Revision — Chapter [8]

How to Solve RLC Circuit ?

① find initial condition ($V_c(0^-)$ $i_L(0^-)$)

② After changing the switch, determine if it's Parallel or Series

- **Parallel RLC Circuit** :-

(بالزبط ← مهم)

$$\alpha = \frac{1}{2RC} \quad , \quad w_0 = \frac{1}{\sqrt{LC}}$$

① if $\alpha^2 > w_0^2 \Rightarrow$ overdamping

$$V(t) = Ae^{s_1 t} + Be^{s_2 t}$$

$$s_{1,2} = -\alpha \mp \sqrt{\alpha^2 - w_0^2}$$

② if $w_0^2 = \alpha_0^2 \Rightarrow$ Critically damped

$$V(t) = Ae^{-\alpha t} + Bt\,e^{-\alpha t}$$

③ if $\alpha^2 < w_0^2 \Rightarrow$ under damping

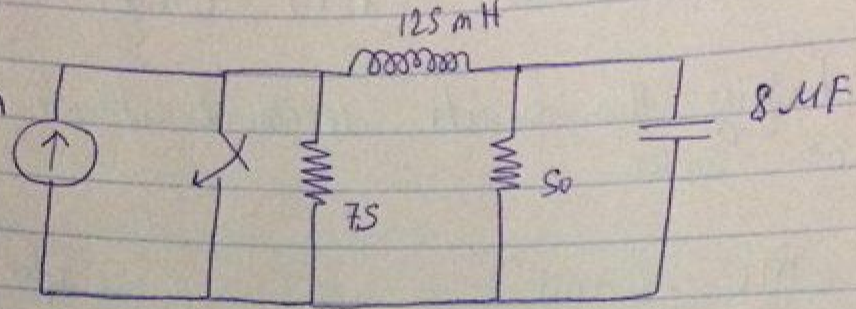
$$V(t) = e^{-\alpha t}\left[A\cos w_d t + B\sin w_d t\right]$$

$$w_d = \sqrt{w_0^2 - \alpha^2}$$

Series RLC circuit

$\alpha = \frac{R}{2L}$, the equations depend on $i(t)$

Ex

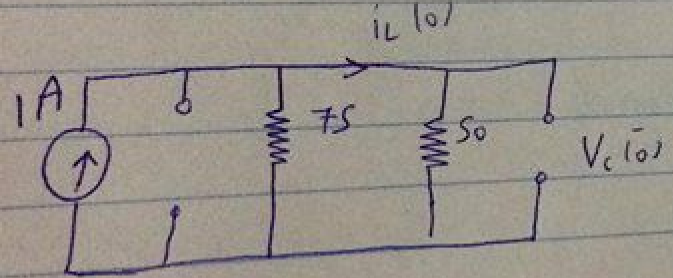


find $i_L(t)$?

Solution

* نبحث الحل عند الحلول بعد الشرط ① الابتدائية أولاً $i_L(\bar{0})$, $V_c(\bar{0})$

At $t<0$, $t=\bar{0}$ Inductor: short CKT
Capacitor: open CKT

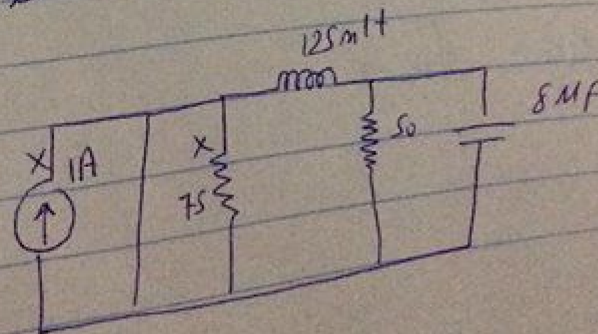


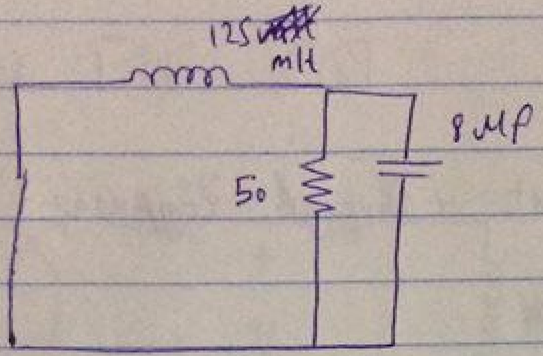
$i_L(\bar{0}) = \frac{75}{125} * 1 = 0.6\ A$

$V_c(\bar{0}) = 50 * \frac{6}{10} = 30$ Volt

② نفحص هل هي Series أو Parallel
عند $t>0$ ونجد $V(t)$ أو $I(t)$

At $t>0$ ⇒





Parallel ⇒ find $V(t)$

$\alpha = \frac{1}{2RC} = \frac{1}{2*50*8*10^{-6}} = 1250$

$w = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{125*10^{-3}*8*10^{-6}}} = 1000$

$\alpha^2 > w^2$ ⇒ overdamp

$V(t) = Ae^{s_1 t} + Be^{s_2 t}$

ناقص

$S_{1,2} = -\alpha \mp \sqrt{\alpha^2 - w_0^2}$ ⇒ $-1250 \mp \sqrt{562500}$ (750)

-1250 ∓ 750

الصح

$S_1 = -2000 \;,\; S_2 = -500$

⇒ $-2500 \;,\; -10000$ خطأ

$V(t) = Ae^{-2500t} + Be^{-1000t}$

$L\frac{di}{dt} = V_L(t)$

$i(t) = \frac{1}{L}\int v(t)\,dt - i(0)$

$V(0) = 30$ ⇒ $A + B = 30$

2nd equation

$0 = i_c + i_c + i_R$

ناقص

$0 = C\frac{dv(t)}{dt} + \frac{V(t)}{R} + \frac{1}{L}\int_0^{t=0} V(t)\,dt - i(0)$

$0 = C\frac{dV(0)}{dt} + \frac{V(0)}{R} + 0 - 0.6$ (0.6)

$0 = C\frac{dV(0)}{dt}$ ⇒ $\frac{dV(0)}{dt} = 0$

$\frac{dv(t)}{dt} = -2500A - 10000B = 0$

$A = 40$

$B = -10$

$V(t) = 40e^{-2500t} - 10e^{-10000t}$

$i(t)$ = Natural component + forced component

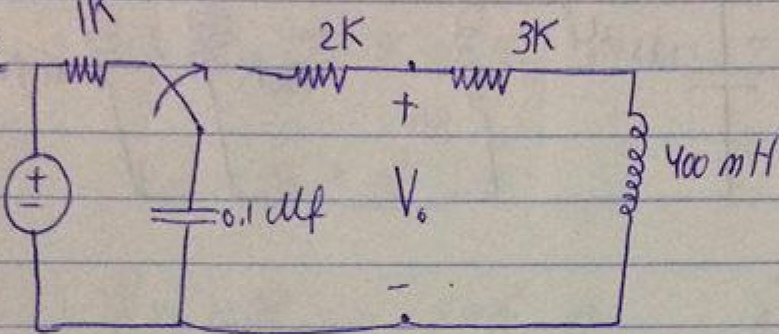
$$i_L(t) = \frac{1}{L}\int_0^t V(t)\,dt$$

$$= \frac{1}{L}\left[\frac{-40}{4000}e^{-2500t} + \frac{10}{10000}e^{-10000t}\right]$$

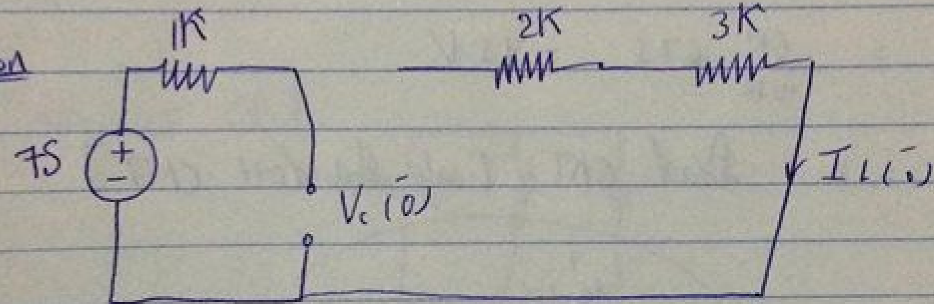
$$i_L(t) = -0.64e^{-2500t} + 0.04e^{-10000t}$$

Series RLC CKT

Ex



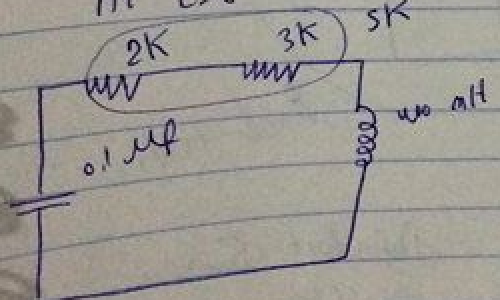
Solution



At $t<0$

$I_L(0^-) = 0$ $V_c(0^-) = 75$ Volt

At $t>0$



Series

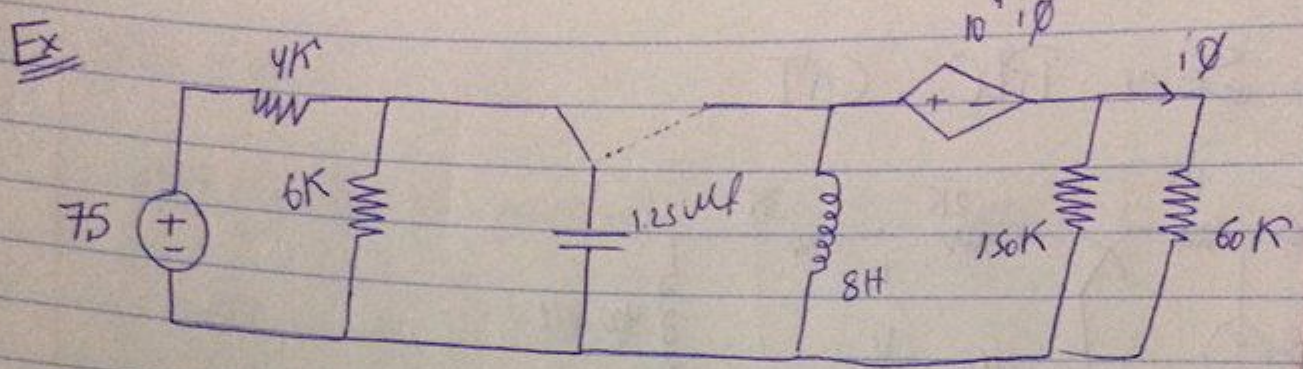
$\alpha = \frac{R}{2L} = 6250$

$\omega_0 = \frac{1}{\sqrt{LC}} = 5000$

] overdamp

$i(t) = \ldots$

Ex

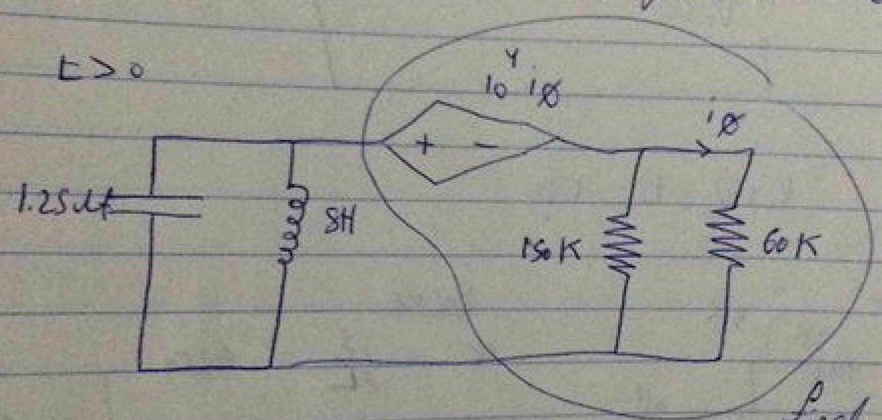


Solution $t < 0$, $t = 0^-$

$V_C(0^-) = \frac{6K}{10K} * 75 = 45\,V$

$i_L(0^-) = 0$ Dead CKT [only Dependent CKT]

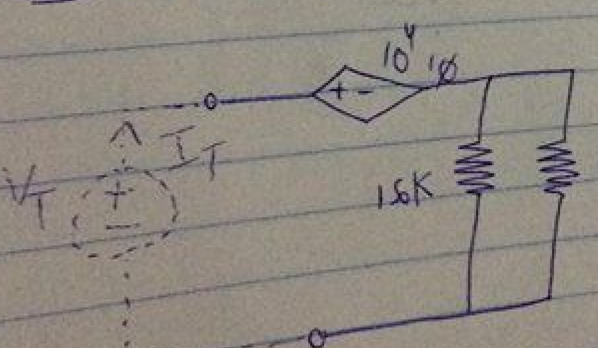
$t > 0$



Find Rth

Note to have RLC CKT

Dependent Source ⇒ use Test Source



$-V_T + 10^4 i_\phi + 150*10^3 (I_T - I_\phi) = 0$

$i_\phi = \frac{150}{210} I_T$ CDR

$V_T = 10^4 I_\phi + 150 * 10^3 (I_T - I_\phi)$

$\Rightarrow V_T = 7.1 * 10^3 I_T + (0.28 I_T)(150 * 10^3)$

$V_T = 50 * 10^3 I_T$

$\frac{V_T}{I_T} = R = 50 * 10^3 \Omega$

$\Rightarrow$ our CKT