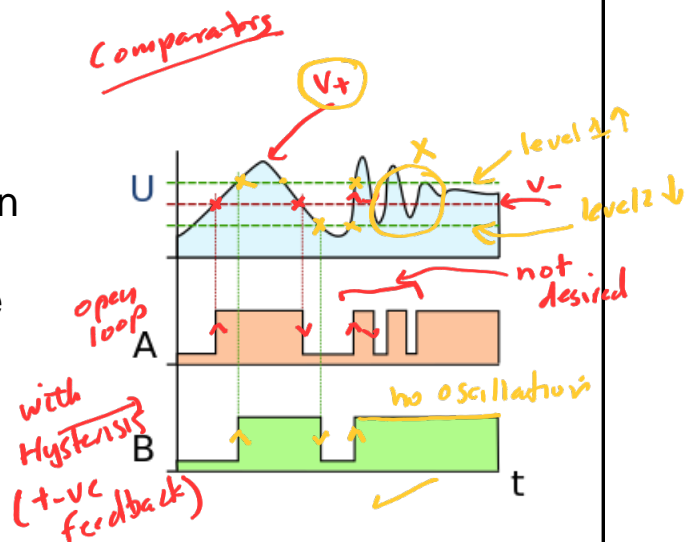
Schmitt trigger devices are typically used in signal conditioning applications to remove noise from signals used in digital circuits, particularly mechanical switch bounce.

They are also used in closed loop negative feedback configurations to implement relaxation oscillators, used in function generators and switching power supplies.

The output of a Schmitt trigger (B) and a comparator (A), when a noisy signal (U) is applied.

The green dotted lines are the circuit's switching thresholds.

The Schmitt trigger tends to remove noise from the signal.



*comparator with hysteresis*

**Schmitt trigger**

This is a comparator circuit and the output is  $V_o = \pm V_{sat}$   
Analysis: step 1

let  $V_o = +V_{sat}$

$$V_+ = \frac{R_2}{R_1 + R_2} (+V_{sat}) = V_{UT} \text{ - upper threshold voltage}$$

in order for  $V_o$  to be  $+V_{sat}$

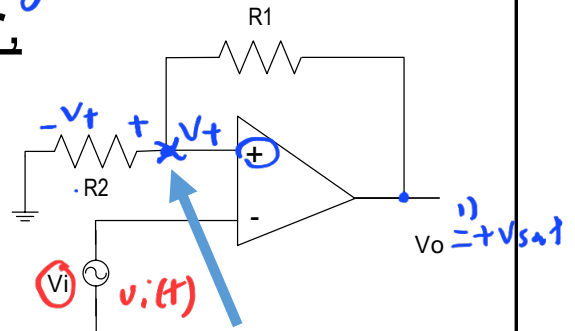
$$V_d > 0$$

$$V_d = V(+) - V(-) > 0$$

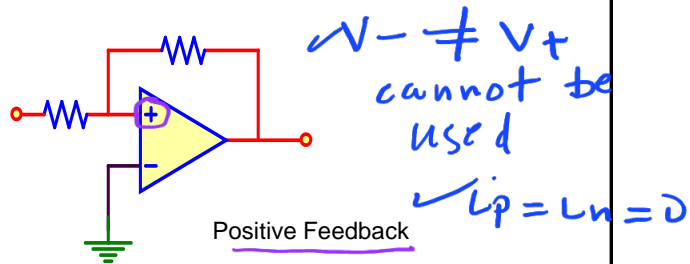
$$\frac{R_2}{R_1 + R_2} + V_{sat} > V_i$$

when  $V_{UT} > V_i \Rightarrow V_o = +V_{sat}$

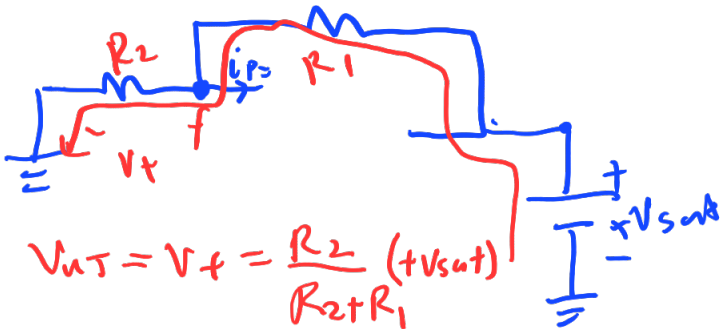
But when  $V_i > V_{UT} \Rightarrow V_o$  switches to  $-V_{sat}$



R1 is Fed back from output to (+) input  
 This is called positive feedback



1) let  $V_o = +V_{sat}$

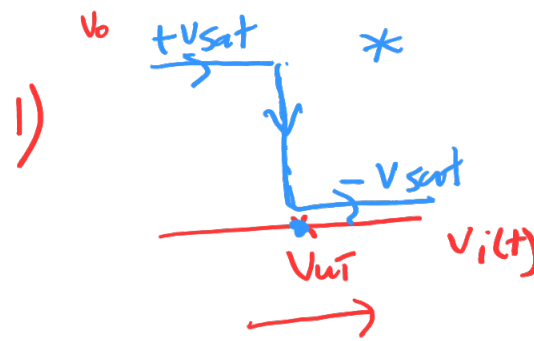


if  $V_+ > V_-$

$$[V_{UT} > V_i(t)] \Rightarrow V_o = +V_{sat}$$

→ as long as  $V_i(t) < V_{UT}$   
 $V_o = +V_{sat}$

→ if  $V_i(t)$  increases such that  $V_i(t) > V_{UT}$   
 $V_o = -V_{sat}$



## Schmitt trigger

### Analysis: step 2

let  $V_o = -V_{sat}$

$$V = \frac{R_2}{R_1 + R_2} (-V_{sat}) = V_{LT} \text{ - Lower threshold voltage}$$

in order for  $V_o$  to be  $-V_{sat}$

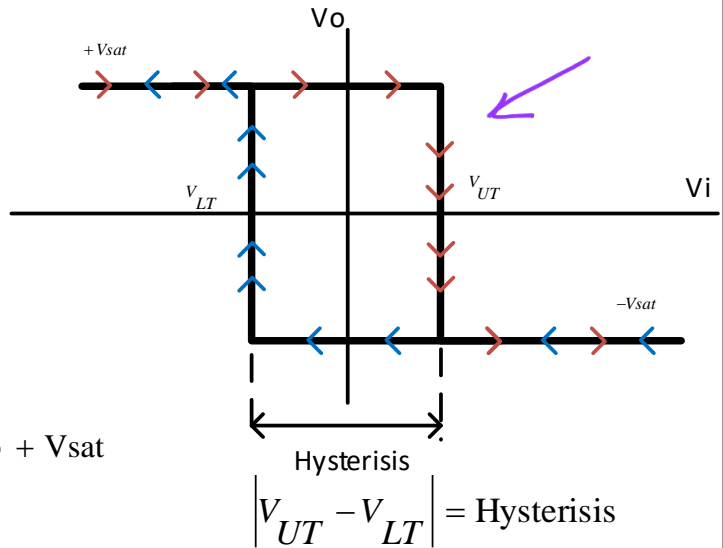
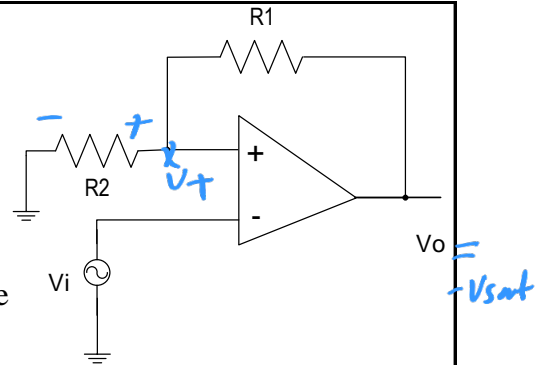
$$V_d < 0$$

$$V_d = V(+) - V(-) < 0$$

$$\frac{R_2}{R_1 + R_2} - V_{sat} < V_i$$

when  $V_{LT} < V_i \Rightarrow V_o = -V_{sat}$

But when  $V_i < V_{LT} \Rightarrow V_o$  switches to  $+V_{sat}$



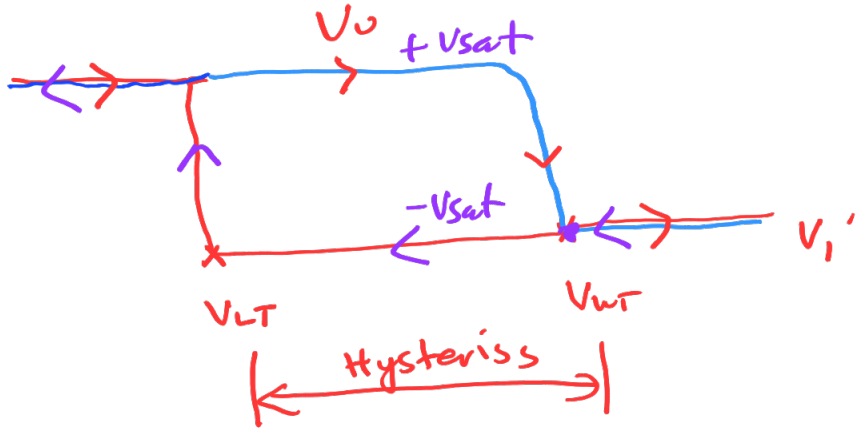
$$V_+ - V_- < 0 \Rightarrow V_o = -V_{sat}$$

$$\downarrow$$

$$V_{LT} - V_i(t) < 0 \Rightarrow V_{LT} < V_i(t)$$

- as long as  $V_i(t) > V_{LT} \Rightarrow V_o = -V_{sat}$
- if  $V_i(t)$  decreases below  $V_{LT} \Rightarrow V_o = +V_{sat}$





End of L 24

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### Schmitt trigger

calculate

Example: Find and sketch  $V_o(t)$  and the plot of  $V_o=f(V_i)$

Solution: this is a Schmitt trigger and

$$V_o = \pm V_{sat}$$

1) let  $V_o = +V_{sat} = 10V$   
in order for  $V_o$  to be  $+V_{sat}$

$$V_d > 0$$

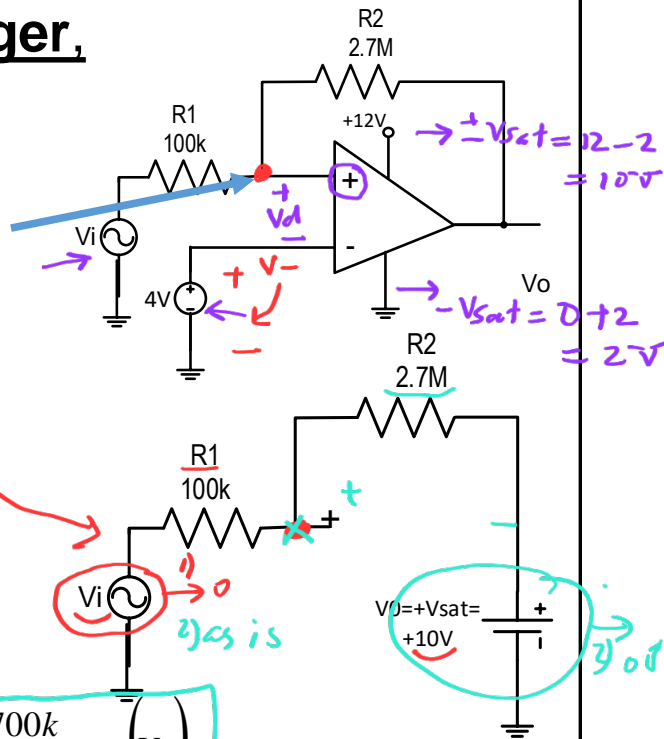
$$V_d = V(+) - V(-) > 0$$

$$V(+) > V(-)$$

$$V(-) = 4V$$

$$V(+) = \frac{100k}{(2700 + 100)k} (+V_{sat}) + \frac{2700k}{(2700 + 100)k} (V_i)$$

$$= \frac{100k}{(2700 + 100)k} (+10V) + \frac{2700k}{(2700 + 100)k} (V_i) > 4V$$



$V_i(t) \rightarrow$  charging voltage

Find  $v_i$ ?

$V_+$

$V_-$

$V_{UT}$

$$\frac{100k}{(2700+100)k}(10V) + \frac{2700k}{(2700+100)k}(V_i) > 4V$$

$$(V_i) > \left[ 4V - \left( \frac{100k}{(2700+100)k}(10V) \right) \right] \left[ \frac{(2700+100)k}{2700k} \right] \Rightarrow V_i > 3.777V$$

when  $V_i > 3.777 \Rightarrow V_o = +V_{sat}$ ; But when  $V_i < 3.777 \Rightarrow V_o$  switches to  $-V_{sat}$

2) let  $V_o = -V_{sat} = (0 + 2) = 2V$

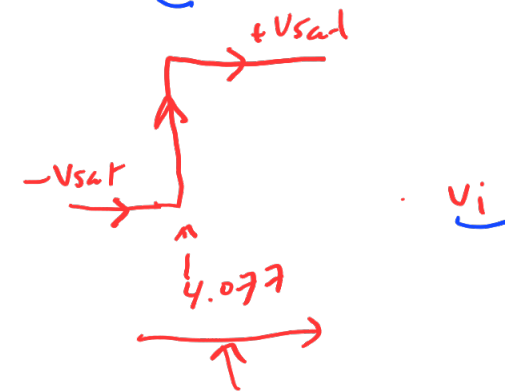
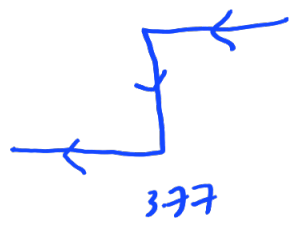
in order for  $V_o$  to be  $-V_{sat} \Rightarrow V_d < 0$ ;  $\therefore V(+) < V(-)$

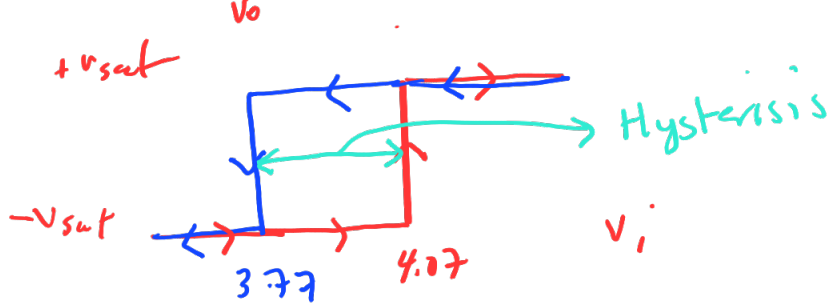
$$V(+) = \frac{100k}{(2700+100)k}(-V_{sat}) + \frac{2700k}{(2700+100)k}(V_i)$$

$$\frac{100k}{(2700+100)k}(-V_{sat}) + \frac{2700k}{(2700+100)k}(V_i) < 4V$$

$$(V_i) < \left[ 4V - \left( \frac{100k}{(2700+100)k}(2V) \right) \right] \left[ \frac{(2700+100)k}{2700k} \right] \Rightarrow V_i < 4.074V$$

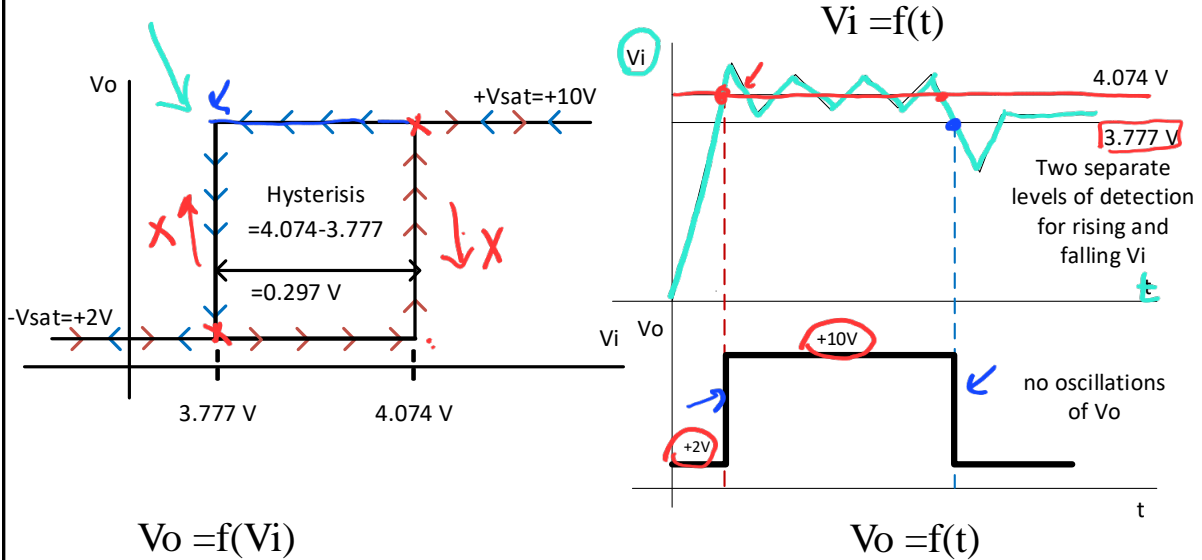
when  $V_i < 4.074V \Rightarrow V_o = -V_{sat}$ ; But when  $V_i > 4.074V \Rightarrow V_o$  switches to  $+V_{sat}$





**Conclusion**

- 1) when  $V_i$  is decreasing, as long as  $V_i$  is  $> 3.777V \Rightarrow V_o = +V_{sat}$  ;  
 but when  $V_i$  becomes  $< 3.777 \Rightarrow V_o$  switches to  $-V_{sat}$
- 2) when  $V_i$  is increasing, as long as  $V_i$  is  $< 4.074V \Rightarrow V_o = -V_{sat}$  ;  
 but when  $V_i$  becomes  $> 4.074 \Rightarrow V_o$  switches to  $+V_{sat}$



## Comparator without Hysteresis

**Example: Find and sketch  $V_o(t)$  and the plot of  $V_o=f(V_i)$**

**Solution: this is a comparator and**

$$V_o = \pm V_{sat}$$

1) in order for  $V_o$  to be  $+V_{sat}$

$$V_d > 0 \text{ and } V_d = V(+)-V(-) > 0$$

$$V(+)>V(-)$$

$$V(-) = 4V; V(+)=V_i$$

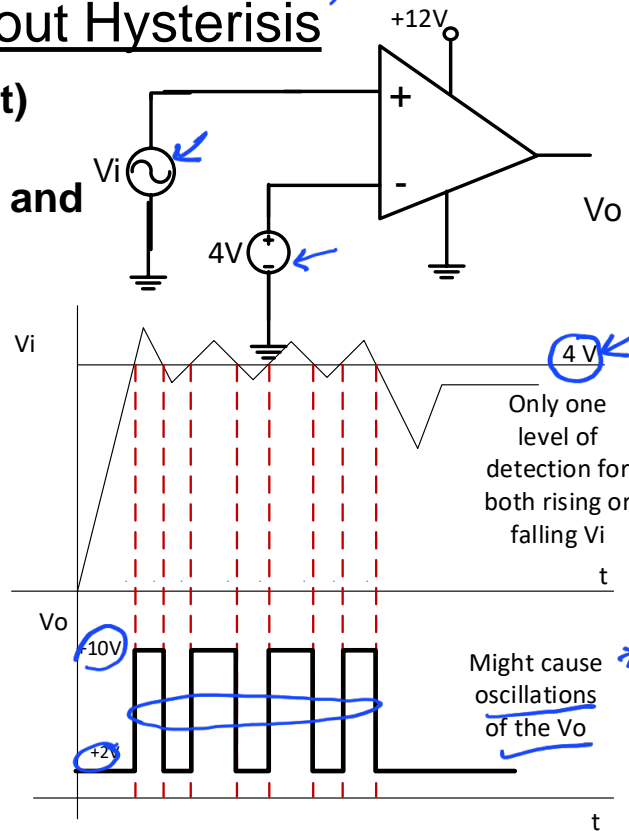
$$V_i > 4V$$

2) in order for  $V_o$  to be  $-V_{sat} = 2V$

$$V_d < 0 \text{ and } V_d = V(+)-V(-) < 0$$

$$V(+)<V(-)$$

$$V_i < 4V$$





$$\rightarrow i_c(t) = C \frac{dV_c(t)}{dt}$$

## Integrator $V_i(t) \int V_i(t) \rightarrow$ time domain

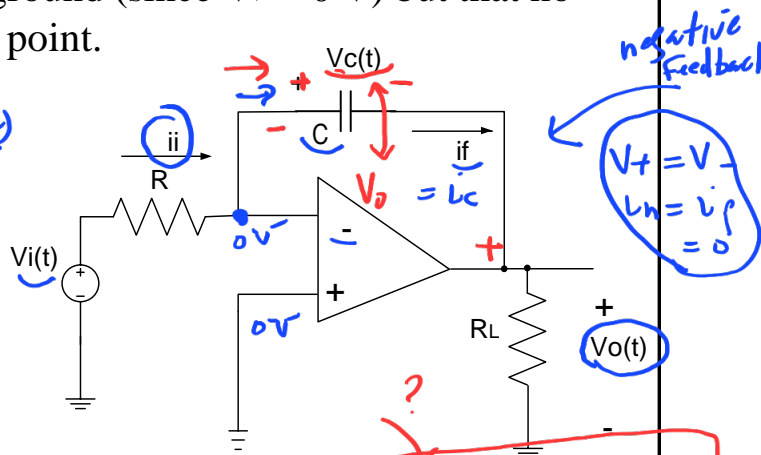
- So far, the input and feedback components have been resistors. If the feedback component used is a capacitor, the resulting connection is called an *integrator*.
- Recall that virtual ground means that we can consider the voltage at the junction of  $R$  and  $X_c$  to be ground (since  $V_+ = 0$  V) but that no current goes into ground at that point.

$$i_i = i_f = \frac{V_i(t)}{R} = i_c = C \frac{dV_c(t)}{dt}$$

$$V_c(t) = \frac{1}{C} \int_0^t i_f(t) dt$$

$$V_o = -V_c(t)$$

$$V_o = -\frac{1}{C} \int_0^t \frac{V_i(t)}{R} dt = -\frac{1}{RC} \int_0^t V_i(t) dt$$



$$i_c(t) = C \frac{dV_c(t)}{dt} = \frac{V_i(t)}{R}$$

## Differentiator $\rightarrow$ time domain

A differentiator, while not as useful as the circuit forms covered above, the differentiator does provide a useful operation, the resulting relation for the circuit being

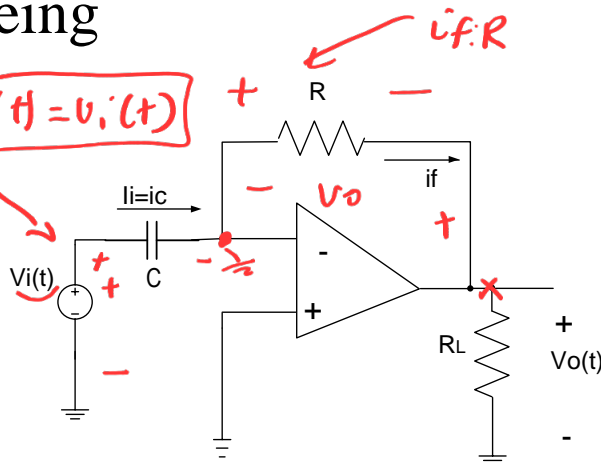
$$i_i = i_f = i_C = C \frac{dV_i(t)}{dt}$$

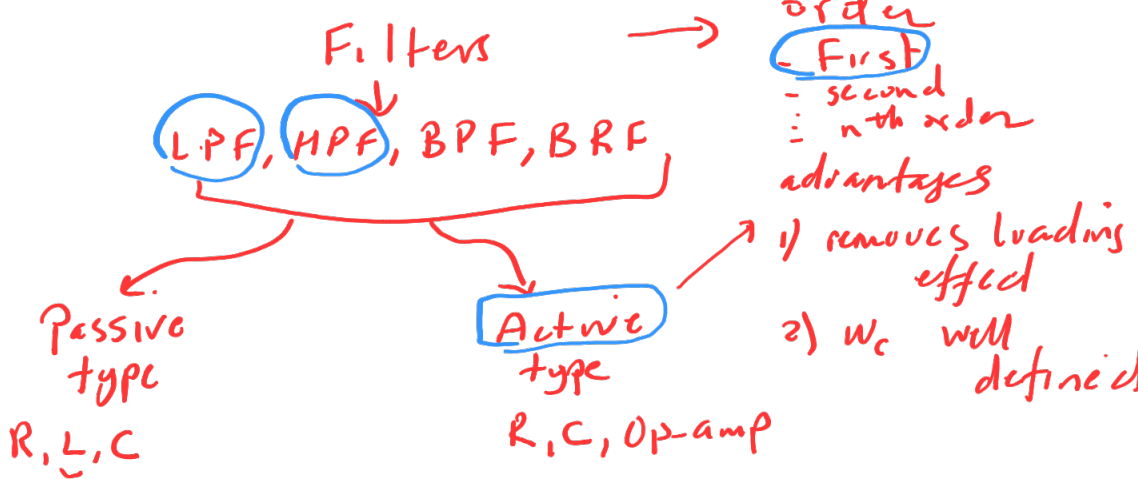
$$V_c(t) = \frac{1}{C} \int_0^t i_f(t) dt$$

$$V_o = -i_f(t)R$$

$$V_o = - \left( C \frac{dV_i(t)}{dt} \right) (R) = -RC \frac{dV_i(t)}{dt}$$

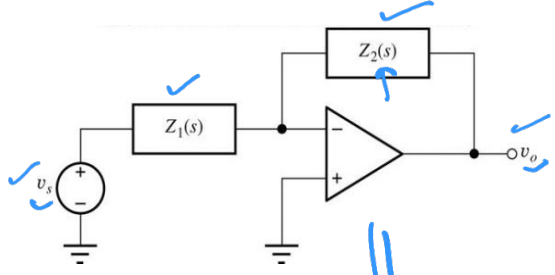
$$V_c(t) = V_i(t)$$





### The Active Low-pass Filter

The gain analysis of this inverting amplifier. Note  $s = j\omega$ .



$$A_v = \frac{\tilde{v}_o(j\omega)}{\tilde{v}_s(j\omega)} = -\frac{Z_2(j\omega)}{Z_1(j\omega)}$$

$Z_1(j\omega) = R_1$

$$Z_2(j\omega) = \frac{R_2 \cdot \frac{1}{j\omega C}}{R_2 + \frac{1}{j\omega C}} = \frac{R_2}{j\omega C R_2 + 1}$$

$$A_v = -\frac{R_2}{R_1} \frac{1}{(1 + j\omega C R_2)}$$

$$A_v = \frac{-K}{1 + \frac{j\omega}{\omega_c}}$$

$\omega_c = \frac{1}{R_2 C}$

$$\Rightarrow K = \frac{R_2}{R_1} \leftarrow \text{gain (dc gain)}$$

$$\omega_c = 2\pi f_c = \frac{1}{R_2 C} \therefore f_c = \frac{1}{2\pi R_2 C}$$

& cut-off frequency:

$\omega$  rad/sec  
 $f$  (hertz)

$\omega \rightarrow f$  ?  
 $\omega = 2\pi f$   
 $\omega_c = 2\pi f_c$

take the Magnitude →

$$A_v = \frac{-K}{1 + \frac{j\omega}{\omega_c}}$$

← complex number  
↓ phasor representation

**|mag|** < **phase** or exponential

Assume  $R=1$

$$|A_v| = \frac{|-1|}{|1 + \frac{j\omega}{\omega_c}|} = \frac{1}{\sqrt{1 + (\frac{\omega}{\omega_c})^2}}$$

phase



$$\omega_c = \frac{1}{R_2 C} \leftarrow \text{known}$$

$|A_v(j\omega)| \rightarrow$  drawing



### Frequency Response (Bode Plot)

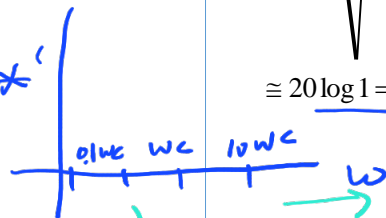
$\rightarrow$  log scale  $\omega$   
 $\rightarrow$  Decibel scale

Magnitude Plot (Magnitude in decibels vs log of frequency)

$$A_{dB} = 20 \log |H(j\omega)| = \frac{K}{|1 + (\frac{f}{f_c})^2|} \text{ dB}$$

$$|H(j\omega)| = |A_v(j\omega)|$$

$$|A_v(j\omega)| = \frac{K}{\sqrt{1 + (\frac{\omega}{\omega_c})^2}}$$



at  $\omega = 0.1\omega_c$

$$20 \log \frac{1}{\sqrt{1 + (\frac{0.1\omega_c}{\omega_c})^2}} = 20 \log \frac{1}{\sqrt{1 + 0.01}} \approx 20 \log 1 = 0 \text{ dB}$$

at  $\omega = \omega_c$

$$20 \log \frac{1}{\sqrt{1 + (\frac{\omega_c}{\omega_c})^2}} = 20 \log \frac{1}{\sqrt{1 + 1}} = \frac{V_o}{V_{in}} \text{ dB}$$

$$= 20 \log |0.707| = -3 \text{ dB}$$

at  $\omega = 10\omega_c$

$$20 \log \frac{1}{\sqrt{1 + (\frac{10\omega_c}{\omega_c})^2}} = 20 \log \frac{1}{\sqrt{1 + 100}} = 20 \log 0.1 = -20 \text{ dB}$$

cut-off frequency  $\equiv$  corner frequency  $\equiv$  -3dB frequency

@  $\omega = 100\omega_c$

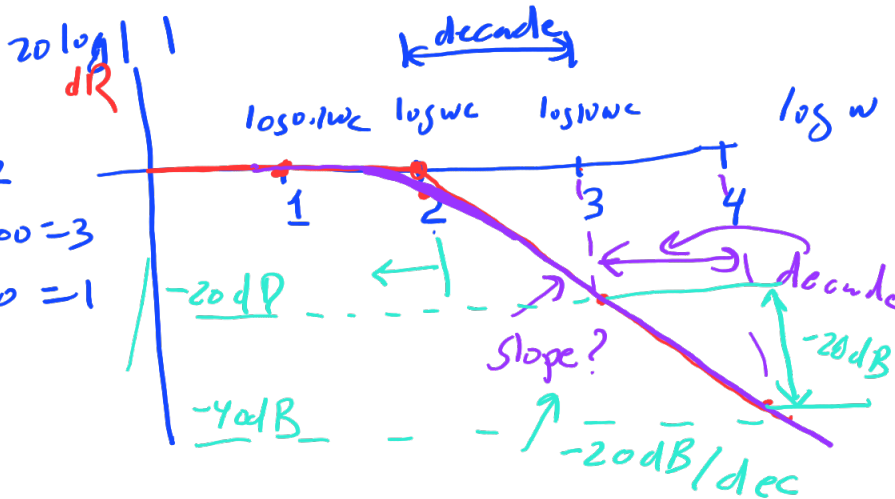
$$= \frac{1}{\sqrt{1 + (100)^2}}$$

$$= 20 \log 0.01$$

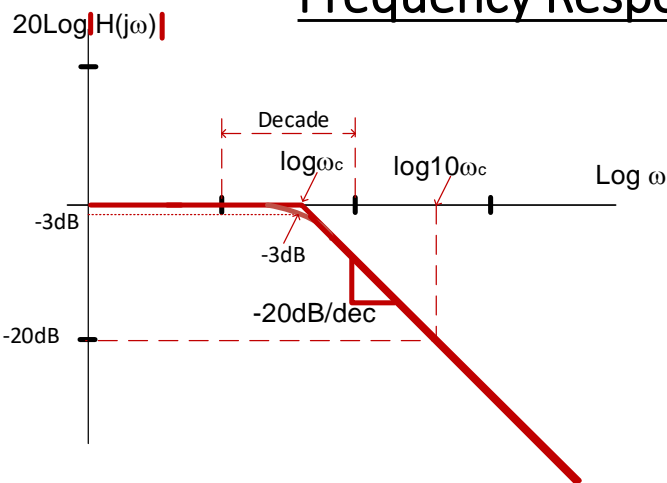
$$= -40 \text{ dB}$$

$\omega$	$ A_v $	$= \frac{V_o}{V_i} \Rightarrow V_o = V_i \times A_v$
$0.1\omega_c$	0 dB	$V_i = 5 \sin 1000t$
$\omega_c$	-3 dB	$V_o = ?$
$10\omega_c$	-20 dB $\leftarrow 0.1$	$= 0.5 \sin 1000t$
$100\omega_c$	-40 dB $\leftarrow 0.01$	

assume  $\omega_c = 100 \rightarrow \log 100 = 2$   
 $10\omega_c = 1000 \rightarrow \log 1000 = 3$   
 $0.1\omega_c = 10 \rightarrow \log 10 = 1$



## Frequency Response (Bode Plot)



### Summary

at  $\omega = 0.1\omega_c$   
 $\text{Log } \omega = 20\log 1 = 0\text{dB}$   
 at  $\omega = \omega_c$   
 $= 20\log 0.707 = -3\text{dB}$   
 at  $\omega = 10\omega_c$   
 $= 20\log 0.1 = -20\text{dB}$   
 at  $\omega = 100\omega_c$   
 $= 20\log 0.01 = -40\text{dB}$

- At frequencies below  $\omega_c$ , the amplifier is an inverting amplifier with gain set by the ratio of resistors  $R_2$  and  $R_1$ .
- At frequencies above  $\omega_c$  the amplifier response “rolls off” at -20dB/decade.
- Notice that cutoff frequency and gain can be independently set.

End of L25

$\omega = 1000$

$$A_v = \frac{v_o}{v_i} = \frac{1}{\sqrt{1 + (\frac{\omega C}{R_2})^2}} = 0.1$$

$$\frac{v_o}{5} = 0.1$$

$$v_o = 5 \times 0.1 = 0.5$$

### Active Low-pass Filter: Example

- Problem:** Design an active low-pass filter
- Given Data:**  $A_v = 40 \text{ dB}$ ,  $R_{in} = 5 \text{ k}\Omega$ ,  $f_c = 2 \text{ kHz}$
- Assumptions:** Ideal op amp, specified gain represents the desired low-frequency gain.
- Analysis:**  $|A_v| = 10^{40\text{dB}/20\text{dB}} = 100$

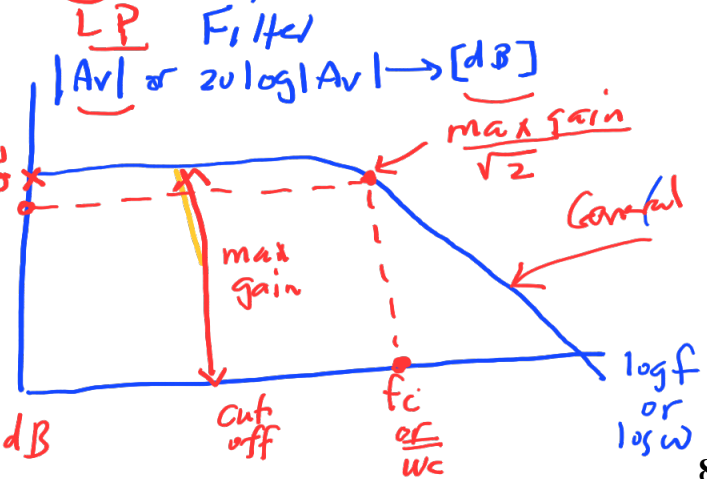
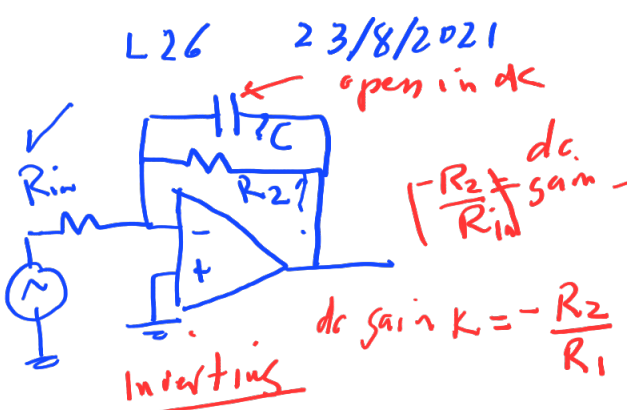
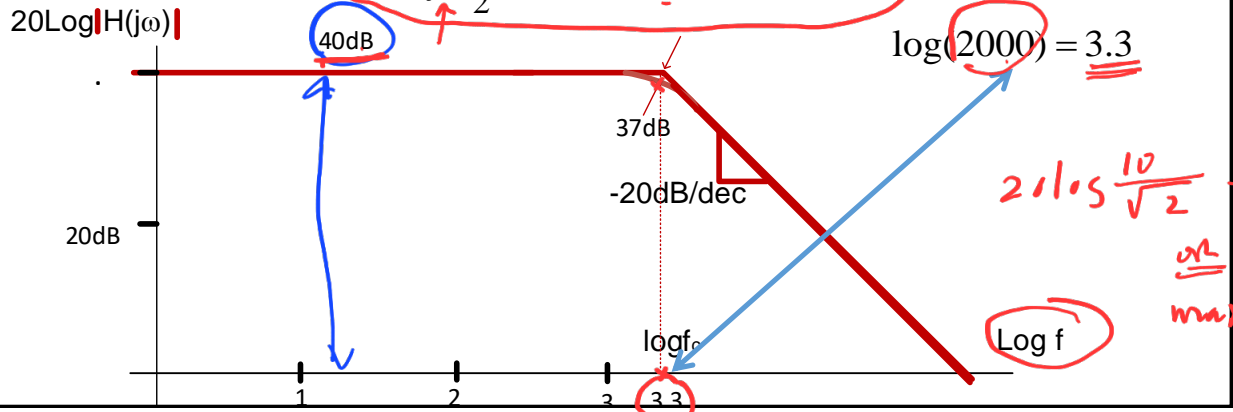
$20 \log |A_v| = 40 \text{ dB}$

$$\omega_c = \frac{1}{R_2 C} = 2\pi f_c$$

Input resistance is controlled by  $R_1$  and voltage gain is set by  $R_2/R_1$ .  
The cutoff frequency is then set by  $C$ .

$$R_1 = R_{in} = 5 \text{ k}\Omega \quad \text{and} \quad |A_v| = \frac{R_2}{R_1} \Rightarrow R_2 = 100 R_1 = 500 \text{ k}\Omega$$

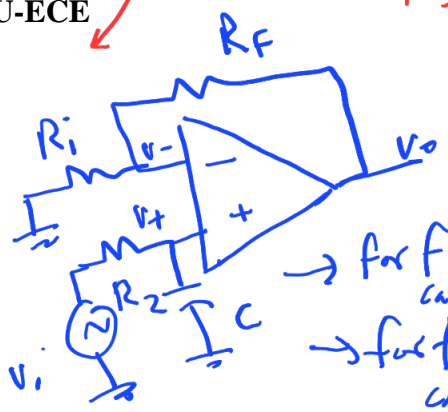
$$C = \frac{1}{2\pi f_c R_2} = \frac{1}{2\pi (2 \text{ kHz})(500 \text{ k}\Omega)} = 159 \text{ pF}$$



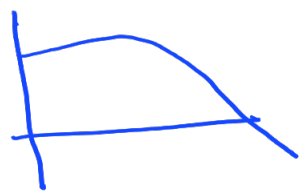
$20 \log |A_v| = 40 \text{ dB}$

$105 |Av| = \frac{40}{20} = 2 \Rightarrow |Av| = 10^2 = 100$

$R_2 = 100 R_{in} = 500k\Omega$   
 $100 = \frac{R_2}{R_{in}}$

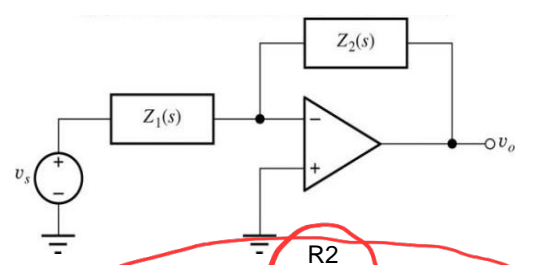


for  $f=0 \Rightarrow$  cap-open  $V_o = (1 + \frac{R_F}{R_i}) \cdot V_{i,c}$   
 for  $f \rightarrow \infty \Rightarrow$  cap short  $V_o = 0$   
 $K =$



### The Active High-pass Filter

The gain analysis of this inverting amplifier.



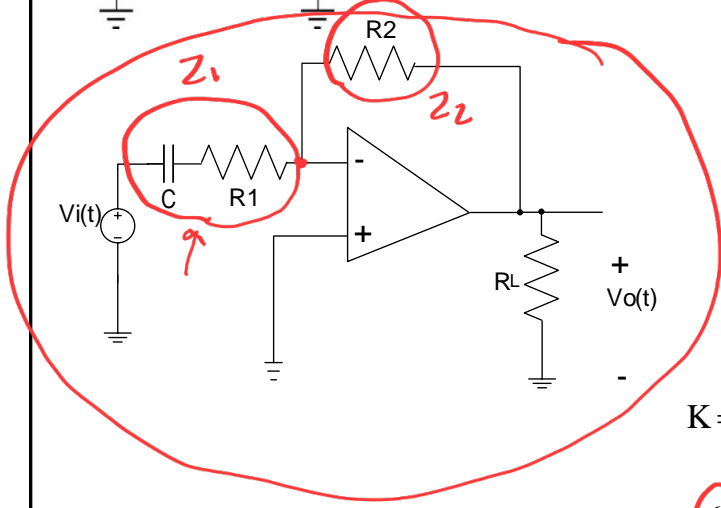
$A_v = \frac{\tilde{v}_o(j\omega)}{\tilde{v}_s(j\omega)} = -\frac{Z_2(j\omega)}{Z_1(j\omega)}$   
 $Z_2(j\omega) = R_2$

$Z_1(j\omega) = R_1 + \frac{1}{j\omega C}$

$A_v = -\frac{R_2}{(R_1 + \frac{1}{j\omega C})}$

$A_v = -\frac{R_2 / R_1}{(1 + \frac{1}{j\omega R_1 C})}$

$A_v = \frac{-K}{1 + \frac{c}{j\omega}}$



$K = \frac{R_2}{R_1}$  high frequency gain

$\omega_c = 2\pi f_c = \frac{1}{R_1 C} \therefore f_c = \frac{1}{2\pi R_1 C}$

$|Av| = \frac{K}{1 + (\frac{\omega_c}{\omega})^2}$

1st order LPF

$A_v = \frac{K}{\sqrt{1 + (\frac{\omega}{\omega_c})^2}}$

1st order HPF

$|Av| = \frac{K}{\sqrt{1 + (\frac{\omega_c}{\omega})^2}}$

assume  $K=1$

for  $\omega = \omega_c \Rightarrow 20 \log |A_v| = 20 \log \frac{1}{\sqrt{1+1}} = -3 \text{ dB}$

for  $\omega = 0.1 \omega_c \Rightarrow 20 \log \frac{1}{\sqrt{1+(\frac{\omega_c}{0.1 \omega_c})^2}} =$   
 $= 20 \log \frac{1}{\sqrt{101}} \approx 20 \log 0.1 = -20 \text{ dB}$

for  $\omega = 0.01 \omega_c \Rightarrow 20 \log \frac{1}{\sqrt{10001}} = 20 \log 0.01 = -40 \text{ dB}$

for  $\omega = 10 \omega_c \Rightarrow 20 \log \frac{1}{\sqrt{1.01}} \approx 20 \log 1 = 0 \text{ dB}$

## Frequency Response (Bode Plot)

Magnitude Plot (Magnitude in decibels vs log of frequency)

$$A_{\text{dB}} = 20 \log |H(j\omega)|$$

$$|H(j\omega)| = |A_v(j\omega)|$$

$$= \frac{K}{\sqrt{1 + \left(\frac{\omega_c}{\omega}\right)^2}}$$

at  $\omega = 0.1 \omega_c$

$$20 \log \frac{1}{\sqrt{1 + \left(\frac{\omega_c}{0.1 \omega_c}\right)^2}} = 20 \log \frac{1}{\sqrt{1+100}}$$

$$\approx 20 \log 0.1 = -20 \text{ dB}$$

at  $\omega = \omega_c$

$$20 \log \frac{1}{\sqrt{1 + \left(\frac{\omega_c}{\omega_c}\right)^2}} = 20 \log \frac{1}{\sqrt{1+1}}$$

$$= 20 \log 0.707 = -3 \text{ dB}$$

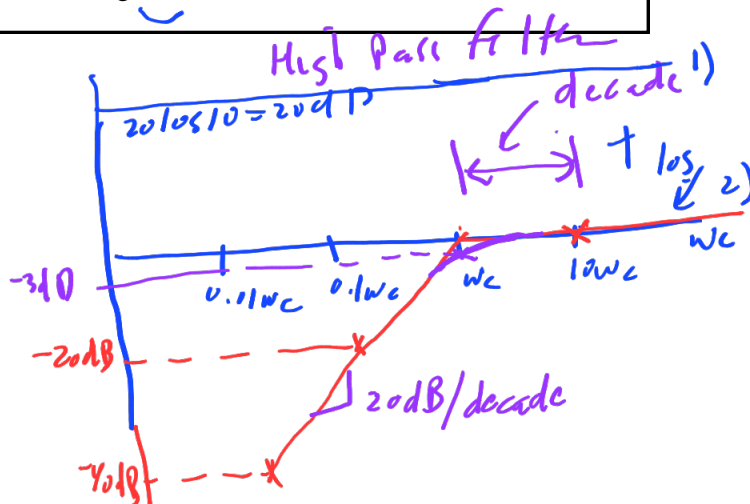
at  $\omega = 0.01 \omega_c$

$$20 \log \frac{1}{\sqrt{1 + \left(\frac{\omega_c}{0.01 \omega_c}\right)^2}} = 20 \log \frac{1}{\sqrt{1+10000}}$$

$$= 20 \log 0.01 = -40 \text{ dB}$$

$\omega$	$20 \log  A_v  \text{ [dB]}$	$K=10$
$10 \omega_c$	0 dB	20 dB
$\omega_c$	-3 dB	17 dB
$0.1 \omega_c$	-20 dB	0 dB
$0.01 \omega_c$	-40 dB	-20 dB

+20dB





$20 \log 10 - 20 \log 100$

for  $K=10$

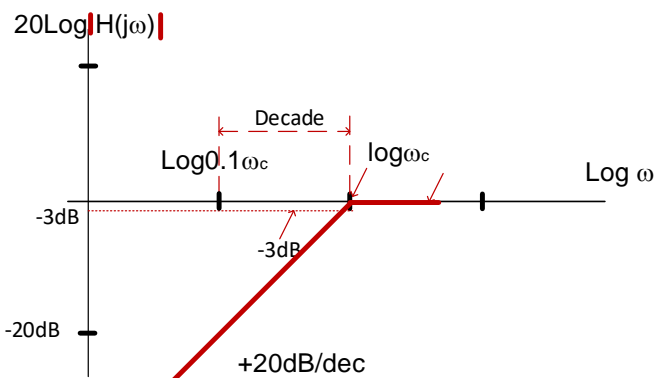
$20 \log \frac{10}{100} = 20 \text{ dB} - 40 \text{ dB}$   
 $= -20 \text{ dB}$

for  $\omega = 0.1 \omega_c \Rightarrow 20 \log \frac{10}{\sqrt{101}} \times 20 \log 1$   
 $= 0 \text{ dB}$

$\omega = 0.01 \omega_c \Rightarrow 20 \log \frac{10}{100}$   
 $= -20 \text{ dB}$

$\omega = \omega_c \Rightarrow 20 \log \frac{10}{\sqrt{2}}$   
 $= 17 \text{ dB}$

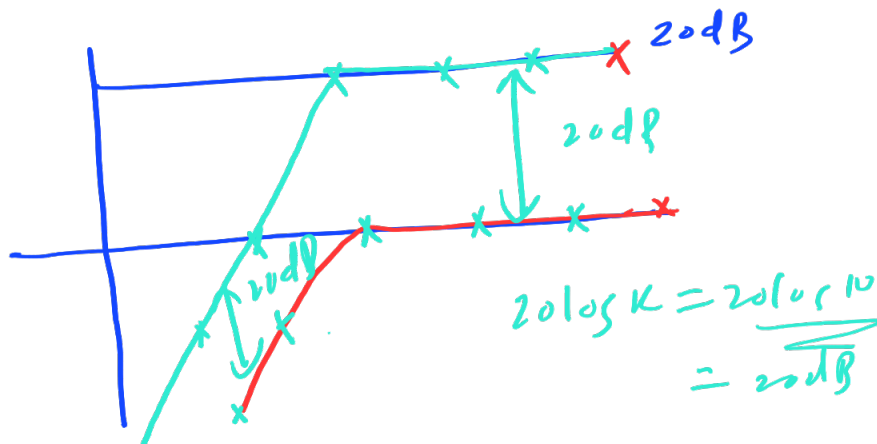
### Frequency Response (Bode Plot)



#### Summary

- at  $\omega = 0.1 \omega_c$   
 $= 20 \log 0.1 = -20 \text{ dB}$
- at  $\omega = \omega_c$   
 $= 20 \log 0.707 = -3 \text{ dB}$
- at  $\omega = 0.01 \omega_c$   
 $= 20 \log 0.01 = -40 \text{ dB}$
- at  $\omega = 0$   
 $= 20 \log 1 = 0 \text{ dB}$

- At frequencies above  $\omega_c$ , the amplifier is an inverting amplifier with gain set by the ratio of resistors  $R_2$  and  $R_1$ .
- At frequencies below  $\omega_c$ , the amplifier response "rolls off" at  $-20 \text{ dB/decade}$ .
- Notice that cutoff frequency and gain can be independently set.

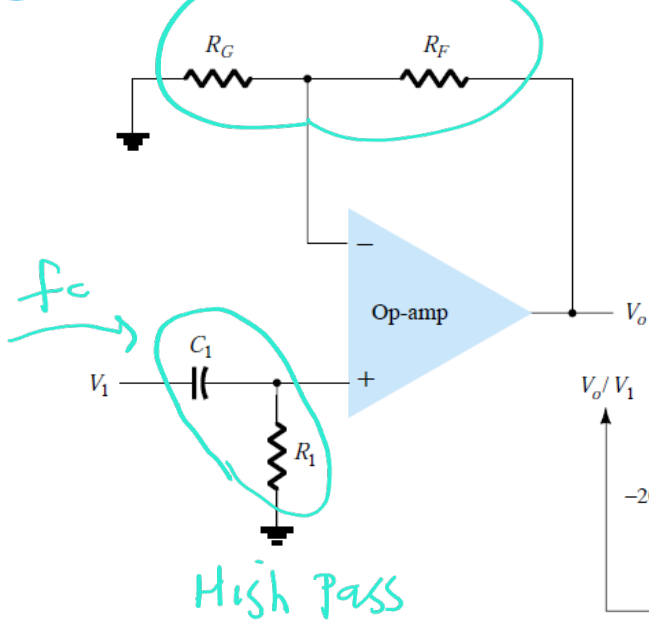


*LPF non-inverting*

at a cutoff frequency of

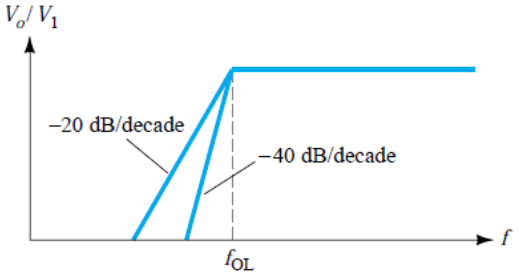
$$A_V = 1 + \frac{R_F}{R_G} = K$$
$$f_c = f_{OH} = \frac{1}{2\pi R_1 C_1}$$

### High-Pass Active Filter



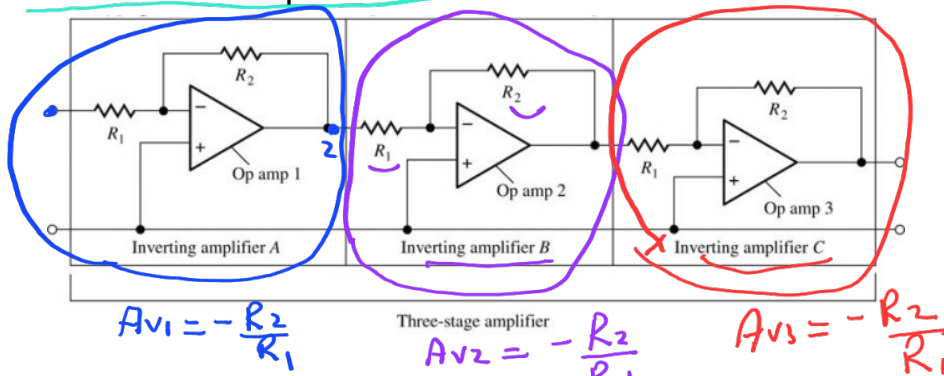
$$f_c = f_{OL} = \frac{1}{2\pi R_1 C_1}$$

$$K = 1 + \frac{R_F}{R_G}$$

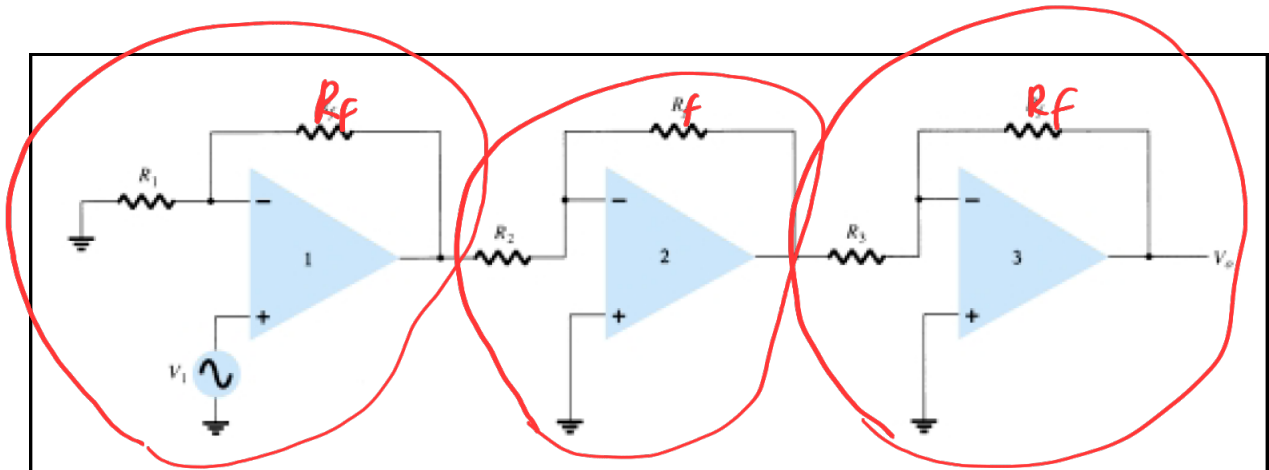


$$A_V = A_{V1} \cdot A_{V2} \cdot A_{V3}$$

## Cascaded Amplifiers



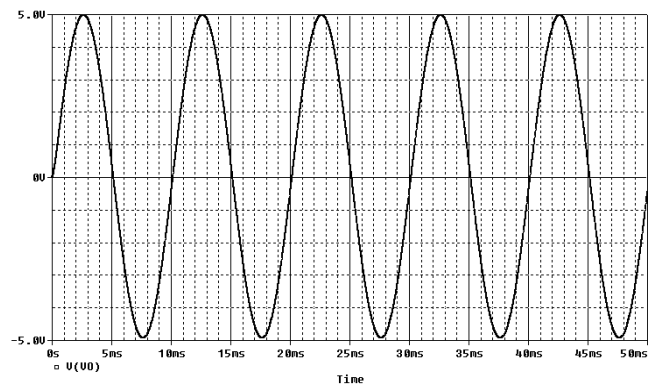
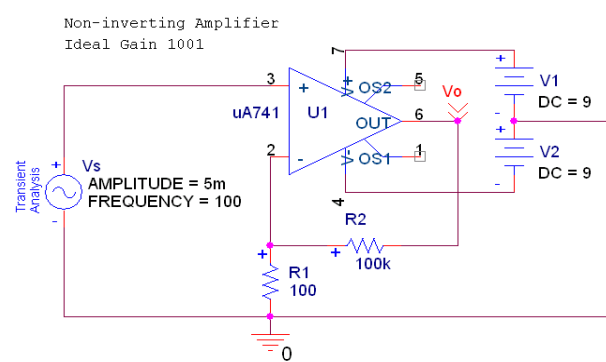
- Connecting several amplifiers in cascade (output of one stage connected to the input of the next) can meet design specifications not met by a single amplifier.
- Each amplifier stage is built using an op amp with parameters  $A$ ,  $R_{id}$ ,  $R_o$ , called open loop parameters, that describe the op amp with no external elements.
- $A_v$ ,  $R_{in}$ ,  $R_{out}$  are closed loop parameters that can be used to describe each closed-loop op amp stage with its feedback network, as well as the overall composite (cascaded) amplifier.
- The gain of each stage can be calculated separately, then the total gain is found by multiplying the resulting gains

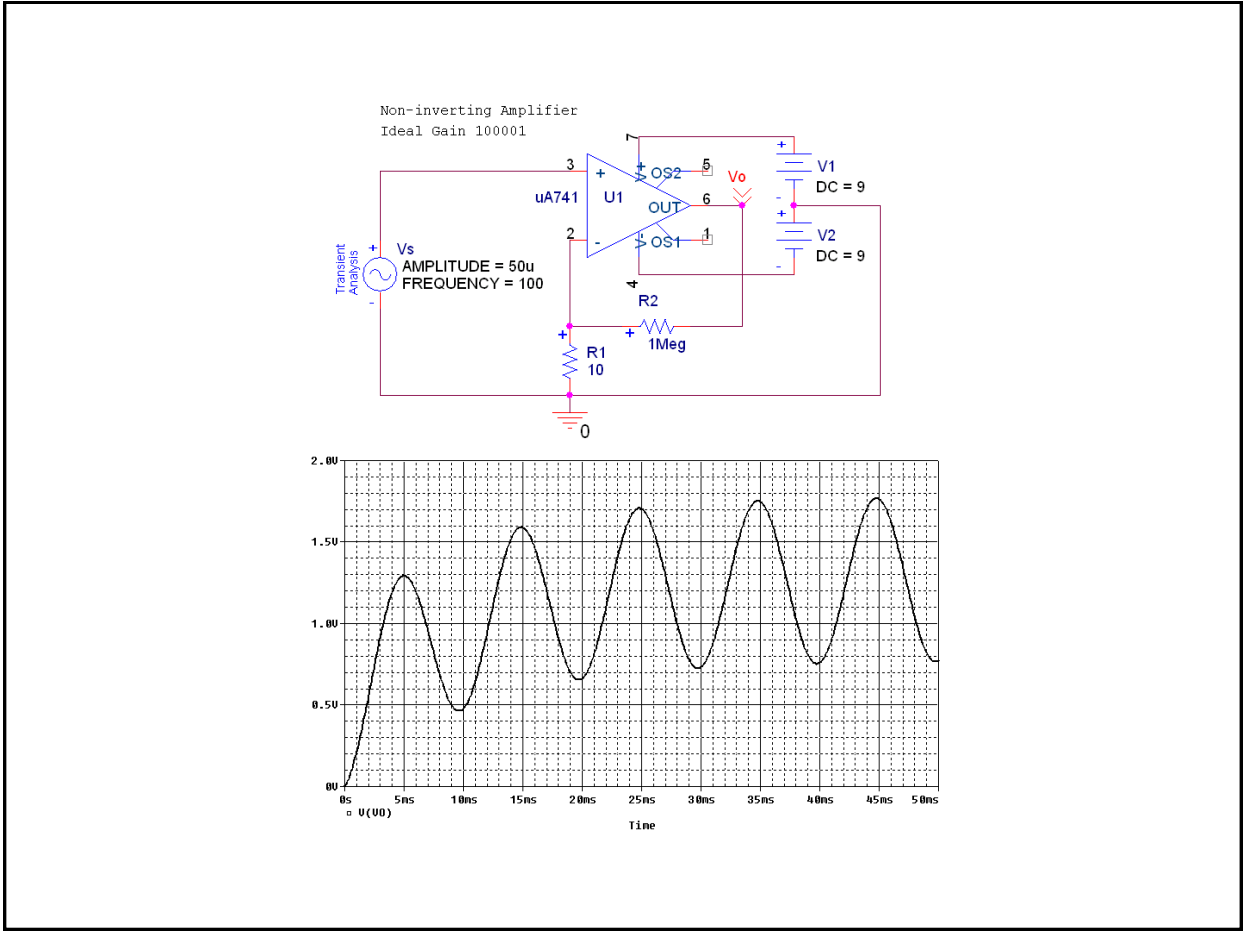


$$A = A_1 A_2 A_3$$

where  $A_1 = 1 + R_f/R_1$ ,  $A_2 = -R_f/R_2$ , and  $A_3 = -R_f/R_3$ .

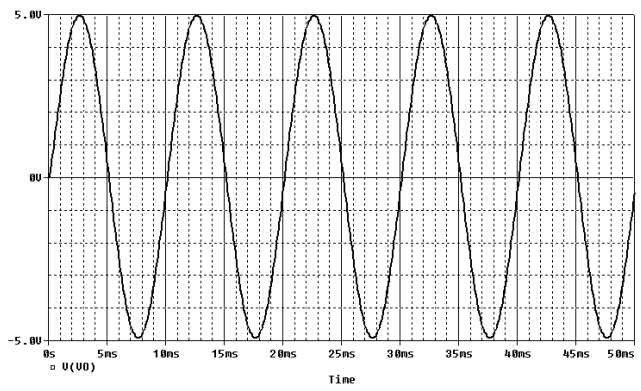
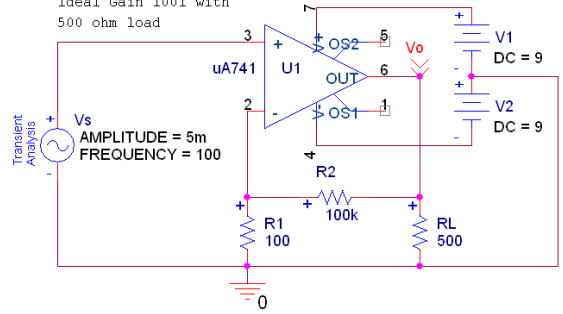
# Example PSpice Simulations of Non-inverting Amplifier Circuits

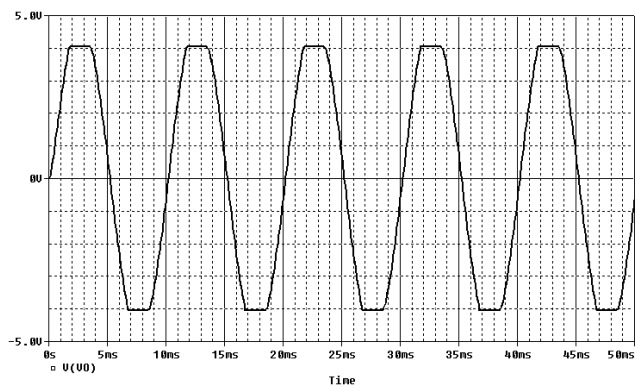
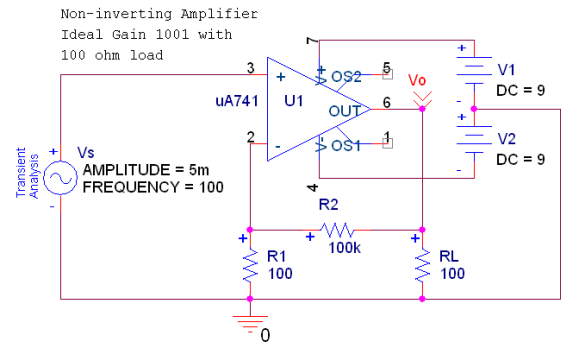




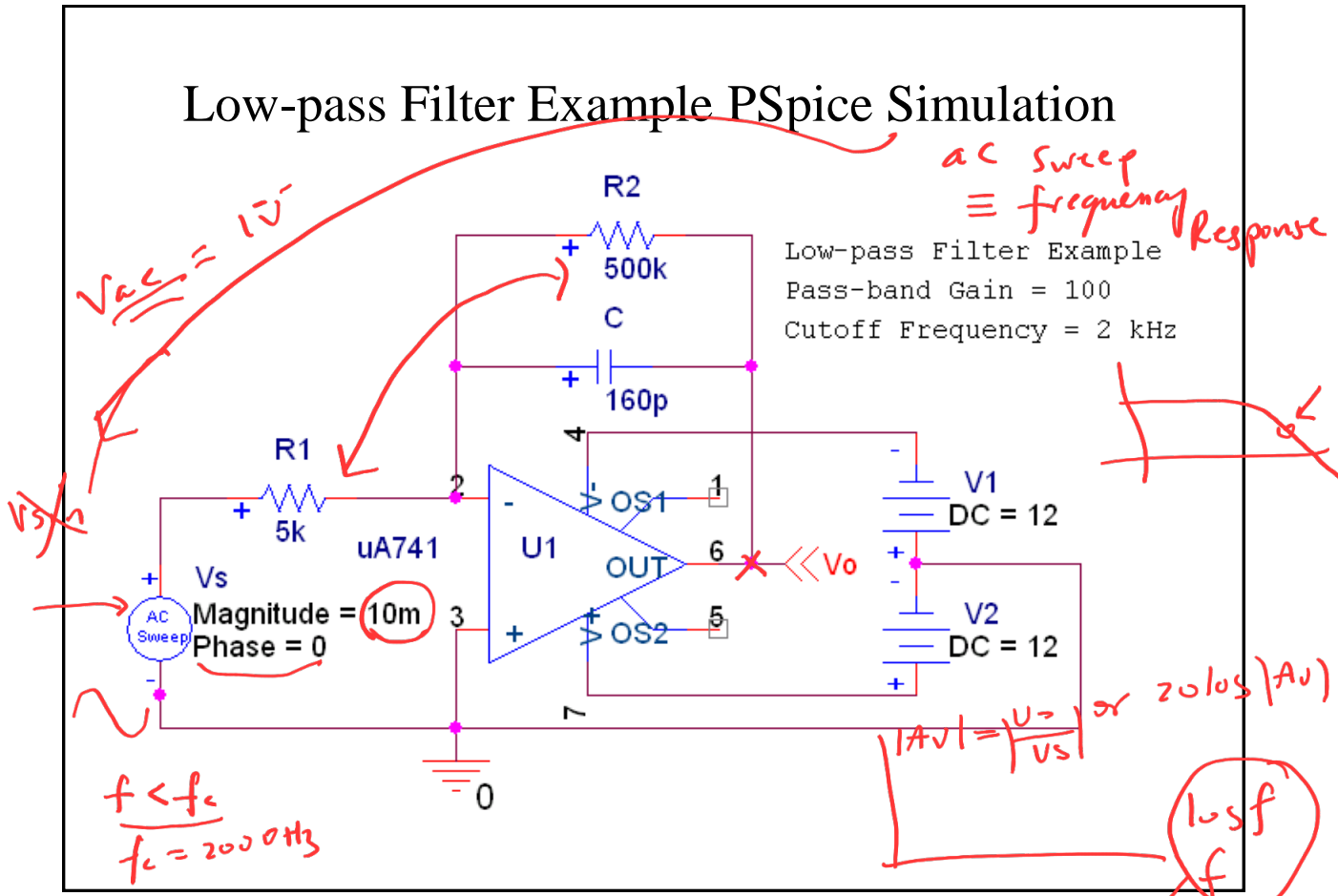


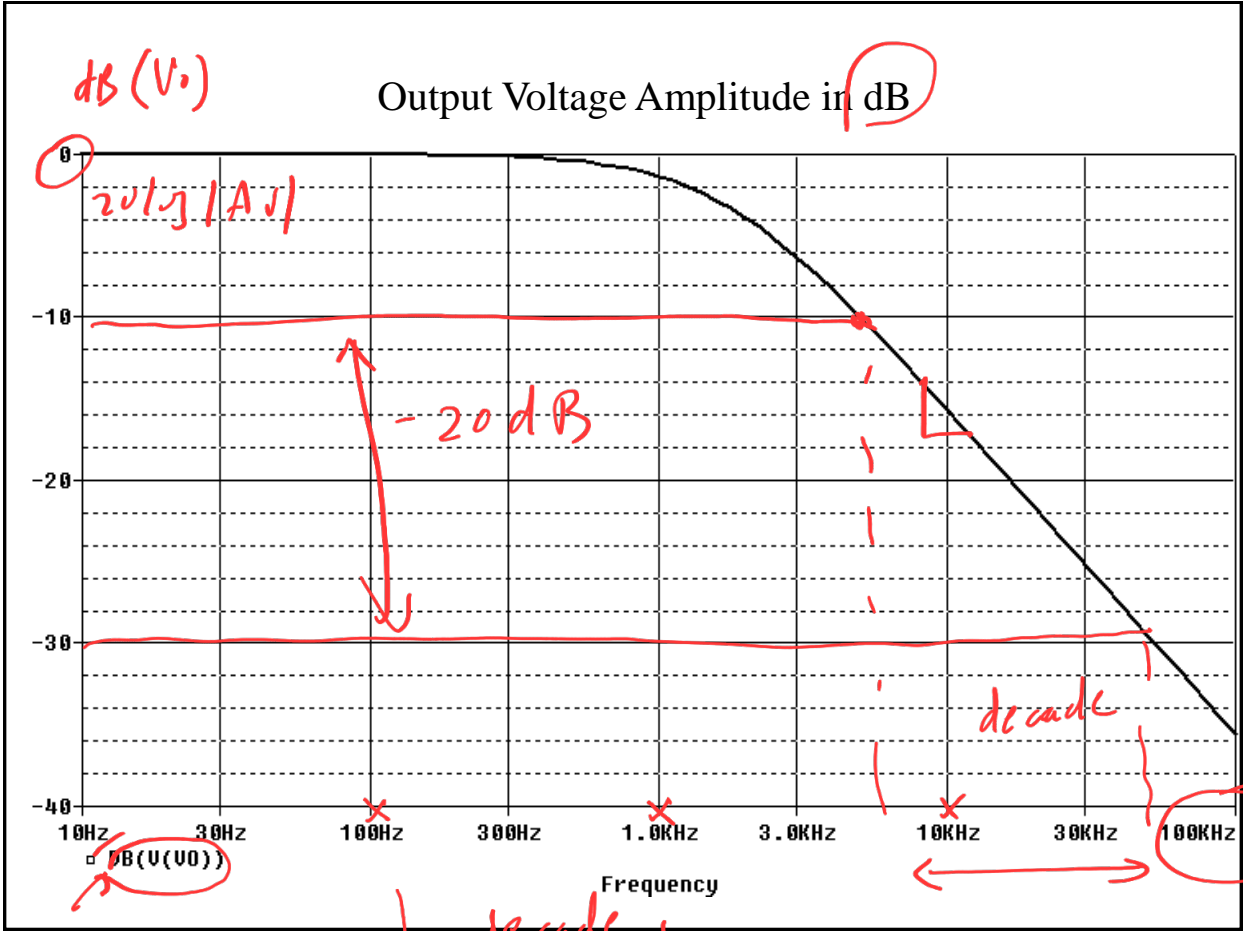
Non-inverting Amplifier  
Ideal Gain 1001 with  
500 ohm load

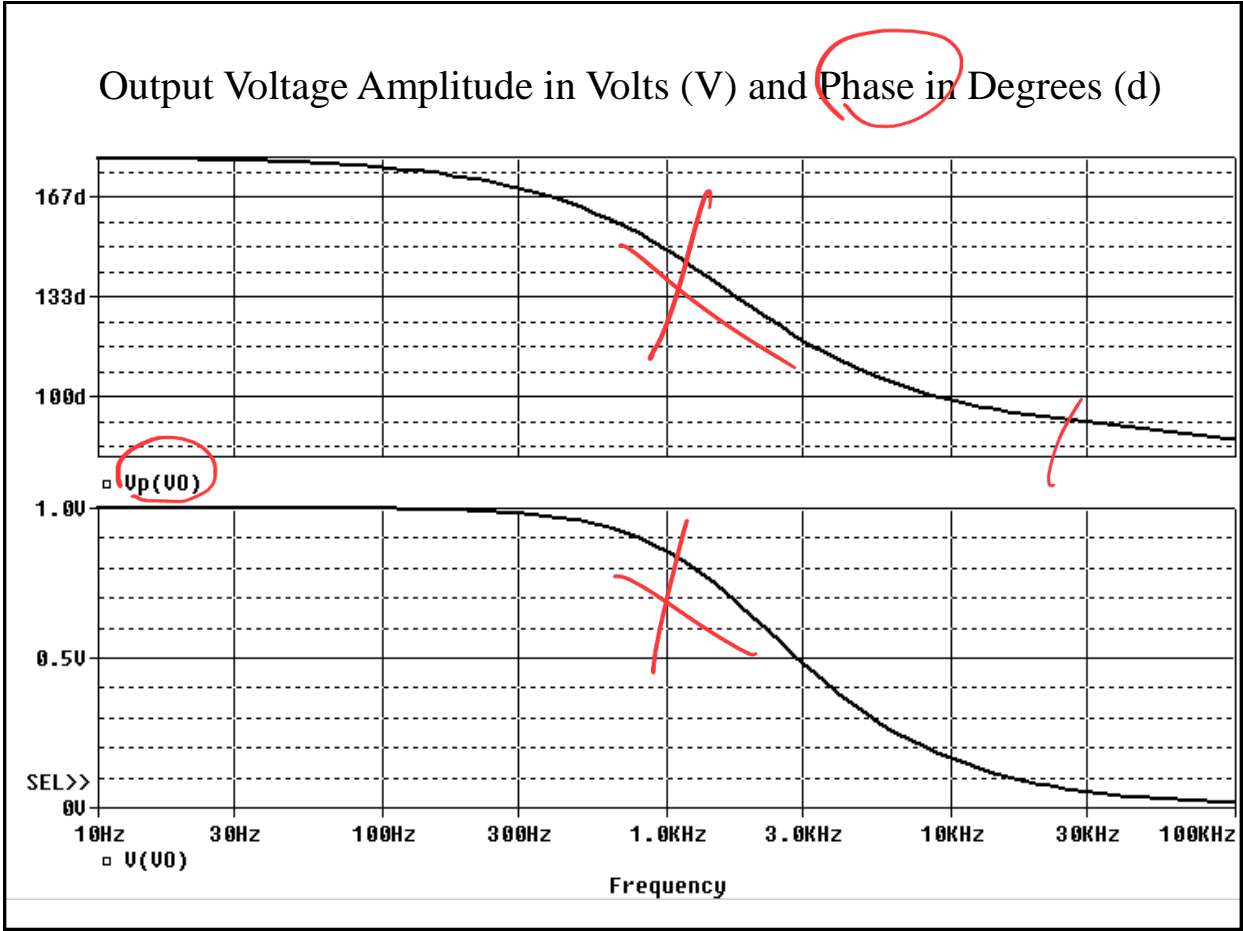




## Low-pass Filter Example PSpice Simulation







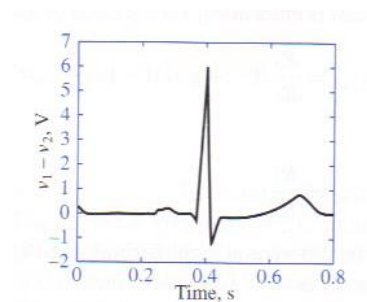
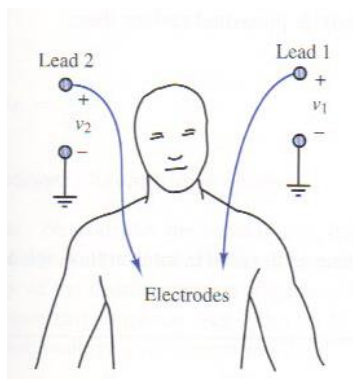
Following Material is for  
Reference Only

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## Applications of Op-Amps

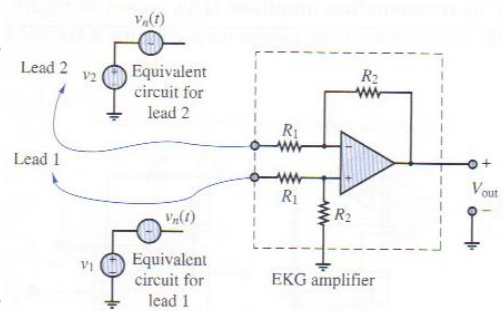
### Electrocardiogram (EKG) Amplification •

- Need to measure difference in voltage from lead 1 and lead 2 •
- 60 Hz interference from electrical equipment •

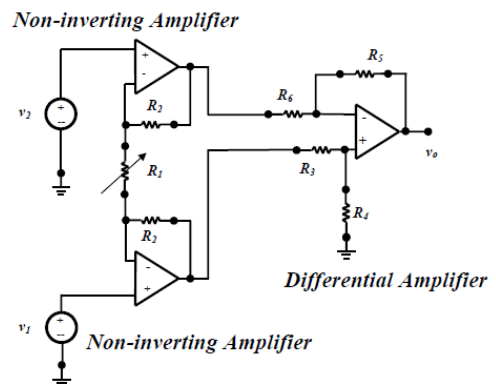


## Applications of Op-Amps

- Simple EKG circuit •
- Uses differential amplifier •
- to cancel common mode signal and amplify differential mode signal

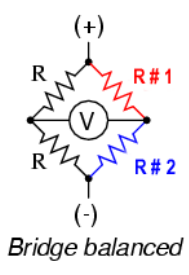
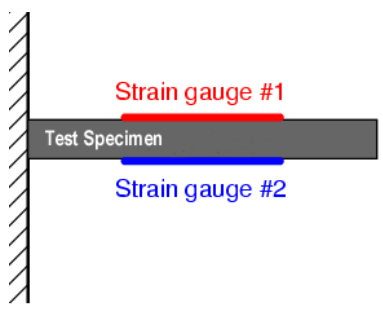


- Realistic EKG circuit •
- Uses two non-inverting •
- amplifiers to first amplify voltage from each lead, followed by differential amplifier
- Forms an •
- “instrumentation amplifier”

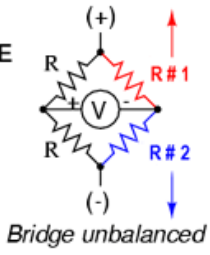
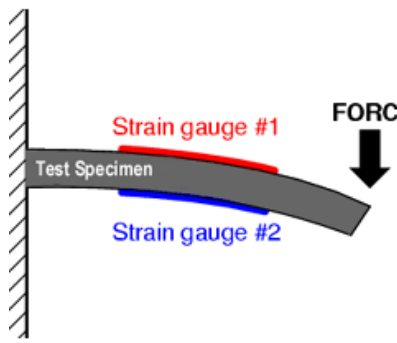




# Strain Gauge



Use a Wheatstone bridge to determine the strain of an element by measuring the change in resistance of a strain gauge

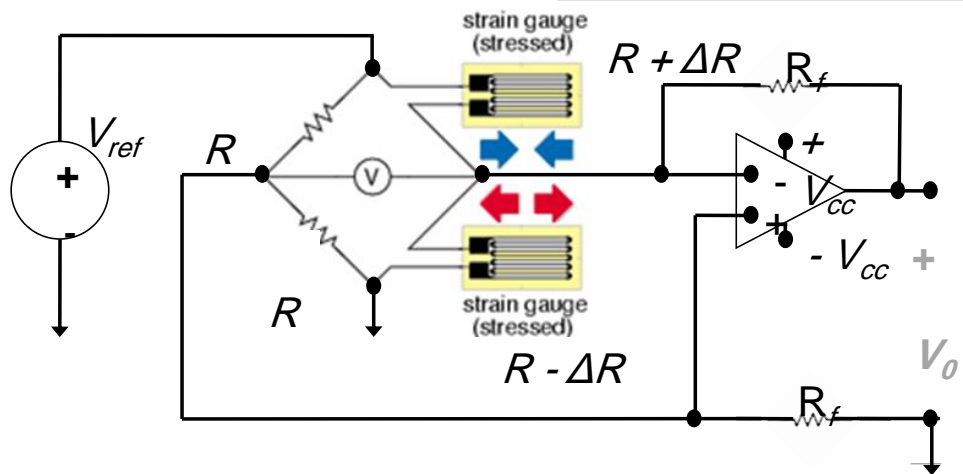


(No strain) Balanced Bridge  
 $R \#1 = R \#2$   
(Strain) Unbalanced Bridge  
 $R \#1 \neq R \#2$

Strain Gauge

Half-Bridge Arrangement

Op amp used to amplify output from strain gauge

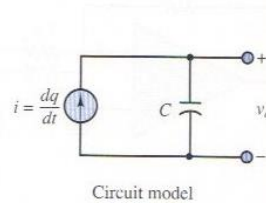
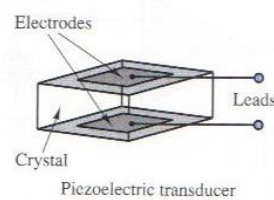


Using KCL at the inverting and non-inverting terminals of the op amp we find that  $\rightarrow V_o \sim \frac{2\Delta R(R_f)}{R^2}$

## Applications of Op-Amps

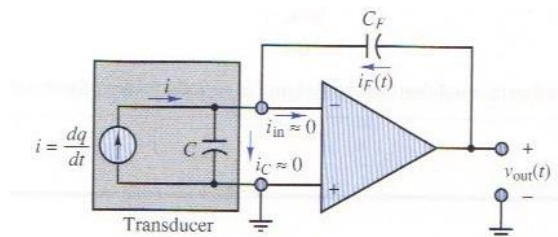
### Piezoelectric Transducer •

- Used to measure force, pressure, acceleration •
- Piezoelectric crystal generates an electric charge in •
- response to deformation



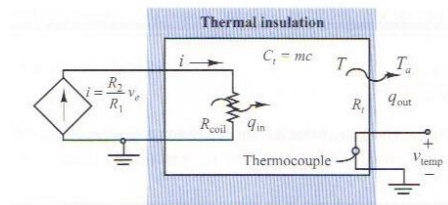
### Use Charge Amplifier •

- Just an integrator op-amp circuit •



## Applications of Op-Amps

- Example of PI Control:  
 Temperature Control
- Thermal System we wish  
 to automatically control  
 the temperature of:



- Block Diagram of Control  
 System:

