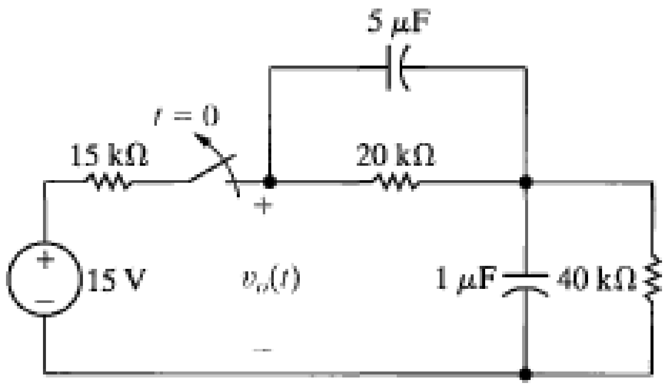
Assessment 7.4

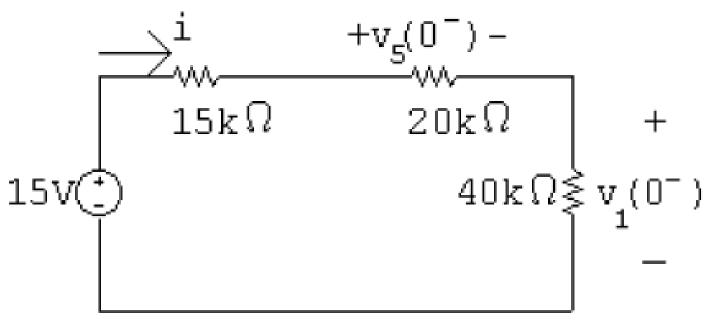
The switch in the circuit shown has been closed for a long time before being opened at $t = 0$.

Find $v_o(t)$ for $t > 0$.



Solution

This circuit is actually two RC circuits in series, and the requested voltage, v_o, is the sum of the voltage drops for the two RC circuits. The circuit for $t < 0$ is shown below:



Find the current in the loop and use it to find the initial voltage drops across the two RC circuits:

$$i = \frac{15}{75{,}000} = 0.2\,\text{mA}, \qquad v_5(0^-) = 4\,\text{V}, \qquad v_1(0^-) = 8\,\text{V}$$

There are two time constants in the circuit, one for each RC subcircuit. τ_5 is the time constant for the $5\,\mu\text{F} - 20\,\text{k}\Omega$ subcircuit, and τ_1 is the time constant for the $1\,\mu\text{F} - 40\,\text{k}\Omega$ subcircuit:

$\tau_5 = (20 \times 10^3)(5 \times 10^{-6}) = 100\,\text{ms}; \qquad \tau_1 = (40 \times 10^3)(1 \times 10^{-6}) = 40\,\text{ms}$

Therefore,

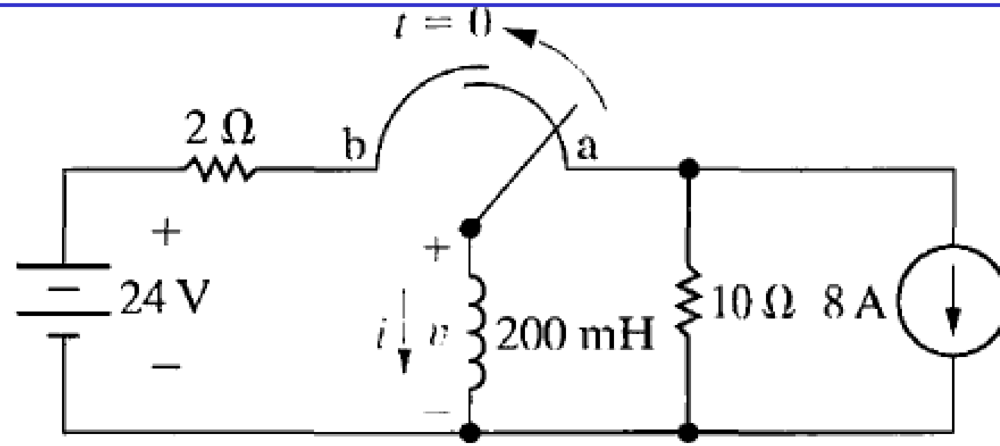
$v_5(t) = v_5(0^-)e^{-t/\tau_5} = 4e^{-t/0.1} = 4e^{-10t}\,\text{V}, \quad t \geq 0$

$v_1(t) = v_1(0^-)e^{-t/\tau_1} = 8e^{-t/0.04} = 8e^{-25t}\,\text{V}, \quad t \geq 0$

Finally,

$v_o(t) = v_1(t) + v_5(t) = [8e^{-25t} + 4e^{-10t}]\,\text{V}, \qquad t \geq 0$

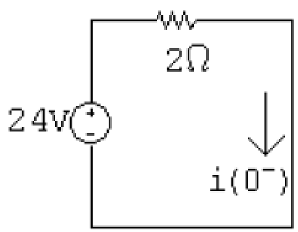
Assessment 7.5



Assume that the switch in the circuit shown has been in position b for a long time, and at $t = 0$ it moves to position

Find : *i(t)*, $t > 0$; and *v(t)*, $t > 0_{+}$.

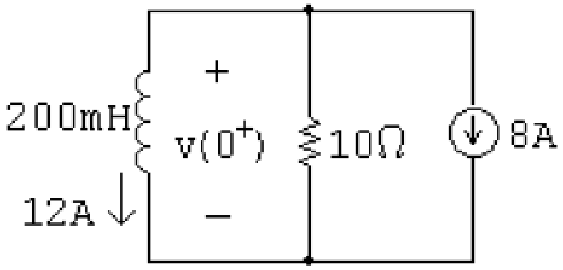
Solution Use the circuit at $t < 0$, shown below, to calculate the initial current in the inductor:



$$i(0^-) = 24/2 = 12\,\text{A} = i(0^+)$$

Note that $i(0^-) = i(0^+)$ because the current in an inductor is continuous.

Use the circuit at $t = 0^+$, shown below, to calculate the voltage drop across the inductor at 0^+. Note that this is the same as the voltage drop across the $10\,\Omega$ resistor, which has current from two sources — 8 A from the current source and 12 A from the initial current through the inductor.

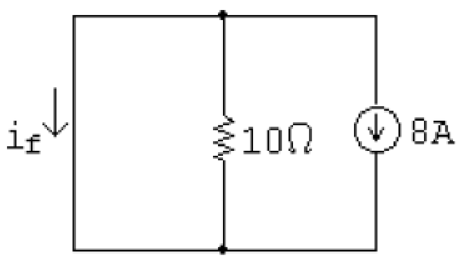


$$v(0^+) = -10(8 + 12) = -200\,\text{V}$$

To calculate the time constant we need the equivalent resistance seen by the inductor for $t > 0$. Only the $10\,\Omega$ resistor is connected to the inductor for $t > 0$. Thus,

$$\tau = L/R = (200 \times 10^{-3}/10) = 20\,\text{ms}$$

To find $i(t)$, we need to find the final value of the current in the inductor. When the switch has been in position a for a long time, the circuit reduces to the one below:



Note that the inductor behaves as a short circuit and all of the current from the 8 A source flows through the short circuit. Thus,

$i_f = -8\,\mathrm{A}$

Now,

$$i(t) = i_f + [i(0^+) - i_f]e^{-t/\tau} = -8 + [12 - (-8)]e^{-t/0.02}$$
$$= -8 + 20e^{-50t}\,\mathrm{A}, \quad t \geq 0$$

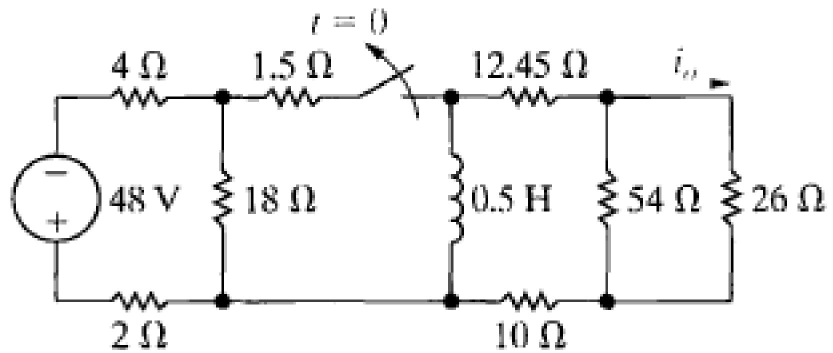
To find $v(t)$, use the relationship between voltage and current for an inductor:

$$v(t) = L\frac{di(t)}{dt} = (200 \times 10^{-3})(-50)(20e^{-50t}) = -200e^{-50t}\,\mathrm{V}, \qquad t \geq 0^+$$

Problem 7.6

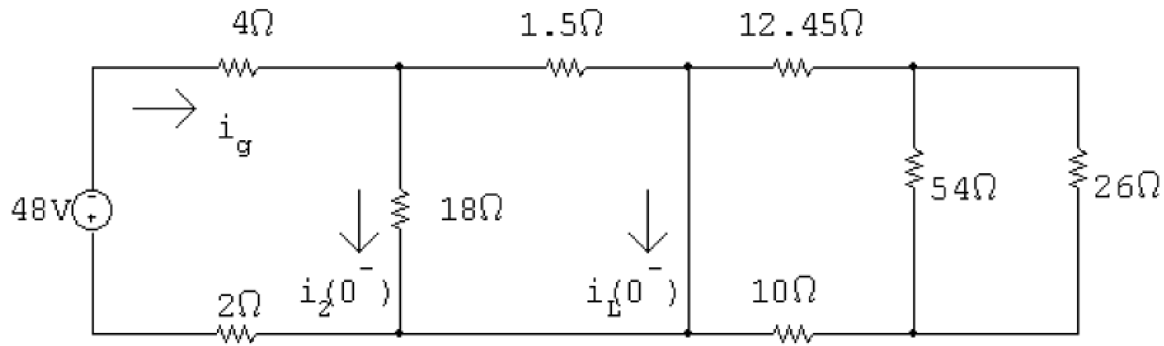
The switch in the circuit has been closed a long time. At $t = 0$ it is opened.

Find $i_0(t)$ for $t > 0$.



Solution

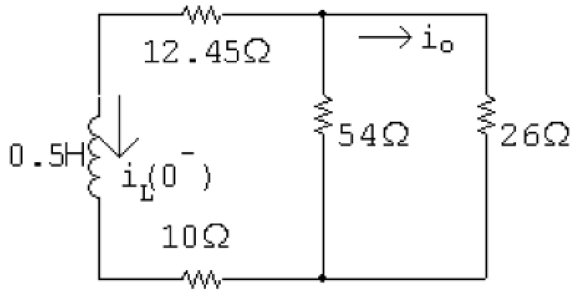
For $t < 0$



$$i_g = \frac{-48}{6 + (18\|1.5)} = -6.5\,\text{A}$$

$$i_L(0^-) = \frac{18}{18 + 1.5}(-6.5) = -6\,\text{A} = i_L(0^+)$$

For $t > 0$



$$i_L(t) = i_L(0^+)e^{-t/\tau}\,\text{A}, \qquad t \ge 0$$

$$\tau = \frac{L}{R} = \frac{0.5}{10 + 12.45 + (54\|26)} = 0.0125\,\text{s}; \qquad \frac{1}{\tau} = 80$$

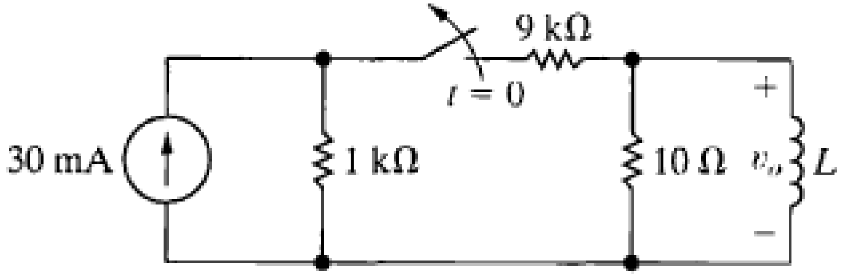
$$i_L(t) = -6e^{-80t}\,\text{A}, \qquad t \ge 0$$

$$i_o(t) = \frac{54}{80}(-i_L(t)) = \frac{54}{80}(6e^{-80t}) = 4.05e^{-80t}\,\text{V}, \qquad t \ge 0^+$$

Problem 7.10

In the circuit ,the switch has been closed for a long time before opening at $t = 0$.

Find the value of L so that $v_0(t)$ equals to 0.5 $v_0(0+)$ when $t = 1$ ms



Solution

$$v_o(t) = v_o(0^+)e^{-t/\tau}$$

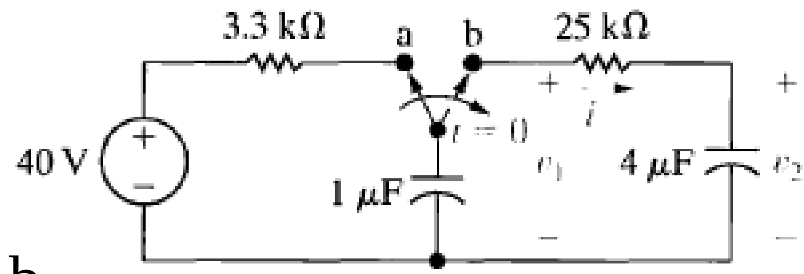
$$\therefore \quad v_o(0^+)e^{-10^{-3}/\tau} = 0.5v_o(0^+)$$

$$\therefore \quad e^{10^{-3}/\tau} = 2$$

$$\therefore \quad \tau = \frac{L}{R} = \frac{10^{-3}}{\ln 2}$$

$$\therefore \quad L = \frac{10 \times 10^{-3}}{\ln 2} = 14.43\,\mathrm{mH}$$

Problem 7.23



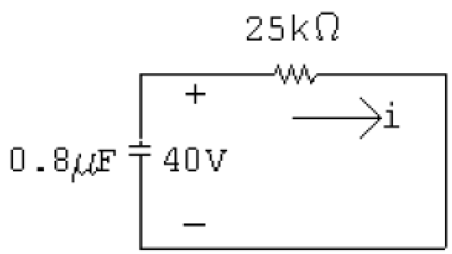
The switch in the circuit has been in position a for a long time and $v_2 = 0$ V. At $t = 0$, the switch is thrown to position b. Calculate

i, v_1 and v_2 for $t > 0+$.

Solution

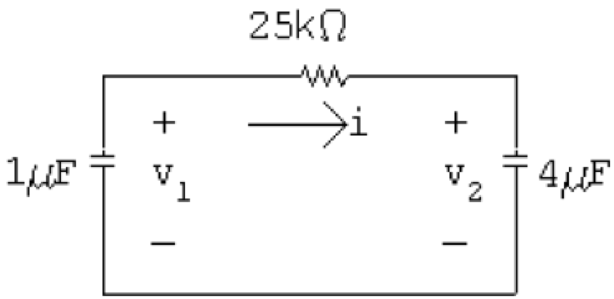
P 7.23 **[a]** $v_1(0^-) = v_1(0^+) = 40\,\text{V}$ $\qquad v_2(0^+) = 0$

$$C_{\text{eq}} = (1)(4)/5 = 0.8\,\mu\text{F}$$



$$\tau = (25 \times 10^3)(0.8 \times 10^{-6}) = 20\text{ms}; \qquad \frac{1}{\tau} = 50$$

$$i = \frac{40}{25{,}000} e^{-50t} = 1.6e^{-50t}\,\text{mA}, \qquad t \geq 0^{+}$$



$$v_1 = \frac{-1}{10^{-6}} \int_0^t 1.6 \times 10^{-3} e^{-50x}\,dx + 40 = 32e^{-50t} + 8\,\text{V}, \qquad t \geq 0$$

$$v_2 = \frac{1}{4 \times 10^{-6}} \int_0^t 1.6 \times 10^{-3} e^{-50x}\,dx + 0 = -8e^{-50t} + 8\,\text{V}, \qquad t \geq 0$$

Answer

i(t)= $1.6e^{-50t}\,\text{mA}, \qquad t \geq 0^{+}$

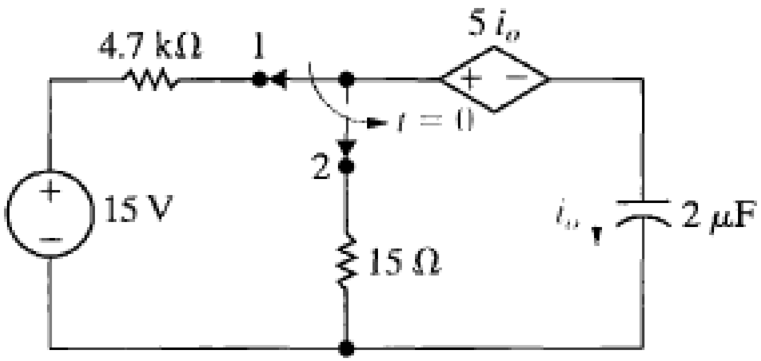
V1(t)= $32e^{-50t} + 8\,\text{V}, \qquad t \geq 0$

V2(t)= $-8e^{-50t} + 8\,\text{V}, \qquad t \geq 0$

Problem 7.30

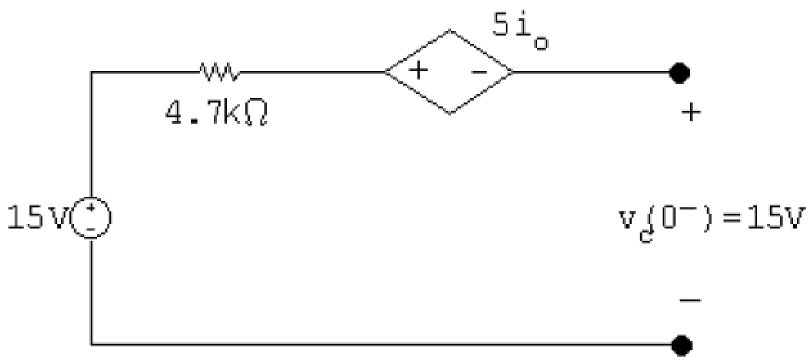
The switch in the circuit has been in position 1 for a long time before moving to position 2 at $t = 0$.

Find $i_o(t)$ for $t > 0+$.

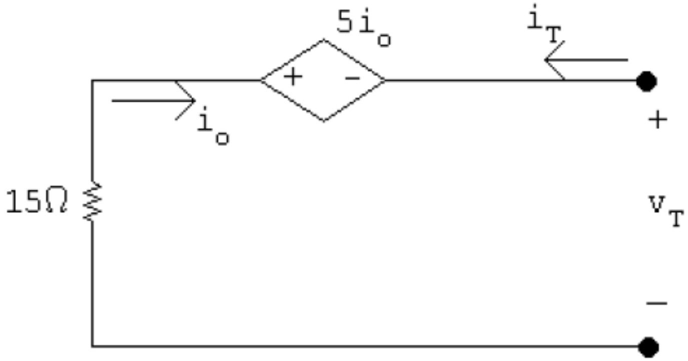


Solution

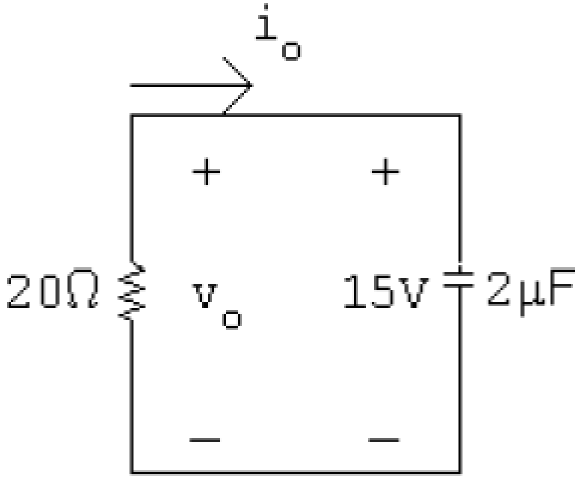
$t < 0$



$t > 0$



$$v_T = -5i_o - 15i_o = -20i_o = 20i_T \quad \therefore \quad R_{\text{Th}} = \frac{v_T}{i_T} = 20\,\Omega$$



$$\tau = RC = 40\,\mu\text{s}; \qquad \frac{1}{\tau} = 25{,}000$$

$$v_o = 15e^{-25{,}000t}\,\text{V}, \qquad t \geq 0$$

$$i_o = -\frac{v_o}{20} = -0.75e^{-25{,}000t}\,\text{A}, \qquad t \geq 0^+$$

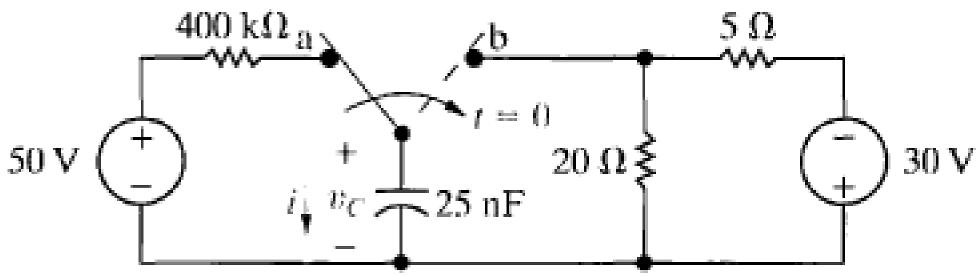
Problem 7.55

Assume that the switch in the circuit has been in position a for a long time and that at $t = 0$ it is moved to position b.

Find

(a) v_C, $t > 0$;

(b) i, $t > 0+$.



$$v_c(0^+) = 50\,\text{V}$$

Use voltage division to find the final value of voltage:

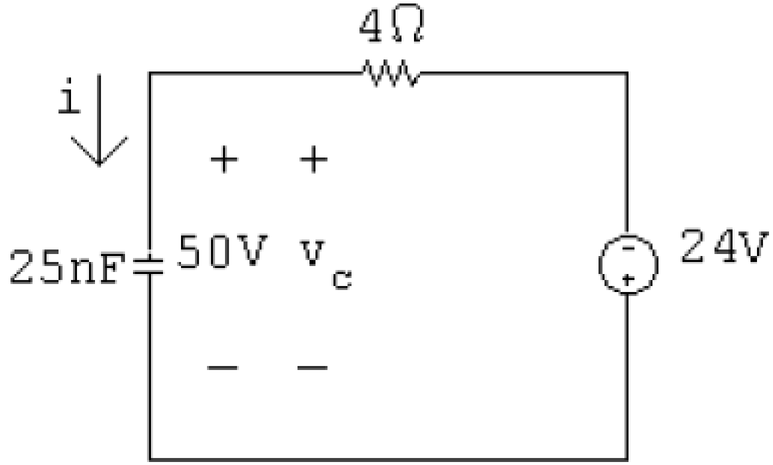
$$v_c(\infty) = \frac{20}{20+5}(-30) = -24\,\text{V}$$

Find the Thévenin equivalent with respect to the terminals of the capacitor:

$$V_{\mathrm{Th}} = -24\,\mathrm{V}, \qquad R_{\mathrm{Th}} = 20\|5 = 4\,\Omega,$$

Therefore $\tau = R_{\mathrm{eq}}C = 4(25 \times 10^{-9}) = 0.1\,\mu\mathrm{s}$

The simplified circuit for $t > 0$ is:



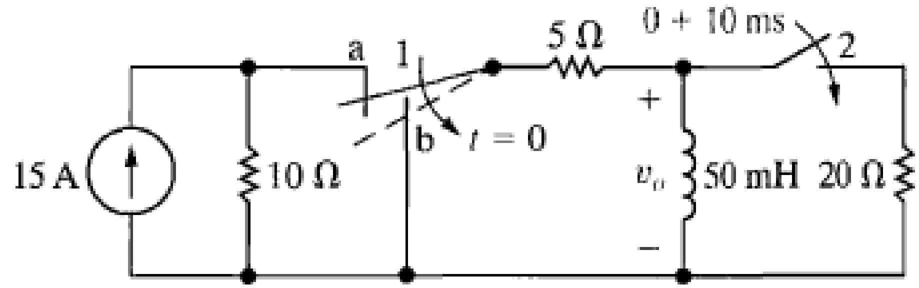
$$i(0^+) = \frac{-24 - 50}{4} = -18.5\,\mathrm{A}$$

$$v_c = v_c(\infty) + [v_c(0^+) - v_c(\infty)]e^{-t/\tau}$$

$$= -24 + [50 - (-24)]e^{-t/\tau} = -24 + 74e^{-10^7 t}\,\text{V}, \qquad t \geq 0$$

$$i = C\frac{dv_c}{dt} = (25 \times 10^{-9})(-10^7)(74e^{-10^7 t}) = -18.5e^{-10^7 t}\,\text{A}, \qquad t \geq 0^+$$

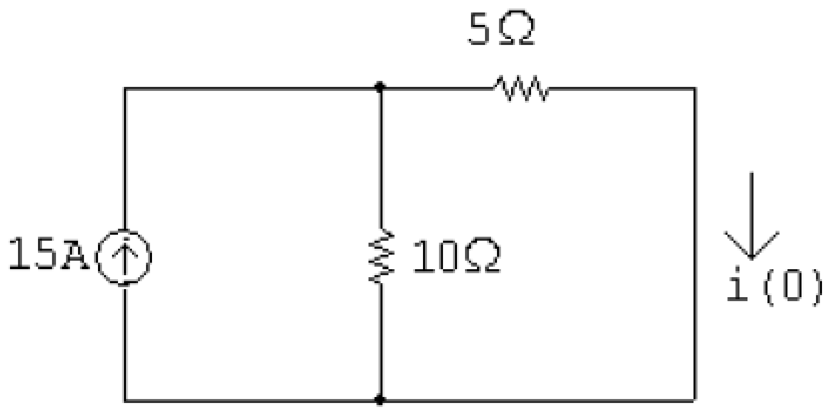
Problem 7.71



The action of the two switches in the circuit seen in is as follows.
For $t < 0$, switch 1 is in position a and switch 2 is open.
This state has existed for a long time.
At $t = 0$, switch 1 moves instantaneously from position a to position b, while switch 2 remains open. Ten milliseconds after switch 1 operates, switch 2 closes, remains closed for 10 ms and then opens.

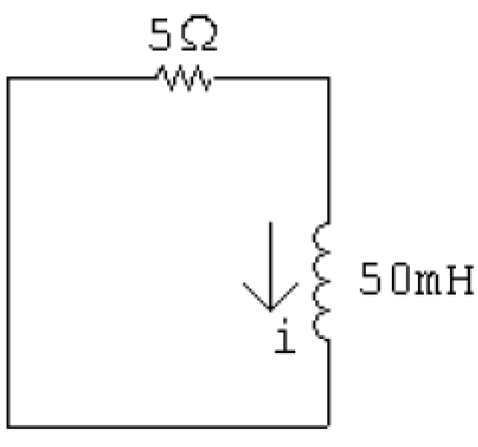
Find $v_o(t)$ for t > 20 ms.

For $t < 0$:



$$i(0) = \frac{10}{15}(15) = 10\,\text{A}$$

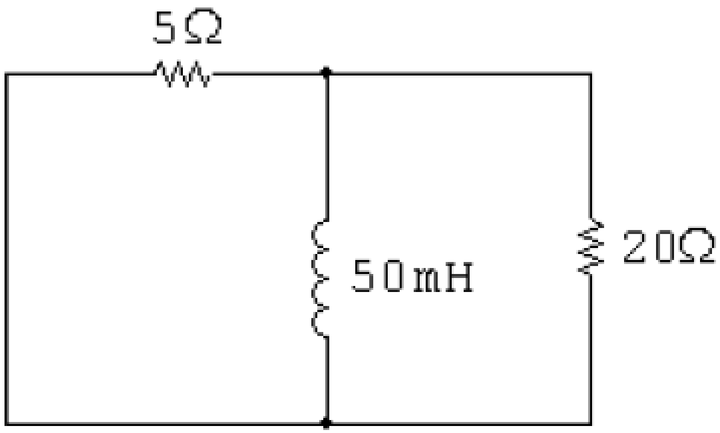
$0 \leq t \leq 10\,\text{ms}$:



$$i = 10e^{-100t}\,\text{A}$$

$$i(10\,\text{ms}) = 10e^{-1} = 3.68\,\text{A}$$

$10\,\text{ms} \le t \le 20\,\text{ms}$:



$$R_{\text{eq}} = \frac{(5)(20)}{25} = 4\,\Omega$$

$$\frac{1}{\tau} = \frac{R}{L} = \frac{4}{50 \times 10^{-3}} = 80$$

$$i = 3.68e^{-80(t-0.01)}\ \text{A}$$

$20\,\text{ms} \le t < \infty$:

$$i(20\,\text{ms}) = 3.68e^{-80(0.02-0.01)} = 1.65\,\text{A}$$

$$i = 1.65e^{-100(t-0.02)}\ \text{A}$$

$$v_o = L\frac{di}{dt}; \qquad L = 50\,\text{mH}$$

$$\frac{di}{dt} = 1.65(-100)e^{-100(t-0.02)} = -165e^{-100(t-0.02)}$$

$$v_o = (50 \times 10^{-3})(-165)e^{-100(t-0.02)}$$

$$= -8.26e^{-100(t-0.02)}\,\mathrm{V}, \qquad t > 20^{+}\,\mathrm{ms}$$

$$v_o(25\,\mathrm{ms}) = -8.26e^{-100(0.025-0.02)} = -5.013\,\mathrm{V}$$