1

Circuit Variables

Assessment Problems

AP 1.1 Use a product of ratios to convert two-thirds the speed of light from meters per second to miles per second:

$$\left(\frac{2}{3}\right)\frac{3\times10^8\text{ m}}{1\text{ s}}\cdot\frac{100\text{ cm}}{1\text{ m}}\cdot\frac{1\text{ in}}{2.54\text{ cm}}\cdot\frac{1\text{ ft}}{12\text{ in}}\cdot\frac{1\text{ mile}}{5280\text{ feet}}=\frac{124{,}274.24\text{ miles}}{1\text{ s}}$$

Now set up a proportion to determine how long it takes this signal to travel 1100 miles:

$$\frac{124{,}274.24\text{ miles}}{1\text{ s}}=\frac{1100\text{ miles}}{x\text{ s}}$$

Therefore,

$$x=\frac{1100}{124{,}274.24}=0.00885=8.85\times10^{-3}\text{ s}=8.85\text{ ms}$$

AP 1.2 To solve this problem we use a product of ratios to change units from dollars/year to dollars/millisecond. We begin by expressing \$10 billion in scientific notation:

$$\$100\text{ billion }=\$100\times10^9$$

Now we determine the number of milliseconds in one year, again using a product of ratios:

$$\frac{1\text{ year}}{365.25\text{ days}}\cdot\frac{1\text{ day}}{24\text{ hours}}\cdot\frac{1\text{ hour}}{60\text{ mins}}\cdot\frac{1\text{ min}}{60\text{ secs}}\cdot\frac{1\text{ sec}}{1000\text{ ms}}=\frac{1\text{ year}}{31.5576\times10^9\text{ ms}}$$

Now we can convert from dollars/year to dollars/millisecond, again with a product of ratios:

$$\frac{\$100\times10^9}{1\text{ year}}\cdot\frac{1\text{ year}}{31.5576\times10^9\text{ ms}}=\frac{100}{31.5576}=\$3.17/\text{ms}$$

AP 1.3 Remember from Eq. (1.2), current is the time rate of change of charge, or $i = \frac{dq}{dt}$ In this problem, we are given the current and asked to find the total charge. To do this, we must integrate Eq. (1.2) to find an expression for charge in terms of current:

$$q(t) = \int_0^t i(x)\,dx$$

We are given the expression for current, i, which can be substituted into the above expression. To find the total charge, we let $t \to \infty$ in the integral. Thus we have

$$q_{\text{total}} = \int_0^\infty 20e^{-5000x}\,dx = \frac{20}{-5000}e^{-5000x}\Big|_0^\infty = \frac{20}{-5000}(e^{-\infty} - e^0)$$

$$= \frac{20}{-5000}(0-1) = \frac{20}{5000} = 0.004 \text{ C} = 4000\,\mu\text{C}$$

AP 1.4 Recall from Eq. (1.2) that current is the time rate of change of charge, or $i = \frac{dq}{dt}$. In this problem we are given an expression for the charge, and asked to find the maximum current. First we will find an expression for the current using Eq. (1.2):

$$i = \frac{dq}{dt} = \frac{d}{dt}\left[\frac{1}{\alpha^2} - \left(\frac{t}{\alpha} + \frac{1}{\alpha^2}\right)e^{-\alpha t}\right]$$

$$= \frac{d}{dt}\left(\frac{1}{\alpha^2}\right) - \frac{d}{dt}\left(\frac{t}{\alpha}e^{-\alpha t}\right) - \frac{d}{dt}\left(\frac{1}{\alpha^2}e^{-\alpha t}\right)$$

$$= 0 - \left(\frac{1}{\alpha}e^{-\alpha t} - \alpha\frac{t}{\alpha}e^{-\alpha t}\right) - \left(-\alpha\frac{1}{\alpha^2}e^{-\alpha t}\right)$$

$$= \left(-\frac{1}{\alpha} + t + \frac{1}{\alpha}\right)e^{-\alpha t}$$

$$= te^{-\alpha t}$$

Now that we have an expression for the current, we can find the maximum value of the current by setting the first derivative of the current to zero and solving for t:

$$\frac{di}{dt} = \frac{d}{dt}(te^{-\alpha t}) = e^{-\alpha t} + t(-\alpha)e^{\alpha t} = (1-\alpha t)e^{-\alpha t} = 0$$

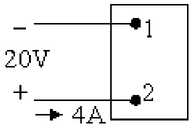
Since $e^{-\alpha t}$ never equals 0 for a finite value of t, the expression equals 0 only when $(1 - \alpha t) = 0$. Thus, $t = 1/\alpha$ will cause the current to be maximum. For this value of t, the current is

$$i = \frac{1}{\alpha}e^{-\alpha/\alpha} = \frac{1}{\alpha}e^{-1}$$

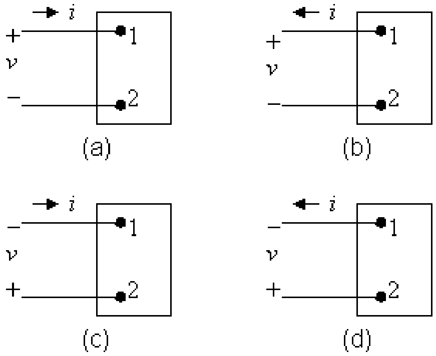
Remember in the problem statement, $\alpha = 0.03679$. Using this value for α,

$$i = \frac{1}{0.03679}e^{-1} \cong 10 \text{ A}$$

AP 1.5 Start by drawing a picture of the circuit described in the problem statement:



Also sketch the four figures from Fig. 1.6:



[**a**] Now we have to match the voltage and current shown in the first figure with the polarities shown in Fig. 1.6. Remember that 4A of current entering Terminal 2 is the same as 4A of current leaving Terminal 1. We get

$$\text{(a) } v = -20\,\text{V}, \qquad i = -4\,\text{A}; \quad \text{(b) } v = -20\,\text{V}, \qquad i = 4\,\text{A}$$

$$\text{(c) } v = 20\,\text{V}, \qquad i = -4\,\text{A}; \qquad \text{(d) } v = 20\,\text{V}, \qquad i = 4\,\text{A}$$

[**b**] Using the reference system in Fig. 1.6(a) and the passive sign convention, $p = vi = (-20)(-4) = 80\,\text{W}$. Since the power is greater than 0, the box is absorbing power.

[**c**] From the calculation in part (b), the box is absorbing 80 W.

AP 1.6 [**a**] Applying the passive sign convention to the power equation using the voltage and current polarities shown in Fig. 1.5, $p = vi$. To find the time at which the power is maximum, find the first derivative of the power with respect to time, set the resulting expression equal to zero, and solve for time:

$$p = (80{,}000te^{-500t})(15te^{-500t}) = 120 \times 10^4 t^2 e^{-1000t}$$

$$\frac{dp}{dt} = 240 \times 10^4 te^{-1000t} - 120 \times 10^7 t^2 e^{-1000t} = 0$$

Therefore,

$$240 \times 10^4 - 120 \times 10^7 t = 0$$

Solving,

$$t = \frac{240 \times 10^4}{120 \times 10^7} = 2 \times 10^{-3} = 2 \text{ ms}$$

[**b**] The maximum power occurs at 2 ms, so find the value of the power at 2 ms:

$$p(0.002) = 120 \times 10^4 (0.002)^2 e^{-2} = 649.6 \text{ mW}$$

[**c**] From Eq. (1.3), we know that power is the time rate of change of energy, or $p = dw/dt$. If we know the power, we can find the energy by integrating Eq. (1.3). To find the total energy, the upper limit of the integral is infinity:

$$\begin{aligned} w_{\text{total}} &= \int_0^\infty 120 \times 10^4 x^2 e^{-1000x}\, dx \\ &= \frac{120 \times 10^4}{(-1000)^3} e^{-1000x}[(-1000)^2 x^2 - 2(-1000)x + 2)\Big|_0^\infty \\ &= 0 - \frac{120 \times 10^4}{(-1000)^3} e^0 (0 - 0 + 2) = 2.4 \text{ mJ} \end{aligned}$$

AP 1.7 At the Oregon end of the line the current is leaving the upper terminal, and thus entering the lower terminal where the polarity marking of the voltage is negative. Thus, using the passive sign convention, $p = -vi$. Substituting the values of voltage and current given in the figure,

$$p = -(800 \times 10^3)(1.8 \times 10^3) = -1440 \times 10^6 = -1440 \text{ MW}$$

Thus, because the power associated with the Oregon end of the line is negative, power is being generated at the Oregon end of the line and transmitted by the line to be delivered to the California end of the line.

Chapter Problems

P 1.1 [a] We can set up a ratio to determine how long it takes the bamboo to grow $10\,\mu$m First, recall that 1 mm = $10^3\mu$m. Let's also express the rate of growth of bamboo using the units mm/s instead of mm/day. Use a product of ratios to perform this conversion:

$$\frac{250\text{ mm}}{1\text{ day}}\cdot\frac{1\text{ day}}{24\text{ hours}}\cdot\frac{1\text{ hour}}{60\text{ min}}\cdot\frac{1\text{ min}}{60\text{ sec}}=\frac{250}{(24)(60)(60)}=\frac{10}{3456}\text{ mm/s}$$

Use a ratio to determine the time it takes for the bamboo to grow $10\,\mu$m:

$$\frac{10/3456\times10^{-3}\text{ m}}{1\text{ s}}=\frac{10\times10^{-6}\text{ m}}{x\text{ s}}\qquad\text{so}\qquad x=\frac{10\times10^{-6}}{10/3456\times10^{-3}}=3.456\text{ s}$$

[b] $$\frac{1\text{ cell length}}{3.456\text{ s}}\cdot\frac{3600\text{ s}}{1\text{ hr}}\cdot\frac{(24)(7)\text{ hr}}{1\text{ week}}=175{,}000\text{ cell lengths/week}$$

P 1.2 Volume = area × thickness

Convert values to millimeters, noting that 10 $\text{m}^2=10^6\text{ mm}^2$

$$10^6=(10\times10^6)(\text{thickness})$$

$$\Rightarrow\ \text{thickness}=\frac{10^6}{10\times10^6}=0.10\text{ mm}$$

P 1.3 $$\frac{(260\times10^6)(540)}{10^9}=104.4\text{ gigawatt-hours}$$

P 1.4 [a] $$\frac{20{,}000\text{ photos}}{(11)(15)(1)\text{ mm}^3}=\frac{x\text{ photos}}{1\text{ mm}^3}$$

$$x=\frac{(20{,}000)(1)}{(11)(15)(1)}=121\text{ photos}$$

[b] $$\frac{16\times2^{30}\text{ bytes}}{(11)(15)(1)\text{ mm}^3}=\frac{x\text{ bytes}}{(0.2)^3\text{ mm}^3}$$

$$x=\frac{(16\times2^{30})(0.008)}{(11)(15)(1)}=832{,}963\text{ bytes}$$

P 1.5 $$\frac{(480)(320)\text{ pixels}}{1\text{ frame}}\cdot\frac{2\text{ bytes}}{1\text{ pixel}}\cdot\frac{30\text{ frames}}{1\text{ sec}}=9.216\times10^6\text{ bytes/sec}$$

$$(9.216\times10^6\text{ bytes/sec})(x\text{ secs})=32\times2^{30}\text{ bytes}$$

$$x=\frac{32\times2^{30}}{9.216\times10^6}=3728\text{ sec}=62\text{ min}\approx1\text{ hour of video}$$

P 1.6 $$(4\text{ cond.})\cdot(845\text{ mi})\cdot\frac{5280\text{ ft}}{1\text{ mi}}\cdot\frac{2526\text{ lb}}{1000\text{ ft}}\cdot\frac{1\text{ kg}}{2.2\text{ lb}} = 20.5\times10^6\text{ kg}$$

P 1.7 $$w = qV = (1.6022\times10^{-19})(6) = 9.61\times10^{-19} = 0.961\text{ aJ}$$

P 1.8 $$n = \frac{35\times10^{-6}\text{ C/s}}{1.6022\times10^{-19}\text{ C/elec}} = 2.18\times10^{14}\text{ elec/s}$$

P 1.9 $$\text{C/m}^3 = \frac{1.6022\times10^{-19}\text{ C}}{1\text{ electron}}\times\frac{10^{29}\text{ electrons}}{1\text{ m}^3} = 1.6022\times10^{10}\text{ C/m}^3$$

$$\text{Cross-sectional area of wire} = (0.4\times10^{-2}\text{ m})(16\times10^{-2}\text{ m}) = 6.4\times10^{-4}\text{ m}^2$$

$$\text{C/m} = (1.6022\times10^{10}\text{C/m}^3)(6.4\times10^{-4}\text{m}^2) = 10.254\times10^6\text{ C/m}$$

$$\text{Therefore, } i\left(\frac{\text{C}}{\text{sec}}\right) = (10.254\times10^6)\left(\frac{\text{C}}{\text{m}}\right)\times\text{avg vel}\left(\frac{\text{m}}{\text{s}}\right)$$

$$\text{Thus, average velocity } = \frac{i}{10.254\times10^6} = \frac{1600}{10.254\times10^6} = 156.04\,\mu\text{m/s}$$

P 1.10 First we use Eq. (1.2) to relate current and charge:

$$i = \frac{dq}{dt} = 20\cos 5000t$$

Therefore, $dq = 20\cos 5000t\,dt$

To find the charge, we can integrate both sides of the last equation. Note that we substitute x for q on the left side of the integral, and y for t on the right side of the integral:

$$\int_{q(0)}^{q(t)} dx = 20\int_0^t \cos 5000y\,dy$$

We solve the integral and make the substitutions for the limits of the integral, remembering that $\sin 0 = 0$:

$$q(t) - q(0) = 20\frac{\sin 5000y}{5000}\Bigg|_0^t = \frac{20}{5000}\sin 5000t - \frac{20}{5000}\sin 5000(0) = \frac{20}{5000}\sin 5000t$$

But $q(0) = 0$ by hypothesis, i.e., the current passes through its maximum value at $t = 0$, so $q(t) = 4\times10^{-3}\sin 5000t\,\text{C} = 4\sin 5000t\,\text{mC}$

P 1.11 **[a]** In Car A, the current i is in the direction of the voltage drop across the 12 V battery(the current i flows into the + terminal of the battery of Car A). Therefore using the passive sign convention,

$p = vi = (30)(12) = 360$ W.

Since the power is positive, the battery in Car A is absorbing power, so Car A must have the "dead" battery.

[b] $w(t) = \int_0^t p\,dx; \qquad 1 \text{ min} = 60 \text{ s}$

$$w(60) = \int_0^{60} 360\,dx$$

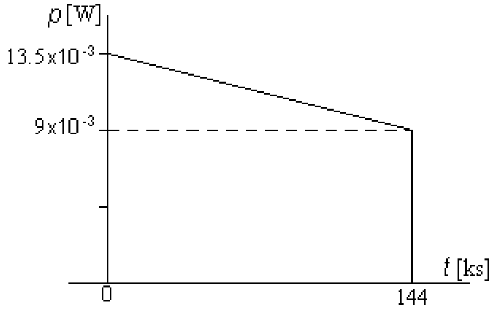
$$w = 360(60-0) = 360(60) = 21{,}600 \text{ J} = 21.6 \text{ kJ}$$

P 1.12 $p = (12)(100 \times 10^{-3}) = 1.2$ W; $\qquad 4 \text{ hr} \cdot \dfrac{3600 \text{ s}}{1 \text{ hr}} = 14{,}400$ s

$$w(t) = \int_0^t p\,dt \qquad w(14{,}400) = \int_0^{14{,}400} 1.2\,dt = 1.2(14{,}400) = 17.28 \text{ kJ}$$

P 1.13 $p = vi; \qquad w = \int_0^t p\,dx$

Since the energy is the area under the power vs. time plot, let us plot p vs. t.



Note that in constructing the plot above, we used the fact that 40 hr = 144,000 s = 144 ks

$$p(0) = (1.5)(9 \times 10^{-3}) = 13.5 \times 10^{-3} \text{ W}$$

$$p(144 \text{ ks}) = (1)(9 \times 10^{-3}) = 9 \times 10^{-3} \text{ W}$$

$$w = (9 \times 10^{-3})(144 \times 10^3) + \frac{1}{2}(13.5 \times 10^{-3} - 9 \times 10^{-3})(144 \times 10^3) = 1620 \text{ J}$$

P 1.14 Assume we are standing at box A looking toward box B. Then, using the passive sign convention $p = -vi$, since the current i is flowing into the − terminal of the voltage v. Now we just substitute the values for v and i into the equation for power. Remember that if the power is positive, B is absorbing power, so the power must be flowing from A to B. If the power is negative, B is generating power so the power must be flowing from B to A.

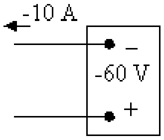
[a] $p = -(125)(10) = -1250$ W 1250 W from B to A

[b] $p = -(-240)(5) = 1200$ W 1200 W from A to B

[c] $p = -(480)(-12) = 5760$ W 5760 W from A to B

[d] $p = -(-660)(-25) = -16{,}500$ W 16,500 W from B to A

P 1.15 **[a]**



$p = vi = (40)(-10) = -400$ W
Power is being delivered by the box.

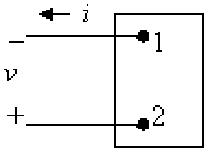
[b] Entering

[c] Gaining

P 1.16 **[a]** $p = vi = (-60)(-10) = 600$ W, so power is being absorbed by the box.

[b] Entering

[c] Losing



P 1.17 **[a]** $p = vi = (0.05e^{-1000t})(75 - 75e^{-1000t}) = (3.75e^{-1000t} - 3.75e^{-2000t})$ W

$$\frac{dp}{dt} = -3750e^{-1000t} + 7500e^{-2000t} = 0 \qquad \text{so} \qquad 2e^{-2000t} = e^{-1000t}$$

$$2 = e^{1000t} \qquad \text{so} \qquad \ln 2 = 1000t \qquad \text{thus} \qquad p \text{ is maximum at } t = 693.15\,\mu\text{s}$$

$$p_{\max} = p(693.15\,\mu\text{s}) = 937.5 \text{ mW}$$

[b] $$w = \int_0^\infty [3.75e^{-1000t} - 3.75e^{-2000t}]\,dt = \left[\frac{3.75}{-1000}e^{-1000t} - \frac{3.75}{-2000}e^{-2000t}\Big|_0^\infty\right]$$

$$= \frac{3.75}{1000} - \frac{3.75}{2000} = 1.875 \text{ mJ}$$

P 1.18 **[a]** $p = vi = 0.25e^{-3200t} - 0.5e^{-2000t} + 0.25e^{-800t}$
$p(625\,\mu\text{s}) = 42.2$ mW

[b]
$$\begin{aligned} w(t) &= \int_0^t (0.25e^{-3200t} - 0.5e^{-2000t} + 0.25e^{-800t}) \\ &= 140.625 - 78.125e^{-3200t} + 250e^{-2000t} - 312.5e^{-800t}\mu\text{J} \\ w(625\,\mu\text{s}) &= 12.14\,\mu\text{J} \end{aligned}$$

[**c**] $w_{\text{total}} = 140.625\,\mu\text{J}$

P 1.19 [**a**] $0\text{ s} \le t < 1\text{ s}$:

$$v = 5\text{ V}; \quad i = 20t\text{ A}; \quad p = 100t\text{ W}$$

$1\text{ s} < t \le 3\text{ s}$:

$$v = 0\text{ V}; \quad i = 20\text{ A}; \quad p = 0\text{ W}$$

$3\text{ s} \le t < 5\text{ s}$:

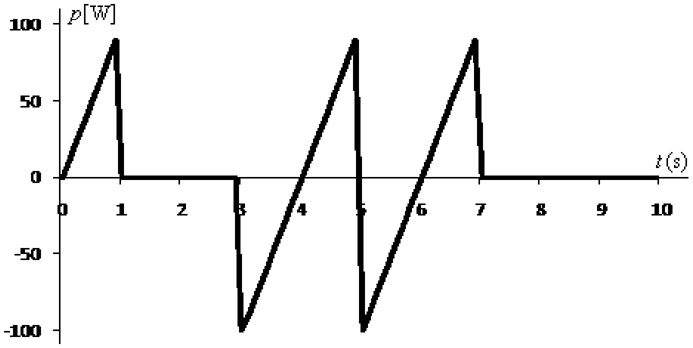
$$v = -5\text{ V}; \quad i = 80 - 20t\text{ A}; \quad p = 100t - 400\text{ W}$$

$5\text{ s} < t \le 7\text{ s}$:

$$v = 5\text{ V}; \quad i = 20t - 120\text{ A}; \quad p = 100t - 600\text{ W}$$

$t > 7\text{ s}$:

$$v = 0\text{ V}; \quad i = 20\text{ A}; \quad p = 0\text{ W}$$



[**b**] Calculate the area under the curve from zero up to the desired time:

$$\begin{aligned} w(1) &= \tfrac{1}{2}(1)(100) = 50\text{ J} \\ w(6) &= \tfrac{1}{2}(1)(100) - \tfrac{1}{2}(1)(100) + \tfrac{1}{2}(1)(100) - \tfrac{1}{2}(1)(100) = 0\text{ J} \\ w(10) &= w(6) + \tfrac{1}{2}(1)(100) = 50\text{ J} \end{aligned}$$

P 1.20 [**a**] $v(10\text{ ms}) = 400e^{-1}\sin 2 = 133.8\text{ V}$
$i(10\text{ ms}) = 5e^{-1}\sin 2 = 1.67\text{ A}$
$p(10\text{ ms}) = vi = 223.80\text{ W}$

[b]
$$\begin{aligned}
p &= vi = 2000e^{-200t}\sin^2 200t \\
&= 2000e^{-200t}\left[\frac{1}{2} - \frac{1}{2}\cos 400t\right] \\
&= 1000e^{-200t} - 1000e^{-200t}\cos 400t \\
w &= \int_0^{\infty} 1000e^{-200t}\,dt - \int_0^{\infty} 1000e^{-200t}\cos 400t\,dt \\
&= 1000\left.\frac{e^{-200t}}{-200}\right|_0^{\infty} \\
&\quad -1000\left.\left\{\frac{e^{-200t}}{(200)^2+(400)^2}\left[-200\cos 400t + 400\sin 400t\right]\right\}\right|_0^{\infty} \\
&= 5 - 1000\left[\frac{200}{4\times 10^4 + 16\times 10^4}\right] = 5 - 1 \\
w &= 4 \text{ J}
\end{aligned}$$

P 1.21 **[a]**

$$\begin{aligned}
p &= vi = [16{,}000t + 20)e^{-800t}][(128t + 0.16)e^{-800t}] \\
&= 2048\times 10^3 t^2 e^{-1600t} + 5120te^{-1600t} + 3.2e^{-1600t} \\
&= 3.2e^{-1600t}[640{,}000t^2 + 1600t + 1] \\
\frac{dp}{dt} &= 3.2\{e^{-1600t}[1280\times 10^3 t + 1600] - 1600e^{-1600t}[640{,}000t^2 + 1600t + 1]\} \\
&= -3.2e^{-1600t}[128\times 10^4(800t^2 + t)] = -409.6\times 10^4 e^{-1600t}t(800t+1)
\end{aligned}$$

Therefore, $\frac{dp}{dt} = 0$ when $t = 0$
so p_{max} occurs at $t = 0$.

[b]
$$\begin{aligned}
p_{\text{max}} &= 3.2e^{-0}[0 + 0 + 1] \\
&= 3.2 \text{ W}
\end{aligned}$$

[c]
$$\begin{aligned}
w &= \int_0^t p dx \\
\frac{w}{3.2} &= \int_0^t 640{,}000x^2 e^{-1600x}\,dx + \int_0^t 1600xe^{-1600x}\,dx + \int_0^t e^{-1600x}\,dx \\
&= \left.\frac{640{,}000e^{-1600x}}{-4096\times 10^6}[256\times 10^4 x^2 + 3200x + 2]\right|_0^t + \\
&\quad \left.\frac{1600e^{-1600x}}{256\times 10^4}(-1600x - 1)\right|_0^t + \left.\frac{e^{-1600x}}{-1600}\right|_0^t
\end{aligned}$$

When $t \rightarrow \infty$ all the upper limits evaluate to zero, hence
$$\frac{w}{3.2} = \frac{(640{,}000)(2)}{4096\times 10^6} + \frac{1600}{256\times 10^4} + \frac{1}{1600}$$
$w = 10^{-3} + 2\times 10^{-3} + 2\times 10^{-3} = 5$ mJ.

P 1.22 **[a]**

$$\begin{aligned} p &= vi \\ &= 400 \times 10^3 t^2 e^{-800t} + 700te^{-800t} + 0.25e^{-800t} \\ &= e^{-800t}[400{,}000t^2 + 700t + 0.25] \\ \frac{dp}{dt} &= \{e^{-800t}[800 \times 10^3 t + 700] - 800e^{-800t}[400{,}000t^2 + 700t + 0.25]\} \\ &= [-3{,}200{,}000t^2 + 2400t + 5]100e^{-800t} \end{aligned}$$

Therefore, $\frac{dp}{dt} = 0$ when $3{,}200{,}000t^2 - 2400t - 5 = 0$
so p_{max} occurs at $t = 1.68$ ms.

[b]

$$\begin{aligned} p_{\text{max}} &= [400{,}000(.00168)^2 + 700(.00168) + 0.25]e^{-800(.00168)} \\ &= 666 \text{ mW} \end{aligned}$$

[c]

$$\begin{aligned} w &= \int_0^t p dx \\ w &= \int_0^t 400{,}000x^2 e^{-800x}\,dx + \int_0^t 700xe^{-800x}\,dx + \int_0^t 0.25e^{-800x}\,dx \\ &= \frac{400{,}000e^{-800x}}{-512 \times 10^6}[64 \times 10^4 x^2 + 1600x + 2]\Big|_0^t + \\ & \quad \frac{700e^{-800x}}{64 \times 10^4}(-800x - 1)\Big|_0^t + 0.25\frac{e^{-800x}}{-800}\Big|_0^t \end{aligned}$$

When $t = \infty$ all the upper limits evaluate to zero, hence
$w = \frac{(400{,}000)(2)}{512 \times 10^6} + \frac{700}{64 \times 10^4} + \frac{0.25}{800} = 2.97$ mJ.

P 1.23 **[a]** $p = vi = 2000\cos(800\pi t)\sin(800\pi t) = 1000\sin(1600\pi t)$ W

Therefore, $p_{\text{max}} = 1000$ W

[b] $p_{\text{max}}(\text{extracting}) = 1000$ W

[c]

$$\begin{aligned} p_{\text{avg}} &= \frac{1}{2.5 \times 10^{-3}}\int_0^{2.5\times 10^{-3}} 1000\sin(1600\pi t)\,dt \\ &= 4 \times 10^5 \left[\frac{-\cos 1600\pi t}{1600\pi}\right]_0^{2.5\times 10^{-3}} = \frac{250}{\pi}[1 - \cos 4\pi] = 0 \end{aligned}$$

[d]

$$\begin{aligned} p_{\text{avg}} &= \frac{1}{15.625 \times 10^{-3}}\int_0^{15.625\times 10^{-3}} 1000\sin(1600\pi t)\,dt \\ &= 64 \times 10^3 \left[\frac{-\cos 1600\pi t}{1600\pi}\right]_0^{15.625\times 10^{-3}} = \frac{40}{\pi}[1 - \cos 25\pi] = 25.46 \text{ W} \end{aligned}$$

P 1.24 **[a]**

$$\begin{aligned} q &= \text{area under } i \text{ vs. } t \text{ plot} \\ &= \left[\tfrac{1}{2}(5)(4) + (10)(4) + \tfrac{1}{2}(8)(4) + (8)(6) + \tfrac{1}{2}(3)(6)\right] \times 10^3 \\ &= [10 + 40 + 16 + 48 + 9]10^3 = 123{,}000 \text{ C} \end{aligned}$$

[b]
$$w = \int p\,dt = \int vi\,dt$$
$$v = 0.2\times10^{-3}t+9 \qquad 0\le t\le 15 \text{ ks}$$

$0 \le t \le 4000s$

$$\begin{aligned} i &= 15-1.25\times10^{-3}t \\ p &= 135-8.25\times10^{-3}t-0.25\times10^{-6}t^2 \\ w_1 &= \int_0^{4000}(135-8.25\times10^{-3}t-0.25\times10^{-6}t^2)\,dt \\ &= (540-66-5.3333)10^3 = 468.667 \text{ kJ} \end{aligned}$$

$4000 \le t \le 12{,}000$

$$\begin{aligned} i &= 12-0.5\times10^{-3}t \\ p &= 108-2.1\times10^{-3}t-0.1\times10^{-6}t^2 \\ w_2 &= \int_{4000}^{12{,}000}(108-2.1\times10^{-3}t-0.1\times10^{-6}t^2)\,dt \\ &= (864-134.4-55.467)10^3 = 674.133 \text{ kJ} \end{aligned}$$

$12{,}000 \le t \le 15{,}000$

$$\begin{aligned} i &= 30-2\times10^{-3}t \\ p &= 270-12\times10^{-3}t-0.4\times10^{-6}t^2 \\ w_3 &= \int_{12{,}000}^{15{,}000}(270-12\times10^{-3}t-0.4\times10^{-6}t^2)\,dt \\ &= (810-486-219.6)10^3 = 104.4 \text{ kJ} \\ w_T &= w_1+w_2+w_3 = 468.667+674.133+104.4 = 1247.2 \text{ kJ} \end{aligned}$$

P 1.25 **[a]** We can find the time at which the power is a maximum by writing an expression for $p(t) = v(t)i(t)$, taking the first derivative of $p(t)$ and setting it to zero, then solving for t. The calculations are shown below:

$$\begin{aligned} p &= 0 \quad t<0, \qquad p=0 \quad t>40 \text{ s} \\ p &= vi = t(1-0.025t)(4-0.2t) = 4t-0.3t^2+0.005t^3 \text{ W} \qquad 0\le t\le 40 \text{ s} \\ \frac{dp}{dt} &= 4-0.6t+0.015t^2 = 0.015(t^2-40t+266.67) \\ \frac{dp}{dt} &= 0 \qquad \text{when } t^2-40t+266.67=0 \\ t_1 &= 8.453 \text{ s}; \qquad t_2 = 31.547 \text{ s} \end{aligned}$$

(using the polynomial solver on your calculator)

$$\begin{aligned} p(t_1) &= 4(8.453)-0.3(8.453)^2+0.005(8.453)^3 = 15.396 \text{ W} \\ p(t_2) &= 4(31.547)-0.3(31.547)^2+0.005(31.547)^3 = -15.396 \text{ W} \end{aligned}$$

Therefore, maximum power is being delivered at $t = 8.453$ s.

[b] The maximum power was calculated in part (a) to determine the time at which the power is maximum: $p_{\max} = 15.396$ W (delivered)

[c] As we saw in part (a), the other "maximum" power is actually a minimum, or the maximum negative power. As we calculated in part (a), maximum power is being extracted at $t = 31.547$ s.

[d] This maximum extracted power was calculated in part (a) to determine the time at which power is maximum: $p_{\text{max}} = 15.396$ W (extracted)

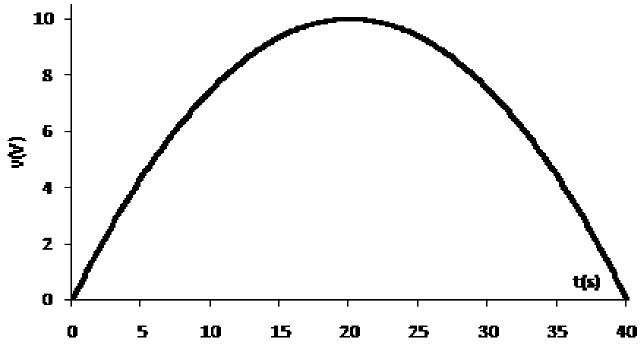
[e] $w = \int_0^t p dx = \int_0^t (4x - 0.3x^2 + 0.005x^3)dx = 2t^2 - 0.1t^3 + 0.00125t^4$

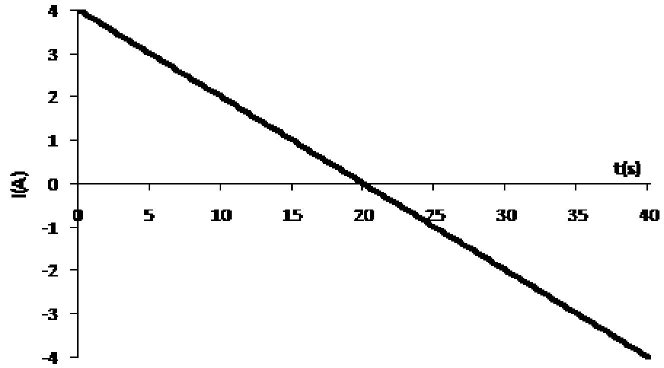
$$w(0) = 0\text{ J} \qquad w(30) = 112.5\text{ J}$$

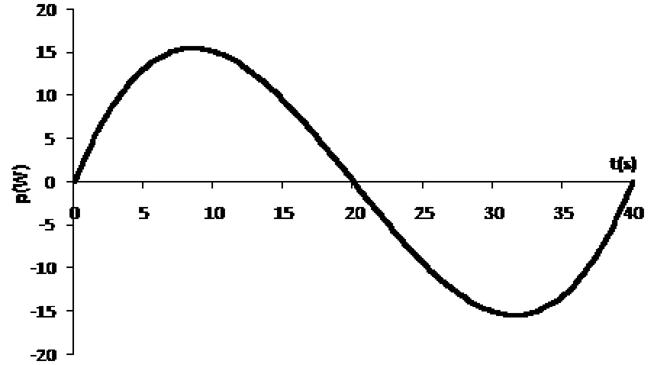
$$w(10) = 112.5\text{ J} \qquad w(40) = 0\text{ J}$$

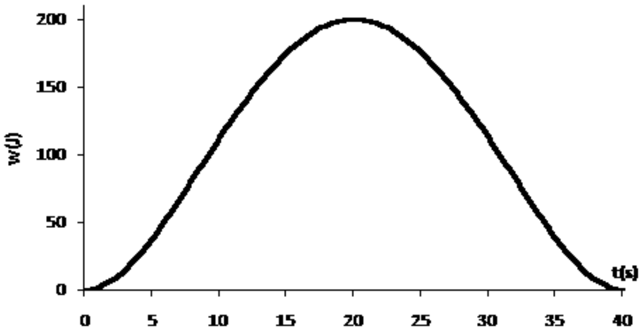
$$w(20) = 200\text{ J}$$

To give you a feel for the quantities of voltage, current, power, and energy and their relationships among one another, they are plotted below:









P 1.26 We use the passive sign convention to determine whether the power equation is $p = vi$ or $p = -vi$ and substitute into the power equation the values for v and i, as shown below:

$$\begin{aligned}
p_{\rm a} &= v_{\rm a}i_{\rm a} = (150 \times 10^3)(0.6 \times 10^{-3}) = 90 \text{ W} \\
p_{\rm b} &= v_{\rm b}i_{\rm b} = (150 \times 10^3)(-1.4 \times 10^{-3}) = -210 \text{ W} \\
p_{\rm c} &= -v_{\rm c}i_{\rm c} = -(100 \times 10^3)(-0.8 \times 10^{-3}) = 80 \text{ W} \\
p_{\rm d} &= v_{\rm d}i_{\rm d} = (250 \times 10^3)(-0.8 \times 10^{-3}) = -200 \text{ W} \\
p_{\rm e} &= -v_{\rm e}i_{\rm e} = -(300 \times 10^3)(-2 \times 10^{-3}) = 600 \text{ W} \\
p_{\rm f} &= v_{\rm f}i_{\rm f} = (-300 \times 10^3)(1.2 \times 10^{-3}) = -360 \text{ W}
\end{aligned}$$

Remember that if the power is positive, the circuit element is absorbing power, whereas is the power is negative, the circuit element is developing power. We can add the positive powers together and the negative powers together — if the power balances, these power sums should be equal:
$\sum P_{\rm dev} = 210 + 200 + 360 = 770$ W;
$\sum P_{\rm abs} = 90 + 80 + 600 = 770$ W
Thus, the power balances and the total power developed in the circuit is 770 W.

P 1.27

$$\begin{aligned}
p_{\rm a} &= -v_{\rm a}i_{\rm a} = -(990)(-0.0225) = 22.275 \text{ W} \\
p_{\rm b} &= -v_{\rm b}i_{\rm b} = -(600)(-0.03) = 18 \text{ W} \\
p_{\rm c} &= v_{\rm c}i_{\rm c} = (300)(0.06) = 18 \text{ W} \\
p_{\rm d} &= v_{\rm d}i_{\rm d} = (105)(0.0525) = 5.5125 \text{ W} \\
p_{\rm e} &= -v_{\rm e}i_{\rm e} = -(-120)(0.03) = 3.6 \text{ W} \\
p_{\rm f} &= v_{\rm f}i_{\rm f} = (165)(0.0825) = 13.6125 \text{ W} \\
p_{\rm g} &= -v_{\rm g}i_{\rm g} = -(585)(0.0525) = -30.7125 \text{ W} \\
p_{\rm h} &= v_{\rm h}i_{\rm h} = (-585)(0.0825) = -48.2625 \text{ W}
\end{aligned}$$

Therefore,

$$\sum P_{\rm abs} = 22.275 + 18 + 18 + 5.5125 + 3.6 + 13.6125 = 81 \text{ W}$$

$$\sum P_{\text{del}} = 30.7125 + 48.2625 = 78.975 \text{ W}$$

$$\sum P_{\text{abs}} \neq \sum P_{\text{del}}$$

Thus, the interconnection does not satisfy the power check.

P 1.28 **[a]** From the diagram and the table we have

$$\begin{aligned}
p_{\text{a}} &= -v_{\text{a}}i_{\text{a}} = -(46.16)(-6) = -276.96 \text{ W} \\
p_{\text{b}} &= v_{\text{b}}i_{\text{b}} = (14.16)(4.72) = 66.8352 \text{ W} \\
p_{\text{c}} &= v_{\text{c}}i_{\text{c}} = (-32)(-6.4) = 204.8 \text{ W} \\
p_{\text{d}} &= -v_{\text{d}}i_{\text{d}} = -(22)(1.28) = -28.16 \text{ W} \\
p_{\text{e}} &= -v_{\text{e}}i_{\text{e}} = -(33.6)(1.68) = -56.448 \text{ W} \\
p_{\text{f}} &= v_{\text{f}}i_{\text{f}} = (66)(-0.4) = -26.4 \text{ W} \\
p_{\text{g}} &= v_{\text{g}}i_{\text{g}} = (2.56)(1.28) = 3.2768 \text{ W} \\
p_{\text{h}} &= -v_{\text{h}}i_{\text{h}} = -(-0.4)(0.4) = 0.16 \text{ W} \\
\sum P_{\text{del}} &= 276.96 + 28.16 + 56.448 + 26.4 = 387.968 \text{ W} \\
\sum P_{\text{abs}} &= 66.8352 + 204.8 + 3.2768 + 0.16 = 275.072 \text{ W}
\end{aligned}$$

Therefore, $\sum P_{\text{del}} \neq \sum P_{\text{abs}}$ and the subordinate engineer is correct.

[b] The difference between the power delivered to the circuit and the power absorbed by the circuit is

$$-387.986 + 275.072 = -112.896 \text{ W}$$

One-half of this difference is −56.448 W, so it is likely that p_{e} is in error. Either the voltage or the current probably has the wrong sign. (In Chapter 2, we will discover that using KCL at the node connecting components b, c, and e, the current i_{e} should be −1.68 A, not 1.68 A!) If the sign of p_{e} is changed from negative to positive, we can recalculate the power delivered and the power absorbed as follows:

$$\begin{aligned}
\sum P_{\text{del}} &= 276.96 + 28.16 + 26.4 = 331.52 \text{ W} \\
\sum P_{\text{abs}} &= 66.8352 + 204.8 + 56.448 + 3.2768 + 0.16 = 331.52 \text{ W}
\end{aligned}$$

Now the power delivered equals the power absorbed and the power balances for the circuit.

P 1.29 **[a]** From an examination of reference polarities, elements a, e, f, and h use a + sign in the power equation, so would be expected to absorb power. Elements b, c, d, and g use a − sign in the power equation, so would be expected to supply power.

[b]
$$\begin{aligned}
p_{\rm a} &= v_{\rm a}i_{\rm a} = (5)(2\times 10^{-3}) = 10 \text{ mW}\\
p_{\rm b} &= -v_{\rm b}i_{\rm b} = -(1)(3\times 10^{-3}) = -3 \text{ mW}\\
p_{\rm c} &= -v_{\rm c}i_{\rm c} = -(7)(-2\times 10^{-3}) = 14 \text{ mW}\\
p_{\rm d} &= -v_{\rm d}i_{\rm d} = -(-9)(1\times 10^{-3}) = 9 \text{ mW}\\
p_{\rm e} &= v_{\rm e}i_{\rm e} = (-20)(5\times 10^{-3}) = -100 \text{ mW}\\
p_{\rm f} &= v_{\rm f}i_{\rm f} = (20)(2\times 10^{-3}) = 40 \text{ mW}\\
p_{\rm g} &= -v_{\rm g}i_{\rm g} = -(-3)(-2\times 10^{-3}) = -6 \text{ mW}\\
p_{\rm h} &= v_{\rm h}i_{\rm h} = (-12)(-3\times 10^{-3}) = 36 \text{ mW}
\end{aligned}$$

$\sum P_{\rm abs} = 10 + 14 + 9 + 40 + 36 = 109$ mW

$\sum P_{\rm del} = 3 + 100 + 6 = 109$ mW

Thus, 109 mW of power is delivered and 109 mW of power is absorbed, and the power balances.

[c] Looking at the calculated power values, elements a, c, d, f, and h have positive power, so are absorbing, while elements b, e, and g have negative power so are supplying. These answers are different from those in part (a) because the voltages and currents used in the power equation are not all positive numbers.

P 1.30
$$\begin{aligned}
p_{\rm a} &= -v_{\rm a}i_{\rm a} = -(1.6)(0.080) = -128 \text{ mW}\\
p_{\rm b} &= -v_{\rm b}i_{\rm b} = -(2.6)(0.060) = -156 \text{ mW}\\
p_{\rm c} &= v_{\rm c}i_{\rm c} = (-4.2)(-0.050) = 210 \text{ mW}\\
p_{\rm d} &= -v_{\rm d}i_{\rm d} = -(1.2)(0.020) = -24 \text{ mW}\\
p_{\rm e} &= v_{\rm e}i_{\rm e} = (1.8)(0.030) = 54 \text{ mW}\\
p_{\rm f} &= -v_{\rm f}i_{\rm f} = -(-1.8)(-0.040) = -72 \text{ mW}\\
p_{\rm g} &= v_{\rm g}i_{\rm g} = (-3.6)(-0.030) = 108 \text{ mW}\\
p_{\rm h} &= v_{\rm h}i_{\rm h} = (3.2)(-0.020) = -64 \text{ mW}\\
p_{\rm j} &= -v_{\rm j}i_{\rm j} = -(-2.4)(0.030) = 72 \text{ mW}
\end{aligned}$$

$\sum P_{\rm del} = 128 + 156 + 24 + 72 + 64 = 444$ mW

$\sum P_{\rm abs} = 210 + 54 + 108 + 72 = 444$ mW

Therefore, $\sum P_{\rm del} = \sum P_{\rm abs} = 444$ mW

Thus, the interconnection satisfies the power check.

P 1.31
$$\begin{aligned}
p_a &= v_a i_a = (120)(-10) = -1200 \text{ W}\\
p_b &= -v_b i_b = -(120)(9) = -1080 \text{ W}\\
p_c &= v_c i_c = (10)(10) = 100 \text{ W}\\
p_d &= -v_d i_d = -(10)(-1) = 10 \text{ W}\\
p_e &= v_e i_e = (-10)(-9) = 90 \text{ W}\\
p_f &= -v_f i_f = -(-100)(5) = 500 \text{ W}\\
p_g &= v_g i_g = (120)(4) = 480 \text{ W}\\
p_h &= v_h i_h = (-220)(-5) = 1100 \text{ W}
\end{aligned}$$

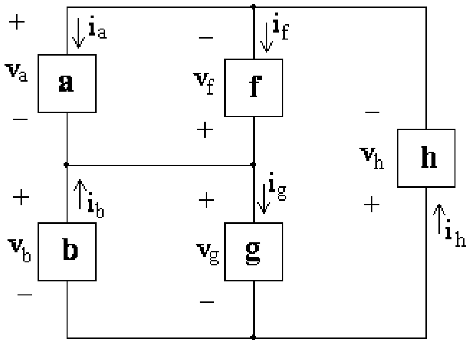
$\sum P_{\text{del}} = 1200 + 1080 = 2280$ W

$\sum P_{\text{abs}} = 100 + 10 + 90 + 500 + 480 + 1100 = 2280$ W

Therefore, $\sum P_{\text{del}} = \sum P_{\text{abs}} = 2280$ W

Thus, the interconnection now satisfies the power check.

P 1.32 [a] The revised circuit model is shown below:



[b] The expression for the total power in this circuit is

$$v_a i_a - v_b i_b - v_f i_f + v_g i_g + v_h i_h$$

$$= (120)(-10) - (120)(10) - (-120)(3) + 120 i_g + (-240)(-7) = 0$$

Therefore,

$$120 i_g = 1200 + 1200 - 360 - 1680 = 360$$

so

$$i_g = \frac{360}{120} = 3 \text{ A}$$

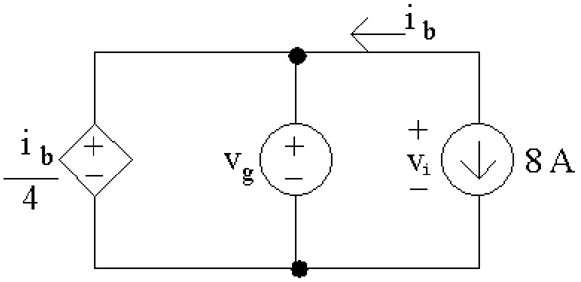
Thus, if the power in the modified circuit is balanced the current in component g is 3 A.

2

Circuit Elements

Assessment Problems

AP 2.1



[a] Note that the current i_b is in the same circuit branch as the 8 A current source; however, i_b is defined in the opposite direction of the current source. Therefore,

$$i_\mathrm{b} = -8\,\mathrm{A}$$

Next, note that the dependent voltage source and the independent voltage source are in parallel with the same polarity. Therefore, their voltages are equal, and

$$v_\mathrm{g} = \frac{i_\mathrm{b}}{4} = \frac{-8}{4} = -2\,\mathrm{V}$$

[b] To find the power associated with the 8 A source, we need to find the voltage drop across the source, v_i. Note that the two independent sources are in parallel, and that the voltages v_g and v_1 have the same polarities, so these voltages are equal:

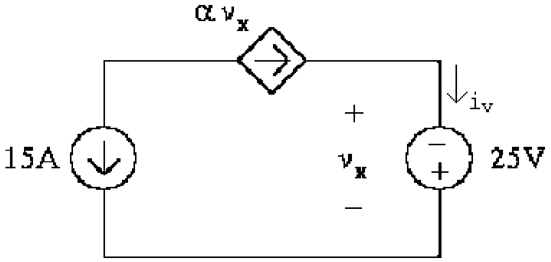
$$v_i = v_g = -2\,\mathrm{V}$$

Using the passive sign convention,

$$p_s = (8\,\mathrm{A})(v_i) = (8\,\mathrm{A})(-2\,\mathrm{V}) = -16\,\mathrm{W}$$

Thus the current source generated 16 W of power.

AP 2.2



[a] Note from the circuit that $v_x = -25$ V. To find α note that the two current sources are in the same branch of the circuit but their currents flow in opposite directions. Therefore

$\alpha v_x = -15\,\text{A}$

Solve the above equation for α and substitute for v_x,

$$\alpha = \frac{-15\,\text{A}}{v_x} = \frac{-15\,\text{A}}{-25\,\text{V}} = 0.6\ \text{A/V}$$

[b] To find the power associated with the voltage source we need to know the current, i_v. Note that this current is in the same branch of the circuit as the dependent current source and these two currents flow in the same direction. Therefore, the current i_v is the same as the current of the dependent source:

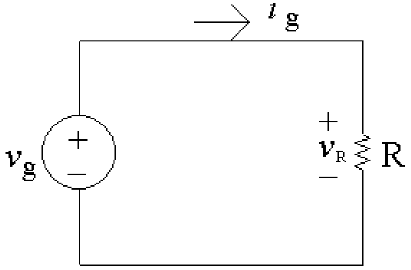
$i_v = \alpha v_x = (0.6)(-25) = -15\,\text{A}$

Using the passive sign convention,

$p_s = -(i_v)(25\,\text{V}) = -(-15\,\text{A})(25\,\text{V}) = 375\,\text{W}.$

Thus the voltage source dissipates 375 W.

AP 2.3



[a] The resistor and the voltage source are in parallel and the resistor voltage and the voltage source have the same polarities. Therefore these two voltages are the same:

$v_R = v_g = 1\,\text{kV}$

Note from the circuit that the current through the resistor is $i_g = 5$ mA. Use Ohm's law to calculate the value of the resistor:

$$R = \frac{v_R}{i_g} = \frac{1\,\text{kV}}{5\,\text{mA}} = 200\,\text{k}\Omega$$

Using the passive sign convention to calculate the power in the resistor,

$$p_R = (v_R)(i_g) = (1\,\text{kV})(5\,\text{mA}) = 5\,\text{W}$$

The resistor is dissipating 5 W of power.

[b] Note from part (a) the $v_R = v_g$ and $i_R = i_g$. The power delivered by the source is thus

$$p_{\text{source}} = -v_g i_g \qquad \text{so} \qquad v_g = -\frac{p_{\text{source}}}{i_g} = -\frac{-3\,\text{W}}{75\,\text{mA}} = 40\,\text{V}$$

Since we now have the value of both the voltage and the current for the resistor, we can use Ohm's law to calculate the resistor value:

$$R = \frac{v_g}{i_g} = \frac{40\,\text{V}}{75\,\text{mA}} = 533.33\,\Omega$$

The power absorbed by the resistor must equal the power generated by the source. Thus,

$$p_R = -p_{\text{source}} = -(-3\,\text{W}) = 3\,\text{W}$$

[c] Again, note the $i_R = i_g$. The power dissipated by the resistor can be determined from the resistor's current:

$$p_R = R(i_R)^2 = R(i_g)^2$$

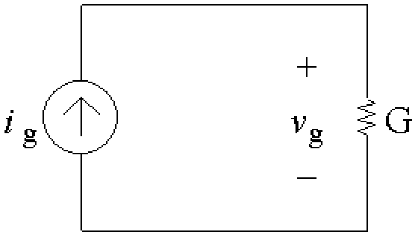
Solving for i_g,

$$i_g^2 = \frac{p_r}{R} = \frac{480\,\text{mW}}{300\,\Omega} = 0.0016 \qquad \text{so} \qquad i_g = \sqrt{0.0016} = 0.04\,\text{A} = 40\,\text{mA}$$

Then, since $v_R = v_g$

$$v_R = Ri_R = Ri_g = (300\,\Omega)(40\,\text{mA}) = 12\,\text{V} \qquad \text{so} \qquad v_g = 12\,\text{V}$$

AP 2.4



[a] Note from the circuit that the current through the conductance G is i_g, flowing from top to bottom, because the current source and the conductance are in the same branch of the circuit so must have the same

current. The voltage drop across the current source is v_g, positive at the top, because the current source and the conductance are also in parallel so must have the same voltage. From a version of Ohm's law,

$$v_g = \frac{i_g}{G} = \frac{0.5\,\text{A}}{50\,\text{mS}} = 10\,\text{V}$$

Now that we know the voltage drop across the current source, we can find the power delivered by this source:

$$p_{\text{source}} = -v_g i_g = -(10)(0.5) = -5\,\text{W}$$

Thus the current source delivers 5 W to the circuit.

[b] We can find the value of the conductance using the power, and the value of the current using Ohm's law and the conductance value:

$$p_g = Gv_g^2 \qquad \text{so} \qquad G = \frac{p_g}{v_g^2} = \frac{9}{15^2} = 0.04\,\text{S} = 40\,\text{mS}$$

$$i_g = Gv_g = (40\,\text{mS})(15\,\text{V}) = 0.6\,\text{A}$$

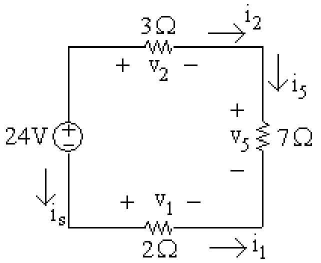
[c] We can find the voltage from the power and the conductance, and then use the voltage value in Ohm's law to find the current:

$$p_g = Gv_g^2 \qquad \text{so} \qquad v_g^2 = \frac{p_g}{G} = \frac{8\,\text{W}}{200\,\mu\text{S}} = 40{,}000$$

Thus $\quad v_g = \sqrt{40{,}000} = 200\,\text{V}$

$$i_g = Gv_g = (200\,\mu\text{S})(200\,\text{V}) = 0.04\,\text{A} = 40\,\text{mA}$$

AP 2.5 **[a]** Redraw the circuit with all of the voltages and currents labeled for every circuit element.



Write a KVL equation clockwise around the circuit, starting below the voltage source:

$$-24\,\text{V} + v_2 + v_5 - v_1 = 0$$

Next, use Ohm's law to calculate the three unknown voltages from the three currents:

$$v_2 = 3i_2; \qquad v_5 = 7i_5; \qquad v_1 = 2i_1$$

A KCL equation at the upper right node gives $i_2 = i_5$; a KCL equation at the bottom right node gives $i_5 = -i_1$; a KCL equation at the upper left node gives $i_s = -i_2$. Now replace the currents i_1 and i_2 in the Ohm's law equations with i_5:

$$v_2 = 3i_2 = 3i_5; \qquad v_5 = 7i_5; \qquad v_1 = 2i_1 = -2i_5$$

Now substitute these expressions for the three voltages into the first equation:

$$24 = v_2 + v_5 - v_1 = 3i_5 + 7i_5 - (-2i_5) = 12i_5$$

Therefore $i_5 = 24/12 = 2$ A

[b] $v_1 = -2i_5 = -2(2) = -4\,\text{V}$

[c] $v_2 = 3i_5 = 3(2) = 6\,\text{V}$

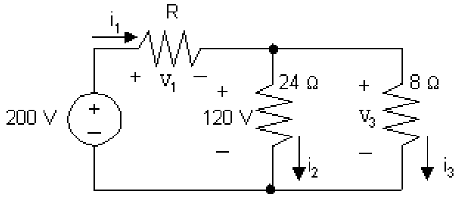
[d] $v_5 = 7i_5 = 7(2) = 14\,\text{V}$

[e] A KCL equation at the lower left node gives $i_s = i_1$. Since $i_1 = -i_5$, $i_s = -2\,\text{A}$. We can now compute the power associated with the voltage source:

$$p_{24} = (24)i_s = (24)(-2) = -48\,\text{W}$$

Therefore 24 V source is delivering 48 W.

AP 2.6 Redraw the circuit labeling all voltages and currents:



We can find the value of the unknown resistor if we can find the value of its voltage and its current. To start, write a KVL equation clockwise around the right loop, starting below the 24 Ω resistor:

$$-120\,\text{V} + v_3 = 0$$

Use Ohm's law to calculate the voltage across the 8 Ω resistor in terms of its current:

$$v_3 = 8i_3$$

Substitute the expression for v_3 into the first equation:

$$-120\,\text{V} + 8i_3 = 0 \qquad \text{so} \qquad i_3 = \frac{120}{8} = 15\,\text{A}$$

Also use Ohm's law to calculate the value of the current through the 24 Ω resistor:

$$i_2 = \frac{120\,\mathrm{V}}{24\,\Omega} = 5\,\mathrm{A}$$

Now write a KCL equation at the top middle node, summing the currents leaving:

$$-i_1 + i_2 + i_3 = 0 \qquad \text{so} \qquad i_1 = i_2 + i_3 = 5 + 15 = 20\,\mathrm{A}$$

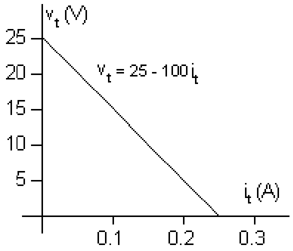
Write a KVL equation clockwise around the left loop, starting below the voltage source:

$$-200\,\mathrm{V} + v_1 + 120\,\mathrm{V} = 0 \qquad \text{so} \qquad v_1 = 200 - 120 = 80\,\mathrm{V}$$

Now that we know the values of both the voltage and the current for the unknown resistor, we can use Ohm's law to calculate the resistance:

$$\mathrm{R} = \frac{v_1}{i_1} = \frac{80}{20} = 4\,\Omega$$

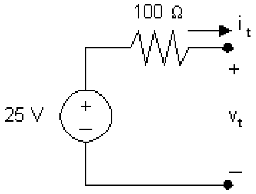
AP 2.7 **[a]** Plotting a graph of v_t versus i_t gives



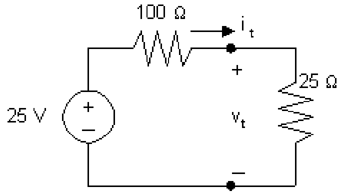
Note that when $i_t = 0$, $v_t = 25$ V; therefore the voltage source must be 25 V. Since the plot is a straight line, its slope can be used to calculate the value of resistance:

$$R = \frac{\Delta v}{\Delta i} = \frac{25 - 0}{0.25 - 0} = \frac{25}{0.25} = 100\,\Omega$$

A circuit model having the same $v - i$ characteristic is a 25 V source in series with a 100Ω resistor, as shown below:



[b] Draw the circuit model from part (a) and attach a 25 Ω resistor:



To find the power delivered to the 25 Ω resistor we must calculate the current through the 25 Ω resistor. Do this by first using KCL to recognize that the current in each of the components is i_t, flowing in a clockwise direction. Write a KVL equation in the clockwise direction, starting below the voltage source, and using Ohm's law to express the voltage drop across the resistors in the direction of the current i_t flowing through the resistors:

$$-25\,\text{V} + 100i_t + 25i_t = 0 \qquad \text{so} \qquad 125i_t = 25 \qquad \text{so} \qquad i_t = \frac{25}{125} = 0.2\,\text{A}$$

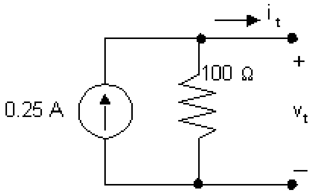
Thus, the power delivered to the 25 Ω resistor is

$$p_{25} = (25)i_t^2 = (25)(0.2)^2 = 1\,\text{W}.$$

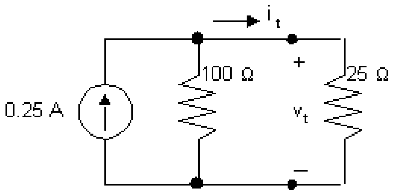
AP 2.8 **[a]** From the graph in Assessment Problem 2.7(a), we see that when $v_t = 0$, $i_t = 0.25\,\text{A}$. Therefore the current source must be 0.25 A. Since the plot is a straight line, its slope can be used to calculate the value of resistance:

$$R = \frac{\Delta v}{\Delta i} = \frac{25 - 0}{0.25 - 0} = \frac{25}{0.25} = 100\,\Omega$$

A circuit model having the same $v - i$ characteristic is a 0.25 A current source in parallel with a 100Ω resistor, as shown below:



[b] Draw the circuit model from part (a) and attach a 25 Ω resistor:



Note that by writing a KVL equation around the right loop we see that the voltage drop across both resistors is v_t. Write a KCL equation at the top center node, summing the currents leaving the node. Use Ohm's law to specify the currents through the resistors in terms of the voltage drop across the resistors and the value of the resistors.

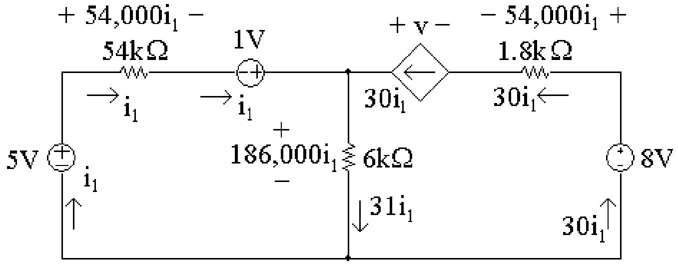
$$-0.25 + \frac{v_t}{100} + \frac{v_t}{25} = 0, \qquad \text{so} \qquad 5v_t = 25, \qquad \text{thus} \qquad v_t = 5\,\text{V}$$

$$p_{25} = \frac{v_t^2}{25} = 1\,\mathrm{W}.$$

AP 2.9 First note that we know the current through all elements in the circuit except the 6 kΩ resistor (the current in the three elements to the left of the 6 kΩ resistor is i_1; the current in the three elements to the right of the 6 kΩ resistor is $30i_1$). To find the current in the 6 kΩ resistor, write a KCL equation at the top node:

$$i_1 + 30i_1 = i_{6\mathrm{k}} = 31i_1$$

We can then use Ohm's law to find the voltages across each resistor in terms of i_1. The results are shown in the figure below:



[**a**] To find i_1, write a KVL equation around the left-hand loop, summing voltages in a clockwise direction starting below the 5V source:

$$-5\,\mathrm{V} + 54{,}000i_1 - 1\,\mathrm{V} + 186{,}000i_1 = 0$$

Solving for i_1

$$54{,}000i_1 + 186{,}000i_1 = 6\,\mathrm{V} \qquad \text{so} \qquad 240{,}000i_1 = 6\,\mathrm{V}$$

Thus,

$$i_1 = \frac{6}{240{,}000} = 25\,\mu\mathrm{A}$$

[**b**] Now that we have the value of i_1, we can calculate the voltage for each component except the dependent source. Then we can write a KVL equation for the right-hand loop to find the voltage v of the dependent source. Sum the voltages in the clockwise direction, starting to the left of the dependent source:

$$+v - 54{,}000i_1 + 8\,\mathrm{V} - 186{,}000i_1 = 0$$

Thus,

$$v = 240{,}000i_1 - 8\,\mathrm{V} = 240{,}000(25 \times 10^{-6}) - 8\,\mathrm{V} = 6\,\mathrm{V} - 8\,\mathrm{V} = -2\,\mathrm{V}$$

We now know the values of voltage and current for every circuit element.

Let's construct a power table:

Element	**Current (μA)**	**Voltage (V)**	**Power Equation**	**Power (μW)**
5 V	25	5	$p = -vi$	−125
54 kΩ	25	1.35	$p = Ri^2$	33.75
1 V	25	1	$p = -vi$	−25
6 kΩ	775	4.65	$p = Ri^2$	3603.75
Dep. source	750	−2	$p = -vi$	1500
1.8 kΩ	750	1.35	$p = Ri^2$	1012.5
8 V	750	8	$p = -vi$	−6000

[**c**] The total power generated in the circuit is the sum of the negative power values in the power table:

$$-125\,\mu\text{W} + -25\,\mu\text{W} + -6000\,\mu\text{W} = -6150\,\mu\text{W}$$

Thus, the total power generated in the circuit is 6150 μW.

[**d**] The total power absorbed in the circuit is the sum of the positive power values in the power table:

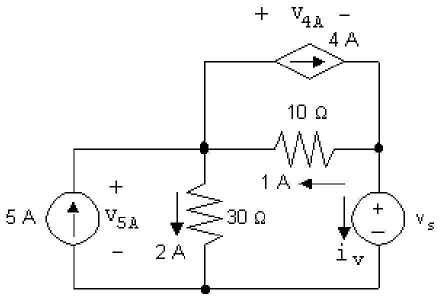
$$33.75\,\mu\text{W} + 3603.75\,\mu\text{W} + 1500\,\mu\text{W} + 1012.5\,\mu\text{W} = 6150\,\mu\text{W}$$

Thus, the total power absorbed in the circuit is 6150 μW.

AP 2.10 Given that $i_\phi = 2$ A, we know the current in the dependent source is $2i_\phi = 4$ A. We can write a KCL equation at the left node to find the current in the 10 Ω resistor. Summing the currents leaving the node,

$$-5\,\text{A} + 2\,\text{A} + 4\,\text{A} + i_{10\Omega} = 0 \qquad \text{so} \qquad i_{10\Omega} = 5\,\text{A} - 2\,\text{A} - 4\,\text{A} = -1\,\text{A}$$

Thus, the current in the 10 Ω resistor is 1 A, flowing right to left, as seen in the circuit below.



[**a**] To find v_s, write a KVL equation, summing the voltages counter-clockwise around the lower right loop. Start below the voltage source.

$$-v_s + (1\,\text{A})(10\,\Omega) + (2\,\text{A})(30\,\Omega) = 0 \qquad \text{so} \qquad v_s = 10\,\text{V} + 60\,\text{V} = 70\,\text{V}$$

[**b**] The current in the voltage source can be found by writing a KCL equation at the right-hand node. Sum the currents leaving the node

$$-4\,\text{A} + 1\,\text{A} + i_v = 0 \qquad \text{so} \qquad i_v = 4\,\text{A} - 1\,\text{A} = 3\,\text{A}$$

The current in the voltage source is 3 A, flowing top to bottom. The power associated with this source is

$$p = vi = (70\,\text{V})(3\,\text{A}) = 210\,\text{W}$$

Thus, 210 W are absorbed by the voltage source.

[**c**] The voltage drop across the independent current source can be found by writing a KVL equation around the left loop in a clockwise direction:

$$-v_{5A} + (2\,\text{A})(30\,\Omega) = 0 \qquad \text{so} \qquad v_{5A} = 60\,\text{V}$$

The power associated with this source is

$$p = -v_{5A}i = -(60\,\text{V})(5\,\text{A}) = -300\,\text{W}$$

This source thus delivers 300 W of power to the circuit.

[**d**] The voltage across the controlled current source can be found by writing a KVL equation around the upper right loop in a clockwise direction:

$$+v_{4A} + (10\,\Omega)(1\,\text{A}) = 0 \qquad \text{so} \qquad v_{4A} = -10\,\text{V}$$

The power associated with this source is

$$p = v_{4A}i = (-10\,\text{V})(4\,\text{A}) = -40\,\text{W}$$

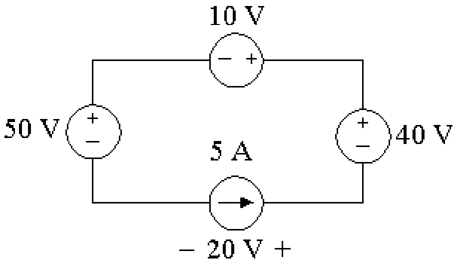
This source thus delivers 40 W of power to the circuit.

[**e**] The total power dissipated by the resistors is given by

$$(i_{30\Omega})^2(30\,\Omega) + (i_{10\Omega})^2(10\,\Omega) = (2)^2(30\,\Omega) + (1)^2(10\,\Omega) = 120 + 10 = 130\,\text{W}$$

Problems

P 2.1 The interconnect is valid since the voltage sources can all carry 5 A of current supplied by the current source, and the current source can carry the voltage drop required by the interconnection. Note that the branch containing the 10 V, 40 V, and 5 A sources must have the same voltage drop as the branch containing the 50 V source, so the 5 A current source must have a voltage drop of 20 V, positive at the right. The voltages and currents are summarize in the circuit below:



$$
\begin{aligned}
P_{50V} &= (50)(5) = 250 \text{ W} \quad \text{(abs)} \\
P_{10V} &= (10)(5) = 50 \text{ W} \quad \text{(abs)} \\
P_{40V} &= -(40)(5) = -200 \text{ W} \quad \text{(dev)} \\
P_{5A} &= -(20)(5) = -100 \text{ W} \quad \text{(dev)}
\end{aligned}
$$

$\sum P_{\text{dev}} = 300$ W

P 2.2 The interconnection is not valid. Note that the 10 V and 20 V sources are both connected between the same two nodes in the circuit. If the interconnection was valid, these two voltage sources would supply the same voltage drop between these two nodes, which they do not.

P 2.3 [**a**] Yes, independent voltage sources can carry the 5 A current required by the connection; independent current source can support any voltage required by the connection, in this case 5 V, positive at the bottom.

[**b**]

20 V source:	absorbing
15 V source:	developing (delivering)
5 A source:	developing (delivering)

[**c**]

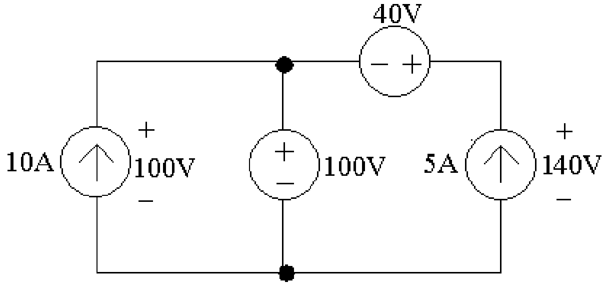
$$
\begin{aligned}
P_{20V} &= (20)(5) = 100 \text{ W} \quad \text{(abs)} \\
P_{15V} &= -(15)(5) = -75 \text{ W} \quad \text{(dev/del)} \\
P_{5A} &= -(5)(5) = -25 \text{ W} \quad \text{(dev/del)}
\end{aligned}
$$

$\sum P_{\text{abs}} = \sum P_{\text{del}} = 100$ W

[d] The interconnection is valid, but in this circuit the voltage drop across the 5 A current source is 35 V, positive at the top; 20 V source is developing (delivering), the 15 V source is developing (delivering), and the 5 A source is absorbing:

$$P_{20\text{V}} = -(20)(5) = -100\text{ W} \quad \text{(dev/del)}$$
$$P_{15\text{V}} = -(15)(5) = -75\text{ W} \quad \text{(dev/del)}$$
$$P_{5\text{A}} = (35)(5) = 175\text{ W} \quad \text{(abs)}$$
$$\sum P_{\text{abs}} = \sum P_{\text{del}} = 175\text{ W}$$

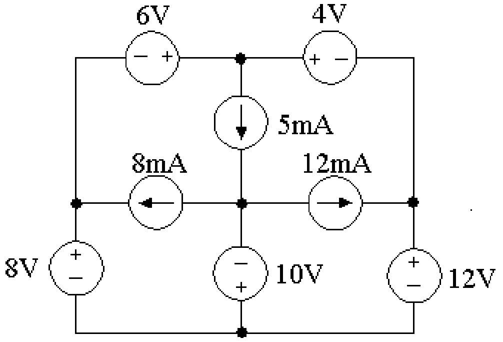
P 2.4 The interconnection is valid. The 10 A current source has a voltage drop of 100 V, positive at the top, because the 100 V source supplies its voltage drop across a pair of terminals shared by the 10 A current source. The right hand branch of the circuit must also have a voltage drop of 100 V from the left terminal of the 40 V source to the bottom terminal of the 5 A current source, because this branch shares the same terminals as the 100 V source. This means that the voltage drop across the 5 A current source is 140 V, positive at the top. Also, the two voltage sources can carry the current required of the interconnection. This is summarized in the figure below:



From the values of voltage and current in the figure, the power supplied by the current sources is calculated as follows:

$$P_{10\text{A}} = -(100)(10) = -1000\text{ W} \quad \text{(1000 W supplied)}$$
$$P_{5\text{A}} = -(140)(5) = -700\text{ W} \quad \text{(700 W supplied)}$$
$$\sum P_{\text{dev}} = 1700\text{ W}$$

P 2.5



The interconnection is invalid. The voltage drop between the top terminal and the bottom terminal on the left hand side is due to the 6 V and 8 V sources, giving a total voltage drop between these terminals of 14 V. But the voltage drop between the top terminal and the bottom terminal on the right hand side is due to the 4 V and 12 V sources, giving a total voltage drop between these two terminals of 16 V. The voltage drop between any two terminals in a valid circuit must be the same, so the interconnection is invalid.

P 2.6 The interconnection is valid, since the voltage sources can carry the 20 mA current supplied by the current source, and the current sources can carry whatever voltage drop is required by the interconnection. In particular, note the the voltage drop across the three sources in the right hand branch must be the same as the voltage drop across the 15 mA current source in the middle branch, since the middle and right hand branches are connected between the same two terminals. In particular, this means that

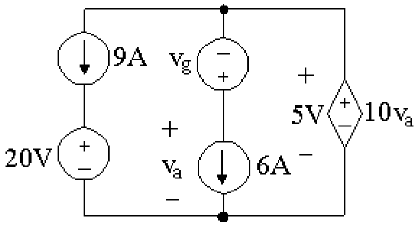
v_1(the voltage drop across the middle branch)

$$= -20\text{V} + 60\text{V} - v_2$$

Hence any combination of v_1 and v_2 such that $v_1 + v_2 = 40\,\text{V}$ is a valid solution.

P 2.7 The interconnection is invalid. In the middle branch, the value of the current i_Δ must be -25 A, since the 25 A current source supplies current in this branch in the direction opposite the direction of the current i_Δ. Therefore, the voltage supplied by the dependent voltage source in the left hand branch is $6(-25) = -150$ V. This gives a voltage drop from the top terminal to the bottom terminal in the left hand branch of $50 - (-150) = 200$ V. But the voltage drop between these same terminals in the right hand branch is 250 V, due to the voltage source in that branch. Therefore, the interconnection is invalid.

P 2.8



First, $10v_a = 5$ V, so $v_a = 0.5$ V. Then recognize that each of the three branches is connected between the same two nodes, so each of these branches must have the same voltage drop. The voltage drop across the middle branch is 5 V, and since $v_a = 0.5$ V, $v_g = 0.5 - 5 = -4.5$ V. Also, the voltage drop across the left branch is 5 V, so $20 + v_{9\text{A}} = 5$ V, and $v_{9\text{A}} = -15$ V, where $v_{9\text{A}}$ is positive at the top. Note that the current through the 20 V source must be 9 A, flowing from top to bottom, and the current through the v_{g} is 6 A flowing from top to bottom. Let's find the power associated with the left and middle branches:

$p_{9\text{A}} = (9)(-15) = -135\,\text{W}$

$p_{20\text{V}} = (9)(20) = 180\,\text{W}$

$p_{v_g} = -(6)(-4.5) = 27\,\text{W}$

$p_{6\text{A}} = (6)(0.5) = 3\,\text{W}$

Since there is only one component left, we can find the total power:

$p_{\text{total}} = -135 + 180 + 27 + 3 + p_{\text{ds}} = 75 + p_{\text{ds}} = 0$

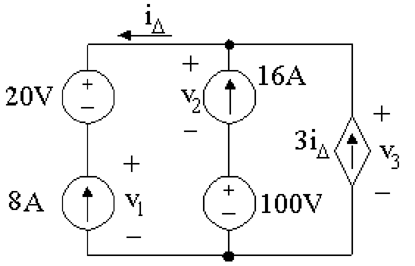
so p_{ds} must equal −75 W.

Therefore,

$$\sum P_{\text{dev}} = \sum P_{\text{abs}} = 210\,\text{W}$$

P 2.9 [a] Yes, each of the voltage sources can carry the current required by the interconnection, and each of the current sources can carry the voltage drop required by the interconnection. (Note that $i_\Delta = -8$ A.)

[b] No, because the voltage drop between the top terminal and the bottom terminal cannot be determined. For example, define v_1, v_2, and v_3 as shown:

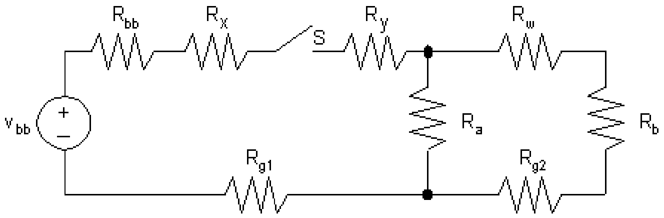


The voltage drop across the left branch, the center branch, and the right branch must be the same, since these branches are connected at the same two terminals. This requires that

$20 + v_1 = v_2 + 100 = v_3$

But this equation has three unknown voltages, so the individual voltages cannot be determined, and thus the power of the sources cannot be determined.

P 2.10 [a]



[b] V_{bb} = no-load voltage of battery

R_{bb} = internal resistance of battery

R_x = resistance of wire between battery and switch

R_y = resistance of wire between switch and lamp A

R_{a} = resistance of lamp A

R_{b} = resistance of lamp B

R_w = resistance of wire between lamp A and lamp B

R_{g1} = resistance of frame between battery and lamp A

R_{g2} = resistance of frame between lamp A and lamp B

S = switch

P 2.11 Since we know the device is a resistor, we can use Ohm's law to calculate the resistance. From Fig. P2.11(a),

$$v = Ri \qquad \text{so} \qquad R = \frac{v}{i}$$

Using the values in the table of Fig. P2.11(b),

$$R = \frac{-108}{-0.004} = \frac{-54}{-0.002} = \frac{54}{0.002} = \frac{108}{0.004} = \frac{162}{0.006} = 27\,\mathrm{k}\Omega$$

Note that this value is found in Appendix H.

P 2.12 The resistor value is the ratio of the power to the square of the current: $R = \frac{p}{i^2}$. Using the values for power and current in Fig. P2.12(b),

$$\frac{5.5\times10^{-3}}{(50\times10^{-6})^2} = \frac{22\times10^{-3}}{(100\times10^{-6})^2} = \frac{49.5\times10^{-3}}{(150\times10^{-6})^2} = \frac{88\times10^{-3}}{(200\times10^{-6})^2}$$

$$= \frac{137.5\times10^{-3}}{(250\times10^{-6})^2} = \frac{198\times10^{-3}}{(300\times10^{-6})^2} = 2.2\ \mathrm{M}\Omega$$

Note that this is a value from Appendix H.

P 2.13 Since we know the device is a resistor, we can use the power equation. From Fig. P2.13(a),

$$p = vi = \frac{v^2}{R} \qquad \text{so} \qquad R = \frac{v^2}{p}$$

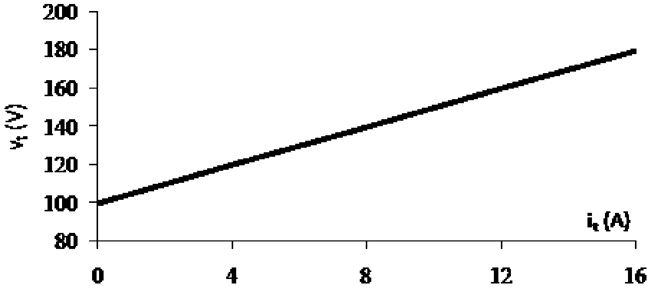
Using the values in the table of Fig. P2.13(b)

$$R = \frac{(-10)^2}{17.86 \times 10^{-3}} = \frac{(-5)^2}{4.46 \times 10^{-3}} = \frac{(5)^2}{4.46 \times 10^{-3}} = \frac{(10)^2}{17.86 \times 10^{-3}}$$

$$= \frac{(15)^2}{40.18 \times 10^{-3}} = \frac{(20)^2}{71.43 \times 10^{-3}} \approx 5.6\,\text{k}\Omega$$

Note that this value is found in Appendix H.

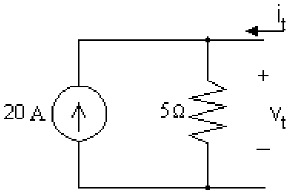
P 2.14 **[a]** Plot the v—i characteristic:



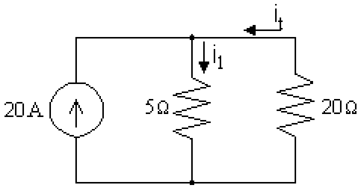
From the plot:

$$R = \frac{\Delta v}{\Delta i} = \frac{(180 - 100)}{(16 - 0)} = 5\,\Omega$$

When $i_t = 0$, $v_t = 100$ V; therefore the ideal current source must have a current of 100/5 = 20 A



[b] We attach a 20 Ω resistor to the device model developed in part (a):



Write a KCL equation at the top node:

$20 + i_t = i_1$

Write a KVL equation for the right loop, in the direction of the two currents, using Ohm's law:

$5i_1 + 20i_t = 0$

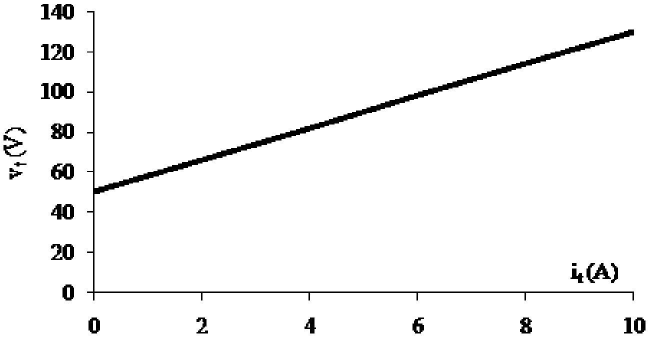
Combining the two equations and solving,

$5(20 + i_t) + 20i_t = 0 \qquad \text{so} \qquad 25i_t = -100; \qquad \text{thus} \qquad i_t = -4\,\text{A}$

Now calculate the power dissipated by the resistor:

$p_{20\,\Omega} = 20i_t^2 = 20(-4)^2 = 320\,\text{W}$

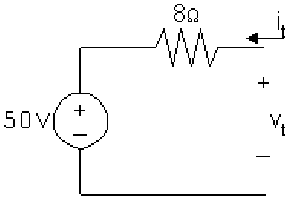
P 2.15 **[a]** Plot the $v - i$ characteristic



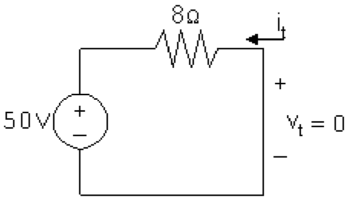
From the plot:

$$R = \frac{\Delta v}{\Delta i} = \frac{(130 - 50)}{(10 - 0)} = 8\,\Omega$$

When $i_t = 0$, $v_t = 50$ V; therefore the ideal voltage source has a voltage of 50 V.



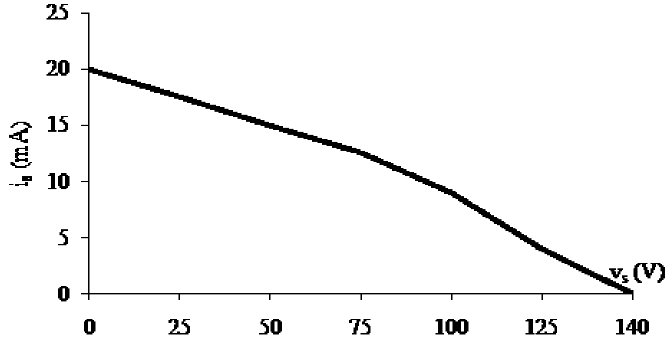
[b]



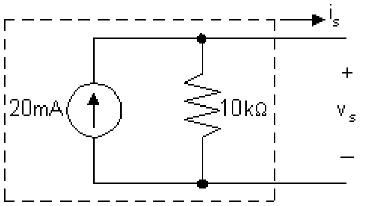
When $v_t = 0$, $\qquad i_t = \dfrac{-50}{8} = -6.25\text{A}$

Note that this result can also be obtained by extrapolating the $v - i$ characteristic to $v_t = 0$.

P 2.16 [a]

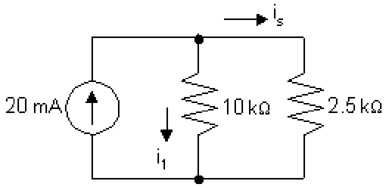


[b] $\Delta v = 25\text{V}; \quad \Delta i = 2.5 \text{ mA}; \quad R = \frac{\Delta v}{\Delta i} = 10 \text{ k}\Omega$



[c] $10{,}000 i_1 = 2500 i_s, \qquad i_1 = 0.25 i_s$

$0.02 = i_1 + i_s = 1.25 i_s, \qquad i_s = 16 \text{ mA}$

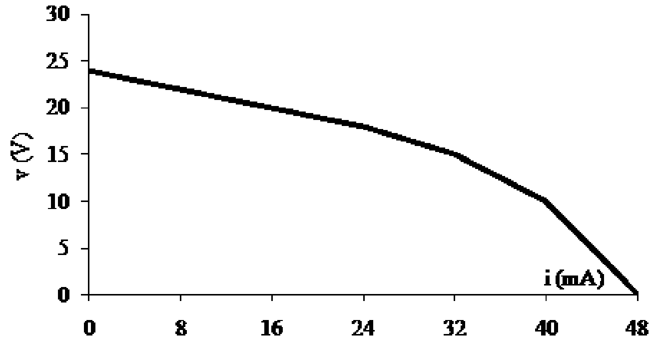


[d] $v_s(\text{open circuit}) = (20 \times 10^{-3})(10 \times 10^3) = 200$ V

[e] The open circuit voltage can be found in the table of values (or from the plot) as the value of the voltage v_s when the current $i_s = 0$. Thus, $v_s(\text{open circuit}) = 140$ V (from the table)

[f] Linear model cannot predict the nonlinear behavior of the practical current source.

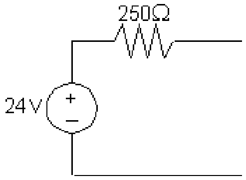
P 2.17 [a] Begin by constructing a plot of voltage versus current:



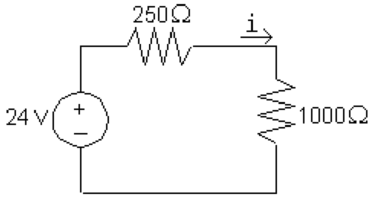
[b] Since the plot is linear for $0 \le i_s \le 24$ mA amd since $R = \Delta v/\Delta i$, we can calculate R from the plotted values as follows:

$$R = \frac{\Delta v}{\Delta i} = \frac{24 - 18}{0.024 - 0} = \frac{6}{0.024} = 250\,\Omega$$

We can determine the value of the ideal voltage source by considering the value of v_s when $i_s = 0$. When there is no current, there is no voltage drop across the resistor, so all of the voltage drop at the output is due to the voltage source. Thus the value of the voltage source must be 24 V. The model, valid for $0 \le i_s \le 24$ mA, is shown below:



[c] The circuit is shown below:

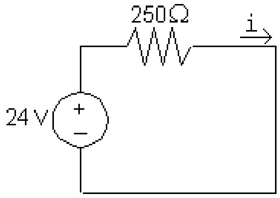


Write a KVL equation in the clockwise direction, starting below the voltage source. Use Ohm's law to express the voltage drop across the resistors in terms of the current i:

$$-24\,\text{V} + 250i + 1000i = 0 \qquad \text{so} \qquad 1250i = 24\,\text{V}$$

$$\text{Thus,} \qquad i = \frac{24\,\text{V}}{1250\,\Omega} = 19.2\,\text{mA}$$

[d] The circuit is shown below:



Write a KVL equation in the clockwise direction, starting below the voltage source. Use Ohm's law to express the voltage drop across the resistors in terms of the current i:

$$-24\,\text{V} + 250i = 0 \qquad \text{so} \qquad 250i = 24\,\text{V}$$

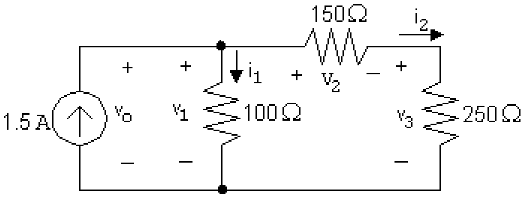
$$\text{Thus,} \qquad i = \frac{24\,\text{V}}{250\,\Omega} = 96\,\text{mA}$$

[**e**] The short circuit current can be found in the table of values (or from the plot) as the value of the current i_s when the voltage $v_s = 0$. Thus,

$i_{sc} = 48\,\text{mA}$ (from table)

[**f**] The plot of voltage versus current constructed in part (a) is not linear (it is piecewise linear, but not linear for all values of i_s). Since the proposed circuit model is a linear model, it cannot be used to predict the nonlinear behavior exhibited by the plotted data.

P 2.18



[**a**] Write a KCL equation at the top node:

$-1.5 + i_1 + i_2 = 0$ so $i_1 + i_2 = 1.5$

Write a KVL equation around the right loop:

$-v_1 + v_2 + v_3 = 0$

From Ohm's law,

$v_1 = 100i_1, \quad v_2 = 150i_2, \quad v_3 = 250i_2$

Substituting,

$-100i_1 + 150i_2 + 250i_2 = 0$ so $-100i_1 + 400i_2 = 0$

Solving the two equations for i_1 and i_2 simultaneously,

$i_1 = 1.2\,\text{A}$ and $i_2 = 0.3\,\text{A}$

[**b**] Write a KVL equation clockwise around the left loop:

$-v_o + v_1 = 0$ but $v_1 = 100i_1 = 100(1.2) = 120\,\text{V}$

So $v_o = v_1 = 120\,\text{V}$

[**c**] Calculate power using $p = vi$ for the source and $p = Ri^2$ for the resistors:

$p_{\text{source}} = -v_o(1.5) = -(120)(1.5) = -180\,\text{W}$

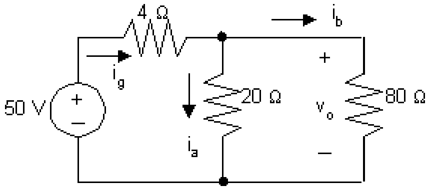
$p_{100\Omega} = 1.2^2(100) = 144\,\text{W}$

$p_{150\Omega} = 0.3^2(150) = 13.5\,\text{W}$

$p_{250\Omega} = 0.3^2(250) = 22.5\,\text{W}$

$\sum P_{\text{dev}} = 180\,\text{W} \qquad \sum P_{\text{abs}} = 144 + 13.5 + 22.5 = 180\,\text{W}$

P 2.19 [**a**]



$$\begin{aligned} 20i_{\rm a} &= 80i_{\rm b} \qquad i_g = i_{\rm a} + i_{\rm b} = 5i_{\rm b} \\ i_{\rm a} &= 4i_{\rm b} \\ 50 &= 4i_g + 80i_{\rm b} = 20i_{\rm b} + 80i_{\rm b} = 100i_{\rm b} \\ i_{\rm b} &= 0.5 \text{ A, therefore, } i_{\rm a} = 2 \text{ A} \quad \text{and} \quad i_g = 2.5 \text{ A} \end{aligned}$$

[**b**] $i_{\rm b} = 0.5$ A

[**c**] $v_o = 80i_{\rm b} = 40$ V

[**d**]
$$\begin{aligned} p_{4\Omega} &= i_g^2(4) = 6.25(4) = 25 \text{ W} \\ p_{20\Omega} &= i_{\rm a}^2(20) = (4)(20) = 80 \text{ W} \\ p_{80\Omega} &= i_{\rm b}^2(80) = 0.25(80) = 20 \text{ W} \end{aligned}$$

[**e**] $p_{50\rm V}$ (delivered) $= 50i_g = 125$ W
Check:

$$\sum P_{\rm dis} = 25 + 80 + 20 = 125\,\text{W}$$

$$\sum P_{\rm del} = 125\,\text{W}$$

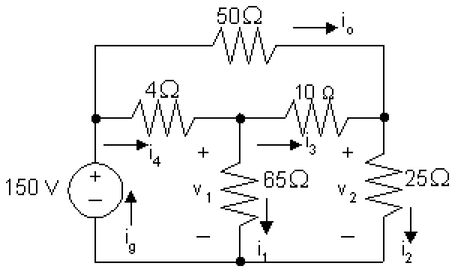
P 2.20 [**a**] Use KVL for the right loop to calculate the voltage drop across the right-hand branch v_o. This is also the voltage drop across the middle branch, so once v_o is known, use Ohm's law to calculate i_o:

$$\begin{aligned} v_o &= 1000i_{\rm a} + 4000i_{\rm a} + 3000i_{\rm a} = 8000i_{\rm a} = 8000(0.002) = 16 \text{ V} \\ 16 &= 2000i_o \\ i_o &= \frac{16}{2000} = 8 \text{ mA} \end{aligned}$$

[**b**] KCL at the top node: $i_g = i_{\rm a} + i_o = 0.002 + 0.008 = 0.010 \text{ A} = 10$ mA.

[**c**] The voltage drop across the source is v_0, seen by writing a KVL equation for the left loop. Thus,
$p_g = -v_o i_{\rm g} = -(16)(0.01) = -0.160 \text{ W} = -160$ mW.
Thus the source delivers 160 mW.

P 2.21 **[a]**



$$v_2 = 150 - 50(1) = 100\text{V}$$

$$i_2 = \frac{v_2}{25} = 4\text{A}$$

$$i_3 + 1 = i_2, \qquad i_3 = 4 - 1 = 3\text{A}$$

$$v_1 = 10i_3 + 25i_2 = 10(3) + 25(4) = 130\text{V}$$

$$i_1 = \frac{v_1}{65} = \frac{130}{65} = 2\text{A}$$

Note also that

$$i_4 = i_1 + i_3 = 2 + 3 = 5\,\text{A}$$

$$i_g = i_4 + i_o = 5 + 1 = 6\,\text{A}$$

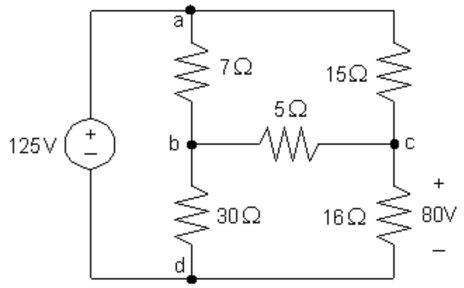
[b]

$$\begin{aligned} p_{4\Omega} &= 5^2(4) = 100\ \text{W} \\ p_{50\Omega} &= 1^2(50) = 50\ \text{W} \\ p_{65\Omega} &= 2^2(65) = 260\ \text{W} \\ p_{10\Omega} &= 3^2(10) = 90\ \text{W} \\ p_{25\Omega} &= 4^2(25) = 400\ \text{W} \end{aligned}$$

[c] $\sum P_{\text{dis}} = 100 + 50 + 260 + 90 + 400 = 900\,\text{W}$

$$P_{\text{dev}} = 150i_g = 150(6) = 900\,\text{W}$$

P 2.22 **[a]**



$$i_{\text{cd}} = 80/16 = 5\,\text{A}$$

$v_{\rm ac} = 125 - 80 = 45 \qquad \text{so} \qquad i_{\rm ac} = 45/15 = 3\,\text{A}$

$i_{\rm ac} + i_{\rm bc} = i_{\rm cd} \qquad \text{so} \qquad i_{\rm bc} = 5 - 3 = 2\,\text{A}$

$v_{\rm ab} = 15i_{\rm ac} - 5i_{\rm bc} = 15(3) - 5(2) = 35\,\text{V} \qquad \text{so} \qquad i_{\rm ab} = 35/7 = 5\,\text{A}$

$i_{\rm bd} = i_{\rm ab} - i_{\rm bc} = 5 - 2 = 3\,\text{A}$

Calculate the power dissipated by the resistors using the equation $p_R = Ri_R^2$:

$p_{7\Omega} = (7)(5)^2 = 175\,\text{W} \qquad p_{30\Omega} = (30)(3)^2 = 270\,\text{W}$

$p_{15\Omega} = (15)(3)^2 = 135\,\text{W} \qquad p_{16\Omega} = (16)(5)^2 = 400\,\text{W}$

$p_{5\Omega} = (5)(2)^2 = 20\,\text{W}$

[b] Calculate the current through the voltage source:

$i_{\rm ad} = -i_{\rm ab} - i_{\rm ac} = -5 - 3 = -8\,\text{A}$

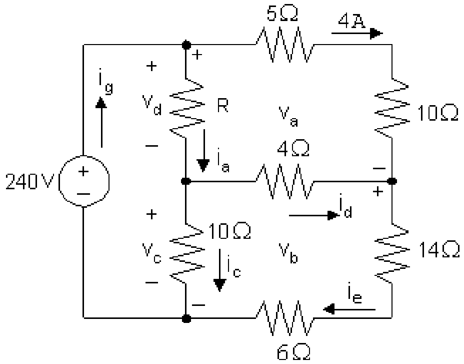
Now that we have both the voltage and the current for the source, we can calculate the power supplied by the source:

$p_g = 125(-8) = -1000\,\text{W} \qquad \text{thus} \qquad p_g\ \text{(supplied)} = 1000\,\text{W}$

[c] $\sum P_{\rm dis} = 175 + 270 + 135 + 400 + 20 = 1000\,\text{W}$
Therefore,

$\sum P_{\rm supp} = \sum P_{\rm dis}$

P 2.23 **[a]**

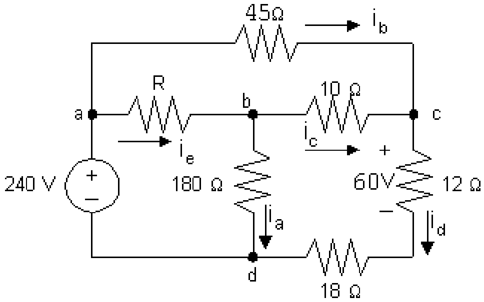


$v_a = (5 + 10)(4) = 60\,\text{V}$
$-240 + v_a + v_b = 0 \qquad \text{so} \qquad v_b = 240 - v_a = 240 - 60 = 180\,\text{V}$
$i_e = v_b/(14 + 6) = 180/20 = 9\,\text{A}$
$i_d = i_e - 4 = 9 - 4 = 5\,\text{A}$
$v_c = 4i_d + v_b = 4(5) + 180 = 200\,\text{V}$
$i_c = v_c/10 = 200/10 = 20\,\text{A}$
$v_d = 240 - v_c = 240 - 200 = 40\,\text{V}$
$i_a = i_d + i_c = 5 + 20 = 25\,\text{A}$
$R = v_d/i_a = 40/25 = 1.6\,\Omega$

[b] $i_g = i_a + 4 = 25 + 4 = 29$ A

p_g (supplied) $= (240)(29) = 6960$ W

P 2.24



$i_d = 60/12 = 5\,\mathrm{A}$; therefore, $v_{cd} = 60 + 18(5) = 150\,\mathrm{V}$

$-240 + v_{ac} + v_{cd} = 0$; therefore, $v_{ac} = 240 - 150 = 90\,\mathrm{V}$

$i_b = v_{ac}/45 = 90/45 = 2\,\mathrm{A}$; therefore, $i_c = i_d - i_b = 5 - 2 = 3\,\mathrm{A}$

$v_{bd} = 10i_c + v_{cd} = 10(3) + 150 = 180\,\mathrm{V}$;

therefore, $i_a = v_{bd}/180 = 180/180 = 1\,\mathrm{A}$

$i_e = i_a + i_c = 1 + 3 = 4\,\mathrm{A}$

$-240 + v_{ab} + v_{bd} = 0$ therefore, $v_{ab} = 240 - 180 = 60\,\mathrm{V}$

$R = v_{ab}/i_e = 60/4 = 15\,\Omega$

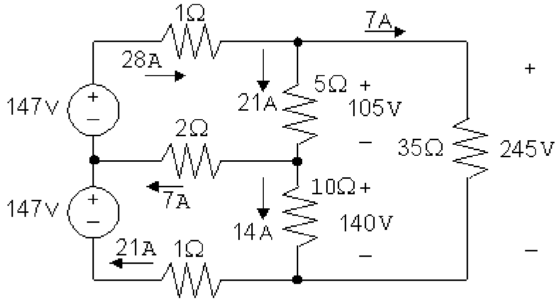
CHECK: $i_g = i_b + i_e = 2 + 4 = 6$ A

$$p_{\text{dev}} = (240)(6) = 1440 \text{ W}$$

$$\sum P_{\text{dis}} = 1^2(180) + 4^2(15) + 3^2(10) + 5^2(12) + 5^2(18) + 2^2(45)$$

$$= 1440 \text{ W (CHECKS)}$$

P 2.25 **[a]** Start by calculating the voltage drops due to the currents i_1 and i_2. Then use KVL to calculate the voltage drop across and $35\,\Omega$ resistor, and Ohm's law to find the current in the $35\,\Omega$ resistor. Finally, KCL at each of the middle three nodes yields the currents in the two sources and the current in the middle $2\,\Omega$ resistor. These calculations are summarized in the figure below:



$$p_{147\text{(top)}} = -(147)(28) = -4116 \text{ W}$$

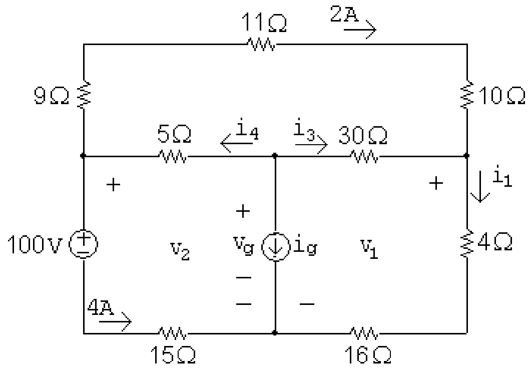
$$p_{147\text{(bottom)}} = -(147)(21) = -3087 \text{ W}$$

[b]

$$\begin{aligned}\sum P_{\text{dis}} &= (28)^2(1) + (7)^2(2) + (21)^2(1) + (21)^2(5) + (14)^2(10) + (7)^2(35)\\ &= 784 + 98 + 441 + 2205 + 1960 + 1715 = 7203 \text{ W}\\ \sum P_{\text{sup}} &= 4116 + 3087 = 7203 \text{ W}\end{aligned}$$

Therefore, $\sum P_{\text{dis}} = \sum P_{\text{sup}} = 7203$ W

P 2.26 **[a]**



$v_2 = 100 + 4(15) = 160\,\text{V}; \qquad v_1 = 160 - (9 + 11 + 10)(2) = 100\,\text{V}$

$i_1 = \dfrac{v_1}{4 + 16} = \dfrac{100}{20} = 5\,\text{A}; \qquad i_3 = i_1 - 2 = 5 - 2 = 3\,\text{A}$

$v_g = v_1 + 30i_3 = 100 + 30(3) = 190\,\text{V}$

$i_4 = 2 + 4 = 6\,\text{A}$

$i_g = -i_4 - i_3 = -6 - 3 = -9\,\text{A}$

[b] Calculate power using the formula $p = Ri^2$:

$p_{9\,\Omega} = (9)(2)^2 = 36\,\text{W}; \qquad p_{11\,\Omega} = (11)(2)^2 = 44\,\text{W}$

$p_{10\,\Omega} = (10)(2)^2 = 40\,\text{W}; \qquad p_{5\,\Omega} = (5)(6)^2 = 180\,\text{W}$

$p_{30\,\Omega} = (30)(3)^2 = 270\,\text{W}; \qquad p_{4\,\Omega} = (4)(5)^2 = 100\,\text{W}$

$p_{16\,\Omega} = (16)(5)^2 = 400\,\text{W}; \qquad p_{15\,\Omega} = (15)(4)^2 = 240\,\text{W}$

[c] $v_g = 190\,\text{V}$

[d] Sum the power dissipated by the resistors:

$\sum p_{\text{diss}} = 36 + 44 + 40 + 180 + 270 + 100 + 400 + 240 = 1310\,\text{W}$

The power associated with the sources is

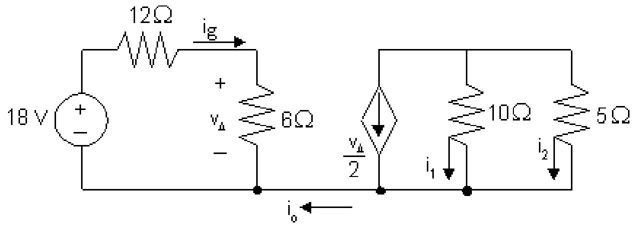
$p_{\text{volt-source}} = (100)(4) = 400\,\text{W}$

$p_{\text{curr-source}} = v_g i_g = (190)(-9) = -1710\,\text{W}$

Thus the total power dissipated is 1310 + 400 = 1710 W and the total power developed is 1710 W, so the power balances.

P 2.27 [**a**] $i_o = 0$ because no current can exist in a single conductor connecting two parts of a circuit.

[**b**]



$$18 = (12 + 6)i_g \qquad i_g = 1 \text{ A}$$

$$v_\Delta = 6i_g = 6\text{V} \qquad v_\Delta/2 = 3 \text{ A}$$

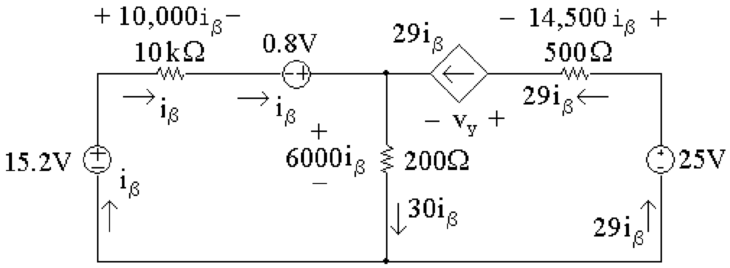
$$10i_1 = 5i_2, \text{ so } i_1 + 2i_1 = -3 \text{ A; therefore, } i_1 = -1 \text{ A}$$

[**c**] $i_2 = 2i_1 = -2$ A.

P 2.28 First note that we know the current through all elements in the circuit except the 200 Ω resistor (the current in the three elements to the left of the 200 Ω resistor is i_β; the current in the three elements to the right of the 200 Ω resistor is $29i_\beta$). To find the current in the 200 Ω resistor, write a KCL equation at the top node:

$$i_\beta + 29i_\beta = i_{200\,\Omega} = 30i_\beta$$

We can then use Ohm's law to find the voltages across each resistor in terms of i_β. The results are shown in the figure below:



[**a**] To find i_β, write a KVL equation around the left-hand loop, summing voltages in a clockwise direction starting below the 15.2V source:

$$-15.2\,\text{V} + 10{,}000i_1 - 0.8\,\text{V} + 6000i_\beta = 0$$

Solving for i_β

$$10{,}000i_\beta + 6000i_\beta = 16\,\text{V} \qquad \text{so} \qquad 16{,}000i_\beta = 16\,\text{V}$$

Thus,

$$i_\beta = \frac{16}{16{,}000} = 1\,\mathrm{mA}$$

Now that we have the value of i_β, we can calculate the voltage for each component except the dependent source. Then we can write a KVL equation for the right-hand loop to find the voltage v_y of the dependent source. Sum the voltages in the clockwise direction, starting to the left of the dependent source:

$$-v_y - 14{,}500 i_\beta + 25\,\mathrm{V} - 6000 i_\beta = 0$$

Thus,

$$v_y = 25\,\mathrm{V} - 20{,}500 i_\beta = 25\,\mathrm{V} - 20{,}500(10^{-3}) = 25\,\mathrm{V} - 20.5\,\mathrm{V} = 4.5\,\mathrm{V}$$

[**b**] We now know the values of voltage and current for every circuit element. Let's construct a power table:

Element	**Current (mA)**	**Voltage (V)**	**Power Equation**	**Power (mW)**
15.2 V	1	15.2	$p = -vi$	−15.2
10 kΩ	1	10	$p = Ri^2$	10
0.8 V	1	0.8	$p = -vi$	−0.8
200 Ω	30	6	$p = Ri^2$	180
Dep. source	29	4.5	$p = vi$	130.5
500 Ω	29	14.5	$p = Ri^2$	420.5
25 V	29	25	$p = -vi$	−725

The total power generated in the circuit is the sum of the negative power values in the power table:

$$-15.2\,\mathrm{mW} + -0.8\,\mathrm{mW} + -725\,\mathrm{mW} = -741\,\mathrm{mW}$$

Thus, the total power generated in the circuit is 741 mW. The total power absorbed in the circuit is the sum of the positive power values in the power table:

$$10\,\mathrm{mW} + 180\,\mathrm{mW} + 130.5\,\mathrm{mW} + 420.5\,\mathrm{mW} = 741\,\mathrm{mW}$$

Thus, the total power absorbed in the circuit is 741 mW and the power in the circuit balances.

P 2.29 $40i_2 + \frac{5}{40} + \frac{5}{10} = 0; \quad i_2 = -15.625\ \mathrm{mA}$

$$v_1 = 80i_2 = -1.25\ \mathrm{V}$$

$$25i_1 + \frac{(-1.25)}{20} + (-0.015625) = 0; \quad i_1 = 3.125 \text{ mA}$$

$$v_g = 60i_1 + 260i_1 = 320i_1$$

Therefore, $v_g = 1$ V.

P 2.30 **[a]** $-50 - 20i_\sigma + 18i_\Delta = 0$

$$-18i_\Delta + 5i_\sigma + 40i_\sigma = 0 \quad \text{so} \quad 18i_\Delta = 45i_\sigma$$

Therefore, $\quad -50 - 20i_\sigma + 45i_\sigma = 0, \quad$ so $\quad i_\sigma = 2\,\text{A}$

$$18i_\Delta = 45i_\sigma = 90; \ \text{so } i_\Delta = 5\,\text{A}$$

$$v_o = 40i_\sigma = 80\,\text{V}$$

[b] i_g = current out of the positive terminal of the 50 V source

v_d = voltage drop across the $8i_\Delta$ source

$$i_g = i_\Delta + i_\sigma + 8i_\Delta = 9i_\Delta + i_\sigma = 47\,\text{A}$$

$$v_d = 80 - 20 = 60\,\text{V}$$

$$\begin{aligned}
\sum P_\text{gen} &= 50i_g + 20i_\sigma i_g = 50(47) + 20(2)(47) = 4230 \text{ W} \\
\sum P_\text{diss} &= 18i_\Delta^2 + 5i_\sigma(i_g - i_\Delta) + 40i_\sigma^2 + 8i_\Delta v_d + 8i_\Delta(20) \\
&= (18)(25) + 10(47-5) + 4(40) + 40(60) + 40(20) \\
&= 4230 \text{ W; Therefore,} \\
\sum P_\text{gen} &= \sum P_\text{diss} = 4230 \text{ W}
\end{aligned}$$

P 2.31 $i_E - i_B - i_C = 0$

$$i_C = \beta i_B \qquad \text{therefore } i_E = (1+\beta)i_B$$

$$i_2 = -i_B + i_1$$

$$V_o + i_E R_E - (i_1 - i_B)R_2 = 0$$

$$-i_1R_1 + V_{CC} - (i_1 - i_B)R_2 = 0 \quad \text{or} \quad i_1 = \frac{V_{CC} + i_BR_2}{R_1 + R_2}$$

$$V_o + i_ER_E + i_BR_2 - \frac{V_{CC} + i_BR_2}{R_1 + R_2}R_2 = 0$$

Now replace i_E by $(1+\beta)i_B$ and solve for i_B. Thus

$$i_B = \frac{[V_{CC}R_2/(R_1 + R_2)] - V_o}{(1+\beta)R_E + R_1R_2/(R_1 + R_2)}$$

P 2.32 Here is Equation 2.25:

$$i_{\rm B}=\frac{(V_{\rm CC}R_2)/(R_1+R_2)-V_0}{(R_1R_2)/(R_1+R_2)+(1+\beta)R_{\rm E}}$$

$$\frac{V_{CC}R_2}{R_1+R_2}=\frac{(10)(60{,}000)}{100{,}000}=6\text{V}$$

$$\frac{R_1R_2}{R_1+R_2}=\frac{(40{,}000)(60{,}000)}{100{,}000}=24\ \text{k}\Omega$$

$$i_B=\frac{6-0.6}{24{,}000+50(120)}=\frac{5.4}{30{,}000}=0.18\ \text{mA}$$

$$i_C=\beta i_B=(49)(0.18)=8.82\ \text{mA}$$

$$i_E=i_C+i_B=8.82+0.18=9\ \text{mA}$$

$$v_{3d}=(0.009)(120)=1.08\text{V}$$

$$v_{bd}=V_o+v_{3d}=1.68\text{V}$$

$$i_2=\frac{v_{bd}}{R_2}=\frac{1.68}{60{,}000}=28\,\mu\text{A}$$

$$i_1=i_2+i_B=28+180=208\,\mu\text{A}$$

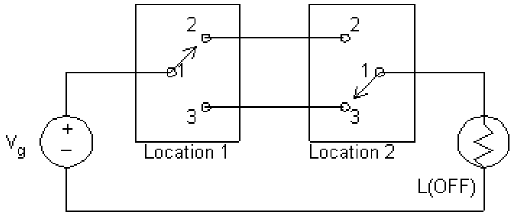
$$v_{\rm ab}=40{,}000(208\times10^{-6})=8.32\,\text{V}$$

$$i_{CC}=i_C+i_1=8.82+0.208=9.028\ \text{mA}$$

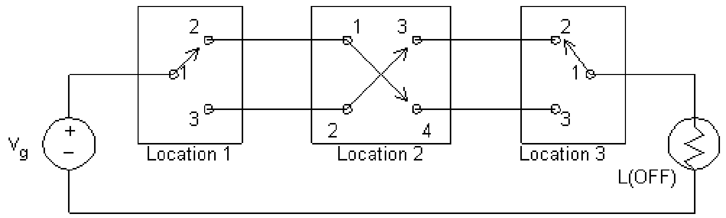
$$v_{13}+(8.82\times10^{-3})(750)+1.08=10\ \text{V}$$

$$v_{13}=2.305\,\text{V}$$

P 2.33 [**a**]



[b]



P 2.34 **[a]** From the simplified circuit model, using Ohm's law and KVL:

$$400i + 50i + 200i - 250 = 0 \qquad \text{so} \qquad i = 250/650 = 385 \text{ mA}$$

This current is nearly enough to stop the heart, according to Table 2.1, so a warning sign should be posted at the 250 V source.

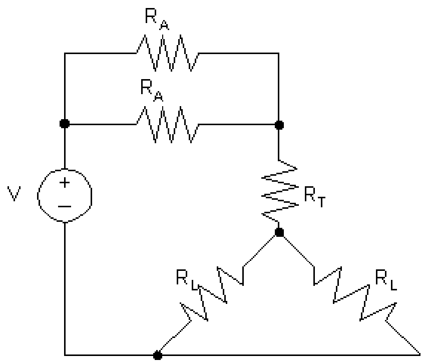
[b] The closest value from Appendix H to 400 Ω is 390 Ω; the closest value from Appendix H to 50 Ω is 47 Ω. There are two possibilites for replacing the 200 Ω resistor with a value from Appendix H – 180 Ω and 220 Ω. We calculate the resulting current for each of these possibilities, and determine which current is closest to 385 mA:

$$390i + 47i + 180i - 250 = 0 \qquad \text{so} \qquad i = 250/617 = 405.2 \text{ mA}$$

$$390i + 47i + 220i - 250 = 0 \qquad \text{so} \qquad i = 250/657 = 380.5 \text{ mA}$$

Therefore, choose the 220 Ω resistor to replace the 200 Ω resistor in the model.

P 2.35



P 2.36 **[a]** $p = i^2R$

$$p_{\text{arm}} = \left(\frac{250}{650}\right)^2 (400) = 59.17\,\text{W}$$

$$p_{\text{leg}} = \left(\frac{250}{650}\right)^2 (200) = 29.59\,\text{W}$$

$$p_{\text{trunk}} = \left(\frac{250}{650}\right)^2 (50) = 7.40\,\text{W}$$

[b] $\left(\frac{dT}{dt}\right)_{\text{arm}} = \frac{2.39 \times 10^{-4} p_{\text{arm}}}{4} = 35.36 \times 10^{-4}\,^\circ\text{ C/s}$

$$t_{\text{arm}} = \frac{5}{35.36} \times 10^4 = 1414.23 \text{ s or } 23.57 \text{ min}$$

$$\left(\frac{dT}{dt}\right)_{\text{leg}} = \frac{2.39 \times 10^{-4}}{10} P_{\text{leg}} = 7.07 \times 10^{-4\circ} \text{ C/s}$$

$$t_{\text{leg}} = \frac{5 \times 10^4}{7.07} = 7{,}071.13 \text{ s or } 117.85 \text{ min}$$

$$\left(\frac{dT}{dt}\right)_{\text{trunk}} = \frac{2.39 \times 10^{-4}(7.4)}{25} = 0.707 \times 10^{-4}\,^\circ \text{ C/s}$$

$$t_{\text{trunk}} = \frac{5 \times 10^4}{0.707} = 70{,}711.30 \text{ s or } 1{,}178.52 \text{ min}$$

[c] They are all much greater than a few minutes.

P 2.37 **[a]** $R_{\text{arms}} = 400 + 400 = 800\,\Omega$

$$i_{\text{letgo}} = 50 \text{ mA (minimum)}$$

$$v_{\text{min}} = (800)(50) \times 10^{-3} = 40\,\text{V}$$

[b] No, 12/800 = 15 mA. Note this current is sufficient to give a perceptible shock.

P 2.38 $R_{\text{space}} = 1 \text{ M}\Omega$

$$i_{\text{space}} = 3 \text{ mA}$$

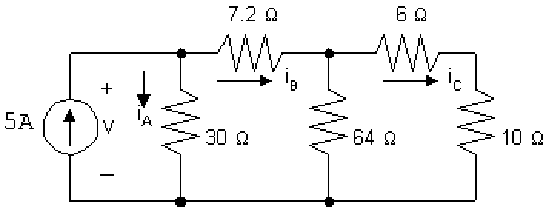
$$v = i_{\text{space}} R_{\text{space}} = 3000\,\text{V}.$$

3

Simple Resistive Circuits

Assessment Problems

AP 3.1



Start from the right hand side of the circuit and make series and parallel combinations of the resistors until one equivalent resistor remains. Begin by combining the 6 Ω resistor and the 10 Ω resistor in series:

$$6\,\Omega + 10\,\Omega = 16\,\Omega$$

Now combine this 16 Ω resistor in parallel with the 64 Ω resistor:

$$16\,\Omega \| 64\,\Omega = \frac{(16)(64)}{16+64} = \frac{1024}{80} = 12.8\,\Omega$$

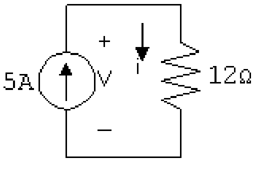
This equivalent 12.8 Ω resistor is in series with the 7.2 Ω resistor:

$$12.8\,\Omega + 7.2\,\Omega = 20\,\Omega$$

Finally, this equivalent 20 Ω resistor is in parallel with the 30 Ω resistor:

$$20\,\Omega \| 30\,\Omega = \frac{(20)(30)}{20+30} = \frac{600}{50} = 12\,\Omega$$

Thus, the simplified circuit is as shown:



[a] With the simplified circuit we can use Ohm's law to find the voltage across both the current source and the $12\,\Omega$ equivalent resistor:

$$v = (12\,\Omega)(5\text{ A}) = 60\text{ V}$$

[b] Now that we know the value of the voltage drop across the current source, we can use the formula $p = -vi$ to find the power associated with the source:

$$p = -(60\text{ V})(5\text{ A}) = -300\text{ W}$$

Thus, the source delivers 300 W of power to the circuit.

[c] We now can return to the original circuit, shown in the first figure. In this circuit, $v = 60$ V, as calculated in part (a). This is also the voltage drop across the $30\,\Omega$ resistor, so we can use Ohm's law to calculate the current through this resistor:

$$i_A = \frac{60\text{ V}}{30\,\Omega} = 2\text{ A}$$

Now write a KCL equation at the upper left node to find the current i_B:

$$-5\text{ A} + i_A + i_B = 0 \qquad \text{so} \qquad i_B = 5\text{ A} - i_A = 5\text{ A} - 2\text{ A} = 3\text{ A}$$

Next, write a KVL equation around the outer loop of the circuit, using Ohm's law to express the voltage drop across the resistors in terms of the current through the resistors:

$$-v + 7.2i_B + 6i_C + 10i_C = 0$$

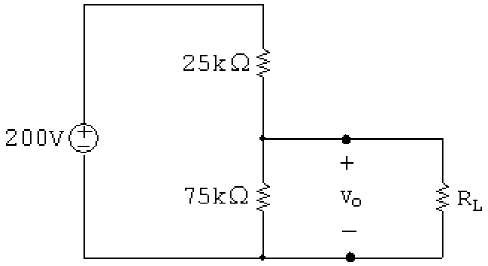
$$\text{So} \qquad 16i_C = v - 7.2i_B = 60\text{ V} - (7.2)(3) = 38.4\text{ V}$$

$$\text{Thus} \qquad i_C = \frac{38.4}{16} = 2.4\text{ A}$$

Now that we have the current through the $10\,\Omega$ resistor we can use the formula $p = Ri^2$ to find the power:

$$p_{10\,\Omega} = (10)(2.4)^2 = 57.6\text{ W}$$

AP 3.2



[a] We can use voltage division to calculate the voltage v_o across the 75 kΩ resistor:

$$v_o(\text{no load}) = \frac{75{,}000}{75{,}000 + 25{,}000}(200\text{ V}) = 150\text{ V}$$

[b] When we have a load resistance of 150 kΩ then the voltage v_o is across the parallel combination of the 75 kΩ resistor and the 150 kΩ resistor. First, calculate the equivalent resistance of the parallel combination:

$$75\text{ k}\Omega \| 150\text{ k}\Omega = \frac{(75{,}000)(150{,}000)}{75{,}000 + 150{,}000} = 50{,}000\,\Omega = 50\text{ k}\Omega$$

Now use voltage division to find v_o across this equivalent resistance:

$$v_o = \frac{50{,}000}{50{,}000 + 25{,}000}(200\text{ V}) = 133.3\text{ V}$$

[c] If the load terminals are short-circuited, the 75 kΩ resistor is effectively removed from the circuit, leaving only the voltage source and the 25 kΩ resistor. We can calculate the current in the resistor using Ohm's law:

$$i = \frac{200\text{ V}}{25\text{ k}\Omega} = 8\text{ mA}$$

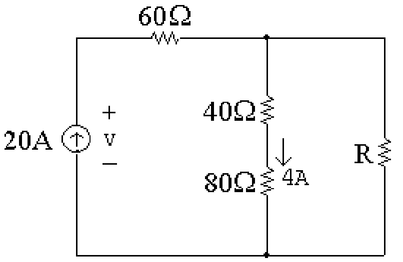
Now we can use the formula $p = Ri^2$ to find the power dissipated in the 25 kΩ resistor:

$$p_{25k} = (25{,}000)(0.008)^2 = 1.6\text{ W}$$

[d] The power dissipated in the 75 kΩ resistor will be maximum at no load since v_o is maximum. In part (a) we determined that the no-load voltage is 150 V, so be can use the formula $p = v^2/R$ to calculate the power:

$$p_{75k}(\text{max}) = \frac{(150)^2}{75{,}000} = 0.3\text{ W}$$

AP 3.3



[a] We will write a current division equation for the current throught the 80Ω resistor and use this equation to solve for R:

$$i_{80\Omega} = \frac{R}{R + 40\,\Omega + 80\,\Omega}(20\text{ A}) = 4\text{ A} \qquad \text{so} \qquad 20R = 4(R + 120)$$

$$\text{Thus} \qquad 16R = 480 \qquad \text{and} \qquad R = \frac{480}{16} = 30\,\Omega$$

[b] With $R = 30\,\Omega$ we can calculate the current through R using current division, and then use this current to find the power dissipated by R, using the formula $p = Ri^2$:

$$i_R = \frac{40 + 80}{40 + 80 + 30}(20\text{ A}) = 16\text{ A} \qquad \text{so} \qquad p_R = (30)(16)^2 = 7680\text{ W}$$

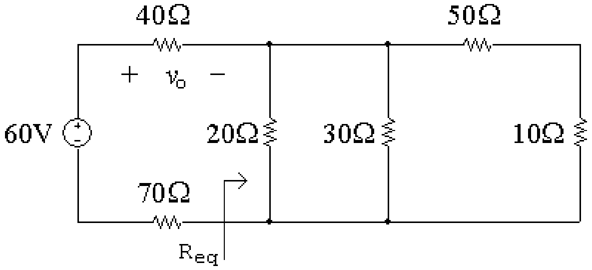
[c] Write a KVL equation around the outer loop to solve for the voltage v, and then use the formula $p = -vi$ to calculate the power delivered by the current source:

$$-v + (60\,\Omega)(20\text{ A}) + (30\,\Omega)(16\text{ A}) = 0 \qquad \text{so} \qquad v = 1200 + 480 = 1680\text{ V}$$

Thus, $\quad p_{\text{source}} = -(1680\text{ V})(20\text{ A}) = -33{,}600\text{ W}$

Thus, the current source generates 33,600 W of power.

AP 3.4



[a] First we need to determine the equivalent resistance to the right of the 40 Ω and 70 Ω resistors:

$$R_{\text{eq}} = 20\,\Omega\|30\,\Omega\|(50\,\Omega + 10\,\Omega) \qquad \text{so} \qquad \frac{1}{R_{\text{eq}}} = \frac{1}{20\,\Omega} + \frac{1}{30\,\Omega} + \frac{1}{60\,\Omega} = \frac{1}{10\,\Omega}$$

Thus, $\quad R_{\text{eq}} = 10\,\Omega$

Now we can use voltage division to find the voltage v_o:

$$v_o = \frac{40}{40 + 10 + 70}(60\text{ V}) = 20\text{ V}$$

[b] The current through the 40 Ω resistor can be found using Ohm's law:

$$i_{40\Omega} = \frac{v_o}{40} = \frac{20\text{ V}}{40\,\Omega} = 0.5\text{ A}$$

This current flows from left to right through the 40 Ω resistor. To use current division, we need to find the equivalent resistance of the two parallel branches containing the 20 Ω resistor and the 50 Ω and 10 Ω resistors:

$$20\,\Omega\|(50\,\Omega + 10\,\Omega) = \frac{(20)(60)}{20 + 60} = 15\,\Omega$$

Now we use current division to find the current in the 30 Ω branch:

$$i_{30\Omega} = \frac{15}{15 + 30}(0.5\text{ A}) = 0.16667\text{ A} = 166.67\text{ mA}$$

[c] We can find the power dissipated by the 50 Ω resistor if we can find the current in this resistor. We can use current division to find this current

from the current in the 40 Ω resistor, but first we need to calculate the equivalent resistance of the 20 Ω branch and the 30 Ω branch:

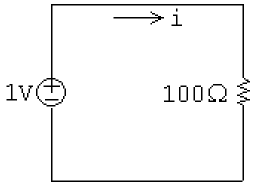
$$20\,\Omega \| 30\,\Omega = \frac{(20)(30)}{20+30} = 12\,\Omega$$

Current division gives:

$$i_{50\Omega} = \frac{12}{12+50+10}(0.5\text{ A}) = 0.08333\text{ A}$$

Thus, $p_{50\Omega} = (50)(0.08333)^2 = 0.34722\text{ W} = 347.22\text{ mW}$

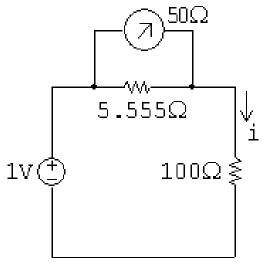
AP 3.5 [**a**]



We can find the current i using Ohm's law:

$$i = \frac{1\text{ V}}{100\,\Omega} = 0.01\text{ A} = 10\text{ mA}$$

[**b**]

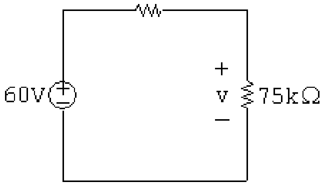


$R_m = 50\,\Omega \| 5.555\,\Omega = 5\,\Omega$

We can use the meter resistance to find the current using Ohm's law:

$$i_{\text{meas}} = \frac{1\text{ V}}{100\,\Omega + 5\,\Omega} = 0.009524 = 9.524\text{ mA}$$

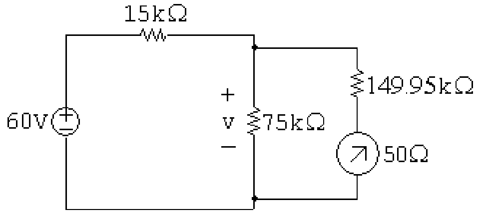
AP 3.6 [**a**]



Use voltage division to find the voltage v:

$$v = \frac{75{,}000}{75{,}000+15{,}000}(60\text{ V}) = 50\text{ V}$$

[b]



The meter resistance is a series combination of resistances:

$R_m = 149{,}950 + 50 = 150{,}000\,\Omega$

We can use voltage division to find v, but first we must calculate the equivalent resistance of the parallel combination of the 75 kΩ resistor and the voltmeter:

$$75{,}000\,\Omega \| 150{,}000\,\Omega = \frac{(75{,}000)(150{,}000)}{75{,}000 + 150{,}000} = 50 \text{ k}\Omega$$

Thus, $$v_{\text{meas}} = \frac{50{,}000}{50{,}000 + 15{,}000}(60 \text{ V}) = 46.15 \text{ V}$$

AP 3.7 **[a]** Using the condition for a balanced bridge, the products of the opposite resistors must be equal. Therefore,

$$100R_x = (1000)(150) \qquad \text{so} \qquad R_x = \frac{(1000)(150)}{100} = 1500\,\Omega = 1.5 \text{ k}\Omega$$

[b] When the bridge is balanced, there is no current flowing through the meter, so the meter acts like an open circuit. This places the following branches in parallel: The branch with the voltage source, the branch with the series combination R_1 and R_3 and the branch with the series combination of R_2 and R_x. We can find the current in the latter two branches using Ohm's law:

$$i_{R_1,R_3} = \frac{5 \text{ V}}{100\,\Omega + 150\,\Omega} = 20 \text{ mA}; \qquad i_{R_2,R_x} = \frac{5 \text{ V}}{1000 + 1500} = 2 \text{ mA}$$

We can calculate the power dissipated by each resistor using the formula $p = Ri^2$:

$$p_{100\Omega} = (100\,\Omega)(0.02 \text{ A})^2 = 40 \text{ mW}$$

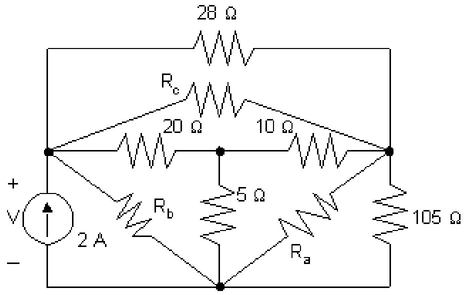
$$p_{150\Omega} = (150\,\Omega)(0.02 \text{ A})^2 = 60 \text{ mW}$$

$$p_{1000\Omega} = (1000\,\Omega)(0.002 \text{ A})^2 = 4 \text{ mW}$$

$$p_{1500\Omega} = (1500\,\Omega)(0.002 \text{ A})^2 = 6 \text{ mW}$$

Since none of the power dissipation values exceeds 250 mW, the bridge can be left in the balanced state without exceeding the power-dissipating capacity of the resistors.

AP 3.8 Convert the three Y-connected resistors, $20\,\Omega$, $10\,\Omega$, and $5\,\Omega$ to three Δ-connected resistors R_a, R_b, and R_c. To assist you the figure below has both the Y-connected resistors and the Δ-connected resistors

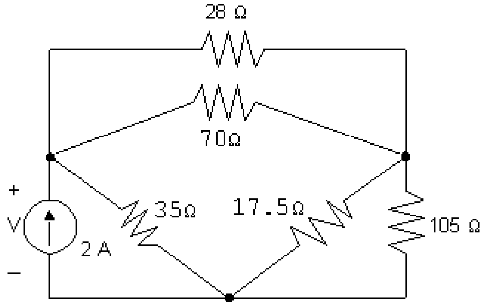


$$R_\mathrm{a} = \frac{(5)(10) + (5)(20) + (10)(20)}{20} = 17.5\,\Omega$$

$$R_\mathrm{b} = \frac{(5)(10) + (5)(20) + (10)(20)}{10} = 35\,\Omega$$

$$R_\mathrm{c} = \frac{(5)(10) + (5)(20) + (10)(20)}{5} = 70\,\Omega$$

The circuit with these new Δ-connected resistors is shown below:



From this circuit we see that the $70\,\Omega$ resistor is parallel to the $28\,\Omega$ resistor:

$$70\,\Omega \| 28\,\Omega = \frac{(70)(28)}{70 + 28} = 20\,\Omega$$

Also, the $17.5\,\Omega$ resistor is parallel to the $105\,\Omega$ resistor:

$$17.5\,\Omega \| 105\,\Omega = \frac{(17.5)(105)}{17.5 + 105} = 15\,\Omega$$

Once the parallel combinations are made, we can see that the equivalent $20\,\Omega$ resistor is in series with the equivalent $15\,\Omega$ resistor, giving an equivalent resistance of $20\,\Omega + 15\,\Omega = 35\,\Omega$. Finally, this equivalent $35\,\Omega$ resistor is in parallel with the other $35\,\Omega$ resistor:

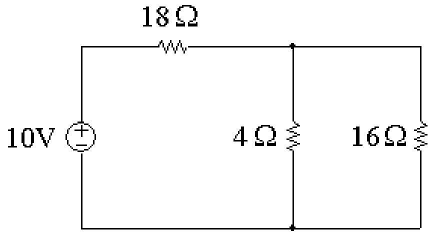
$$35\,\Omega \| 35\,\Omega = \frac{(35)(35)}{35 + 35} = 17.5\,\Omega$$

Thus, the resistance seen by the 2 A source is $17.5\,\Omega$, and the voltage can be calculated using Ohm's law:

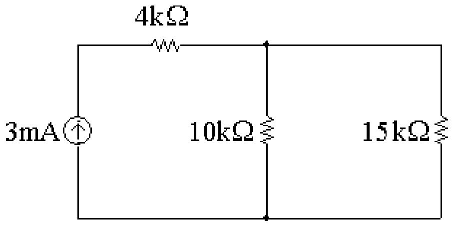
$$v = (17.5\,\Omega)(2\text{ A}) = 35\text{ V}$$

Problems

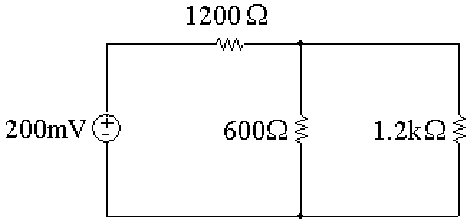
P 3.1 [**a**] The 6 kΩ and 12 kΩ resistors are in series, as are the 9 kΩ and 7 kΩ resistors. The simplified circuit is shown below:



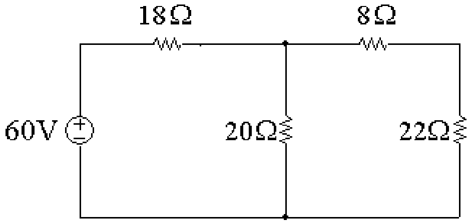
[**b**] The 3 kΩ, 5 kΩ, and 7 kΩ resistors are in series. The simplified circuit is shown below:



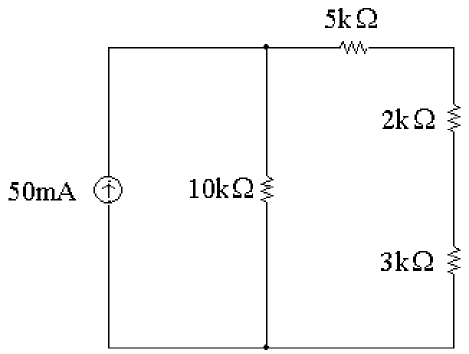
[**c**] The 300 Ω, 400 Ω, and 500 Ω resistors are in series. The simplified circuit is shown below:



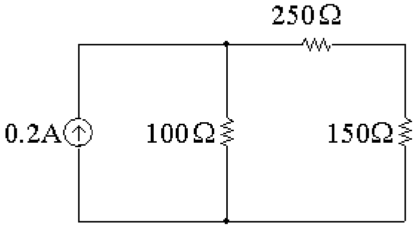
P 3.2 [**a**] The 10 Ω and 40 Ω resistors are in parallel, as are the 100 Ω and 25 Ω resistors. The simplified circuit is shown below:



[**b**] The 9 kΩ, 18 kΩ, and 6 kΩ resistors are in parallel. The simplified circuit is shown below:



[**c**] The 600 Ω, 200 Ω, and 300 Ω resistors are in parallel. The simplified circuit is shown below:



P 3.3 Always work from the side of the circuit furthest from the source. Remember that the current in all series-connected circuits is the same, and that the voltage drop across all parallel-connected resistors is the same.

[**a**] $R_{\text{eq}} = 6 + 12 + [4\|(9 + 7)] = 6 + 12 + 4\|16 = 6 + 12 + 3.2 = 21.2\,\Omega$

[**b**] $R_{\text{eq}} = 4\text{ k} + [10\text{ k}\|(3\text{ k} + 5\text{ k} + 7\text{ k})] = 4\text{ k} + 10\text{ k}\|15\text{ k} = 4\text{ k} + 6\text{ k} = 10\ \text{k}\Omega$

[**c**] $R_{\text{eq}} = 300 + 400 + 500 + (600\|1200) = 300 + 400 + 500 + 400 = 1600\,\Omega$

P 3.4 Always work from the side of the circuit furthest from the source. Remember that the current in all series-connected circuits is the same, and that the voltage drop across all parallel-connected resistors is the same.

[**a**] $R_{\text{eq}} = 18 + [100\|25\|(10\|40 + 22)] = 18 + [100\|25\|(8 + 22)]$

$$= 18 + [100\|25\|30] = 18 + 12 = 30\,\Omega$$

[**b**] $R_{\text{eq}} = 10\text{ k}\|[5\text{ k} + 2\text{ k} + (9\text{ k}\|18\text{ k}\|6\text{ k})] = 10\text{ k}\|[5\text{ k} + 2\text{ k} + 3\text{ k}]$

$$= 10\text{ k}\|10\text{ k} = 5\ \text{k}\Omega$$

[**c**] $R_{\text{eq}} = 600\|200\|300\|(250 + 150) = 600\|200\|300\|400 = 80\,\Omega$

P 3.5 [**a**] $R_{\text{ab}} = 10 + (5\|20) + 6 = 10 + 4 + 6 = 20\,\Omega$

[**b**] $R_{\text{ab}} = 30\text{ k}\|60\text{ k}\|[20\text{ k} + (200\text{ k}\|50\text{ k})] = 30\text{ k}\|60\text{ k}\|(20\text{ k} + 40\text{ k})$

$$= 30\text{ k}\|60\text{ k}\|60\text{ k} = 15\ \text{k}\Omega$$

P 3.6 **[a]** $60\|20 = 1200/80 = 15\,\Omega$ $\quad 12\|24 = 288/36 = 8\,\Omega$

$15 + 8 + 7 = 30\,\Omega$ $\quad 30\|120 = 3600/150 = 24\,\Omega$

$R_{\text{ab}} = 15 + 24 + 25 = 64\,\Omega$

[b] $35 + 40 = 75\,\Omega$ $\quad 75\|50 = 3750/125 = 30\,\Omega$

$30 + 20 = 50\,\Omega$ $\quad 50\|75 = 3750/125 = 30\,\Omega$

$30 + 10 = 40\,\Omega$ $\quad 40\|60 + 9\|18 = 24 + 6 = 30\,\Omega$

$30\|30 = 15\,\Omega$ $\quad R_{\text{ab}} = 10 + 15 + 5 = 30\,\Omega$

[c] $50 + 30 = 80\,\Omega$ $\quad 80\|20 = 16\,\Omega$

$16 + 14 = 30\,\Omega$ $\quad 30 + 24 = 54\,\Omega$

$54\|27 = 18\,\Omega$ $\quad 18 + 12 = 30\,\Omega$

$30\|30 = 15\,\Omega$ $\quad R_{\text{ab}} = 3 + 15 + 2 = 20\,\Omega$

P 3.7 **[a]** For circuit (a)

$$R_{\text{ab}} = 4\|(3 + 7 + 2) = 4\|12 = 3\,\Omega$$

For circuit (b)

$$R_{\text{ab}} = 6 + 2 + [8\|(7 + 5\|2.5\|7.5\|5\|(9 + 6))] = 6 + 2 + 8\|(7 + 1)$$

$$= 6 + 2 + 4 = 12\,\Omega$$

For circuit (c)

$$144\|(4 + 12) = 14.4\,\Omega$$

$$14.4 + 5.6 = 20\,\Omega$$

$$20\|12 = 7.5\,\Omega$$

$$7.5 + 2.5 = 10\,\Omega$$

$$10\|15 = 6\,\Omega$$

$$14 + 6 + 10 = 30\,\Omega$$

$$R_{\text{ab}} = 30\|60 = 20\,\Omega$$

[b] $P_a = \dfrac{15^2}{3} = 75$ W

$$P_b = \frac{48^2}{12} = 192 \text{ W}$$

$$P_c = 5^2(20) = 500 \text{ W}$$

P 3.8 **[a]** $p_{4\Omega} = i_s^2 4 = (12)^2 4 = 576$ W $\quad p_{18\Omega} = (4)^2 18 = 288$ W

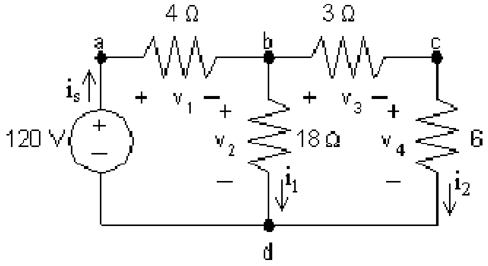
$p_{3\Omega} = (8)^2 3 = 192$ W $\quad p_{6\Omega} = (8)^2 6 = 384$ W

[b] $p_{120\text{V}}(\text{delivered}) = 120 i_s = 120(12) = 1440$ W

[c] $p_{\text{diss}} = 576 + 288 + 192 + 384 = 1440$ W

P 3.9 **[a]** From Ex. 3-1: $i_1 = 4$ A, $\quad i_2 = 8$ A, $\quad i_s = 12$ A

at node b: $-12 + 4 + 8 = 0$, $\quad$ at node d: $12 - 4 - 8 = 0$



[b] $v_1 = 4 i_s = 48$ V $\quad v_3 = 3 i_2 = 24$ V

$v_2 = 18 i_1 = 72$ V $\quad v_4 = 6 i_2 = 48$ V

loop abda: $-120 + 48 + 72 = 0$,
loop bcdb: $-72 + 24 + 48 = 0$,
loop abcda: $-120 + 48 + 24 + 48 = 0$

P 3.10 $R_{\text{eq}} = 10\|[6 + 5\|(8 + 12)] = 10\|(6 + 5\|20) = 10\|(6 + 4) = 5\,\Omega$

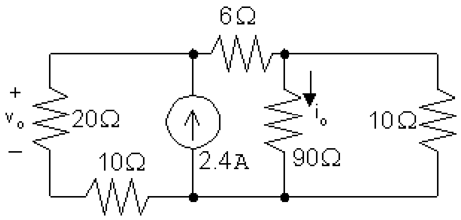
$v_{10\text{A}} = v_{10\Omega} = (10\ \text{A})(5\,\Omega) = 50\ \text{V}$

Using voltage division:

$$v_{5\Omega} = \frac{5\|(8 + 12)}{6 + 5\|(8 + 12)}(50) = \frac{4}{6 + 4}(50) = 20\ \text{V}$$

Thus, $p_{5\Omega} = \dfrac{v_{5\Omega}^2}{5} = \dfrac{20^2}{5} = 80\ \text{W}$

P 3.11 **[a]**



$R_{\text{eq}} = (10 + 20)\|[12 + (90\|10)] = 30\|15 = 10\,\Omega$

$v_{2.4\text{A}} = 10(2.4) = 24$ V

$$v_o = v_{20\Omega} = \frac{20}{10+20}(24) = 16 \text{ V}$$

$$v_{90\Omega} = \frac{90\|10}{6+(90\|10)}(24) = \frac{9}{15}(24) = 14.4 \text{ V}$$

$$i_o = \frac{14.4}{90} = 0.16 \text{ A}$$

[**b**] $p_{6\Omega} = \dfrac{(v_{2.4\text{A}} - v_{90\Omega})^2}{6} = \dfrac{(24-14.4)^2}{6} = 15.36 \text{ W}$

[**c**] $p_{2.4\text{A}} = -(2.4)(24) = -57.6 \text{ W}$

Thus the power developed by the current source is 57.6 W.

P 3.12 [**a**] $R + R = 2R$

[**b**] $R + R + R + \cdots + R = nR$

[**c**] $R + R = 2R = 3000 \quad \text{so} \quad R = 1500 = 1.5 \text{ k}\Omega$

This is a resistor from Appendix H.

[**d**] $nR = 4000; \qquad \text{so if } n = 4, \quad R = 1 \text{ k}\Omega$

This is a resistor from Appendix H.

P 3.13 [**a**] $R_{\text{eq}} = R\|R = \dfrac{R^2}{2R} = \dfrac{R}{2}$

[**b**]
$$\begin{aligned} R_{\text{eq}} &= R\|R\|R\|\cdots\|R \qquad (n\ R\text{'s}) \\ &= R\|\frac{R}{n-1} \\ &= \frac{R^2/(n-1)}{R + R/(n-1)} = \frac{R^2}{nR} = \frac{R}{n} \end{aligned}$$

[**c**] $\dfrac{R}{2} = 5000 \quad \text{so} \quad R = 10 \text{ k}\Omega$

This is a resistor from Appendix H.

[**d**] $\dfrac{R}{n} = 4000 \quad \text{so} \quad R = 4000n$

If $n = 3 \qquad r = 4000(3) = 12 \text{ k}\Omega$

This is a resistor from Appendix H. So put three 12k resistors in parallel to get 4kΩ.

P 3.14 $4 = \dfrac{20R_2}{R_2+40} \quad \text{so} \quad R_2 = 10\,\Omega$

$$3 = \frac{20R_\text{e}}{40+R_\text{e}} \quad \text{so} \quad R_\text{e} = \frac{120}{17}\,\Omega$$

Thus, $\dfrac{120}{17} = \dfrac{10R_\text{L}}{10+R_\text{L}} \quad \text{so} \quad R_\text{L} = 24\,\Omega$

P 3.15 **[a]** $v_o = \dfrac{160(3300)}{(4700+3300)} = 66\ \text{V}$

[b] $i = 160/8000 = 20\ \text{mA}$

$$P_{R_1} = (400 \times 10^{-6})(4.7 \times 10^3) = 1.88\ \text{W}$$

$$P_{R_2} = (400 \times 10^{-6})(3.3 \times 10^3) = 1.32\ \text{W}$$

[c] Since R_1 and R_2 carry the same current and $R_1 > R_2$ to satisfy the voltage requirement, first pick R_1 to meet the 0.5 W specification

$$i_{R_1} = \frac{160-66}{R_1}, \qquad \text{Therefore,} \ \left(\frac{94}{R_1}\right)^2 R_1 \leq 0.5$$

Thus, $R_1 \geq \dfrac{94^2}{0.5}$ or $R_1 \geq 17{,}672\,\Omega$

Now use the voltage specification:

$$\frac{R_2}{R_2 + 17{,}672}(160) = 66$$

Thus, $R_2 = 12{,}408\,\Omega$

P 3.16 **[a]** $v_o = \dfrac{40R_2}{R_1+R_2} = 8 \quad \text{so} \quad R_1 = 4R_2$

Let $R_\text{e} = R_2 \| R_\text{L} = \dfrac{R_2 R_\text{L}}{R_2 + R_\text{L}}$

$$v_o = \frac{40R_\text{e}}{R_1 + R_\text{e}} = 7.5 \quad \text{so} \quad R_1 = 4.33R_\text{e}$$

Then, $4R_2 = 4.33R_\text{e} = \dfrac{4.33(3600R_2)}{3600 + R_2}$

Thus, $R_2 = 300\,\Omega$ and $R_1 = 4(300) = 1200\,\Omega$

[b] The resistor that must dissipate the most power is R_1, as it has the largest resistance and carries the same current as the parallel combination of R_2 and the load resistor. The power dissipated in R_1 will be maximum when the voltage across R_1 is maximum. This will occur when the voltage divider has a resistive load. Thus,

$$v_{R_1} = 40 - 7.5 = 32.5\ \text{V}$$

$$p_{R_1} = \frac{32.5^2}{1200} = 880.2\ \text{m W}$$

Thus the minimum power rating for all resistors should be 1 W.

P 3.17 Refer to the solution to Problem 3.16. The voltage divider will reach the maximum power it can safely dissipate when the power dissipated in R_1 equals 1 W. Thus,

$$\frac{v_{R_1}^2}{1200} = 1 \quad \text{so} \quad v_{R_1} = 34.64 \text{ V}$$

$$v_o = 40 - 34.64 = 5.36 \text{ V}$$

$$\text{So, } \frac{40R_\text{e}}{1200 + R_\text{e}} = 5.36 \quad \text{and} \quad R_\text{e} = 185.68\,\Omega$$

$$\text{Thus, } \frac{(300)R_\text{L}}{300 + R_\text{L}} = 185.68 \quad \text{and} \quad R_\text{L} = 487.26\,\Omega$$

The minimum value for R_L from Appendix H is $560\,\Omega$.

P 3.18 Begin by using the relationships among the branch currents to express all branch currents in terms of i_4:

$$i_1 = 2i_2 = 2(2i_3) = 4(2i_4)$$

$$i_2 = 2i_3 = 2(2i_4)$$

$$i_3 = 2i_4$$

Now use KCL at the top node to relate the branch currents to the current supplied by the source.

$$i_1 + i_2 + i_3 + i_4 = 1 \text{ mA}$$

Express the branch currents in terms of i_4 and solve for i_4:

$$1 \text{ mA} = 8i_4 + 4i_4 + 2i_4 + i_4 = 15i_4 \quad \text{so} \quad i_4 = \frac{0.001}{15} \text{ A}$$

Since the resistors are in parallel, the same voltage, 1 V appears across each of them. We know the current and the voltage for R_4 so we can use Ohm's law to calculate R_4:

$$R_4 = \frac{v_g}{i_4} = \frac{1 \text{ V}}{(1/15) \text{ mA}} = 15 \text{ k}\Omega$$

Calculate i_3 from i_4 and use Ohm's law as above to find R_3:

$$i_3 = 2i_4 = \frac{0.002}{15} \text{ A} \quad \therefore \quad R_3 = \frac{v_g}{i_3} = \frac{1 \text{ V}}{(2/15) \text{ mA}} = 7.5 \text{ k}\Omega$$

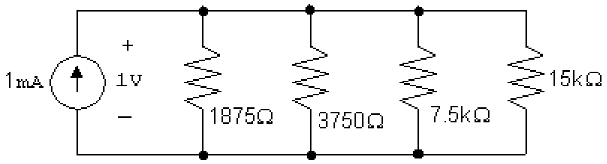
Calculate i_2 from i_4 and use Ohm's law as above to find R_2:

$$i_2 = 4i_4 = \frac{0.004}{15}\ \text{A} \qquad \therefore\ \ R_2 = \frac{v_g}{i_2} = \frac{1\ \text{V}}{(4/15)\ \text{mA}} = 3750\,\Omega$$

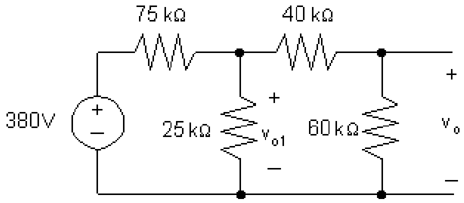
Calculate i_1 from i_4 and use Ohm's law as above to find R_1:

$$i_1 = 8i_4 = \frac{0.008}{15}\ \text{A} \qquad \therefore\ \ R_1 = \frac{v_g}{i_1} = \frac{1\ \text{V}}{(8/15)\ \text{mA}} = 1875\,\Omega$$

The resulting circuit is shown below:



P 3.19 [**a**]



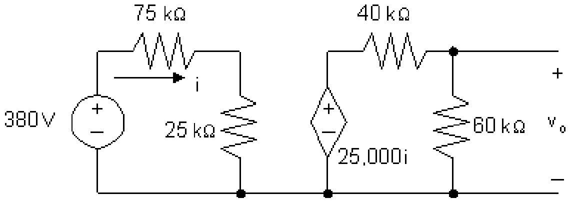
$$40\ \text{k}\Omega + 60\ \text{k}\Omega = 100\ \text{k}\Omega$$

$$25\ \text{k}\Omega \| 100\ \text{k}\Omega = 20\ \text{k}\Omega$$

$$v_{o1} = \frac{20{,}000}{(75{,}000 + 20{,}000)}(380) = 80\ \text{V}$$

$$v_o = \frac{60{,}000}{(100{,}000)}(v_{o1}) = 48\ \text{V}$$

[**b**]



$$i = \frac{380}{100{,}000} = 3.8\ \text{mA}$$

$$25{,}000i = 95\ \text{V}$$

$$v_o = \frac{60{,}000}{100{,}000}(95) = 57\ \text{V}$$

[c] It removes loading effect of second voltage divider on the first voltage divider. Observe that the open circuit voltage of the first divider is

$$v'_{o1} = \frac{25{,}000}{(100{,}000)}(380) = 95 \text{ V}$$

Now note this is the input voltage to the second voltage divider when the current controlled voltage source is used.

P 3.20 $$\frac{(24)^2}{R_1 + R_2 + R_3} = 80, \qquad \text{Therefore, } R_1 + R_2 + R_3 = 7.2\,\Omega$$

$$\frac{(R_1 + R_2)24}{(R_1 + R_2 + R_3)} = 12$$

Therefore, $2(R_1 + R_2) = R_1 + R_2 + R_3$

Thus, $R_1 + R_2 = R_3$; $\quad 2R_3 = 7.2$; $\quad R_3 = 3.6\,\Omega$

$$\frac{R_2(24)}{R_1 + R_2 + R_3} = 5$$

$$4.8R_2 = R_1 + R_2 + 3.6 = 7.2$$

Thus, $R_2 = 1.5\,\Omega$; $\quad R_1 = 7.2 - R_2 - R_3 = 2.1\,\Omega$

P 3.21 [a] Let v_o be the voltage across the parallel branches, positive at the upper terminal, then

$$i_g = v_oG_1 + v_oG_2 + \cdots + v_oG_N = v_o(G_1 + G_2 + \cdots + G_N)$$

It follows that $\quad v_o = \dfrac{i_g}{(G_1 + G_2 + \cdots + G_N)}$

The current in the k^{th} branch is $\quad i_k = v_oG_k$; $\qquad$ Thus,

$$i_k = \frac{i_gG_k}{[G_1 + G_2 + \cdots + G_N]}$$

[b] $$i_5 = \frac{40(0.2)}{2 + 0.2 + 0.125 + 0.1 + 0.05 + 0.025} = 3.2 \text{ A}$$

P 3.22 [a] At no load: $\quad v_o = kv_s = \dfrac{R_2}{R_1 + R_2}v_s.$

At full load: $\quad v_o = \alpha v_s = \dfrac{R_\text{e}}{R_1 + R_\text{e}}v_s, \qquad$ where $R_\text{e} = \dfrac{R_oR_2}{R_o + R_2}$

$$\text{Therefore} \quad k = \frac{R_2}{R_1+R_2} \quad \text{and} \quad R_1 = \frac{(1-k)}{k}R_2$$

$$\alpha = \frac{R_\text{e}}{R_1+R_\text{e}} \quad \text{and} \quad R_1 = \frac{(1-\alpha)}{\alpha}R_\text{e}$$

$$\text{Thus} \quad \left(\frac{1-\alpha}{\alpha}\right)\left[\frac{R_2R_o}{R_o+R_2}\right] = \frac{(1-k)}{k}R_2$$

Solving for R_2 yields $R_2 = \dfrac{(k-\alpha)}{\alpha(1-k)}R_o$

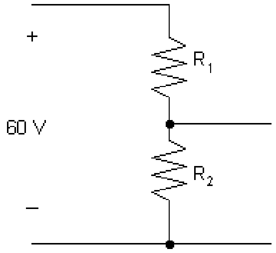
Also, $R_1 = \dfrac{(1-k)}{k}R_2 \quad \therefore \quad R_1 = \dfrac{(k-\alpha)}{\alpha k}R_o$

[b]

$$R_1 = \left(\frac{0.05}{0.68}\right)R_o = 2.5\ \text{k}\Omega$$

$$R_2 = \left(\frac{0.05}{0.12}\right)R_o = 14.167\ \text{k}\Omega$$

[c]



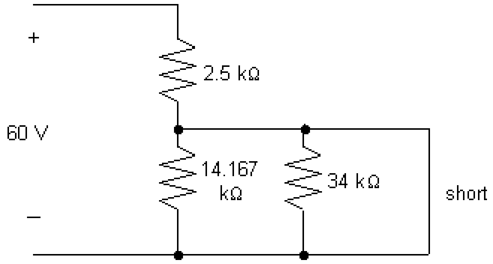
Maximum dissipation in R_2 occurs at no load, therefore,

$$P_{R_2(\text{max})} = \frac{[(60)(0.85)]^2}{14{,}167} = 183.6\ \text{mW}$$

Maximum dissipation in R_1 occurs at full load.

$$P_{R_1(\text{max})} = \frac{[60-0.80(60)]^2}{2500} = 57.60\ \text{mW}$$

[d]



$$P_{R_1} = \frac{(60)^2}{2500} = 1.44\ \text{W} = 1440\ \text{mW}$$

$$P_{R_2} = \frac{(0)^2}{14{,}167} = 0\ \text{W}$$

P 3.23 [**a**] The equivalent resistance of the circuit to the right of the 18 Ω resistor is

$$100\|25\|[(40\|10)+22] = 100\|25\|30 = 12\,\Omega$$

Thus by voltage division,

$$v_{18} = \frac{18}{18+12}(60) = 36 \text{ V}$$

[**b**] The current in the 18 Ω resistor can be found from its voltage using Ohm's law:

$$i_{18} = \frac{36}{18} = 2 \text{ A}$$

[**c**] The current in the 18 Ω resistor divides among three branches – one containing 100 Ω, one containing 25 Ω and one containing $(22 + 40\|10) = 30\,\Omega$. Using current division,

$$i_{25} = \frac{100\|25\|30}{25}(i_{18}) = \frac{12}{25}(2) = 0.96 \text{ A}$$

[**d**] The voltage drop across the 25 Ω resistor can be found using Ohm's law:

$$v_{25} = 25i_{25} = 25(0.96) = 24 \text{ V}$$

[**e**] The voltage v_{25} divides across the 22 Ω resistor and the equivalent resistance $40\|10 = 8\,\Omega$. Using voltage division,

$$v_{10} = \frac{8}{8+22}(24) = 6.4 \text{ V}$$

P 3.24 [**a**] The equivalent resistance to the right of the 10 kΩ resistor is $5\text{ k} + 2\text{ k} + [9\text{ k}\|18\text{ k}\|6\text{ k})] = 10\text{ k}\Omega$. Therefore,

$$i_{10\text{k}} = \frac{10\text{ k}\|10\text{ k}}{10\text{ k}}(0.050) = 25 \text{ mA}$$

[**b**] The voltage drop across the 10 kΩ resistor can be found using Ohm's law:

$$v_{10\text{k}} = (10,000)i_{10\text{k}} = (10,000)(0.025) = 250 \text{ V}$$

[**c**] The voltage $v_{10\text{k}}$ drops across the 5 kΩ resistor, the 2 kΩ resistor and the equivalent resistance of the 9 kΩ, 18 kΩ and 6 kΩ resistors in parallel. Thus, using voltage division,

$$v_{6\text{k}} = \frac{2\text{ k}}{5\text{ k} + 2\text{ k} + [9\text{ k}\|18\text{ k}\|6\text{ k}]}(250) = \frac{2}{10}(250) = 50 \text{ V}$$

[**d**] The current through the 2 kΩ resistor can be found from its voltage using Ohm's law:

$$i_{2\text{k}} = \frac{v_{2\text{k}}}{2000} = \frac{50}{2000} = 25 \text{ mA}$$

[e] The current through the 2 kΩ resistor divides among the 9 kΩ, 18 kΩ, and 6 kΩ. Using current division,

$$i_{18\text{k}} = \frac{9\text{ k}\|18\text{ k}\|6\text{ k}}{18\text{ k}}(0.025) = \frac{3}{18}(0.025) = 4.167\text{ mA}$$

P 3.25 The equivalent resistance of the circuit to the right of the 90 Ω resistor is

$$R_{\text{eq}} = [(150\|75) + 40]\|(30 + 60) = 90\|90 = 45\,\Omega$$

Use voltage division to find the voltage drop between the top and bottom nodes:

$$v_{\text{Req}} = \frac{45}{45 + 90}(3) = 1\text{ V}$$

Use voltage division again to find v_1 from v_{Req}:

$$v_1 = \frac{150\|75}{150\|75 + 40}(1) = \frac{50}{90}(1) = \frac{5}{9}\text{ V}$$

Use voltage division one more time to find v_2 from v_{Req}:

$$v_2 = \frac{30}{30 + 60}(1) = \frac{1}{3}\text{ V}$$

P 3.26 $$i_{10\text{k}} = \frac{(18)(15\text{ k})}{40\text{ k}} = 6.75\text{ mA}$$

$$v_{15\text{k}} = -(6.75\text{ m})(15\text{ k}) = -101.25\text{ V}$$

$$i_{3\text{k}} = 18\text{ m} - 6.75\text{ m} = 11.25\text{ mA}$$

$$v_{12\text{k}} = -(12\text{ k})(11.25\text{ m}) = -135\text{ V}$$

$$v_o = -101.25 - (-135) = 33.75\text{ V}$$

P 3.27 **[a]** $$v_{6\text{k}} = \frac{6}{6 + 2}(18) = 13.5\text{ V}$$

$$v_{3\text{k}} = \frac{3}{3 + 9}(18) = 4.5\text{ V}$$

$$v_x = v_{6\text{k}} - v_{3\text{k}} = 13.5 - 4.5 = 9\text{ V}$$

[b] $v_{6\text{k}} = \frac{6}{8}(V_s) = 0.75V_s$

$$v_{3\text{k}} = \frac{3}{12}(V_s) = 0.25V_s$$

$$v_x = (0.75V_s) - (0.25V_s) = 0.5V_s$$

P 3.28 $5\,\Omega\|20\,\Omega = 4\,\Omega; \qquad 4\,\Omega + 6\,\Omega = 10\,\Omega; \qquad 10\|(15 + 12 + 13) = 8\,\Omega;$

Therefore, $i_g = \frac{125}{2 + 8} = 12.5$ A

$$i_{6\Omega} = \frac{8}{6 + 4}(12.5) = 10 \text{ A}; \quad i_o = \frac{5\|20}{20}(10) = 2 \text{ A}$$

P 3.29 **[a]** The equivalent resistance seen by the voltage source is

$$60\|[8 + 30\|(4 + 80\|20)] = 60\|[8 + 30\|20] = 60\|20 = 15\,\Omega$$

Thus,

$$i_g = \frac{300}{15} = 20 \text{ A}$$

[b] Use current division to find the current in the $8\,\Omega$ division:

$$\frac{15}{20}(20) = 15 \text{ A}$$

Use current division again to find the current in the $30\,\Omega$ resistor:

$$i_{30} = \frac{12}{30}(15) = 6 \text{ A}$$

Thus,

$$p_{30} = (6)^2(30) = 1080 \text{ W}$$

P 3.30 **[a]** The voltage across the $9\,\Omega$ resistor is $1(12 + 6) = 18$ V.

The current in the $9\,\Omega$ resistor is $18/9 = 2$ A. The current in the $2\,\Omega$ resistor is $1 + 2 = 3$ A. Therefore, the voltage across the $24\,\Omega$ resistor is $(2)(3) + 18 = 24$ V.

The current in the $24\,\Omega$ resistor is 1 A. The current in the $3\,\Omega$ resistor is $1 + 2 + 1 = 4$ A. Therefore, the voltage across the $72\,\Omega$ resistor is $24 + 3(4) = 36$ V.

The current in the $72\,\Omega$ resistor is $36/72 = 0.5$ A.

The $20\,\Omega\|5\,\Omega$ resistors are equivalent to a $4\,\Omega$ resistor. The current in this equivalent resistor is $0.5 + 1 + 3 = 4.5$ A. Therefore the voltage across the $108\,\Omega$ resistor is $36 + 4.5(4) = 54$ V.

The current in the $108\,\Omega$ resistor is $54/108 = 0.5$ A. The current in the $1.2\,\Omega$ resistor is $4.5 + 0.5 = 5$ A. Therefore,

$$v_g = (1.2)(5) + 54 = 60 \text{ V}$$

[b] The current in the 20 Ω resistor is

$$i_{20} = \frac{(4.5)(4)}{20} = \frac{18}{20} = 0.9 \text{ A}$$

Thus, the power dissipated by the 20 Ω resistor is

$$p_{20} = (0.9)^2(20) = 16.2 \text{ W}$$

P 3.31 For all full-scale readings the total resistance is

$$R_V + R_{\text{movement}} = \frac{\text{full-scale reading}}{10^{-3}}$$

We can calculate the resistance of the movement as follows:

$$R_{\text{movement}} = \frac{20 \text{ mV}}{1 \text{ mA}} = 20\,\Omega$$

Therefore, $R_V = 1000$ (full-scale reading) $- 20$

[a] $R_V = 1000(50) - 20 = 49{,}980\,\Omega$

[b] $R_V = 1000(5) - 20 = 4980\,\Omega$

[c] $R_V = 1000(0.25) - 20 = 230\,\Omega$

[d] $R_V = 1000(0.025) - 20 = 5\,\Omega$

P 3.32 **[a]** $v_{\text{meas}} = (50 \times 10^{-3})[15\|45\|(4980 + 20)] = 0.5612 \text{ V}$

[b] $v_{\text{true}} = (50 \times 10^{-3})(15\|45) = 0.5625 \text{ V}$

$$\%\text{ error} = \left(\frac{0.5612}{0.5625} - 1\right) \times 100 = -0.224\%$$

P 3.33 The measured value is $60\|20.1 = 15.05618\,\Omega$.

$$i_g = \frac{50}{(15.05618 + 10)} = 1.995526 \text{ A}; \qquad i_{\text{meas}} = \frac{60}{80.1}(1.996) = 1.494768 \text{ A}$$

The true value is $60\|20 = 15\,\Omega$.

$$i_g = \frac{50}{(15 + 10)} = 2 \text{ A}; \qquad i_{\text{true}} = \frac{60}{80}(2) = 1.5 \text{ A}$$

$$\%\text{error} = \left[\frac{1.494768}{1.5} - 1\right] \times 100 = -0.34878\% \approx -0.35\%$$

P 3.34 Begin by using current division to find the actual value of the current i_o:

$$i_{\text{true}} = \frac{15}{15+45}(50 \text{ mA}) = 12.5 \text{ mA}$$

$$i_{\text{meas}} = \frac{15}{15+45+0.1}(50 \text{ mA}) = 12.4792 \text{ mA}$$

$$\% \text{ error } = \left[\frac{12.4792}{12.5} - 1\right] 100 = -0.166389\% \approx -0.17\%$$

P 3.35 [**a**] The model of the ammeter is an ideal ammeter in parallel with a resistor whose resistance is given by

$$R_m = \frac{100\,\text{mV}}{2\,\text{mA}} = 50\,\Omega.$$

We can calculate the current through the real meter using current division:

$$i_m = \frac{(25/12)}{50+(25/12)}(i_{\text{meas}}) = \frac{25}{625}(i_{\text{meas}}) = \frac{1}{25}i_{\text{meas}}$$

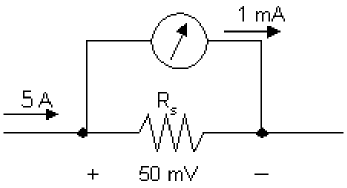
[**b**] At full scale, $i_{\text{meas}} = 5$ A and $i_{\text{m}} = 2$ mA so $5 - 0.002 = 4998$ mA flows throught the resistor R_{A}:

$$R_{\text{A}} = \frac{100 \text{ mV}}{4998 \text{ mA}} = \frac{100}{4998}\,\Omega$$

$$i_m = \frac{(100/4998)}{50+(100/4998)}(i_{\text{meas}}) = \frac{1}{2500}(i_{\text{meas}})$$

[**c**] Yes

P 3.36



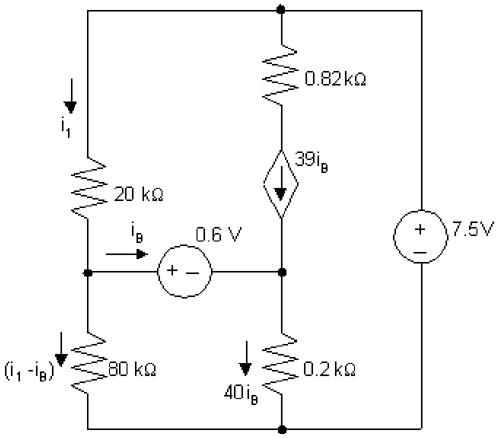
$$\text{Original meter:} \quad R_{\text{e}} = \frac{50 \times 10^{-3}}{5} = 0.01\,\Omega$$

$$\text{Modified meter:} \quad R_{\text{e}} = \frac{(0.02)(0.01)}{0.03} = 0.00667\,\Omega$$

$$\therefore \quad (I_{\text{fs}})(0.00667) = 50 \times 10^{-3}$$

$$\therefore \quad I_{\text{fs}} = 7.5 \text{ A}$$

P 3.37 [**a**]



$$20 \times 10^3 i_1 + 80 \times 10^3 (i_1 - i_B) = 7.5$$

$$80 \times 10^3 (i_1 - i_B) = 0.6 + 40 i_B (0.2 \times 10^3)$$

$$\therefore \quad 100 i_1 - 80 i_B = 7.5 \times 10^{-3}$$

$$80 i_1 - 88 i_B = 0.6 \times 10^{-3}$$

Calculator solution yields $i_B = 225\,\mu A$

[**b**] With the insertion of the ammeter the equations become

$$100 i_1 - 80 i_B = 7.5 \times 10^{-3} \quad \text{(no change)}$$

$$80 \times 10^3 (i_1 - i_B) = 10^3 i_B + 0.6 + 40 i_B (200)$$

$$80 i_1 - 89 i_B = 0.6 \times 10^{-3}$$

Calculator solution yields $i_B = 216\,\mu A$

[**c**] % error $= \left(\frac{216}{225} - 1\right) 100 = -4\%$

P 3.38 The current in the shunt resistor at full-scale deflection is $i_A = i_{\text{fullscale}} = 2 \times 10^{-3}$ A. The voltage across R_A at full-scale deflection is always 50 mV; therefore,

$$R_A = \frac{50 \times 10^{-3}}{i_{\text{fullscale}} - 2 \times 10^{-3}} = \frac{50}{1000 i_{\text{fullscale}} - 2}$$

[**a**] $R_A = \frac{50}{10{,}000 - 2} = 5.001 \ \text{m}\Omega$

[**b**] $R_A = \frac{50}{1000 - 2} = 50.1 \ \text{m}\Omega$

[**c**] $R_A = \frac{50}{50 - 2} = 1.042 \ \text{m}\Omega$

[d] $R_\text{A} = \dfrac{50}{2-2} = \infty$ (open circuit)

P 3.39 At full scale the voltage across the shunt resistor will be 50 mV; therefore the power dissipated will be

$$P_\text{A} = \frac{(50 \times 10^{-3})^2}{R_\text{A}}$$

$$\text{Therefore } R_\text{A} \geq \frac{(50 \times 10^{-3})^2}{0.5} = 5\text{ m}\Omega$$

Otherwise the power dissipated in R_A will exceed its power rating of 0.5 W When $R_\text{A} = 5$ mΩ, the shunt current will be

$$i_\text{A} = \frac{50 \times 10^{-3}}{5 \times 10^{-3}} = 10\text{ A}$$

The measured current will be $i_\text{meas} = 10 + 0.001 = 10.001$ A
∴ Full-scale reading for practical purposes is 10 A.

P 3.40 $R_\text{meter} = R_m + R_\text{movement} = \dfrac{750\text{ V}}{1.5\text{ mA}} = 500\text{ k}\Omega$

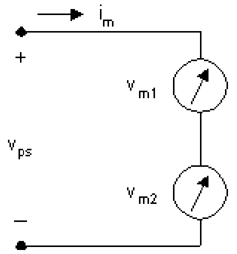
$$v_\text{meas} = (25\text{ k}\Omega \| 125\text{ k}\Omega \| 50\text{ k}\Omega)(30\text{ mA}) = (20\text{ k}\Omega)(30\text{ mA}) = 600\text{ V}$$

$$v_\text{true} = (25\text{ k}\Omega \| 125\text{ k}\Omega)(30\text{ mA}) = (20.83\text{ k}\Omega)(30\text{ mA}) = 625\text{ V}$$

$$\%\text{ error } = \left(\frac{600}{625} - 1\right) 100 = -4\%$$

P 3.41 **[a]** Since the unknown voltage is greater than either voltmeter's maximum reading, the only possible way to use the voltmeters would be to connect them in series.

[b]



$$R_{m1} = (300)(900) = 270\text{ k}\Omega; \qquad R_{m2} = (150)(1200) = 180\text{ k}\Omega$$

$$\therefore\ R_{m1} + R_{m2} = 450\text{ k}\Omega$$

$$i_{1\ \max} = \frac{300}{270} \times 10^{-3} = 1.11\ \text{mA}; \qquad i_{2\ \max} = \frac{150}{180} \times 10^{-3} = 0.833\ \text{mA}$$

$$\therefore\ \ i_{\max} = 0.833\ \text{mA since meters are in series}$$

$$v_{\max} = (0.833 \times 10^{-3})(270 + 180)10^3 = 375\ \text{V}$$

Thus the meters can be used to measure the voltage.

[c] $i_m = \dfrac{320}{450 \times 10^3} = 0.711\ \text{mA}$

$$v_{m1} = (0.711)(270) = 192\ \text{V}; \qquad v_{m2} = (0.711)(180) = 128\ \text{V}$$

P 3.42 The current in the series-connected voltmeters is

$$i_m = \frac{205.2}{270{,}000} = \frac{136.8}{180{,}000} = 0.76\ \text{mA}$$

$$v_{50\ \text{k}\Omega} = (0.76 \times 10^{-3})(50{,}000) = 38\ \text{V}$$

$$V_{\text{power supply}} = 205.2 + 136.8 + 38 = 380\ \text{V}$$

P 3.43 **[a]** $v_{\text{meter}} = 180\ \text{V}$

[b] $R_{\text{meter}} = (100)(200) = 20\ \text{k}\Omega$

$$20\|70 = 15.555556\ \text{k}\Omega$$

$$v_{\text{meter}} = \frac{180}{35.555556} \times 15.555556 = 78.75\ \text{V}$$

[c] $20\|20 = 10\ \text{k}\Omega$

$$v_{\text{meter}} = \frac{180}{80}(10) = 22.5\ \text{V}$$

[d] $v_{\text{meter a}} = 180\ \text{V}$

$$v_{\text{meter b}} + v_{\text{meter c}} = 101.26\ \text{V}$$

No, because of the loading effect.

P 3.44 From the problem statement we have

$$50 = \frac{V_s(10)}{10 + R_s} \qquad (1) \quad V_s \text{ in mV; } R_s \text{ in M}\Omega$$

$$48.75 = \frac{V_s(6)}{6 + R_s} \qquad (2)$$

[a] From Eq (1) $10 + R_s = 0.2V_s$

$$\therefore \quad R_s = 0.2V_s - 10$$

Substituting into Eq (2) yields

$$48.75 = \frac{6V_s}{0.2V_s - 4} \quad \text{or} \quad V_s = 52 \text{ mV}$$

[b] From Eq (1)

$$50 = \frac{520}{10 + R_s} \quad \text{or} \quad 50R_s = 20$$

So $R_s = 400$ kΩ

P 3.45 **[a]**

$$R_1 = (100/2)10^3 = 50 \text{ k}\Omega$$

$$R_2 = (10/2)10^3 = 5 \text{ k}\Omega$$

$$R_3 = (1/2)10^3 = 500\,\Omega$$

[b] Let i_{a} = actual current in the movement

i_{d} = design current in the movement

$$\text{Then \% error } = \left(\frac{i_{\text{a}}}{i_{\text{d}}} - 1\right)100$$

For the 100 V scale:

$$i_{\text{a}} = \frac{100}{50{,}000 + 25} = \frac{100}{50{,}025}, \qquad i_{\text{d}} = \frac{100}{50{,}000}$$

$$\frac{i_{\text{a}}}{i_{\text{d}}} = \frac{50{,}000}{50{,}025} = 0.9995 \qquad \text{\% error } = (0.9995 - 1)100 = -0.05\%$$

For the 10 V scale:

$$\frac{i_{\text{a}}}{i_{\text{d}}} = \frac{5000}{5025} = 0.995 \qquad \text{\% error } = (0.995 - 1.0)100 = -0.4975\%$$

For the 1 V scale:

$$\frac{i_{\text{a}}}{i_{\text{d}}} = \frac{500}{525} = 0.9524 \qquad \text{\% error } = (0.9524 - 1.0)100 = -4.76\%$$

P 3.46 **[a]** $R_{\text{movement}} = 50\,\Omega$

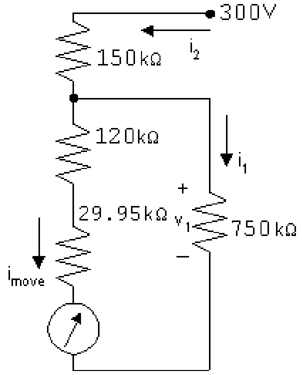
$$R_1 + R_{\text{movement}} = \frac{30}{1 \times 10^{-3}} = 30 \text{ k}\Omega \quad \therefore \quad R_1 = 29{,}950\,\Omega$$

$$R_2 + R_1 + R_{\text{movement}} = \frac{150}{1 \times 10^{-3}} = 150 \text{ k}\Omega \quad \therefore \quad R_2 = 120 \text{ k}\Omega$$

$$R_3 + R_2 + R_1 + R_{\text{movement}} = \frac{300}{1 \times 10^{-3}} = 300\ \text{k}\Omega$$

$$\therefore\quad R_3 = 150\ \text{k}\Omega$$

[b]



$$v_1 = (0.96\ \text{m})(150\ \text{k}) = 144\ \text{V}$$

$$i_{\text{move}} = \frac{144}{120 + 29.95 + 0.05} = 0.96\ \text{mA}$$

$$i_1 = \frac{144}{750\ \text{k}} = 0.192\ \text{mA}$$

$$i_2 = i_{\text{move}} + i_1 = 0.96\ \text{m} + 0.192\ \text{m} = 1.152\ \text{mA}$$

$$v_{\text{meas}} = v_x = 144 + 150i_2 = 316.8\ \text{V}$$

[c] $v_1 = 150$ V; $i_2 = 1\ \text{m} + 0.20\ \text{m} = 1.20$ mA

$$i_1 = 150/750{,}000 = 0.20\ \text{mA}$$

$$\therefore\quad v_{\text{meas}} = v_x = 150 + (150\ \text{k})(1.20\ \text{m}) = 330\ \text{V}$$

P 3.47 **[a]** $R_{\text{meter}} = 300\ \text{k}\Omega + 600\ \text{k}\Omega \| 200\ \text{k}\Omega = 450\ \text{k}\Omega$

$$450 \| 360 = 200\ \text{k}\Omega$$

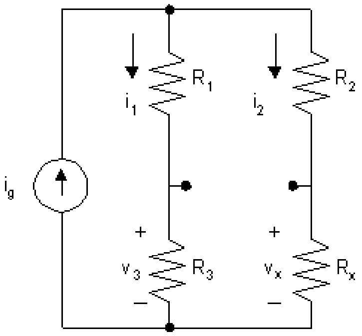
$$V_{\text{meter}} = \frac{200}{240}(600) = 500\ \text{V}$$

[b] What is the percent error in the measured voltage?

$$\text{True value } = \frac{360}{400}(600) = 540\ \text{V}$$

$$\%\ \text{error } = \left(\frac{500}{540} - 1\right) 100 = -7.41\%$$

P 3.48 Since the bridge is balanced, we can remove the detector without disturbing the voltages and currents in the circuit.



It follows that

$$i_1 = \frac{i_g(R_2 + R_x)}{R_1 + R_2 + R_3 + R_x} = \frac{i_g(R_2 + R_x)}{\sum R}$$

$$i_2 = \frac{i_g(R_1 + R_3)}{R_1 + R_2 + R_3 + R_x} = \frac{i_g(R_1 + R_3)}{\sum R}$$

$$v_3 = R_3 i_1 = v_x = i_2 R_x$$

$$\therefore \quad \frac{R_3 i_g(R_2 + R_x)}{\sum R} = \frac{R_x i_g(R_1 + R_3)}{\sum R}$$

$$\therefore \quad R_3(R_2 + R_x) = R_x(R_1 + R_3)$$

From which $R_x = \dfrac{R_2 R_3}{R_1}$

P 3.49 Note the bridge structure is balanced, that is $15 \times 5 = 3 \times 25$, hence there is no current in the 5 kΩ resistor. It follows that the equivalent resistance of the circuit is

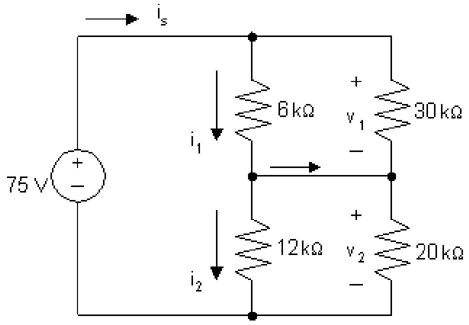
$$R_{\text{eq}} = 750 + (15{,}000 + 3000)\|(25{,}000 + 5000) = 750 + 11{,}250 = 12 \text{ k}\Omega$$

The source current is $192/12{,}000 = 16$ mA.
The current down through the branch containing the 15 kΩ and 3 kΩ resistors is

$$i_{3\text{k}} = \frac{11{,}250}{18{,}000}(0.016) = 10 \text{ mA}$$

$$\therefore \quad p_{3\text{k}} = 3000(0.01)^2 = 0.3 \text{ W}$$

P 3.50 Redraw the circuit, replacing the detector branch with a short circuit.



$$6\ \text{k}\Omega \| 30\ \text{k}\Omega = 5\ \text{k}\Omega$$

$$12\ \text{k}\Omega \| 20\ \text{k}\Omega = 7.5\ \text{k}\Omega$$

$$i_s = \frac{75}{12{,}500} = 6\ \text{mA}$$

$$v_1 = 0.006(5000) = 30\ \text{V}$$

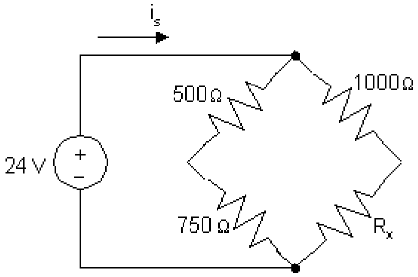
$$v_2 = 0.006(7500) = 45\ \text{V}$$

$$i_1 = \frac{30}{6000} = 5\ \text{mA}$$

$$i_2 = \frac{45}{12{,}000} = 3.75\ \text{mA}$$

$$i_\text{d} = i_1 - i_2 = 1.25\ \text{mA}$$

P 3.51 **[a]**



The condition for a balanced bridge is that the product of the opposite resistors must be equal:

$$(500)(R_x) = (1000)(750) \qquad \text{so} \qquad R_x = \frac{(1000)(750)}{500} = 1500\,\Omega$$

[**b**] The source current is the sum of the two branch currents. Each branch current can be determined using Ohm's law, since the resistors in each branch are in series and the voltage drop across each branch is 24 V:

$$i_s = \frac{24\ \text{V}}{500\,\Omega + 750\,\Omega} + \frac{24\ \text{V}}{1000\,\Omega + 1500\,\Omega} = 28.8\ \text{mA}$$

[**c**] We can use Ohm's law to find the current in each branch:

$$i_{\text{left}} = \frac{24}{500 + 750} = 19.2\ \text{mA}$$

$$i_{\text{right}} = \frac{24}{1000 + 1500} = 9.6\ \text{mA}$$

Now we can use the formula $p = Ri^2$ to find the power dissipated by each resistor:

$$p_{500} = (500)(0.0192)^2 = 184.32\ \text{mW} \qquad p_{750} = (750)(0.0192)^2 = 276.18\ \text{mW}$$

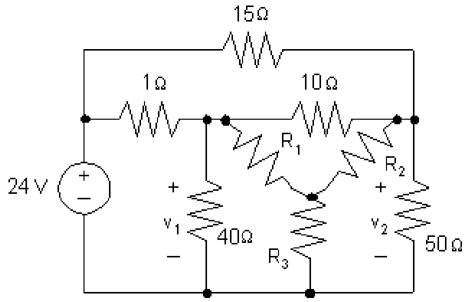
$$p_{1000} = (1000)(0.0096)^2 = 92.16\ \text{mW} \qquad p_{1500} = (1500)(0.0096)^2 = 138.24\ \text{mW}$$

Thus, the 750 Ω resistor absorbs the most power; it absorbs 276.48 mW of power.

[**d**] From the analysis in part (c), the 1000 Ω resistor absorbs the least power; it absorbs 92.16 mW of power.

P 3.52 In order that all four decades (1, 10, 100, 1000) that are used to set R_3 contribute to the balance of the bridge, the ratio R_2/R_1 should be set to 0.001.

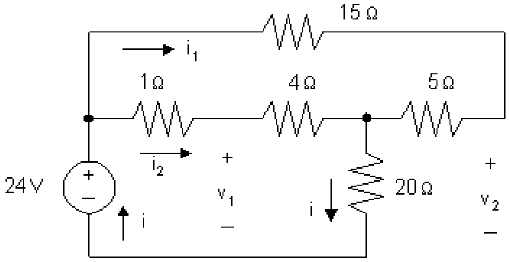
P 3.53 Begin by transforming the Δ-connected resistors $(10\,\Omega, 40\,\Omega, 50\,\Omega)$ to Y-connected resistors. Both the Y-connected and Δ-connected resistors are shown below to assist in using Eqs. 3.44 – 3.46:



Now use Eqs. 3.44 – 3.46 to calculate the values of the Y-connected resistors:

$$R_1 = \frac{(40)(10)}{10 + 40 + 50} = 4\,\Omega; \quad R_2 = \frac{(10)(50)}{10 + 40 + 50} = 5\,\Omega; \quad R_3 = \frac{(40)(50)}{10 + 40 + 50} = 20\,\Omega$$

The transformed circuit is shown below:



The equivalent resistance seen by the 24 V source can be calculated by making series and parallel combinations of the resistors to the right of the 24 V source:

$$R_{\text{eq}} = (15+5)\|(1+4)+20 = 20\|5+20 = 4+20 = 24\,\Omega$$

Therefore, the current i in the 24 V source is given by

$$i = \frac{24\text{ V}}{24\,\Omega} = 1\text{ A}$$

Use current division to calculate the currents i_1 and i_2. Note that the current i_1 flows in the branch containing the $15\,\Omega$ and $5\,\Omega$ series connected resistors, while the current i_2 flows in the parallel branch that contains the series connection of the $1\,\Omega$ and $4\,\Omega$ resistors:

$$i_1 = \frac{4}{15+5}(i) = \frac{4}{20}(1\text{ A}) = 0.2\text{ A}, \qquad \text{and} \qquad i_2 = 1\text{ A} - 0.2\text{ A} = 0.8\text{ A}$$

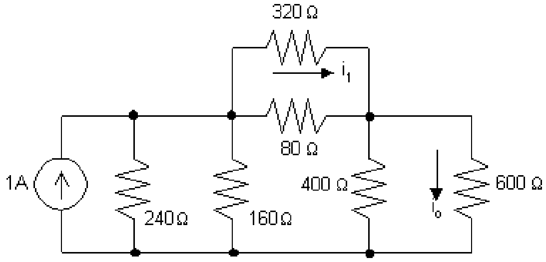
Now use KVL and Ohm's law to calculate v_1. Note that v_1 is the sum of the voltage drop across the $4\,\Omega$ resistor, $4i_2$, and the voltage drop across the $20\,\Omega$ resistor, $20i$:

$$v_1 = 4i_2 + 20i = 4(0.8\text{ A}) + 20(1\text{ A}) = 3.2 + 20 = 23.2\text{ V}$$

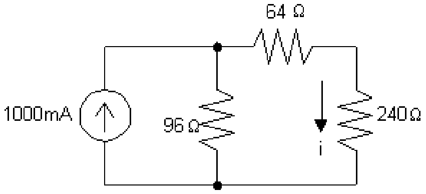
Finally, use KVL and Ohm's law to calculate v_2. Note that v_2 is the sum of the voltage drop across the $5\,\Omega$ resistor, $5i_1$, and the voltage drop across the $20\,\Omega$ resistor, $20i$:

$$v_2 = 5i_1 + 20i = 5(0.2\text{ A}) + 20(1\text{ A}) = 1 + 20 = 21\text{ V}$$

P 3.54 [**a**] After the 20 Ω—100 Ω—50 Ω wye is replaced by its equivalent delta, the circuit reduces to



Now the circuit can be reduced to

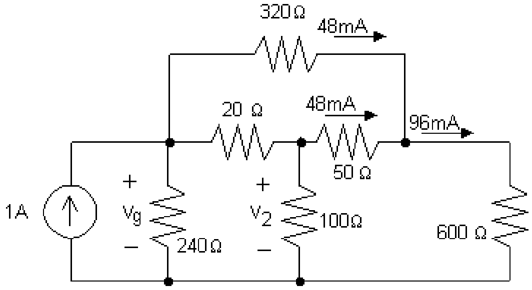


$$i = \frac{96}{400}(1000) = 240 \text{ mA}$$

$$i_o = \frac{400}{1000}(240) = 96 \text{ mA}$$

[**b**] $i_1 = \dfrac{80}{400}(240) = 48 \text{ mA}$

[**c**] Now that i_o and i_1 are known return to the original circuit



$$v_2 = (50)(0.048) + (600)(0.096) = 60 \text{ V}$$

$$i_2 = \frac{v_2}{100} = \frac{60}{100} = 600 \text{ mA}$$

[**d**] $v_g = v_2 + 20(0.6 + 0.048) = 60 + 12.96 = 72.96$ V

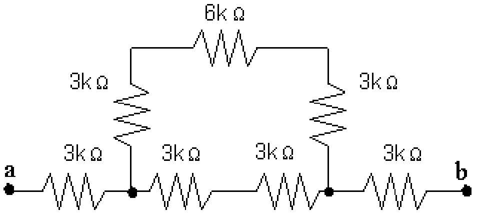
$$p_g = -(v_g)(1) = -72.96 \text{ W}$$

Thus the current source delivers 72.96 W.

P 3.55 The top of the pyramid can be replaced by a resistor equal to

$$R_1 = \frac{(18)(9)}{27} = 6 \text{ k}\Omega$$

The lower left and right deltas can be replaced by wyes. Each resistance in the wye equals 3 kΩ. Thus our circuit can be reduced to



Now the 12 kΩ in parallel with 6 kΩ reduces to 4 kΩ.

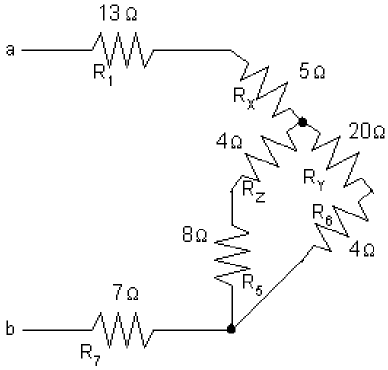
$$\therefore \quad R_{\text{ab}} = 3\text{ k} + 4\text{ k} + 3\text{ k} = 10\text{ k}\Omega$$

P 3.56 [a] Calculate the values of the Y-connected resistors that are equivalent to the $10\,\Omega, 40\,\Omega$, and 50Ω Δ-connected resistors:

$$R_X = \frac{(10)(50)}{10+40+50} = 5\,\Omega; \qquad R_Y = \frac{(50)(40)}{10+40+50} = 20\,\Omega;$$

$$R_Z = \frac{(10)(40)}{10+40+50} = 4\,\Omega$$

Replacing the R_2—R_3—R_4 delta with its equivalent Y gives



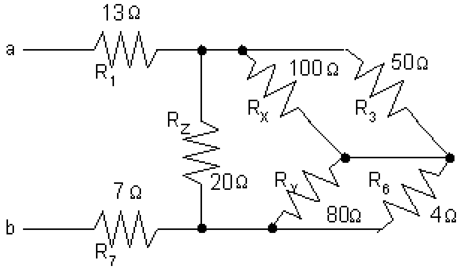
Now calculate the equivalent resistance R_{ab} by making series and parallel combinations of the resistors:

$$R_{\text{ab}} = 13 + 5 + [(8+4)\|(20+4)] + 7 = 33\,\Omega$$

[b] Calculate the values of the Δ-connected resistors that are equivalent to the $10\,\Omega, 8\,\Omega$, and $40\,\Omega$ Y-connected resistors:

$$\begin{aligned} R_X &= \frac{(10)(8)+(8)(40)+(10)(40)}{8} = \frac{800}{8} = 100\,\Omega \\ R_Y &= \frac{(10)(8)+(8)(40)+(10)(40)}{10} = \frac{800}{10} = 80\,\Omega \\ R_Z &= \frac{(10)(8)+(8)(40)+(10)(40)}{40} = \frac{800}{40} = 20\,\Omega \end{aligned}$$

Replacing the R_2, R_4, R_5 wye with its equivalent Δ gives



Make series and parallel combinations of the resistors to find the equivalent resistance R_{ab}:

$$100\,\Omega\|50\,\Omega = 33.33\,\Omega; \qquad 80\,\Omega\|4\,\Omega = 3.81\,\Omega$$

$$\therefore \quad 20\|(33.33 + 3.81) = 13\,\Omega$$

$$\therefore \quad R_{\text{ab}} = 13 + 13 + 7 = 33\,\Omega$$

[c] Convert the delta connection R_4—R_5—R_6 to its equivalent wye.
Convert the wye connection R_3—R_4—R_6 to its equivalent delta.

P 3.57 **[a]** Convert the upper delta to a wye.

$$R_1 = \frac{(50)(50)}{200} = 12.5\,\Omega$$

$$R_2 = \frac{(50)(100)}{200} = 25\,\Omega$$

$$R_3 = \frac{(100)(50)}{200} = 25\,\Omega$$

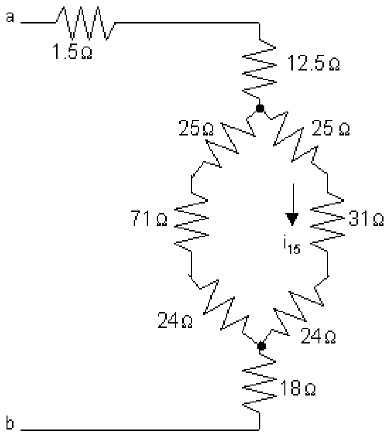
Convert the lower delta to a wye.

$$R_4 = \frac{(60)(80)}{200} = 24\,\Omega$$

$$R_5 = \frac{(60)(60)}{200} = 18\,\Omega$$

$$R_6 = \frac{(80)(60)}{200} = 24\,\Omega$$

Now redraw the circuit using the wye equivalents.



$$R_{\rm ab} = 1.5 + 12.5 + \frac{(120)(80)}{200} + 18 = 14 + 48 + 18 = 80\,\Omega$$

[b] When $v_{\rm ab} = 400$ V

$$i_g = \frac{400}{80} = 5 \text{ A}$$

$$i_{31} = \frac{48}{80}(5) = 3 \text{ A}$$

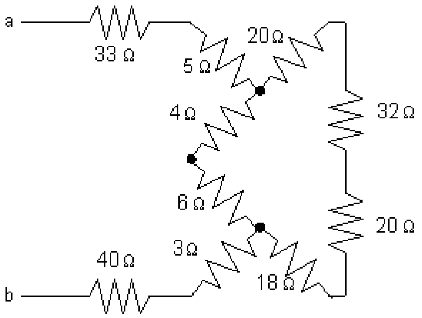
$$p_{31\Omega} = (31)(3)^2 = 279 \text{ W}$$

P 3.58 Replace the upper and lower deltas with the equivalent wyes:

$$R_{\rm 1U} = \frac{(10)(50)}{100} = 5\,\Omega;\, R_{\rm 2U} = \frac{(50)(40)}{100} = 20\,\Omega;\, R_{\rm 3U} = \frac{(10)(40)}{100} = 4\,\Omega$$

$$R_{\rm 1L} = \frac{(10)(60)}{100} = 6\,\Omega;\, R_{\rm 2L} = \frac{(60)(30)}{100} = 18\,\Omega;\, R_{\rm 3L} = \frac{(10)(30)}{100} = 3\,\Omega$$

The resulting circuit is shown below:



Now make series and parallel combinations of the resistors:

$$(4 + 6)\|(20 + 32 + 20 + 18) = 10\|90 = 9\,\Omega$$

$$R_{\rm ab} = 33 + 5 + 9 + 3 + 40 = 90\,\Omega$$

P 3.59 $8 + 12 = 20\,\Omega$

$20\|60 = 15\,\Omega$

$15 + 20 = 35\,\Omega$

$35\|140 = 28\,\Omega$

$28 + 22 = 50\,\Omega$

$50\|75 = 30\,\Omega$

$30 + 10 = 40\,\Omega$

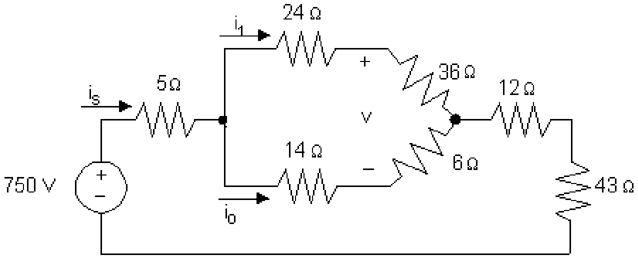
$i_g = 240/40 = 6$ A

$i_o = (6)(50)/125 = 2.4$ A

$i_{140\Omega} = (6 - 2.4)(35)/175 = 0.72$ A

$p_{140\Omega} = (0.72)^2(140) = 72.576$ W

P 3.60 **[a]** Replace the 60—120—20 Ω delta with a wye equivalent to get



$$i_s = \frac{750}{5 + (24 + 36)\|(14 + 6) + 12 + 43} = \frac{750}{75} = 10 \text{ A}$$

$$i_1 = \frac{(24 + 36)\|(14 + 6)}{24 + 36}(10) = \frac{15}{60}(10) = 2.5 \text{ A}$$

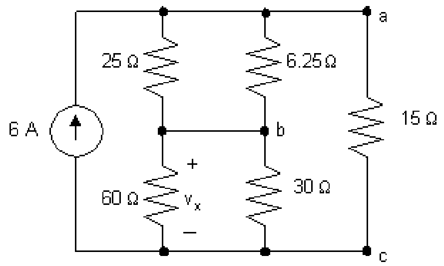
[b] $i_o = 10 - 2.5 = 7.5$ A

$$v = 36i_1 - 6i_o = 36(2.5) - 6(7.5) = 45 \text{ V}$$

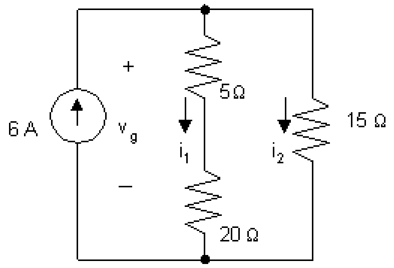
[c] $i_2 = i_o + \dfrac{v}{60} = 7.5 + \dfrac{45}{60} = 8.25$ A

[d] $P_{\text{supplied}} = (750)(10) = 7500$ W

P 3.61



$25\|6.25 = 5\,\Omega \qquad\qquad 60\|30 = 20\,\Omega$



$$i_1 = \frac{(6)(15)}{(40)} = 2.25\ \text{A}; \qquad v_x = 20i_1 = 45\ \text{V}$$

$$v_g = 25i_1 = 56.25\ \text{V}$$

$$v_{6.25} = v_g - v_x = 11.25\ \text{V}$$

$$P_{\text{device}} = \frac{11.25^2}{6.25} + \frac{45^2}{30} + \frac{56.25^2}{15} = 298.6875\ \text{W}$$

P 3.62 **[a]** Subtracting Eq. 3.42 from Eq. 3.43 gives

$R_1 - R_2 = (R_cR_b - R_cR_a)/(R_a + R_b + R_c)$.

Adding this expression to Eq. 3.41 and solving for R_1 gives

$R_1 = R_cR_b/(R_a + R_b + R_c)$.

To find R_2, subtract Eq. 3.43 from Eq. 3.41 and add this result to Eq. 3.42. To find R_3, subtract Eq. 3.41 from Eq. 3.42 and add this result to Eq. 3.43.

[b] Using the hint, Eq. 3.43 becomes

$$R_1 + R_3 = \frac{R_b[(R_2/R_3)R_b + (R_2/R_1)R_b]}{(R_2/R_1)R_b + R_b + (R_2/R_3)R_b} = \frac{R_b(R_1 + R_3)R_2}{(R_1R_2 + R_2R_3 + R_3R_1)}$$

Solving for R_b gives $R_b = (R_1R_2 + R_2R_3 + R_3R_1)/R_2$. To find R_a: First use Eqs. 3.44–3.46 to obtain the ratios $(R_1/R_3) = (R_c/R_a)$ or

$R_c = (R_1/R_3)R_a$ and $(R_1/R_2) = (R_b/R_a)$ or $R_b = (R_1/R_2)R_a$. Now use these relationships to eliminate R_b and R_c from Eq. 3.42. To find R_c, use Eqs. 3.44–3.46 to obtain the ratios $R_b = (R_3/R_2)R_c$ and $R_a = (R_3/R_1)R_c$. Now use the relationships to eliminate R_b and R_a from Eq. 3.41.

P 3.63

$$\begin{aligned} G_a &= \frac{1}{R_a} = \frac{R_1}{R_1R_2 + R_2R_3 + R_3R_1} \\ &= \frac{1/G_1}{(1/G_1)(1/G_2) + (1/G_2)(1/G_3) + (1/G_3)(1/G_1)} \\ &= \frac{(1/G_1)(G_1G_2G_3)}{G_1 + G_2 + G_3} = \frac{G_2G_3}{G_1 + G_2 + G_3} \end{aligned}$$

Similar manipulations generate the expressions for G_b and G_c.

P 3.64 **[a]** $R_{ab} = 2R_1 + \dfrac{R_2(2R_1 + R_L)}{2R_1 + R_2 + R_L} = R_L$

Therefore $\quad 2R_1 - R_L + \dfrac{R_2(2R_1 + R_L)}{2R_1 + R_2 + R_L} = 0$

Thus $\quad R_L^2 = 4R_1^2 + 4R_1R_2 = 4R_1(R_1 + R_2)$

When $R_{ab} = R_L$, the current into terminal a of the attenuator will be v_i/R_L

Using current division, the current in the R_L branch will be

$$\frac{v_i}{R_L} \cdot \frac{R_2}{2R_1 + R_2 + R_L}$$

Therefore $\quad v_o = \dfrac{v_i}{R_L} \cdot \dfrac{R_2}{2R_1 + R_2 + R_L} R_L$

and $\quad \dfrac{v_o}{v_i} = \dfrac{R_2}{2R_1 + R_2 + R_L}$

[b] $(600)^2 = 4(R_1 + R_2)R_1$

$9 \times 10^4 = R_1^2 + R_1R_2$

$\dfrac{v_o}{v_i} = 0.6 = \dfrac{R_2}{2R_1 + R_2 + 600}$

$\therefore \quad 1.2R_1 + 0.6R_2 + 360 = R_2$

$0.4R_2 = 1.2R_1 + 360$

$R_2 = 3R_1 + 900$

$\therefore \quad 9 \times 10^4 = R_1^2 + R_1(3R_1 + 900) = 4R_1^2 + 900R_1$

$\therefore \quad R_1^2 + 225R_1 - 22{,}500 = 0$

$$R_1 = -112.5 \pm \sqrt{(112.5)^2 + 22{,}500} = -112.5 \pm 187.5$$

$$\therefore \quad R_1 = 75\,\Omega$$

$$\therefore \quad R_2 = 3(75) + 900 = 1125\,\Omega$$

[c] From Appendix H, choose $R_1 = 68\,\Omega$ and $R_2 = 1.2\ \text{k}\Omega$. For these values,

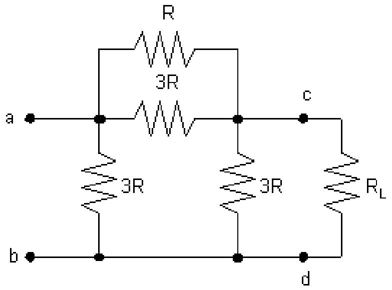
$$R_{\text{ab}} = R_{\text{L}} = \sqrt{(4)(68)(68 + 1200)} = 587.3\,\Omega$$

$$\%\ \text{error} = \left(\frac{587.3}{600} - 1\right) 100 = -2.1\%$$

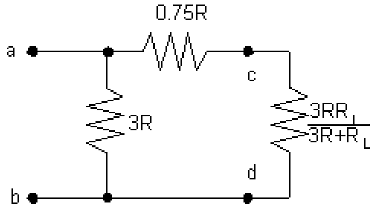
$$\frac{v_o}{v_i} = \frac{1200}{2(68) + 1200 + 587.3} = 0.624$$

$$\%\ \text{error} = \left(\frac{0.624}{0.6} - 1\right) 100 = 4\%$$

P 3.65 **[a]** After making the Y-to-Δ transformation, the circuit reduces to



Combining the parallel resistors reduces the circuit to



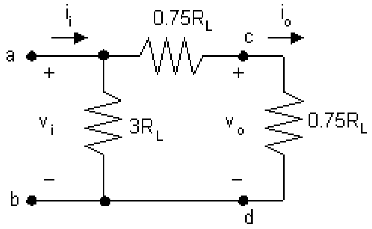
$$\text{Now note:} \quad 0.75R + \frac{3RR_{\text{L}}}{3R + R_{\text{L}}} = \frac{2.25R^2 + 3.75RR_{\text{L}}}{3R + R_{\text{L}}}$$

$$\text{Therefore} \quad R_{\text{ab}} = \frac{3R\left(\dfrac{2.25R^2 + 3.75RR_{\text{L}}}{3R + R_{\text{L}}}\right)}{3R + \left(\dfrac{2.25R^2 + 3.75RR_{\text{L}}}{3R + R_{\text{L}}}\right)} = \frac{3R(3R + 5R_{\text{L}})}{15R + 9R_{\text{L}}}$$

$$\text{If } R = R_{\text{L}},\ \text{we have} \quad R_{\text{ab}} = \frac{3R_{\text{L}}(8R_{\text{L}})}{24R_{\text{L}}} = R_{\text{L}}$$

$$\text{Therefore} \quad R_{\text{ab}} = R_{\text{L}}$$

[b] When $R = R_L$, the circuit reduces to



$$i_o = \frac{i_i(3R_L)}{4.5R_L} = \frac{1}{1.5}i_i = \frac{1}{1.5}\frac{v_i}{R_L}, \qquad v_o = 0.75R_L i_o = \frac{1}{2}v_i,$$

$$\text{Therefore} \quad \frac{v_o}{v_i} = 0.5$$

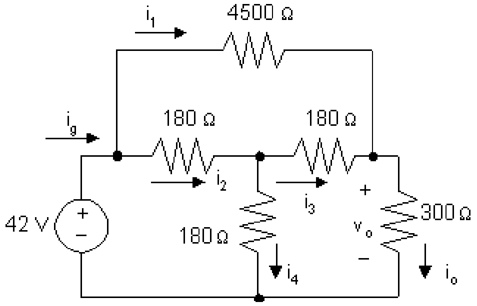
P 3.66 **[a]** $3.5(3R - R_L) = 3R + R_L$

$$10.5R - 1050 = 3R + 300$$

$$7.5R = 1350, \qquad R = 180\,\Omega$$

$$R_2 = \frac{2(180)(300)^2}{3(180)^2 - (300)^2} = 4500\,\Omega$$

[b]



$$v_o = \frac{v_i}{3.5} = \frac{42}{3.5} = 12 \text{ V}$$

$$i_o = \frac{12}{300} = 40 \text{ mA}$$

$$i_1 = \frac{42 - 12}{4500} = \frac{30}{4500} = 6.67 \text{ mA}$$

$$i_g = \frac{42}{300} = 140 \text{ mA}$$

$$i_2 = 140 - 6.67 = 133.33 \text{ mA}$$

$$i_3 = 40 - 6.67 = 33.33 \text{ mA}$$

$$i_4 = 133.33 - 33.33 = 100 \text{ mA}$$

$$p_{4500\text{ top}} = (6.67 \times 10^{-3})^2(4500) = 0.2\text{ W}$$

$$p_{180\text{ left}} = (133.33 \times 10^{-3})^2(180) = 3.2\text{ W}$$

$$p_{180\text{ right}} = (33.33 \times 10^{-3})^2(180) = 0.2\text{ W}$$

$$p_{180\text{ vertical}} = (100 \times 10^{-3})^2(180) = 0.48\text{ W}$$

$$p_{300\text{ load}} = (40 \times 10^{-3})^2(300) = 0.48\text{ W}$$

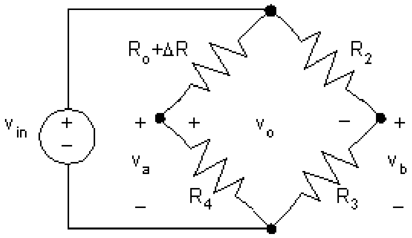
The 180 Ω resistor carrying i_2

[c] $p_{180\text{ left}} = 3.2$ W

[d] Two resistors dissipate minimum power – the 4500 Ω resistor and the 180 Ω resistor carrying i_3.

[e] They both dissipate 0.2 W.

P 3.67 **[a]**



$$v_a = \frac{v_{in}R_4}{R_o + R_4 + \Delta R}$$

$$v_b = \frac{R_3}{R_2 + R_3}v_{in}$$

$$v_o = v_a - v_b = \frac{R_4 v_{in}}{R_o + R_4 + \Delta R} - \frac{R_3}{R_2 + R_3}v_{in}$$

When the bridge is balanced,

$$\frac{R_4}{R_o + R_4}v_{in} = \frac{R_3}{R_2 + R_3}v_{in}$$

$$\therefore \quad \frac{R_4}{R_o + R_4} = \frac{R_3}{R_2 + R_3}$$

$$\begin{aligned} \text{Thus,} \qquad v_o &= \frac{R_4 v_{in}}{R_o + R_4 + \Delta R} - \frac{R_4 v_{in}}{R_o + R_4} \\ &= R_4 v_{in}\left[\frac{1}{R_o + R_4 + \Delta R} - \frac{1}{R_o + R_4}\right] \\ &= \frac{R_4 v_{in}(-\Delta R)}{(R_o + R_4 + \Delta R)(R_o + R_4)} \\ &\approx \frac{-(\Delta R)R_4 v_{in}}{(R_o + R_4)^2}, \qquad \text{since } \Delta R << R_4 \end{aligned}$$

[b] $\Delta R = 0.03R_o$

$$R_o = \frac{R_2R_4}{R_3} = \frac{(1000)(5000)}{500} = 10{,}000\,\Omega$$

$$\Delta R = (0.03)(10^4) = 300\,\Omega$$

$$\therefore \quad v_o \approx \frac{-300(5000)(6)}{(15{,}000)^2} = -40 \text{ mV}$$

[c]
$$\begin{aligned} v_o &= \frac{-(\Delta R)R_4v_{\text{in}}}{(R_o + R_4 + \Delta R)(R_o + R_4)} \\ &= \frac{-300(5000)(6)}{(15{,}300)(15{,}000)} \\ &= -39.2157 \text{ mV} \end{aligned}$$

P 3.68 **[a]** approx value $= \dfrac{-(\Delta R)R_4v_{\text{in}}}{(R_o + R_4)^2}$

$$\text{true value } = \frac{-(\Delta R)R_4v_{\text{in}}}{(R_o + R_4 + \Delta R)(R_o + R_4)}$$

$$\therefore \quad \frac{\text{approx value}}{\text{true value}} = \frac{(R_o + R_4 + \Delta R)}{(R_o + R_4)}$$

$$\therefore \quad \%\text{ error } = \left[\frac{R_o + R_4}{R_o + R_4 + \Delta R} - 1\right] \times 100 = \frac{-\Delta R}{R_o + R_4} \times 100$$

Note that in the above expression, we take the ratio of the true value to the approximate value because both values are negative.

But $R_o = \dfrac{R_2R_4}{R_3}$

$$\therefore \quad \%\text{ error } = \frac{-R_3\Delta R}{R_4(R_2 + R_3)}$$

[b] % error $= \dfrac{-(500)(300)}{(5000)(1500)} \times 100 = -2\%$

P 3.69 $\dfrac{\Delta R(R_3)(100)}{(R_2 + R_3)R_4} = 0.5$

$$\frac{\Delta R(500)(100)}{(1500)(5000)} = 0.5$$

$$\therefore \quad \Delta R = 75\,\Omega$$

$$\%\text{ change } = \frac{75}{10{,}000} \times 100 = 0.75\%$$

P 3.70 **[a]** From Eq 3.64 we have

$$\left(\frac{i_1}{i_2}\right)^2 = \frac{R_2^2}{R_1^2(1+2\sigma)^2}$$

Substituting into Eq 3.63 yields

$$R_2 = \frac{R_2^2}{R_1^2(1+2\sigma)^2}R_1$$

Solving for R_2 yields

$$R_2 = (1+2\sigma)^2 R_1$$

[b] From Eq 3.67 we have

$$\frac{i_1}{i_\mathrm{b}} = \frac{R_2}{R_1 + R_2 + 2R_\mathrm{a}}$$

But $R_2 = (1+2\sigma)^2 R_1$ and $R_\mathrm{a} = \sigma R_1$ therefore

$$\frac{i_1}{i_\mathrm{b}} = \frac{(1+2\sigma)^2 R_1}{R_1 + (1+2\sigma)^2 R_1 + 2\sigma R_1} = \frac{(1+2\sigma)^2}{(1+2\sigma) + (1+2\sigma)^2}$$

$$= \frac{1+2\sigma}{2(1+\sigma)}$$

It follows that

$$\left(\frac{i_1}{i_\mathrm{b}}\right)^2 = \frac{(1+2\sigma)^2}{4(1+\sigma)^2}$$

Substituting into Eq 3.66 gives

$$R_\mathrm{b} = \frac{(1+2\sigma)^2 R_\mathrm{a}}{4(1+\sigma)^2} = \frac{(1+2\sigma)^2 \sigma R_1}{4(1+\sigma)^2}$$

P 3.71 From Eq 3.69

$$\frac{i_1}{i_3} = \frac{R_2 R_3}{D}$$

But $D = (R_1 + 2R_\mathrm{a})(R_2 + 2R_\mathrm{b}) + 2R_\mathrm{b}R_2$

where $R_\mathrm{a} = \sigma R_1; R_2 = (1+2\sigma)^2 R_1$ and $R_\mathrm{b} = \dfrac{(1+2\sigma)^2 \sigma R_1}{4(1+\sigma)^2}$

Therefore D can be written as

$$\begin{aligned}
D &= (R_1 + 2\sigma R_1)\left[(1+2\sigma)^2 R_1 + \frac{2(1+2\sigma)^2\sigma R_1}{4(1+\sigma)^2}\right] + \\
&\quad 2(1+2\sigma)^2 R_1\left[\frac{(1+2\sigma)^2\sigma R_1}{4(1+\sigma)^2}\right] \\
&= (1+2\sigma)^3 R_1^2\left[1 + \frac{\sigma}{2(1+\sigma)^2} + \frac{(1+2\sigma)\sigma}{2(1+\sigma)^2}\right] \\
&= \frac{(1+2\sigma)^3 R_1^2}{2(1+\sigma)^2}\{2(1+\sigma)^2 + \sigma + (1+2\sigma)\sigma\} \\
&= \frac{(1+2\sigma)^3 R_1^2}{(1+\sigma)^2}\{1 + 3\sigma + 2\sigma^2\}
\end{aligned}$$

$$D = \frac{(1+2\sigma)^4 R_1^2}{(1+\sigma)}$$

$$\begin{aligned}
\therefore\quad \frac{i_1}{i_3} &= \frac{R_2 R_3(1+\sigma)}{(1+2\sigma)^4 R_1^2} \\
&= \frac{(1+2\sigma)^2 R_1 R_3(1+\sigma)}{(1+2\sigma)^4 R_1^2} \\
&= \frac{(1+\sigma)R_3}{(1+2\sigma)^2 R_1}
\end{aligned}$$

When this result is substituted into Eq 3.69 we get

$$R_3 = \frac{(1+\sigma)^2 R_3^2 R_1}{(1+2\sigma)^4 R_1^2}$$

Solving for R_3 gives

$$R_3 = \frac{(1+2\sigma)^4 R_1}{(1+\sigma)^2}$$

P 3.72 From the dimensional specifications, calculate σ and R_3:

$$\sigma = \frac{y}{x} = \frac{0.025}{1} = 0.025; \qquad R_3 = \frac{V_{\text{dc}}^2}{p} = \frac{12^2}{120} = 1.2\,\Omega$$

Calculate R_1 from R_3 and σ:

$$R_1 = \frac{(1+\sigma)^2}{(1+2\sigma)^4} R_3 = 1.0372\,\Omega$$

Calculate R_a, R_b, and R_2:

$$R_a = \sigma R_1 = 0.0259\,\Omega \qquad R_b = \frac{(1+2\sigma)^2\sigma R_1}{4(1+\sigma)^2} = 0.0068\,\Omega$$

$R_2 = (1+2\sigma)^2 R_1 = 1.1435\,\Omega$

Using symmetry,

$R_4 = R_2 = 1.1435\,\Omega \qquad R_5 = R_1 = 1.0372\,\Omega$

$R_c = R_b = 0.0068\,\Omega \qquad R_d = R_a = 0.0259\,\Omega$

Test the calculations by checking the power dissipated, which should be 120 W/m. Calculate D, then use Eqs. (3.58)-(3.60) to calculate i_b, i_1, and i_2:

$D = (R_1 + 2R_a)(R_2 + 2R_b) + 2R_2 R_b = 1.2758$

$$i_b = \frac{V_{\rm dc}(R_1 + R_2 + 2R_a)}{D} = 21\text{ A}$$

$$i_1 = \frac{V_{\rm dc} R_2}{D} = 10.7561\text{ A} \qquad i_2 = \frac{V_{\rm dc}(R_1 + 2R_a)}{D} = 10.2439\text{ A}$$

It follows that $i_b^2 R_b = 3$ W and the power dissipation per meter is $3/0.025 = 120$ W/m. The value of $i_1^2 R_1 = 120$ W/m. The value of $i_2^2 R_2 = 120$ W/m. Finally, $i_1^2 R_a = 3$ W/m.

P 3.73 From the solution to Problem 3.72 we have $i_{\rm b} = 21$ A and $i_3 = 10$ A. By symmetry $i_{\rm c} = 21$ A thus the total current supplied by the 12 V source is $21 + 21 + 10$ or 52 A. Therefore the total power delivered by the source is $p_{12\,\rm V}$ (del) $= (12)(52) = 624$ W. We also have from the solution that $p_{\rm a} = p_{\rm b} = p_{\rm c} = p_{\rm d} = 3$ W. Therefore the total power delivered to the vertical resistors is $p_{\rm V} = (8)(3) = 24$ W. The total power delivered to the five horizontal resistors is $p_{\rm H} = 5(120) = 600$ W.

$$\therefore \quad \sum p_{\rm diss} = p_{\rm H} + p_{\rm V} = 624\text{ W} = \sum p_{\rm del}$$

P 3.74 **[a]** $\sigma = 0.03/1.5 = 0.02$

Since the power dissipation is 200 W/m the power dissipated in R_3 must be 200(1.5) or 300 W. Therefore

$$R_3 = \frac{12^2}{300} = 0.48\,\Omega$$

From Table 3.1 we have

$$R_1 = \frac{(1+\sigma)^2 R_3}{(1+2\sigma)^4} = 0.4269\,\Omega$$

$R_a = \sigma R_1 = 0.0085\,\Omega$

$$R_2 = (1+2\sigma)^2 R_1 = 0.4617\,\Omega$$

$$R_b = \frac{(1+2\sigma)^2 \sigma R_1}{4(1+\sigma)^2} = 0.0022\,\Omega$$

Therefore

$$R_4 = R_2 = 0.4617\,\Omega \qquad R_5 = R_1 = 0.4269\,\Omega$$

$$R_c = R_b = 0.0022\,\Omega \qquad R_d = R_a = 0.0085\,\Omega$$

[b] $D = [0.4269 + 2(0.0085)][0.4617 + 2(0.0022)] + 2(0.4617)(0.0022) = 0.2090$

$$i_1 = \frac{V_{\rm dc} R_2}{D} = 26.51 \text{ A}$$

$$i_1^2 R_1 = 300 \text{ W or } 200 \text{ W/m}$$

$$i_2 = \frac{R_1 + 2R_a}{D} V_{\rm dc} = 25.49 \text{ A}$$

$$i_2^2 R_2 = 300 \text{ W or } 200 \text{ W/m}$$

$$i_1^2 R_{\rm a} = 6 \text{ W or } 200 \text{ W/m}$$

$$i_{\rm b} = \frac{R_1 + R_2 + 2R_a}{D} V_{\rm dc} = 52 \text{ A}$$

$$i_{\rm b}^2 R_{\rm b} = 6 \text{ W or } 200 \text{ W/m}$$

$$i_{\rm source} = 52 + 52 + \frac{12}{0.48} = 129 \text{ A}$$

$$p_{\rm del} = 12(129) = 1548 \text{ W}$$

$$p_H = 5(300) = 1500 \text{ W}$$

$$p_{\rm V} = 8(6) = 48 \text{ W}$$

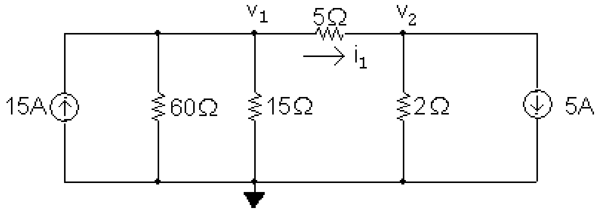
$$\sum p_{\rm del} = \sum p_{\rm diss} = 1548 \text{ W}$$

4

Techniques of Circuit Analysis

Assessment Problems

AP 4.1 [**a**] Redraw the circuit, labeling the reference node and the two node voltages:



The two node voltage equations are

$$-15 + \frac{v_1}{60} + \frac{v_1}{15} + \frac{v_1 - v_2}{5} = 0$$

$$5 + \frac{v_2}{2} + \frac{v_2 - v_1}{5} = 0$$

Place these equations in standard form:

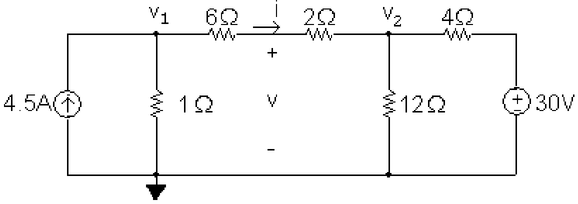
$$\begin{aligned} v_1\left(\frac{1}{60} + \frac{1}{15} + \frac{1}{5}\right) &+ v_2\left(-\frac{1}{5}\right) &= 15 \\ v_1\left(-\frac{1}{5}\right) &+ v_2\left(\frac{1}{2} + \frac{1}{5}\right) &= -5 \end{aligned}$$

Solving, $v_1 = 60$ V and $v_2 = 10$ V;
Therefore, $i_1 = (v_1 - v_2)/5 = 10$ A

[**b**] $p_{15\text{A}} = -(15\text{ A})v_1 = -(15\text{ A})(60\text{ V}) = -900\text{ W} = 900\text{ W(delivered)}$

[**c**] $p_{5\text{A}} = (5\text{ A})v_2 = (5\text{ A})(10\text{ V}) = 50\text{ W} = -50\text{ W(delivered)}$

AP 4.2 Redraw the circuit, choosing the node voltages and reference node as shown:



The two node voltage equations are:

$$
\begin{aligned}
-4.5 + \frac{v_1}{1} + \frac{v_1 - v_2}{6+2} &= 0 \\
\frac{v_2}{12} + \frac{v_2 - v_1}{6+2} + \frac{v_2 - 30}{4} &= 0
\end{aligned}
$$

Place these equations in standard form:

$$
\begin{aligned}
v_1\left(1 + \frac{1}{8}\right) \quad + \quad v_2\left(-\frac{1}{8}\right) &= 4.5 \\
v_1\left(-\frac{1}{8}\right) \quad + \quad v_2\left(\frac{1}{12} + \frac{1}{8} + \frac{1}{4}\right) &= 7.5
\end{aligned}
$$

Solving, $v_1 = 6$ V $\qquad v_2 = 18$ V

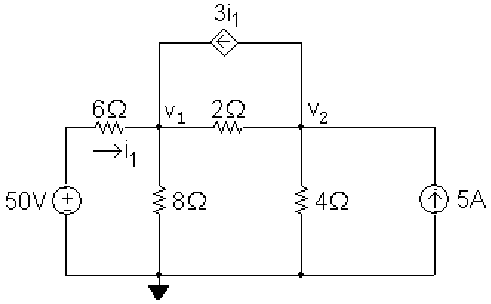
To find the voltage v, first find the current i through the series-connected $6\,\Omega$ and $2\,\Omega$ resistors:

$$i = \frac{v_1 - v_2}{6+2} = \frac{6 - 18}{8} = -1.5 \text{ A}$$

Using a KVL equation, calculate v:

$$v = 2i + v_2 = 2(-1.5) + 18 = 15 \text{ V}$$

AP 4.3 **[a]** Redraw the circuit, choosing the node voltages and reference node as shown:



The node voltage equations are:

$$
\begin{aligned}
\frac{v_1 - 50}{6} + \frac{v_1}{8} + \frac{v_1 - v_2}{2} - 3i_1 &= 0 \\
-5 + \frac{v_2}{4} + \frac{v_2 - v_1}{2} + 3i_1 &= 0
\end{aligned}
$$

The dependent source requires the following constraint equation:

$$i_1 = \frac{50 - v_1}{6}$$

Place these equations in standard form:

$$\begin{array}{lllllll}
v_1\left(\frac{1}{6}+\frac{1}{8}+\frac{1}{2}\right) & + & v_2\left(-\frac{1}{2}\right) & + & i_1(-3) & = & \frac{50}{6} \\
v_1\left(-\frac{1}{2}\right) & + & v_2\left(\frac{1}{4}+\frac{1}{2}\right) & + & i_1(3) & = & 5 \\
v_1\left(\frac{1}{6}\right) & + & v_2(0) & + & i_1(1) & = & \frac{50}{6}
\end{array}$$

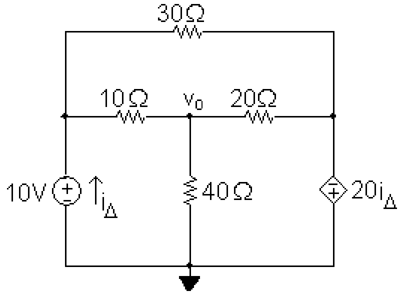
Solving, $v_1 = 32$ V; $v_2 = 16$ V; $i_1 = 3$ A

Using these values to calculate the power associated with each source:

$$\begin{aligned}
p_{50\text{V}} = -50i_1 &= -150 \text{ W} \\
p_{5\text{A}} = -5(v_2) &= -80 \text{ W} \\
p_{3i_1} = 3i_1(v_2 - v_1) &= -144 \text{ W}
\end{aligned}$$

[**b**] All three sources are delivering power to the circuit because the power computed in (a) for each of the sources is negative.

AP 4.4 Redraw the circuit and label the reference node and the node at which the node voltage equation will be written:



The node voltage equation is

$$\frac{v_o}{40} + \frac{v_o - 10}{10} + \frac{v_o + 20i_\Delta}{20} = 0$$

The constraint equation required by the dependent source is

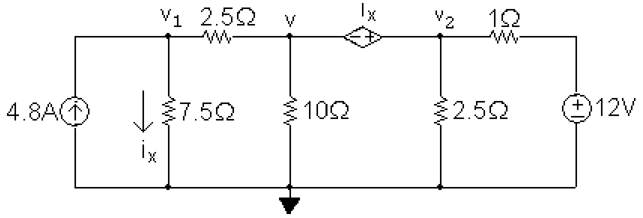
$$i_\Delta = i_{10\,\Omega} + i_{30\,\Omega} = \frac{10 - v_o}{10} + \frac{10 + 20i_\Delta}{30}$$

Place these equations in standard form:

$$\begin{array}{lllll}
v_o\left(\frac{1}{40}+\frac{1}{10}+\frac{1}{20}\right) & + & i_\Delta(1) & = & 1 \\
v_o\left(\frac{1}{10}\right) & + & i_\Delta\left(1-\frac{20}{30}\right) & = & 1+\frac{10}{30}
\end{array}$$

Solving, $i_\Delta = -3.2$ A and $v_o = 24$ V

AP 4.5 Redraw the circuit identifying the three node voltages and the reference node:



Note that the dependent voltage source and the node voltages v and v_2 form a supernode. The v_1 node voltage equation is

$$\frac{v_1}{7.5} + \frac{v_1 - v}{2.5} - 4.8 = 0$$

The supernode equation is

$$\frac{v - v_1}{2.5} + \frac{v}{10} + \frac{v_2}{2.5} + \frac{v_2 - 12}{1} = 0$$

The constraint equation due to the dependent source is

$$i_x = \frac{v_1}{7.5}$$

The constraint equation due to the supernode is

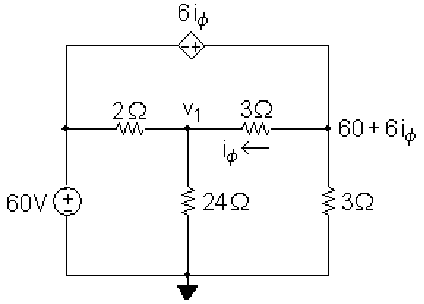
$$v + i_x = v_2$$

Place this set of equations in standard form:

$$\begin{array}{lllllllll}
v_1\left(\frac{1}{7.5} + \frac{1}{2.5}\right) & + & v\left(-\frac{1}{2.5}\right) & + & v_2(0) & + & i_x(0) & = & 4.8 \\
v_1\left(-\frac{1}{2.5}\right) & + & v\left(\frac{1}{2.5} + \frac{1}{10}\right) & + & v_2\left(\frac{1}{2.5} + 1\right) & + & i_x(0) & = & 12 \\
v_1\left(-\frac{1}{7.5}\right) & + & v(0) & + & v_2(0) & + & i_x(1) & = & 0 \\
v_1(0) & + & v(1) & + & v_2(-1) & + & i_x(1) & = & 0
\end{array}$$

Solving this set of equations gives $v_1 = 15$ V, $v_2 = 10$ V, $i_x = 2$ A, and $v = 8$ V.

AP 4.6 Redraw the circuit identifying the reference node and the two unknown node voltages. Note that the right-most node voltage is the sum of the 60 V source and the dependent source voltage.



The node voltage equation at v_1 is

$$\frac{v_1 - 60}{2} + \frac{v_1}{24} + \frac{v_1 - (60 + 6i_\phi)}{3} = 0$$

The constraint equation due to the dependent source is

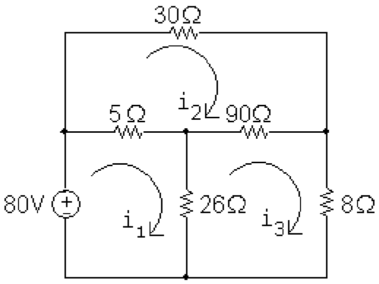
$$i_\phi = \frac{60 + 6i_\phi - v_1}{3}$$

Place these two equations in standard form:

$$\begin{aligned} v_1\left(\frac{1}{2} + \frac{1}{24} + \frac{1}{3}\right) &+ i_\phi(-2) &= 30 + 20 \\ v_1\left(\frac{1}{3}\right) &+ i_\phi(1 - 2) &= 20 \end{aligned}$$

Solving, $i_\phi = -4$ A and $v_1 = 48$ V

AP 4.7 **[a]** Redraw the circuit identifying the three mesh currents:



The mesh current equations are:

$$\begin{aligned} -80 + 5(i_1 - i_2) + 26(i_1 - i_3) &= 0 \\ 30i_2 + 90(i_2 - i_3) + 5(i_2 - i_1) &= 0 \\ 8i_3 + 26(i_3 - i_1) + 90(i_3 - i_2) &= 0 \end{aligned}$$

Place these equations in standard form:

$$\begin{aligned}31i_1 - 5i_2 - 26i_3 &= 80\\ -5i_1 + 125i_2 - 90i_3 &= 0\\ -26i_1 - 90i_2 + 124i_3 &= 0\end{aligned}$$

Solving,

$i_1 = 5$ A; $\quad i_2 = 2$ A; $\quad i_3 = 2.5$ A

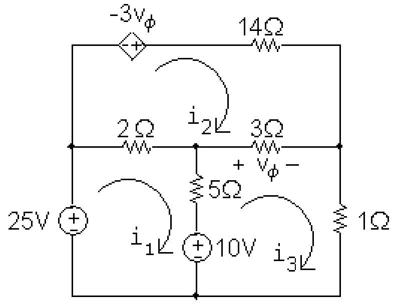
$p_{80V} = -(80)i_1 = -(80)(5) = -400$ W

Therefore the 80 V source is delivering 400 W to the circuit.

[b] $p_{8\Omega} = (8)i_3^2 = 8(2.5)^2 = 50$ W, so the $8\,\Omega$ resistor dissipates 50 W.

AP 4.8 **[a]** $b = 8, \quad n = 6, \quad b - n + 1 = 3$

[b] Redraw the circuit identifying the three mesh currents:



The three mesh-current equations are

$$\begin{aligned}-25 + 2(i_1 - i_2) + 5(i_1 - i_3) + 10 &= 0\\ -(-3v_\phi) + 14i_2 + 3(i_2 - i_3) + 2(i_2 - i_1) &= 0\\ 1i_3 - 10 + 5(i_3 - i_1) + 3(i_3 - i_2) &= 0\end{aligned}$$

The dependent source constraint equation is

$v_\phi = 3(i_3 - i_2)$

Place these four equations in standard form:

$$\begin{aligned}7i_1 - 2i_2 - 5i_3 + 0v_\phi &= 15\\ -2i_1 + 19i_2 - 3i_3 + 3v_\phi &= 0\\ -5i_1 - 3i_2 + 9i_3 + 0v_\phi &= 10\\ 0i_1 + 3i_2 - 3i_3 + 1v_\phi &= 0\end{aligned}$$

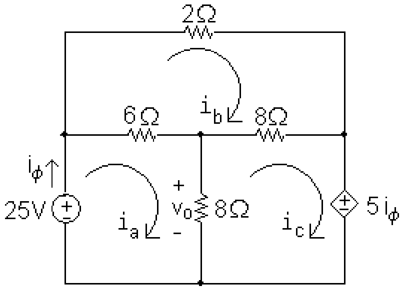
Solving

$i_1 = 4$ A; $\quad i_2 = -1$ A; $\quad i_3 = 3$ A; $\quad v_\phi = 12$ V

$p_{\text{ds}} = -(-3v_\phi)i_2 = 3(12)(-1) = -36$ W

Thus, the dependent source is delivering 36 W, or absorbing −36 W.

AP 4.9 Redraw the circuit identifying the three mesh currents:



The mesh current equations are:

$$\begin{aligned}-25 + 6(i_\mathrm{a} - i_\mathrm{b}) + 8(i_\mathrm{a} - i_\mathrm{c}) &= 0\\ 2i_\mathrm{b} + 8(i_\mathrm{b} - i_\mathrm{c}) + 6(i_\mathrm{b} - i_\mathrm{a}) &= 0\\ 5i_\phi + 8(i_\mathrm{c} - i_\mathrm{a}) + 8(i_\mathrm{c} - i_\mathrm{b}) &= 0\end{aligned}$$

The dependent source constraint equation is $i_\phi = i_\mathrm{a}$. We can substitute this simple expression for i_ϕ into the third mesh equation and place the equations in standard form:

$$\begin{aligned}14i_\mathrm{a} - 6i_\mathrm{b} - 8i_\mathrm{c} &= 25\\ -6i_\mathrm{a} + 16i_\mathrm{b} - 8i_\mathrm{c} &= 0\\ -3i_\mathrm{a} - 8i_\mathrm{b} + 16i_\mathrm{c} &= 0\end{aligned}$$

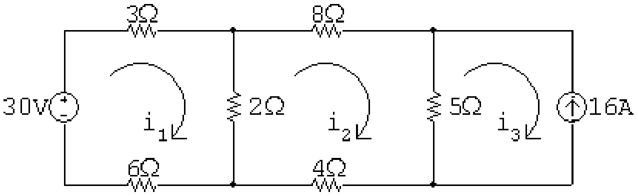
Solving,

$i_\mathrm{a} = 4$ A; $\qquad i_\mathrm{b} = 2.5$ A; $\qquad i_\mathrm{c} = 2$ A

Thus,

$v_o = 8(i_\mathrm{a} - i_\mathrm{c}) = 8(4 - 2) = 16$ V

AP 4.10 Redraw the circuit identifying the mesh currents:



Since there is a current source on the perimeter of the i_3 mesh, we know that $i_3 = -16$ A. The remaining two mesh equations are

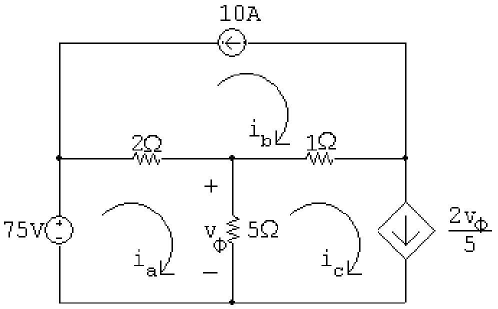
$$\begin{aligned}-30 + 3i_1 + 2(i_1 - i_2) + 6i_1 &= 0\\ 8i_2 + 5(i_2 + 16) + 4i_2 + 2(i_2 - i_1) &= 0\end{aligned}$$

Place these equations in standard form:

$$\begin{aligned} 11i_1 - 2i_2 &= 30 \\ -2i_1 + 19i_2 &= -80 \end{aligned}$$

Solving: $i_1 = 2$ A, $i_2 = -4$ A, $i_3 = -16$ A
The current in the $2\,\Omega$ resistor is $i_1 - i_2 = 6$ A $\therefore$ $p_{2\,\Omega} = (6)^2(2) = 72$ W
Thus, the $2\,\Omega$ resistors dissipates 72 W.

AP 4.11 Redraw the circuit and identify the mesh currents:



There are current sources on the perimeters of both the i_b mesh and the i_c mesh, so we know that

$$i_\mathrm{b} = -10 \text{ A}; \qquad i_\mathrm{c} = \frac{2v_\phi}{5}$$

The remaining mesh current equation is

$$-75 + 2(i_\mathrm{a} + 10) + 5(i_\mathrm{a} - 0.4v_\phi) = 0$$

The dependent source requires the following constraint equation:

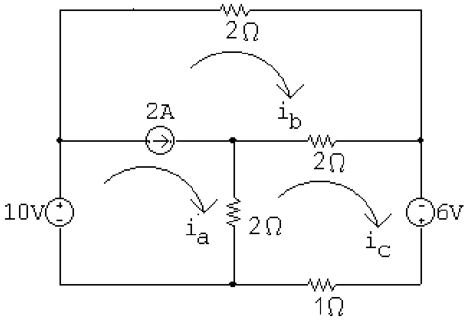
$$v_\phi = 5(i_\mathrm{a} - i_\mathrm{c}) = 5(i_\mathrm{a} - 0.4v_\phi)$$

Place the mesh current equation and the dependent source equation is standard form:

$$\begin{aligned} 7i_\mathrm{a} - 2v_\phi &= 55 \\ 5i_\mathrm{a} - 3v_\phi &= 0 \end{aligned}$$

Solving: $i_\mathrm{a} = 15$ A; $i_\mathrm{b} = -10$ A; $i_\mathrm{c} = 10$ A; $v_\phi = 25$ V
Thus, $i_\mathrm{a} = 15$ A.

AP 4.12 Redraw the circuit and identify the mesh currents:



The 2 A current source is shared by the meshes $i_{\rm a}$ and $i_{\rm b}$. Thus we combine these meshes to form a supermesh and write the following equation:

$$-10 + 2i_{\rm b} + 2(i_{\rm b} - i_{\rm c}) + 2(i_{\rm a} - i_{\rm c}) = 0$$

The other mesh current equation is

$$-6 + 1i_{\rm c} + 2(i_{\rm c} - i_{\rm a}) + 2(i_{\rm c} - i_{\rm b}) = 0$$

The supermesh constraint equation is

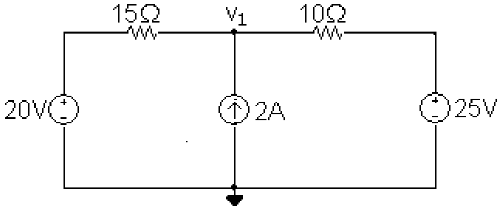
$$i_{\rm a} - i_{\rm b} = 2$$

Place these three equations in standard form:

$$\begin{aligned} 2i_{\rm a} + 4i_{\rm b} - 4i_{\rm c} &= 10 \\ -2i_{\rm a} - 2i_{\rm b} + 5i_{\rm c} &= 6 \\ i_{\rm a} - i_{\rm b} + 0i_{\rm c} &= 2 \end{aligned}$$

Solving, $i_{\rm a} = 7$ A; $i_{\rm b} = 5$ A; $i_{\rm c} = 6$ A
Thus, $p_{1\,\Omega} = i_{\rm c}^2(1) = (6)^2(1) = 36$ W

AP 4.13 Redraw the circuit and identify the reference node and the node voltage v_1:



The node voltage equation is

$$\frac{v_1 - 20}{15} - 2 + \frac{v_1 - 25}{10} = 0$$

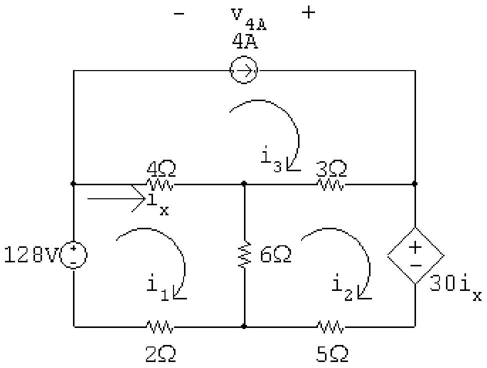
Rearranging and solving,

$$v_1\left(\frac{1}{15}+\frac{1}{10}\right) = 2+\frac{20}{15}+\frac{25}{10} \qquad \therefore \quad v_1 = 35 \text{ V}$$

$$p_{2A} = -35(2) = -70 \text{ W}$$

Thus the 2 A current source delivers 70 W.

AP 4.14 Redraw the circuit and identify the mesh currents:



There is a current source on the perimeter of the i_3 mesh, so $i_3 = 4$ A. The other two mesh current equations are

$$\begin{aligned} -128 + 4(i_1-4) + 6(i_1-i_2) + 2i_1 &= 0 \\ 30i_x + 5i_2 + 6(i_2-i_1) + 3(i_2-4) &= 0 \end{aligned}$$

The constraint equation due to the dependent source is

$$i_x = i_1 - i_3 = i_1 - 4$$

Substitute the constraint equation into the second mesh equation and place the resulting two mesh equations in standard form:

$$\begin{aligned} 12i_1 - 6i_2 &= 144 \\ 24i_1 + 14i_2 &= 132 \end{aligned}$$

Solving,

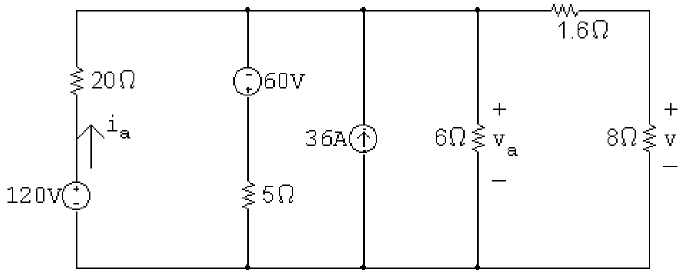
$$i_1 = 9 \text{ A}; \qquad i_2 = -6 \text{ A}; \qquad i_3 = 4 \text{ A}; \qquad i_x = 9-4 = 5 \text{ A}$$

$$\therefore \quad v_{4\text{A}} = 3(i_3 - i_2) - 4i_x = 10 \text{ V}$$

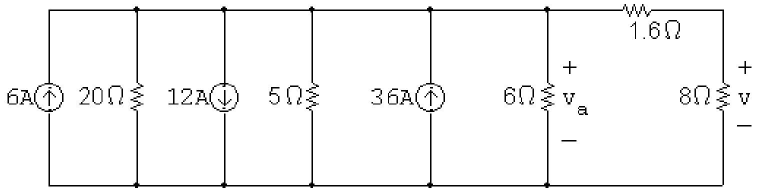
$$p_{4A} = -v_{4\text{A}}(4) = -(10)(4) = -40 \text{ W}$$

Thus, the 2 A current source delivers 40 W.

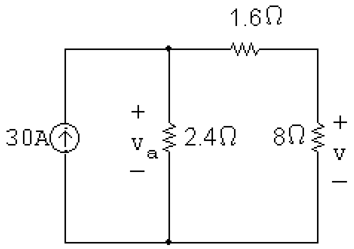
AP 4.15 [**a**] Redraw the circuit with a helpful voltage and current labeled:

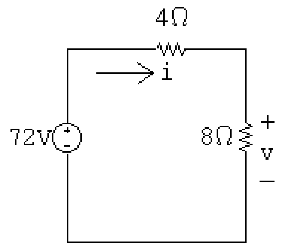


Transform the 120 V source in series with the 20 Ω resistor into a 6 A source in parallel with the 20 Ω resistor. Also transform the −60 V source in series with the 5 Ω resistor into a −12 A source in parallel with the 5 Ω resistor. The result is the following circuit:



Combine the three current sources into a single current source, using KCL, and combine the 20 Ω, 5 Ω, and 6 Ω resistors in parallel. The resulting circuit is shown on the left. To simplify the circuit further, transform the resulting 30 A source in parallel with the 2.4 Ω resistor into a 72 V source in series with the 2.4 Ω resistor. Combine the 2.4 Ω resistor in series with the 1.6 Ω resisor to get a very simple circuit that still maintains the voltage v. The resulting circuit is on the right.





Use voltage division in the circuit on the right to calculate v as follows:

$$v = \frac{8}{12}(72) = 48 \text{ V}$$

[**b**] Calculate i in the circuit on the right using Ohm's law:

$$i = \frac{v}{8} = \frac{48}{8} = 6 \text{ A}$$

Now use i to calculate v_{a} in the circuit on the left:

$$v_{\text{a}} = 6(1.6 + 8) = 57.6 \text{ V}$$

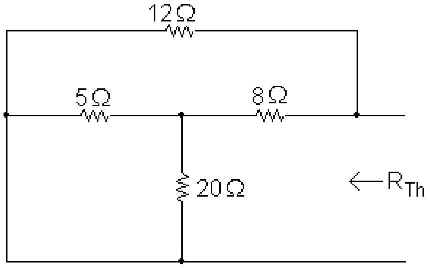
Returning back to the original circuit, note that the voltage v_{a} is also the voltage drop across the series combination of the 120 V source and 20 Ω resistor. Use this fact to calculate the current in the 120 V source, i_a:

$$i_a = \frac{120 - v_a}{20} = \frac{120 - 57.6}{20} = 3.12 \text{ A}$$

$$p_{120V} = -(120)i_a = -(120)(3.12) = -374.40 \text{ W}$$

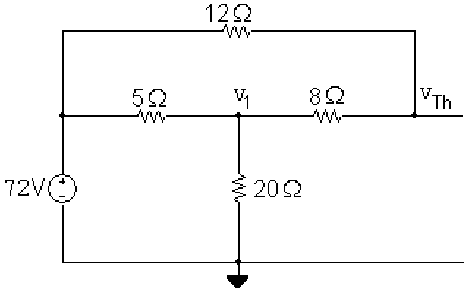
Thus, the 120 V source delivers 374.4 W.

AP 4.16 To find R_{Th}, replace the 72 V source with a short circuit:



Note that the $5\,\Omega$ and $20\,\Omega$ resistors are in parallel, with an equivalent resistance of $5\|20 = 4\,\Omega$. The equivalent $4\,\Omega$ resistance is in series with the $8\,\Omega$ resistor for an equivalent resistance of $4 + 8 = 12\,\Omega$. Finally, the $12\,\Omega$ equivalent resistance is in parallel with the $12\,\Omega$ resistor, so $R_{Th} = 12\|12 = 6\,\Omega$.

Use node voltage analysis to find v_{Th}. Begin by redrawing the circuit and labeling the node voltages:



The node voltage equations are

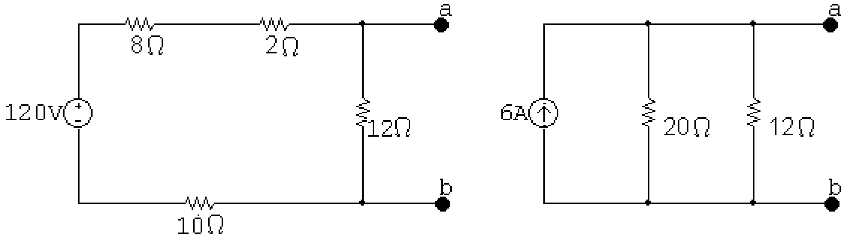
$$\begin{aligned} \frac{v_1 - 72}{5} + \frac{v_1}{20} + \frac{v_1 - v_{Th}}{8} &= 0 \\ \frac{v_{Th} - v_1}{8} + \frac{v_{Th} - 72}{12} &= 0 \end{aligned}$$

Place these equations in standard form:

$$\begin{aligned} v_1\left(\frac{1}{5} + \frac{1}{20} + \frac{1}{8}\right) + v_{Th}\left(-\frac{1}{8}\right) &= \frac{72}{5} \\ v_1\left(-\frac{1}{8}\right) + v_{Th}\left(\frac{1}{8} + \frac{1}{12}\right) &= 6 \end{aligned}$$

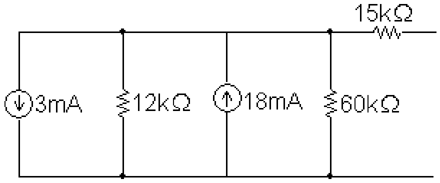
Solving, $v_1 = 60$ V and $v_{Th} = 64.8$ V. Therefore, the Thévenin equivalent circuit is a 64.8 V source in series with a $6\,\Omega$ resistor.

AP 4.17 We begin by performing a source transformation, turning the parallel combination of the 15 A source and 8 Ω resistor into a series combination of a 120 V source and an 8 Ω resistor, as shown in the figure on the left. Next, combine the 2 Ω, 8 Ω and 10 Ω resistors in series to give an equivalent 20 Ω resistance. Then transform the series combination of the 120 V source and the 20 Ω equivalent resistance into a parallel combination of a 6 A source and a 20 Ω resistor, as shown in the figure on the right.

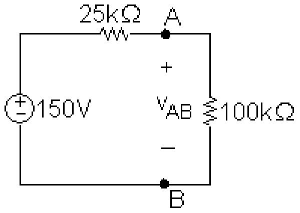


Finally, combine the 20 Ω and 12 Ω parallel resistors to give $R_N = 20\|12 = 7.5\,\Omega$. Thus, the Norton equivalent circuit is the parallel combination of a 6 A source and a 7.5 Ω resistor.

AP 4.18 Find the Thévenin equivalent with respect to A, B using source transformations. To begin, convert the series combination of the −36 V source and 12 kΩ resistor into a parallel combination of a −3 mA source and 12 kΩ resistor. The resulting circuit is shown below:



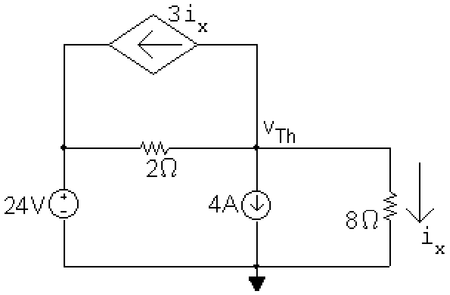
Now combine the two parallel current sources and the two parallel resistors to give a $-3 + 18 = 15$ mA source in parallel with a 12 k$\|$60 k= 10 kΩ resistor. Then transform the 15 mA source in parallel with the 10 kΩ resistor into a 150 V source in series with a 10 kΩ resistor, and combine this 10 kΩ resistor in series with the 15 kΩ resistor. The Thévenin equivalent is thus a 150 V source in series with a 25 kΩ resistor, as seen to the left of the terminals A,B in the circuit below.



Now attach the voltmeter, modeled as a 100 kΩ resistor, to the Thévenin equivalent and use voltage division to calculate the meter reading v_{AB}:

$$v_{AB} = \frac{100{,}000}{125{,}000}(150) = 120 \text{ V}$$

AP 4.19 Begin by calculating the open circuit voltage, which is also $v_{\rm Th}$, from the circuit below:



Summing the currents away from the node labeled $v_{\rm Th}$ We have

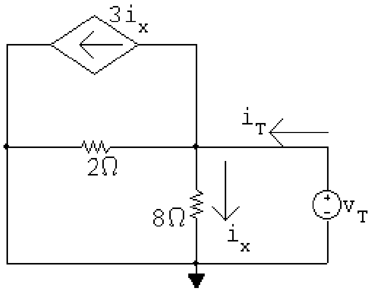
$$\frac{v_{\rm Th}}{8} + 4 + 3i_x + \frac{v_{\rm Th} - 24}{2} = 0$$

Also, using Ohm's law for the $8\,\Omega$ resistor,

$$i_x = \frac{v_{\rm Th}}{8}$$

Substituting the second equation into the first and solving for $v_{\rm Th}$ yields $v_{\rm Th} = 8$ V.

Now calculate $R_{\rm Th}$. To do this, we use the test source method. Replace the voltage source with a short circuit, the current source with an open circuit, and apply the test voltage $v_{\rm T}$, as shown in the circuit below:



Write a KCL equation at the middle node:

$$i_{\rm T} = i_x + 3i_x + v_{\rm T}/2 = 4i_x + v_{\rm T}/2$$

Use Ohm's law to determine i_x as a function of $v_{\rm T}$:

$$i_x = v_{\rm T}/8$$

Substitute the second equation into the first equation:

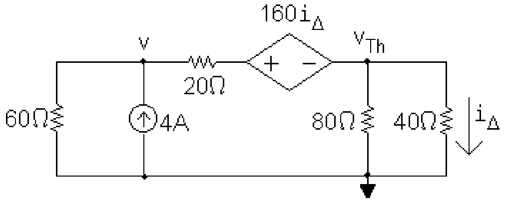
$$i_{\rm T} = 4(v_{\rm T}/8) + v_{\rm T}/2 = v_{\rm T}$$

Thus,

$R_{\text{Th}} = v_{\text{T}}/i_{\text{T}} = 1\,\Omega$

The Thévenin equivalent is an 8 V source in series with a $1\,\Omega$ resistor.

AP 4.20 Begin by calculating the open circuit voltage, which is also v_{Th}, using the node voltage method in the circuit below:



The node voltage equations are

$$\begin{aligned}\frac{v}{60} + \frac{v - (v_{\text{Th}} + 160 i_\Delta)}{20} - 4 &= 0, \\ \frac{v_{\text{Th}}}{40} + \frac{v_{\text{Th}}}{80} + \frac{v_{\text{Th}} + 160 i_\Delta - v}{20} &= 0\end{aligned}$$

The dependent source constraint equation is

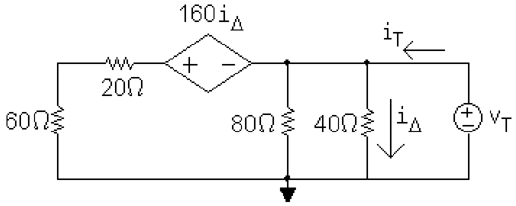
$$i_\Delta = \frac{v_{\text{Th}}}{40}$$

Substitute the constraint equation into the node voltage equations and put the two equations in standard form:

$$\begin{aligned}v\left(\frac{1}{60} + \frac{1}{20}\right) \quad &+ \quad v_{\text{Th}}\left(-\frac{5}{20}\right) &= 4 \\ v\left(-\frac{1}{20}\right) \quad &+ \quad v_{\text{Th}}\left(\frac{1}{40} + \frac{1}{80} + \frac{5}{20}\right) &= 0\end{aligned}$$

Solving, $v = 172.5$ V and $v_{\text{Th}} = 30$ V.

Now use the test source method to calculate the test current and thus R_{Th}. Replace the current source with a short circuit and apply the test source to get the following circuit:



Write a KCL equation at the rightmost node:

$$i_{\text{T}} = \frac{v_{\text{T}}}{80} + \frac{v_{\text{T}}}{40} + \frac{v_{\text{T}} + 160 i_\Delta}{80}$$

The dependent source constraint equation is

$$i_\Delta = \frac{v_T}{40}$$

Substitute the constraint equation into the KCL equation and simplify the right-hand side:

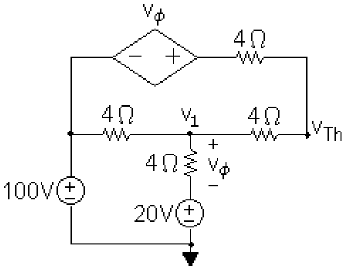
$$i_T = \frac{v_T}{10}$$

Therefore,

$$R_{Th} = \frac{v_T}{i_T} = 10\,\Omega$$

Thus, the Thévenin equivalent is a 30 V source in series with a $10\,\Omega$ resistor.

AP 4.21 First find the Thévenin equivalent circuit. To find v_{Th}, create an open circuit between nodes a and b and use the node voltage method with the circuit below:



The node voltage equations are:

$$\begin{aligned}\frac{v_{Th} - (100 + v_\phi)}{4} + \frac{v_{Th} - v_1}{4} &= 0\\ \frac{v_1 - 100}{4} + \frac{v_1 - 20}{4} + \frac{v_1 - v_{Th}}{4} &= 0\end{aligned}$$

The dependent source constraint equation is

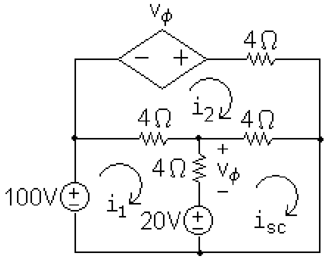
$$v_\phi = v_1 - 20$$

Place these three equations in standard form:

$$\begin{aligned}v_{Th}\left(\frac{1}{4}+\frac{1}{4}\right) &+ v_1\left(-\frac{1}{4}\right) &+ v_\phi\left(-\frac{1}{4}\right) &= 25\\ v_{Th}\left(-\frac{1}{4}\right) &+ v_1\left(\frac{1}{4}+\frac{1}{4}+\frac{1}{4}\right) &+ v_\phi\,(0) &= 30\\ v_{Th}\,(0) &+ v_1\,(1) &+ v_\phi\,(-1) &= 20\end{aligned}$$

Solving, $v_{Th} = 120$ V, $v_1 = 80$ V, and $v_\phi = 60$ V.

Now create a short circuit between nodes a and b and use the mesh current method with the circuit below:



The mesh current equations are

$$\begin{aligned} -100 + 4(i_1 - i_2) + v_\phi + 20 &= 0 \\ -v_\phi + 4i_2 + 4(i_2 - i_{\rm sc}) + 4(i_2 - i_1) &= 0 \\ -20 - v_\phi + 4(i_{\rm sc} - i_2) &= 0 \end{aligned}$$

The dependent source constraint equation is

$$v_\phi = 4(i_1 - i_{\rm sc})$$

Place these four equations in standard form:

$$\begin{aligned} 4i_1 - 4i_2 + 0i_{\rm sc} + v_\phi &= 80 \\ -4i_1 + 12i_2 - 4i_{\rm sc} - v_\phi &= 0 \\ 0i_1 - 4i_2 + 4i_{\rm sc} - v_\phi &= 20 \\ 4i_1 + 0i_2 - 4i_{\rm sc} - v_\phi &= 0 \end{aligned}$$

Solving, $i_1 = 45$ A, $i_2 = 30$ A, $i_{\rm sc} = 40$ A, and $v_\phi = 20$ V. Thus,

$$R_{\rm Th} = \frac{v_{\rm Th}}{i_{\rm sc}} = \frac{120}{40} = 3\,\Omega$$

[a] For maximum power transfer, $R = R_{\rm Th} = 3\,\Omega$

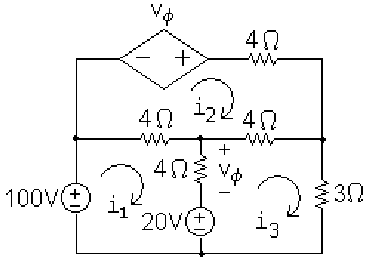
[b] The Thévenin voltage, $v_{\rm Th} = 120$ V, splits equally between the Thévenin resistance and the load resistance, so

$$v_{\rm load} = \frac{120}{2} = 60 \text{ V}$$

Therefore,

$$p_{\max} = \frac{v_{\rm load}^2}{R_{\rm load}} = \frac{60^2}{3} = 1200 \text{ W}$$

AP 4.22 Sustituting the value $R = 3\,\Omega$ into the circuit and identifying three mesh currents we have the circuit below:



The mesh current equations are:

$$\begin{aligned}-100 + 4(i_1 - i_2) + v_\phi + 20 &= 0\\ -v_\phi + 4i_2 + 4(i_2 - i_3) + 4(i_2 - i_1) &= 0\\ -20 - v_\phi + 4(i_3 - i_2) + 3i_3 &= 0\end{aligned}$$

The dependent source constraint equation is

$$v_\phi = 4(i_1 - i_3)$$

Place these four equations in standard form:

$$\begin{aligned}4i_1 - 4i_2 + 0i_3 + v_\phi &= 80\\ -4i_1 + 12i_2 - 4i_3 - v_\phi &= 0\\ 0i_1 - 4i_2 + 7i_3 - v_\phi &= 20\\ 4i_1 + 0i_2 - 4i_3 - v_\phi &= 0\end{aligned}$$

Solving, $i_1 = 30$ A, $i_2 = 20$ A, $i_3 = 20$ A, and $v_\phi = 40$ V.

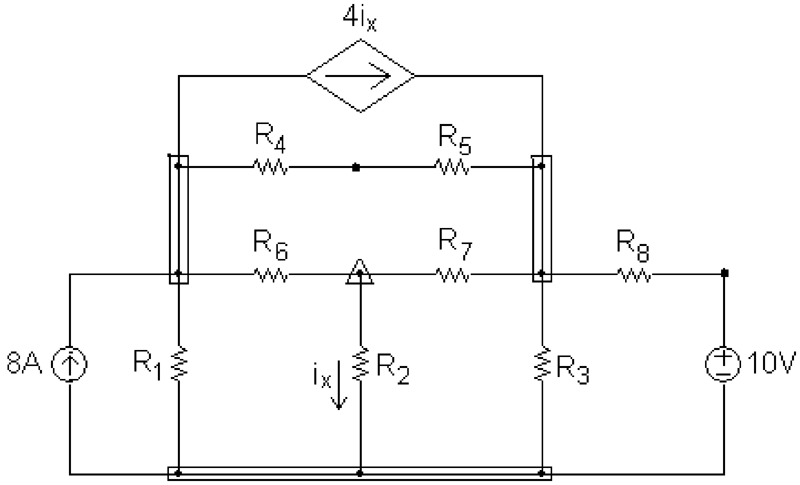
[a] $p_{100\text{V}} = -(100)i_1 = -(100)(30) = -3000$ W. Thus, the 100 V source is delivering 3000 W.

[b] $p_{\text{depsource}} = -v_\phi i_2 = -(40)(20) = -800$ W. Thus, the dependent source is delivering 800 W.

[c] From Assessment Problem 4.21(b), the power delivered to the load resistor is 1200 W, so the load power is $(1200/3800)100 = 31.58\%$ of the combined power generated by the 100 V source and the dependent source.

Problems

P 4.1



[a] 11 branches, 8 branches with resistors, 2 branches with independent sources, 1 branch with a dependent source

[b] The current is unknown in every branch except the one containing the 8 A current source, so the current is unknown in 10 branches.

[c] 9 essential branches – $R_4 - R_5$ forms an essential branch as does $R_8 - 10$ V. The remaining seven branches are essential branches that contain a single element.

[d] The current is known only in the essential branch containing the current source, and is unknown in the remaining 8 essential branches

[e] From the figure there are 6 nodes – three identified by rectangular boxes, two identified with single black dots, and one identified by a triangle.

[f] There are 4 essential nodes, three identified with rectangular boxes and one identified with a triangle

[g] A mesh is like a window pane, and as can be seen from the figure there are 6 window panes or meshes.

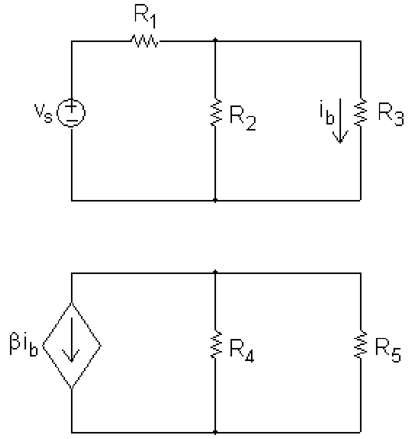
P 4.2 **[a]** From Problem 4.1(d) there are 8 essential branches where the current is unknown, so we need 8 simultaneous equations to describe the circuit.

[b] From Problem 4.1(f), there are 4 essential nodes, so we can apply KCL at $(4 - 1) = 3$ of these essential nodes. There would also be a dependent source constraint equation.

[c] The remaining 4 equations needed to describe the circuit will be derived from KVL equations.

[**d**] We must avoid using the topmost mesh and the leftmost mesh. Each of these meshes contains a current source, and we have no way of determining the voltage drop across a current source.

P 4.3



[**a**] As can be seen from the figure, the circuit has 2 separate parts.

[**b**] There are 5 nodes – the four black dots and the node betweem the voltage source and the resistor R_1.

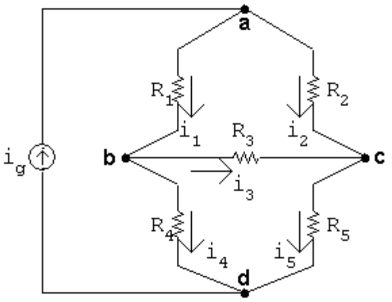
[**c**] There are 7 branches, each containing one of the seven circuit components.

[**d**] When a conductor joins the lower nodes of the two separate parts, there is now only a single part in the circuit. There would now be 4 nodes, because the two lower nodes are now joined as a single node. The number of branches remains at 7, where each branch contains one of the seven individual circuit components.

P 4.4 [**a**] There are six circuit components, five resistors and the current source. Since the current is known only in the current source, it is unknown in the five resistors. Therefore there are **five** unknown currents.

[**b**] There are four essential nodes in this circuit, identified by the dark black dots in Fig. P4.4. At three of these nodes you can write KCL equations that will be independent of one another. A KCL equation at the fourth node would be dependent on the first three. Therefore there are **three** independent KCL equations.

[**c**]



Sum the currents at any three of the four essential nodes a, b, c, and d. Using nodes a, b, and c we get

$-i_g + i_1 + i_2 = 0$

$-i_1 + i_4 + i_3 = 0$

$i_5 - i_2 - i_3 = 0$

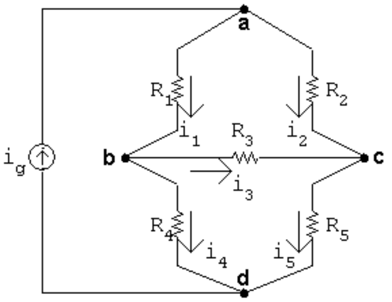
[**d**] There are three meshes in this circuit: one on the left with the components i_g, R_1, and R_4; one on the top right with components R_1, R_2, and R_3; and one on the bottom right with components R_3, R_4, and R_5. We cannot write a KVL equation for the left mesh because we don't know the voltage drop across the current source. Therefore, we can write KVL equations for the two meshes on the right, giving a total of **two** independent KVL equations.

[**e**] Sum the voltages around two independent closed paths, avoiding a path that contains the independent current source since the voltage across the current source is not known. Using the upper and lower meshes formed by the five resistors gives

$R_1 i_1 + R_3 i_3 - R_2 i_2 = 0$

$R_3 i_3 + R_5 i_5 - R_4 i_4 = 0$

P 4.5



[**a**] At node a: $-i_g + i_1 + i_2 = 0$

At node b: $-i_1 + i_3 + i_4 = 0$

At node c: $-i_2 - i_3 + i_5 = 0$

At node d: $i_g - i_4 - i_5 = 0$

[**b**] There are many possible solutions. For example, solve the equations at nodes a and d for i_g:

$$i_g = i_4 + i_5 \qquad i_g = i_1 + i_2 \qquad \text{so} \qquad i_1 + i_2 = i_4 + i_5$$

Solve this expression for i_1:

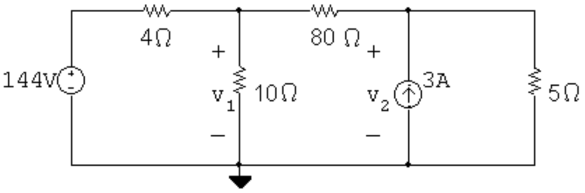
$$i_1 = i_4 + i_5 - i_2$$

Substitute this expression for i_1 into the equation for node b:

$$-(i_4 + i_5 - i_2) + i_3 + i_4 = 0 \qquad \text{so} \qquad -i_2 - i_3 + i_5 = 0$$

The result above is the equation at node c.

P 4.6

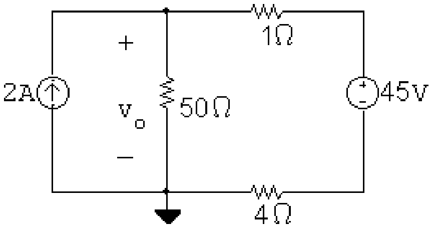


$$\frac{v_1 - 144}{4} + \frac{v_1}{10} + \frac{v_1 - v_2}{80} = 0 \qquad \text{so} \qquad 29v_1 - v_2 = 2880$$

$$-3 + \frac{v_2 - v_1}{80} + \frac{v_2}{5} = 0 \qquad \text{so} \qquad -v_1 + 17v_2 = 240$$

Solving, $v_1 = 100$ V; $\qquad v_2 = 20$ V

P 4.7



$$-2 + \frac{v_o}{50} + \frac{v_o - 45}{1+4} = 0$$

$v_o = 50$ V

$p_{2\text{A}} = -(50)(2) = -100$ W (delivering)

The 2 A source extracts −100 W from the circuit, because it delivers 100 W to the circuit.

P 4.8 $-6 + \frac{v_1}{40} + \frac{v_1 - v_2}{8} = 0$

$$\frac{v_2 - v_1}{8} + \frac{v_2}{80} + \frac{v_2}{120} + 1 = 0$$

Solving, $v_1 = 120$ V; $v_2 = 96$ V
CHECK:

$$p_{40\Omega} = \frac{(120)^2}{40} = 360 \text{ W}$$

$$p_{8\Omega} = \frac{(120-96)^2}{8} = 72 \text{ W}$$

$$p_{80\Omega} = \frac{(96)^2}{80} = 115.2 \text{ W}$$

$$p_{120\Omega} = \frac{(96)^2}{120} = 76.8 \text{ W}$$

$$p_{6\text{A}} = -(6)(120) = -720 \text{ W}$$

$$p_{1\text{A}} = (1)(96) = 96 \text{ W}$$

$$\sum p_{\text{abs}} = 360 + 72 + 115.2 + 76.8 + 96 = 720 \text{ W}$$

$$\sum p_{\text{dev}} = 720 \text{ W} \quad \text{(CHECKS)}$$

P 4.9 Use the lower terminal of the 25 Ω resistor as the reference node.

$$\frac{v_o - 24}{20 + 80} + \frac{v_o}{25} + 0.04 = 0$$

Solving, $v_o = 4$ V

P 4.10 [**a**] From the solution to Problem 4.9 we know $v_o = 4$ V, therefore

$$p_{40\text{mA}} = 0.04 v_o = 0.16 \text{ W}$$

$$\therefore \; p_{40\text{mA}} \text{ (developed) } = -160 \text{ mW}$$

[**b**] The current into the negative terminal of the 24 V source is

$$i_g = \frac{24 - 4}{20 + 80} = 0.2 \text{ A}$$

$$p_{24\text{V}} = -24(0.2) = -4.8 \text{ W}$$

$$\therefore \; p_{24\text{V}} \text{ (developed) } = 4800 \text{ mW}$$

[c] $p_{20\Omega} = (0.2)^2(20) = 800$ mW

$p_{80\Omega} = (0.2)^2(80) = 3200$ mW

$p_{25\Omega} = (4)^2/25 = 640$ mW

$\sum p_{\text{dev}} = 4800$ mW

$\sum p_{\text{dis}} = 160 + 800 + 3200 + 640 = 4800$ mW

P 4.11 **[a]** $\dfrac{v_0 - 24}{20 + 80} + \dfrac{v_o}{25} + 0.04 = 0; \quad v_o = 4$ V

[b] Let v_x = voltage drop across 40 mA source

$v_x = v_o - (50)(0.04) = 2$ V

$p_{40\text{mA}} = (2)(0.04) = 80$ mW so $p_{40\text{mA}}$ (developed) $= -80$ mW

[c] Let i_g = be the current into the positive terminal of the 24 V source

$i_g = (4 - 24)/100 = -0.2$ A

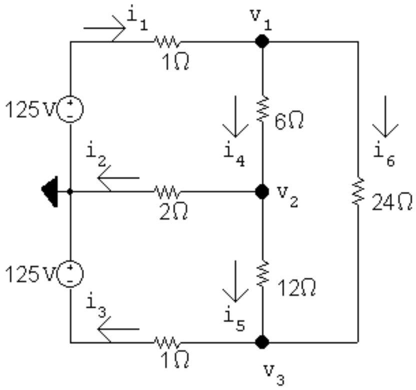
$p_{24\text{V}} = (-0.2)(24) = -4800$ mW so $p_{24\text{V}}$ (developed) $= 4800$ mW

[d] $\sum p_{\text{dis}} = (0.2)^2(20) + (0.2)^2(80) + (4)^2/25 + (0.04)^2(50) + 0.08$

$= 4800$ mW

[e] v_o is independent of any finite resistance connected in series with the 40 mA current source

P 4.12 **[a]**



$$\begin{aligned}
\frac{v_1 - 125}{1} + \frac{v_1 - v_2}{6} + \frac{v_1 - v_3}{24} &= 0 \\
\frac{v_2 - v_1}{6} + \frac{v_2}{2} + \frac{v_2 - v_3}{12} &= 0 \\
\frac{v_3 + 125}{1} + \frac{v_3 - v_2}{12} + \frac{v_3 - v_1}{24} &= 0
\end{aligned}$$

In standard form:

$$\begin{aligned}
v_1\left(\frac{1}{1}+\frac{1}{6}+\frac{1}{24}\right)+v_2\left(-\frac{1}{6}\right)+v_3\left(-\frac{1}{24}\right) &= 125\\
v_1\left(-\frac{1}{6}\right)+v_2\left(\frac{1}{6}+\frac{1}{2}+\frac{1}{12}\right)+v_3\left(-\frac{1}{12}\right) &= 0\\
v_1\left(-\frac{1}{24}\right)+v_2\left(-\frac{1}{12}\right)+v_3\left(\frac{1}{1}+\frac{1}{12}+\frac{1}{24}\right) &= -125
\end{aligned}$$

Solving, $v_1 = 101.24$ V; $v_2 = 10.66$ V; $v_3 = -106.57$ V

Thus, $i_1 = \frac{125 - v_1}{1} = 23.76$ A $\quad i_4 = \frac{v_1 - v_2}{6} = 15.10$ A

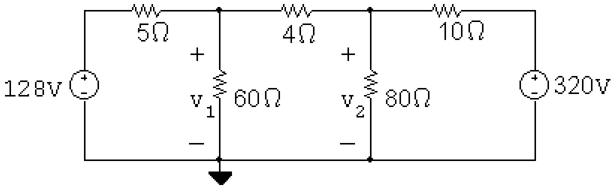
$i_2 = \frac{v_2}{2} = 5.33$ A $\quad i_5 = \frac{v_2 - v_3}{12} = 9.77$ A

$i_3 = \frac{v_3 + 125}{1} = 18.43$ A $\quad i_6 = \frac{v_1 - v_3}{24} = 8.66$ A

[b] $\sum P_{\text{dev}} = 125i_1 + 125i_3 = 5273.09$ W

$\sum P_{\text{dis}} = i_1^2(1) + i_2^2(2) + i_3^2(1) + i_4^2(6) + i_5^2(12) + i_6^2(24) = 5273.09$ W

P 4.13 **[a]**



$$\begin{aligned}
\frac{v_1-128}{5}+\frac{v_1}{60}+\frac{v_1-v_2}{4} &= 0\\
\frac{v_2-v_1}{4}+\frac{v_2}{80}+\frac{v_2-320}{10} &= 0
\end{aligned}$$

In standard form,

$$\begin{aligned}
v_1\left(\frac{1}{5}+\frac{1}{60}+\frac{1}{4}\right)+v_2\left(-\frac{1}{4}\right) &= \frac{128}{5}\\
v_1\left(-\frac{1}{4}\right)+v_2\left(\frac{1}{4}+\frac{1}{80}+\frac{1}{10}\right) &= \frac{320}{10}
\end{aligned}$$

Solving, $v_1 = 162$ V; $v_2 = 200$ V

$$i_a = \frac{128-162}{5} = -6.8 \text{ A}$$

$$i_b = \frac{162}{60} = 2.7 \text{ A}$$

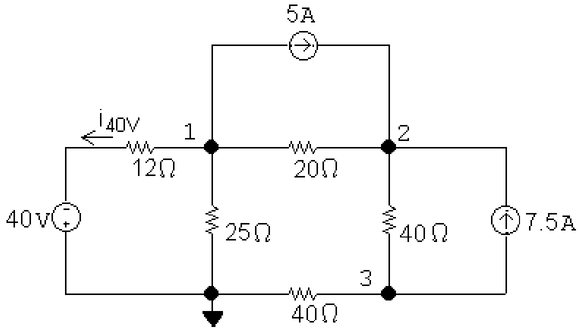
$$i_c = \frac{162-200}{4} = -9.5 \text{ A}$$

$$i_d = \frac{200}{80} = 2.5 \text{ A}$$

$$i_e = \frac{200 - 320}{10} = -12 \text{ A}$$

[b] $p_{128V} = -(128)(-6.8) = 870.4$ W (abs)
$p_{320V} = (320)(-12) = -3840$ W (dev)
Therefore, the total power developed is 3840 W.

P 4.14



$$\frac{v_1 + 40}{12} + \frac{v_1}{25} + \frac{v_1 - v_2}{20} + 5 = 0$$

$$\left[\frac{v_2 - v_1}{20}\right] - 5 + \frac{v_2 - v_1}{40} + -7.5 = 0$$

$$\frac{v_3}{40} + \frac{v_3 - v_2}{40} + 7.5 = 0$$

Solving, $v_1 = -10$ V; $v_2 = 132$ V; $v_3 = -84$ V; $i_{40\text{V}} = \dfrac{-10 + 40}{12} = 2.5$ A

$p_{5\text{A}} = 5(v_1 - v_2) = 5(-10 - 132) = -710$ W (del)

$p_{7.5\text{A}} = (-84 - 132)(7.5) = -1620$ W (del)

$p_{40\text{V}} = -(40)(2.5) = -100$ W (del)

$p_{12\Omega} = (2.5)^2(12) = 75$ W

$$p_{25\Omega} = \frac{v_1^2}{25} = \frac{10^2}{25} = 4 \text{ W}$$

$$p_{20\Omega} = \frac{(v_1 - v_2)^2}{20} = \frac{142^2}{20} = 1008.2 \text{ W}$$

$$p_{40\Omega}(\text{lower}) = \frac{(v_3)^2}{40} = \frac{84^2}{40} = 176.4 \text{ W}$$

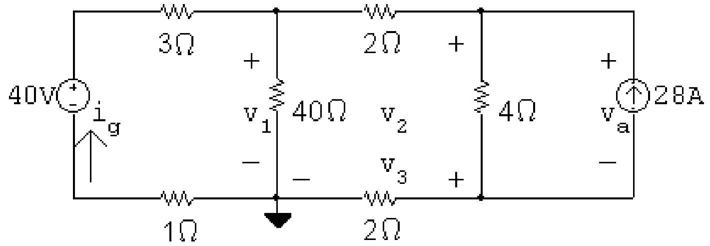
$$p_{40\Omega}(\text{right}) = \frac{(v_2 - v_3)^2}{40} = \frac{216^2}{40} = 1166.4 \text{ W}$$

$$\sum p_{\text{diss}} = 75 + 4 + 1008.2 + 176.4 + 1166.4 = 2430 \text{ W}$$

$$\sum p_{\text{dev}} = 710 + 1620 + 100 = 2430 \text{ W} \qquad \text{(CHECKS)}$$

The total power dissipated in the circuit is 2430 W.

P 4.15 [**a**]



$$\frac{v_1}{40} + \frac{v_1 - 40}{4} + \frac{v_1 - v_2}{2} = 0 \qquad \text{so} \qquad 31v_1 - 20v_2 + 0v_3 = 400$$

$$\frac{v_2 - v_1}{2} + \frac{v_2 - v_3}{4} - 28 = 0 \qquad \text{so} \qquad -2v_1 + 3v_2 - v_3 = 112$$

$$\frac{v_3}{2} + \frac{v_3 - v_2}{4} + 28 = 0 \qquad \text{so} \qquad 0v_1 - v_2 + 3v_3 = -112$$

Solving, $v_1 = 60$ V; $v_2 = 73$ V; $v_3 = -13$ V,

[**b**] $i_g = \dfrac{40 - 60}{4} = -5$ A

$p_g = (40)(-5) = -200$ W

Thus the 40 V source delivers 200 W of power.

P 4.16 [**a**] $\dfrac{v_o - v_1}{R} + \dfrac{v_o - v_2}{R} + \dfrac{v_o - v_3}{R} + \cdots + \dfrac{v_o - v_n}{R} = 0$

$\therefore \quad nv_o = v_1 + v_2 + v_3 + \cdots + v_n$

$\therefore \quad v_o = \dfrac{1}{n}[v_1 + v_2 + v_3 + \cdots + v_n] = \dfrac{1}{n}\sum_{k=1}^{n} v_k$

[**b**] $v_o = \dfrac{1}{3}(100 + 80 - 60) = 40$ V

P 4.17 **[a]** $-25+\dfrac{v_1}{40}+\dfrac{v_1}{160}+\dfrac{v_1-v_2}{10}=0 \quad \text{so} \quad 21v_1-16v_2+0i_\Delta=4000$

$$\frac{v_2-v_1}{10}+\frac{v_2}{20}+\frac{v_2-84i_\Delta}{8}=0 \quad \text{so} \quad -16v_1+44v_2-1680i_\Delta=0$$

$$i_\Delta=\frac{v_1}{160} \quad \text{so} \quad v_1+(0)v_2-160i_\Delta=0$$

Solving, $v_1=352$ V; $v_2=212$ V; $i_\Delta=2.2$ A;

$$i_{\text{depsource}}=\frac{212-84(2.2)}{8}=3.4 \text{ A}$$

$$p_{84i_\Delta}=84(2.2)(3.4)=628.32 \text{ W(abs)}$$

$$p_{25\text{A}}=-25(352)=-8800 \text{ W(del)}$$

$$\therefore \quad p_{\text{dev}}=8800 \text{ W}$$

[b] $\sum p_{\text{abs}}=\dfrac{(352)^2}{40}+\dfrac{(352)^2}{160}+\dfrac{(352-212)^2}{10}+\dfrac{(212)^2}{20}$

$$+(3.4)^2(8)+628.32=8800 \text{ W}$$

$$\therefore \quad \sum p_{\text{dev}}=\sum p_{\text{abs}}=8800 \text{ W}$$

P 4.18 $-3+\dfrac{v_o}{200}+\dfrac{v_o+5i_\Delta}{10}+\dfrac{v_o-80}{20}=0; \quad i_\Delta=\dfrac{v_o-80}{20}$

[a] Solving, $v_o=50$ V

[b] $i_{\text{ds}}=\dfrac{v_o+5i_\Delta}{10}$

$$i_\Delta=(50-80)/20=-1.5 \text{ A}$$

$$\therefore \quad i_{\text{ds}}=4.25 \text{ A}; \quad 5i_\Delta=-7.5 \text{ V}: \quad p_{\text{ds}}=(-5i_\Delta)(i_{\text{ds}})=31.875 \text{ W}$$

[c] $p_{3\text{A}}=-3v_o=-3(50)=-150 \text{ W} \quad \text{(del)}$

$$p_{80\text{V}}=80i_\Delta=80(-1.5)=-120 \text{ W} \quad \text{(del)}$$

$$\sum p_{\text{del}}=150+120=270 \text{ W}$$

CHECK:

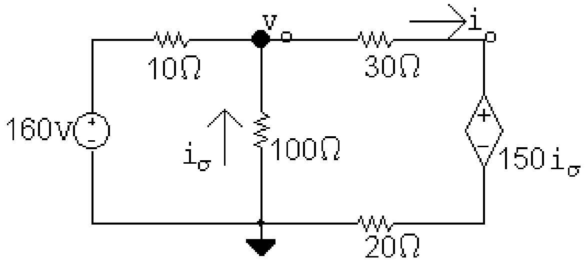
$$p_{200\Omega}=2500/200=12.5 \text{ W}$$

$$p_{20\Omega}=(80-50)^2/20=900/20=45 \text{ W}$$

$$p_{10\Omega}=(4.25)^2(10)=180.625 \text{ W}$$

$$\sum p_{\text{diss}}=31.875+180.625+12.5+45=270 \text{ W}$$

P 4.19



$$\frac{v_o - 160}{10} + \frac{v_o}{100} + \frac{v_o - 150i_\sigma}{50} = 0; \quad i_\sigma = -\frac{v_o}{100}$$

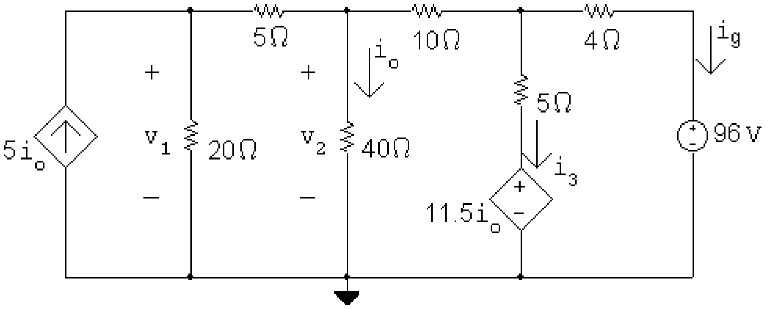
Solving, $v_o = 100$ V; $\quad i_\sigma = -1$ A

$$i_o = \frac{100 - (150)(-1)}{50} = 5 \text{ A}$$

$$p_{150i_\sigma} = 150 i_\sigma i_o = -750 \text{ W}$$

∴ The dependent voltage source delivers 750 W to the circuit.

P 4.20 [**a**]



$$i_o = \frac{v_2}{40}$$

$$-5i_o + \frac{v_1}{20} + \frac{v_1 - v_2}{5} = 0 \qquad \text{so} \qquad 10v_1 - 13v_2 + 0v_3 = 0$$

$$\frac{v_2 - v_1}{5} + \frac{v_2}{40} + \frac{v_2 - v_3}{10} \qquad \text{so} \qquad -8v_1 + 13v_2 - 4v_3 = 0$$

$$\frac{v_3 - v_2}{10} + \frac{v_3 - 11.5i_o}{5} + \frac{v_3 - 96}{4} = 0 \qquad \text{so} \qquad 0v_1 - 63v_2 + 220v_3 = 9600$$

Solving, $v_1 = 156$ V; $\quad v_2 = 120$ V; $\quad v_3 = 78$ V

[**b**] $i_o = \dfrac{v_2}{40} = \dfrac{120}{40} = 3$ A

$$i_3 = \frac{v_3 - 11.5i_o}{5} = \frac{78 - 11.5(3)}{5} = 8.7 \text{ A}$$

$$i_g = \frac{78 - 96}{4} = -4.5 \text{ A}$$

$$p_{5i_o} = -5i_o v_1 = -5(3)(156) = -2340 \text{ W(dev)}$$

$$p_{11.5i_o} = 11.5 i_o i_3 = 11.5(3)(8.7) = 300.15 \text{ W(abs)}$$

$$p_{96V} = 96(-4.5) = -432 \text{ W(dev)}$$

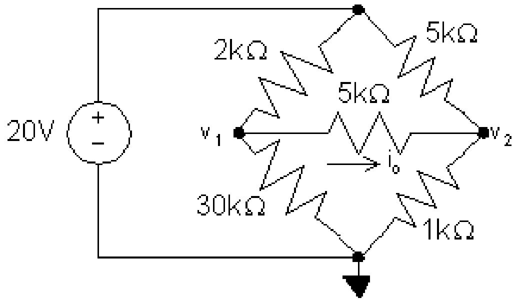
$$\sum p_{\text{dev}} = 2340 + 432 = 2772 \text{ W}$$

CHECK

$$\sum p_{\text{dis}} = \frac{156^2}{20} + \frac{(156-120)^2}{5} + \frac{120^2}{40} + \frac{(120-78)^2}{50}$$
$$+(8.7)^2(5) + (4.5)^2(4) + 300.15 = 2772 \text{ W}$$

$$\therefore \quad \sum p_{\text{dev}} = \sum p_{\text{dis}} = 2772 \text{ W}$$

P 4.21

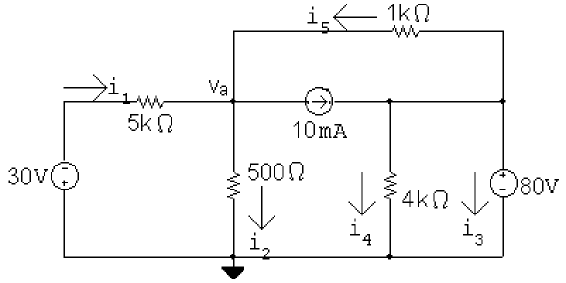


$$\frac{v_1}{30{,}000} + \frac{v_1 - v_2}{5000} + \frac{v_1 - 20}{2000} = 0 \qquad \text{so} \qquad 22v_1 - 6v_2 = 300$$
$$\frac{v_2}{1000} + \frac{v_2 - v_1}{5000} + \frac{v_2 - 20}{5000} = 0 \qquad \text{so} \qquad -v_1 + 7v_2 = 20$$

Solving, $v_1 = 15$ V; $v_2 = 5$ V

Thus, $i_o = \dfrac{v_1 - v_2}{5000} = 2$ mA

P 4.22 **[a]**



There is only one node voltage equation:

$$\frac{v_\mathrm{a}+30}{5000}+\frac{v_\mathrm{a}}{500}+\frac{v_\mathrm{a}-80}{1000}+0.01=0$$

Solving,

$v_\mathrm{a}+30+10v_\mathrm{a}+5v_\mathrm{a}-400+50=0 \qquad$ so $\qquad 16v_\mathrm{a}=320$

$\therefore \qquad v_\mathrm{a}=20$ V

Calculate the currents:

$$\begin{aligned} i_1 &= (-30-20)/5000=-10\text{ mA}\\ i_2 &= 20/500=40\text{ mA}\\ i_4 &= 80/4000=20\text{ mA}\\ i_5 &= (80-20)/1000=60\text{ mA}\end{aligned}$$

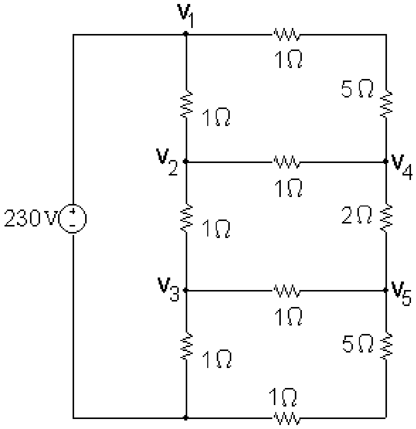
$i_3+i_4+i_5-10\text{ mA}=0$ so $i_3=0.01-0.02-0.06=-0.07=-70$ mA

[b]
$$\begin{aligned} p_{30\mathrm{V}} &= (30)(-0.01)=-0.3\text{ W}\\ p_{10\mathrm{mA}} &= (20-80)(0.01)=-0.6\text{ W}\\ p_{80\mathrm{V}} &= (80)(-0.07)=-5.6\text{ W}\\ p_{5\mathrm{k}} &= (-0.01)^2(5000)=0.5\text{ W}\\ p_{500\Omega} &= (0.04)^2(500)=0.8\text{ W}\\ p_{1\mathrm{k}} &= (80-20)^2/(1000)=3.6\text{ W}\\ p_{4\mathrm{k}} &= (80)^2/(4000)=1.6\text{ W}\end{aligned}$$

$\sum p_\mathrm{abs}=0.5+0.8+3.6+1.6=6.5$ W

$\sum p_\mathrm{del}=0.3+0.6+5.6=6.5$ W (checks!)

P 4.23 **[a]**



$$\frac{v_2-230}{1}+\frac{v_2-v_4}{1}+\frac{v_2-v_3}{1}=0 \qquad \text{so} \qquad 3v_2-1v_3-1v_4+0v_5=230$$

$$\frac{v_3-v_2}{1}+\frac{v_3}{1}+\frac{v_3-v_5}{1}=0 \qquad \text{so} \qquad -1v_2+3v_3+0v_4-1v_5=0$$

$$\frac{v_4 - v_2}{1} + \frac{v_4 - 230}{6} + \frac{v_4 - v_5}{2} = 0 \qquad \text{so} \qquad -12v_2 + 0v_3 + 20v_4 - 6v_5 = 460$$

$$\frac{v_5 - v_3}{1} + \frac{v_5}{6} + \frac{v_5 - v_4}{2} = 0 \qquad \text{so} \qquad 0v_2 - 12v_3 - 6v_4 + 20v_5 = 0$$

Solving, $v_2 = 150$ V; $v_3 = 80$ V; $v_4 = 140$ V; $v_5 = 90$ V

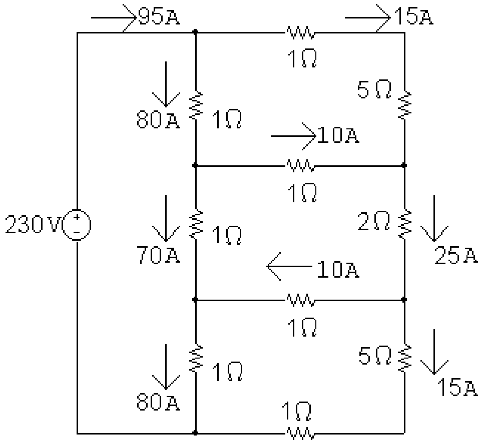
$$i_{2\Omega} = \frac{v_4 - v_5}{2} = \frac{140 - 90}{2} = 25 \text{ A}$$

$$p_{2\Omega} = (25)^2(2) = 1250 \text{ W}$$

[b]
$$\begin{aligned} i_{230\text{V}} &= \frac{v_1 - v_2}{1} + \frac{v_1 - v_4}{6} \\ &= \frac{230 - 150}{1} + \frac{230 - 140}{6} = 80 + 15 = 95 \text{ A} \end{aligned}$$

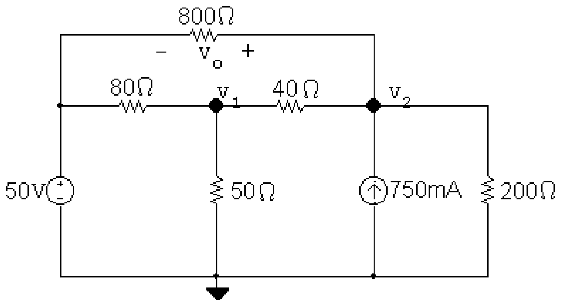
$$p_{230\text{V}} = (230)(95) = 21{,}850 \text{ W}$$

Check:



$$\begin{aligned} \sum P_{\text{dis}} &= (80)^2(1) + (70)^2(1) + (80)^2(1) + (15)^2(6) + (10)^2(1) \\ &\quad + (10)^2(1) + (25)^2(2) + (15)^2(6) = 21{,}850 \text{ W} \end{aligned}$$

P 4.24



The two node voltage equations are:

$$\begin{aligned}\frac{v_1 - 50}{80} + \frac{v_1}{50} + \frac{v_1 - v_2}{40} &= 0\\ \frac{v_2 - v_1}{40} - 0.75 + \frac{v_2}{200} + \frac{v_2 - 50}{800} &= 0\end{aligned}$$

Place these equations in standard form:

$$\begin{aligned} v_1\left(\frac{1}{80}+\frac{1}{50}+\frac{1}{40}\right) &+ v_2\left(-\frac{1}{40}\right) &= \frac{50}{80}\\ v_1\left(-\frac{1}{40}\right) &+ v_2\left(\frac{1}{40}+\frac{1}{200}+\frac{1}{800}\right) &= 0.75+\frac{50}{800}\end{aligned}$$

Solving, $v_1 = 34$ V; $v_2 = 53.2$ V.

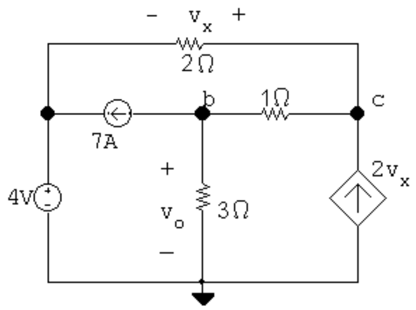
Thus, $v_o = v_2 - 50 = 53.2 - 50 = 3.2$ V.

POWER CHECK:

$$\begin{aligned} i_g &= (50-34)/80 + (50-53.2)/800 = 196\text{ m A}\\ p_{50\text{V}} &= -(50)(0.196) = -9.8\text{ W}\\ p_{80\Omega} &= (50-34)^2/80 = 3.2\text{ W}\\ p_{800\Omega} &= (50-53.2)^2/800 = 12.8\text{ m W}\\ p_{40\Omega} &= (53.2-34)^2/40 = 9.216\text{ W}\\ p_{50\Omega} &= 34^2/50 = 23.12\text{ W}\\ p_{200\Omega} &= 53.2^2/200 = 14.1512\text{ W}\\ p_{0.75\text{A}} &= -(53.2)(0.75) = -39.9\text{ W}\end{aligned}$$

$\sum p_{\text{abs}} = 3.2 + .0128 + 9.216 + 23.12 + 14.1512 = 49.7\text{ W} = \sum p_{\text{del}} =$
$9.8 + 39.9 = 49.7$

P 4.25



The two node voltage equations are:

$$\begin{aligned}7 + \frac{v_\text{b}}{3} + \frac{v_\text{b} - v_\text{c}}{1} &= 0\\ -2v_x + \frac{v_\text{c} - v_\text{b}}{1} + \frac{v_\text{c} - 4}{2} &= 0\end{aligned}$$

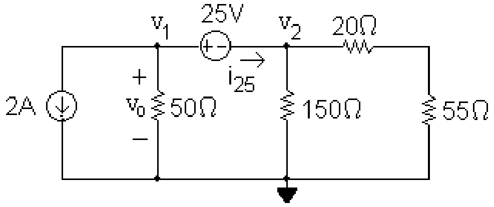
The constraint equation for the dependent source is:
$v_x = v_c - 4$

Place these equations in standard form:

$$\begin{array}{lclclcl} v_b\left(\frac{1}{3}+1\right) & + & v_c(-1) & + & v_x(0) & = & -7 \\ v_b(-1) & + & v_c\left(1+\frac{1}{2}\right) & + & v_x(-2) & = & \frac{4}{2} \\ v_b(0) & + & v_c(1) & + & v_x(-1) & = & 4 \end{array}$$

Solving, $v_c = 9\ \text{V}, v_x = 5\ \text{V},$ and $v_o = v_b = 1.5\ \text{V}$

P 4.26 **[a]**



This circuit has a supernode includes the nodes v_1, v_2 and the 25 V source. The supernode equation is

$$2 + \frac{v_1}{50} + \frac{v_2}{150} + \frac{v_2}{75} = 0$$

The supernode constraint equation is

$$v_1 - v_2 = 25$$

Place these two equations in standard form:

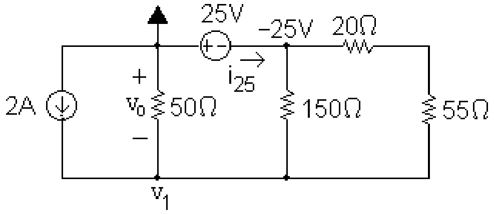
$$\begin{array}{lclcl} v_1\left(\frac{1}{50}\right) & + & v_2\left(\frac{1}{150}+\frac{1}{75}\right) & = & -2 \\ v_1(1) & + & v_2(-1) & = & 25 \end{array}$$

Solving, $v_1 = -37.5$ V and $v_2 = -62.5$ V, so $v_o = v_1 = -37.5$ V.

$$p_{2A} = (2)v_o = (2)(-37.5) = -75\ \ \text{W}$$

The 2 A source delivers 75 W.

[b]



This circuit now has only one non-reference essential node where the voltage is not known – note that it is not a supernode. The KCL equation at v_1 is

$$-2+\frac{v_1}{50}+\frac{v_1+25}{150}+\frac{v_1+25}{75}=0$$

Solving, $v_1 = 37.5$ V so $v_0 = -v_1 = -37.5$ V.

$$p_{2\text{A}} = (2)v_o = (2)(-37.5) = -75 \ \text{W}$$

The 2 A source delivers 75 W.

[**c**] The choice of a reference node in part (b) resulted in one simple KCL equation, while the choice of a reference node in part (a) resulted in a supernode KCL equation and a second supernode constraint equation. Both methods give the same result but the choice of reference node in part (b) yielded fewer equations to solve, so is the preferred method.

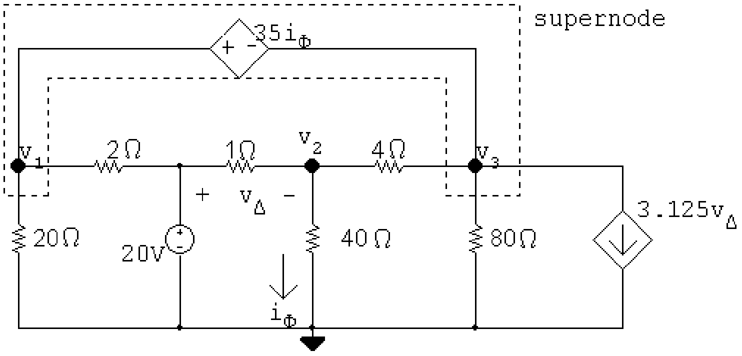
P 4.27 Place $5v_\Delta$ inside a supernode and use the lower node as a reference. Then

$$\frac{v_\Delta-15}{10}+\frac{v_\Delta}{2}+\frac{v_\Delta-5v_\Delta}{20}+\frac{v_\Delta-5v_\Delta}{40}=0$$

$$12v_\Delta = 60; \qquad v_\Delta = 5 \ \text{V}$$

$$v_o = v_\Delta - 5v_\Delta = -4(5) = -20 \ \text{V}$$

P 4.28



Node equations:

$$\frac{v_1}{20}+\frac{v_1-20}{2}+\frac{v_3-v_2}{4}+\frac{v_3}{80}+3.125v_\Delta=0$$

$$\frac{v_2}{40}+\frac{v_2-v_3}{4}+\frac{v_2-20}{1}=0$$

Constraint equations:

$$v_\Delta = 20 - v_2$$

$v_1 - 35i_\phi = v_3$

$i_\phi = v_2/40$

Solving, $v_1 = -20.25$ V; $v_2 = 10$ V; $v_3 = -29$ V

Let i_g be the current delivered by the 20 V source, then

$$i_g = \frac{20 - (20.25)}{2} + \frac{20 - 10}{1} = 30.125 \text{ A}$$

$$p_g \text{ (delivered) } = 20(30.125) = 602.5 \text{ W}$$

P 4.29 For the given values of v_3 and v_4:

$$v_\Delta = 120 - v_3 = 120 - 108 = 12 \text{ V}$$

$$i_\phi = \frac{v_4 - v_3}{8} = \frac{81.6 - 108}{8} = -3.3 \text{ A}$$

$$\frac{40}{3} i_\phi = -44 \text{ V}$$

$$v_1 = v_4 + \frac{40}{3} i_\phi = 81.6 - 44 = 37.6 \text{ V}$$

Let i_a be the current from right to left through the dependent voltage source:

$$i_a = \frac{v_1}{20} + \frac{v_1 - v_2}{4} = 1.88 - 20.6 = -18.72 \text{ A}$$

Let i_b be the current supplied by the 120 V source:

$$i_b = \frac{120 - 37.6}{4} + \frac{120 - 108}{2} = 20.6 + 6 = 26.6 \text{ A}$$

Then

$$p_{120\text{V}} = -(120)(26.6) = -3192 \text{ W}$$

$$p_{\text{CCVS}} = [(40/3)(-3.3)](-18.72) = -823.68 \text{ W}$$

$$p_{\text{VCVS}} = (81.6)[1.75(12)] = 1713.6 \text{ W}$$

$$\therefore \sum p_{\text{dev}} = 3192 + 823.68 = 4015.68 \text{ W}$$

The total power dissipated by the resistors is

$$p_R = \frac{(37.6)^2}{2} + \frac{(82.4)^2}{4} + \frac{(12)^2}{2} + \frac{(108)^2}{40}$$

$$= +(3.3)^2(8) + \frac{(81.6)^2}{80} = 2302.08 \text{ W}$$

$$\therefore \quad \sum p_{\text{diss}} = 2302.08 + 1713.6 = 4015.68 \text{ W}$$

Thus, $\sum p_{\text{dev}} = \sum p_{\text{diss}}$; Agree with analyst

P 4.30 From Eq. 4.16, $i_B = v_c/(1+\beta)R_E$

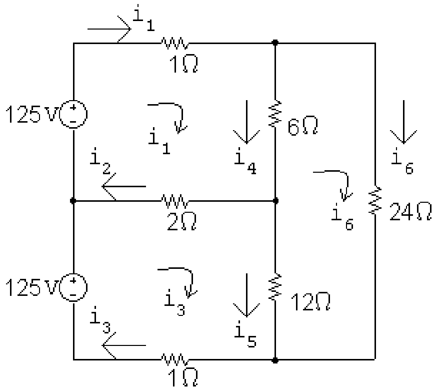
From Eq. 4.17, $i_B = (v_b - V_o)/(1+\beta)R_E$

From Eq. 4.19,

$$i_B = \frac{1}{(1+\beta)R_E}\left[\frac{V_{CC}(1+\beta)R_ER_2 + V_oR_1R_2}{R_1R_2+(1+\beta)R_E(R_1+R_2)} - V_o\right]$$

$$= \frac{V_{CC}R_2 - V_o(R_1+R_2)}{R_1R_2+(1+\beta)R_E(R_1+R_2)} = \frac{[V_{CC}R_2/(R_1+R_2)] - V_o}{[R_1R_2/(R_1+R_2)]+(1+\beta)R_E}$$

P 4.31 **[a]**



The three mesh current equations are:

$$-125 + 1i_1 + 6(i_1 - i_6) + 2(i_1 - i_3) = 0$$

$$24i_6 + 12(i_6 - i_3) + 6(i_6 - i_1) = 0$$

$$-125 + 2(i_3 - i_1) + 12(i_3 - i_6) + 1i_3 = 0$$

Place these equations in standard form:

$$\begin{aligned}
i_1(1+6+2)+i_3(-2)+i_6(-6) &= 125\\
i_1(-6)+i_3(-12)+i_6(24+12+6) &= 0\\
i_1(-2)+i_3(2+12+1)+i_6(-12) &= 125
\end{aligned}$$

Solving, $i_1 = 23.76$ A; $i_3 = 18.43$ A; $i_6 = 8.66$ A

Now calculate the remaining branch currents:

$$\begin{aligned}
i_2 &= i_1 - i_3 = 5.33 \text{ A}\\
i_4 &= i_1 - i_6 = 15.10 \text{ A}\\
i_5 &= i_3 - i_6 = 9.77 \text{ A}
\end{aligned}$$

[b] $p_{\text{sources}} = p_{\text{top}} + p_{\text{bottom}} = -(125)(23.76) - (125)(18.43)$

$$= -2970 - 2304 = -5274 \text{ W}$$

Thus, the power developed in the circuit is 5274 W.

Now calculate the power absorbed by the resistors:

$$p_{1\text{top}} = (23.76)^2(1) = 564.54 \text{ W}$$

$$p_2 = (5.33)^2(2) = 56.82 \text{ W}$$

$$p_{1\text{bot}} = (18.43)^2(1) = 339.66 \text{ W}$$

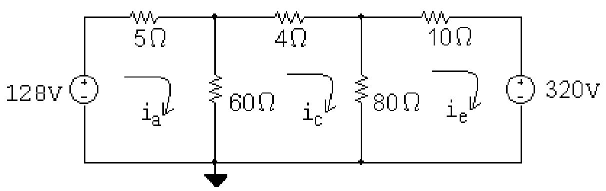
$$p_6 = (15.10)^2(6) = 1368.06 \text{ W}$$

$$p_{12} = (9.77)^2(12) = 1145.43 \text{ W}$$

$$p_{24} = (8.66)^2(24) = 1799.89 \text{ W}$$

The power absorbed by the resistors is
$564.54 + 56.82 + 339.66 + 1368.06 + 1145.43 + 1799.89 = 5274$ W so the power balances.

P 4.32 **[a]**



The three mesh current equations are:

$$\begin{aligned}-128 + 5i_{\rm a} + 60(i_{\rm a} - i_{\rm c}) &= 0\\ 4i_{\rm c} + 80(i_{\rm c} - i_{\rm e}) + 60(i_{\rm c} - i_{\rm a}) &= 0\\ 320 + 80(i_{\rm e} - i_{\rm c}) + 10i_{\rm e} &= 0\end{aligned}$$

Place these equations in standard form:

$$\begin{aligned}i_{\rm a}(5+60) + i_{\rm c}(-60) + i_{\rm e}(0) &= 128\\ i_{\rm a}(-60) + i_{\rm c}(4+80+60) + i_{\rm e}(-80) &= 0\\ i_{\rm a}(0) + i_{\rm c}(-80) + i_{\rm e}(80+10) &= -320\end{aligned}$$

Solving, $i_{\rm a} = -6.8$ A; $i_{\rm c} = -9.5$ A; $i_{\rm e} = -12$ A
Now calculate the remaining branch currents:

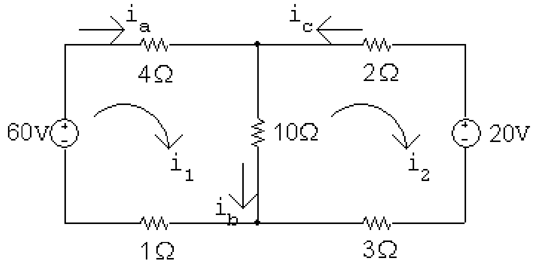
$$\begin{aligned}i_{\rm b} &= i_{\rm a} - i_{\rm c} = 2.7 \text{ A}\\ i_{\rm d} &= i_{\rm c} - i_{\rm e} = 2.5 \text{ A}\end{aligned}$$

[b] $p_{128\rm V} = -(128)i_a = -(128)(-6.8) = 870.4$ W (abs)

$p_{320\rm V} = (320)i_e = (320)(-12) = -3840$ W (dev)

Thus, the power developed in the circuit is 3840 W. Note that the resistors cannot develop power!

P 4.33 **[a]**



$60 = 15i_1 - 10i_2$

$-20 = -10i_1 + 15i_2$

Solving, $i_1 = 5.6$ A; $i_2 = 2.4$ A

$i_{\rm a} = i_1 = 5.6$ A; $i_{\rm b} = i_1 - i_2 = 3.2$ A; $i_{\rm c} = -i_2 = -2.4$ A

[b] If the polarity of the 60 V source is reversed, we have

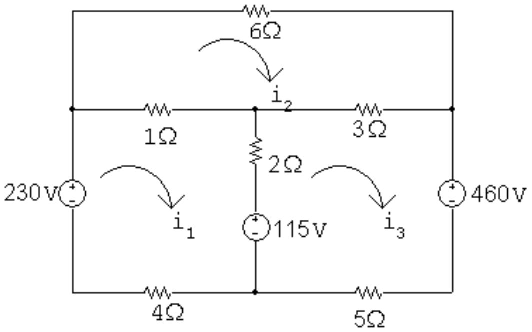
$-60 = 15i_1 - 10i_2$

$$-20 = -10i_1 + 15i_2$$

$$i_1 = -8.8 \text{ A} \quad \text{and} \quad i_2 = -7.2 \text{ A}$$

$$i_a = i_1 = -8.8 \text{ A}; \quad i_b = i_1 - i_2 = -1.6 \text{ A}; \quad i_c = -i_2 = 7.2 \text{ A}$$

P 4.34 **[a]**



$$230 - 115 = 7i_1 - 1i_2 - 2i_3$$

$$0 = -1i_1 + 10i_2 - 3i_3$$

$$115 - 460 = -2i_1 - 3i_2 + 10i_3$$

Solving, $i_1 = 4.4$ A; $\quad i_2 = -10.6$ A; $\quad i_3 = -36.8$ A

$$p_{230} = -230i_1 = -1012 \text{ W(del)}$$

$$p_{115} = 115(i_1 - i_3) = 4738 \text{ W(abs)}$$

$$p_{460} = 460i_3 = -16{,}928 \text{ W(del)}$$

$$\therefore \quad \sum p_{\text{dev}} = 17{,}940 \text{ W}$$

[b] $p_{6\Omega} = (10.6)^2(6) = 674.16$ W

$$p_{1\Omega} = (15)^2(1) = 225 \text{ W}$$

$$p_{3\Omega} = (26.2)^2(3) = 2059.32 \text{ W}$$

$$p_{2\Omega} = (41.2)^2(2) = 3394.88 \text{ W}$$

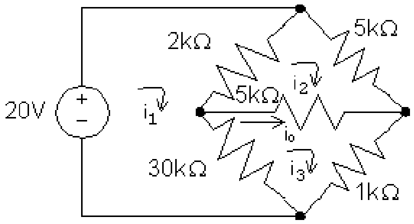
$$p_{4\Omega} = (4.4)^2(4) = 77.44 \text{ W}$$

$$p_{5\Omega} = (36.8)^2(5) = 6771.2 \text{ W}$$

$$\therefore \quad \sum p_{\text{abs}} = 4738 + 674.16 + 225 + 2059.32 + 3394.88$$

$$+77.44 + 6771.2 = 17{,}940 \text{ W}$$

P 4.35



The three mesh current equations are:

$$-20 + 2000(i_1 - i_2) + 30{,}000(i_1 - i_3) = 0$$

$$5000i_2 + 5000(i_2 - i_3) + 2000(i_2 - i_1) = 0$$

$$1000i_3 + 30{,}000(i_3 - i_1) + 5000(i_3 - i_2) = 0$$

Place these equations in standard form:

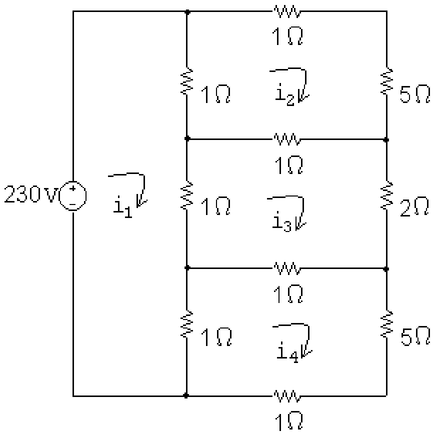
$$i_1(32{,}000) + i_2(-2000) + i_3(-30{,}000) = 20$$

$$i_1(-2000) + i_2(12{,}000) + i_3(-5000) = 0$$

$$i_1(-30{,}000) + i_2(-5000) + i_3(36{,}000) = 0$$

Solving, $i_1 = 5.5$ mA; $i_2 = 3$ mA; $i_3 = 5$ mA
Thus, $i_o = i_3 - i_2 = 2$ mA.

P 4.36 [**a**]



The four mesh current equations are:

$$\begin{aligned}
-230+1(i_1-i_2)+1(i_1-i_3)+1(i_1-i_4) &= 0\\
6i_2+1(i_2-i_3)+1(i_2-i_1) &= 0\\
2i_3+1(i_3-i_4)+1(i_3-i_1)+1(i_3-i_2) &= 0\\
6i_4+1(i_4-i_1)+1(i_4-i_3) &= 0
\end{aligned}$$

Place these equations in standard form:

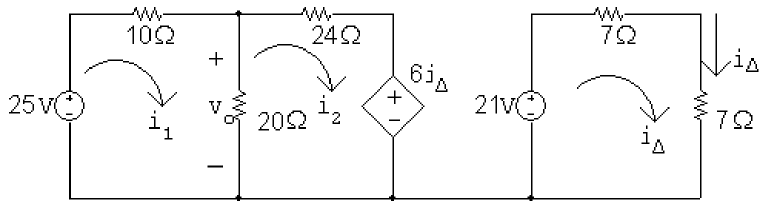
$$\begin{aligned}
i_1(3)+i_2(-1)+i_3(-1)+i_4(-1) &= 230\\
i_1(-1)+i_2(8)+i_3(-1)+i_4(0) &= 0\\
i_1(-1)+i_2(-1)+i_3(5)+i_4(-1) &= 0\\
i_1(-1)+i_2(0)+i_3(-1)+i_4(8) &= 0
\end{aligned}$$

Solving, $i_1 = 95$ A; $\quad i_2 = 15$ A; $\quad i_3 = 25$ A; $\quad i_4 = 15$ A
The power absorbed by the $5\,\Omega$ resistor is

$$p_5 = i_3^2(2) = (25)^2(2) = 1250 \text{ W}$$

[b] $p_{230} = -(230)i_1 = -(230)(95) = -21{,}850 \text{ W}$

P 4.37 **[a]**



$$25 = 30i_1 - 20i_2 + 0i_\Delta$$

$$0 = -20i_1 + 44i_2 + 6i_\Delta$$

$$21 = 0i_1 + 0i_2 + 14i_\Delta$$

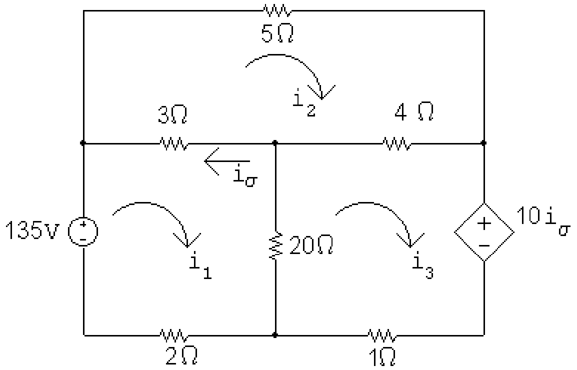
Solving, $i_1 = 1$ A; $\quad i_2 = 0.25$ A; $\quad i_\Delta = 1.5$ A

$$v_o = 20(i_1 - i_2) = 20(0.75) = 15 \text{ V}$$

[b] $p_{6i_\Delta} = 6i_\Delta i_2 = (6)(1.5)(0.25) = 2.25$ W (abs)

$$\therefore \quad p_{6i_\Delta} \text{ (deliver) } = -2.25 \text{ W}$$

P 4.38



$$-135 + 25i_1 - 3i_2 - 20i_3 + 0i_\sigma = 0$$

$$-3i_1 + 12i_2 - 4i_3 + 0i_\sigma = 0$$

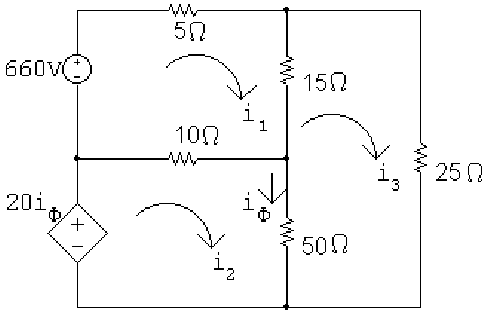
$$-20i_1 - 4i_2 + 25i_3 + 10i_\sigma = 0$$

$$1i_1 - 1i_2 + 0i_3 + 1i_\sigma = 0$$

Solving, $i_1 = 64.8$ A $\quad i_2 = 39$ A $\quad i_3 = 68.4$ A $\quad i_\sigma = -25.8$ A

$$p_{20\Omega} = (68.4 - 64.8)^2(20) = 259.2 \text{ W}$$

P 4.39



$$660 = 30i_1 - 10i_2 - 15i_3$$

$$20i_\phi = -10i_1 + 60i_2 - 50i_3$$

$$0 = -15i_1 - 50i_2 + 90i_3$$

$$i_\phi = i_2 - i_3$$

Solving, $i_1 = 42$ A; $\quad i_2 = 27$ A; $\quad i_3 = 22$ A; $\quad i_\phi = 5$ A

$$20i_\phi = 100 \text{ V}$$

$p_{20i_\phi} = -100i_2 = -100(27) = -2700$ W

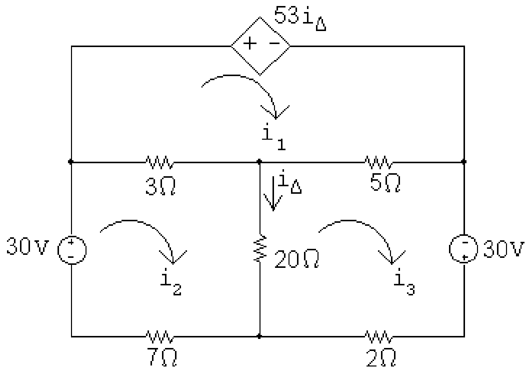
$\therefore\ \ p_{20i_\phi}$ (developed) $= 2700$ W

CHECK:

$p_{660\text{V}} = -660(42) = -27{,}720$ W (dev)

$$\begin{aligned}\therefore\ \sum P_{\text{dev}} &= 27{,}720 + 2700 = 30{,}420 \text{ W}\\ \sum P_{\text{dis}} &= (42)^2(5) + (22)^2(25) + (20)^2(15) + (5)^2(50) +\\ &\quad (15)^2(10)\\ &= 30{,}420 \text{ W}\end{aligned}$$

P 4.40



Mesh equations:

$53i_\Delta + 8i_1 - 3i_2 - 5i_3 = 0$

$0i_\Delta - 3i_1 + 30i_2 - 20i_3 = 30$

$0i_\Delta - 5i_1 - 20i_2 + 27i_3 = 30$

Constraint equations:

$i_\Delta = i_2 - i_3$

Solving, $i_1 = 110$ A; $i_2 = 52$ A; $i_3 = 60$ A; $i_\Delta = -8$ A

$p_{\text{depsource}} = 53i_\Delta i_1 = (53)(-8)(110) = -46{,}640$ W

Therefore, the dependent source is developing 46,640 W.

CHECK:

$p_{30\text{V}} = -30i_2 = -1560$ W (left source)

$p_{30\text{V}} = -30i_3 = -1800$ W (right source)

$\sum p_{\text{dev}} = 46{,}640 + 1560 + 1800 = 50$ k W

$p_{3\Omega} = (110 - 52)^2(3) = 10{,}092$ W

$p_{5\Omega} = (110 - 60)^2(5) = 12{,}500$ W

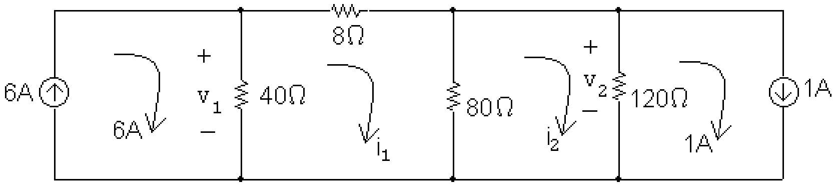
$p_{20\Omega} = (-8)^2(20) = 1280$ W

$p_{7\Omega} = (52)^2(7) = 18{,}928$ W

$p_{2\Omega} = (60)^2(2) = 7200$ W

$\sum p_{\text{diss}} = 10{,}092 + 12{,}500 + 1280 + 18{,}928 + 7200 = 50$ kW

P 4.41



Mesh equations:

$128i_1 - 80i_2 = 240$

$-80i_1 + 200i_2 = 120$

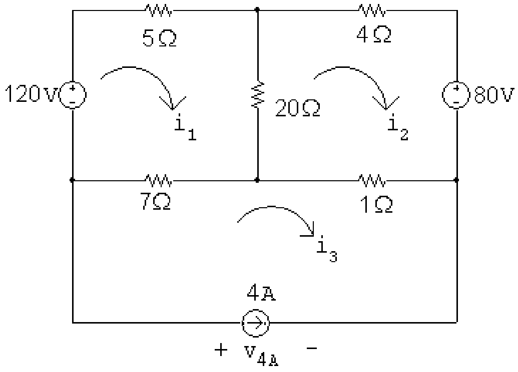
Solving,

$i_1 = 3$ A; $\qquad i_2 = 1.8$ A

Therefore,

$v_1 = 40(6 - 3) = 120$ V; $\qquad v_2 = 120(1.8 - 1) = 96$ V

P 4.42



$$120 = 32i_1 - 20i_2 - 7i_3$$

$$-80 = -20i_1 + 25i_2 - 1i_3$$

$$-4 = 0i_1 + 0i_2 + 1i_3$$

Solving, $i_1 = 1.55$ A; $i_2 = -2.12$ A; $i_3 = -4$ A

[a] $v_{4\text{A}} = 7(-4 - 1.55) + 1(-4 + 2.12)$

$= -40.73$ V

$p_{4\text{A}} = 4v_{4\text{A}} = 4(-40.73) = -162.92$ W

Therefore, the 4 A source delivers 162.92 W.

[b] $p_{120\text{V}} = -120(1.55) = -186$ W

$p_{80\text{V}} = -80(-2.12) = 169.6$ W

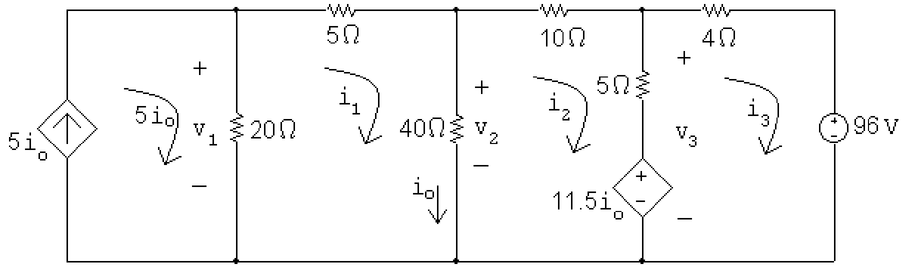
Therefore, the total power delivered is $162.92 + 186 + 169.6 = 518.52$ W

[c] $\sum p_{\text{resistors}} = (1.55)^2(5) + (2.12)^2(4) + (3.67)^2(20) + (5.55)^2(7) + (1.88)^2(1)$

$= 518.52$ W

$\sum p_{\text{abs}} = 518.52 \text{ W} = \sum p_{\text{del}}$ (CHECKS)

P 4.43 **[a]**



Mesh equations:

$$65i_1 - 40i_2 + 0i_3 - 100i_o = 0$$

$$-40i_1 + 55i_2 - 5i_3 + 11.5i_o = 0$$

$$0i_1 - 5i_2 + 9i_3 - 11.5i_o = 0$$

$$-1i_1 + 1i_2 + 0i_3 + 1i_o = 0$$

Solving,

$i_1 = 7.2$ A; $i_2 = 4.2$ A; $i_3 = -4.5$ A; $i_o = 3$ A

Therefore,

$v_1 = 20[5(3) - 7.2] = 156$ V; $v_2 = 40(7.2 - 4.2) = 120$ V

$v_3 = 5(4.2 + 4.5) + 11.5(3) = 78$ V

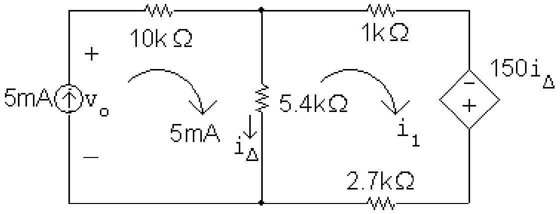
[b] $p_{5i_o} = -5i_o v_1 = -5(3)(156) = -2340$ W

$p_{11.5i_o} = 11.5i_o(i_2 - i_3) = 11.5(3)(4.2 + 4.5) = 300.15$ W

$p_{96\text{V}} = 96i_3 = 96(-4.5) = -432$ W

Thus, the total power dissipated in the circuit, which equals the total power developed in the circuit is 2340 + 432 = 2772 W.

P 4.44 **[a]**

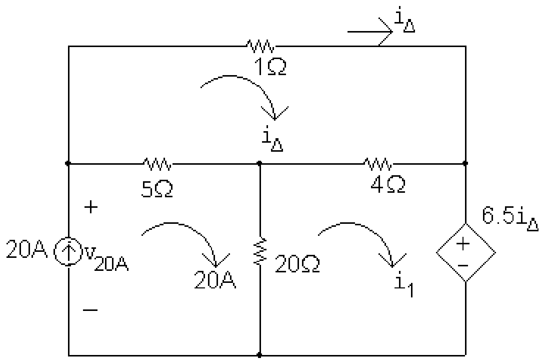


The mesh current equation for the right mesh is:
$5400(i_1 - 0.005) + 3700i_1 - 150(0.005 - i_1) = 0$
Solving, $9250i_1 = 27.75$ $\therefore$ $i_1 = 3$ mA
Then, $i_\Delta = 5 - 3 = 2$ mA

[b] $v_o = (0.005)(10{,}000) + (5400)(0.002) = 60.8$ V
$p_{5\text{mA}} = -(60.8)(0.005) = -304$ mW
Thus, the 5 mA source delivers 304 mW

[c] $p_{\text{dep source}} = -150i_\Delta i_1 = (-150)(0.002)(0.003) = -0.9$ mW
The dependent source delivers 0.9 mW.

P 4.45



Mesh equations:

$$10i_\Delta - 4i_1 = 0$$

$$-4i_\Delta + 24i_1 + 6.5i_\Delta = 400$$

Solving, $i_1 = 15$ A; $\qquad i_\Delta = 16$ A

$$v_{20\text{A}} = 1i_\Delta + 6.5i_\Delta = 7.5(16) = 120 \text{ V}$$

$$p_{20\text{A}} = -20v_{20\text{A}} = -(20)(120) = -2400 \text{ W (del)}$$

$$p_{6.5i_\Delta} = 6.5i_\Delta i_1 = (6.5)(16)(15) = 1560 \text{ W (abs)}$$

Therefore, the independent source is developing 2400 W, all other elements are absorbing power, and the total power developed is thus 2400 W.
CHECK:

$$p_{1\Omega} = (16)^2(1) = 256 \text{ W}$$

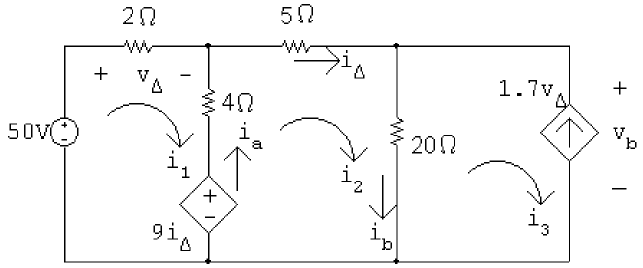
$$p_{5\Omega} = (20 - 16)^2(5) = 80 \text{ W}$$

$$p_{4\Omega} = (1)^2(4) = 4 \text{ W}$$

$$p_{20\Omega} = (20 - 15)^2(20) = 500 \text{ W}$$

$$\sum p_{\text{abs}} = 1560 + 256 + 80 + 4 + 500 = 2400 \text{ W (CHECKS)}$$

P 4.46 **[a]**



Mesh equations:

$$-50 + 6i_1 - 4i_2 + 9i_\Delta = 0$$

$$-9i_\Delta - 4i_1 + 29i_2 - 20i_3 = 0$$

Constraint equations:

$$i_\Delta = i_2; \qquad i_3 = -1.7v_\Delta; \qquad v_\Delta = 2i_1$$

Solving, $i_1 = -5$ A; $i_2 = 16$ A; $i_3 = 17$ A; $v_\Delta = -10$ V

$$9i_\Delta = 9(16) = 144 \text{ V}$$

$$i_\text{a} = i_2 - i_1 = 21 \text{ A}$$

$$i_\text{b} = i_2 - i_3 = -1 \text{ A}$$

$$v_\text{b} = 20i_\text{b} = -20 \text{ V}$$

$$p_{50\text{V}} = -50i_1 = 250 \text{ W (absorbing)}$$

$$p_{9i_\Delta} = -i_\text{a}(9i_\Delta) = -(21)(144) = -3024 \text{ W (delivering)}$$

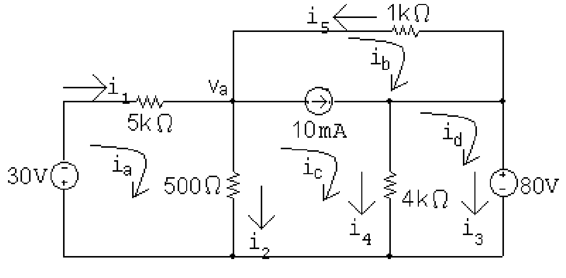
$$p_{1.7\text{V}} = -1.7v_\Delta v_\text{b} = i_3 v_\text{b} = (17)(-20) = -340 \text{ W (delivering)}$$

[b] $\sum P_\text{dev} = 3024 + 340 = 3364$ W

$$\sum P_\text{dis} = 250 + (-5)^2(2) + (21)^2(4) + (16)^2(5) + (-1)^2(20)$$

$$= 3364 \text{ W}$$

P 4.47 **[a]**



Supermesh equations:

$1000i_b + 4000(i_c - i_d) + 500(i_c - i_a) = 0$

$i_c - i_b = 0.01$

Two remaining mesh equations:

$5500i_a - 500i_c = -30$

$4000i_d - 4000i_c = -80$

In standard form,

$-500i_a + 1000i_b + 4500i_c - 4000i_d = 0$

$0i_a - 1i_b + 1i_c + 0i_d = 0.01$

$5500i_a + 0i_b - 500i_c + 0i_d = -30$

$0i_a + 0i_b - 4000i_c + 4000i_d = -80$

Solving:

$i_a = -10$ mA; $i_b = -60$ mA; $i_c = -50$ mA; $i_d = -70$ mA

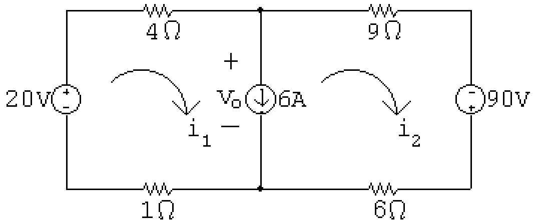
Then,

$i_1 = i_a = -10$ mA; $i_2 = i_a - i_c = 40$ mA; $i_3 = i_d = -70$ mA

[b] $p_{\text{sources}} = 30(-0.01) + [1000(-0.06)](0.01) + 80(-0.07) = -6.5$ W

$$p_{\text{resistors}} = 1000(0.06)^2 + 5000(0.01)^2 + 500(0.04)^2$$
$$+4000(-0.05 + 0.07)^2 = 6.5 \text{ W}$$

P 4.48



$-20 + 4i_1 + 9i_2 - 90 + 6i_2 + 1i_1 = 0; \quad i_1 - i_2 = 6$

Solving, $i_1 = 10$ A; $i_2 = 4$ A

$p_{20V} = -20i_1 = -200$ W (diss)

$p_{4\,\Omega} = (10)^2(4) = 400$ W

$p_{1\,\Omega} = (10)^2(1) = 100\ \text{W}$

$p_{9\,\Omega} = (4)^2(9) = 144\ \text{W}$

$p_{6\,\Omega} = (4)^2(6) = 96\ \text{W}$

$v_o = 9(4) - 90 + 6(4) = -30\ \text{V}$

$p_{6A} = 6v_o = -180\ \text{W}$

$p_{90V} = -90i_2 = -360\ \text{W}$

$\sum p_{\text{dev}} = 200 + 180 + 360 = 740\ \text{W}$

$\sum p_{\text{diss}} = 400 + 100 + 144 + 96 = 740\ \text{W}$

Thus the total power dissipated is 740 W.

P 4.49 **[a]** Summing around the supermesh used in the solution to Problem 4.48 gives

$-60 + 4i_1 + 9i_2 - 90 + 6i_2 + 1i_1 = 0; \qquad i_1 - i_2 = 6$

$\therefore\ \ i_1 = 12\ \text{A}; \qquad i_2 = 6\ \text{A}$

$p_{60\text{V}} = -60(12) = -720\ \text{W (del)}$

$v_o = 9(6) - 90 + 6(6) = 0\ \text{V}$

$p_{6\text{A}} = 6v_o = 0\ \text{W}$

$p_{90\text{V}} = -90i_2 = -540\ \text{W (del)}$

$\sum p_{\text{diss}} = (12)^2(4 + 1) + (6)^2(9 + 6) = 1260\ \text{W}$

$\sum p_{\text{dev}} = 720 + 0 + 540 = 1260\ \text{W} = \sum p_{\text{diss}}$

[b] With 6 A current source replaced with a short circuit

$5i_1 = 60; \qquad 15i_2 = 90$

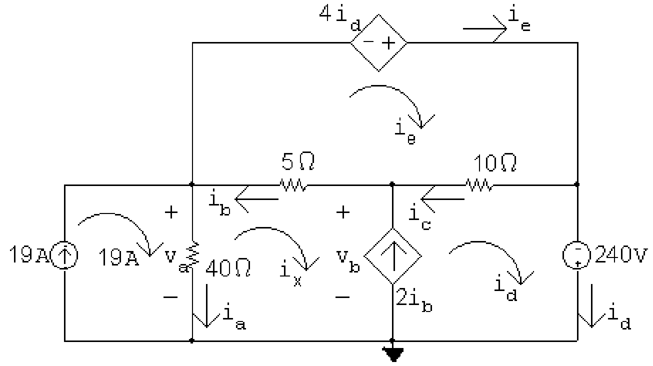
Solving,

$i_1 = 12\ \text{A}, \qquad i_2 = 6\ \text{A}$

$\therefore\ \ \sum P_{\text{sources}} = -(60)(12) - (90)(6) = -1260\ \text{W}$

[c] A 6 A source with zero terminal voltage is equivalent to a short circuit carrying 6 A.

P 4.50 [**a**]



$$-4i_d + 10(i_e - i_d) + 5(i_e - i_x) = 0$$

$$5(i_x - i_e) + 10(i_d - i_e) - 240 + 40(i_x - 19) = 0$$

$$i_d - i_x = 2i_b = 2(i_e - i_x)$$

Solving, $i_d = 10$ A; $i_e = 18$ A; $i_x = 26$ A

$i_a = 19 - i_x = -7$ A; $i_b = i_e - i_x = -8$ A; $i_c = i_e - i_d = 8$ A;

[**b**] $v_\mathrm{a} = 40i_\mathrm{a} = -280$ V; $v_\mathrm{b} = 5i_b + 40i_a = -320$ V

$$p_{19\mathrm{A}} = -19v_a = 5320 \text{ W}$$

$$p_{4i_d} = -4i_d i_e = -720 \text{ W}$$

$$p_{2i_a} = -2i_b v_b = -5120 \text{ W}$$

$$p_{240\mathrm{V}} = -240i_d = -2400 \text{ W}$$

$$p_{40\Omega} = (7)^2(40) = 1960 \text{ W} =$$

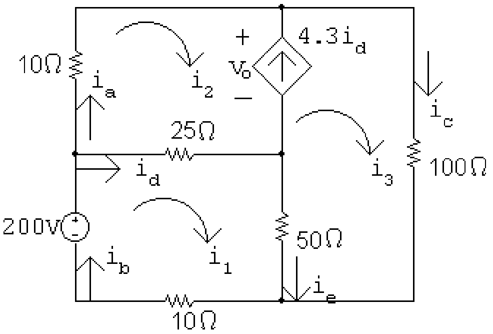
$$p_{5\Omega} = (8)^2(5) = 320 \text{ W}$$

$$p_{10\Omega} = (8)^2(10) = 640 \text{ W}$$

$$\sum P_\mathrm{gen} = 720 + 5120 + 2400 = 8240 \text{ W}$$

$$\sum P_\mathrm{diss} = 5320 + 1960 + 320 + 640 = 8240 \text{ W}$$

P 4.51 [**a**]



$$200 = 85i_1 - 25i_2 - 50i_3$$

$$0 = -75i_1 + 35i_2 + 150i_3 \qquad \text{(supermesh)}$$

$$i_3 - i_2 = 4.3(i_1 - i_2)$$

Solving, $i_1 = 4.6$ A; $\quad i_2 = 5.7$ A; $\quad i_3 = 0.97$ A

$i_\text{a} = i_2 = 5.7$ A; $\quad i_\text{b} = i_1 = 4.6$ A

$i_\text{c} = i_3 = 0.97$ A; $\quad i_\text{d} = i_1 - i_2 = -1.1$ A

$i_\text{e} = i_1 - i_3 = 3.63$ A

[**b**] $10i_2 + v_o + 25(i_2 - i_1) = 0$

$$\therefore \quad v_o = -57 - 27.5 = -84.5 \text{ V}$$

$$p_{4.3i_\text{d}} = -v_o(4.3i_\text{d}) = -(-84.5)(4.3)(-1.1) = -399.685 \text{ W(dev)}$$

$$p_{200\text{V}} = -200(4.6) = -920 \text{ W(dev)}$$

$$\begin{aligned}
\sum P_\text{dev} &= 1319.685 \text{ W} \\
\sum P_\text{dis} &= (5.7)^2 10 + (1.1)^2(25) + (0.97)^2 100 + (4.6)^2(10) + \\
&\quad (3.63)^2(50) \\
&= 1319.685 \text{ W}
\end{aligned}$$

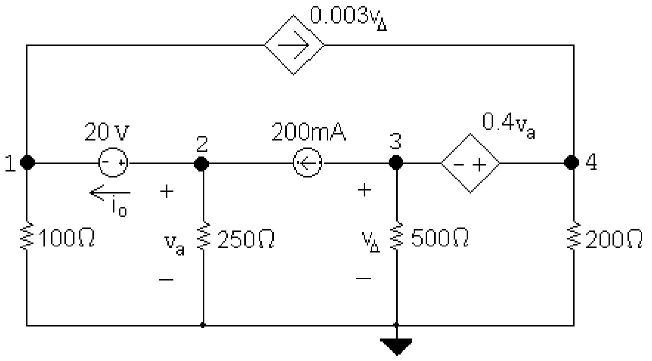
$$\therefore \quad \sum P_\text{dev} = \sum P_\text{dis} = 1319.685 \text{ W}$$

P 4.52 [**a**] The node voltage method requires summing the currents at two supernodes in terms of four node voltages and using two constraint equations to reduce the system of equations to two unknowns. If the connection at the bottom of the circuit is used as the reference node, then the voltages controlling the dependent sources are node voltages. This makes it easy to formulate the constraint equations. The current in the 20 V source is obtained by summing the currents at either terminal of the source.

The mesh current method requires summing the voltages around the two meshes not containing current sources in terms of four mesh currents. In addition the voltages controlling the dependent sources must be expressed in terms of the mesh currents. Thus the constraint equations are more complicated, and the reduction to two equations and two unknowns involves more algebraic manipulation. The current in the 20 V source is found by subtracting two mesh currents.

Because the constraint equations are easier to formulate in the node voltage method, it is the preferred approach.

[b]



Node voltage equations:

$$\frac{v_1}{100} + 0.003v_\Delta + \frac{v_2}{250} - 0.2 = 0$$

$$0.2 + \frac{v_3}{100} + \frac{v_4}{200} - 0.003v_\Delta = 0$$

Constraints:

$$v_2 = v_a; \qquad v_3 = v_\Delta; \qquad v_4 - v_3 = 0.4v_a; \qquad v_2 - v_1 = 20$$

Solving, $v_1 = 24$ V; $v_2 = 44$ V; $v_3 = -72$ V; $v_4 = -54$ V.

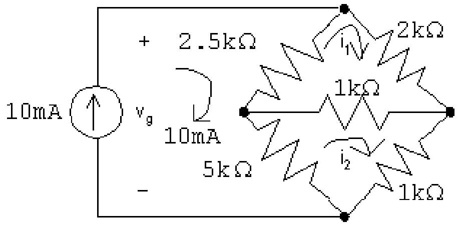
$$i_o = 0.2 - \frac{v_2}{250} = 24 \text{ m A}$$

$$p_{20\text{V}} = 20(0.024) = 480 \text{ m W}$$

Thus, the 20 V source absorbs 480 mW.

P 4.53 **[a]** There are three unknown node voltages and only two unknown mesh currents. Use the mesh current method to minimize the number of simultaneous equations.

[b]



The mesh current equations:

$$2500(i_1 - 0.01) + 2000i_1 + 1000(i_1 - i_2) = 0$$

$$5000(i_2 - 0.01) + 1000(i_2 - i_1) + 1000i_2 = 0$$

Place the equations in standard form:

$$i_1(2500 + 2000 + 1000) + i_2(-1000) = 25$$

$$i_1(-1000) + i_2(5000 + 1000 + 1000) = 50$$

Solving, $i_1 = 6$ mA; $\qquad i_2 = 8$ mA

Find the power in the 1 kΩ resistor:

$i_{1\text{k}} = i_1 - i_2 = -2$ m A

$p_{1\text{k}} = (-0.002)^2(1000) = 4$ mW

[c] No, the voltage across the 10 A current source is readily available from the mesh currents, and solving two simultaneous mesh-current equations is less work than solving three node voltage equations.

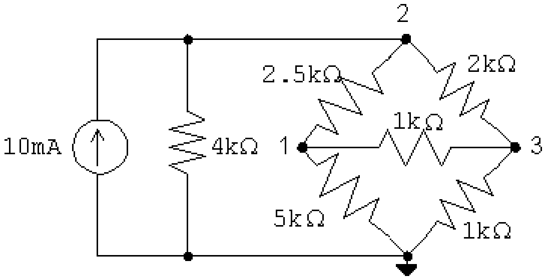
[d] $v_g = 2000i_1 + 1000i_2 = 12 + 8 = 20$ V

$p_{10\text{mA}} = -(20)(0.01) = -200$ m W

Thus the 10 mA source develops 200 mW.

P 4.54 [a] There are three unknown node voltages and three unknown mesh currents, so the number of simultaneous equations required is the same for both methods. The node voltage method has the advantage of having to solve the three simultaneous equations for one unknown voltage provided the connection at either the top or bottom of the circuit is used as the reference node. Therefore recommend the node voltage method.

[b]



The node voltage equations are:

$$\begin{aligned}
\frac{v_1}{5000} + \frac{v_1 - v_2}{2500} + \frac{v_1 - v_3}{1000} &= 0\\
-0.01 + \frac{v_2}{4000} + \frac{v_2 - v_1}{2500} + \frac{v_2 - v_3}{2000} &= 0\\
\frac{v_3 - v_1}{1000} + \frac{v_3 - v_2}{2000} + \frac{v_3}{1000} &= 0
\end{aligned}$$

Put the equations in standard form:

$$\begin{aligned}
v_1\left(\frac{1}{5000} + \frac{1}{2500} + \frac{1}{1000}\right) + v_2\left(-\frac{1}{2500}\right) + v_3\left(-\frac{1}{1000}\right) &= 0\\
v_1\left(-\frac{1}{2500}\right) + v_2\left(\frac{1}{4000} + \frac{1}{2500} + \frac{1}{2000}\right) + v_3\left(-\frac{1}{2000}\right) &= 0.01\\
v_1\left(-\frac{1}{1000}\right) + v_2\left(-\frac{1}{2000}\right) + v_3\left(\frac{1}{2000} + \frac{1}{1000} + \frac{1}{1000}\right) &= 0
\end{aligned}$$

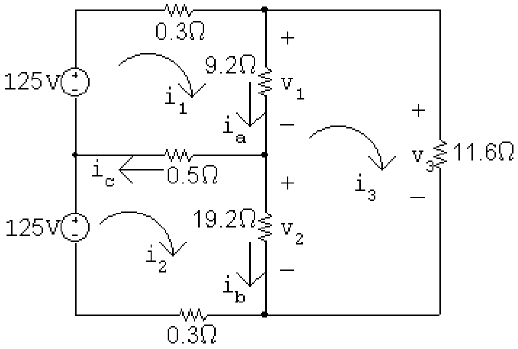
Solving, $v_1 = 6.67$ V; $v_2 = 13.33$ V; $v_3 = 5.33$ V

$p_{10\text{m}} = -(13.33)(0.01) = -133.33$ m W

Therefore, the 10 mA source is developing 133.33 mW

P 4.55 **[a]** Both the mesh-current method and the node-voltage method require three equations. The mesh-current method is a bit more intuitive due to the presence of the voltage sources. We choose the mesh-current method, although technically it is a toss-up.

[b]



$$\begin{aligned}
125 &= 10i_1 - 0.5i_2 - 9.2i_3\\
125 &= -0.5i_1 + 20i_2 - 19.2i_3\\
0 &= -9.2i_1 - 19.2i_2 + 40i_3
\end{aligned}$$

Solving, $i_1 = 32.25$ A; $i_2 = 26.29$ A; $i_3 = 20.04$ A

$$\begin{aligned}
v_1 &= 9.2(i_1 - i_3) = 112.35 \text{ V}\\
v_2 &= 19.2(i_2 - i_3) = 120.09 \text{ V}\\
v_3 &= 11.6i_3 = 232.44 \text{ V}
\end{aligned}$$

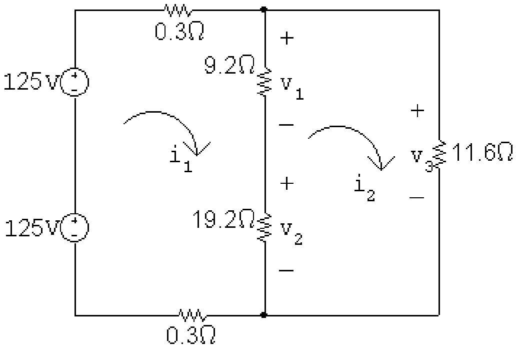
[c] $$p_{R1} = (i_1 - i_3)^2(9.2) = 1371.93 \text{ W}$$
$$p_{R2} = (i_2 - i_3)^2(19.2) = 751.13 \text{ W}$$
$$p_{R3} = i_3^2(11.6) = 4657.52 \text{ W}$$

[d] $$\sum p_{\text{dev}} = 125(i_1 + i_2) = 7317.72 \text{ W}$$

$$\sum p_{\text{load}} = 6780.58 \text{ W}$$

$$\% \text{ delivered } = \frac{6780.58}{7317.72} \times 100 = 92.66\%$$

[e]



$$250 = 29i_1 - 28.4i_2$$

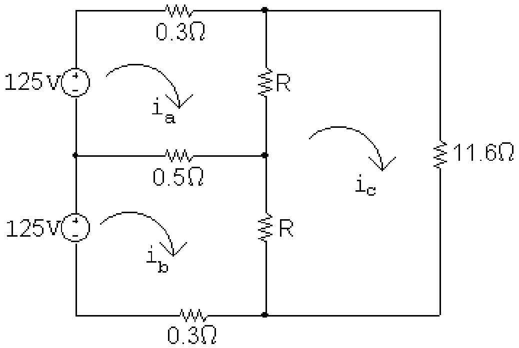
$$0 = -28.4i_1 + 40i_2$$

Solving, $i_1 = 28.29$ A; $i_2 = 20.09$ A

$$i_1 - i_2 = 8.2 \text{ A}$$
$$v_1 = (8.2)(9.2) = 75.44 \text{ V}$$
$$v_2 = (8.2)(19.2) = 157.44 \text{ V}$$

Note v_1 is low and v_2 is high. Therefore, loads designed for 125 V would not function properly, and could be damaged.

P 4.56



The mesh current equations:

$$125 = (R+0.8)i_{\rm a} - 0.5i_{\rm b} - Ri_{\rm c}$$

$$125 = -0.5i_{\rm a} + (R+0.8)i_{\rm b} - Ri_{\rm c}$$

$$\therefore\ (R+0.8)i_{\rm a} - 0.5i_{\rm b} - Ri_{\rm c} = -0.5i_{\rm a} + (R+0.8)i_{\rm b} - Ri_{\rm c}$$

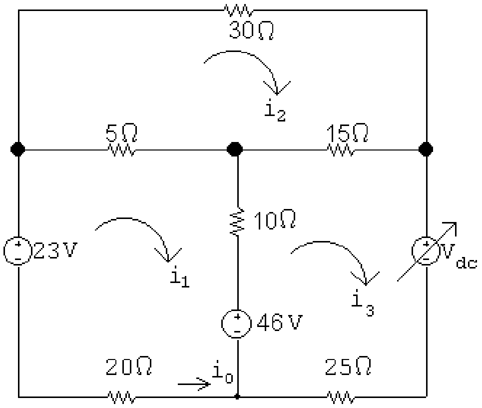
$$\therefore\ (R+0.8)i_{\rm a} - 0.5i_{\rm b} = -0.5i_{\rm a} + (R+0.8)i_{\rm b}$$

$$\therefore\ (R+1.3)i_{\rm a} = (R+1.3)i_{\rm b}$$

Thus

$i_{\rm a} = i_{\rm b}$ so $i_o = i_{\rm b} - i_{\rm a} = 0$

P 4.57 **[a]**



Write the mesh current equations. Note that if $i_o = 0$, then $i_1 = 0$:

$$-23 + 5(-i_2) + 10(-i_3) + 46 = 0$$

$$30i_2 + 15(i_2 - i_3) + 5i_2 = 0$$

$$V_{\rm dc} + 25i_3 - 46 + 10i_3 + 15(i_3 - i_2) = 0$$

Place the equations in standard form:

$$i_2(-5) + i_3(-10) + V_{\rm dc}(0) = -23$$

$$i_2(30+15+5) + i_3(-15) + V_{\rm dc}(0) = 0$$

$$i_2(-15) + i_3(25+10+15) + V_{\rm dc}(1) = 46$$

Solving, $i_2 = 0.6$ A; $i_3 = 2$ A; $V_{\rm dc} = -45$ V

Thus, the value of $V_{\rm dc}$ required to make $i_o = 0$ is -45 V.

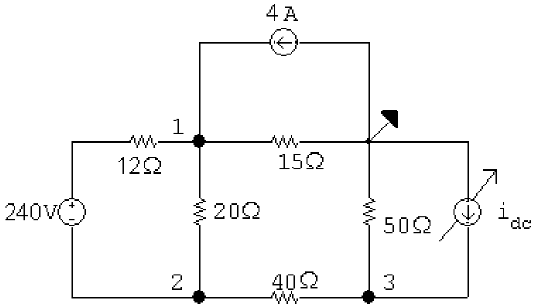
[b] Calculate the power:

$$\begin{aligned}
p_{23\text{V}} &= -(23)(0) = 0 \text{ W}\\
p_{46\text{V}} &= -(46)(2) = -92 \text{ W}\\
p_{\text{Vdc}} &= (-45)(2) = -90 \text{ W}\\
p_{30\Omega} &= (30)(0.6)^2 = 10.8 \text{ W}\\
p_{5\Omega} &= (5)(0.6)^2 = 1.8 \text{ W}\\
p_{15\Omega} &= (15)(2-0.6)^2 = 29.4 \text{ W}\\
p_{10\Omega} &= (10)(2)^2 = 40 \text{ W}\\
p_{20\Omega} &= (20)(0)^2 = 0 \text{ W}\\
p_{25\Omega} &= (25)(2)^2 = 100 \text{ W}
\end{aligned}$$

$$\sum p_{\text{dev}} = 92 + 90 = 182 \text{ W}$$

$$\sum p_{\text{dis}} = 10.8 + 1.8 + 29.4 + 40 + 0 + 100 = 182 \text{ W(checks)}$$

P 4.58 Choose the reference node so that a node voltage is identical to the voltage across the 4 A source; thus:



Since the 4 A source is developing 0 W, v_1 must be 0 V.

Since v_1 is known, we can sum the currents away from node 1 to find v_2; thus:

$$\frac{0-(240+v_2)}{12} + \frac{0-v_2}{20} + \frac{0}{15} - 4 = 0$$

$$\therefore \quad v_2 = -180 \text{ V}$$

Now that we know v_2 we sum the currents away from node 2 to find v_3; thus:

$$\frac{v_2+240-0}{12} + \frac{v_2-0}{20} + \frac{v_2-v_3}{40} = 0$$

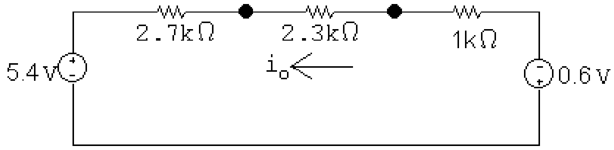
$$\therefore \quad v_3 = -340 \text{ V}$$

Now that we know v_3 we sum the currents away from node 3 to find i_{dc}; thus:

$$\frac{v_3}{50} + \frac{v_3 - v_2}{40} = i_{\text{dc}}$$

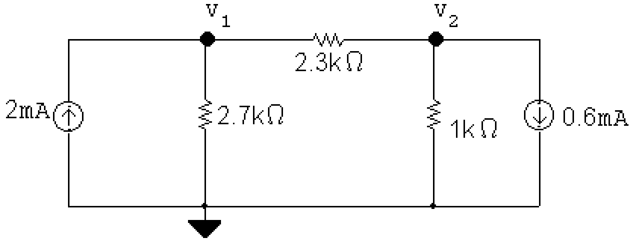
$$\therefore \quad i_{\text{dc}} = -10.8 \text{ A}$$

P 4.59 **[a]** Apply source transformations to both current sources to get



$$i_o = \frac{-(5.4 + 0.6)}{2700 + 2300 + 1000} = -1 \text{ mA}$$

[b]



The node voltage equations:

$$-2 \times 10^{-3} + \frac{v_1}{2700} + \frac{v_1 - v_2}{2300} = 0$$

$$\frac{v_2}{1000} + \frac{v_2 - v_1}{2300} + 0.6 \times 10^{-3} = 0$$

Place these equations in standard form:

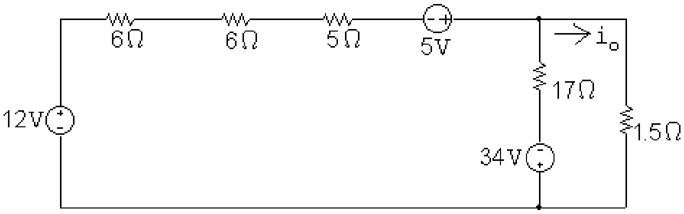
$$v_1\left(\frac{1}{2700} + \frac{1}{2300}\right) + v_2\left(-\frac{1}{2300}\right) = 2 \times 10^{-3}$$

$$v_1\left(-\frac{1}{2300}\right) + v_2\left(\frac{1}{1000} + \frac{1}{2300}\right) = -0.6 \times 10^{-3}$$

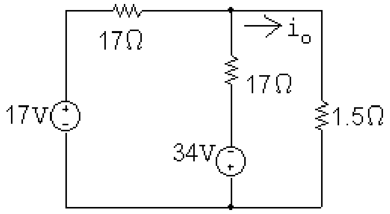
Solving, $v_1 = 2.7$ V; $v_2 = 0.4$ V

$$\therefore \quad i_o = \frac{v_2 - v_1}{2300} = -1 \text{ mA}$$

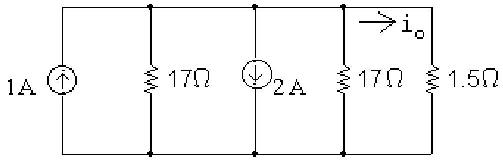
P 4.60 **[a]** Applying a source transformation to each current source yields



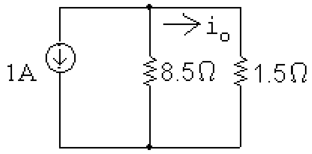
Now combine the 12 V and 5 V sources into a single voltage source and the 6 Ω, 6 Ω and 5 Ω resistors into a single resistor to get



Now use a source transformation on each voltage source, thus

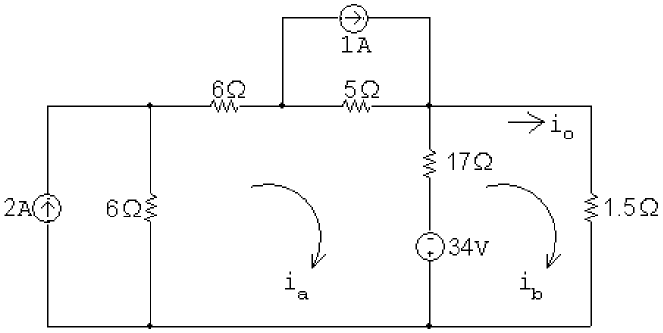


which can be reduced to



$$\therefore \quad i_o = -\frac{8.5}{10}(1) = -0.85 \text{ A}$$

[b]

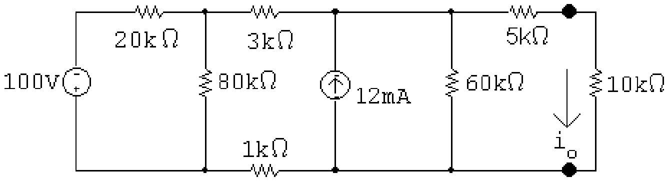


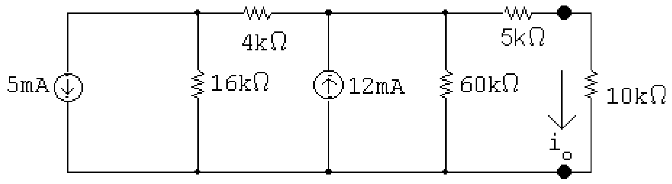
$$34i_{\rm a} - 17i_{\rm b} = 12 + 5 + 34 = 51$$

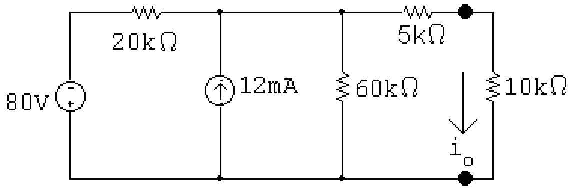
$$-17i_{\rm a} + 18.5i_{\rm b} = -34$$

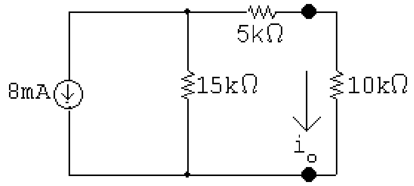
Solving, $i_{\rm b} = -0.85 \text{ A} = i_o$

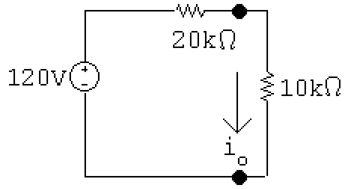
P 4.61 **[a]**





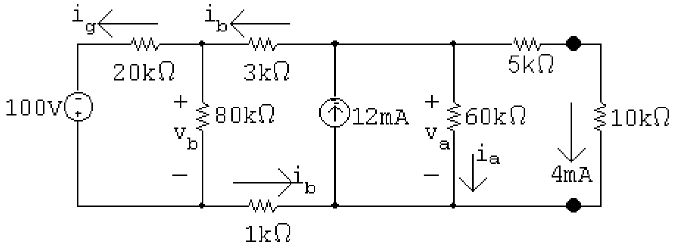






$$i_o = \frac{120}{30{,}000} = 4\ \text{mA}$$

[b]

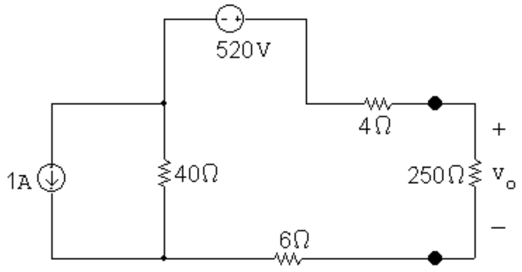


$$\begin{aligned}
v_\text{a} &= (15{,}000)(0.004) = 60\ \text{V} \\
i_\text{a} &= \frac{v_\text{a}}{60{,}000} = 1\ \text{mA} \\
i_\text{b} &= 12 - 1 - 4 = 7\ \text{mA} \\
v_\text{b} &= 60 - (0.007)(4000) = 32\ \text{V} \\
i_g &= 0.007 - \frac{32}{80{,}000} = 6.6\ \text{mA} \\
p_{100\text{V}} &= -(100)(6.6 \times 10^{-3}) = -660\ \text{mW}
\end{aligned}$$

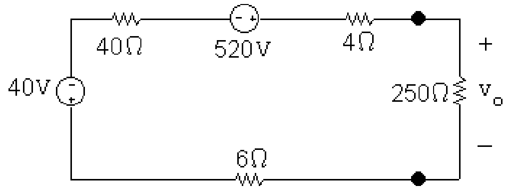
Check:

$$\begin{aligned}
p_{12\text{mA}} &= -(60)(12\times 10^{-3}) = -720 \text{ mW}\\
\sum P_{\text{dev}} &= 660+720 = 1380 \text{ mW}\\
\sum P_{\text{dis}} &= (20{,}000)(6.6\times 10^{-3})^2 + (80{,}000)(0.4\times 10^{-3})^2 + (4000)(7\times 10^{-3})^2\\
&\quad +(60{,}000)(1\times 10^{-3})^2 + (15{,}000)(4\times 10^{-3})^2\\
&= 1380 \text{ mW}
\end{aligned}$$

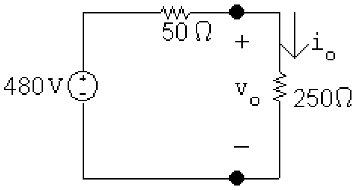
P 4.62 [**a**] First remove the 16 Ω and 260 Ω resistors:



Next use a source transformation to convert the 1 A current source and 40 Ω resistor:

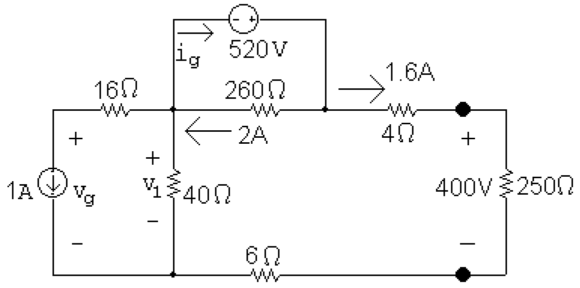


which simplifies to



$$\therefore \quad v_o = \frac{250}{300}(480) = 400 \text{ V}$$

[**b**] Return to the original circuit with $v_o = 400$ V:



$$i_g = \frac{520}{260} + 1.6 = 3.6 \text{ A}$$

$$p_{520\text{V}} = -(520)(3.6) = -1872 \text{ W}$$

Therefore, the 520 V source is developing 1872 W.

[c] $v_1 = -520 + 1.6(4 + 250 + 6) = -104 \text{ V}$

$$v_g = v_1 - 1(16) = -104 - 16 = -120 \text{ V}$$

$$p_{1\text{A}} = (1)(-120) = -120 \text{ W}$$

Therefore the 1 A source is developing 120 W.

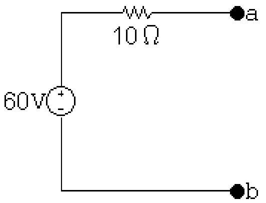
[d] $\sum p_{\text{dev}} = 1872 + 120 = 1992 \text{ W}$

$$\sum p_{\text{diss}} = (1)^2(16) + \frac{(104)^2}{40} + \frac{(520)^2}{260} + (1.6)^2(260) = 1992 \text{ W}$$

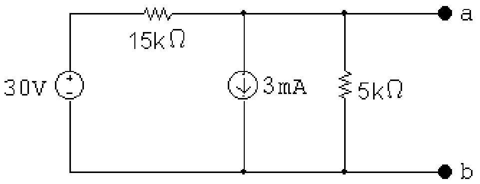
$$\therefore \sum p_{\text{diss}} = \sum p_{\text{dev}}$$

P 4.63 $v_{\text{Th}} = \frac{30}{40}(80) = 60 \text{ V}$

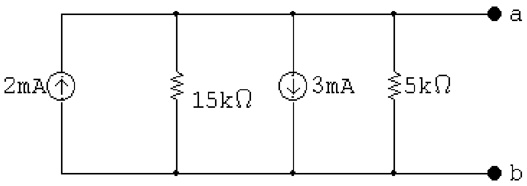
$$R_{\text{Th}} = 2.5 + \frac{(30)(10)}{40} = 10\,\Omega$$



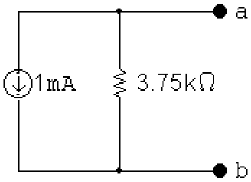
P 4.64 First we make the observation that the 10 mA current source and the 10 kΩ resistor will have no influence on the behavior of the circuit with respect to the terminals a,b. This follows because they are in parallel with an ideal voltage source. Hence our circuit can be simplified to



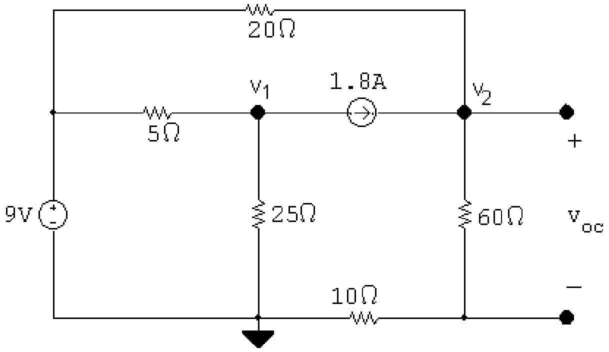
or



Therefore the Norton equivalent is



P 4.65 [**a**] Open circuit:

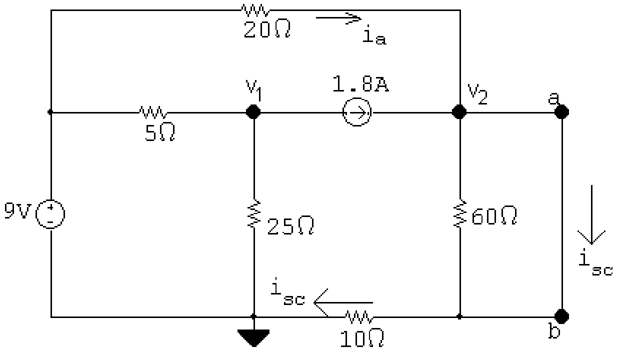


$$\frac{v_2 - 9}{20} + \frac{v_2}{70} - 1.8 = 0$$

$$v_2 = 35 \text{ V}$$

$$v_{\rm Th} = \frac{60}{70} v_2 = 30 \text{ V}$$

Short circuit:



$$\frac{v_2 - 9}{20} + \frac{v_2}{10} - 1.8 = 0$$

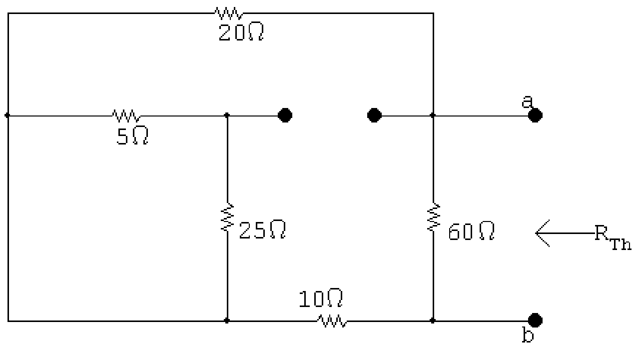
$$\therefore \quad v_2 = 15 \text{ V}$$

$$i_{\rm a} = \frac{9 - 15}{20} = -0.3 \text{ A}$$

$$i_{\rm sc} = 1.8 - 0.3 = 1.5 \text{ A}$$

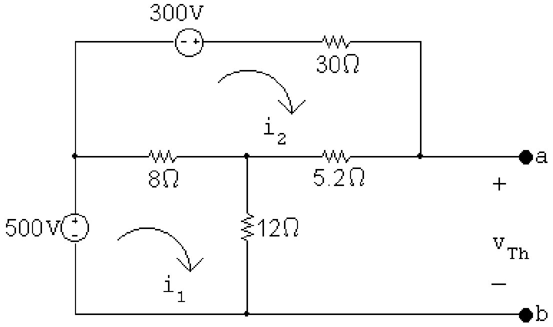
$$R_{\rm Th} = \frac{30}{1.5} = 20\,\Omega$$

[b]



$R_{\text{Th}} = (20 + 10\|60 = 20\,\Omega$ (CHECKS)

P 4.66 After making a source transformation the circuit becomes



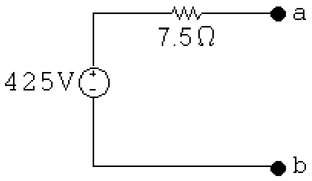
$500 = 20i_1 - 8i_2$

$300 = -8i_1 + 43.2i_2$

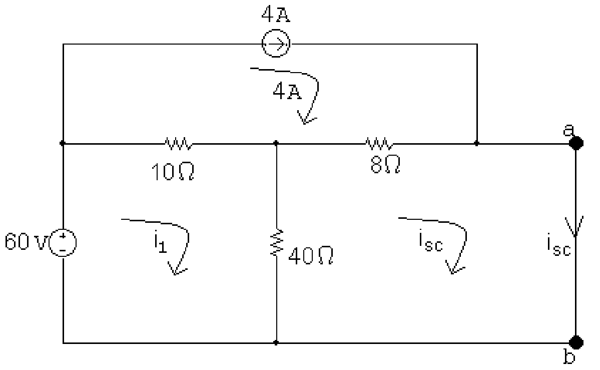
$\therefore \quad i_1 = 30$ A and $i_2 = 12.5$ A

$v_{\text{Th}} = 12i_1 + 5.2i_2 = 425$ V

$R_{\text{Th}} = (8\|12 + 5.2)\|30 = 7.5\,\Omega$



P 4.67

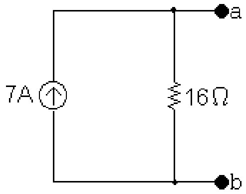


$$50i_1 - 40i_{\text{sc}} = 60 + 40$$

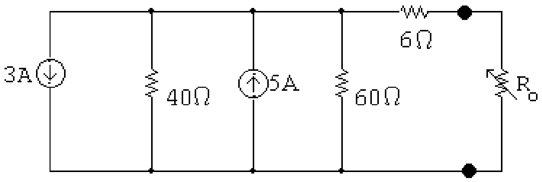
$$-40i_1 + 48i_{\text{scs}} = 32$$

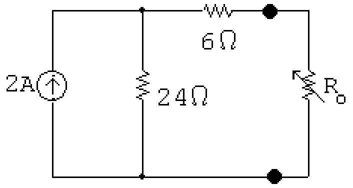
Solving, $\quad i_{\text{sc}} = 7$ A

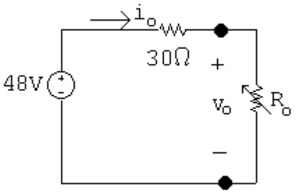
$$R_{\text{Th}} = 8 + \frac{(10)(40)}{50} = 16\,\Omega$$



P 4.68 First, find the Thévenin equivalent with respect to R_o.

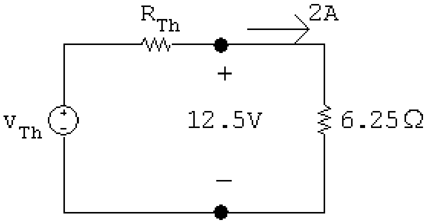




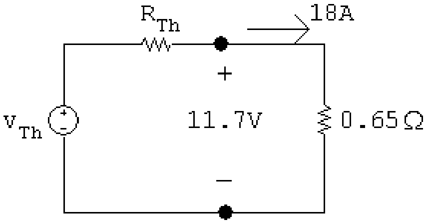


$R_o(\Omega)$	i_o(A)	v_o(V)
10	1.2	12
15	1.067	16
22	0.923	20.31
33	0.762	25.14
47	0.623	29.30
68	0.490	33.31

P 4.69



$12.5 = v_{\text{Th}} - 2R_{\text{Th}}$



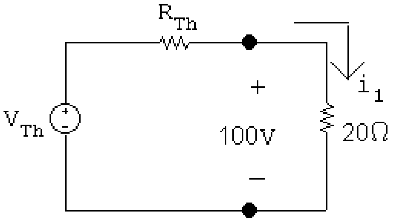
$11.7 = v_{\text{Th}} - 18R_{\text{Th}}$

Solving the above equations for V_{Th} and R_{Th} yields

$v_{\text{Th}} = 12.6$ V, $\qquad R_{\text{Th}} = 50$ mΩ

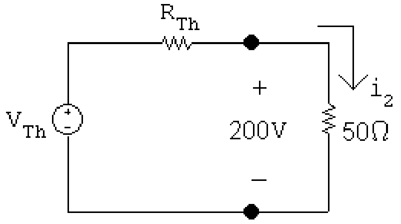
$\therefore\quad I_N = 252$ A, $\qquad R_N = 50$ mΩ

P 4.70



$i_1 = 100/20 = 5$ A

$100 = v_{\text{Th}} - 5R_{\text{Th}}, \qquad v_{\text{Th}} = 100 + 5R_{\text{Th}}$

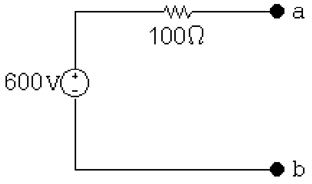


$i_2 = 200/50 = 4 \text{ A}$

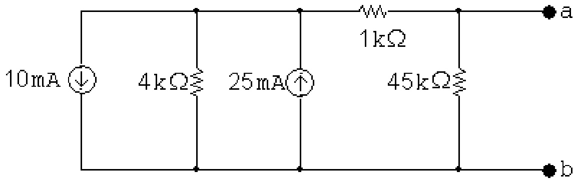
$200 = v_{\text{Th}} - 4R_{\text{Th}}, \qquad v_{\text{Th}} = 200 + 4R_{\text{Th}}$

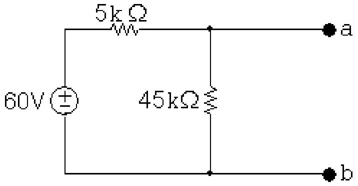
$\therefore \quad 100 + 5R_{\text{Th}} = 200 + 4R_{\text{Th}} \qquad \text{so} \qquad R_{\text{Th}} = 100\Omega$

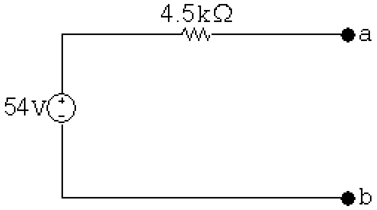
$v_{\text{Th}} = 100 + 500 = 600 \text{ V}$



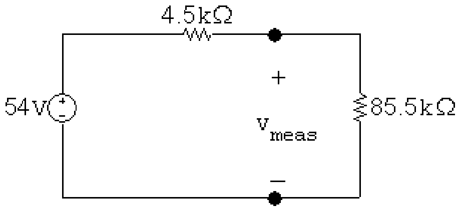
P 4.71 **[a]** First, find the Thévenin equivalent with respect to a,b using a succession of source transformations.







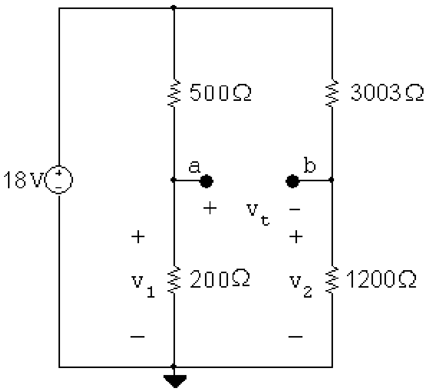
$\therefore \quad v_{\text{Th}} = 54 \text{ V} \qquad R_{\text{Th}} = 4.5 \text{ k}\Omega$



$$v_{\text{meas}} = \frac{54}{90}(85.5) = 51.3 \text{ V}$$

[b] %error $= \left(\frac{51.3 - 54}{54}\right) \times 100 = -5\%$

P 4.72

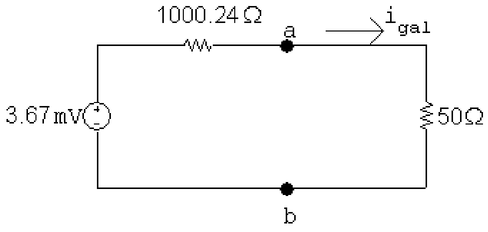


$$v_1 = \frac{200}{700}(18) = 5.143 \text{ V}$$

$$v_2 = \frac{1200}{4203}(18) = 5.139 \text{ V}$$

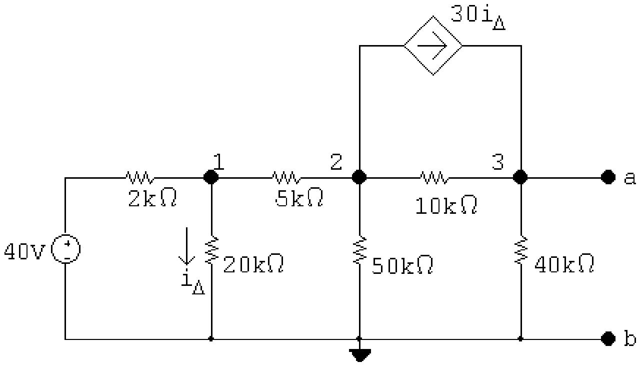
$$v_{\text{Th}} = v_1 - v_2 = 5.143 - 5.139 = 3.67 \text{ mV}$$

$$R_{\text{Th}} = \frac{(500)(200)}{700} + \frac{(3003)(1200)}{4203} = 1000.24\,\Omega$$



$$i_{\text{gal}} = \frac{3.67 \times 10^{-3}}{1050.24} = 3.5\,\mu\text{A}$$

P 4.73



The node voltage equations are:

$$\frac{v_1 - 40}{2000} + \frac{v_1}{20{,}000} + \frac{v_1 - v_2}{5000} = 0$$

$$\frac{v_2 - v_1}{5000} + \frac{v_2}{50{,}000} + \frac{v_2 - v_3}{10{,}000} + 30\frac{v_1}{20{,}000} = 0$$

$$\frac{v_3 - v_2}{10{,}000} + \frac{v_3}{40{,}000} - 30\frac{v_1}{20{,}000} = 0$$

In standard form:

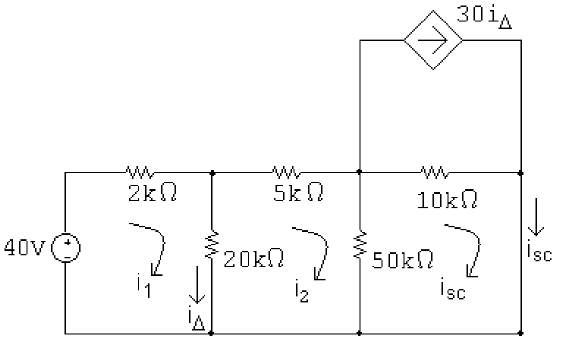
$$v_1\left(\frac{1}{2000} + \frac{1}{20{,}000} + \frac{1}{5000}\right) + v_2\left(-\frac{1}{5000}\right) + v_3(0) = \frac{40}{2000}$$

$$v_1\left(-\frac{1}{5000} + \frac{30}{20{,}000}\right) + v_2\left(\frac{1}{5000} + \frac{1}{50{,}000} + \frac{1}{10{,}000}\right) + v_3\left(-\frac{1}{10{,}000}\right) = 0$$

$$v_1\left(-\frac{30}{20{,}000}\right) + v_2\left(-\frac{1}{10{,}000}\right) + v_3\left(\frac{1}{10{,}000} + \frac{1}{40{,}000}\right) = 0$$

Solving, $v_1 = 24$ V; $v_2 = -10$ V; $v_3 = 280$ V

$V_{\text{Th}} = v_3 = 280$ V



The mesh current equations are:

$$\begin{aligned} -40 + 2000i_1 + 20{,}000(i_1 - i_2) &= 0 \\ 5000i_2 + 50{,}000(i_2 - i_{\rm sc}) + 20{,}000(i_2 - i_1) &= 0 \\ 50{,}000(i_{\rm sc} - i_2) + 10{,}000(i_{\rm sc} - 30i_\Delta) &= 0 \end{aligned}$$

The constraint equation is:

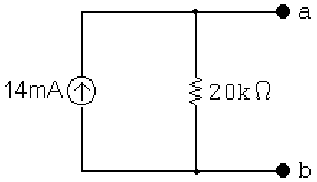
$i_\Delta = i_1 - i_2$

Put these equations in standard form:

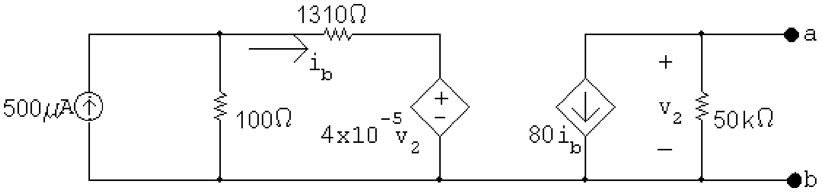
$$\begin{aligned} i_1(22{,}000) + i_2(-20{,}000) + i_{\rm sc}(0) + i_\Delta(0) &= 40 \\ i_1(-20{,}000) + i_2(75{,}000) + i_{\rm sc}(-50{,}000) + i_\Delta(0) &= 0 \\ i_1(0) + i_2(-50{,}000) + i_{\rm sc}(60{,}000) + i_\Delta(-300{,}000) &= 0 \\ i_1(-1) + i_2(1) + i_{\rm sc}(0) + i_\Delta(1) &= 0 \end{aligned}$$

Solving, $i_1 = 13.6$ mA; $i_2 = 12.96$ mA; $i_{\rm sc} = 14$ mA; $i_\Delta = 640\,\mu$A

$$R_{\rm Th} = \frac{280}{0.014} = 20\ \text{k}\Omega$$



P 4.74



OPEN CIRCUIT

$v_2 = -80i_b(50 \times 10^3) = -40 \times 10^5 i_b$

$4 \times 10^{-5} v_2 = -160i_b$

$1310i_b + 4 \times 10^{-5} v_2 = 1310i_b - 160i_b = 1150i_b$

So $1150i_b$ is the voltage across the $100\,\Omega$ resistor.

From KCL at the top left node, $500\,\mu\text{A} = \dfrac{1150i_b}{100} + i_b = 12.5i_b$

$$\therefore \quad i_b = \frac{500 \times 10^{-6}}{12.5} = 40\,\mu\text{A}$$

$$v_{\text{Th}} = -40 \times 10^5 (40 \times 10^{-6}) = -160 \text{ V}$$

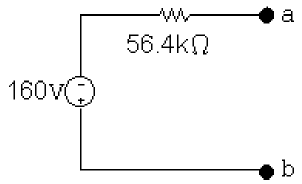
SHORT CIRCUIT

$$v_2 = 0; \qquad i_{\text{sc}} = -80 i_b$$

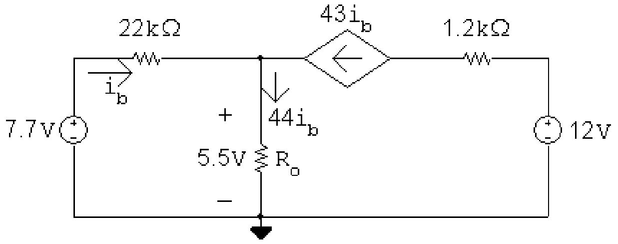
$$i_b = \frac{100}{100 + 1310}(500 \times 10^{-6}) = 35.46\,\mu\text{A}$$

$$i_{\text{sc}} = -80(35.46) = -2837\,\mu\text{A}$$

$$R_{\text{Th}} = \frac{-160}{-2837 \times 10^{-6}} = 56.4 \text{ k}\Omega$$



P 4.75 **[a]** Use source transformations to simplify the left side of the circuit.



$$i_b = \frac{7.7 - 5.5}{22{,}000} = 0.1 \text{ mA}$$

Let $\quad R_o = R_{\text{meter}} \| 1.3 \text{ k}\Omega = 5.5/4.4 = 1.25 \text{ k}\Omega$

$$\therefore \quad \frac{(R_{\text{meter}})(1.3)}{R_{\text{metere}} + 1.3} = 1.25; \qquad R_{\text{meter}} = \frac{(1.25)(1.3)}{0.05} = 32.5 \text{ k}\Omega$$

[b] Actual value of v_e:

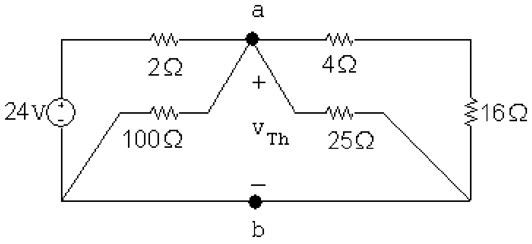
$$i_b = \frac{7.7}{22 + (44)(1.3)} = 0.0972 \text{ mA}$$

$$v_e = 44 i_b (1.3) = 5.56 \text{ V}$$

$$\% \text{ error } = \left(\frac{5.5 - 5.56}{5.56}\right) \times 100 = -1.1\%$$

P 4.76 [a] Find the Thévenin equivalent with respect to the terminals of the ammeter. This is most easily done by first finding the Thévenin with respect to the terminals of the $4.8\,\Omega$ resistor.

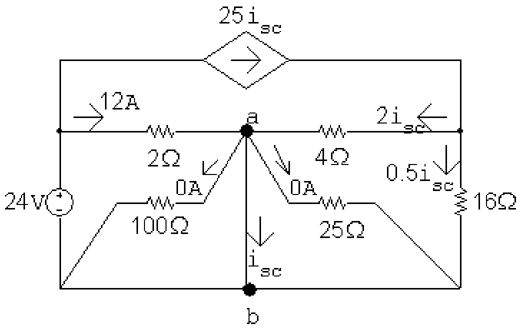
Thévenin voltage: note i_ϕ is zero.



$$\frac{v_{\rm Th}}{100} + \frac{v_{\rm Th}}{25} + \frac{v_{\rm Th}}{20} + \frac{v_{\rm Th} - 16}{2} = 0$$

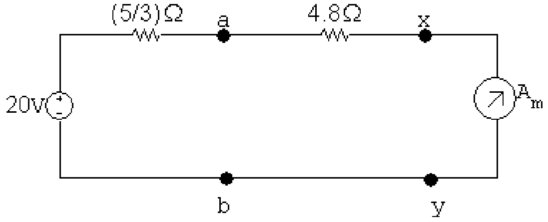
Solving, $v_{\rm Th} = 20$ V.

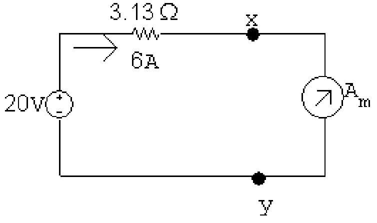
Short-circuit current:



$$i_{\rm sc} = 12 + 2i_{\rm sc}, \qquad \therefore \quad i_{\rm sc} = -12 \text{ A}$$

$$R_{\rm Th} = \frac{20}{-12} = -(5/3)\,\Omega$$

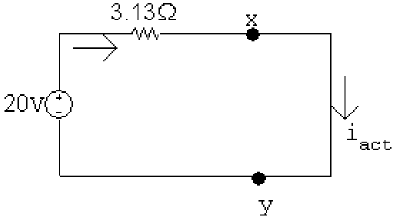




$$R_{\rm total} = \frac{20}{6} = 3.33\,\Omega$$

$$R_{\rm meter} = 3.33 - 3.13 = 0.2\,\Omega$$

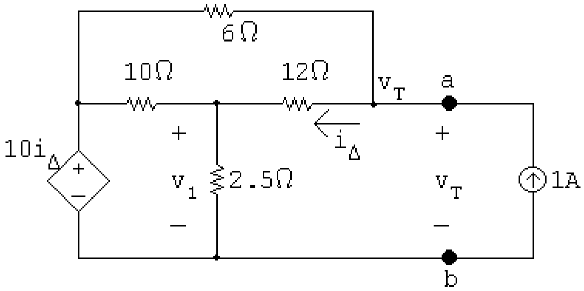
[**b**] Actual current:



$$i_{\text{actual}} = \frac{20}{3.13} = 6.38 \text{ A}$$

$$\% \text{ error } = \frac{6 - 6.38}{6.38} \times 100 = -6\%$$

P 4.77 $V_{\text{Th}} = 0$, since circuit contains no independent sources.



$$\frac{v_1 - 10i_\Delta}{10} + \frac{v_1}{2.5} + \frac{v_1 - v_{\text{T}}}{12} = 0$$

$$\frac{v_{\text{T}} - v_1}{12} + \frac{v_{\text{T}} - 10i_\Delta}{6} - 1 = 0$$

$$i_\Delta = \frac{v_{\text{T}} - v_1}{12}$$

In standard form:

$$v_1\left(\frac{1}{10} + \frac{1}{2.5} + \frac{1}{12}\right) + v_{\text{T}}\left(-\frac{1}{12}\right) + i_\Delta(-1) = 0$$

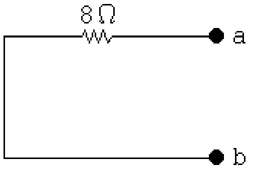
$$v_1\left(-\frac{1}{12}\right) + v_{\text{T}}\left(\frac{1}{12} + \frac{1}{6}\right) + i_\Delta\left(-\frac{10}{6}\right) = 1$$

$$v_1(1) + v_{\text{T}}(-1) + i_\Delta(12) = 0$$

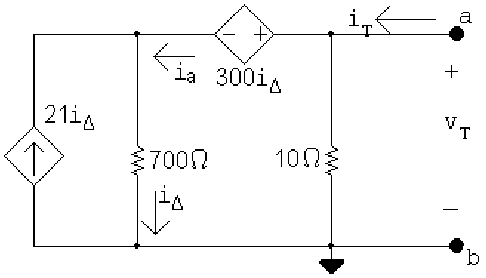
Solving,

$$v_1 = 2 \text{ V}; \qquad v_{\text{T}} = 8 \text{ V}; \qquad i_\Delta = 0.5 \text{ A}$$

$$\therefore \quad R_{\text{Th}} = \frac{v_{\text{T}}}{1\ \text{A}} = 8\,\Omega$$



P 4.78 $V_{\text{Th}} = 0$ since there are no independent sources in the circuit. Thus we need only find R_{Th}.



$$i_{\text{T}} = \frac{v_{\text{T}}}{10} + i_{\text{a}}$$

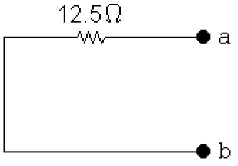
$$i_{\text{a}} = i_\Delta - 21i_\Delta = -20i_\Delta$$

$$i_\Delta = \frac{v_{\text{T}} - 300i_\Delta}{700}, \qquad 1000i_\Delta = v_{\text{T}}$$

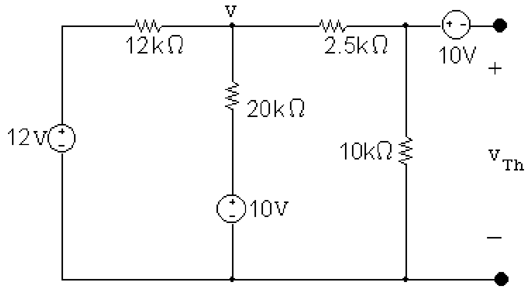
$$\therefore \quad i_{\text{T}} = \frac{v_{\text{T}}}{10} - 20\frac{v_{\text{T}}}{1000} = 0.08v_{\text{T}}$$

$$\frac{v_{\text{T}}}{i_{\text{T}}} = 1/0.08 = 12.5\,\Omega$$

$$\therefore \quad R_{\text{Th}} = 12.5\,\Omega$$



P 4.79 [**a**]

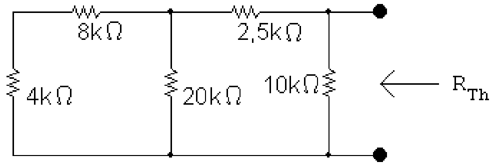


$$\frac{v-12}{12{,}000}+\frac{v-10}{20{,}000}+\frac{v}{12{,}500}=0$$

Solving, $v = 7.03125$ V

$$v_{10k}=\frac{10{,}000}{12{,}500}(7.03125)=5.625 \text{ V}$$

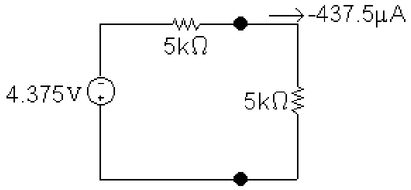
$\therefore \quad V_{Th}=v-10=-4.375$ V



$R_{Th}=[(12{,}000\|20{,}000)+2500]=5$ kΩ

$R_o=R_{Th}=5$ kΩ

[**b**]



$$p_{\max}=(-437.5\times10^{-6})^2(5000)=957\,\mu\text{W}$$

[**c**] The resistor closest to 5 kΩ from Appendix H has a value of 4.7 kΩ. Use voltage division to find the voltage drop across this load resistor, and use the voltage to find the power delivered to it:

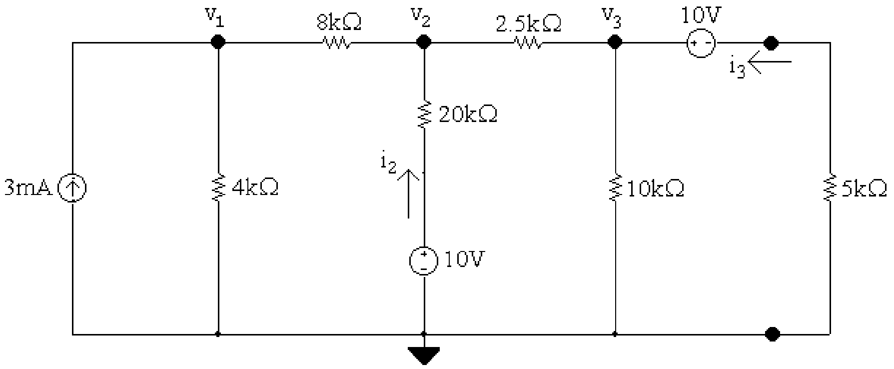
$$v_{4.7k}=\frac{4700}{4700+5000}(-4.375)=-2.12 \text{ V}$$

$$p_{4.7k}=\frac{(-2.12)^2}{4700}=956.12\,\mu\text{W}$$

The percent error between the maximum power and the power delivered to the best resistor from Appendix H is

$$\%\text{ error}=\left(\frac{956}{957}-1\right)(100)=-0.1\%$$

P 4.80 Write KCL equations at each of the labeled nodes, place them in standard form, and solve:



At v_1: $$-3\times10^{-3}+\frac{v_1}{4000}+\frac{v_1-v_2}{8000}=0$$

At v_2: $$\frac{v_2-v_1}{8000}+\frac{v_2-10}{20{,}000}+\frac{v_2-v_3}{2500}=0$$

At v_3: $$\frac{v_3-v_2}{2500}+\frac{v_3}{10{,}000}+\frac{v_3-10}{5000}=0$$

Standard form:

$$v_1\left(\frac{1}{4000}+\frac{1}{8000}\right)+v_2\left(-\frac{1}{8000}\right)+v_3(0)=0.003$$

$$v_1\left(-\frac{1}{8000}\right)+v_2\left(\frac{1}{8000}+\frac{1}{20{,}000}+\frac{1}{2500}\right)+v_3\left(-\frac{1}{2500}\right)=\frac{10}{20{,}000}$$

$$v_1(0)+v_2\left(-\frac{1}{2500}\right)+v_3\left(\frac{1}{2500}+\frac{1}{10{,}000}+\frac{1}{5000}\right)=\frac{10}{5000}$$

Calculator solution:

$v_1 = 10.890625\ \text{V}$ $\quad v_2 = 8.671875\ \text{V}$ $\quad v_3 = 7.8125\ \text{V}$

Calculate currents:

$$i_2=\frac{10-v_2}{20{,}000}=66.40625\,\mu\ \text{A} \qquad i_3=\frac{10-v_3}{5000}=437.5\,\mu\ \text{A}$$

Calculate power delivered by the sources:

$$p_{3\text{mA}}=(3\times10^{-3})v_1=(3\times10^{-3})(10.890625)=32.671875\ \text{mW}$$

$$p_{\text{10Vmiddle}} = i_2(10) = (66.40625 \times 10^{-6})(10) = 0.6640625 \text{ mW}$$

$$p_{\text{10Vtop}} = i_3(10) = (437.5 \times 10^{-6})(10) = 4.375 \text{ mW}$$

$$p_{\text{deliveredtotal}} = 32.671875 + 0.6640625 + 4.375 = 37.7109375 \text{ mW}$$

Calculate power absorbed by the 5 kΩ resistor and the percentage power:

$$p_{\text{5k}} = i_3^2(5000) = (437.5 \times 10^{-6})^2(5000) = 0.95703125 \text{ mW}$$

$$\% \text{ delivered to } R_o: \quad \frac{0.95793125}{37.7109375}(100) = 2.54\%$$

P 4.81 [**a**] Since $0 \leq R_o \leq \infty$ maximum power will be delivered to the 6 Ω resistor when $R_o = 0$.

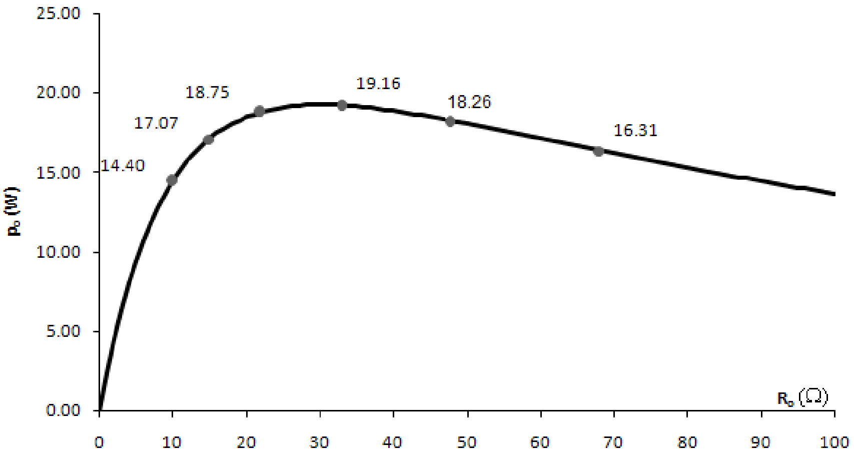
[**b**] $P = \dfrac{30^2}{6} = 150$ W

P 4.82 [**a**] From the solution to Problem 4.68 we have

$R_o(\Omega)$	$p_o(\text{W})$
10	14.4
15	17.07
22	18.75
33	19.16
47	18.26
68	16.31

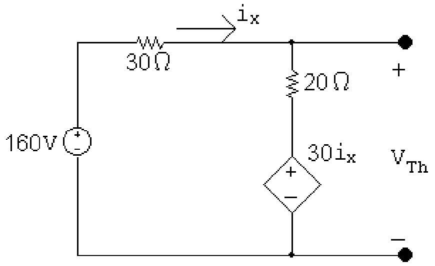
The 33 Ω resistor dissipates the most power, because its value is closest to the Thévenin equivalent resistance of the circuit.

[**b**]



[c] $R_o = 33\,\Omega$, $p_o = 19.16$ W. Compare this to $R_o = R_{\text{Th}} = 30\,\Omega$, which then gives the maximum power delivered to the load, p_o (max) $= 19.2$ W.

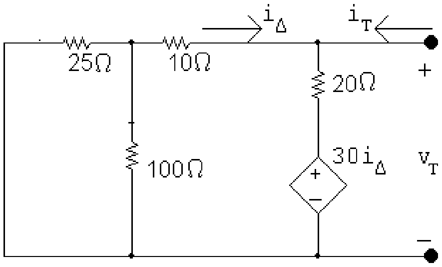
P 4.83 We begin by finding the Thévenin equivalent with respect to R_o. After making a couple of source transformations the circuit simplifies to



$$i_\Delta = \frac{160 - 30i_\Delta}{50}; \qquad i_\Delta = 2 \text{ A}$$

$$V_{\text{Th}} = 20i_\Delta + 30i_\Delta = 50i_\Delta = 100 \text{ V}$$

Using the test-source method to find the Thévenin resistance gives

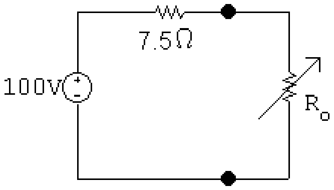


$$i_{\text{T}} = \frac{v_{\text{T}}}{30} + \frac{v_{\text{T}} - 30(-v_{\text{T}}/30)}{20}$$

$$\frac{i_{\text{T}}}{v_{\text{T}}} = \frac{1}{30} + \frac{1}{10} = \frac{4}{30} = \frac{2}{15}$$

$$R_{\text{Th}} = \frac{v_{\text{T}}}{i_{\text{T}}} = \frac{15}{2} = 7.5\,\Omega$$

Thus our problem is reduced to analyzing the circuit shown below.



$$p = \left(\frac{100}{7.5 + R_o}\right)^2 R_o = 250$$

$$\frac{10^4}{R_o^2 + 15R_o + 56.25}R_o = 250$$

$$\frac{10^4 R_o}{250} = R_o^2 + 15R_o + 56.25$$

$$40R_o = R_o^2 + 15R_o + 56.25$$

$$R_o^2 - 25R_o + 56.25 = 0$$

$$R_o = 12.5 \pm \sqrt{156.25 - 56.25} = 12.5 \pm 10$$

$$R_o = 22.5\,\Omega$$

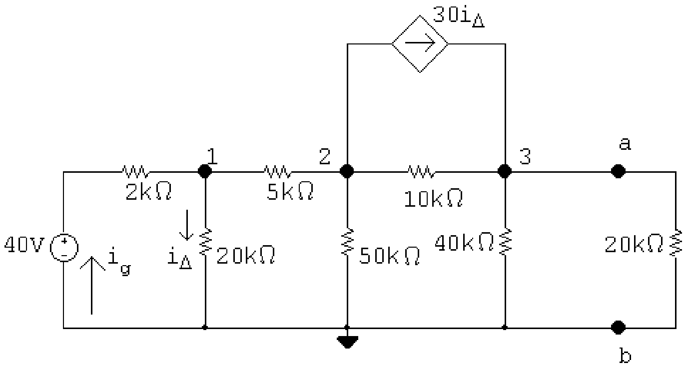
$$R_o = 2.5\,\Omega$$

P 4.84 **[a]** From the solution of Problem 4.73 we have $R_{\text{Th}} = 20$ kΩ and $V_{\text{Th}} = 280$ V. Therefore

$$R_o = R_{\text{Th}} = 20\text{ k}\Omega$$

[b] $p = \dfrac{(140)^2}{20{,}000} = 980$ mW

[c]



The node voltage equations are:

$$\begin{aligned}
\frac{v_1 - 40}{2000} + \frac{v_1}{20{,}000} + \frac{v_1 - v_2}{5000} &= 0 \\
\frac{v_2 - v_1}{5000} + \frac{v_2}{50{,}000} + \frac{v_2 - v_3}{10{,}000} + 30i_\Delta &= 0 \\
\frac{v_3 - v_2}{10{,}000} + \frac{v_3}{40{,}000} - 30i_\Delta + \frac{v_3}{20{,}000} &= 0
\end{aligned}$$

The dependent source constraint equation is:

$$i_\Delta = \frac{v_1}{20{,}000}$$

Place these equations in standard form:

$$v_1\left(\frac{1}{2000}+\frac{1}{20{,}000}+\frac{1}{5000}\right)+v_2\left(-\frac{1}{5000}\right)+v_3(0)+i_\Delta(0)=\frac{40}{2000}$$

$$v_1\left(-\frac{1}{4000}\right)+v_2\left(\frac{1}{4000}+\frac{1}{50{,}000}+\frac{1}{10{,}000}\right)+v_3\left(-\frac{1}{10{,}000}\right)+i_\Delta(30)=0$$

$$v_1(0)+v_2\left(-\frac{1}{10{,}000}\right)+v_3\left(\frac{1}{10{,}000}+\frac{1}{40{,}000}+\frac{1}{20{,}000}\right)+i_\Delta(-30)=0$$

$$v_1\left(\frac{-1}{20{,}000}\right)+v_2(0)+v_3(0)+i_\Delta(1)=0$$

Solving, $v_1 = 18.4$ V; $v_2 = -31$ V; $v_3 = 140$ V; $i_\Delta = 920\,\mu$A

Calculate the power:

$$i_g = \frac{40-18.4}{2000} = 10.8 \text{ mA}$$

$$p_{40\text{V}} = -(40)(10.8\times 10^{-3}) = -432 \text{ mW}$$

$$p_{\text{dep source}} = (v_2 - v_3)(30 i_\Delta) = -4719.6 \text{ mW}$$

$$\sum p_{\text{dev}} = 432 + 4719.6 = 5151.6 \text{ mW}$$

$$\% \text{ delivered } = \frac{980\times 10^{-3}}{5151.6\times 10^{-3}} \times 100 = 19.02\%$$

[d] There are two resistor values in Appendix H that fit the criterion – 18 kΩ and 22 kΩ. Let's use the Thévenin equivalent circuit to calculate the power delivered to each in turn, first by calculating the current through the load resistor and then using the current to calculate to power delivered to the load:

$$i_{18\text{k}} = \frac{280}{20{,}000+18{,}000} = 7.368 \text{ m A}$$

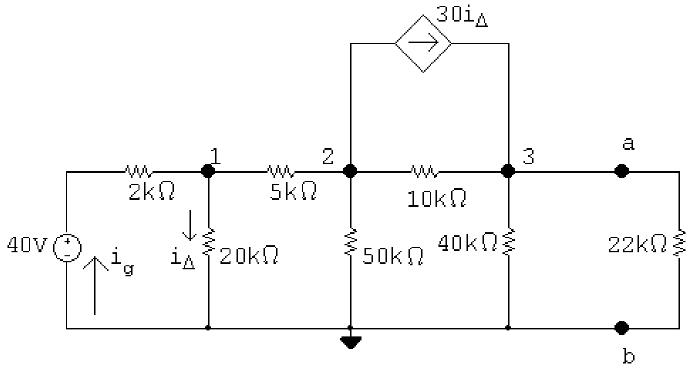
$$p_{18\text{k}} = (7.368)^2(18{,}000) = 977.17 \text{ m W}$$

$$i_{22\text{k}} = \frac{280}{20{,}000+22{,}000} = 6.667 \text{ m A}$$

$$p_{22\text{k}} = (6.667)^2(22{,}000) = 977.88 \text{ m W}$$

We select the 22 kΩ resistor, as the power delivered to it is closer to the maximum power of 980 mW.

[e] Now substitute the 22 kΩ resistor into the original circuit and calculate the power developed by the sources in this circuit:



The node voltage equations are:

$$\frac{v_1 - 40}{2000} + \frac{v_1}{20{,}000} + \frac{v_1 - v_2}{5000} = 0$$

$$\frac{v_2 - v_1}{5000} + \frac{v_2}{50{,}000} + \frac{v_2 - v_3}{10{,}000} + 30i_\Delta = 0$$

$$\frac{v_3 - v_2}{10{,}000} + \frac{v_3}{40{,}000} - 30i_\Delta + \frac{v_3}{22{,}000} = 0$$

The dependent source constraint equation is:

$$i_\Delta = \frac{v_1}{20{,}000}$$

Place these equations in standard form:

$$v_1\left(\frac{1}{2000} + \frac{1}{20{,}000} + \frac{1}{5000}\right) + v_2\left(-\frac{1}{5000}\right) + v_3(0) + i_\Delta(0) = \frac{40}{2000}$$

$$v_1\left(-\frac{1}{5000}\right) + v_2\left(\frac{1}{5000} + \frac{1}{50{,}000} + \frac{1}{10{,}000}\right) + v_3\left(-\frac{1}{10{,}000}\right) + i_\Delta(30) = 0$$

$$v_1(0) + v_2\left(-\frac{1}{10{,}000}\right) + v_3\left(\frac{1}{10{,}000} + \frac{1}{40{,}000} + \frac{1}{22{,}000}\right) + i_\Delta(-30) = 0$$

$$v_1\left(\frac{-1}{20{,}000}\right) + v_2(0) + v_3(0) + i_\Delta(1) = 0$$

Solving, $v_1 = 18.67$ V; $v_2 = -30$ V; $v_3 = 146.67$ V; $i_\Delta = 933.3\,\mu$A

Calculate the power:

$$i_g = \frac{40 - 18.67}{2000} = 10.67 \text{ mA}$$

$$p_{40\text{V}} = -(40)(10.67 \times 10^{-3}) = -426.67 \text{ mW}$$

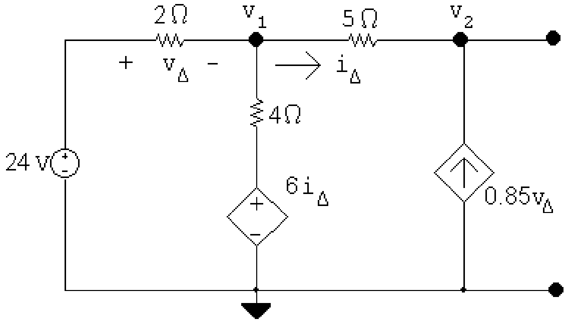
$$p_{\text{dep source}} = (v_2 - v_3)(30i_\Delta) = -4946.67 \text{ mW}$$

$$\sum p_{\text{dev}} = 426.67 + 4946.67 = 5373.33 \text{ mW}$$

$$p_L = (146.67)^2/22{,}000 = 977.78 \text{ mW}$$

$$\% \text{ delivered } = \frac{977.78 \times 10^{-3}}{5373.33 \times 10^{-3}} \times 100 = 18.20\%$$

P 4.85 [**a**] Open circuit voltage



Node voltage equations:

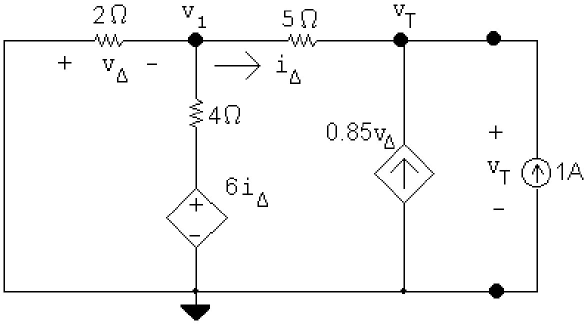
$$\frac{v_1 - 24}{2} + \frac{v_1 - 6i_\Delta}{4} + \frac{v_1 - v_2}{5} = 0$$

$$\frac{v_2 - v_1}{5} - 0.85v_\Delta = 0$$

Constraint equations:

$$i_\Delta = \frac{v_1 - v_2}{5}; \qquad v_\Delta = 24 - v_1$$

Solving, $v_2 = 84\text{ V} = v_{\text{Th}}$
Thévenin resistance using a test source:



$$\frac{v_1}{2} + \frac{v_1 - 6i_\Delta}{4} + \frac{v_1 - v_T}{5} = 0$$

$$\frac{v_T - v_1}{5} - 0.85v_\Delta - 1 = 0$$

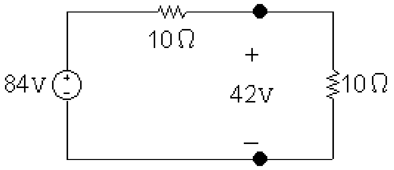
$$i_\Delta = \frac{v_1 - v_T}{5}; \qquad v_\Delta = -v_1$$

Solving, $v_T = 10$

$$R_{\text{Th}} = \frac{v_T}{1} = 10\,\Omega$$

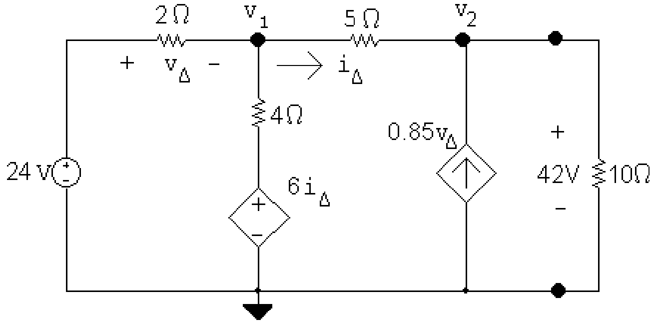
$$\therefore \quad R_o = R_{\text{Th}} = 10\,\Omega$$

[**b**]



$$p_{\text{max}} = \frac{(42)^2}{10} = 176.4 \text{ W}$$

[**c**]



$$\frac{v_1 - 24}{2} + \frac{v_1 - 6i_\Delta}{4} + \frac{v_1 - 42}{5} = 0$$

$$i_\Delta = \frac{v_1 - 42}{5}$$

Solving, $v_1 = 12$ V; $i_\Delta = -6$ A; $v_\Delta = 24 - v_1 = 24 - 12 = 12$ V

$$i_{24\text{V}} = \frac{24 - v_1}{2} = 6 \text{ A}$$

$$p_{24\text{V}} = -24 i_{24\text{V}} = -24(6) = -144 \text{ W}$$

$$i_{\text{CCVS}} = \frac{v_1 - 6i_\Delta}{4} = 12 \text{ A}$$

$$p_{\text{CCVS}} = [6(-6)](12) = -432 \text{ W}$$

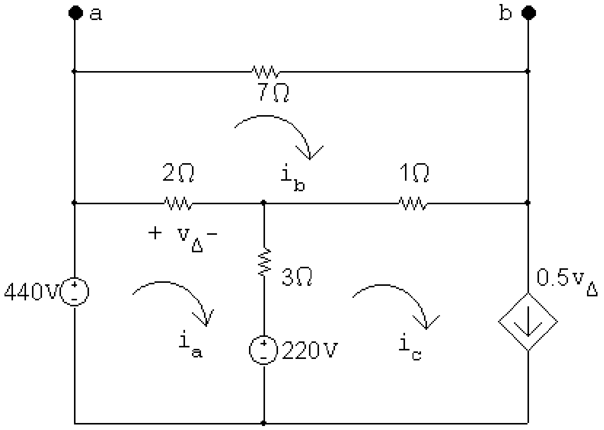
$$p_{\text{VCCS}} = -[0.85(12)](42) = -428.4 \text{ W}$$

$$\sum p_{\text{dev}} = 144 + 432 + 428.4 = 1004.4 \text{ W}$$

$$\% \text{ delivered } = \frac{176.4}{1004.4} \times 100 = 17.56\%$$

P 4.86 Find the Thévenin equivalent with respect to the terminals of R_o.

Open circuit voltage:



$$(440 - 220) = 5i_a - 2i_b - 3i_c$$

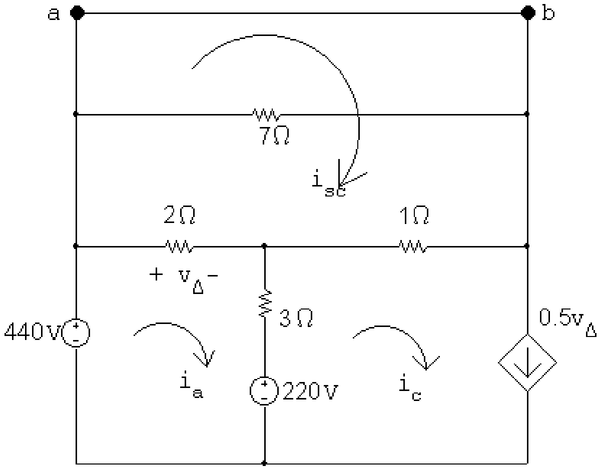
$$0 = -2i_a + 10i_b - 1i_c$$

$$i_c = 0.5v_\Delta; \qquad v_\Delta = 2(i_a - i_b)$$

Solving, $i_b = 26.4$ A

$$\therefore \quad v_{\text{Th}} = 7i_b = 184.8 \text{ V}$$

Short circuit current:



$$440 - 220 = 5i_a - 2i_{\text{sc}} - 3i_c$$

$$0 = -2i_a + 3i_{\text{sc}} - 1i_c$$

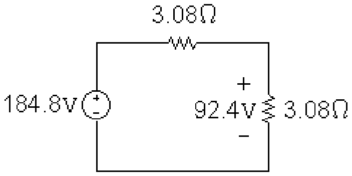
$$i_c = 0.5v_\Delta; \qquad v_\Delta = 2(i_a - i_{\text{sc}})$$

Solving, $i_{\rm sc} = 60$ A; $\quad i_a = 80$ A; $\quad i_c = 20$ A

$R_{\rm Th} = v_{\rm Th}/i_{\rm sc} = 184.8/60 = 3.08\,\Omega$

$R_o = 3.08\,\Omega$

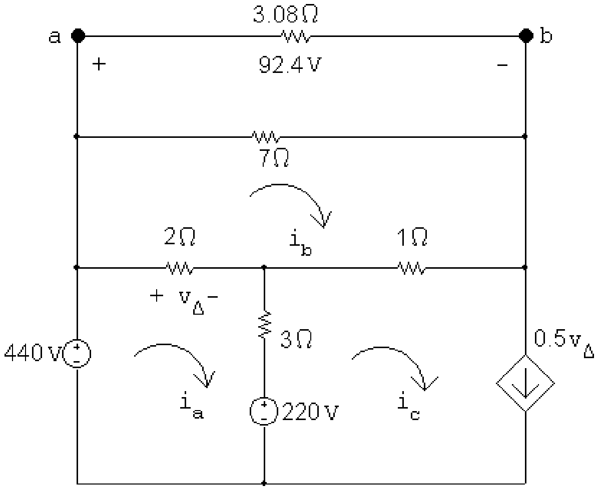
Therefore, the Thévenin equivalent circuit configured for maximum power to the load is



From this circuit,

$$p_{\max} = \frac{(92.4)^2}{3.08} = 2772 \text{ W}$$

With R_o equal to 3.08 Ω the original circuit becomes



$440 - 220 = 5i_a - 2i_b - 3i_c$

$i_c = 0.5v_\Delta; \qquad v_\Delta = 2(i_a - i_b)$

$-92.4 = -2i_a + 3i_b - 1i_c$

Solving, $i_a = 88.4$ A; $\qquad i_b = 43.2$ A; $\qquad i_c = 45.2$ A

$v_\Delta = 2(88.4 - 43.2) = 90.4$ V

$p_{440\rm V} = -(440)(88.4) = -38{,}896$ W

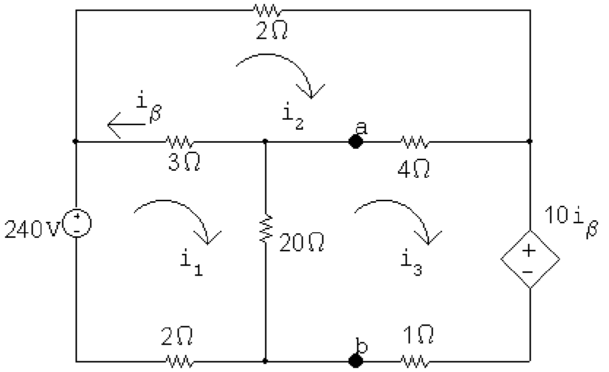
$$p_{220\text{V}} = (220)(88.4 - 45.2) = 9504 \ \text{W}$$

$$p_{\text{dep.source}} = (440 - 92.4)[0.5(90.4)] = 15{,}711.52 \ \text{W}$$

Therefore, only the 440 V source supplies power to the circuit, and the power supplied is 38,896 W.

$$\% \text{ delivered } = \frac{2772}{38{,}896} = 7.13\%$$

P 4.87 **[a]** Find the Thévenin equivalent with respect to the terminals of R_{L}.
Open circuit voltage:



The mesh current equations are:

$$\begin{aligned} -240 + 3(i_1 - i_2) + 20(i_1 - i_3) + 2i_1 &= 0 \\ 2i_2 + 4(i_2 - i_3) + 3(i_2 - i_1) &= 0 \\ 10i_\beta + 1i_3 + 20(i_3 - i_1) + 4(i_3 - i_2) &= 0 \end{aligned}$$

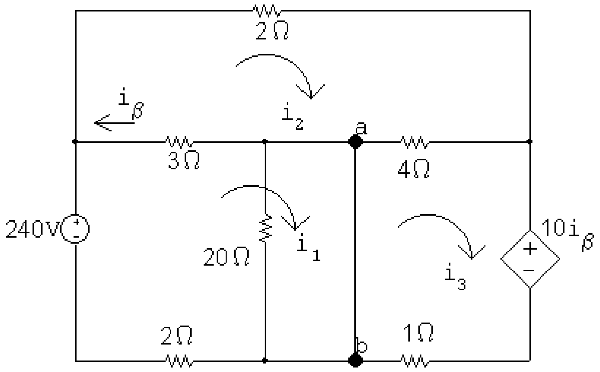
The dependent source constraint equation is:
$i_\beta = i_2 - i_1$

Place these equations in standard form:

$$\begin{aligned} i_1(3 + 20 + 2) + i_2(-3) + i_3(-20) + i_\beta(0) &= 240 \\ i_1(-3) + i_2(2 + 4 + 3) + i_3(-4) + i_\beta(0) &= 0 \\ i_1(-20) + i_2(-4) + i_3(1 + 20 + 4) + i_\beta(10) &= 0 \\ i_1(-1) + i_2(1) + i_3(0) + i_\beta(-1) &= 0 \end{aligned}$$

Solving, $i_1 = 99.6$ A; $i_2 = 78$ A; $i_3 = 100.8$ A; $i_\beta = 21.6$ A
$V_{\text{Th}} = 20(i_1 - i_3) = -24$ V

Short-circuit current:



The mesh current equations are:

$$\begin{aligned} -240 + 3(i_1 - i_2) + 2i_1 &= 0 \\ 2i_2 + 4(i_2 - i_3) + 3(i_2 - i_1) &= 0 \\ 10i_\beta + 1i_3 + 4(i_3 - i_2) &= 0 \end{aligned}$$

The dependent source constraint equation is:

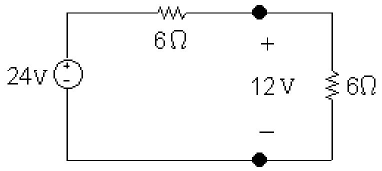
$i_\beta = i_2 - i_1$

Place these equations in standard form:

$$\begin{aligned} i_1(3+2) + i_2(-3) + i_3(0) + i_\beta(0) &= 240 \\ i_1(-3) + i_2(2+4+3) + i_3(-4) + i_\beta(0) &= 0 \\ i_1(0) + i_2(-4) + i_3(4+1) + i_\beta(10) &= 0 \\ i_1(-1) + i_2(1) + i_3(0) + i_\beta(-1) &= 0 \end{aligned}$$

Solving, $i_1 = 92$ A; $i_2 = 73.33$ A; $i_3 = 96$ A; $i_\beta = 18.67$ A

$i_{\text{sc}} = i_1 - i_3 = -4$ A; $\quad R_{\text{Th}} = \dfrac{V_{\text{Th}}}{i_{\text{sc}}} = \dfrac{-24}{-4} = 6\,\Omega$

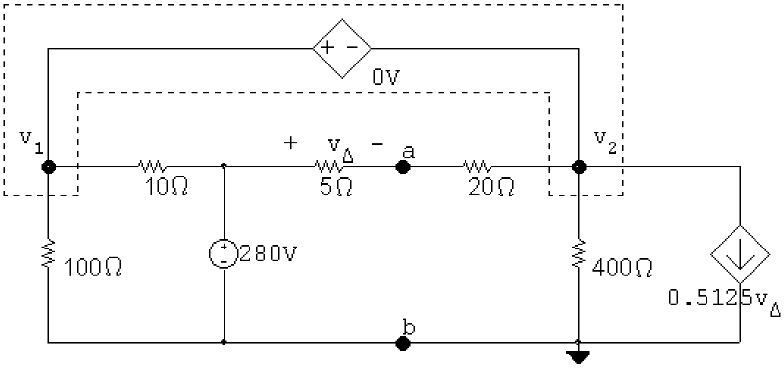


$R_{\text{L}} = R_{\text{Th}} = 6\,\Omega$

[b] $p_{\max} = \dfrac{12^2}{6} = 24$ W

P 4.88 **[a]** First find the Thévenin equivalent with respect to R_o.

Open circuit voltage: $i_\phi = 0$; $50i_\phi = 0$



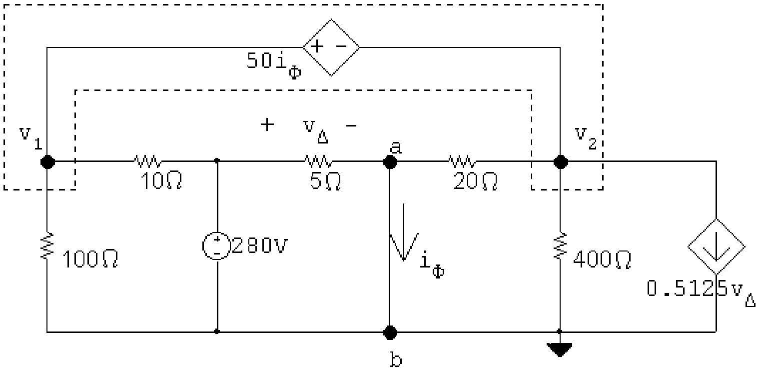
$$\frac{v_1}{100} + \frac{v_1 - 280}{10} + \frac{v_1 - 280}{25} + \frac{v_1}{400} + 0.5125v_\Delta = 0$$

$$v_\Delta = \frac{(280 - v_1)}{25}5 = 56 - 0.2v_1$$

$$v_1 = 210 \text{ V}; \qquad v_\Delta = 14 \text{ V}$$

$$V_{\text{Th}} = 280 - v_\Delta = 280 - 14 = 266 \text{ V}$$

Short circuit current



$$\frac{v_1}{100} + \frac{v_1 - 280}{10} + \frac{v_2}{20} + \frac{v_2}{400} + 0.5125(280) = 0$$

$$v_\Delta = 280 \text{ V}$$

$$v_2 + 50i_\phi = v_1$$

$$i_\phi = \frac{280}{5} + \frac{v_2}{20} = 56 + 0.05v_2$$

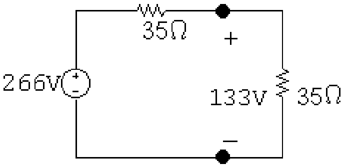
$$v_2 = -968 \text{ V}; \qquad v_1 = -588 \text{ V}$$

$$i_\phi = i_{\text{sc}} = 56 + 0.05(-968) = 7.6 \text{ A}$$

$$R_{\text{Th}} = V_{\text{Th}}/i_{\text{sc}} = 266/7.6 = 35\,\Omega$$

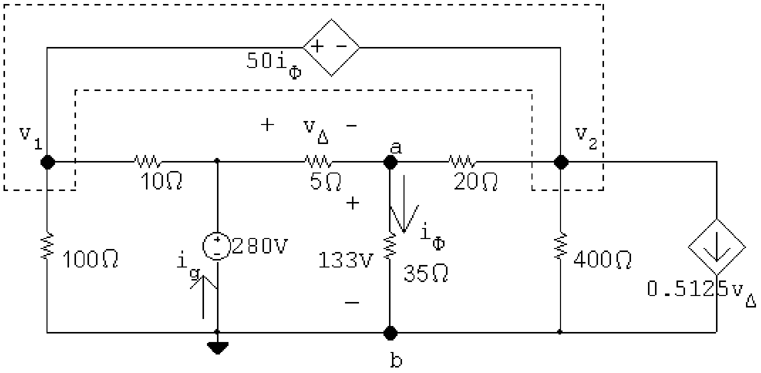
$$\therefore \quad R_o = 35\,\Omega$$

[**b**]



$p_{\max} = (133)^2/35 = 505.4$ W

[**c**]



$$\frac{v_1}{100} + \frac{v_1 - 280}{10} + \frac{v_2 - 133}{20} + \frac{v_2}{400} + 0.5125(280 - 133) = 0$$

$v_2 + 50i_\phi = v_1$; $i_\phi = 133/35 = 3.8$ A

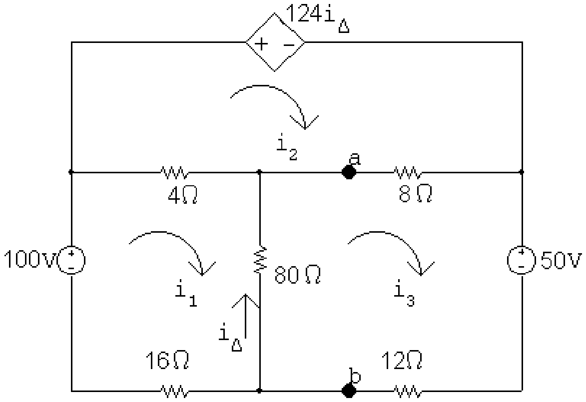
Therefore, $v_1 = -189$ V and $v_2 = -379$ V; thus,

$$i_g = \frac{280 - 133}{5} + \frac{280 + 189}{10} = 76.30 \text{ A}$$

$p_{280\text{V}}$ (dev) $= (280)(76.3) = 21{,}364$ W

P 4.89 [**a**] We begin by finding the Thévenin equivalent with respect to the terminals of R_o.

Open circuit voltage



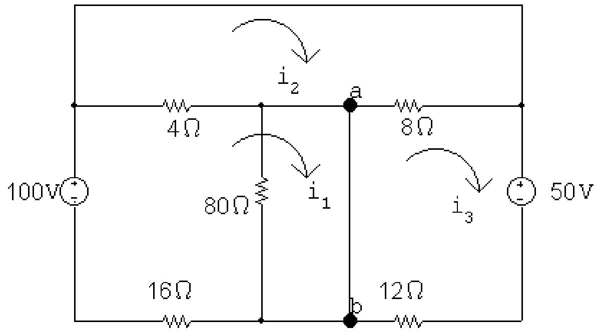
The mesh current equations are:

$$\begin{aligned}
-100 + 4(i_1 - i_2) + 80(i_1 - i_3) + 16i_1 &= 0\\
124i_\Delta + 8(i_2 - i_3) + 4(i_2 - i_1) &= 0\\
50 + 12i_3 + 80(i_3 - i_1) + 8(i_3 - i_2) &= 0
\end{aligned}$$

The constraint equation is:
$i_\Delta = i_3 - i_1$
Place these equations in standard form:

$$\begin{aligned}
i_1(4 + 80 + 16) + i_2(-4) + i_3(-80) + i_\Delta(0) &= 100\\
i_1(-4) + i_2(8 + 4) + i_3(-8) + i_\Delta(124) &= 0\\
i_1(-80) + i_2(-8) + i_3(12 + 80 + 8) + i_\Delta(0) &= -50\\
i_1(1) + i_2(0) + i_3(-1) + i_\Delta(1) &= 0
\end{aligned}$$

Solving, $i_1 = 4.7$ A; $i_2 = 10.5$ A; $i_3 = 4.1$ A; $i_\Delta = -0.6$ A
Also, $V_{\text{Th}} = v_{\text{ab}} = -80i_\Delta = 48$ V
Now find the short-circuit current.



Note with the short circuit from a to b that i_Δ is zero, hence $124i_\Delta$ is also zero.

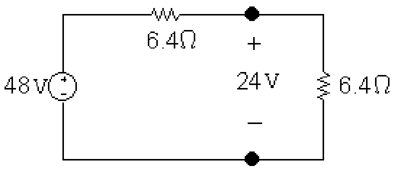
The mesh currents are:

$$\begin{aligned}
-100 + 4(i_1 - i_2) + 16i_1 &= 0\\
8(i_2 - i_3) + 4(i_2 - i_1) &= 0\\
50 + 12i_3 + 8(i_3 - i_2) &= 0
\end{aligned}$$

Place these equations in standard form:

$$\begin{aligned}
i_1(4 + 16) + i_2(-4) + i_3(0) &= 100\\
i_1(-4) + i_2(8 + 4) + i_3(-8) &= 0\\
i_1(0) + i_2(-8) + i_3(12 + 8) &= -50
\end{aligned}$$

Solving, $i_1 = 5$ A; $i_2 = 0$ A; $i_3 = -2.5$ A
Then, $i_{\text{sc}} = i_1 - i_3 = 7.5$ A
$R_{\text{Th}} = 48/7.5 = 6.4\,\Omega$



For maximum power transfer $R_o = R_{\text{Th}} = 6.4\,\Omega$

[b] $p_{\max} = \dfrac{24^2}{6.4} = 90$ W

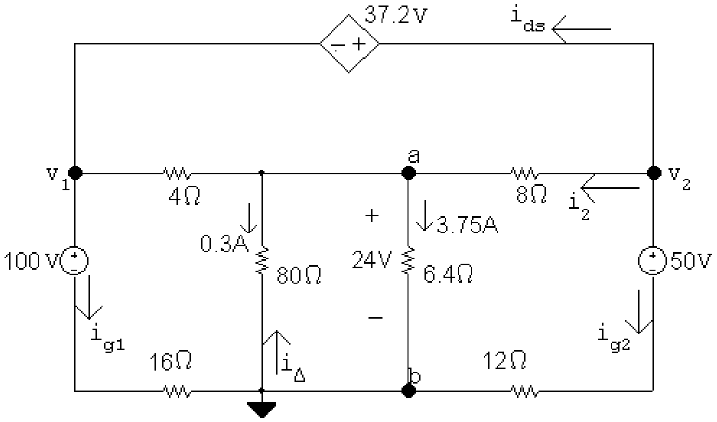
[c] The resistor from Appendix H that is closest to the Thévenin resistance is 10 Ω. To calculate the power delivered to a 10 Ω load resistor, calculate the current using the Thévenin circuit and use it to find the power delivered to the load resistor:

$$i_{10} = \frac{48}{6.4 + 10} = 2.927 \text{ A}$$

$$p_{10} = 10(2.927)^2 = 85.7 \text{ W}$$

Thus, using a 10 Ω resistor selected from Appendix H will cause 85.7 W of power to be delivered to the load, compared to the maximum power of 90 W that will be delivered if a 6.4 Ω resistor is used.

P 4.90 From the solution of Problem 4.89 we know that when R_o is 6.4 Ω, the voltage across R_o is 24 V, positive at the upper terminal. Therefore our problem reduces to the analysis of the following circuit. In constructing the circuit we have used the fact that i_Δ is −0.3 A, and hence $124i_\Delta$ is −37.2 V.



Using the node voltage method to find v_1 and v_2 yields

$$4.05 + \frac{24 - v_1}{4} + \frac{24 - v_2}{8} = 0$$

$$2v_1 + v_2 = 104.4; \qquad v_1 + 37.2 = v_2$$

Solving, $v_1 = 22.4$ V; $\qquad v_2 = 59.6$ V.

It follows that

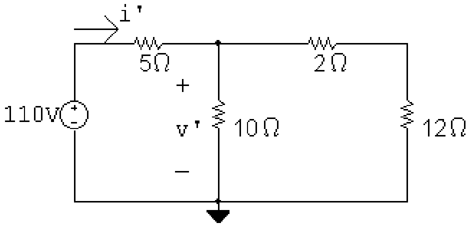
$$
\begin{aligned}
i_{g_1} &= \frac{22.4-100}{16} = -4.85 \text{ A} \\
i_{g_2} &= \frac{59.6-50}{12} = 0.8 \text{ A} \\
i_2 &= \frac{59.6-24}{8} = 4.45 \text{ A} \\
i_{\text{ds}} &= -4.45-0.8 = -5.25 \text{ A} \\
p_{100\text{V}} &= 100 i_{g_1} = -485 \text{ W} \\
p_{50\text{V}} &= 50 i_{g_2} = 40 \text{ W} \\
p_{\text{ds}} &= 37.2 i_{\text{ds}} = -195.3 \text{ W}
\end{aligned}
$$

$$\therefore \quad \sum p_{\text{dev}} = 485 + 195.3 = 680.3 \text{ W}$$

$$\therefore \quad \% \text{ delivered } = \frac{90}{680.3}(100) = 13.23\%$$

$\therefore$ 13.23% of developed power is delivered to load

P 4.91 **[a]** 110 V source acting alone:

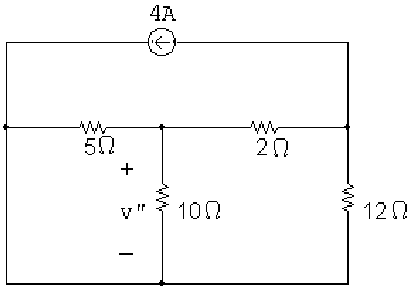


$$R_e = \frac{10(14)}{24} = \frac{35}{6}\,\Omega$$

$$i' = \frac{110}{5+35/6} = \frac{132}{13} \text{ A}$$

$$v' = \left(\frac{35}{6}\right)\left(\frac{132}{13}\right) = \frac{770}{13} \text{ V} = 59.231 \text{ V}$$

4 A source acting alone:

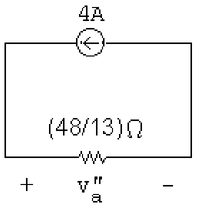


$$5\,\Omega \| 10\,\Omega = 50/15 = 10/3\,\Omega$$

$10/3 + 2 = 16/3\,\Omega$

$16/3\|12 = 48/13\,\Omega$

Hence our circuit reduces to:



It follows that

$v''_a = 4(48/13) = (192/13)$ V

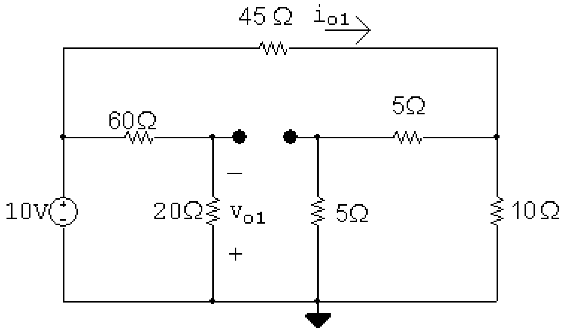
and

$$v'' = \frac{-v''_a}{(16/3)}(10/3) = -\frac{5}{8}v''_a = -(120/13)\text{ V} = -9.231\text{ V}$$

$$\therefore \qquad v = v' + v'' = \frac{770}{13} - \frac{120}{13} = 50\text{ V}$$

[b] $p = \dfrac{v^2}{10} = 250$ W

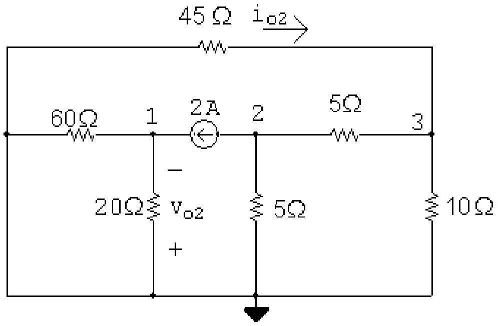
P 4.92 Voltage source acting alone:



$$i_{o1} = \frac{10}{45 + (5+5)\|10} = \frac{10}{45+5} = 0.2\text{ A}$$

$$v_{o1} = \frac{20}{20+60}(-10) = -2.5\text{ V}$$

Current source acting alone:



$$\frac{v_2}{5} + 2 + \frac{v_2 - v_3}{5} = 0$$

$$\frac{v_3}{10} + \frac{v_3 - v_2}{5} + \frac{v_3}{45} = 0$$

Solving, $v_2 = -7.25 \text{ V} = v_{o2}$; $\qquad v_3 = -4.5$ V

$$i_{o2} = -\frac{v_3}{45} = -0.1 \text{ A}$$

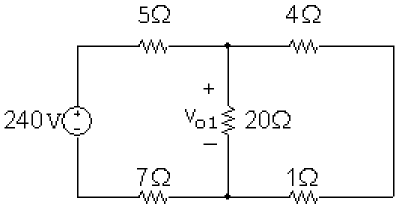
$$i_{20} = \frac{60\|20}{20}(2) = 1.5 \text{ A}$$

$$v_{o2} = -20 i_{20} = -20(1.5) = -30 \text{ V}$$

$$\therefore \quad v_o = v_{o1} + v_{o2} = -2.5 - 30 = -32.5 \text{ V}$$

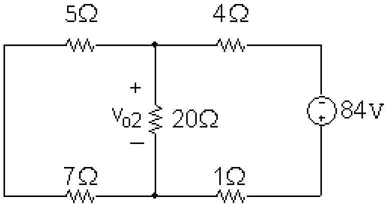
$$i_o = i_{o1} + i_{o2} = 0.2 + 0.1 = 0.3 \text{ A}$$

P 4.93 240 V source acting alone:



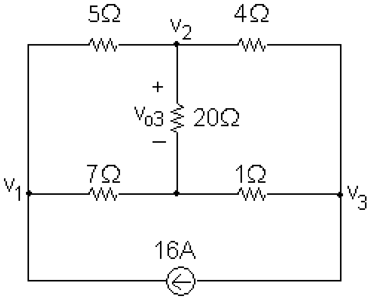
$$v_{o1} = \frac{20\|5}{5 + 7 + 20\|5}(240) = 60 \text{ V}$$

84 V source acting alone:



$$v_{o2} = \frac{20\|12}{1+4+20\|12}(-84) = -50.4 \text{ V}$$

16 A current source acting alone:



$$\frac{v_1 - v_2}{5} + \frac{v_1}{7} - 16 = 0$$

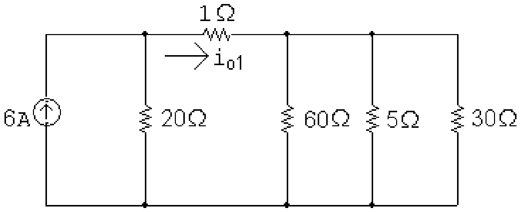
$$\frac{v_2 - v_1}{5} + \frac{v_2}{20} + \frac{v_2 - v_3}{4} = 0$$

$$\frac{v_3 - v_2}{4} + \frac{v_3}{1} + 16 = 0$$

Solving, $v_2 = 18.4\ \text{V} = v_{o3}$. Therefore,

$$v_o = v_{o1} + v_{o2} + v_{o3} = 60 - 50.4 + 18.4 = 28 \text{ V}$$

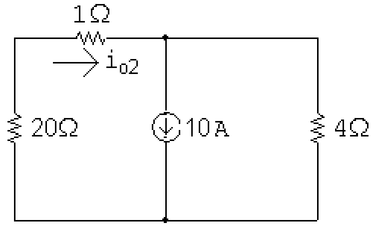
P 4.94 6 A source:



$$30\,\Omega\|5\,\Omega\|60\,\Omega = 4\,\Omega$$

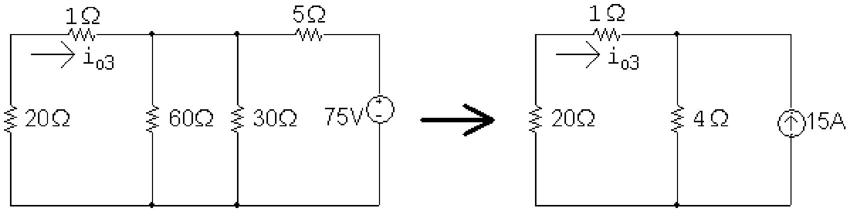
$$\therefore \quad i_{o1} = \frac{20}{20+5}(6) = 4.8 \text{ A}$$

10 A source:



$$i_{o2} = \frac{4}{25}(10) = 1.6 \text{ A}$$

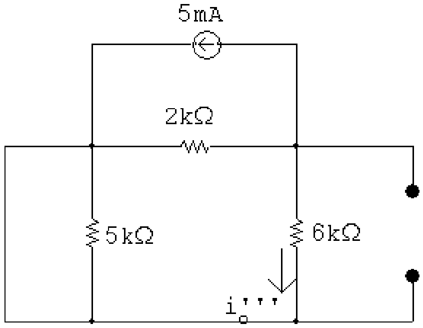
75 V source:



$$i_{o3} = -\frac{4}{25}(15) = -2.4 \text{ A}$$

$$i_o = i_{o1} + i_{o2} + i_{o3} = 4.8 + 1.6 - 2.4 = 4 \text{ A}$$

P 4.95 [**a**] By hypothesis $i_o' + i_o'' = 3$ mA.



$$i_o''' = -5\frac{(2)}{(8)} = -1.25 \text{ mA}; \qquad \therefore \quad i_o = 3.5 - 1.25 = 2.25 \text{ mA}$$

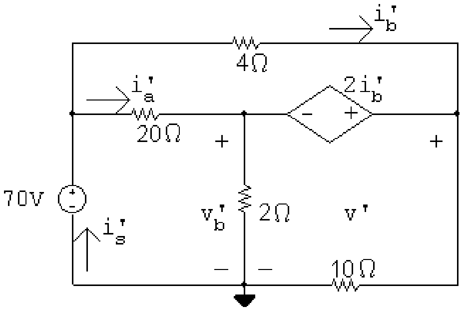
[**b**] With all three sources in the circuit write a single node voltage equation.

$$\frac{v_b}{6} + \frac{v_b - 8}{2} + 5 - 10 = 0$$

$$\therefore \quad v_b = 13.5 \text{ V}$$

$$i_o = \frac{v_b}{6} = 2.25 \text{ mA}$$

P 4.96 70-V source acting alone:



$$v' = 70 - 4i_b'$$

$$i_s' = \frac{v_b'}{2} + \frac{v'}{10} = i_a' + i_b'$$

$$70 = 20i_a' + v_b'$$

$$i_a' = \frac{70 - v_b'}{20}$$

$$\therefore \qquad i_b' = \frac{v_b'}{2} + \frac{v'}{10} - \frac{70 - v_b'}{20} = \frac{11}{20}v_b' + \frac{v'}{10} - 3.5$$

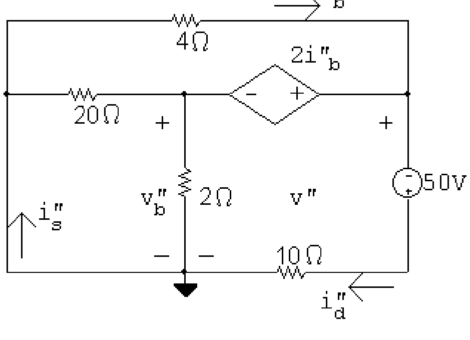
$$v' = v_b' + 2i_b'$$

$$\therefore \;\; v_b' = v' - 2i_b'$$

$$\therefore \;\; i_b' = \frac{11}{20}(v' - 2i_b') + \frac{v'}{10} - 3.5 \quad \text{or} \quad i_b' = \frac{13}{42}v' - \frac{70}{42}$$

$$\therefore \;\; v' = 70 - 4\left(\frac{13}{42}v' - \frac{70}{42}\right) \quad \text{or} \quad v' = \frac{3220}{94} = \frac{1610}{47}\text{ V} = 34.255\text{ V}$$

50-V source acting alone:



$$v'' = -4i_b''$$

$$v'' = v_b'' + 2i_b''$$

$$v'' = -50 + 10i_d''$$

$$\therefore \quad i_d'' = \frac{v'' + 50}{10}$$

$$i_s'' = \frac{v_b''}{2} + \frac{v'' + 50}{10}$$

$$i_b'' = \frac{v_b''}{20} + i_s'' = \frac{v_b''}{20} + \frac{v_b''}{2} + \frac{v'' + 50}{10} = \frac{11}{20}v_b'' + \frac{v'' + 50}{10}$$

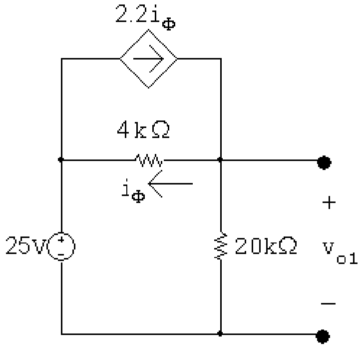
$$v_b'' = v'' - 2i_b''$$

$$\therefore \quad i_b'' = \frac{11}{20}(v'' - 2i_b'') + \frac{v'' + 50}{10} \qquad \text{or} \qquad i_b'' = \frac{13}{42}v'' + \frac{100}{42}$$

$$\text{Thus,} \quad v'' = -4\left(\frac{13}{42}v'' + \frac{100}{42}\right) \qquad \text{or} \qquad v'' = -\frac{200}{47}\text{ V} = -4.255\text{ V}$$

$$\text{Hence,} \quad v = v' + v'' = \frac{1610}{47} - \frac{200}{47} = \frac{1410}{47} = 30\text{ V}$$

P 4.97 Voltage source acting alone:

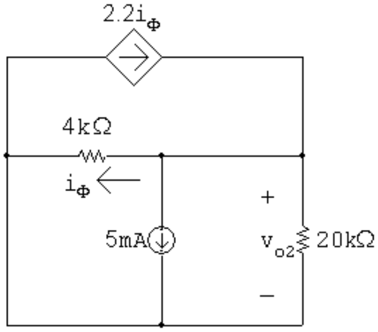


$$\frac{v_{o1} - 25}{4000} + \frac{v_{o1}}{20{,}000} - 2.2\left(\frac{v_{o1} - 25}{4000}\right) = 0$$

$$\text{Simplifying} \qquad 5v_{o1} - 125 + v_{o1} - 11v_{o1} + 275 = 0$$

$$\therefore \quad v_{o1} = 30\text{ V}$$

Current source acting alone:



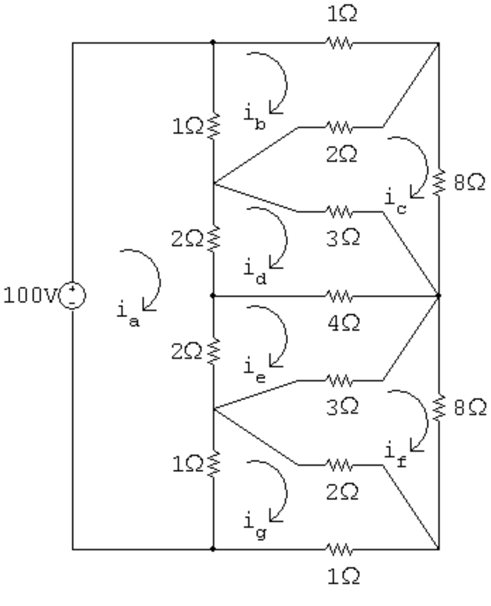
$$\frac{v_{o2}}{4000} + \frac{v_{o2}}{20{,}000} + 0.005 - 2.2\left(\frac{v_{o2}}{4000}\right) = 0$$

Simplifying $\qquad 5v_{o2} + v_{o2} + 100 - 11v_{o2} = 0$

$\therefore \quad v_{o2} = 20$ V

$v_o = v_{o1} + v_{o2} = 30 + 20 = 50$ V

P 4.98



$$\begin{aligned}
100 &= 6i_a - 1i_b + 0i_c - 2i_d - 2i_e + 0i_f - 1i_g \\
0 &= -1i_a + 4i_b - 2i_c + 0i_d + 0i_e + 0i_f + 0i_g \\
0 &= 0i_a - 2i_b + 13i_c - 3i_d + 0i_e + 0i_f + 0i_g \\
0 &= -2i_a + 0i_b - 3i_c + 9i_d - 4i_e + 0i_f + 0i_g \\
0 &= -2i_a + 0i_b + 0i_c - 4i_d + 9i_e - 3i_f + 0i_g \\
0 &= 0i_a + 0i_b + 0i_c + 0i_d - 3i_e + 13i_f - 2i_g \\
0 &= -1i_a + 0i_b + 0i_c + 0i_d + 0i_e - 2i_f + 4i_g
\end{aligned}$$

A computer solution yields

$i_a = 30$ A; $\quad i_e = 15$ A;

$i_b = 10$ A; $\quad i_f = 5$ A;

$i_c = 5$ A; $\quad i_g = 10$ A;

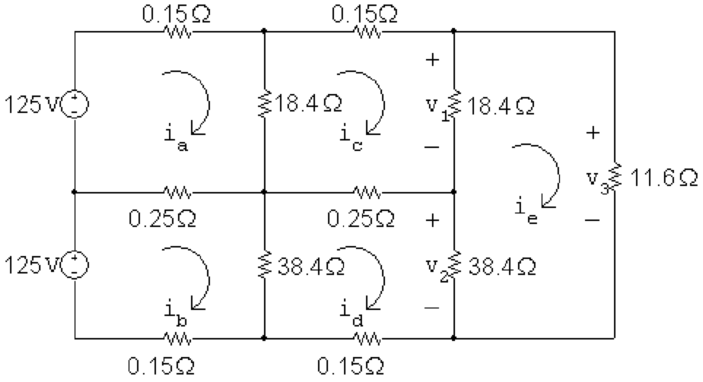
$i_d = 15$ A

$$\therefore \quad i = i_d - i_e = 0 \text{ A}$$

CHECK:
$$p_{\text{1T}} = p_{\text{1B}} = (i_b)^2 = (i_g)^2 = 100 \text{ W}$$
$$p_{\text{1L}} = (i_a - i_b)^2 = (i_a - i_g)^2 = 400 \text{ W}$$
$$p_{\text{2C}} = 2(i_b - i_c)^2 = (i_g - i_f)^2 = 50 \text{ W}$$
$$p3 = 3(i_c - i_d)^2 = 3(i_e - i_f)^2 = 300 \text{ W}$$
$$p4 = 4(i_d - i_e)^2 = 0 \text{ W}$$
$$p8 = 8(i_c)^2 = 8(i_f)^2 = 200 \text{ W}$$
$$p_{\text{2L}} = 2(i_a - i_d)^2 = 2(i_a - i_e)^2 = 450 \text{ W}$$

$$\begin{aligned}\sum p_{\text{abs}} &= 100 + 400 + 50 + 200 + 300 + 450 + 0 + 450 + 300 + \\ &\quad 200 + 50 + 400 + 100 = 3000 \text{ W}\\ \sum p_{\text{gen}} &= 100 i_a = 100(30) = 3000 \text{ W (CHECKS)}\end{aligned}$$

P 4.99



The mesh equations are:

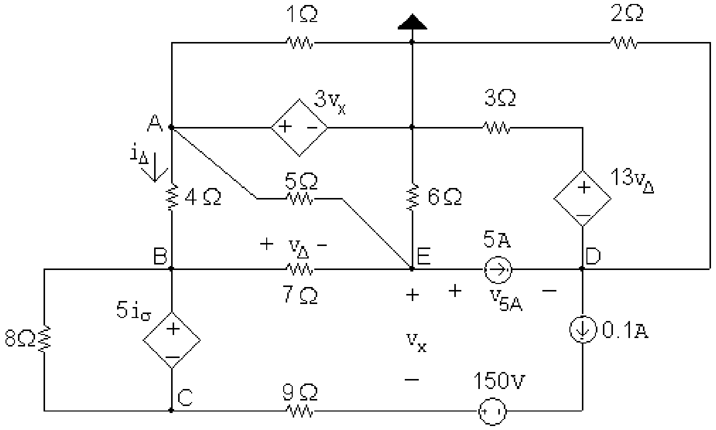
$$\begin{aligned}
-125 + 0.15i_{\text{a}} + 18.4(i_{\text{a}} - i_{\text{c}}) + 0.25(i_{\text{a}} - i_{\text{b}}) &= 0\\
-125 + 0.25(i_{\text{b}} - i_{\text{a}}) + 38.4(i_{\text{b}} - i_{\text{d}}) + 0.15i_{\text{b}} &= 0\\
0.15i_{\text{c}} + 18.4(i_{\text{c}} - i_{\text{e}}) + 0.25(i_{\text{c}} - i_{\text{d}}) + 18.4(i_{\text{c}} - i_{\text{a}}) &= 0\\
0.15i_{\text{d}} + 38.4(i_{\text{d}} - i_{\text{b}}) + 0.25(i_{\text{d}} - i_{\text{c}}) + 38.4(i_{\text{d}} - i_{\text{e}}) &= 0\\
11.6i_{\text{e}} + 38.4(i_{\text{e}} - i_{\text{d}}) + 18.4(i_{\text{e}} - i_{\text{c}}) &= 0
\end{aligned}$$

Place these equations in standard form:

$$\begin{aligned}
i_{\rm a}(18.8) + i_{\rm b}(-0.25) + i_{\rm c}(-18.4) + i_{\rm d}(0) + i_{\rm e}(0) &= 125\\
i_{\rm a}(-0.25) + i_{\rm b}(38.8) + i_{\rm c}(0) + i_{\rm d}(-38.4) + i_{\rm e}(0) &= 125\\
i_{\rm a}(-18.4) + i_{\rm b}(0) + i_{\rm c}(37.2) + i_{\rm d}(-0.25) + i_{\rm e}(-18.4) &= 0\\
i_{\rm a}(0) + i_{\rm b}(-38.4) + i_{\rm c}(-0.25) + i_{\rm d}(77.2) + i_{\rm e}(-38.4) &= 0\\
i_{\rm a}(0) + i_{\rm b}(0) + i_{\rm c}(-18.4) + i_{\rm d}(-38.4) + i_{\rm e}(68.4) &= 0
\end{aligned}$$

Solving,
$i_{\rm a} = 32.77$ A; $i_{\rm b} = 26.46$ A; $i_{\rm c} = 26.33$ A; $i_{\rm d} = 23.27$ A; $i_{\rm e} = 20.14$ A
Find the requested voltages:
$v_1 = 18.4(i_{\rm c} - i_{\rm e}) = 113.90$ V
$v_2 = 38.4(i_{\rm d} - i_{\rm e}) = 120.19$ V
$v_3 = 11.6i_{\rm e} = 233.62$ V

P 4.100



KCL equations at nodes B, D, and E:

$$\frac{v_{\rm B} - v_{\rm A}}{4} + \frac{v_{\rm B} - v_{\rm E}}{7} - 0.1 = 0$$

$$0.1 + \frac{v_{\rm D}}{2} + \frac{v_{\rm D} + 13v_\Delta}{3} - 5 = 0$$

$$\frac{v_{\rm E} - v_{\rm B}}{7} + \frac{v_{\rm E} - v_{\rm A}}{5} + \frac{v_{\rm E}}{6} + 5 = 0$$

Multiply the first equation by 28, the second by 6, and the third by 42 to get

$$-7v_{\rm A} + 11v_{\rm B} - 4v_{\rm E} = 2.8$$

$$5v_{\rm D} + 26v_\Delta = 29.4$$

$$-8.4v_{\rm A} - 6v_{\rm B} + 21.4v_{\rm E} = -210$$

Constraint equations:

$$v_{\rm A} = 3v_x; \qquad v_x = v_{\rm E} - v_{\rm C} - 0.9; \qquad v_\Delta = v_{\rm B} - v_{\rm E}$$

$$v_\sigma = \frac{v_{\rm A} - v_{\rm B}}{4} = 0.25v_{\rm A} - 0.25v_{\rm B}; \qquad 5i_\sigma = v_{\rm B} = v_{\rm C}$$

Use the constraint equations to solve for $v_{\rm A}, v_{\rm B}$ and v_Δ in terms of $v_{\rm C}$ and $v_{\rm E}$:

$$v_{\rm A} = 3v_{\rm E} - 3v_{\rm C} - 2.7$$

$$v_{\rm B} = \frac{15}{9}v_{\rm E} - \frac{11}{9}v_{\rm C} - 1.5$$

$$v_\Delta = \frac{6}{9}v_{\rm E} - \frac{11}{9}v_{\rm C} - 1.5$$

Substitute these three expressions into the previous three equations to yield:

$$68v_{\rm C} + 0v_{\rm D} - 60v_{\rm E} = 3.6$$

$$-286v_{\rm C} + 45v_{\rm D} + 156v_{\rm E} = 615.6$$

$$292.8v_{\rm C} + 0v_{\rm D} - 124.2v_{\rm E} = -2175.12$$

Solving,

$$v_{\rm C} = -14.3552 \text{ V}; \qquad v_{\rm D} = -20.9474 \text{ V}; \qquad v_{\rm C} = 16.3293 \text{ V}$$

From the circuit diagram,

$$p_{5\rm A} = 5v_{5\rm A} = 5(v_{\rm E} - v_{\rm D}) = 23.09 \text{ W}$$

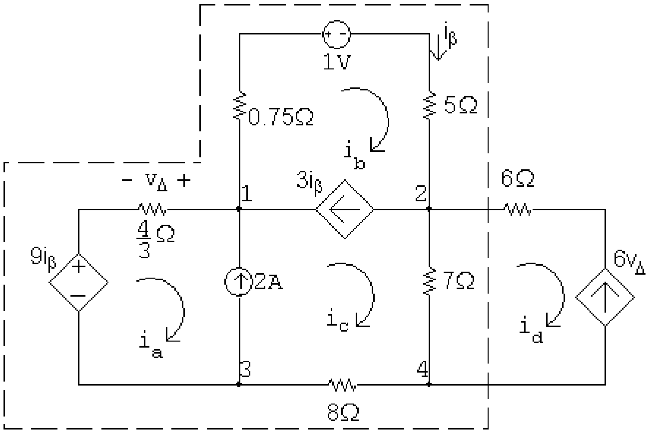
Therefore the 5 A source is absorbing 23.09 W of power.

P 4.101 **[a]** In studying the circuit in Fig. P4.101 we note it contains six meshes and six essential nodes. Further study shows that by replacing the parallel resistors with their equivalent values the circuit reduces to four meshes and four essential nodes as shown in the following diagram.

The node Voltage approach will require solving three node Voltage equations along with equations involving v_Δ and i_β.

The mesh-current approach will require writing one supermesh equation plus three constraint equations involving the three current sources. Thus at the outset we know the supermesh equation can be reduced to a single unknown current. Since we are interested in the power developed by the 1 V source, we will retain the mesh current i_b and eliminate the mesh currents i_a, i_c And i_d.

The supermesh is denoted by the dashed line in the following figure.



[b] Summing the voltages around the supermesh yields

$$-9i_\beta + \frac{4}{3}i_\mathrm{a} + 0.75i_\mathrm{b} + 1 + 5i_\mathrm{b} + 7(i_\mathrm{c} - i_\mathrm{d}) + 8i_\mathrm{c} = 0$$

Note that $i_\beta = i_\mathrm{b}$; make that substitution and multiply the equation by 12:

$$-108i_\mathrm{b} + 16i_\mathrm{a} + 9i_\mathrm{b} + 12 + 60i_\mathrm{b} + 84(i_\mathrm{c} - i_\mathrm{d}) + 96i_\mathrm{c} = 0$$

or

$$16i_\mathrm{a} - 39i_\mathrm{b} + 180i_\mathrm{c} - 84i_\mathrm{d} = -12$$

Use the following constraints:

$$i_\mathrm{a} - i_\mathrm{c} = -2; \qquad i_\mathrm{b} - i_\mathrm{c} = 3i_\mathrm{b}$$

$$\therefore \quad i_\mathrm{a} = -2 + i_\mathrm{c} = -2 - 2i_\mathrm{b}$$

Therefore,

$$16(-2 - 2i_\mathrm{b}) - 39i_\mathrm{b} + 180(-2i_\mathrm{b}) - 84i_\mathrm{d} = -12$$

so

$$-431i_\mathrm{b} - 84i_\mathrm{d} = 20$$

Finally use the following constraint:

$$i_\mathrm{d} = -6v_\Delta = -6\left(-\frac{4}{3}i_\mathrm{a}\right) = 8i_\mathrm{a} = -16 - 16i_\mathrm{b}$$

Thus,

$$-431i_\mathrm{b} - 84(-16 - 16i_\mathrm{b}) = 20$$

so

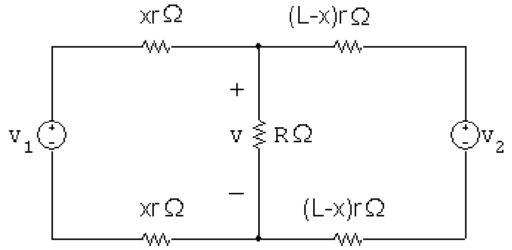
$913 i_b = -1324$ and $i_b = -1.45$ A

Finally,

$p_{1V} = 1 i_b = -1.45$ W

The 1 V source delivers 1.45 W of power.

P 4.102 **[a]**



$$\frac{v - v_1}{2xr} + \frac{v}{R} + \frac{v - v_2}{2r(L-x)} = 0$$

$$v\left[\frac{1}{2xr} + \frac{1}{R} + \frac{1}{2r(L-x)}\right] = \frac{v_1}{2xr} + \frac{v_2}{2r(L-x)}$$

$$v = \frac{v_1 RL + xR(v_2 - v_1)}{RL + 2rLx - 2rx^2}$$

[b] Let $D = RL + 2rLx - 2rx^2$

$$\frac{dv}{dx} = \frac{(RL + 2rLx - 2rx^2)R(v_2 - v_1) - [v_1 RL + xR(v_2 - v_1)]2r(L - 2x)}{D^2}$$

$$\frac{dv}{dx} = 0 \quad \text{when numerator is zero.}$$

The numerator simplifies to

$$x^2 + \frac{2Lv_1}{(v_2 - v_1)}x + \frac{RL(v_2 - v_1) - 2rv_1L^2}{2r(v_2 - v_1)} = 0$$

Solving for the roots of the quadratic yields

$$x = \frac{L}{v_2 - v_1}\left\{-v_1 \pm \sqrt{v_1 v_2 - \frac{R}{2rL}(v_2 - v_1)^2}\right\}$$

[c] $$x = \frac{L}{v_2 - v_1}\left\{-v_1 \pm \sqrt{v_1 v_2 - \frac{R}{2rL}(v_1 - v_2)^2}\right\}$$

$v_2 = 1200$ V, $v_1 = 1000$ V, $L = 16$ km

$r = 5 \times 10^{-5}\,\Omega/m$; $R = 3.9\,\Omega$

$$\frac{L}{v_2 - v_1} = \frac{16{,}000}{1200 - 1000} = 80; \qquad v_1 v_2 = 1.2 \times 10^6$$

$$\frac{R}{2rL}(v_1 - v_2)^2 = \frac{3.9(-200)^2}{(10 \times 10^{-5})(16 \times 10^3)} = 0.975 \times 10^5$$

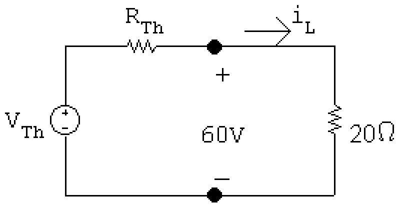
$$x = 80\{-1000 \pm \sqrt{1.2 \times 10^6 - 0.0975 \times 10^6}\}$$

$$= 80\{-1000 \pm 1050\} = 80(50) = 4000 \text{ m}$$

[d]

$$\begin{aligned} v_{\min} &= \frac{v_1 RL + R(v_2 - v_1)x}{RL + 2rLx - 2rx^2} \\ &= \frac{(1000)(3.9)(16 \times 10^3) + 3.9(200)(4000)}{(3.9)(16{,}000) + 10 \times 10^{-5}(16{,}000)(4000) - 10 \times 10^{-5}(16 \times 10^6)} \\ &= 975 \text{ V} \end{aligned}$$

P 4.103 **[a]**



$$v_{\text{oc}} = V_{\text{Th}} = 75 \text{ V}; \qquad i_L = \frac{60}{20} = 3 \text{ A}; \qquad i_L = \frac{75 - 60}{R_{\text{Th}}} = \frac{15}{R_{\text{Th}}}$$

Therefore $R_{\text{Th}} = \dfrac{15}{3} = 5\,\Omega$

[b] $i_L = \dfrac{v_o}{R_L} = \dfrac{V_{\text{Th}} - v_o}{R_{\text{Th}}}$

Therefore $R_{\text{Th}} = \dfrac{V_{\text{Th}} - v_o}{v_o/R_L} = \left(\dfrac{V_{\text{Th}}}{v_o} - 1\right) R_L$

P 4.104 $\dfrac{dv_1}{dI_{g1}} = \dfrac{-R_1[R_2(R_3 + R_4) + R_3R_4]}{(R_1 + R_2)(R_3 + R_4) + R_3R_4}$

$$\frac{dv_1}{dI_{g2}} = \frac{R_1R_3R_4}{(R_1 + R_2)(R_3 + R_4) + R_3R_4}$$

$$\frac{dv_2}{dI_{g1}} + \frac{-R_1R_3R_4}{(R_1 + R_2)(R_3 + R_4) + R_3R_4}$$

$$\frac{dv_2}{dI_{g2}} = \frac{R_3R_4(R_1 + R_2)}{(R_1 + R_2)(R_3 + R_4) + R_3R_4}$$

P 4.105 From the solution to Problem 4.104 we have

$$\frac{dv_1}{dI_{g1}} = \frac{-25[5(125) + 3750]}{30(125) + 3750} = -\frac{175}{12} \text{ V/A} = -14.5833 \text{ V/A}$$

and

$$\frac{dv_2}{dI_{g1}} = \frac{-(25)(50)(75)}{30(125) + 3750} = -12.5 \text{ V/A}$$

By hypothesis, $\Delta I_{g1} = 11 - 12 = -1$ A

$$\therefore \quad \Delta v_1 = (-\frac{175}{12})(-1) = \frac{175}{12} = 14.583 \text{ V}$$

Thus, $v_1 = 25 + 14.583 = 39.583$ V
Also,

$$\Delta v_2 = (-12.5)(-1) = 12.5 \text{ V}$$

Thus, $v_2 = 90 + 12.5 = 102.5$ V
The PSpice solution is

$$v_1 = 39.583 \text{ V}$$

and

$$v_2 = 102.5 \text{ V}$$

These values are in agreement with our predicted values.

P 4.106 From the solution to Problem 4.104 we have

$$\frac{dv_1}{dI_{g2}} = \frac{(25)(50)(75)}{30(125) + 3750} = 12.5 \text{ V/A}$$

and

$$\frac{dv_2}{dI_{g2}} = \frac{(50)(75)(30)}{30(125) + 3750} = 15 \text{ V/A}$$

By hypothesis, $\Delta I_{g2} = 17 - 16 = 1$ A

$$\therefore \quad \Delta v_1 = (12.5)(1) = 12.5 \text{ V}$$

Thus, $v_1 = 25 + 12.5 = 37.5$ V

Also,

$$\Delta v_2 = (15)(1) = 15 \text{ V}$$

Thus, $v_2 = 90 + 15 = 105$ V
The PSpice solution is

$$v_1 = 37.5 \text{ V}$$

and

$$v_2 = 105 \text{ V}$$

These values are in agreement with our predicted values.

P 4.107 From the solutions to Problems 4.104 — 4.106 we have

$$\frac{dv_1}{dI_{g1}} = -\frac{175}{12} \text{ V/A}; \qquad \frac{dv_1}{dI_{g2}} = 12.5 \text{ V/A}$$

$$\frac{dv_2}{dI_{g1}} = -12.5 \text{ V/A}; \qquad \frac{dv_2}{dI_{g2}} = 15 \text{ V/A}$$

By hypothesis,

$$\Delta I_{g1} = 11 - 12 = -1 \text{ A}$$

$$\Delta I_{g2} = 17 - 16 = 1 \text{ A}$$

Therefore,

$$\Delta v_1 = \frac{175}{12} + 12.5 = 27.0833 \text{ V}$$

$$\Delta v_2 = 12.5 + 15 = 27.5 \text{ V}$$

Hence

$$v_1 = 25 + 27.0833 = 52.0833 \text{ V}$$

$$v_2 = 90 + 27.5 = 117.5 \text{ V}$$

The PSpice solution is

$$v_1 = 52.0830 \text{ V}$$

and

$$v_2 = 117.5 \text{ V}$$

These values are in agreement with our predicted values.

P 4.108 By hypothesis,

$$\Delta R_1 = 27.5 - 25 = 2.5\,\Omega$$

$$\Delta R_2 = 4.5 - 5 = -0.5\,\Omega$$

$$\Delta R_3 = 55 - 50 = 5\,\Omega$$

$$\Delta R_4 = 67.5 - 75 = -7.5\,\Omega$$

So

$$\Delta v_1 = 0.5833(2.5) - 5.417(-0.5) + 0.45(5) + 0.2(-7.5) = 4.9168 \text{ V}$$

$$\therefore \quad v_1 = 25 + 4.9168 = 29.9168 \text{ V}$$

$$\Delta v_2 = 0.5(2.5) + 6.5(-0.5) + 0.54(5) + 0.24(-7.5) = -1.1 \text{ V}$$

$$\therefore \quad v_2 = 90 - 1.1 = 88.9 \text{ V}$$

The PSpice solution is

$$v_1 = 29.6710 \text{ V}$$

and

$$v_2 = 88.5260 \text{ V}$$

Note our predicted values are within a fraction of a volt of the actual values.

5

The Operational Amplifier

Assessment Problems

AP 5.1 **[a]** This is an inverting amplifier, so

$$v_o = (-R_f/R_i)v_s = (-80/16)v_s, \qquad \text{so} \qquad v_o = -5v_s$$

v_s(V)	0.4	2.0	3.5	−0.6	−1.6	−2.4
v_o(V)	−2.0	−10.0	−15.0	3.0	8.0	10.0

Two of the values, 3.5 V and −2.4 V, cause the op amp to saturate.

[b] Use the negative power supply value to determine the largest input voltage:

$$-15 = -5v_s, \quad v_s = 3\ \text{V}$$

Use the positive power supply value to determine the smallest input voltage:

$$10 = -5v_s, \qquad v_s = -2\ \text{V}$$

Therefore $\quad -2 \leq v_s \leq 3\ \text{V}$

AP 5.2 From Assessment Problem 5.1

$$v_o = (-R_f/R_i)v_s = (-R_x/16{,}000)v_s = (-R_x/16{,}000)(-0.640)$$

$$= 0.64R_x/16{,}000 = 4 \times 10^{-5}R_x$$

Use the negative power supply value to determine one limit on the value of R_x:

$$4 \times 10^{-5}R_x = -15 \qquad \text{so} \qquad R_x = -15/4 \times 10^{-5} = -375\,\text{k}\Omega$$

Since we cannot have negative resistor values, the lower limit for R_x is 0. Now use the positive power supply value to determine the upper limit on the value of R_x:

$$4 \times 10^{-5} R_x = 10 \qquad \text{so} \qquad R_x = 10/4 \times 10^{-5} = 250\,\text{k}\Omega$$

Therefore,

$$0 \le R_x \le 250\,\text{k}\Omega$$

AP 5.3 **[a]** This is an inverting summing amplifier so

$$v_o = (-R_f/R_\text{a})v_\text{a} + (-R_f/R_\text{b})v_\text{b} = -(250/5)v_\text{a} - (250/25)v_\text{b} = -50v_\text{a} - 10v_\text{b}$$

Substituting the values for v_a and v_b:

$$v_o = -50(0.1) - 10(0.25) = -5 - 2.5 = -7.5 \text{ V}$$

[b] Substitute the value for v_b into the equation for v_o from part (a) and use the negative power supply value:

$$v_o = -50v_\text{a} - 10(0.25) = -50v_\text{a} - 2.5 = -10 \text{ V}$$

$$\text{Therefore} \quad 50v_\text{a} = 7.5, \quad \text{so} \quad v_\text{a} = 0.15 \text{ V}$$

[c] Substitute the value for v_a into the equation for v_o from part (a) and use the negative power supply value:

$$v_o = -50(0.10) - 10v_\text{b} = -5 - 10v_\text{b} = -10 \text{ V};$$

$$\text{Therefore} \quad 10v_\text{b} = 5, \quad \text{so} \quad v_\text{b} = 0.5 \text{ V}$$

[d] The effect of reversing polarity is to change the sign on the v_b term in each equation from negative to positive.
Repeat part (a):

$$v_o = -50v_\text{a} + 10v_\text{b} = -5 + 2.5 = -2.5 \text{ V}$$

Repeat part (b):

$$v_o = -50v_\text{a} + 2.5 = -10 \text{ V}; \qquad 50v_\text{a} = 12.5, \quad v_\text{a} = 0.25 \text{ V}$$

Repeat part (c), using the value of the positive power supply:

$$v_o = -5 + 10v_\text{b} = 15 \text{ V}; \qquad 10v_\text{b} = 20; \qquad v_\text{b} = 2.0 \text{ V}$$

AP 5.4 **[a]** Write a node voltage equation at v_n; remember that for an ideal op amp, the current into the op amp at the inputs is zero:

$$\frac{v_n}{4500} + \frac{v_n - v_o}{63{,}000} = 0$$

Solve for v_o in terms of v_n by multiplying both sides by 63,000 and collecting terms:

$14v_n + v_n - v_o = 0$ so $v_o = 15v_n$

Now use voltage division to calculate v_p. We can use voltage division because the op amp is ideal, so no current flows into the non-inverting input terminal and the 400 mV divides between the 15 kΩ resistor and the R_x resistor:

$$v_p = \frac{R_x}{15{,}000 + R_x}(0.400)$$

Now substitute the value $R_x = 60$ kΩ:

$$v_p = \frac{60{,}000}{15{,}000 + 60{,}000}(0.400) = 0.32 \text{ V}$$

Finally, remember that for an ideal op amp, $v_n = v_p$, so substitute the value of v_p into the equation for v_0

$v_o = 15v_n = 15v_p = 15(0.32) = 4.8$ V

[b] Substitute the expression for v_p into the equation for v_o and set the resulting equation equal to the positive power supply value:

$$v_o = 15\left(\frac{0.4R_x}{15{,}000 + R_x}\right) = 5$$

$15(0.4R_x) = 5(15{,}000 + R_x)$ so $R_x = 75\,\text{k}\Omega$

AP 5.5 **[a]** Since this is a difference amplifier, we can use the expression for the output voltage in terms of the input voltages and the resistor values given in Eq. 5.22:

$$v_o = \frac{20(60)}{10(24)}v_\text{b} - \frac{50}{10}v_\text{a}$$

Simplify this expression and subsitute in the value for v_b:

$v_o = 5(v_\text{b} - v_\text{a}) = 20 - 5v_\text{a}$

Set this expression for v_o to the positive power supply value:

$20 - 5v_\text{a} = 10$ V so $v_\text{a} = 2$ V

Now set the expression for v_o to the negative power supply value:

$20 - 5v_\text{a} = -10$ V so $v_\text{a} = 6$ V

Therefore $2 \leq v_\text{a} \leq 6$ V

[**b**] Begin as before by substituting the appropriate values into Eq. 5.22:

$$v_o = \frac{8(60)}{10(12)}v_\text{b} - 5v_\text{a} = 4v_\text{b} - 5v_\text{a}$$

Now substitute the value for v_b:

$$v_o = 4(4) - 5v_\text{a} = 16 - 5v_\text{a}$$

Set this expression for v_o to the positive power supply value:

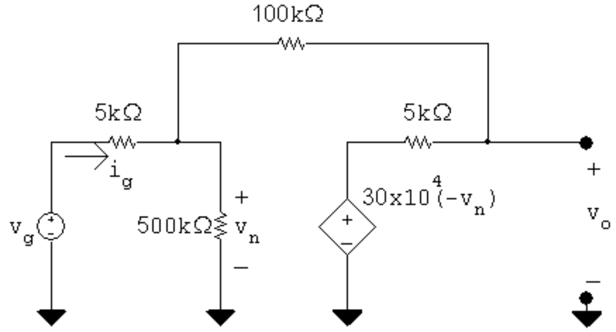
$$16 - 5v_\text{a} = 10\ \text{V} \quad \text{so} \quad v_\text{a} = 1.2\ \text{V}$$

Now set the expression for v_o to the negative power supply value:

$$16 - 5v_\text{a} = -10\ \text{V} \quad \text{so} \quad v_\text{a} = 5.2\ \text{V}$$

Therefore $1.2 \leq v_\text{a} \leq 5.2\ \text{V}$

AP 5.6 [**a**] Replace the op amp with the more realistic model of the op amp from Fig. 5.15:



Write the node voltage equation at the left hand node:

$$\frac{v_n}{500{,}000} + \frac{v_n - v_g}{5000} + \frac{v_n - v_o}{100{,}000} = 0$$

Multiply both sides by 500,000 and simplify:

$$v_n + 100v_n - 100v_g + 5v_n - 5v_0 = 0 \quad \text{so} \quad 21.2v_n - v_o = 20v_g$$

Write the node voltage equation at the right hand node:

$$\frac{v_o - 300{,}000(-v_n)}{5000} + \frac{v_o - v_n}{100{,}000} = 0$$

Multiply through by 100,000 and simplify:

$$20v_o + 6 \times 10^6 v_n + v_o - v_n = 0 \quad \text{so} \quad 6 \times 10^6 v_n + 21v_o = 0$$

Use Cramer's method to solve for v_o:

$$\Delta = \begin{vmatrix} 21.2 & -1 \\ 6 \times 10^6 & 21 \end{vmatrix} = 6{,}000{,}445.2$$

$$N_o = \begin{vmatrix} 21.2 & 20v_g \\ 6 \times 10^6 & 0 \end{vmatrix} = -120 \times 10^6 v_g$$

$$v_o = \frac{N_o}{\Delta} = -19.9985 v_g; \qquad \text{so } \frac{v_o}{v_g} = -19.9985$$

[b] Use Cramer's method again to solve for v_n:

$$N_1 = \begin{vmatrix} 20v_g & -1 \\ 0 & 21 \end{vmatrix} = 420 v_g$$

$$v_n = \frac{N_1}{\Delta} = 6.9995 \times 10^{-5} v_g$$

$$v_g = 1\ \text{V}, \qquad v_n = 69.995\,\mu\ \text{V}$$

[c] The resistance seen at the input to the op amp is the ratio of the input voltage to the input current, so calculate the input current as a function of the input voltage:

$$i_g = \frac{v_g - v_n}{5000} = \frac{v_g - 6.9995 \times 10^{-5} v_g}{5000}$$

Solve for the ratio of v_g to i_g to get the input resistance:

$$R_g = \frac{v_g}{i_g} = \frac{5000}{1 - 6.9995 \times 10^{-5}} = 5000.35\,\Omega$$

[d] This is a simple inverting amplifier configuration, so the voltage gain is the ratio of the feedback resistance to the input resistance:

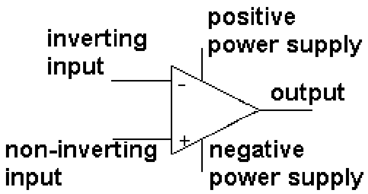
$$\frac{v_o}{v_g} = -\frac{100{,}000}{5000} = -20$$

Since this is now an ideal op amp, the voltage difference between the two input terminals is zero; since $v_p = 0$, $v_n = 0$
Since there is no current into the inputs of an ideal op amp, the resistance seen by the input voltage source is the input resistance:

$$R_g = 5000\,\Omega$$

Problems

P 5.1 **[a]** The five terminals of the op amp are identified as follows:



[b] The input resistance of an ideal op amp is infinite, which constrains the value of the input currents to 0. Thus, $i_n = 0$ A.

[c] The open-loop voltage gain of an ideal op amp is infinite, which constrains the difference between the voltage at the two input terminals to 0. Thus, $(v_p - v_n) = 0$.

[d] Write a node voltage equation at v_n:

$$\frac{v_n + 3}{5000} + \frac{v_n - v_o}{15{,}000} = 0$$

But $v_p = 0$ and $v_n = v_p = 0$. Thus,

$$\frac{3}{5000} - \frac{v_o}{15{,}000} = 0 \quad \text{so} \quad v_o = 9 \text{ V}$$

P 5.2 $v_o = -(0.5 \times 10^{-3})(10{,}000) = -5$ V

$$\therefore \quad i_o = \frac{v_o}{5000} = \frac{-5}{5000} = -1\,\text{mA}$$

P 5.3 $$\frac{v_\text{b} - v_\text{a}}{20{,}000} + \frac{v_\text{b} - v_o}{100{,}000} = 0, \qquad \text{therefore} \quad v_o = 6v_\text{b} - 5v_\text{a}$$

[a] $v_\text{a} = 4$ V, $v_\text{b} = 0$ V, $v_o = -15$ V (sat)

[b] $v_\text{a} = 2$ V, $v_\text{b} = 0$ V, $v_o = -10$ V

[c] $v_\text{a} = 2$ V, $v_\text{b} = 1$ V, $v_o = -4$ V

[d] $v_\text{a} = 1$ V, $v_\text{b} = 2$ V, $v_o = 7$ V

[e] $v_\text{a} = 1.5$ V, $v_\text{b} = 4$ V, $v_o = 15$ V (sat)

[f] If $v_\text{b} = 1.6$ V, $v_o = 9.6 - 5v_\text{a} = \pm 15$

$$\therefore \quad -1.08 \text{ V} \le v_\text{a} \le 4.92 \text{ V}$$

P 5.4 $$v_p = \frac{3000}{3000+6000}(3) = 1\ \text{V} = v_n$$

$$\frac{v_n - 5}{10{,}000} + \frac{v_n - v_o}{5000} = 0$$

$$(1-5) + 2(1 - v_o) = 0$$

$$v_o = -1.0\ \text{V}$$

$$i_\text{L} = \frac{v_o}{4000} = -\frac{1}{4000} = -250 \times 10^{-6}$$

$$i_\text{L} = -250\,\mu\text{A}$$

P 5.5 Since the current into the inverting input terminal of an ideal op-amp is zero, the voltage across the 2.2 MΩ resistor is $(2.2 \times 10^6)(3.5 \times 10^{-6})$ or 7.7 V. Therefore the voltmeter reads 7.7 V.

P 5.6 **[a]** $$i_2 = \frac{150 \times 10^{-3}}{2000} = 75\,\mu\text{A}$$

$$v_1 = -40 \times 10^3 i_2 = -3\,\text{V}$$

[b] $$\frac{v_1}{20{,}000} + \frac{v_1}{40{,}000} + \frac{v_1 - v_o}{50{,}000} = 0$$

$$\therefore\ \ v_o = 4.75 v_1 = -14.25\ \text{V}$$

[c] $i_2 = 75\,\mu\text{A}$, (from part [a])

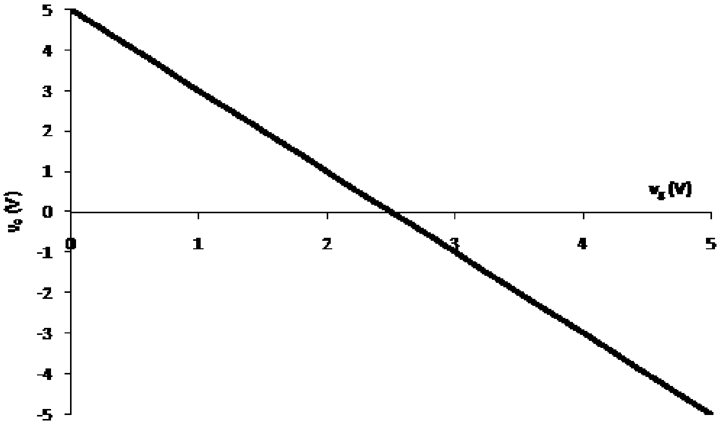
[d] $$i_o = \frac{-v_o}{25{,}000} + \frac{v_1 - v_o}{50{,}000} = 795\,\mu\ \text{A}$$

P 5.7 **[a]** First, note that $v_n = v_p = 2.5$ V

Let v_{o1} equal the voltage output of the op-amp. Then

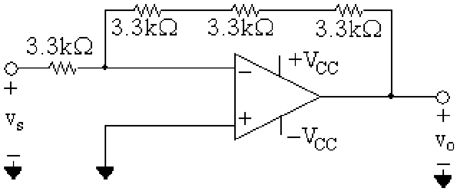
$$\frac{2.5 - v_g}{5000} + \frac{2.5 - v_{o1}}{10{,}000} = 0, \qquad \therefore\ \ v_{o1} = 7.5 - 2v_g$$

Also note that $v_{o1} - 2.5 = v_o$, $\qquad \therefore\ \ v_o = 5 - 2v_g$

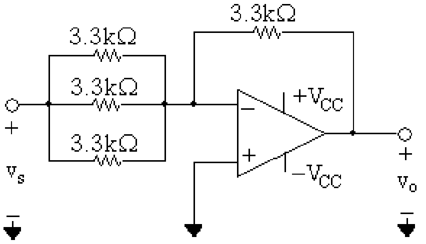


[b] Yes, the circuit designer is correct!

P 5.8 **[a]** The gain of an inverting amplifier is the ratio of the feedback resistor to the input resistor. If the gain of the inverting amplifier is to be 3, the feedback resistor must be 3 times as large as the input resistor. There are many possible designs that use a resistor value chosen from Appendix H. We present two here that use 3.3 kΩ resistors. Use a single 3.3 kΩ resistor as the input resistor, and use three 3.3 kΩ resistors in series as the feedback resistor to give a total of 9.9 kΩ.

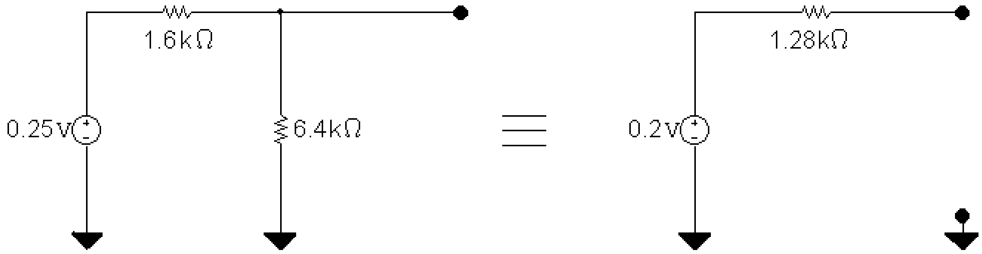


Alternately, use a single 3.3 kΩ resistor as the feedback resistor and use three 3.3 kΩ resistors in parallel as the input resistor to give a total of 1.1 kΩ.



[b] To amplify a 5 V signal without saturating the op amp, the power supply voltages must be greater than or equal to the product of the input voltage and the amplifier gain. Thus, the power supplies should have a magnitude of (5)(3) = 15 V.

P 5.9 **[a]** Replace the combination of v_g, 1.6 kΩ, and the 6.4 kΩ resistors with its Thévenin equivalent.



$$\text{Then}\quad v_o = \frac{-[12+\sigma 50]}{1.28}(0.20)$$

At saturation $v_o = -5\ \text{V}$; therefore

$$-\left(\frac{12+\sigma 50}{1.28}\right)(0.2) = -5, \quad \text{or} \quad \sigma = 0.4$$

Thus for $0 \le \sigma \le 0.40$ the operational amplifier will not saturate.

[b] When $\sigma = 0.272, \qquad v_o = \dfrac{-(12 + 13.6)}{1.28}(0.20) = -4\ \text{V}$

Also $\dfrac{v_o}{10} + \dfrac{v_o}{25.6} + i_o = 0$

$$\therefore\ \ i_o = -\frac{v_o}{10} - \frac{v_o}{25.6} = \frac{4}{10} + \frac{4}{25.6}\,\text{mA} = 556.25\,\mu\text{A}$$

P 5.10 **[a]** Let v_Δ be the voltage from the potentiometer contact to ground. Then

$$\frac{0 - v_g}{2000} + \frac{0 - v_\Delta}{50{,}000} = 0$$

$$-25v_g - v_\Delta = 0, \qquad \therefore\ \ v_\Delta = -25(40 \times 10^{-3}) = -1\ \text{V}$$

$$\frac{v_\Delta}{\alpha R_\Delta} + \frac{v_\Delta - 0}{50{,}000} + \frac{v_\Delta - v_o}{(1-\alpha)R_\Delta} = 0$$

$$\frac{v_\Delta}{\alpha} + 2v_\Delta + \frac{v_\Delta - v_o}{1-\alpha} = 0$$

$$v_\Delta\left(\frac{1}{\alpha} + 2 + \frac{1}{1-\alpha}\right) = \frac{v_o}{1-\alpha}$$

$$\therefore\ \ v_o = -1\left[1 + 2(1-\alpha) + \frac{(1-\alpha)}{\alpha}\right]$$

When $\alpha = 0.2, \qquad v_o = -1(1 + 1.6 + 4) = -6.6\ \text{V}$

When $\alpha = 1, \qquad v_o = -1(1 + 0 + 0) = -1\ \text{V}$

$$\therefore\ \ -6.6\ \text{V} \le v_o \le -1\ \text{V}$$

[b] $-1\left[1 + 2(1-\alpha) + \dfrac{(1-\alpha)}{\alpha}\right] = -7$

$$\alpha + 2\alpha(1-\alpha) + (1-\alpha) = 7\alpha$$

$$\alpha + 2\alpha - 2\alpha^2 + 1 - \alpha = 7\alpha$$

$$\therefore\ \ 2\alpha^2 + 5\alpha - 1 = 0 \quad \text{so} \quad \alpha \cong 0.186$$

P 5.11 $v_o = -\left[\dfrac{R_\text{f}}{4000}(0.2) + \dfrac{R_\text{f}}{5000}(0.15) + \dfrac{R_\text{f}}{20{,}000}(0.4)\right]$

$$-6 = -0.1 \times 10^{-3} R_\text{f}; \qquad R_\text{f} = 60\,\text{k}\Omega; \qquad \therefore\ \ 0 \le R_\text{f} \le 60\,\text{k}\Omega$$

P 5.12 **[a]** This circuit is an example of an inverting summing amplifier.

[b] $v_o = -\frac{220}{44}v_{\rm a} - \frac{220}{27.5}v_{\rm b} - \frac{220}{80}v_{\rm c} = -5 - 12 + 11 = -6 \text{ V}$

[c] $v_o = -6 - 8v_{\rm b} = \pm 10$

$$\therefore\quad v_{\rm b} = -0.5 \text{ V} \quad \text{when} \quad v_o = 10 \text{ V};$$

$$v_{\rm b} = 2 \text{ V} \quad \text{when} \quad v_o = -10 \text{ V}$$

$$\therefore\quad -0.5 \text{ V} \le v_{\rm b} \le 2 \text{ V}$$

P 5.13 We want the following expression for the output voltage:

$$v_o = -(3v_a + 5v_b + 4v_c + 2v_d)$$

This is an inverting summing amplifier, so each input voltage is amplified by a gain that is the ratio of the feedback resistance to the resistance in the forward path for the input voltage. Pick a feedback resistor with divisors of 3, 5, 4, and 2 – say 60 kΩ:

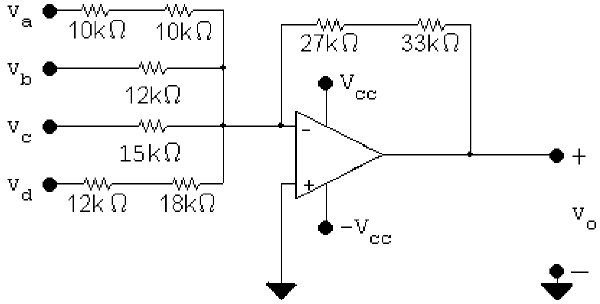
$$v_o = -\left[\frac{60\text{k}}{R_{\rm a}}v_{\rm a} + \frac{60\text{k}}{R_{\rm b}}v_{\rm b} + \frac{60\text{k}}{R_{\rm c}}v_{\rm c} + \frac{60\text{k}}{R_{\rm d}}v_{\rm d}\right]$$

Solve for each input resistance value to yield the desired gain:

$$\therefore\quad R_{\rm a} = 60{,}000/3 = 20\,\text{k}\Omega \qquad R_{\rm c} = 60{,}000/4 = 15\,\text{k}\Omega$$

$$R_{\rm b} = 60{,}000/5 = 12\,\text{k}\Omega \qquad R_{\rm d} = 60{,}000/2 = 30\,\text{k}\Omega$$

Now create the 5 resistor values needed from the realistic resistor values in Appendix H. Note that $R_{\rm b} = 12\,\text{k}\Omega$ and $R_{\rm c} = 15\,\text{k}\Omega$ are already values from Appendix H. Create $R_{\rm f} = 60\,\text{k}\Omega$ by combining 27 kΩ and 33 kΩ in series. Create $R_{\rm a} = 20\,\text{k}\Omega$ by combining two 10 kΩ resistors in series. Create $R_{\rm d} = 30\,\text{k}\Omega$ by combining 18 kΩ and 12 kΩ in series. Of course there are many other acceptable possibilities. The final circuit is shown here:



P 5.14 [a] Write a KCL equation at the inverting input to the op amp:

$$\frac{v_{\rm d} - v_{\rm a}}{40{,}000} + \frac{v_{\rm d} - v_{\rm b}}{22{,}000} + \frac{v_{\rm d} - v_{\rm c}}{100{,}000} + \frac{v_{\rm d}}{352{,}000} + \frac{v_{\rm d} - v_o}{220{,}000} = 0$$

Multiply through by 220,000, plug in the values of input voltages, and rearrange to solve for v_o:

$$v_o = 220{,}000\left(\frac{4}{40{,}000}+\frac{-1}{22{,}000}+\frac{-5}{100{,}000}\right.$$

$$\left.+\frac{8}{352{,}000}+\frac{8}{220{,}000}\right) = 14\ \text{V}$$

[**b**] Write a KCL equation at the inverting input to the op amp. Use the given values of input voltages in the equation:

$$\frac{8-v_\text{a}}{40{,}000}+\frac{8-9}{22{,}000}+\frac{8-13}{100{,}000}+\frac{8}{352{,}000}+\frac{8-v_o}{220{,}000}=0$$

Simplify and solve for v_o:

$$44-5.5v_\text{a}-10-11+5+8-v_o=0 \quad \text{so} \quad v_o = 36-5.5v_\text{a}$$

Set v_o to the positive power supply voltage and solve for v_a:

$$36-5.5v_\text{a}=15 \qquad \therefore \qquad v_\text{a}=3.818\ \text{V}$$

Set v_o to the negative power supply voltage and solve for v_a:

$$36-5.5v_\text{a}=-15 \qquad \therefore \qquad v_\text{a}=9.273\ \text{V}$$

Therefore,

$$3.818\ \text{V} \le v_\text{a} \le 9.273\ \text{V}$$

P 5.15 [**a**] $$\frac{8-4}{40{,}000}+\frac{8-9}{22{,}000}+\frac{8-13}{100{,}000}+\frac{8}{352{,}000}+\frac{8-v_0}{R_f}=0$$

$$\frac{8-v_o}{R_f}=-2.7272\times10^{-5} \quad \text{so} \quad R_f=\frac{8-v_o}{-2.727\times10^{-5}}$$

For $v_o = 15$ V, $R_f = 256.7\,\text{k}\Omega$

For $v_o = -15$ V, $R_f < 0$ so this solution is not possible.

[**b**] $$i_o = -(i_f+i_{10\text{k}}) = -\left[\frac{15-8}{256.7\times10^3}+\frac{15}{10{,}000}\right] = -1527\,\mu\text{A}$$

P 5.16 [**a**] The circuit shown is a non-inverting amplifier.

[**b**] We assume the op amp to be ideal, so $v_n = v_p = 3$ V. Write a KCL equation at v_n:

$$\frac{3}{40{,}000}+\frac{3-v_o}{80{,}000}=0$$

Solving,

$$v_o = 9\ \text{V}.$$

P 5.17 **[a]** This circuit is an example of the non-inverting amplifier.

[b] Use voltage division to calculate v_p:

$$v_p = \frac{10{,}000}{10{,}000 + 30{,}000} v_s = \frac{v_s}{4}$$

Write a KCL equation at $v_n = v_p = v_s/4$:

$$\frac{v_s/4}{4000} + \frac{v_s/4 - v_o}{28{,}000} = 0$$

Solving,

$$v_o = 7v_s/4 + v_s/4 = 2v_s$$

[c] $2v_s = 8 \qquad \text{so} \qquad v_s = 4\ \text{V}$

$2v_s = -12 \qquad \text{so} \qquad v_s = -6\ \text{V}$

Thus, $-6\ \text{V} \leq v_s \leq 4\ \text{V}$.

P 5.18 **[a]** $v_p = v_n = \dfrac{68}{80} v_g = 0.85 v_g$

$$\therefore \quad \frac{0.85v_g}{30{,}000} + \frac{0.85v_g - v_o}{63{,}000} = 0;$$

$$\therefore \quad v_o = 2.635v_g = 2.635(4), \qquad v_o = 10.54\ \text{V}$$

[b] $v_o = 2.635v_g = \pm 12$

$v_g = \pm 4.55\ \text{V}, \qquad -4.55 \leq v_g \leq 4.55\ \text{V}$

[c] $\dfrac{0.85v_g}{30{,}000} + \dfrac{0.85v_g - v_o}{R_\text{f}} = 0$

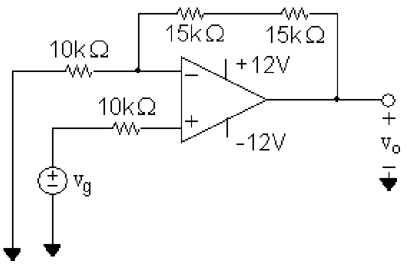
$$\left(\frac{0.85R_\text{f}}{30{,}000} + 0.85\right) v_g = v_o = \pm 12$$

$$\therefore \quad 1.7R_\text{f} + 51 = \pm 360; \qquad 1.7R_\text{f} = 360 - 51; \qquad R_\text{f} = 181.76\,\text{k}\Omega$$

P 5.19 **[a]** From the equation for the non-inverting amplifier,

$$\frac{R_s + R_f}{R_s} = 4 \qquad \text{so} \qquad R_s + R_f = 4R_s \qquad \text{and therefore} \qquad R_f = 3R_s$$

Choose $R_f = 30\,\text{k}\Omega$ and implement this choice from components in Appendix H by combining two $15\,\text{k}\Omega$ resistors in series. Choose $R_s = R_g = 10\,\text{k}\Omega$, which is a component in Appendix H. The resulting non-inverting amplifier circuit is shown here:



[b] $v_o = 4v_g = 12 \qquad \text{so} \qquad v_g = 3\ \text{V}$

$v_o = 4v_g = -12 \qquad \text{so} \qquad v_g = -3\ \text{V}$

Therefore,

$-3\ \text{V} \leq v_g \leq 3\ \text{V}$

P 5.20 [a] This circuit is an example of a non-inverting summing amplifier.

[b] Write a KCL equation at v_p and solve for v_p in terms of v_s:

$$\frac{v_p - v_s}{15{,}000} + \frac{v_p - 6}{30{,}000} = 0$$

$2v_p - 2v_s + v_p - 6 = 0 \qquad \text{so} \qquad v_p = 2v_s/3 + 2$

Now write a KCL equation at v_n and solve for v_o:

$$\frac{v_n}{20{,}000} + \frac{v_n - v_o}{60{,}000} = 0 \qquad \text{so} \qquad v_o = 4v_n$$

Since we assume the op amp is ideal, $v_n = v_p$. Thus,

$v_o = 4(2v_s/3 + 2) = 8v_s/3 + 8$

[c] $8v_s/3 + 8 = 16 \qquad \text{so} \qquad v_s = 3\ \text{V}$

$8v_s/3 + 8 = -12 \qquad \text{so} \qquad v_s = -7.5\ \text{V}$

Thus, $-7.5\ \text{V} \leq v_s \leq 3$ V.

P 5.21 [a] This is a non-inverting summing amplifier.

[b] $$\frac{v_p - v_\text{a}}{13 \times 10^3} + \frac{v_p - v_\text{b}}{27 \times 10^3} = 0$$

$\therefore \quad 40v_p = 27v_\text{a} + 13v_\text{b} \qquad \text{so} \qquad v_p = 0.675v_a + 0.325v_b$

$$\frac{v_n}{11{,}000} + \frac{v_n - v_o}{110{,}000} = 0$$

$\therefore \quad v_o = 11v_n = 11v_p = 11(0.675v_a + 0.325v_b)$

$= 11[0.675(0.8) + 0.325(0.4)] = 7.37\ \text{V}$

[c] $v_p = v_n = \dfrac{v_o}{11} = 0.667\ \text{V}$

$$i_\text{a} = \frac{v_\text{a} - v_p}{13 \times 10^3} = 10\,\mu\text{A}$$

$$i_\text{b} = \frac{v_\text{b} - v_p}{27 \times 10^3} = -10\,\mu\text{A}$$

[d] 7.425 for v_a; 3.575 for v_b

P 5.22 **[a]** $\dfrac{v_p - v_\text{a}}{R_\text{a}} + \dfrac{v_p - v_\text{b}}{R_\text{b}} + \dfrac{v_p - v_\text{c}}{R_\text{c}} = 0$

$$\therefore \quad v_p = \frac{R_\text{b}R_\text{c}}{D}v_\text{a} + \frac{R_\text{a}R_\text{c}}{D}v_\text{b} + \frac{R_\text{a}R_\text{b}}{D}v_\text{c}$$

where $D = R_\text{b}R_\text{c} + R_\text{a}R_\text{c} + R_\text{a}R_\text{b}$

$$\frac{v_n}{20{,}000} + \frac{v_n - v_o}{100{,}000} = 0$$

$$\left(\frac{100{,}000}{20{,}000} + 1\right) v_n = 6v_n = v_o$$

$$\therefore \quad v_o = \frac{6R_\text{b}R_\text{c}}{D}v_\text{a} + \frac{6R_\text{a}R_\text{c}}{D}v_\text{b} + \frac{6R_\text{a}R_\text{b}}{D}v_\text{c}$$

By hypothesis,

$$\frac{6R_\text{b}R_\text{c}}{D} = 1; \qquad \frac{6R_\text{a}R_\text{c}}{D} = 2; \qquad \frac{6R_\text{a}R_\text{b}}{D} = 3$$

Then

$$\frac{6R_\text{a}R_\text{b}/D}{6R_\text{a}R_\text{c}/D} = \frac{3}{2} \qquad \text{so} \qquad R_\text{b} = 1.5R_\text{c}$$

But from the circuit

$$R_\text{b} = 15\,\text{k}\Omega \qquad \text{so} \qquad R_\text{c} = 10\,\text{k}\Omega$$

Similarly,

$$\frac{6R_\text{b}R_\text{c}/D}{6R_\text{a}R_\text{b}/D} = \frac{1}{3} \qquad \text{so} \qquad 3R_\text{c} = R_\text{a}$$

Thus,

$$R_\text{a} = 30\,\text{k}\Omega$$

[b] $v_o = 1(0.7) + 2(0.4) + 3(1.1) = 4.8\ \text{V}$

$$v_n = v_o/6 = 0.8\ \text{V} = v_p$$

$$i_\text{a} = \frac{v_\text{a} - v_p}{30{,}000} = \frac{0.7 - 0.8}{30{,}000} = -3.33\,\mu\text{A}$$

$$i_{\rm b} = \frac{v_{\rm b} - v_p}{15{,}000} = \frac{0.4 - 0.8}{15{,}000} = -26.67\,\mu{\rm A}$$

$$i_{\rm c} = \frac{v_{\rm c} - v_p}{10{,}000} = \frac{1.1 - 0.8}{10{,}000} = 30\,\mu{\rm A}$$

Check:

$$i_{\rm a} + i_{\rm b} + i_{\rm c} = 0? \qquad -3.33 - 26.67 + 30 = 0 \text{ (checks)}$$

P 5.23 **[a]** $$\frac{v_p - v_{\rm a}}{R_{\rm a}} + \frac{v_p - v_{\rm b}}{R_{\rm b}} + \frac{v_p - v_{\rm c}}{R_{\rm c}} + \frac{v_p}{R_g} = 0$$

$$\therefore \quad v_p = \frac{R_{\rm b}R_{\rm c}R_g}{D}v_{\rm a} + \frac{R_{\rm a}R_{\rm c}R_g}{D}v_{\rm b} + \frac{R_{\rm a}R_{\rm b}R_g}{D}v_{\rm c}$$

where $D = R_{\rm b}R_{\rm c}R_g + R_{\rm a}R_{\rm c}R_g + R_{\rm a}R_{\rm b}R_g + R_{\rm a}R_{\rm b}R_{\rm c}$

$$\frac{v_n}{R_s} + \frac{v_n - v_o}{R_{\rm f}} = 0$$

$$v_n\left(\frac{1}{R_s} + \frac{1}{R_{\rm f}}\right) = \frac{v_o}{R_{\rm f}}$$

$$\therefore \quad v_o = \left(1 + \frac{R_{\rm f}}{R_s}\right)v_n = kv_n$$

where $k = \left(1 + \frac{R_{\rm f}}{R_s}\right)$

$$v_p = v_n$$

$$\therefore \quad v_o = kv_p$$

or

$$v_o = \frac{kR_gR_{\rm b}R_{\rm c}}{D}v_{\rm a} + \frac{kR_gR_{\rm a}R_{\rm c}}{D}v_{\rm b} + \frac{kR_gR_{\rm a}R_{\rm b}}{D}v_{\rm c}$$

$$\frac{kR_gR_{\rm b}R_{\rm c}}{D} = 6 \qquad \frac{kR_gR_{\rm a}R_{\rm c}}{D} = 3 \qquad \frac{kR_gR_{\rm a}R_{\rm b}}{D} = 4$$

$$\therefore \quad \frac{R_{\rm b}}{R_{\rm a}} = \frac{6}{3} = 2 \qquad \frac{R_{\rm c}}{R_{\rm b}} = \frac{3}{4} = 0.75 \qquad \frac{R_{\rm c}}{R_{\rm a}} = \frac{6}{4} = 1.5$$

Since $R_{\rm a} = 1\,{\rm k}\Omega \qquad R_{\rm b} = 2\,{\rm k}\Omega \qquad R_{\rm c} = 1.5\,{\rm k}\Omega$

$$\therefore \quad D = [(2)(1.5)(3) + (1)(1.5)(3) + (1)(2)(3) + (1)(2)(1.5)] \times 10^9 = 22.5 \times 10^9$$

$$\frac{k(3)(2)(1.5) \times 10^9}{22.5 \times 10^9} = 6$$

$$k = \frac{135 \times 10^9}{9 \times 10^9} = 15$$

$$\therefore \quad 15 = 1 + \frac{R_\text{f}}{R_s}$$

$$\frac{R_\text{f}}{R_s} = 14$$

$$R_\text{f} = (14)(15{,}000) = 210\,\text{k}\Omega$$

[b] $v_o = 6(0.5) + 3(2.5) + 4(1) = 14$ V

$$v_n = v_p = \frac{14.5}{15} = 0.967 \text{ V}$$

$$i_\text{a} = \frac{0.5 - 0.967}{1000} = -466.67\,\mu\text{A}$$

$$i_\text{b} = \frac{2.5 - 0.967}{2000} = 766.67\,\mu\text{A}$$

$$i_\text{c} = \frac{1 - 0.967}{1500} = 22.22\,\mu\text{A}$$

$$i_g = \frac{0.967}{3000} = 322.22\,\mu\text{A}$$

$$i_s = \frac{v_n}{15{,}000} = \frac{0.967}{15{,}000} = 64.44\,\mu\text{A}$$

P 5.24 **[a]** Assume v_a is acting alone. Replacing v_b with a short circuit yields $v_p = 0$, therefore $v_n = 0$ and we have

$$\frac{0 - v_\text{a}}{R_\text{a}} + \frac{0 - v_o'}{R_\text{b}} + i_n = 0, \qquad i_n = 0$$

Therefore

$$\frac{v_o'}{R_\text{b}} = -\frac{v_\text{a}}{R_\text{a}}, \qquad v_o' = -\frac{R_\text{b}}{R_\text{a}} v_\text{a}$$

Assume v_b is acting alone. Replace v_a with a short circuit. Now

$$v_p = v_n = \frac{v_\text{b} R_\text{d}}{R_\text{c} + R_\text{d}}$$

$$\frac{v_n}{R_\text{a}} + \frac{v_n - v_o''}{R_\text{b}} + i_n = 0, \qquad i_n = 0$$

$$\left(\frac{1}{R_\text{a}} + \frac{1}{R_\text{b}}\right)\left(\frac{R_\text{d}}{R_\text{c} + R_\text{d}}\right) v_\text{b} - \frac{v_o''}{R_\text{b}} = 0$$

$$v_o'' = \left(\frac{R_\text{b}}{R_\text{a}} + 1\right)\left(\frac{R_\text{d}}{R_\text{c} + R_\text{d}}\right) v_\text{b} = \frac{R_\text{d}}{R_\text{a}}\left(\frac{R_\text{a} + R_\text{b}}{R_\text{c} + R_\text{d}}\right) v_\text{b}$$

$$v_o = v_o' + v_o'' = \frac{R_\text{d}}{R_\text{a}}\left(\frac{R_\text{a} + R_\text{b}}{R_\text{c} + R_\text{d}}\right) v_\text{b} - \frac{R_\text{b}}{R_\text{a}} v_\text{a}$$

[b] $\frac{R_{\rm d}}{R_{\rm a}}\left(\frac{R_{\rm a}+R_{\rm b}}{R_{\rm c}+R_{\rm d}}\right)=\frac{R_{\rm b}}{R_{\rm a}},\qquad$ therefore $\quad R_{\rm d}(R_{\rm a}+R_{\rm b})=R_{\rm b}(R_{\rm c}+R_{\rm d})$

$R_{\rm d}R_{\rm a}=R_{\rm b}R_{\rm c},\qquad$ therefore $\quad \frac{R_{\rm a}}{R_{\rm b}}=\frac{R_{\rm c}}{R_{\rm d}}$

When $\frac{R_{\rm d}}{R_{\rm a}}\left(\frac{R_{\rm a}+R_{\rm b}}{R_{\rm c}+R_{\rm d}}\right)=\frac{R_{\rm b}}{R_{\rm a}}$

Eq. (5.22) reduces to $\quad v_o=\frac{R_{\rm b}}{R_{\rm a}}v_{\rm b}-\frac{R_{\rm b}}{R_{\rm a}}v_{\rm a}=\frac{R_{\rm b}}{R_{\rm a}}(v_{\rm b}-v_{\rm a}).$

P 5.25 **[a]** $v_o=\frac{R_{\rm d}(R_{\rm a}+R_{\rm b})}{R_{\rm a}(R_{\rm c}+R_{\rm d})}v_{\rm b}-\frac{R_{\rm b}}{R_{\rm a}}v_{\rm a}=\frac{120(24+75)}{24(130+120)}(5)-\frac{75}{24}(8)$

$v_o=9.9-25=-15.1$ V

[b] $\frac{v_1-8}{24{,}000}+\frac{v_1-15.1}{75{,}000}=0\qquad$ so $\qquad v_1=2.4$ V

$$i_{\rm a}=\frac{8-2.4}{24{,}000}=233\,\mu\text{ A}$$

$$R_{\rm ina}=\frac{v_{\rm a}}{i_{\rm a}}=\frac{8}{233\times10^{-6}}=34.3\,\text{k}\Omega$$

[c] $R_{\rm in\,b}=R_{\rm c}+R_{\rm d}=250\,\text{k}\Omega$

P 5.26 Use voltage division to find v_p:

$$v_p=\frac{2000}{2000+8000}(5)=1\text{ V}$$

Write a KCL equation at v_n and solve it for v_o:

$\frac{v_n-v_a}{5000}+\frac{v_n-v_o}{R_f}=0\qquad$ so $\qquad\left(\frac{R_f}{5000}+1\right)v_n-\frac{R_f}{5000}v_a=v_o$

Since the op amp is ideal, $v_n=v_p=1$V, so

$$v_o=\left(\frac{R_f}{5000}+1\right)-\frac{R_f}{5000}v_a$$

To satisfy the equation,

$\left(\frac{R_f}{5000}+1\right)=5\qquad$ and $\qquad\frac{R_f}{5000}=4$

Thus, $R_f=20$ kΩ.

P 5.27 $$v_p = \frac{v_\mathrm{b} R_\mathrm{b}}{R_\mathrm{a} + R_\mathrm{b}} = v_n$$

$$\frac{v_n - v_\mathrm{a}}{4700} + \frac{v_n - v_o}{R_\mathrm{f}} = 0$$

$$v_n\left(\frac{R_\mathrm{f}}{4700} + 1\right) - \frac{v_\mathrm{a} R_\mathrm{f}}{4700} = v_o$$

$$\therefore \quad \left(\frac{R_\mathrm{f}}{4700} + 1\right) \frac{R_\mathrm{b}}{R_\mathrm{a} + R_\mathrm{b}} v_\mathrm{b} - \frac{R_\mathrm{f}}{4700} v_\mathrm{a} = v_o$$

$$\therefore \quad \frac{R_\mathrm{f}}{4700} = 10; \qquad R_\mathrm{f} = 47\,\mathrm{k}\Omega \qquad \text{(a value from Appendix H)}$$

$$R_\mathrm{a} + R_\mathrm{b} = 220\,\mathrm{k}\Omega$$

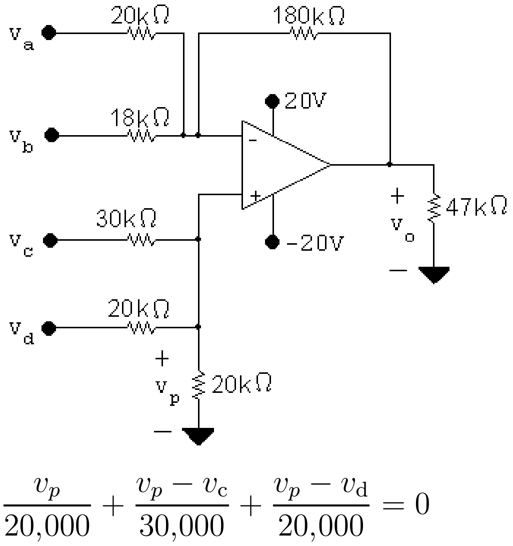
Thus,

$$\left(1 + \frac{47}{4700}\right)\left(\frac{R_\mathrm{b}}{220{,}000}\right) = 10$$

$$\therefore \quad R_\mathrm{b} = 200\,\mathrm{k}\Omega \qquad \text{and} \qquad R_\mathrm{a} = 220 - 200 = 20\,\mathrm{k}\Omega$$

Use two $100\,\mathrm{k}\Omega$ resistors in series for R_b and use two $10\,\mathrm{k}\Omega$ resistors in series for R_a.

P 5.28 **[a]**



$$\frac{v_p}{20{,}000} + \frac{v_p - v_\mathrm{c}}{30{,}000} + \frac{v_p - v_\mathrm{d}}{20{,}000} = 0$$

$$\therefore \quad 8v_p = 2v_\mathrm{c} + 3v_\mathrm{d} = 8v_n$$

$$\frac{v_n - v_a}{20{,}000} + \frac{v_n - v_b}{18{,}000} + \frac{v_n - v_o}{180{,}000} = 0$$

$$\begin{aligned} \therefore\quad v_o &= 20v_n - 9v_a - 10v_b \\ &= 20[(1/4)v_c + (3/8)v_d] - 9v_a - 10v_b \\ &= 20(0.75 + 1.5) - 9(1) - 10(2) = 16\ \text{V} \end{aligned}$$

[b] $v_o = 5v_c + 30 - 9 - 20 = 5v_c + 1$

$\pm 20 = 5v_c + 1$

$\therefore\quad v_b = -4.2\ \text{V} \qquad \text{and} \qquad v_b = 3.8\ \text{V}$

$\therefore\quad -4.2\ \text{V} \le v_b \le 3.8\ \text{V}$

P 5.29 $v_p = 1000i_b$

$$\frac{1000i_b}{R_a} + \frac{1000i_b - v_o}{R_f} - i_a = 0$$

$$\therefore\qquad 1000i_b\left(\frac{1}{R_a} + \frac{1}{R_f}\right) - i_a = \frac{v_o}{R_f}$$

$$\therefore\qquad 1000i_b\left(1 + \frac{R_f}{R_a}\right) - R_f i_a = v_o$$

By hypopthesis, $v_o = 5000(i_b - i_a)$. Therefore,

$R_f = 5\,\text{k}\Omega$ (use two $10\,\text{k}\Omega$ resistors in parallel)

$$1000\left(1 + \frac{R_f}{R_a}\right) = 5000 \qquad \text{so} \qquad R_a = 1250\,\Omega$$

To construct the $1250\,\Omega$ resistor, combine a $1.2\,\text{k}\Omega$ resistor in series with a parallel combination of two $100\,\Omega$ resistors.

P 5.30 $v_o = \dfrac{R_d(R_a + R_b)}{R_a(R_c + R_d)}v_b - \dfrac{R_b}{R_a}v_a$

By hypothesis: $R_b/R_a = 4$; $R_c + R_d = 470\,\text{k}\Omega$; $\dfrac{R_d(R_a + R_b)}{R_a(R_c + R_d)} = 3$

$$\therefore\quad \frac{R_d}{R_a}\frac{(R_a + 4R_a)}{470{,}000} = 3 \qquad \text{so} \qquad R_d = 282\,\text{k}\Omega; \quad R_c = 188\,\text{k}\Omega$$

Create $R_\text{d} = 282\,\text{k}\Omega$ by combining a $270\,\text{k}\Omega$ resistor and a $12\,\text{k}\Omega$ resistor in series. Create $R_\text{c} = 188\,\text{k}\Omega$ by combining a $120\,\text{k}\Omega$ resistor and a $68\,\text{k}\Omega$ resistor in series. Also, when $v_o = 0$ we have

$$\frac{v_n - v_\text{a}}{R_\text{a}} + \frac{v_n}{R_\text{b}} = 0$$

$$\therefore \quad v_n\left(1 + \frac{R_\text{a}}{R_\text{b}}\right) = v_\text{a}; \qquad v_n = 0.8v_\text{a}$$

$$i_\text{a} = \frac{v_\text{a} - 0.8v_\text{a}}{R_\text{a}} = 0.2\frac{v_\text{a}}{R_\text{a}}; \qquad R_\text{in} = \frac{v_\text{a}}{i_\text{a}} = 5R_\text{a} = 22\,\text{k}\Omega$$

$$\therefore \quad R_\text{a} = 4.4\,\text{k}\Omega; \qquad R_\text{b} = 17.6\,\text{k}\Omega$$

Create $R_\text{a} = 4.4\,\text{k}\Omega$ by combining two $2.2\,\text{k}\Omega$ resistors in series. Create $R_\text{b} = 17.6\,\text{k}\Omega$ by combining a $12\,\text{k}\Omega$ resistor and a $5.6\,\text{k}\Omega$ resistor in series.

P 5.31 $v_p = \frac{1500}{9000}(-18) = -3 \text{ V} = v_n$

$$\frac{-3+18}{1600} + \frac{-3 - v_o}{R_\text{f}} = 0$$

$$\therefore \quad v_o = 0.009375R_\text{f} - 3$$

$$v_o = 9 \text{ V}; \qquad R_\text{f} = 1280\,\Omega$$

$$v_o = -9 \text{ V}; \qquad R_\text{f} = -640\,\Omega$$

But $R_\text{f} \geq 0, \qquad \therefore \quad R_\text{f} = 1.28\,\text{k}\Omega$

P 5.32 [a] $v_p = \frac{\alpha R_\text{g}}{\alpha R_\text{g} + (R_\text{g} - \alpha R_\text{g})}v_\text{g}$

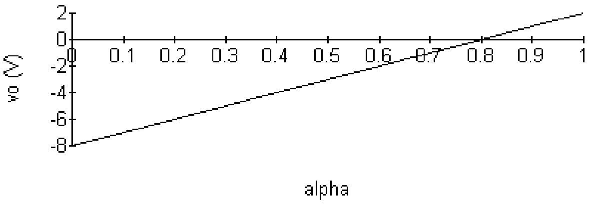
$$v_n = v_p = \alpha v_g$$

$$\frac{v_n - v_g}{R_1} + \frac{v_n - v_o}{R_\text{f}} = 0$$

$$(v_n - v_g)\frac{R_\text{f}}{R_1} + v_n - v_o = 0$$

$$\begin{aligned} v_o &= \left(1 + \frac{R_\text{f}}{R_\text{g}}\right)\alpha v_\text{g} - \frac{R_\text{f}}{R_1}v_\text{g} \\ &= (\alpha v_g - v_g)4 + \alpha v_g \\ &= [(\alpha - 1)4 + \alpha]v_g \\ &= (5\alpha - 4)v_g \\ &= (5\alpha - 4)(2) = 10\alpha - 8 \end{aligned}$$

α	v_o	α	v_o	α	v_o
0.0	−8 V	0.4	−4 V	0.8	0 V
0.1	−7 V	0.5	−3 V	0.9	1 V
0.2	−6 V	0.6	−2 V	1.0	2 V
0.3	−5 V	0.7	−1 V		



[**b**] Rearranging the equation for v_o from (a) gives

$$v_o = \left(\frac{R_f}{R_1} + 1\right) v_g\alpha + -\left(\frac{R_f}{R_1}\right) v_g$$

Therefore,

$$\text{slope } = \left(\frac{R_f}{R_1} + 1\right) v_g; \qquad \text{intercept } = -\left(\frac{R_f}{R_1}\right) v_g$$

[**c**] Using the equations from (b),

$$-6 = \left(\frac{R_f}{R_1} + 1\right) v_g; \qquad 4 = -\left(\frac{R_f}{R_1}\right) v_g$$

Solving,

$$v_g = -2 \text{ V}; \qquad \frac{R_f}{R_1} = 2$$

P 5.33 $$A_{\text{cm}} = \frac{(20)(50) - (50)R_x}{20(50 + R_x)}$$

$$A_{\text{dm}} = \frac{50(20 + 50) + 50(50 + R_x)}{2(20)(50 + R_x)}$$

$$\frac{A_{\text{dm}}}{A_{\text{cm}}} = \frac{R_x + 120}{2(20 - R_x)}$$

$$\therefore \quad \frac{R_x + 120}{2(20 - R_x)} = \pm 1000 \quad \text{for the limits on the value of } R_x$$

If we use +1000 $R_x = 19.93\,\text{k}\Omega$

If we use $-1000 \quad R_x = 20.07\,\text{k}\Omega$

$19.93\,\text{k}\Omega \leq R_x \leq 20.07\,\text{k}\Omega$

P 5.34 [**a**] $A_{\text{dm}} = \dfrac{(24)(26) + (25)(25)}{(2)(1)(25)} = 24.98$

[**b**] $A_{\text{cm}} = \dfrac{(1)(24) - 25(1)}{1(25)} = -0.04$

[**c**] $\text{CMRR} = \left|\dfrac{24.98}{0.04}\right| = 624.50$

P 5.35 [**a**] $v_p = v_s, \qquad v_n = \dfrac{R_1 v_o}{R_1 + R_2}, \qquad v_n = v_p$

$$\text{Therefore} \quad v_o = \left(\frac{R_1 + R_2}{R_1}\right) v_s = \left(1 + \frac{R_2}{R_1}\right) v_s$$

[**b**] $v_o = v_s$

[**c**] Because $v_o = v_s$, thus the output voltage follows the signal voltage.

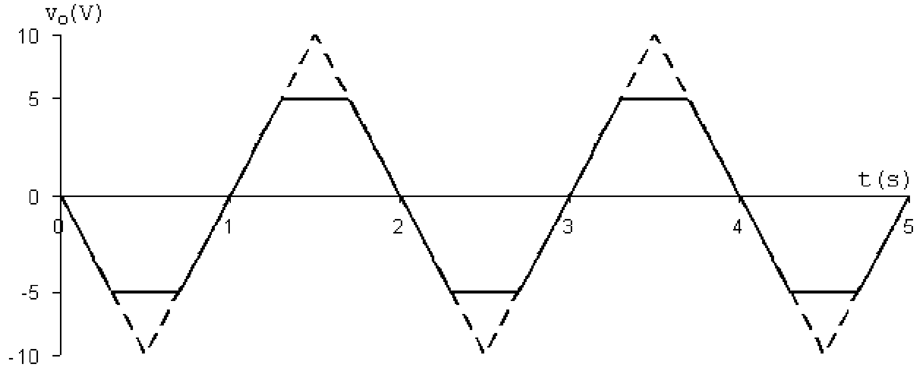
P 5.36 It follows directly from the circuit that $v_o = -(120/7.5)v_g = -16v_g$
From the plot of v_g we have $\quad v_g = 0, \quad t < 0$

$$\begin{aligned}
v_g &= t & 0 \leq t \leq 0.5 \\
v_g &= 1 - t & 0.5 \leq t \leq 1.5 \\
v_g &= t - 2 & 1.5 \leq t \leq 2.5 \\
v_g &= 3 - t & 2.5 \leq t \leq 3.5 \\
v_g &= t - 4 & 3.5 \leq t \leq 4.5, \quad \text{etc.}
\end{aligned}$$

Therefore

$$\begin{aligned}
v_o &= -16t & 0 \leq t \leq 0.5 \\
v_o &= 16t - 16 & 0.5 \leq t \leq 1.5 \\
v_o &= 32 - 16t & 1.5 \leq t \leq 2.5 \\
v_o &= 16t - 48 & 2.5 \leq t \leq 3.5 \\
v_o &= 64 - 16t & 3.5 \leq t \leq 4.5, \quad \text{etc.}
\end{aligned}$$

These expressions for v_o are valid as long as the op amp is not saturated. Since the peak values of v_o are ±5, the output is clipped at ±5. The plot is shown below.



P 5.37 $v_p = \frac{5.6}{8.0}v_g = 0.7v_g = 7\sin(\pi/3)t$ V

$$\frac{v_n}{15{,}000} + \frac{v_n - v_o}{75{,}000} = 0$$

$$6v_n = v_o; \qquad v_n = v_p$$

$$\therefore \quad v_o = 42\sin(\pi/3)t \text{ V} \qquad 0 \le t \le \infty$$

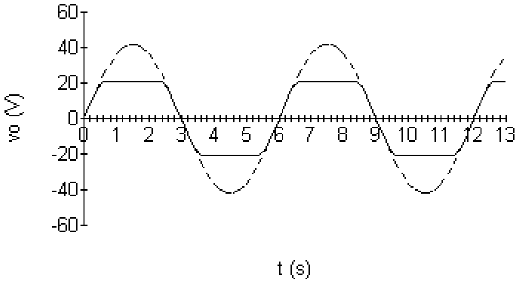
$$v_o = 0 \qquad t \le 0$$

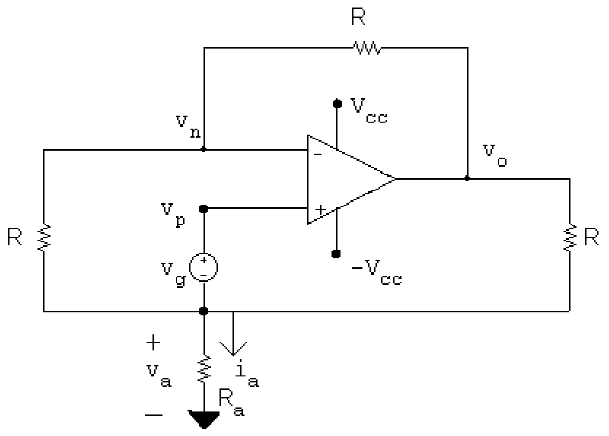
At saturation

$$42\sin\left(\frac{\pi}{3}\right)t = \pm 21; \qquad \sin\frac{\pi}{3}t = \pm 0.5$$

$$\therefore \quad \frac{\pi}{3}t = \frac{\pi}{6}, \quad \frac{5\pi}{6}, \quad \frac{7\pi}{6}, \quad \frac{11\pi}{6}, \quad \text{etc.}$$

$$t = 0.50\,\text{s}, \quad 2.50\,\text{s}, \quad 3.50\,\text{s}, \quad 5.50\,\text{s}, \quad \text{etc.}$$





P 5.38 **[a]**

$$\frac{v_n - v_a}{R} + \frac{v_n - v_o}{R} = 0$$

$$2v_n - v_a = v_o$$

$$\frac{v_a}{R_a} + \frac{v_a - v_n}{R} + \frac{v_a - v_o}{R} = 0$$

$$v_a\left[\frac{1}{R_a} + \frac{2}{R}\right] - \frac{v_n}{R} = \frac{v_o}{R}$$

$$v_a\left(2 + \frac{R}{R_a}\right) - v_n = v_o$$

$$v_n = v_p = v_a + v_g$$

$$\therefore\quad 2v_n - v_a = 2v_a + 2v_g - v_a = v_a + 2v_g$$

$$\therefore\quad v_a - v_o = -2v_g \qquad (1)$$

$$2v_a + v_a\left(\frac{R}{R_a}\right) - v_a - v_g = v_o$$

$$\therefore\quad v_a\left(1 + \frac{R}{R_a}\right) - v_o = v_g \qquad (2)$$

Now combining equations (1) and (2) yields

$$-v_a\frac{R}{R_a} = -3v_g$$

$$\text{or}\quad v_a = 3v_g\frac{R_a}{R}$$

$$\text{Hence}\quad i_a = \frac{v_a}{R_a} = \frac{3v_g}{R} \qquad \text{Q.E.D.}$$

[b] At saturation $v_o = \pm\,\mathrm{V_{cc}}$

$$\therefore \quad v_\mathrm{a} = \pm\,\mathrm{V_{cc}} - 2v_g \qquad (3)$$

and

$$\therefore \quad v_\mathrm{a}\left(1 + \frac{R}{R_\mathrm{a}}\right) = \pm\,\mathrm{V_{cc}} + v_g \qquad (4)$$

Dividing Eq (4) by Eq (3) gives

$$1 + \frac{R}{R_\mathrm{a}} = \frac{\pm\,\mathrm{V_{cc}} + v_g}{\pm\,\mathrm{V_{cc}} - 2v_g}$$

$$\therefore \quad \frac{R}{R_\mathrm{a}} = \frac{\pm\,\mathrm{V_{cc}} + v_g}{\pm\,\mathrm{V_{cc}} - 2v_g} - 1 = \frac{3v_g}{\pm\,\mathrm{V_{cc}} - 2v_g}$$

$$\text{or} \quad R_\mathrm{a} = \frac{(\pm\,\mathrm{V_{cc}} - 2v_g)}{3v_g}R \qquad \text{Q.E.D.}$$

P 5.39 **[a]** $p_{16\,\mathrm{k}\Omega} = \dfrac{(320 \times 10^{-3})^2}{(16 \times 10^3)} = 6.4\,\mu\mathrm{W}$

[b] $v_{16\,\mathrm{k}\Omega} = \left(\dfrac{16}{64}\right)(320) = 80\,\mathrm{mV}$

$$p_{16\,\mathrm{k}\Omega} = \frac{(80 \times 10^{-3})^2}{(16 \times 10^3)} = 0.4\,\mu\mathrm{W}$$

[c] $\dfrac{p_\mathrm{a}}{p_\mathrm{b}} = \dfrac{6.4}{0.4} = 16$

[d] Yes, the operational amplifier serves several useful purposes:

- First, it enables the source to control 16 times as much power delivered to the load resistor. When a small amount of power controls a larger amount of power, we refer to it as *power amplification.*
- Second, it allows the full source voltage to appear across the load resistor, no matter what the source resistance. This is the *voltage follower* function of the operational amplifier.
- Third, it allows the load resistor voltage (and thus its current) to be set without drawing any current from the input voltage source. This is the *current amplification* function of the circuit.

P 5.40 **[a]** Assume the op-amp is operating within its linear range, then

$$i_L = \frac{8}{4000} = 2\,\mathrm{mA}$$

For $R_L = 4\,\mathrm{k}\Omega \qquad v_o = (4 + 4)(2) = 16\ \mathrm{V}$

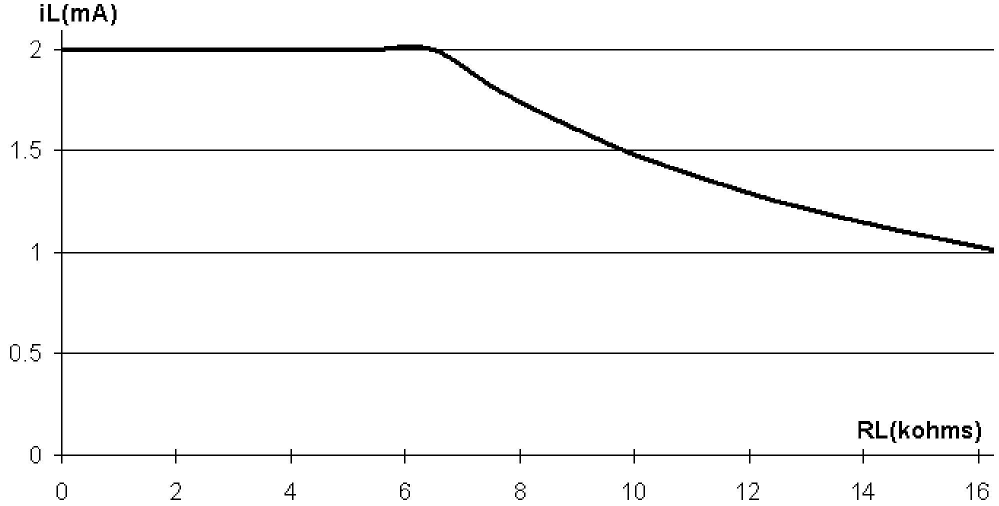
Now since $v_o < 20\ \mathrm{V}$ our assumption of linear operation is correct, therefore

$$i_L = 2\,\mathrm{mA}$$

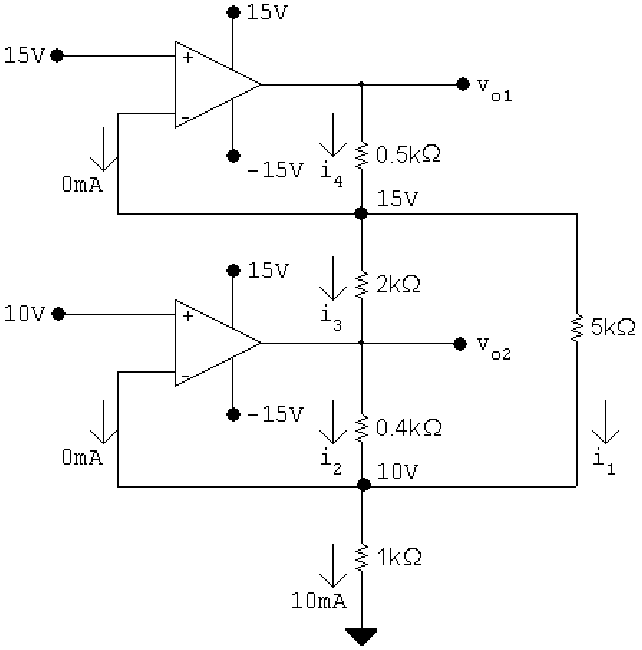
[b] $20 = 2(4 + R_L)$; $\qquad R_L = 6\,\text{k}\Omega$

[c] As long as the op-amp is operating in its linear region i_L is independent of R_L. From (b) we found the op-amp is operating in its linear region as long as $R_L \leq 6\,\text{k}\Omega$. Therefore when $R_L = 6\,\text{k}\Omega$ the op-amp is saturated. We can estimate the value of i_L by assuming $i_p = i_n \ll i_L$. Then $i_L = 20/(4000 + 16{,}000) = 1\,\text{mA}$. To justify neglecting the current into the op-amp assume the drop across the 50 kΩ resistor is negligible, since the input resistance to the op-amp is at least $500\,\text{k}\Omega$. Then $i_p = i_n = (8 - 4)/(500 \times 10^3) = 8\,\mu\text{A}$. But $8\,\mu\text{A} \ll 1\,\text{mA}$, hence our assumption is reasonable.

[d]



P 5.41



$$i_1 = \frac{15 - 10}{5000} = 1\,\text{mA}$$

$$i_2 + i_1 + 0 = 10\,\text{mA}; \qquad i_2 = 9\,\text{mA}$$

$$v_{o2} = 10 + (400)(9) \times 10^{-3} = 13.6 \text{ V}$$

$$i_3 = \frac{15 - 13.6}{2000} = 0.7\,\text{mA}$$

$$i_4 = i_3 + i_1 = 1.7\,\text{mA}$$

$$v_{o1} = 15 + 1.7(0.5) = 15.85 \text{ V}$$

P 5.42 **[a]** Let v_{o1} = output voltage of the amplifier on the left. Let v_{o2} = output voltage of the amplifier on the right. Then

$$v_{o1} = \frac{-47}{10}(1) = -4.7 \text{ V}; \qquad v_{o2} = \frac{-220}{33}(-0.15) = 1.0 \text{ V}$$

$$i_{\text{a}} = \frac{v_{o2} - v_{o1}}{1000} = 5.7\,\text{mA}$$

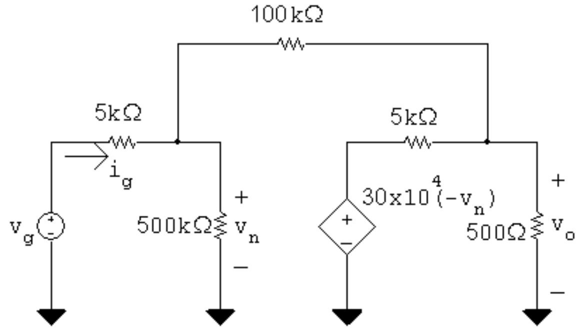
[b] $i_{\text{a}} = 0$ when $v_{o1} = v_{o2}$ so from (a) $v_{o2} = 1$ V

Thus

$$\frac{-47}{10}(v_{\text{L}}) = 1$$

$$v_{\text{L}} = -\frac{10}{47} = -212.77 \text{ mV}$$

P 5.43 **[a]** Replace the op amp with the model from Fig. 5.15:



Write two node voltage equations, one at the left node, the other at the right node:

$$\frac{v_n - v_g}{5000} + \frac{v_n - v_o}{100{,}000} + \frac{v_n}{500{,}000} = 0$$

$$\frac{v_o + 3 \times 10^5 v_n}{5000} + \frac{v_o - v_n}{100{,}000} + \frac{v_o}{500} = 0$$

Simplify and place in standard form:

$$106v_n - 5v_o = 100v_g$$

$$(6 \times 10^6 - 1)v_n + 221v_o = 0$$

Let $v_g = 1$ V and solve the two simultaneous equations:

$$v_o = -19.9844 \text{ V}; \qquad v_n = 736.1\,\mu\text{V}$$

Thus the voltage gain is $v_o/v_g = -19.9844$.

[b] From the solution in part (a), $v_n = 736.1\,\mu$V.

[c] $i_g = \dfrac{v_g - v_n}{5000} = \dfrac{v_g - 736.1 \times 10^{-6} v_g}{5000}$

$$R_g = \frac{v_g}{i_g} = \frac{5000}{1 - 736.1 \times 10^{-6}} = 5003.68\,\Omega$$

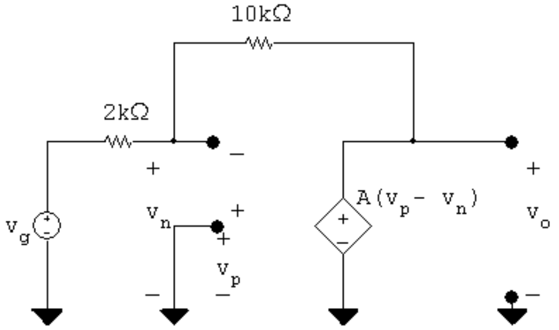
[d] For an ideal op amp, the voltage gain is the ratio between the feedback resistor and the input resistor:

$$\frac{v_o}{v_g} = -\frac{100{,}000}{5000} = -20$$

For an ideal op amp, the difference between the voltages at the input terminals is zero, and the input resistance of the op amp is infinite. Therefore,

$$v_n = v_p = 0 \text{ V}; \qquad R_g = 5000\,\Omega$$

P 5.44 **[a]**



$$\frac{v_n - v_g}{2000} + \frac{v_n - v_o}{10{,}000} = 0$$

$$\therefore \quad v_o = 6v_n - 5v_g$$

Also $\quad v_o = A(v_p - v_n) = -Av_n$

$$\therefore \quad v_n = \frac{-v_o}{A}$$

$$\therefore \quad v_o\left(1 + \frac{6}{A}\right) = -5v_g$$

$$v_o = \frac{-5A}{(6 + A)} v_g$$

[b] $v_o = \dfrac{-5(194)(1)}{200} = -4.85$ V

[c] $v_o = \dfrac{-5}{1 + (6/A)}(1) = -5$ V

[d] $\frac{-5A}{A+6}(1) = -0.99(5) \qquad \text{so} \qquad -5A = -4.95(A+6)$

$\therefore \quad -0.05A = -29.7 \qquad \text{so} \qquad A = 594$

P 5.45 **[a]** $\frac{v_n}{16{,}000} + \frac{v_n - v_g}{800{,}000} + \frac{v_n - v_o}{200{,}000} = 0 \quad \text{or} \quad 55v_n - 4v_o = v_g \qquad \text{Eq (1)}$

$$\frac{v_o}{20{,}000} + \frac{v_o - v_n}{200{,}000} + \frac{v_o - 50{,}000(v_p - v_n)}{8000} = 0$$

$$36v_o - v_n - 125 \times 10^4(v_p - v_n) = 0$$

$$v_p = v_g + \frac{(v_n - v_g)(240)}{800} = (0.7)v_g + (0.3)v_n$$

$$36v_o - v_n - 125 \times 10^4[(0.7)v_g - (0.7)v_n] = 0$$

$$36v_o + 874{,}999v_n = 875{,}000v_g \qquad \text{Eq (2)}$$

Let $v_g = 1$ V and solve Eqs. (1) and (2) simultaneously:

$v_n = 999.446\,\text{mV} \qquad \text{and} \qquad v_o = 13.49\ \text{V}$

$$\therefore \qquad \frac{v_o}{v_g} = 13.49$$

[b] From part (a), $v_n = 999.446$ mV.

$$v_p = (0.7)(1000) + (0.3)(999.446) = 999.834\,\text{mV}$$

[c] $v_p - v_n = 387.78\,\mu\text{V}$

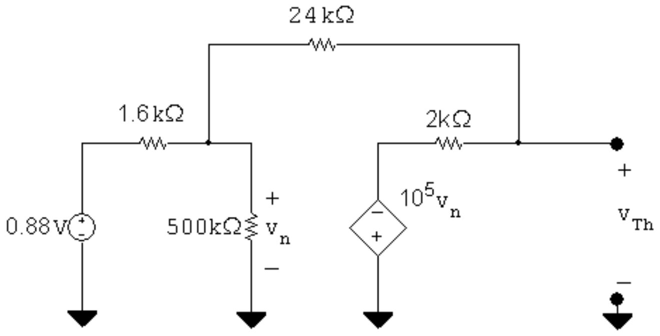
[d] $i_g = \frac{(1000 - 999.83)10^{-3}}{24 \times 10^3} = 692.47\,\text{pA}$

[e] $\frac{v_g}{16{,}000} + \frac{v_g - v_o}{200{,}000} = 0, \qquad \text{since } v_n = v_p = v_g$

$$\therefore \quad v_o = 13.5v_g, \qquad \frac{v_o}{v_g} = 13.5$$

$$v_n = v_p = 1\ \text{V}; \qquad v_p - v_n = 0\ \text{V}; \qquad i_g = 0\ \text{A}$$

P 5.46 **[a]**

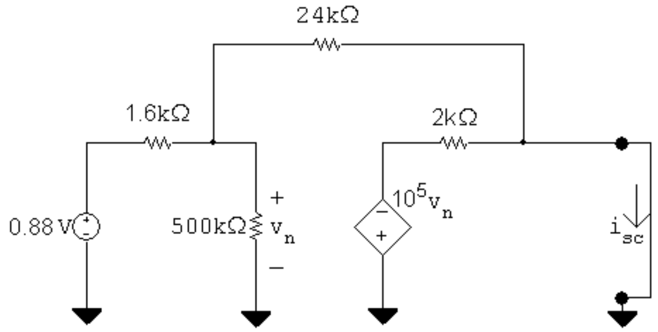


$$\frac{v_n - 0.88}{1600} + \frac{v_n}{500{,}000} + \frac{v_n - v_{\text{Th}}}{24{,}000} = 0$$

$$\frac{v_{\rm Th} + 10^5 v_n}{2000} + \frac{v_{\rm Th} - v_n}{24{,}000} = 0$$

Solving, $v_{\rm Th} = -13.198$ V

Short-circuit current calculation:

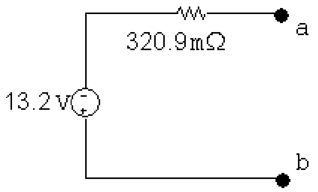


$$\frac{v_n}{500{,}000} + \frac{v_n - 0.88}{1600} + \frac{v_n - 0}{24{,}000} = 0$$

$$\therefore \quad v_n = 0.8225 \text{ V}$$

$$i_{\rm sc} = \frac{v_n}{24{,}000} - \frac{10^5}{2000} v_n = -41.13 \text{ A}$$

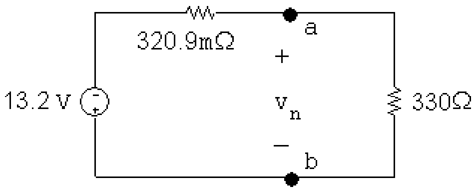
$$R_{\rm Th} = \frac{v_{\rm Th}}{i_{\rm sc}} = 320.9\,{\rm m}\Omega$$



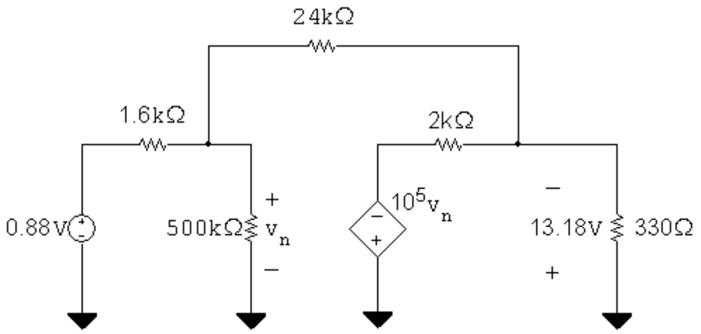
[b] The output resistance of the inverting amplifier is the same as the Thévenin resistance, i.e.,

$$R_o = R_{\rm Th} = 320.9\,{\rm m}\Omega$$

[c]



$$v_o = \left(\frac{330}{330.3209}\right)(-13.2) = -13.18 \text{ V}$$



$$\frac{v_n - 0.88}{1600} + \frac{v_n}{500{,}000} + \frac{v_n + 13.18}{24{,}000} = 0$$

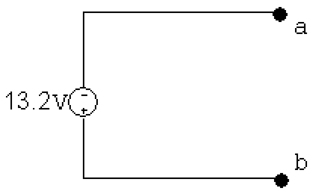
$$\therefore \quad v_n = 942\,\mu\text{V}$$

$$i_g = \frac{0.88 - 942 \times 10^{-6}}{1600} = 549.41\,\mu\text{A}$$

$$R_g = \frac{0.88}{i_g} = 1601.71\,\Omega$$

P 5.47 **[a]** $v_{\text{Th}} = -\dfrac{24{,}000}{1600}(0.88) = -13.2\ \text{V}$

$R_{\text{Th}} = 0,$ since op-amp is ideal



[b] $R_o = R_{\text{Th}} = 0\,\Omega$

[c] $R_g = 1.6\,\text{k}\Omega \quad \text{since} \quad v_n = 0$

P 5.48 From Eq. 5.57,

$$\frac{v_{\text{ref}}}{R + \Delta R} = v_n \left(\frac{1}{R + \Delta R} + \frac{1}{R - \Delta R} + \frac{1}{R_f} \right) - \frac{v_o}{R_f}$$

Substituting Eq. 5.59 for $v_p = v_n$:

$$\frac{v_{\text{ref}}}{R + \Delta R} = \frac{v_{\text{ref}} \left(\frac{1}{R+\Delta R} + \frac{1}{R-\Delta R} + \frac{1}{R_f} \right)}{(R - \Delta R) \left(\frac{1}{R+\Delta R} + \frac{1}{R-\Delta R} + \frac{1}{R_f} \right)} - \frac{v_o}{R_f}$$

Rearranging,

$$\frac{v_o}{R_f} = v_{\text{ref}} \left(\frac{1}{R - \Delta R} - \frac{1}{R + \Delta R} \right)$$

Thus,

$$v_o = v_{\text{ref}}\left(\frac{2\Delta R}{R^2 - \Delta R^2}\right) R_f$$

P 5.49 **[a]** Use Eq. 5.61 to solve for R_f; note that since we are using 1% strain gages, $\Delta = 0.01$:

$$R_f = \frac{v_o R}{2\Delta v_{\text{ref}}} = \frac{(5)(120)}{(2)(0.01)(15)} = 2\,\text{k}\Omega$$

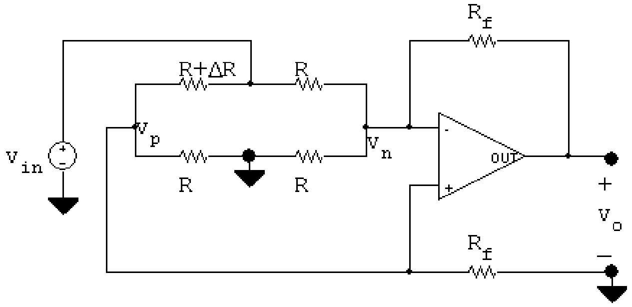
[b] Now solve for Δ given $v_o = 50$ mV:

$$\Delta = \frac{v_o R}{2R_f v_{\text{ref}}} = \frac{(0.05)(120)}{2(2000)(15)} = 100 \times 10^{-6}$$

The change in strain gage resistance that corresponds to a 50 mV change in output voltage is thus

$$\Delta R = \Delta R = (100 \times 10^{-6})(120) = 12\ \text{m}\Omega$$

P 5.50 **[a]**



Let $R_1 = R + \Delta R$

$$\frac{v_p}{R_f} + \frac{v_p}{R} + \frac{v_p - v_{\text{in}}}{R_1} = 0$$

$$\therefore\quad v_p\left[\frac{1}{R_f} + \frac{1}{R} + \frac{1}{R_1}\right] = \frac{v_{\text{in}}}{R_1}$$

$$\therefore\quad v_p = \frac{RR_f v_{\text{in}}}{RR_1 + R_f R_1 + R_f R} = v_n$$

$$\frac{v_n}{R} + \frac{v_n - v_{\text{in}}}{R} + \frac{v_n - v_o}{R_f} = 0$$

$$v_n\left[\frac{1}{R} + \frac{1}{R} + \frac{1}{R_f}\right] - \frac{v_o}{R_f} = \frac{v_{\text{in}}}{R}$$

$$\therefore\quad v_n\left[\frac{R + 2R_f}{RR_f}\right] - \frac{v_{\text{in}}}{R} = \frac{v_o}{R_f}$$

$$\therefore \quad \frac{v_o}{R_f} = \left[\frac{R + 2R_f}{RR_f}\right]\left[\frac{RR_f v_{\text{in}}}{RR_1 + R_f R_1 + R_f R}\right] - \frac{v_{\text{in}}}{R}$$

$$\therefore \quad \frac{v_o}{R_f} = \left[\frac{R + 2R_f}{RR_1 + R_f R_1 + R_f R} - \frac{1}{R}\right] v_{\text{in}}$$

$$\therefore \quad v_o = \frac{[R^2 + 2RR_f - R_1(R + R_f) - RR_f]R_f}{R[R_1(R + R_f) + RR_f]} v_{\text{in}}$$

Now substitute $R_1 = R + \Delta R$ and get

$$v_o = \frac{-\Delta R(R + R_f)R_f v_{\text{in}}}{R[(R + \Delta R)(R + R_f) + RR_f]}$$

If $\Delta R \ll R$

$$v_o \approx \frac{(R + R_f)R_f(-\Delta R)v_{\text{in}}}{R^2(R + 2R_f)}$$

[b] $v_o \approx \dfrac{47 \times 10^4(48 \times 10^4)(-95)15}{10^8(95 \times 10^4)} \approx -3.384 \text{ V}$

[c] $v_o = \dfrac{-95(48 \times 10^4)(47 \times 10^4)15}{10^4[(1.0095)10^4(48 \times 10^4) + 47 \times 10^8]} = -3.368 \text{ V}$

P 5.51 **[a]** $v_o \approx \dfrac{(R + R_f)R_f(-\Delta R)v_{\text{in}}}{R^2(R + 2R_f)}$

$$v_o = \frac{(R + R_f)(-\Delta R)R_f v_{\text{in}}}{R[(R + \Delta R)(R + R_f) + RR_f]}$$

$$\therefore \quad \frac{\text{approx value}}{\text{true value}} = \frac{R[(R + \Delta R)(R + R_f) + RR_f]}{R^2(R + 2R_f)}$$

$$\therefore \quad \text{Error} = \frac{R[(R + \Delta R)(R + R_f) + RR_f] - R^2(R + 2R_f)}{R^2(R + 2R_f)}$$

$$= \frac{\Delta R}{R}\frac{(R + R_f)}{(R + 2R_f)}$$

$$\therefore \quad \%\text{ error} = \frac{\Delta R(R + R_f)}{R(R + 2R_f)} \times 100$$

[b] $\%\text{ error} = \dfrac{95(48 \times 10^4) \times 100}{10^4(95 \times 10^4)} = 0.48\%$

P 5.52 $1 = \dfrac{\Delta R(48 \times 10^4)}{10^4(95 \times 10^4)} \times 100$

$$\therefore \quad \Delta R = \frac{9500}{48} = 197.91667\,\Omega$$

$$\therefore\quad \%\text{ change in } R = \frac{197.19667}{10^4} \times 100 \approx 1.98\%$$

P 5.53 **[a]** It follows directly from the solution to Problem 5.50 that

$$v_o = \frac{[R^2 + 2RR_f - R_1(R + R_f) - RR_f]R_f v_{\text{in}}}{R[R_1(R + R_f) + RR_f]}$$

Now $R_1 = R - \Delta R$. Substituting into the expression gives

$$v_o = \frac{(R + R_f)R_f(\Delta R)v_{\text{in}}}{R[(R - \Delta R)(R + R_f) + RR_f]}$$

Now let $\Delta R \ll R$ and get

$$v_o \approx \frac{(R + R_f)R_f \Delta R v_{\text{in}}}{R^2(R + 2R_f)}$$

[b] It follows directly from the solution to Problem 5.50 that

$$\therefore\quad \frac{\text{approx value}}{\text{true value}} = \frac{R[(R - \Delta R)(R + R_f) + RR_f]}{R^2(R + 2R_f)}$$

$$\therefore\quad \text{Error } = \frac{(R - \Delta R)(R + R_f) + RR_f - R(R + 2R_f)}{R(R + 2R_f)}$$

$$= \frac{-\Delta R(R + R_f)}{R(R + 2R_f)}$$

$$\therefore\quad \%\text{ error } = \frac{-\Delta R(R + R_f)}{R(R + 2R_f)} \times 100$$

[c] $R - \Delta R = 9810\,\Omega \qquad \therefore\quad \Delta R = 10{,}000 - 9810 = 190\,\Omega$

$$\therefore\quad v_o \approx \frac{(48 \times 10^4)(47 \times 10^4)(190)(15)}{10^8(95 \times 10^4)} \approx 6.768\ \text{V}$$

[d] $\%\text{ error } = \dfrac{-190(48 \times 10^4)(100)}{10^4(95 \times 10^4)} = -0.96\%$

6

Inductance, Capacitance, and Mutual Inductance

Assessment Problems

AP 6.1 [a] $i_g = 8e^{-300t} - 8e^{-1200t}\text{A}$

$$v = L\frac{di_g}{dt} = -9.6e^{-300t} + 38.4e^{-1200t}\text{V}, \qquad t > 0^+$$

$$v(0^+) = -9.6 + 38.4 = 28.8\,\text{V}$$

[b] $v = 0 \quad \text{when} \quad 38.4e^{-1200t} = 9.6e^{-300t} \quad \text{or} \quad t = (\ln 4)/900 = 1.54\,\text{ms}$

[c] $p = vi = 384e^{-1500t} - 76.8e^{-600t} - 307.2e^{-2400t}\,\text{W}$

[d] $\frac{dp}{dt} = 0 \quad \text{when} \quad e^{1800t} - 12.5e^{900t} + 16 = 0$

Let $x = e^{900t}$ and solve the quadratic $x^2 - 12.5x + 16 = 0$

$$x = 1.44766, \qquad t = \frac{\ln 1.45}{900} = 411.05\,\mu\text{s}$$

$$x = 11.0523, \qquad t = \frac{\ln 11.05}{900} = 2.67\,\text{ms}$$

p is maximum at $t = 411.05\,\mu$s

[e] $p_{\max} = 384e^{-1.5(0.41105)} - 76.8e^{-0.6(0.41105)} - 307.2e^{-2.4(0.41105)} = 32.72\,\text{W}$

[f] W is max when i is max, i is max when di/dt is zero.

When $di/dt = 0$, $v = 0$, therefore $t = 1.54$ ms.

[g] $i_{\max} = 8[e^{-0.3(1.54)} - e^{-1.2(1.54)}] = 3.78\,\text{A}$

$$w_{\max} = (1/2)(4 \times 10^{-3})(3.78)^2 = 28.6\,\text{mJ}$$

AP 6.2 **[a]** $i = C\frac{dv}{dt} = 24 \times 10^{-6}\frac{d}{dt}[e^{-15{,}000t}\sin 30{,}000t]$

$$= [0.72\cos 30{,}000t - 0.36\sin 30{,}000t]e^{-15{,}000t}\,\text{A}, \qquad i(0^+) = 0.72\,\text{A}$$

[b] $i\left(\frac{\pi}{80}\,\text{ms}\right) = -31.66\,\text{mA}, \quad v\left(\frac{\pi}{80}\,\text{ms}\right) = 20.505\,\text{V},$

$p = vi = -649.23\,\text{mW}$

[c] $w = \left(\frac{1}{2}\right)Cv^2 = 126.13\,\mu\text{J}$

AP 6.3 **[a]** $v = \left(\frac{1}{C}\right)\int_{0^-}^{t} i\,dx + v(0^-)$

$$= \frac{1}{0.6 \times 10^{-6}}\int_{0^-}^{t} 3\cos 50{,}000x\,dx = 100\sin 50{,}000t\,\text{V}$$

[b] $p(t) = vi = [300\cos 50{,}000t]\sin 50{,}000t$

$$= 150\sin 100{,}000t\,\text{W}, \qquad p_{(\text{max})} = 150\,\text{W}$$

[c] $w_{(\text{max})} = \left(\frac{1}{2}\right)Cv_{\text{max}}^2 = 0.30(100)^2 = 3000\,\mu\text{J} = 3\,\text{mJ}$

AP 6.4 **[a]** $L_{\text{eq}} = \frac{60(240)}{300} = 48\,\text{mH}$

[b] $i(0^+) = 3 + -5 = -2\,\text{A}$

[c] $i = \frac{125}{6}\int_{0^+}^{t}(-0.03e^{-5x})\,dx - 2 = 0.125e^{-5t} - 2.125\,\text{A}$

[d] $i_1 = \frac{50}{3}\int_{0^+}^{t}(-0.03e^{-5x})\,dx + 3 = 0.1e^{-5t} + 2.9\,\text{A}$

$i_2 = \frac{25}{6}\int_{0^+}^{t}(-0.03e^{-5x})\,dx - 5 = 0.025e^{-5t} - 5.025\,\text{A}$

$i_1 + i_2 = i$

AP 6.5 $v_1 = 0.5 \times 10^6\int_{0^+}^{t} 240 \times 10^{-6}e^{-10x}\,dx - 10 = -12e^{-10t} + 2\,\text{V}$

$v_2 = 0.125 \times 10^6\int_{0^+}^{t} 240 \times 10^{-6}e^{-10x}\,dx - 5 = -3e^{-10t} - 2\,\text{V}$

$v_1(\infty) = 2\,\text{V}, \qquad v_2(\infty) = -2\,\text{V}$

$W = \left[\frac{1}{2}(2)(4) + \frac{1}{2}(8)(4)\right] \times 10^{-6} = 20\,\mu\text{J}$

AP 6.6 [**a**] Summing the voltages around mesh 1 yields

$$4\frac{di_1}{dt} + 8\frac{d(i_2+i_g)}{dt} + 20(i_1 - i_2) + 5(i_1 + i_g) = 0$$

or

$$4\frac{di_1}{dt} + 25i_1 + 8\frac{di_2}{dt} - 20i_2 = -\left(5i_g + 8\frac{di_g}{dt}\right)$$

Summing the voltages around mesh 2 yields

$$16\frac{d(i_2+i_g)}{dt} + 8\frac{di_1}{dt} + 20(i_2 - i_1) + 780i_2 = 0$$

or

$$8\frac{di_1}{dt} - 20i_1 + 16\frac{di_2}{dt} + 800i_2 = -16\frac{di_g}{dt}$$

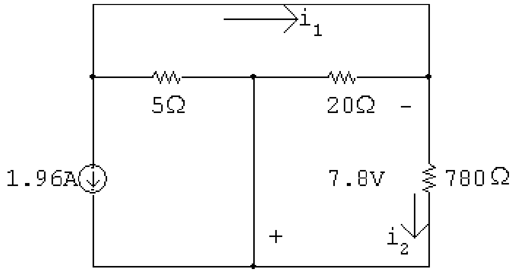
[**b**] From the solutions given in part (b)

$$i_1(0) = -0.4 - 11.6 + 12 = 0; \qquad i_2(0) = -0.01 - 0.99 + 1 = 0$$

These values agree with zero initial energy in the circuit. At infinity,

$$i_1(\infty) = -0.4\text{A}; \qquad i_2(\infty) = -0.01\text{A}$$

When $t = \infty$ the circuit reduces to



$$\therefore \quad i_1(\infty) = -\left(\frac{7.8}{20} + \frac{7.8}{780}\right) = -0.4\text{A}; \quad i_2(\infty) = -\frac{7.8}{780} = -0.01\text{A}$$

From the solutions for i_1 and i_2 we have

$$\frac{di_1}{dt} = 46.40e^{-4t} - 60e^{-5t}$$

$$\frac{di_2}{dt} = 3.96e^{-4t} - 5e^{-5t}$$

Also, $\dfrac{di_g}{dt} = 7.84e^{-4t}$

Thus

$$4\frac{di_1}{dt} = 185.60e^{-4t} - 240e^{-5t}$$

$$25i_1 = -10 - 290e^{-4t} + 300e^{-5t}$$

$$8\frac{di_2}{dt} = 31.68e^{-4t} - 40e^{-5t}$$

$$20i_2 = -0.20 - 19.80e^{-4t} + 20e^{-5t}$$

$$5i_g = 9.8 - 9.8e^{-4t}$$

$$8\frac{di_g}{dt} = 62.72e^{-4t}$$

Test:

$$185.60e^{-4t} - 240e^{-5t} - 10 - 290e^{-4t} + 300e^{-5t} + 31.68e^{-4t} - 40e^{-5t}$$

$$+0.20 + 19.80e^{-4t} - 20e^{-5t} \overset{?}{=} -[9.8 - 9.8e^{-4t} + 62.72e^{-4t}]$$

$$-9.8 + (300 - 240 - 40 - 20)e^{-5t}$$

$$+(185.60 - 290 + 31.68 + 19.80)e^{-4t} \overset{?}{=} -(9.8 + 52.92e^{-4t})$$

$$-9.8 + 0e^{-5t} + (237.08 - 290)e^{-4t} \overset{?}{=} -9.8 - 52.92e^{-4t}$$

$$-9.8 - 52.92e^{-4t} = -9.8 - 52.92e^{-4t} \quad \text{(OK)}$$

Also,

$$8\frac{di_1}{dt} = 371.20e^{-4t} - 480e^{-5t}$$

$$20i_1 = -8 - 232e^{-4t} + 240e^{-5t}$$

$$16\frac{di_2}{dt} = 63.36e^{-4t} - 80e^{-5t}$$

$$800i_2 = -8 - 792e^{-4t} + 800e^{-5t}$$

$$16\frac{di_g}{dt} = 125.44e^{-4t}$$

Test:

$$371.20e^{-4t} - 480e^{-5t} + 8 + 232e^{-4t} - 240e^{-5t} + 63.36e^{-4t} - 80e^{-5t}$$

$$-8 - 792e^{-4t} + 800e^{-5t} \overset{?}{=} -125.44e^{-4t}$$

$$(8 - 8) + (800 - 480 - 240 - 80)e^{-5t}$$

$$+(371.20 + 232 + 63.36 - 792)e^{-4t} \overset{?}{=} -125.44e^{-4t}$$

$$(800 - 800)e^{-5t} + (666.56 - 792)e^{-4t} \overset{?}{=} -125.44e^{-4t}$$

$$-125.44e^{-4t} = -125.44e^{-4t} \quad \text{(OK)}$$

Problems

P 6.1 $0 \le t \le 2\,\mathrm{s}:$

$$i_L = \frac{10^3}{2.5}\int_0^t 3\times 10^{-3}e^{-4x}\,dx + 1 = 1.2\frac{e^{-4x}}{-4}\Big|_0^t + 1$$

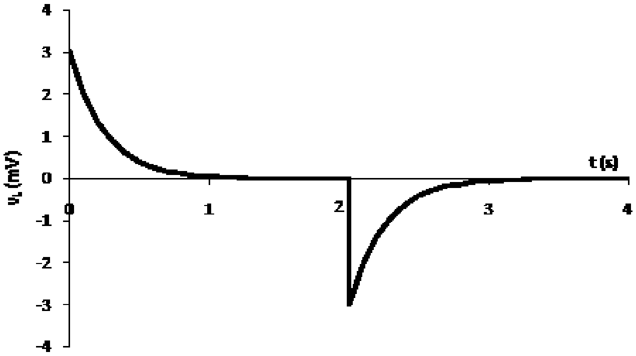
$$= -0.3e^{-4t} + 1.3\,\mathrm{A}, \qquad 0 \le t \le 2\,\mathrm{s}$$

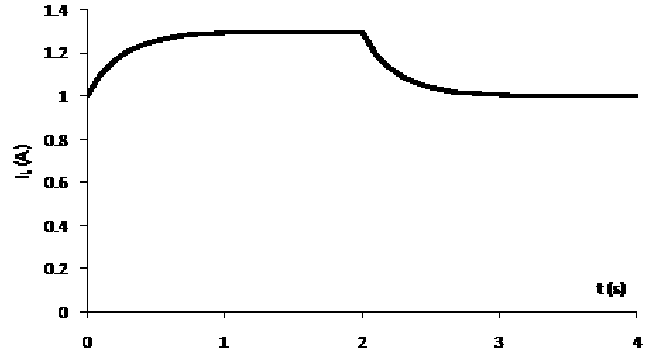
$$i_L(2) = -0.3e^{-8} + 1.3 = 1.3\,\mathrm{A}$$

$t \ge 2\,\mathrm{s}:$

$$i_L = \frac{10^3}{2.5}\int_2^t -3\times 10^{-3}e^{-4(x-2)}\,dx + 1.3 = -1.2\frac{e^{-4(x-2)}}{-4}\Big|_2^t + 1.3$$

$$= 0.3e^{-4(t-2)} + 1\,\mathrm{A}, \qquad t \ge 2\,\mathrm{s}$$





P 6.2 [a] $v = L\dfrac{di}{dt}$

$$= (50\times 10^{-6})(18)[e^{-10t} - 10te^{-10t}] = 900e^{-10t}(1-10t)\,\mu\mathrm{V}$$

[b] $i(200\,\text{ms}) = 18(0.2)(e^{-2}) = 487.21\,\text{mA}$

$$v(200\,\text{ms}) = 900(e^{-2})(1-2) = -121.8\,\mu\text{V}$$

$$p(200\,\text{ms}) = vi = (487.21\times 10^{-3})(-121.8\times 10^{-6}) = -59.34\,\mu\text{W}$$

[c] delivering $59.34\,\mu\text{W}$

[d] $i(200\,\text{ms}) = 487.21\,\text{mA}$ (from part [b])

$$w = \frac{1}{2}Li^2 = \frac{1}{2}(50\times 10^{-6})(0.48721)^2 = 5.93\,\mu\text{J}$$

[e] The energy is a maximum where the current is a maximum:

$$\frac{di_L}{dt} = 0 \quad \text{when} \quad 1 - 10t = 0 \quad \text{or} \quad t = 0.1\,\text{s}$$

$$i_{\text{max}} = 18(0.1)e^{-1} = 662.18\,\text{mA}$$

$$w_{\text{max}} = \frac{1}{2}(50\times 10^{-6})(0.66218)^2 = 10.96\,\mu\text{J}$$

P 6.3 **[a]** $0 \le t \le 2\,\text{ms}:$

$$i = \frac{1}{L}\int_0^t v_s\,dx + i(0) = \frac{10^6}{200}\int_0^t 5\times 10^{-3}\,dx + 0$$

$$= \frac{5000}{200}x\Big|_0^t = 25t\,\text{A}$$

$$2\,\text{ms} \le t < \infty: \qquad i = \frac{10^6}{200}\int_{2\times 10^{-3}}^t (0)\,dx + 2\times 10^{-3} = 50\,\text{mA}$$

[b] $i = 25t\,\text{mA}, \quad 0 \le t \le 2\,\text{ms}; \qquad i = 50\,\text{mA}, \quad t \ge 2\,\text{ms}$

P 6.4 **[a]**

$$\begin{aligned} i &= 0 && t<0\\ i &= 50t\,\text{A} && 0\le t\le 5\,\text{ms}\\ i &= 0.5-50t\,\text{A} && 5\le t\le 10\,\text{ms}\\ i &= 0 && 10\,\text{ms}<t \end{aligned}$$

[b] $v = L\dfrac{di}{dt} = 20\times10^{-3}(50) = 1\,\text{V} \qquad 0\le t\le 5\,\text{ms}$

$v = 20\times10^{-3}(-50) = -1\,\text{V} \qquad 5\le t\le 10\,\text{ms}$

$$\begin{aligned} v &= 0 && t<0\\ v &= 1\,\text{V} && 0<t<5\,\text{ms}\\ v &= -1\,\text{V} && 5<t<10\,\text{ms}\\ v &= 0 && 10\,\text{ms}<t \end{aligned}$$

$p = vi$

$$\begin{aligned} p &= 0 && t<0\\ p &= (50t)(1) = 50t\,\text{W} && 0<t<5\,\text{ms}\\ p &= (0.5-50t)(-1) = 50t-0.5\,\text{W} && 5<t<10\,\text{ms}\\ p &= 0 && 10\,\text{ms}<t\\ w &= 0 && t<0\\ w &= \int_0^t (50x)\,dx = 50\frac{x^2}{2}\Big|_0^t = 25t^2\,\text{J} && 0<t<5\,\text{ms}\\ w &= \int_{0.005}^t (50x-0.5)\,dx + 0.625\times10^{-3}\\ &= 25x^2-0.5x\Big|_{0.005}^t + 0.625\times10^{-3}\\ &= 25t^2-0.5t+2.5\times10^{-3}\,\text{J} && 5<t<10\,\text{ms}\\ w &= 0 && 10\,\text{ms}<t \end{aligned}$$

P 6.5 **[a]** $0\le t\le 1\,\text{s}:$

$v = -100t$

$i = \dfrac{1}{5}\displaystyle\int_0^t -100x\,dx + 0 = -20\frac{x^2}{2}\Big|_0^t$

$i = -10t^2\,\text{A}$

$1\,\text{s} \le t \le 3\,\text{s}:$

$v = 100t - 200$

$i(1) = -10\,\text{A}$

$$\begin{aligned} \therefore\quad i &= \frac{1}{5}\int_1^t (100x - 200)\,dx - 10 \\ &= 20\int_1^t (x-2)\,dx - 10 \\ &= 10t^2 - 40t + 20\,\text{A} \end{aligned}$$

$3\,\text{s} \le t \le 5\,\text{s}:$

$v = 100\,\text{V}$

$i(3) = 90 - 120 + 20 = -10\,\text{A}$

$$\begin{aligned} i &= \frac{1}{5}\int_3^t (100)\,dx - 10 \\ &= 20(t-3) - 10 \\ &= 20t - 70\,\text{A} \end{aligned}$$

$5\,\text{s} \le t \le 6\,\text{s}:$

$v = 600t - 100$

$i(5) = 100 - 70 = 30\,\text{A}$

$$\begin{aligned} i &= \frac{1}{5}\int_5^t (600x - 100)\,dx + 30 \\ &= 20\int_5^t (6-x)\,dx + 30 \\ &= 120t - 600 - 10t^2 + 250 + 30 \\ &= -10t^2 + 120t - 320\,\text{A} \end{aligned}$$

$t \ge 6\,\text{s}:$

$v = 0$

$i(6) = 720 - 360 - 320 = 40\,\text{A}$

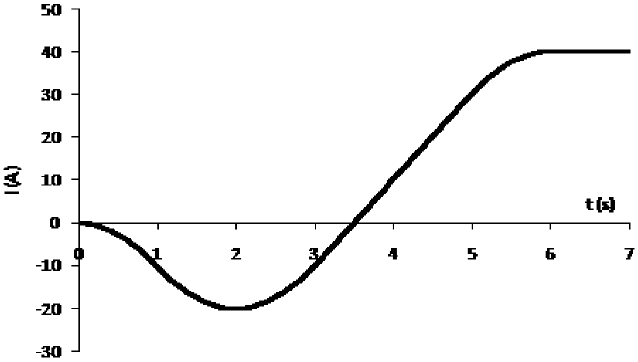
$$\begin{aligned} i &= \frac{1}{5}\int_6^t 0\,dx + 40 \\ &= 40\,\text{A} \end{aligned}$$

[b] $v = 0$ at $t = 2\,\text{s}$ and $t = 6\,\text{s}$

$$i(2) = 10(4) - 40(2) + 20 = -20\,\text{A}$$

$$i(6) = 40\,\text{A}$$

[c]



P 6.6 **[a]** $i(0) = A_1 + A_2 = 0.04$

$$\frac{di}{dt} = -10{,}000A_1e^{-10{,}000t} - 40{,}000A_2e^{-40{,}000t}$$

$$v = -200A_1e^{-10{,}000t} - 800A_2e^{-40{,}000t}\,\text{V}$$

$$v(0) = -200A_1 - 800A_2 = 28$$

Solving, $A_1 = 0.1$ and $A_2 = -0.06$

Thus,

$$i_1 = (100e^{-10{,}000t} - 60e^{-40{,}000t})\,\text{mA} \qquad t \geq 0$$

$$v = -20e^{-10{,}000t} + 48e^{-40{,}000t}\,\text{V}, \qquad t \geq 0$$

[b] $i = 0$ when $100e^{-10{,}000t} = 60e^{-40{,}000t}$

Therefore

$e^{30{,}000t} = 0.6$ so $t = -17.03\,\mu\text{s}$ which is not possible!

$v = 0$ when $20e^{-10{,}000t} = 48e^{-40{,}000t}$

Therefore

$e^{30{,}000t} = 2.4$ so $t = 29.18\,\mu\text{s}$

Thus the power is zero at $t = 29.18\,\mu\text{s}$.

P 6.7 **[a]** From Problem 6.6 we have

$$i = A_1e^{-10{,}000t} + A_2e^{-40{,}000t}\,\text{A}$$

$$v = -20A_1e^{-10{,}000t} + 48A_2e^{-40{,}000t}\,\text{V}$$

$$i(0) = A_1 + A_2 = 0.04$$

$$v(0) = -200A_1 - 800A_2 = -68$$

Solving, $A_1 = -0.06;$ $A_2 = 0.1$
Thus,

$$i = -60e^{-10{,}000t} + 100e^{-40{,}000t}\,\text{mA} \quad t \geq 0$$

$$v = 12e^{-10{,}000t} - 80e^{-40{,}000t}\,\text{V} \quad t \geq 0$$

[b] $i = 0$ when $60e^{-10{,}000t} = 100e^{-40{,}000t}$

$$\therefore \quad e^{30{,}000t} = 5/3 \quad \text{so} \quad t = 17.03\,\mu\text{s}$$

Thus,

$$i > 0 \quad \text{for} \quad 0 \leq t \leq 17.03\,\mu\text{s} \qquad \text{and} \qquad i < 0 \quad \text{for} \quad 17.03\,\mu\text{s} \leq t < \infty$$

$$v = 0 \quad \text{when} \quad 12e^{-10{,}000t} = 80e^{-40{,}000t}$$

$$\therefore \quad e^{30{,}000t} = 20/3 \quad \text{so} \quad t = 63.24\,\mu\text{s}$$

Thus,

$$v < 0 \quad \text{for} \quad 0 \leq t \leq 63.24\,\mu\text{s} \qquad \text{and} \qquad v > 0 \quad \text{for} \quad 63.24\,\mu\text{s} \leq t < \infty$$

Therefore,

$$p < 0 \quad \text{for} \quad 0 \leq t \leq 17.03\,\mu\text{s} \qquad \text{and} \qquad 63.24\,\mu\text{s} \leq t < \infty$$

(inductor delivers energy)

$$p > 0 \quad \text{for} \quad 17.03\,\mu\text{s} \leq t \leq 63.24\,\mu\text{s} \quad \text{(inductor stores energy)}$$

[c] The energy stored at $t = 0$ is

$$w(0) = \frac{1}{2}L[i(0)]^2 = \frac{1}{2}(0.02)(0.04)^2 = 16\,\mu\text{J}$$

$$p = vi = 6e^{-50{,}000t} - 8e^{-80{,}000t} - 0.72e^{-20{,}000t}\,\text{W}$$

For $t > 0$:

$$w = \int_0^\infty 6e^{-50{,}000t}\,dt - \int_0^\infty 8e^{-80{,}000t}\,dt - \int_0^\infty 0.72e^{-20{,}000t}\,dt$$

$$= \frac{6e^{-50{,}000t}}{-50{,}000}\bigg|_0^\infty - \frac{8e^{-80{,}000t}}{-80{,}000}\bigg|_0^\infty - \frac{0.72e^{-20{,}000t}}{-20{,}000}\bigg|_0^\infty$$

$$= (1.2 - 1 - 0.36) \times 10^{-4}$$

$$= -16\,\mu\text{J}$$

Thus, the energy stored equals the energy extracted.

P 6.8 **[a]** $v = L\dfrac{di}{dt}$

$$v = -25 \times 10^{-3}\frac{d}{dt}[10\cos 400t + 5\sin 400t]e^{-200t}$$

$$= -25 \times 10^{-3}(-200e^{-200t}[10\cos 400t + 5\sin 400t]$$

$$+e^{-200t}[-4000\sin 400t + 2000\cos 400t])$$

$$v = -25 \times 10^{-3}e^{-200t}(-1000\sin 400t - 4000\sin 400t)$$

$$= -25 \times 10^{-3}e^{-200t}(-5000\sin 400t)$$

$$= 125e^{-200t}\sin 400t\,\text{V}$$

$$\frac{dv}{dt} = 125(e^{-200t}(400)\cos 400t - 200e^{-200t}\sin 400t)$$

$$= 25{,}000e^{-200t}(2\cos 400t - \sin 400t)\ \text{V/s}$$

$$\frac{dv}{dt} = 0 \qquad \text{when} \qquad 2\cos 400t = \sin 400t$$

$$\therefore\ \tan 400t = 2, \qquad 400t = 1.11; \qquad t = 2.77\,\text{ms}$$

[b] $v(2.77\text{ ms}) = 125e^{-0.55}\sin 1.11 = 64.27\,\text{V}$

P 6.9 **[a]**

$$i = \frac{1000}{20}\int_0^t -50\sin 250x\,dx + 10$$

$$= -2500\frac{-\cos 250x}{250}\bigg|_0^t + 10$$

$$= 10\cos 250t\,\text{A}$$

[b]

$$p = vi = (-50\sin 250t)(10\cos 250t)$$

$$= -500\sin 250t\cos 250t$$

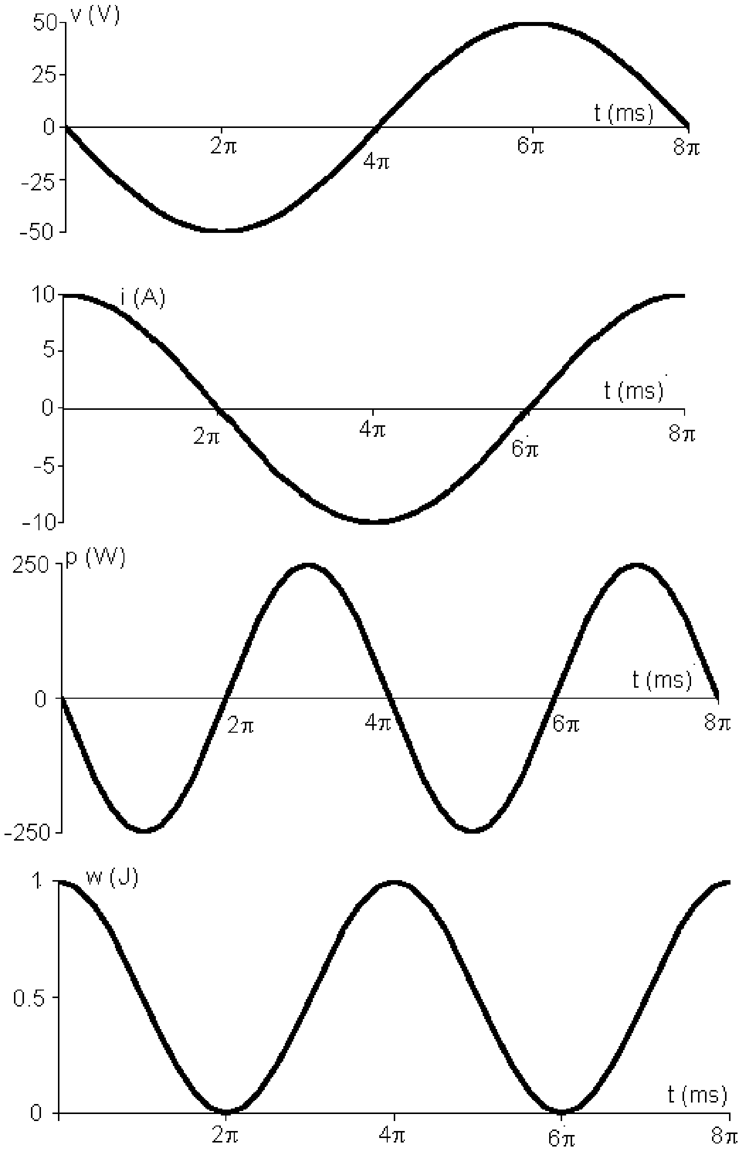
$$p = -250\sin 500t\,\text{W}$$

$$w = \frac{1}{2}Li^2$$

$$= \frac{1}{2}(20 \times 10^{-3})(10\cos 250t)^2$$

$$= 1000\cos^2 250t\,\text{mJ}$$

$$w = (500 + 500\cos 500t)\,\text{mJ}$$



[c] Absorbing power: Delivering power:

Absorbing power:	Delivering power:
$2\pi \le t \le 4\pi$ ms	$0 \le t \le 2\pi$ ms
$6\pi \le t \le 8\pi$ ms	$4\pi \le t \le 6\pi$ ms

P 6.10 $i = (B_1 \cos 4t + B_2 \sin 4t)e^{-t/2}$

$$i(0) = B_1 = 10\,\text{A}$$

$$\frac{di}{dt} = (B_1 \cos 4t + B_2 \sin 4t)(-0.5e^{-t/2}) + e^{-t/2}(-4B_1 \sin 4t + 4B_2 \cos 4t)$$

$$= [(4B_2 - 0.5B_1)\cos 4t - (4B_1 + 0.5B_2)\sin 4t]e^{-t/2}$$

$$v = 4\frac{di}{dt} = [(16B_2 - 2B_1)\cos 4t - (16B_1 + 2B_2)\sin 4t]e^{-t/2}$$

$$v(0) = 60 = 16B_2 - 2B_1 = 16B_2 - 20 \quad \therefore \quad B_2 = 5\,\text{A}$$

Thus,

$$i = (10\cos 4t + 5\sin 4t)e^{-t/2}, \text{A}, \qquad t \geq 0$$

$$v = (60\cos 4t - 170\sin 4t)e^{-t/2}\,\text{V}, \qquad t \geq 0$$

$$i(1) = -26, \text{A}; \qquad v(1) = 54.25\,\text{V}$$

$$p(1) = (-26)(54.25) = -339.57\,\text{W delivering}$$

P 6.11 $p = vi = 40t[e^{-10t} - 10te^{-20t} - e^{-20t}]$

$$\text{W} = \int_0^\infty p\,dx = \int_0^\infty 40x[e^{-10x} - 10xe^{-20x} - e^{-20x}]\,dx = 0.2\,\text{J}$$

This is energy stored in the inductor at $t = \infty$.

P 6.12 **[a]**

$$\begin{aligned} v(20\,\mu\text{s}) &= 12.5 \times 10^9(20 \times 10^{-6})^2 = 5\,\text{V} \quad \text{(end of first interval)} \\ v(20\,\mu\text{s}) &= 10^6(20 \times 10^{-6}) - (12.5)(400) \times 10^{-3} - 10 \\ &= 5\,\text{V (start of second interval)} \\ v(40\,\mu\text{s}) &= 10^6(40 \times 10^{-6}) - (12.5)(1600) \times 10^{-3} - 10 \\ &= 10\,\text{V (end of second interval)} \end{aligned}$$

[b] $p(10\mu\text{s}) = 62.5 \times 10^{12}(10^{-5})^3 = 62.5\,\text{mW}, \qquad v(10\,\mu\text{s}) = 1.25\,\text{V},$

$i(10\mu\text{s}) = 50\,\text{mA}, \qquad p(10\,\mu\text{s}) = vi = (1.25)(50\,\text{m}) = 62.5\,\text{mW (checks)}$

$p(30\,\mu\text{s}) = 437.50\,\text{mW}, \qquad v(30\,\mu\text{s}) = 8.75\,\text{V}, \qquad i(30\,\mu\text{s}) = 0.05\,\text{A}$

$p(30\,\mu\text{s}) = vi = (8.75)(0.05) = 62.5\,\text{mW (checks)}$

[c] $w(10\,\mu\text{s}) = 15.625 \times 10^{12}(10 \times 10^{-6})^4 = 0.15625\,\mu\text{J}$

$w = 0.5Cv^2 = 0.5(0.2 \times 10^{-6})(1.25)^2 = 0.15625\,\mu\text{J}$

$w(30\,\mu\text{s}) = 7.65625\,\mu\text{J}$

$w(30\,\mu\text{s}) = 0.5(0.2 \times 10^{-6})(8.75)^2 = 7.65625\,\mu\text{J}$

P 6.13 For $0 \le t \le 1.6\,\text{s}$:

$$i_L = \frac{1}{5}\int_0^t 3 \times 10^{-3}\,dx + 0 = 0.6 \times 10^{-3}t$$

$$i_L(1.6\,\text{s}) = (0.6 \times 10^{-3})(1.6) = 0.96\,\text{mA}$$

$$R_m = (20)(1000) = 20\,\text{k}\Omega$$

$$v_m(1.6\,\text{s}) = (0.96 \times 10^{-3})(20 \times 10^3) = 19.2\,\text{V}$$

P 6.14 **[a]** $i = \dfrac{400 \times 10^{-3}}{5 \times 10^{-6}}t = 80 \times 10^3 t \qquad 0 \le t \le 5\,\mu\text{s}$

$$i = 400 \times 10^{-3} \qquad 5 \le t \le 20\,\mu\text{s}$$

$$i = \frac{300 \times 10^{-3}}{30 \times 10^{-6}}t - 0.5 = 10^4 t - 0.5 \qquad 20\,\mu\text{s} \le t \le 50\,\mu\text{s}$$

$$\begin{aligned} q &= \int_0^{5\times10^{-6}} 8 \times 10^4 t\,dt + \int_{5\times10^{-6}}^{15\times10^{-6}} 0.4\,dt \\ &= 8 \times 10^4 \frac{t^2}{2}\Big|_0^{5\times10^{-6}} + 0.4(10 \times 10^{-6}) \\ &= 4 \times 10^4(25 \times 10^{-12}) + 4 \times 10^{-6} \\ &= 5\,\mu\text{C} \end{aligned}$$

[b] $v = 4 \times 10^6 \displaystyle\int_0^{5\times10^{-6}} 8 \times 10^4 x\,dx + 4 \times 10^6 \int_{5\times10^{-6}}^{20\times10^{-6}} 0.4\,dx$

$$+ 4 \times 10^6 \int_{20\times10^{-6}}^{30\times10^{-6}} (10^4 x - 0.5)\,dx$$

$$= 4 \times 10^6 \left[8 \times 10^4 \frac{x^2}{2}\Big|_0^{5\times10^{-6}} + 0.4x\Big|_{5\times10^{-6}}^{20\times10^{-6}} + 10^4\frac{x^2}{2}\Big|_{20\times10^{-6}}^{30\times10^{-6}} - 0.5x\Big|_{20\times10^{-6}}^{30\times10^{-6}}\right]$$

$$= 4 \times 10^6[4 \times 10^4(25 \times 10^{-12}) + 0.4(15 \times 10^{-6})$$

$$+ 5000(900 \times 10^{-12} 400 \times 10^{-12}) - 0.5(10 \times 10^{-6})]$$

$$= 18\,\text{V}$$

$$v(30\,\mu\text{s}) = 18\,\text{V}$$

[c] $$v(50\,\mu\text{s}) = 4\times10^{6}[10^{-6} + 6\times10^{-6} + 5000(2500\times10^{-12} - 400\times10^{-12}) - 0.5(30\times10^{-6})]$$

$$= 10\,\text{V}$$

$$w = \frac{1}{2}Cv^2 = \frac{1}{2}(0.25\times10^{-6})(10)^2 = 12.5\,\mu\text{J}$$

P 6.15 **[a]**

$$\begin{aligned} v &= \frac{1}{0.5\times10^{-6}}\int_0^{500\times10^{-6}} 50\times10^{-3}e^{-2000t}\,dt - 20 \\ &= 100\times10^{3}\frac{e^{-2000t}}{-2000}\Bigg|_0^{500\times10^{-6}} -20 \\ &= 50(1-e^{-1}) - 20 = 11.61\,\text{V} \\ w &= \tfrac{1}{2}Cv^2 = \tfrac{1}{2}(0.5)(10^{-6})(11.61)^2 = 33.7\,\mu\text{J} \end{aligned}$$

[b] $v(\infty) = 50 - 20 = 30\text{V}$

$$w(\infty) = \frac{1}{2}(0.5\times10^{-6})(30)^2 = 225\,\mu\text{J}$$

P 6.16 **[a]** $0 \le t \le 10\,\mu\text{s}$

$$C = 0.1\,\mu\text{F} \qquad \frac{1}{C} = 10\times10^{6}$$

$$v = 10\times10^{6}\int_0^t -0.05\,dx + 15$$

$$v = -50\times10^{4}t + 15\,\text{V} \qquad 0\le t\le 10\,\mu\text{s}$$

$$v(10\,\mu\text{s}) = -5 + 15 = 10\,\text{V}$$

[b] $10\,\mu\text{s} \le t \le 20\,\mu\text{s}$

$$v = 10\times10^{6}\int_{10\times10^{-6}}^{t} 0.1\,dx + 10 = 10^{6}t - 10 + 10$$

$$v = 10^{6}t\,\text{V} \qquad 10\le t\le 20\,\mu\text{s}$$

$$v(20\,\mu\text{s}) = 10^{6}(20\times10^{-6}) = 20\,\text{V}$$

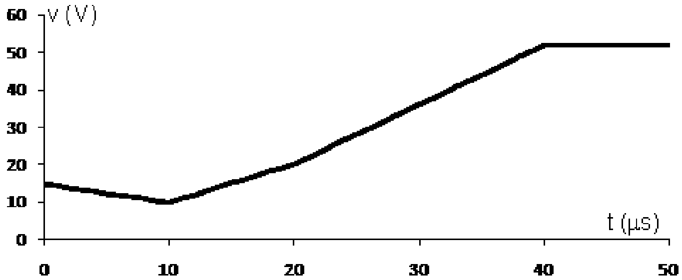
[c] $20\,\mu\text{s} \le t \le 40\,\mu\text{s}$

$$v = 10\times10^{6}\int_{20\times10^{-6}}^{t} 1.6\,dx + 20 = 1.6\times10^{6}t - 32 + 20$$

$$v = 1.6\times10^{6}t - 12\,\text{V}, \qquad 20\,\mu\text{s}\le t\le 40\,\mu\text{s}$$

[d] $40\,\mu\text{s} \leq t < \infty$

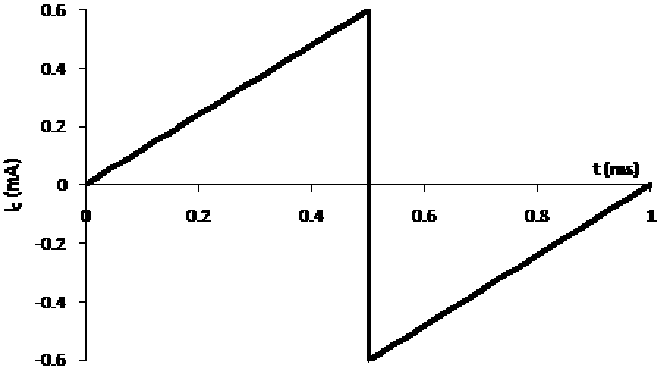
$$v(40\,\mu\text{s}) = 64 - 12 = 52\,\text{V} \qquad 40\,\mu\text{s} \leq t < \infty$$



P 6.17 $i_C = C(dv/dt)$

$$0 < t < 0.5: \qquad i_C = 20 \times 10^{-6}(60)t = 1.2t\,\text{mA}$$

$$0.5 < t < 1: \qquad i_C = 20 \times 10^{-6}(60)(t-1) = 1.2(t-1)\,\text{mA}$$



P 6.18 **[a]** $w(0) = \frac{1}{2}C[v(0)]^2 = \frac{1}{2}(0.20) \times 10^{-6}(150)^2 = 2.25\,\text{mJ}$

[b] $v = (A_1 t + A_2)e^{-5000t}$

$$v(0) = A_2 = 150\,\text{V}$$

$$\frac{dv}{dt} = -5000e^{-5000t}(A_1 t + A_2) + e^{-5000t}(A_1)$$

$$= (-5000A_1 t - 5000A_2 + A_1)e^{-5000t}$$

$$\frac{dv}{dt}(0) = A_1 - 5000A_2$$

$$i = C\frac{dv}{dt}, \qquad i(0) = C\frac{dv(0)}{dt}$$

$$\therefore\quad \frac{dv(0)}{dt} = \frac{i(0)}{C} = \frac{250 \times 10^{-3}}{0.2 \times 10^{-6}} = 1250 \times 10^{3}$$

$$\therefore\quad 1.25 \times 10^{6} = A_1 - 5000(150)$$

$$\text{Thus, } A_1 = 1.25 \times 10^{6} + 75 \times 10^{4} = 2 \times 10^{6}\,\frac{\text{V}}{\text{s}}$$

[c] $v = (2 \times 10^{6}t + 150)e^{-5000t}$

$$i = C\frac{dv}{dt} = 0.2 \times 10^{-6}\frac{d}{dt}(2 \times 10^{6}t + 150)e^{-5000t}$$

$$\begin{aligned} i &= \frac{d}{dt}[(0.4t + 10 \times 30^{-6})e^{-5000t}] \\ &= (0.4t + 30 \times 10^{-6})(-5000)e^{-5000t} + e^{-5000t}(0.4) \\ &= (-2000t - 150 \times 10^{-3} + 0.4)e^{-5000t} \\ &= (0.25 - 2000t)e^{-5000t}\,\text{A}, \qquad t \geq 0 \end{aligned}$$

P 6.19 **[a]** $i = C\frac{dv}{dt} = 0, \quad t < 0$

[b] $i = C\frac{dv}{dt} = 4 \times 10^{-6}\frac{d}{dt}[100 - 40e^{-2000t}(3\cos 1000t + \sin 1000t)]$

$$\begin{aligned} &= 4 \times 10^{-6}[-40(-2000)e^{-2000t}(3\cos 1000t + \sin 1000t) \\ &\quad -40(1000)e^{-2000t}(-3\sin 1000t + \cos 1000t)] \\ &= 0.32e^{-2000t}(3\cos 1000t + \sin 1000t) - 0.16(-3\sin 1000t + \cos 1000t) \\ &= 0.8e^{-2000t}[\cos 1000t + \sin 1000t]\,\text{A}, \quad t \geq 0 \end{aligned}$$

[c] no, $v(0^-) = -20$ V

$v(0^+) = 100 - 40(1)(3) = -20$ V

[d] yes, $i(0^-) = 0$ A

$i(0^+) = 0.8$ A

[e] $v(\infty) = 100\,\text{V}$

$$w = \frac{1}{2}Cv^2 = \frac{1}{2}(4 \times 10^{-6})(100)^2 = 20\,\text{mJ}$$

P 6.20 $30\|20 = 12\,\text{H}$

$80\|(8 + 12) = 16\,\text{H}$

$60\|(14 + 16) = 20\,\text{H}$

$15\|(20 + 10) = 20\,\text{H}$

$L_{\text{ab}} = 5 + 10 = 15\,\text{H}$

P 6.21 $5\|(12+8) = 4\,\text{H}$

$4\|4 = 2\,\text{H}$

$15\|(8+2) = 6\,\text{H}$

$3\|6 = 2\,\text{H}$

$6+2 = 8\,\text{H}$

P 6.22 [**a**] Combine three 1 mH inductors in series to get a 3 mH equivalent inductor.

[**b**] Combine two $100\,\mu$H inductors in parallel to get a $50\,\mu$H inductor. Then combine this parallel pair in series with two more $100\,\mu$H inductors:

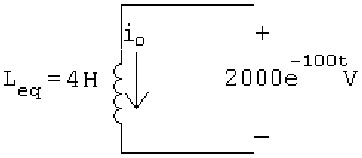
$$100\,\mu\|100\,\mu + 100\,\mu + 100\,\mu = 50\,\mu + 100\,\mu + 100\,\mu = 250\,\mu\text{H}$$

[**c**] Combine two $100\,\mu$H inductors in parallel to get a $50\,\mu$H inductor. Then combine this parallel pair with a $10\,\mu$H inductor in series:

$$100\,\mu\|100\,\mu + 10\,\mu = 50\,\mu + 10\,\mu = 60\,\mu\text{H}$$

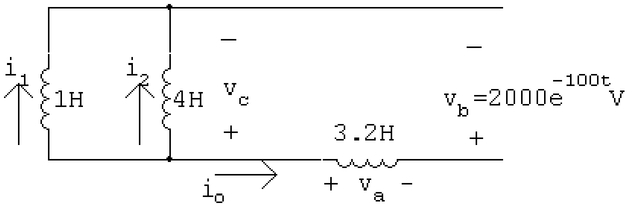
P 6.23 [**a**] $i_o(0) = -i_1(0) - i_2(0) = 6 - 1 = 5\,\text{A}$

[**b**]



$$\begin{aligned} i_o &= -\frac{1}{4}\int_0^t 2000e^{-100x}\,dx + 5 = -500\,\frac{e^{-100x}}{-100}\bigg|_0^t + 5 \\ &= 5(e^{-100t} - 1) + 5 = 5e^{-100t}\,\text{A}, \qquad t \geq 0 \end{aligned}$$

[**c**]



$$\begin{aligned} v_a &= 3.2(-500e^{-100t}) = -1600e^{-100t}\,\text{V} \\ v_c &= v_a + v_b = -1600e^{-100t} + 2000e^{-100t} \\ &= 400e^{-100t}\,\text{V} \\ i_1 &= \frac{1}{1}\int_0^t 400e^{-100x}\,dx - 6 \\ &= -4e^{-100t} + 4 - 6 \\ i_1 &= -4e^{-100t} - 2\,\text{A} \qquad t \geq 0 \end{aligned}$$

[d] $$i_2 = \frac{1}{4}\int_0^t 400e^{-100x}\,dx + 1$$

$$= -e^{-100t} + 2\,\text{A}, \qquad t \geq 0$$

[e] $w(0) = \frac{1}{2}(1)(6)^2 + \frac{1}{2}(4)(1)^2 + \frac{1}{2}(3.2)(5)^2 = 60\,\text{J}$

[f] $w_{\text{del}} = \frac{1}{2}(4)(5)^2 = 50\,\text{J}$

[g] $w_{\text{trapped}} = 60 - 50 = 10\,\text{J}$

or $\quad w_{\text{trapped}} = \frac{1}{2}(1)(2)^2 + \frac{1}{2}(4)(2)^2 + 10\,\text{J}$ (check)

P 6.24 $v_{\text{b}} = 2000e^{-100t}\,\text{V}$

$i_o = 5e^{-100t}\,\text{A}$

$p = 10{,}000e^{-200t}\,\text{W}$

$$w = \int_0^t 10^4 e^{-200x}\,dx = 10{,}000\frac{e^{-200x}}{-200}\bigg|_0^t = 50(1 - e^{-200t})\,\text{W}$$

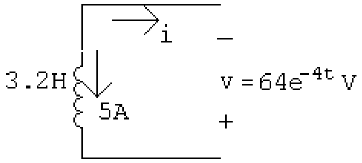
$w_{\text{total}} = 50\,\text{J}$

$80\% w_{\text{total}} = 40\,\text{J}$

Thus,

$50 - 50e^{-200t} = 40; \qquad e^{200t} = 5; \qquad \therefore \quad t = 8.05\,\text{ms}$

P 6.25 **[a]**



$$3.2\frac{di}{dt} = 64e^{-4t} \qquad \text{so} \qquad \frac{di}{dt} = 20e^{-4t}$$

$$\begin{aligned} i(t) &= 20\int_0^t e^{-4x}\,dx - 5 \\ &= 20\frac{e^{-4x}}{-4}\Big|_0^t -5 \\ i(t) &= -5e^{-4t}\,\text{A} \end{aligned}$$

[b] $4\frac{di_1}{dt} = 64e^{-4t}$

$$\begin{aligned} i_1(t) &= 16\int_0^t e^{-4x}\,dx - 10 \\ &= 16\frac{e^{-4x}}{-4}\Big|_0^t -10 \\ i_1(t) &= -4e^{-4t} - 6\,\text{A} \end{aligned}$$

[c] $16\frac{di_2}{dt} = 64e^{-4t} \qquad \text{so} \qquad \frac{di_2}{dt} = 4e^{-4t}$

$$\begin{aligned} i_2(t) &= 4\int_0^t e^{-4x}\,dx + 5 \\ &= 4\frac{e^{-4x}}{-4}\Big|_0^t +5 \\ i_2(t) &= -e^{-4t} + 6\,\text{A} \end{aligned}$$

[d] $p = -vi = (-64e^{-4t})(-5e^{-4t}) = 320e^{-8t}\,\text{W}$

$$\begin{aligned} w &= \int_0^\infty p\,dt = \int_0^\infty 320e^{-8t}\,dt \\ &= 320\frac{e^{-8t}}{-8}\Big|_0^\infty \\ &= 40\,\text{J} \end{aligned}$$

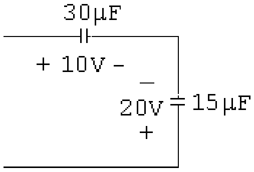
[e] $w = \frac{1}{2}(4)(-10)^2 + \frac{1}{2}(16)(5)^2 = 400\,\text{J}$

[f] $w_{\text{trapped}} = w_{\text{initial}} - w_{\text{delivered}} = 400 - 40 = 360\,\text{J}$

[g] $w_{\text{trapped}} = \frac{1}{2}(4)(-6)^2 + \frac{1}{2}(16)(6)^2 = 360\,\text{J}$ checks

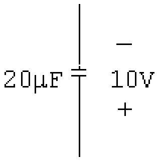
P 6.26 $\frac{1}{C_1} = \frac{1}{48} + \frac{1}{16} = \frac{1}{12};\qquad C_1 = 12\,\mu\text{F}$

$C_2 = 3 + 12 = 15\,\mu\text{F}$

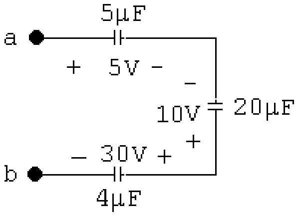


$\frac{1}{C_3} = \frac{1}{30} + \frac{1}{15} = \frac{1}{10};\qquad C_3 = 10\,\mu\text{F}$

$C_4 = 10 + 10 = 20\,\mu\text{F}$

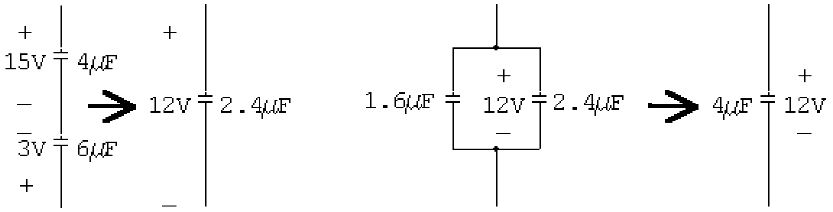


$\frac{1}{C_5} = \frac{1}{5} + \frac{1}{20} + \frac{1}{4} = \frac{1}{2};\qquad C_5 = 2\,\mu\text{F}$

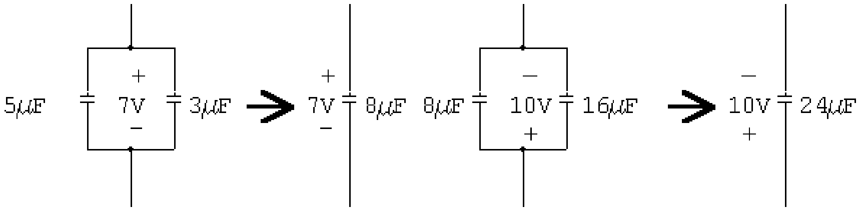


Equivalent capacitance is $2\,\mu$F with an initial voltage drop of +25 V.

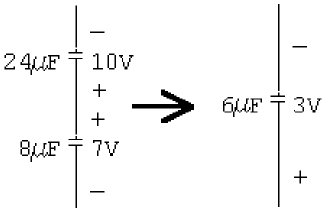
P 6.27 $\frac{1}{4} + \frac{1}{6} = \frac{5}{12}\qquad \therefore\ C_{\text{eq}} = 2.4\,\mu\text{F}$



$\frac{1}{4} + \frac{1}{12} = \frac{4}{12}\qquad \therefore\ C_{\text{eq}} = 3\,\mu\text{F}$



$$\frac{1}{24}+\frac{1}{8}=\frac{4}{24} \qquad \therefore \quad C_{\text{eq}}=6\,\mu\text{F}$$



P 6.28 **[a]** Combine two 220 μF capacitors in series to get a 110 μF capacitor. Then combine the series pair in parallel with another 220 μF capacitor to get 330 μF:

$$(220\,\mu+220\,\mu)\|220\,\mu=110\,\mu\|220\,\mu=330\,\mu\text{F}$$

[b] Create a 1500 nF capacitor as follows:

$$(1\,\mu+1\,\mu)\|1\,\mu=500\,\text{n}\|1000\,\text{n}=1500\,\text{nF}$$

Create a second 1500 nF capacitor using the same three resistors. Place these two 1500 nF in series:

$$1500\,\text{n}+1500\,\text{n}=750\,\text{nF}$$

[c] Combine two 100 pF capacitors in series to get a 50 pF capacitor. Then combine the series pair in parallel with another 100 pF capacitor to get 150 pF:

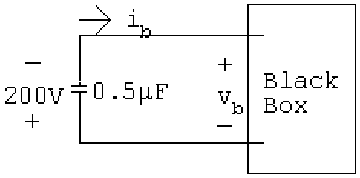
$$(100\text{ p}+100\text{ p})\|100\text{ p}=50\text{ p}\|100\text{ p}=150\text{ pF}$$

P 6.29 $$\frac{1}{C_e}=\frac{1}{1}+\frac{1}{5}+\frac{1}{1.25}=\frac{10}{5}=2$$

$$\therefore \quad C_2=0.5\,\mu\text{F}$$

$$v_{\text{b}}=20-250+30=-200\,\text{V}$$

[a]



$$v_\text{b} = -\frac{10^6}{0.5}\int_0^t -5\times 10^{-3}e^{-50x}\,dx - 200$$

$$= 10{,}000\frac{e^{-50x}}{-50}\Big|_0^t -200$$

$$= -200e^{-50t}\,\text{V}$$

[b] $$v_\text{a} = -\frac{10^6}{0.5}\int_0^t -5\times 10^{-3}e^{-50x}\,dx - 20$$

$$= 20(e^{-50t}-1) - 20$$

$$= 20e^{-50t} - 40\,\text{V}$$

[c] $$v_\text{c} = \frac{10^6}{1.25}\int_0^t -5\times 10^{-3}e^{-50x}\,dx - 30$$

$$= 80(e^{-50t}-1) - 30$$

$$= 80e^{-50t} - 110\,\text{V}$$

[d] $$v_\text{d} = 10^6\int_0^t -5\times 10^{-3}e^{-50x}\,dx + 250$$

$$= 100(e^{-50t}-1) + 250$$

$$= 100e^{-50t} + 150\,\text{V}$$

CHECK: $$v_\text{b} = -v_\text{c} - v_\text{d} - v_\text{a}$$

$$= -200e^{-50t}\,\text{V} \quad \text{(checks)}$$

[e] $$i_1 = 0.2\times 10^{-6}\frac{d}{dt}[100e^{-50t}+150]$$

$$= 0.2\times 10^{-6}(-5000e^{-50t})$$

$$= -e^{-50t}\,\text{mA}$$

[f] $$i_2 = 0.8\times 10^{-6}\frac{d}{dt}[100e^{-50t}+150]$$

$$= -4e^{-50t}\,\text{mA}$$

CHECK: $i_\mathrm{b} = i_1 + i_2 = -5e^{-50t}\,\mathrm{mA}$ (OK)

P 6.30 **[a]**
$$\begin{aligned} w(0) &= \tfrac{1}{2}(0.2\times10^{-6})(250)^2 + \tfrac{1}{2}(0.8\times10^{-6})(250)^2 + \tfrac{1}{2}(5\times10^{-6})(20)^2 \\ &+ \tfrac{1}{2}(1.25\times10^{-6})(30)^2 \\ &= 32{,}812.5\,\mu\mathrm{J} \end{aligned}$$

[b]
$$\begin{aligned} w(\infty) &= \tfrac{1}{2}(5\times10^{-6})(40)^2 + \tfrac{1}{2}(1.25\times10^{-6})(110)^2 + \tfrac{1}{2}(0.2\times10^{-6})(150)^2 \\ &+ \tfrac{1}{2}(0.8\times10^{-6})(150)^2 \\ &= 22{,}812.5\,\mu\mathrm{J} \end{aligned}$$

[c] $w = \dfrac{1}{2}(0.5\times10^{-6})(200)^2 = 10{,}000\,\mu\mathrm{J}$

CHECK: $32{,}812.5 - 22{,}812.5 = 10{,}000\,\mu\mathrm{J}$

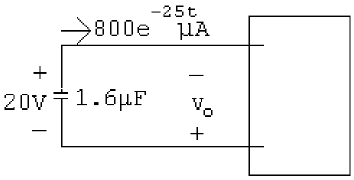
[d] $\%$ delivered $= \dfrac{10{,}000}{32{,}812.5}\times100 = 30.48\%$

[e]
$$\begin{aligned} w &= \int_0^t (-0.005e^{-50x})(-200e^{-50x})\,dx = \int_0^t e^{-100x}\,dx \\ &= 10(1-e^{-100t})\,\mathrm{mJ} \end{aligned}$$

$$\therefore\quad 10^{-2}(1-e^{-100t}) = 7.5\times10^{-3}; \qquad e^{-100t} = 0.25$$

Thus, $t = \dfrac{\ln 4}{100} = 13.86\,\mathrm{ms}.$

P 6.31 **[a]**



$$\begin{aligned} v_o &= \frac{10^6}{1.6}\int_0^t 800\times10^{-6}e^{-25x}\,dx - 20 \\ &= 500\frac{e^{-25x}}{-25}\Big|_0^t -20 \\ &= -20e^{-25t}\,\mathrm{V}, \qquad t\ge0 \end{aligned}$$

[b]
$$\begin{aligned} v_1 &= \frac{10^6}{2}(800\times10^{-6})\frac{e^{-25x}}{-25}\Big|_0^t +5 \\ &= -16e^{-25t}+21\,\mathrm{V}, \qquad t\ge0 \end{aligned}$$

[c] $$v_2 = \frac{10^6}{8}(800\times10^{-6})\frac{e^{-25x}}{-25}\Big|_0^t -25$$

$$= -4e^{-25t} - 21\,\text{V}, \qquad t\geq 0$$

[d] $$p = -vi = -(-20e^{-25t})(800\times10^{-6})e^{-25t}$$

$$= 16\times10^{-3}e^{-50t}$$

$$w = \int_0^\infty 16\times10^{-3}e^{-50t}\,dt$$

$$= 16\times10^{-3}\frac{e^{-50t}}{-50}\Big|_0^\infty$$

$$= -0.32\times10^{-3}(0-1) = 320\,\mu\text{J}$$

[e] $$w = \tfrac{1}{2}(2\times10^{-6})(5)^2 + \tfrac{1}{2}(8\times10^{-6})(25)^2$$

$$= 2525\,\mu\text{J}$$

[f] $w_{\text{trapped}} = w_{\text{initial}} - w_{\text{delivered}} = 2525 - 320 = 2205\,\mu\text{J}$

[g] $$w_{\text{trapped}} = \tfrac{1}{2}(2\times10^{-6})(21)^2 + \tfrac{1}{2}(8\times10^{-6})(-21)^2$$

$$= 2205\,\mu\text{J}$$

P 6.32 From Figure 6.17(a) we have

$$v = \frac{1}{C_1}\int_0^t i\,dx + v_1(0) + \frac{1}{C_2}\int_0^t i\,dx + v_2(0) + \cdots$$

$$v = \left[\frac{1}{C_1} + \frac{1}{C_2} + \cdots\right]\int_0^t i\,dx + v_1(0) + v_2(0) + \cdots$$

Therefore $\dfrac{1}{C_{\text{eq}}} = \left[\dfrac{1}{C_1} + \dfrac{1}{C_2} + \cdots\right], \qquad v_{\text{eq}}(0) = v_1(0) + v_2(0) + \cdots$

P 6.33 From Fig. 6.18(a)

$$i = C_1\frac{dv}{dt} + C_2\frac{dv}{dt} + \cdots = [C_1 + C_2 + \cdots]\frac{dv}{dt}$$

Therefore $C_{\text{eq}} = C_1 + C_2 + \cdots$. Because the capacitors are in parallel, the initial voltage on every capacitor must be the same. This initial voltage would appear on C_{eq}.

P 6.34
$$\begin{aligned}\frac{di_o}{dt} &= (5)\{e^{-2000t}[-8000\sin 4000t + 4000\cos 4000t] \\ &\quad +(-2000e^{-2000t})[2\cos 4000t + \sin 4000t]\} \\ &= e^{-2000t}\{-50{,}000\sin 4000t\}\text{ V}\end{aligned}$$

$$\frac{di_o}{dt}(0^+) = (1)[\sin(0)] = 0$$

$$\therefore \quad 10\times 10^{-3}\frac{di_o}{dt}(0^+) = 0 \qquad \text{so} \qquad v_2(0^+) = 0$$

$$v_1(0^+) = 40i_o(0^+) + v_2(0^+) = 40(10) = 0 = 400\text{ V}$$

P 6.35
$$\begin{aligned}v_c &= -\frac{1}{0.625\times 10^{-6}}\left(\int_0^t 1.5e^{-16{,}000x}\,dx - \int_0^t 0.5e^{-4000x}\,dx\right) - 50 \\ &= 150(e^{-16{,}000t} - 1) - 200(e^{-4000t} - 1) - 50 \\ &= 150e^{-16{,}000t} - 200e^{-4000t}\ \text{ V} \\ v_L &= 25\times 10^{-3}\frac{di_o}{dt} \\ &= 25\times 10^{-3}(-24{,}000e^{-16{,}000t} + 2000e^{-4000t}) \\ &= -600e^{-16{,}000t} + 50e^{-4000t}\ \text{ V} \\ v_o &= v_c - v_L \\ &= (150e^{-16{,}000t} - 200e^{-4000t}) - (-600e^{-16{,}000t} + 50e^{-4000t}) \\ &= 750e^{-16{,}000t} - 250e^{-4000t}\ \text{ V},\ t > 0\end{aligned}$$

P 6.36 **[a]** Rearrange by organizing the equations by di_1/dt, i_1, di_2/dt, i_2 and transfer the i_g terms to the right hand side of the equations. We get

$$4\frac{di_1}{dt} + 25i_1 - 8\frac{di_2}{dt} - 20i_2 = 5i_g - 8\frac{di_g}{dt}$$

$$-8\frac{di_1}{dt} - 20i_1 + 16\frac{di_2}{dt} + 80i_2 = 16\frac{di_g}{dt}$$

[b] From the given solutions we have

$$\frac{di_1}{dt} = -320e^{-5t} + 272e^{-4t}$$

$$\frac{di_2}{dt} = 260e^{-5t} - 204e^{-4t}$$

Thus,

$$4\frac{di_1}{dt} = -1280e^{-5t} + 1088e^{-4t}$$

$$25i_1 = 100 + 1600e^{-5t} - 1700e^{-4t}$$

$$8\frac{di_2}{dt} = 2080e^{-5t} - 1632e^{-4t}$$

$$20i_2 = 20 - 1040e^{-5t} + 1020e^{-4t}$$

$$5i_g = 80 - 80e^{-5t}$$

$$8\frac{di_g}{dt} = 640e^{-5t}$$

Thus,

$$-1280e^{-5t} + 1088e^{-4t} + 100 + 1600e^{-5t} - 1700e^{-4t} - 2080e^{-5t}$$

$$+1632e^{-4t} - 20 + 1040e^{-5t} - 1020e^{-4t} \overset{?}{=} 80 - 80e^{-5t} - 640e^{-5t}$$

$$80 + (1088 - 1700 + 1632 - 1020)e^{-4t}$$

$$+(1600 - 1280 - 2080 + 1040)e^{-5t} \overset{?}{=} 80 - 720e^{-5t}$$

$$80 + (2720 - 2720)e^{-4t} + (2640 - 3360)e^{-5t} = 80 - 720e^{-5t} \quad \text{(OK)}$$

$$8\frac{di_1}{dt} = -2560e^{-5t} + 2176e^{-4t}$$

$$20i_1 = 80 + 1280e^{-5t} - 1360e^{-4t}$$

$$16\frac{di_2}{dt} = 4160e^{-5t} - 3264e^{-4t}$$

$$80i_2 = 80 - 4160e^{-5t} + 4080e^{-4t}$$

$$16\frac{di_g}{dt} = 1280e^{-5t}$$

$$2560e^{-5t} - 2176e^{-4t} - 80 - 1280e^{-5t} + 1360e^{-4t} + 4160e^{-5t} - 3264e^{-4t}$$

$$+80 - 4160e^{-5t} + 4080e^{-4t} \overset{?}{=} 1280e^{-5t}$$

$$(-80 + 80) + (2560 - 1280 + 4160 - 4160)e^{-5t}$$

$$+(1360 - 2176 - 3264 + 4080)e^{-4t} \overset{?}{=} 1280e^{-5t}$$

$$0 + 1280e^{-5t} + 0e^{-4t} = 1280e^{-5t} \quad \text{(OK)}$$

P 6.37 **[a]** Yes, using KVL around the lower right loop

$$v_o = v_{20\Omega} + v_{60\Omega} = 20(i_2 - i_1) + 60i_2$$

[b]
$$\begin{aligned} v_o &= 20(1-52e^{-5t}+51e^{-4t}-4-64e^{-5t}+68e^{-4t})+ \\ &\quad 60(1-52e^{-5t}+51e^{-4t}) \\ &= 20(-3-116e^{-5t}+119e^{-4t})+60-3120e^{-5t}+3060e^{-4t} \\ v_o &= -5440e^{-5t}+5440e^{-4t}\text{ V} \end{aligned}$$

[c]
$$\begin{aligned} v_o &= L_2\frac{d}{dt}(i_g-i_2)+M\frac{di_1}{dt} \\ &= 16\frac{d}{dt}(15+36e^{-5t}-51e^{-4t})+8\frac{d}{dt}(4+64e^{-5t}-68e^{-4t}) \\ &= -2880e^{-5t}+3264e^{-4t}-2560e^{-5t}+2176e^{-4t} \\ v_o &= -5440e^{-5t}+5440e^{-4t}\text{ V} \end{aligned}$$

P 6.38 **[a]**
$$\begin{aligned} v_g &= 5(i_g-i_1)+20(i_2-i_1)+60i_2 \\ &= 5(16-16e^{-5t}-4-64e^{-5t}+68e^{-4t})+ \\ &\quad 20(1-52e^{-5t}+51e^{-4t}-4-64e^{-5t}+68e^{-4t})+ \\ &\quad 60(1-52e^{-5t}+51e^{-4t}) \\ &= 60+5780e^{-4t}-5840e^{-5t}\text{ V} \end{aligned}$$

[b] $v_g(0)=60+5780-5840=0\text{ V}$

[c]
$$\begin{aligned} p_{\text{dev}} &= v_g i_g \\ &= 960+92{,}480e^{-4t}-94{,}400e^{-5t}-92{,}480e^{-9t}+ \\ &\quad 93{,}440e^{-10t}\text{W} \end{aligned}$$

[d] $p_{\text{dev}}(\infty)=960\text{ W}$

[e] $i_1(\infty)=4\text{ A};\quad i_2(\infty)=1\text{ A};\quad i_g(\infty)=16\text{ A};$

$$p_{5\Omega}=(16-4)^2(5)=720\text{ W}$$

$$p_{20\Omega}=3^2(20)=180\text{ W}$$

$$p_{60\Omega}=1^2(60)=60\text{ W}$$

$$\sum p_{\text{abs}}=720+180+60=960\text{ W}$$

$$\therefore\quad \sum p_{\text{dev}}=\sum p_{\text{abs}}=960\text{ W}$$

P 6.39 **[a]** $-2\frac{di_g}{dt}+16\frac{di_2}{dt}+32i_2=0$

$$16\frac{di_2}{dt}+32i_2=2\frac{di_g}{dt}$$

[b] $i_2 = e^{-t} - e^{-2t}\,\text{A}$

$$\frac{di_2}{dt} = -e^{-t} + 2e^{-2t}\,\text{A/s}$$

$$i_g = 8 - 8e^{-t}\,\text{A}$$

$$\frac{di_g}{dt} = 8e^{-t}\,\text{A/s}$$

$$\therefore\quad -16e^{-t} + 32e^{-2t} + 32e^{-t} - 32e^{-2t} = 16e^{-t}$$

[c]

$$\begin{aligned} v_1 &= 4\frac{di_g}{dt} - 2\frac{di_2}{dt} \\ &= 4(8e^{-t}) - 2(-e^{-t} + 2e^{-2t}) \\ &= 34e^{-t} - 4e^{-2t}\,\text{V}, \qquad t > 0 \end{aligned}$$

[d] $v_1(0) = 34 - 4 = 30\,\text{V}$; Also

$$\begin{aligned} v_1(0) &= 4\frac{di_g}{dt}(0) - 2\frac{di_2}{dt}(0) \\ &= 4(8) - 2(-1+2) = 32 - 2 = 30\,\text{V} \end{aligned}$$

Yes, the initial value of v_1 is consistent with known circuit behavior.

P 6.40 **[a]** $v_{\text{ab}} = L_1\frac{di}{dt} + L_2\frac{di}{dt} + M\frac{di}{dt} + M\frac{di}{dt} = (L_1 + L_2 + 2M)\frac{di}{dt}$

It follows that $L_{\text{ab}} = (L_1 + L_2 + 2M)$

[b] $v_{\text{ab}} = L_1\frac{di}{dt} - M\frac{di}{dt} + L_2\frac{di}{dt} - M\frac{di}{dt} = (L_1 + L_2 - 2M)\frac{di}{dt}$

Therefore $L_{\text{ab}} = (L_1 + L_2 - 2M)$

P 6.41 **[a]** $v_{\text{ab}} = L_1\frac{d(i_1 - i_2)}{dt} + M\frac{di_2}{dt}$

$$0 = L_1\frac{d(i_2 - i_1)}{dt} - M\frac{di_2}{dt} + M\frac{d(i_1 - i_2)}{dt} + L_2\frac{di_2}{dt}$$

Collecting coefficients of $[di_1/dt]$ and $[di_2/dt]$, the two mesh-current equations become

$$v_{\text{ab}} = L_1\frac{di_1}{dt} + (M - L_1)\frac{di_2}{dt}$$

and

$$0 = (M - L_1)\frac{di_1}{dt} + (L_1 + L_2 - 2M)\frac{di_2}{dt}$$

Solving for $[di_1/dt]$ gives

$$\frac{di_1}{dt} = \frac{L_1 + L_2 - 2M}{L_1L_2 - M^2}v_{\text{ab}}$$

from which we have

$$v_{\text{ab}} = \left(\frac{L_1L_2 - M^2}{L_1 + L_2 - 2M}\right)\left(\frac{di_1}{dt}\right)$$

$$\therefore \quad L_{\text{ab}} = \frac{L_1L_2 - M^2}{L_1 + L_2 - 2M}$$

[b] If the magnetic polarity of coil 2 is reversed, the sign of M reverses, therefore

$$L_{\text{ab}} = \frac{L_1L_2 - M^2}{L_1 + L_2 + 2M}$$

P 6.42 When the switch is opened the induced voltage is negative at the dotted terminal. Since the voltmeter kicks upscale, the induced voltage across the voltmeter must be positive at its positive terminal. Therefore, the voltage is negative at the negative terminal of the voltmeter.

Thus, the lower terminal of the unmarked coil has the same instantaneous polarity as the dotted terminal. Therefore, place a dot on the lower terminal of the unmarked coil.

P 6.43 **[a]** Dot terminal 1; the flux is up in coil 1-2, and down in coil 3-4. Assign the current into terminal 4; the flux is down in coil 3-4. Therefore, dot terminal 4. Hence, 1 and 4 or 2 and 3.

[b] Dot terminal 2; the flux is up in coil 1-2, and right-to-left in coil 3-4. Assign the current into terminal 4; the flux is right-to-left in coil 3-4. Therefore, dot terminal 4. Hence, 2 and 4 or 1 and 3.

[c] Dot terminal 2; the flux is up in coil 1-2, and right-to-left in coil 3-4. Assign the current into terminal 4; the flux is right-to-left in coil 3-4. Therefore, dot terminal 4. Hence, 2 and 4 or 1 and 3.

[d] Dot terminal 1; the flux is down in coil 1-2, and down in coil 3-4. Assign the current into terminal 4; the flux is down in coil 3-4. Therefore, dot terminal 4. Hence, 1 and 4 or 2 and 3.

P 6.44 **[a]** $W = (0.5)L_1i_1^2 + (0.5)L_2i_2^2 + Mi_1i_2$

$$M = 0.85\sqrt{(18)(32)} = 20.4\,\text{mH}$$

$$W = [9(36) + 16(81) + 20.4(54)] = 2721.6\,\text{mJ}$$

[b] $W = [324 + 1296 + 1101.6] = 2721.6\,\text{mJ}$

[c] $W = [324 + 1296 - 1101.6] = 518.4\,\text{mJ}$

[d] $W = [324 + 1296 - 1101.6] = 518.4\,\text{mJ}$

P 6.45 **[a]** $M = 1.0\sqrt{(18)(32)} = 24\,\text{mH}, \qquad i_1 = 6\,\text{A}$

Therefore $16i_2^2 + 144i_2 + 324 = 0, \qquad i_2^2 + 9i_2 + 20.25 = 0$

$$\text{Therefore} \quad i_2 = -\left(\frac{9}{2}\right) \pm \sqrt{\left(\frac{9}{2}\right)^2 - 20.25} = -4.5 \pm \sqrt{0}$$

Therefore $i_2 = -4.5\,\text{A}$

[b] No, setting W equal to a negative value will make the quantity under the square root sign negative.

P 6.46 **[a]** $k = \dfrac{M}{\sqrt{L_1 L_2}} = \dfrac{22.8}{\sqrt{576}} = 0.95$

[b] $M_{\text{max}} = \sqrt{576} = 24\,\text{mH}$

[c] $\dfrac{L_1}{L_2} = \dfrac{N_1^2\mathcal{P}_1}{N_2^2\mathcal{P}_2} = \left(\dfrac{N_1}{N_2}\right)^2$

$$\therefore \quad \left(\frac{N_1}{N_2}\right)^2 = \frac{60}{9.6} = 6.25$$

$$\frac{N_1}{N_2} = \sqrt{6.25} = 2.5$$

P 6.47 **[a]** $L_1 = N_1^2\mathcal{P}_1; \qquad \mathcal{P}_1 = \dfrac{72 \times 10^{-3}}{6.25 \times 10^4} = 1152\ \text{nWb/A}$

$$\frac{d\phi_{11}}{d\phi_{21}} = \frac{\mathcal{P}_{11}}{\mathcal{P}_{21}} = 0.2; \qquad \mathcal{P}_{21} = 2\mathcal{P}_{11}$$

$$\therefore \quad 1152 \times 10^{-9} = \mathcal{P}_{11} + \mathcal{P}_{21} = 3\mathcal{P}_{11}$$

$$\mathcal{P}_{11} = 192\ \text{nWb/A}; \qquad \mathcal{P}_{21} = 960\ \text{nWb/A}$$

$$M = k\sqrt{L_1 L_2} = (2/3)\sqrt{(0.072)(0.0405)} = 36\,\text{mH}$$

$$N_2 = \frac{M}{N_1\mathcal{P}_{21}} = \frac{36 \times 10^{-3}}{(250)(960 \times 10^{-9})} = 150\ \text{turns}$$

[b] $\mathcal{P}_2 = \dfrac{L_2}{N_2^2} = \dfrac{40.5 \times 10^{-3}}{(150)^2} = 1800\ \text{nWb/A}$

[c] $\mathcal{P}_{11} = 192\ \text{nWb/A}$ [see part (a)]

[d] $\dfrac{\phi_{22}}{\phi_{12}} = \dfrac{\mathcal{P}_{22}}{\mathcal{P}_{12}} = \dfrac{\mathcal{P}_2 - \mathcal{P}_{12}}{\mathcal{P}_{12}} = \dfrac{\mathcal{P}_2}{\mathcal{P}_{12}} - 1$

$$\mathcal{P}_{21} = \mathcal{P}_{21} = 960\ \text{nWb/A}; \qquad \mathcal{P}_2 = 1800\ \text{nWb/A}$$

$$\frac{\phi_{22}}{\phi_{12}} = \frac{1800}{960} - 1 = 0.875$$

P 6.48 **[a]** $L_2 = \left(\frac{M^2}{k^2 L_1}\right) = \frac{(0.09)^2}{(0.75)^2(0.288)} = 50\,\text{mH}$

$$\frac{N_1}{N_2} = \sqrt{\frac{L_1}{L_2}} = \sqrt{\frac{288}{50}} = 2.4$$

[b] $\mathcal{P}_1 = \frac{L_1}{N_1^2} = \frac{0.288}{(1200)^2} = 0.2 \times 10^{-6}$ Wb/A

$$\mathcal{P}_2 = \frac{L_2}{N_2^2} = \frac{0.05}{(500)^2} = 0.2 \times 10^{-6} \text{ Wb/A}$$

P 6.49 $\mathcal{P}_1 = \frac{L_1}{N_1^2} = 2$ nWb/A; $\quad \mathcal{P}_2 = \frac{L_2}{N_2^2} = 2$ nWb/A; $\quad M = k\sqrt{L_1 L_2} = 180\,\mu\text{H}$

$$\mathcal{P}_{12} = \mathcal{P}_{21} = \frac{M}{N_1 N_2} = 1.2 \text{ nWb/A}$$

$$\mathcal{P}_{11} = \mathcal{P}_1 - \mathcal{P}_{21} = 0.8 \text{ nWb/A}$$

P 6.50 **[a]** $\frac{1}{k^2} = \left(1 + \frac{\mathcal{P}_{11}}{\mathcal{P}_{12}}\right)\left(1 + \frac{\mathcal{P}_{22}}{\mathcal{P}_{12}}\right) = \left(1 + \frac{\mathcal{P}_{11}}{\mathcal{P}_{21}}\right)\left(1 + \frac{\mathcal{P}_{22}}{\mathcal{P}_{12}}\right)$

Therefore

$$k^2 = \frac{\mathcal{P}_{12}\mathcal{P}_{21}}{(\mathcal{P}_{21} + \mathcal{P}_{11})(\mathcal{P}_{12} + \mathcal{P}_{22})}$$

Now note that

$$\phi_1 = \phi_{11} + \phi_{21} = \mathcal{P}_{11} N_1 i_1 + \mathcal{P}_{21} N_1 i_1 = N_1 i_1 (\mathcal{P}_{11} + \mathcal{P}_{21})$$

and similarly

$$\phi_2 = N_2 i_2 (\mathcal{P}_{22} + \mathcal{P}_{12})$$

It follows that

$$(\mathcal{P}_{11} + \mathcal{P}_{21}) = \frac{\phi_1}{N_1 i_1}$$

and

$$(\mathcal{P}_{22} + \mathcal{P}_{12}) = \left(\frac{\phi_2}{N_2 i_2}\right)$$

Therefore

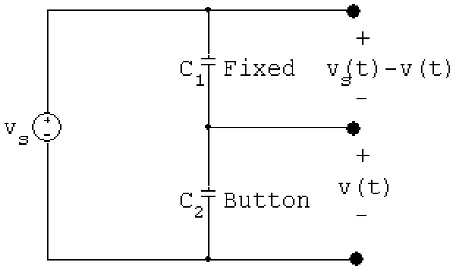
$$k^2 = \frac{(\phi_{12}/N_2 i_2)(\phi_{21}/N_1 i_1)}{(\phi_1/N_1 i_1)(\phi_2/N_2 i_2)} = \frac{\phi_{12}\phi_{21}}{\phi_1 \phi_2}$$

or

$$k = \sqrt{\left(\frac{\phi_{21}}{\phi_1}\right)\left(\frac{\phi_{12}}{\phi_2}\right)}$$

[b] The fractions (ϕ_{21}/ϕ_1) and (ϕ_{12}/ϕ_2) are by definition less than 1.0, therefore $k < 1$.

P 6.51 When the button is not pressed we have



$$C_2\frac{dv}{dt} = C_1\frac{d}{dt}(v_s - v)$$

or

$$(C_1 + C_2)\frac{dv}{dt} = C_1\frac{dv_s}{dt}$$

$$\frac{dv}{dt} = \frac{C_1}{(C_1 + C_2)}\frac{dv_s}{dt}$$

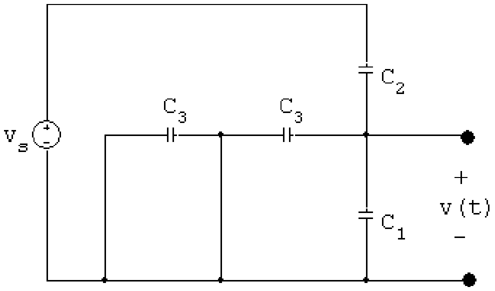
Assuming $C_1 = C_2 = C$

$$\frac{dv}{dt} = 0.5\frac{dv_s}{dt}$$

or

$$v = 0.5v_s(t) + v(0)$$

When the button is pressed we have



$$C_1\frac{dv}{dt} + C_3\frac{dv}{dt} + C_2\frac{d(v - v_s)}{dt} = 0$$

$$\therefore \quad \frac{dv}{dt} = \frac{C_2}{C_1 + C_2 + C_3}\frac{dv_s}{dt}$$

Assuming $C_1 = C_2 = C_3 = C$

$$\frac{dv}{dt} = \frac{1}{3}\frac{dv_s}{dt}$$

$$v = \frac{1}{3}v_s(t) + v(0)$$

Therefore interchanging the fixed capacitor and the button has no effect on the change in $v(t)$.

P 6.52 With no finger touching and equal 10 pF capacitors

$$v(t) = \frac{10}{20}(v_s(t)) + 0 = 0.5v_s(t)$$

With a finger touching

Let C_e = equivalent capacitance of person touching lamp

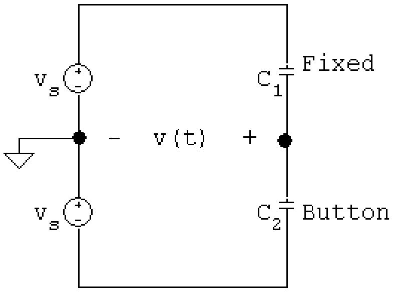
$$C_e = \frac{(10)(100)}{110} = 9.091 \text{ pF}$$

Then $\quad C + C_e = 10 + 9.091 = 19.091$ pF

$$\therefore \quad v(t) = \frac{10}{29.091}v_s = 0.344v_s$$

$$\therefore \quad \Delta v(t) = (0.5 - 0.344)v_s = 0.156v_s$$

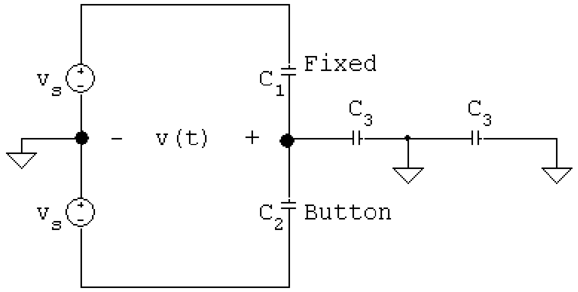
P 6.53 With no finger on the button the circuit is



$$C_1\frac{d}{dt}(v - v_s) + C_2\frac{d}{dt}(v + v_s) = 0$$

when $\quad C_1 = C_2 = C \qquad (2C)\frac{dv}{dt} = 0$

With a finger on the button



$$C_1\frac{d(v-v_s)}{dt}+C_2\frac{d(v+v_s)}{dt}+C_3\frac{dv}{dt}=0$$

$$(C_1+C_2+C_3)\frac{dv}{dt}+C_2\frac{dv_s}{dt}-C_1\frac{dv_s}{dt}=0$$

when $\quad C_1=C_2=C_3=C \qquad (3C)\frac{dv}{dt}=0$

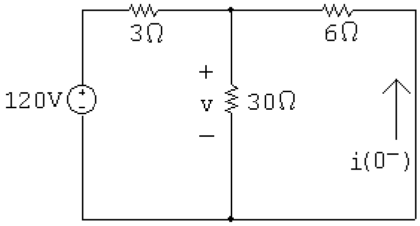
$\therefore$ there is no change in the output voltage of this circuit.

7

Response of First-Order RL and RC Circuits

Assessment Problems

AP 7.1 **[a]** The circuit for $t < 0$ is shown below. Note that the inductor behaves like a short circuit, effectively eliminating the $2\,\Omega$ resistor from the circuit.



First combine the $30\,\Omega$ and $6\,\Omega$ resistors in parallel:
$30\|6 = 5\,\Omega$
Use voltage division to find the voltage drop across the parallel resistors:
$$v = \frac{5}{5+3}(120) = 75\,\text{V}$$
Now find the current using Ohm's law:
$$i(0^-) = -\frac{v}{6} = -\frac{75}{6} = -12.5\,\text{A}$$

[b] $w(0) = \frac{1}{2}Li^2(0) = \frac{1}{2}(8 \times 10^{-3})(12.5)^2 = 625\,\text{mJ}$

[c] To find the time constant, we need to find the equivalent resistance seen by the inductor for $t > 0$. When the switch opens, only the $2\,\Omega$ resistor remains connected to the inductor. Thus,
$$\tau = \frac{L}{R} = \frac{8 \times 10^{-3}}{2} = 4\,\text{ms}$$

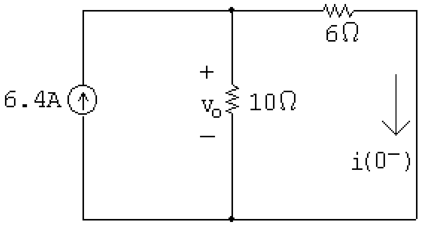
[d] $i(t) = i(0^-)e^{t/\tau} = -12.5e^{-t/0.004} = -12.5e^{-250t}\,\text{A}, \qquad t \geq 0$

[e] $i(5\,\text{ms}) = -12.5e^{-250(0.005)} = -12.5e^{-1.25} = -3.58\,\text{A}$
So $\quad w\,(5\,\text{ms}) = \frac{1}{2}Li^2(5\,\text{ms}) = \frac{1}{2}(8) \times 10^{-3}(3.58)^2 = 51.3\,\text{mJ}$

$w\,(\text{dis}) = 625 - 51.3 = 573.7\,\text{mJ}$

$\%\ \text{dissipated} = \left(\frac{573.7}{625}\right)100 = 91.8\%$

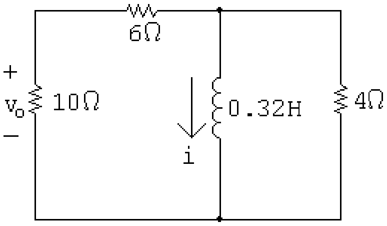
AP 7.2 **[a]** First, use the circuit for $t < 0$ to find the initial current in the inductor:



Using current division,

$i(0^-) = \frac{10}{10+6}(6.4) = 4\,\text{A}$

Now use the circuit for $t > 0$ to find the equivalent resistance seen by the inductor, and use this value to find the time constant:



$R_{\text{eq}} = 4\|(6+10) = 3.2\,\Omega, \quad \therefore \quad \tau = \frac{L}{R_{\text{eq}}} = \frac{0.32}{3.2} = 0.1\,\text{s}$

Use the initial inductor current and the time constant to find the current in the inductor:

$i(t) = i(0^-)e^{-t/\tau} = 4e^{-t/0.1} = 4e^{-10t}\,\text{A}, \quad t \geq 0$

Use current division to find the current in the $10\,\Omega$ resistor:

$$i_o(t) = \frac{4}{4+10+6}(-i) = \frac{4}{20}(-4e^{-10t}) = -0.8e^{-10t}\,\text{A}, \quad t \geq 0^+$$

Finally, use Ohm's law to find the voltage drop across the $10\,\Omega$ resistor:

$v_o(t) = 10i_o = 10(-0.8e^{-10t}) = -8e^{-10t}\,\text{V}, \quad t \geq 0^+$

[b] The initial energy stored in the inductor is

$$w(0) = \frac{1}{2}Li^2(0^-) = \frac{1}{2}(0.32)(4)^2 = 2.56\,\text{J}$$

Find the energy dissipated in the $4\,\Omega$ resistor by integrating the power over all time:

$$v_{4\Omega}(t) = L\frac{di}{dt} = 0.32(-10)(4e^{-10t}) = -12.8e^{-10t}\,\text{V}, \qquad t \geq 0^+$$

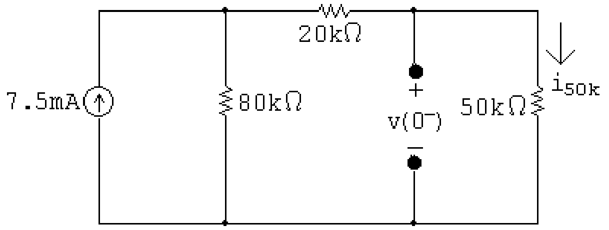
$$p_{4\Omega}(t) = \frac{v_{4\Omega}^2}{4} = 40.96e^{-20t}\,\text{W}, \qquad t \geq 0^+$$

$$w_{4\Omega}(t) = \int_0^\infty 40.96e^{-20t}dt = 2.048\,\mathrm{J}$$

Find the percentage of the initial energy in the inductor dissipated in the $4\,\Omega$ resistor:

$$\%\ \text{dissipated} = \left(\frac{2.048}{2.56}\right)100 = 80\%$$

AP 7.3 **[a]** The circuit for $t < 0$ is shown below. Note that the capacitor behaves like an open circuit.



Find the voltage drop across the open circuit by finding the voltage drop across the $50\,\mathrm{k}\Omega$ resistor. First use current division to find the current through the $50\,\mathrm{k}\Omega$ resistor:

$$i_{50\mathrm{k}} = \frac{80 \times 10^3}{80 \times 10^3 + 20 \times 10^3 + 50 \times 10^3}(7.5 \times 10^{-3}) = 4\,\mathrm{mA}$$

Use Ohm's law to find the voltage drop:
$v(0^-) = (50 \times 10^3)i_{50\mathrm{k}} = (50 \times 10^3)(0.004) = 200\,\mathrm{V}$

[b] To find the time constant, we need to find the equivalent resistance seen by the capacitor for $t > 0$. When the switch opens, only the $50\,\mathrm{k}\Omega$ resistor remains connected to the capacitor. Thus,
$\tau = RC = (50 \times 10^3)(0.4 \times 10^{-6}) = 20\,\mathrm{ms}$

[c] $v(t) = v(0^-)e^{-t/\tau} = 200e^{-t/0.02} = 200e^{-50t}\,\mathrm{V}, \quad t \geq 0$

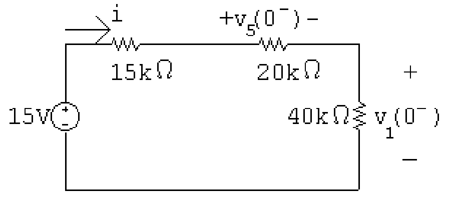
[d] $w(0) = \frac{1}{2}Cv^2 = \frac{1}{2}(0.4 \times 10^{-6})(200)^2 = 8\,\mathrm{mJ}$

[e] $w(t) = \frac{1}{2}Cv^2(t) = \frac{1}{2}(0.4 \times 10^{-6})(200e^{-50t})^2 = 8e^{-100t}\,\mathrm{mJ}$

The initial energy is 8 mJ, so when 75% is dissipated, 2 mJ remains:

$$8 \times 10^{-3}e^{-100t} = 2 \times 10^{-3}, \qquad e^{100t} = 4, \qquad t = (\ln 4)/100 = 13.86\,\mathrm{ms}$$

AP 7.4 **[a]** This circuit is actually two *RC* circuits in series, and the requested voltage, v_o, is the sum of the voltage drops for the two *RC* circuits. The circuit for $t < 0$ is shown below:



Find the current in the loop and use it to find the initial voltage drops across the two RC circuits:

$$i = \frac{15}{75{,}000} = 0.2\,\text{mA}, \qquad v_5(0^-) = 4\,\text{V}, \qquad v_1(0^-) = 8\,\text{V}$$

There are two time constants in the circuit, one for each RC subcircuit. τ_5 is the time constant for the $5\,\mu$F – $20\,\text{k}\Omega$ subcircuit, and τ_1 is the time constant for the $1\,\mu$F – $40\,\text{k}\Omega$ subcircuit:

$$\tau_5 = (20 \times 10^3)(5 \times 10^{-6}) = 100\,\text{ms}; \qquad \tau_1 = (40 \times 10^3)(1 \times 10^{-6}) = 40\,\text{ms}$$

Therefore,

$$v_5(t) = v_5(0^-)e^{-t/\tau_5} = 4e^{-t/0.1} = 4e^{-10t}\,\text{V}, \quad t \geq 0$$

$$v_1(t) = v_1(0^-)e^{-t/\tau_1} = 8e^{-t/0.04} = 8e^{-25t}\,\text{V}, \quad t \geq 0$$

Finally,

$$v_o(t) = v_1(t) + v_5(t) = [8e^{-25t} + 4e^{-10t}]\,\text{V}, \qquad t \geq 0$$

[b] Find the value of the voltage at 60 ms for each subcircuit and use the voltage to find the energy at 60 ms:

$$v_1(60\,\text{ms}) = 8e^{-25(0.06)} \cong 1.79\,\text{V}, \qquad v_5(60\,\text{ms}) = 4e^{-10(0.06)} \cong 2.20\,\text{V}$$

$$w_1(60\,\text{ms}) = \tfrac{1}{2}Cv_1^2(60\,\text{ms}) = \tfrac{1}{2}(1 \times 10^{-6})(1.79)^2 \cong 1.59\,\mu\text{J}$$

$$w_5(60\,\text{ms}) = \tfrac{1}{2}Cv_5^2(60\,\text{ms}) = \tfrac{1}{2}(5 \times 10^{-6})(2.20)^2 \cong 12.05\,\mu\text{J}$$

$$w(60\,\text{ms}) = 1.59 + 12.05 = 13.64\,\mu\text{J}$$

Find the initial energy from the initial voltage:

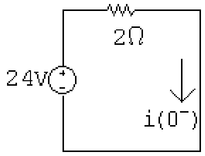
$$w(0) = w_1(0) + w_2(0) = \tfrac{1}{2}(1 \times 10^{-6})(8)^2 + \tfrac{1}{2}(5 \times 10^{-6})(4)^2 = 72\,\mu\text{J}$$

Now calculate the energy dissipated at 60 ms and compare it to the initial energy:

$$w_{\text{diss}} = w(0) - w(60\,\text{ms}) = 72 - 13.64 = 58.36\,\mu\text{J}$$

$$\%\text{ dissipated} = (58.36 \times 10^{-6}/72 \times 10^{-6})(100) = 81.05\,\%$$

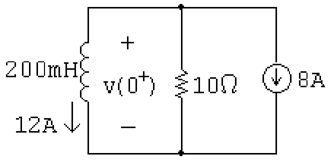
AP 7.5 **[a]** Use the circuit at $t < 0$, shown below, to calculate the initial current in the inductor:



$$i(0^-) = 24/2 = 12\,\text{A} = i(0^+)$$

Note that $i(0^-) = i(0^+)$ because the current in an inductor is continuous.

[b] Use the circuit at $t = 0^+$, shown below, to calculate the voltage drop across the inductor at 0^+. Note that this is the same as the voltage drop across the $10\,\Omega$ resistor, which has current from two sources — 8 A from the current source and 12 A from the initial current through the inductor.

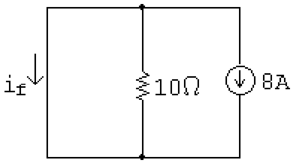


$v(0^+) = -10(8 + 12) = -200\,\text{V}$

[c] To calculate the time constant we need the equivalent resistance seen by the inductor for $t > 0$. Only the $10\,\Omega$ resistor is connected to the inductor for $t > 0$. Thus,

$\tau = L/R = (200 \times 10^{-3}/10) = 20\,\text{ms}$

[d] To find $i(t)$, we need to find the final value of the current in the inductor. When the switch has been in position a for a long time, the circuit reduces to the one below:



Note that the inductor behaves as a short circuit and all of the current from the 8 A source flows through the short circuit. Thus,

$i_f = -8\,\text{A}$

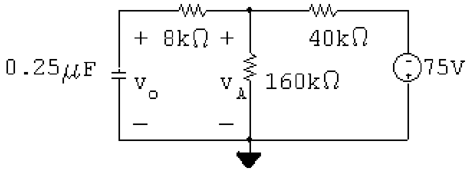
Now,

$$i(t) = i_f + [i(0^+) - i_f]e^{-t/\tau} = -8 + [12 - (-8)]e^{-t/0.02}$$
$$= -8 + 20e^{-50t}\,\text{A}, \quad t \geq 0$$

[e] To find $v(t)$, use the relationship between voltage and current for an inductor:

$$v(t) = L\frac{di(t)}{dt} = (200 \times 10^{-3})(-50)(20e^{-50t}) = -200e^{-50t}\,\text{V}, \qquad t \geq 0^+$$

AP 7.6 **[a]**



From Example 7.6,

$v_o(t) = -60 + 90e^{-100t}\,\text{V}$

Write a KCL equation at the top node and use it to find the relationship between v_o and v_A:

$$\frac{v_A - v_o}{8000} + \frac{v_A}{160{,}000} + \frac{v_A + 75}{40{,}000} = 0$$

$$20v_A - 20v_o + v_A + 4v_A + 300 = 0$$

$$25v_A = 20v_o - 300$$

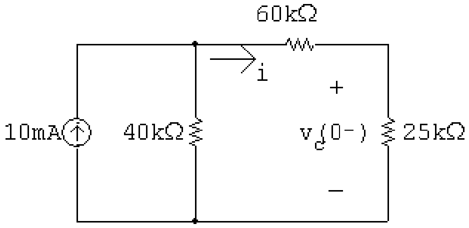
$$v_A = 0.8v_o - 12$$

Use the above equation for v_A in terms of v_o to find the expression for v_A:

$$v_A(t) = 0.8(-60 + 90e^{-100t}) - 12 = -60 + 72e^{-100t}\,\text{V}, \qquad t \geq 0^+$$

[b] $t \geq 0^+$, since there is no requirement that the voltage be continuous in a resistor.

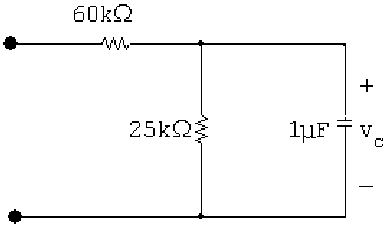
AP 7.7 **[a]** Use the circuit shown below, for $t < 0$, to calculate the initial voltage drop across the capacitor:



$$i = \left(\frac{40 \times 10^3}{125 \times 10^3}\right)(10 \times 10^{-3}) = 3.2\,\text{mA}$$

$$v_c(0^-) = (3.2 \times 10^{-3})(25 \times 10^3) = 80\,\text{V} \quad \text{so} \quad v_c(0^+) = 80\,\text{V}$$

Now use the next circuit, valid for $0 \leq t \leq 10\,\text{ms}$, to calculate $v_c(t)$ for that interval:

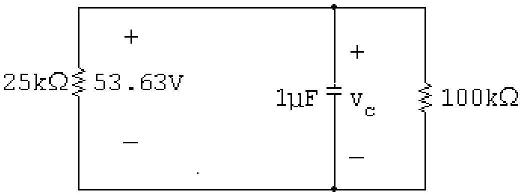


For $\quad 0 \leq t \leq 100\,\text{ms}$:

$$\tau = RC = (25 \times 10^3)(1 \times 10^{-6}) = 25\,\text{ms}$$

$$v_c(t) = v_c(0^-)e^{t/\tau} = 80e^{-40t}\,\text{V} \quad 0 \leq t \leq 10\,\text{ms}$$

[b] Calculate the starting capacitor voltage in the interval $t \geq 10\,\text{ms}$, using the capacitor voltage from the previous interval:
$v_c(0.01) = 80e^{-40(0.01)} = 53.63\,\text{V}$
Now use the next circuit, valid for $t \geq 10\,\text{ms}$, to calculate $v_c(t)$ for that interval:



For $\quad t \geq 10\,\text{ms}$:

$$R_{\text{eq}} = 25\,\text{k}\Omega \| 100\,\text{k}\Omega = 20\,\text{k}\Omega$$

$$\tau = R_{\text{eq}}C = (20 \times 10^3)(1 \times 10^{-6}) = 0.02\,\text{s}$$

Therefore $\quad v_c(t) = v_c(0.01^+)e^{-(t-0.01)/\tau} = 53.63e^{-50(t-0.01)}\,\text{V}, \qquad t \geq 0.01\,\text{s}$

[c] To calculate the energy dissipated in the 25 kΩ resistor, integrate the power absorbed by the resistor over all time. Use the expression $p = v^2/R$ to calculate the power absorbed by the resistor.

$$w_{25\,\text{k}} = \int_0^{0.01} \frac{[80e^{-40t}]^2}{25{,}000}dt + \int_{0.01}^{\infty} \frac{[53.63e^{-50(t-0.01)}]^2}{25{,}000}dt = 2.91\,\text{mJ}$$

[d] Repeat the process in part (c), but recognize that the voltage across this resistor is non-zero only for the second interval:

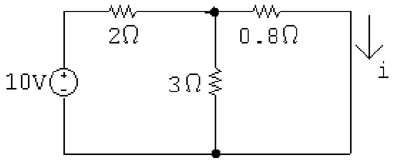
$$w_{100\,\text{k}\Omega} = \int_{0.01}^{\infty} \frac{[53.63e^{-50(t-0.01)}]^2}{100{,}000}dt = 0.29\,\text{mJ}$$

We can check our answers by calculating the initial energy stored in the capacitor. All of this energy must eventually be dissipated by the 25 kΩ resistor and the 100 kΩ resistor.

Check: $w_{\text{stored}} = (1/2)(1 \times 10^{-6})(80)^2 = 3.2\,\text{mJ}$

$$w_{\text{diss}} = 2.91 + 0.29 = 3.2\,\text{mJ}$$

AP 7.8 [a] Prior to switch a closing at $t = 0$, there are no sources connected to the inductor; thus, $i(0^-) = 0$.
At the instant A is closed, $i(0^+) = 0$.
For $0 \leq t \leq 1\,\text{s}$,



The equivalent resistance seen by the 10 V source is $2 + (3\|0.8)$. The current leaving the 10 V source is

$$\frac{10}{2 + (3\|0.8)} = 3.8\,\text{A}$$

The final current in the inductor, which is equal to the current in the 0.8 Ω resistor is

$$I_{\text{F}} = \frac{3}{3 + 0.8}(3.8) = 3\,\text{A}$$

The resistance seen by the inductor is calculated to find the time constant:

$$[(2\|3) + 0.8]\|3\|6 = 1\,\Omega \qquad \tau = \frac{L}{R} = \frac{2}{1} = 2\,\text{s}$$

Therefore,

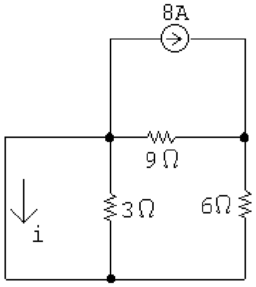
$$i = i_{\text{F}} + [i(0^+) - i_{\text{F}}]e^{-t/\tau} = 3 - 3e^{-0.5t}\,\text{A}, \quad 0 \leq t \leq 1\,\text{s}$$

For part (b) we need the value of $i(t)$ at $t = 1$ s:

$$i(1) = 3 - 3e^{-0.5} = 1.18\,\text{A}$$

.

[b] For $t > 1$ s



Use current division to find the final value of the current:

$$i = \frac{9}{9+6}(-8) = -4.8\,\text{A}$$

The equivalent resistance seen by the inductor is used to calculate the time constant:

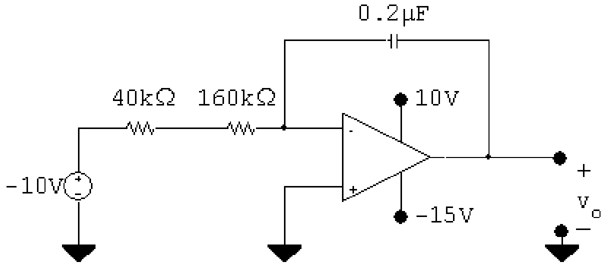
$$3\|(9+6) = 2.5\,\Omega \qquad \tau = \frac{L}{R} = \frac{2}{2.5} = 0.8\,\text{s}$$

Therefore,

$$i = i_{\text{F}} + [i(1^{+}) - i_{\text{F}}]e^{-(t-1)/\tau}$$

$$= -4.8 + 5.98e^{-1.25(t-1)}\,\text{A}, \quad t \geq 1\,\text{s}$$

AP 7.9 $0 \leq t \leq 32\,\text{ms}$:

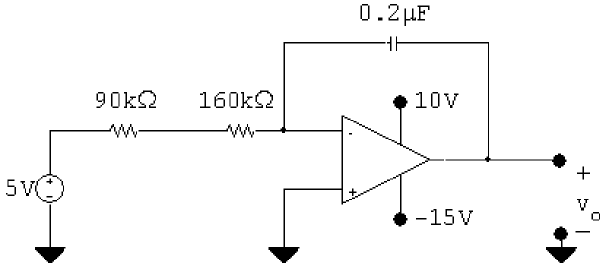


$$v_o = -\frac{1}{RC_f}\int_0^{32\times10^{-3}} -10\,dt + 0 = -\frac{1}{RC_f}(-10t)\Big|_0^{32\times10^{-3}} = -\frac{1}{RC_f}(-320\times10^{-3})$$

$$RC_f = (200\times10^3)(0.2\times10^{-6}) = 40\times10^{-3} \quad \text{so} \quad \frac{1}{RC_f} = 25$$

$$v_o = -25(-320\times10^{-3}) = 8\,\text{V}$$

$t \geq 32\,\text{ms}$:



$$v_o = -\frac{1}{RC_f}\int_{32\times10^{-3}}^{t} 5\,dy + 8 = -\frac{1}{RC_f}(5y)\Big|_{32\times10^{-3}}^{t} +8 = -\frac{1}{RC_f}5(t-32\times10^{-3})+8$$

$$RC_f = (250\times10^3)(0.2\times10^{-6}) = 50\times10^{-3} \quad \text{so} \quad \frac{1}{RC_f} = 20$$

$$v_o = -20(5)(t-32\times10^{-3})+8 = -100t+11.2$$

The output will saturate at the negative power supply value:

$$-15 = -100t + 11.2 \qquad \therefore \qquad t = 262\,\text{ms}$$

AP 7.10 [**a**] Use RC circuit analysis to determine the expression for the voltage at the non-inverting input:

$$v_p = V_f + [V_o - V_f]e^{-t/\tau} = -2 + (0+2)e^{-t/\tau}$$

$$\tau = (160\times10^3)(10\times10^{-9}) = 10^{-3}; \qquad 1/\tau = 625$$

$$v_p = -2 + 2e^{-625t}\,\text{V}; \qquad v_n = v_p$$

Write a KVL equation at the inverting input, and use it to determine v_o:

$$\frac{v_n}{10{,}000} + \frac{v_n - v_o}{40{,}000} = 0$$

$$\therefore \quad v_o = 5v_n = 5v_p = -10 + 10e^{-625t}\,\text{V}$$

The output will saturate at the negative power supply value:

$$-10 + 10e^{-625t} = -5; \quad e^{-625t} = 1/2; \quad t = \ln 2/625 = 1.11\,\text{ms}$$

[**b**] Use RC circuit analysis to determine the expression for the voltage at the non-inverting input:

$$v_p = V_f + [V_o - V_f]e^{-t/\tau} = -2 + (1+2)e^{-625t} = -2 + 3e^{-625t}\,\text{V}$$

The analysis for v_o is the same as in part (a):

$$v_o = 5v_p = -10 + 15e^{-625t}\,\text{V}$$

The output will saturate at the negative power supply value:

$$-10 + 15e^{-625t} = -5; \qquad e^{-625t} = 1/3; \quad t = \ln 3/625 = 1.76\,\text{ms}$$

Problems

P 7.1 **[a]** $R = \dfrac{v}{i} = 25\,\Omega$

[b] $\tau = \dfrac{1}{10} = 100\,\text{ms}$

[c] $\tau = \dfrac{L}{R} = 0.1$

$$L = (0.1)(25) = 2.5\,\text{H}$$

[d] $w(0) = \dfrac{1}{2}L[i(0)]^2 = \dfrac{1}{2}(2.5)(6.4)^2 = 51.2\,\text{J}$

[e] $w_{\text{diss}} = \displaystyle\int_0^t 1024e^{-20x}\,dx = 1024\frac{e^{-20x}}{-20}\Big|_0^t = 51.2(1 - e^{-20t})\,\text{J}$

$$\%\,\text{dissipated} = \frac{51.2(1 - e^{-20t})}{51.2}(100) = 100(1 - e^{-20t})$$

$$\therefore\quad 100(1 - e^{-20t}) = 60 \qquad \text{so} \qquad e^{-20t} = 0.4$$

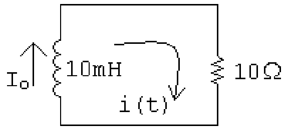
$$\text{Therefore} \quad t = \frac{1}{20}\ln 2.5 = 45.81\,\text{ms}$$

P 7.2 **[a]** Note that there are several different possible solutions to this problem, and the answer to part (c) depends on the value of inductance chosen.

$$R = \frac{L}{\tau}$$

Choose a 10 mH inductor from Appendix H. Then,

$$R = \frac{0.01}{0.001} = 10\,\Omega \quad \text{which is a resistor value from Appendix H.}$$



[b] $i(t) = I_o e^{-t/\tau} = 10e^{-1000t}\,\text{mA}, \qquad t \geq 0$

[c] $w(0) = \dfrac{1}{2}LI_o^2 = \dfrac{1}{2}(0.01)(0.01)^2 = 0.5\,\mu\text{J}$

$$w(t) = \frac{1}{2}(0.01)(0.01e^{-1000t})^2 = 0.5 \times 10^{-6}e^{-2000t}$$

$$\text{So} \quad 0.5 \times 10^{-6}e^{-2000t} = \frac{1}{2}w(0) = 0.25 \times 10^{-6}$$

$$e^{-2000t} = 0.5 \qquad \text{then} \qquad e^{2000t} = 2$$

$$\therefore\quad t = \frac{\ln 2}{2000} = 346.57\,\mu\text{s} \qquad \text{(for a 10 mH inductor)}$$

P 7.3 [**a**] $i_L(0) = \dfrac{125}{50} = 2.5\,\text{A}$

$$i_o(0^+) = \frac{125}{25} - 2.5 = 5 - 2.5 = 2.5\,\text{A}$$

$$i_o(\infty) = \frac{125}{25} = 5\,\text{A}$$

[**b**] $i_L = 2.5e^{-t/\tau}; \qquad \tau = \dfrac{L}{R} = \dfrac{50\times10^{-3}}{25} = 2\,\text{ms}$

$$i_L = 2.5e^{-500t}\,\text{A}$$

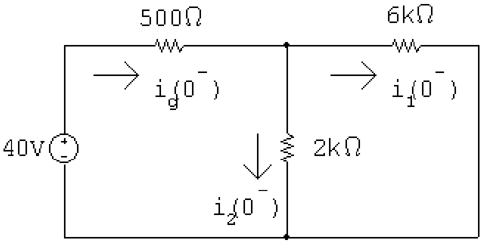
$$i_o = 5 - i_L = 5 - 2.5e^{-500t}\,\text{A}, \qquad t \geq 0^+$$

[**c**] $5 - 2.5e^{-500t} = 3$

$$2 = 2.5e^{-500t}$$

$$e^{500t} = 1.25 \qquad \therefore \quad t = 446.29\,\mu\text{s}$$

P 7.4 [**a**] $t < 0$



$$2\,\text{k}\Omega \| 6\,\text{k}\Omega = 1.5\,\text{k}\Omega$$

Find the current from the voltage source by combining the resistors in series and parallel and using Ohm's law:

$$i_g(0^-) = \frac{40}{(1500 + 500)} = 20\,\text{mA}$$

Find the branch currents using current division:

$$i_1(0^-) = \frac{2000}{8000}(0.02) = 5\,\text{mA}$$

$$i_2(0^-) = \frac{6000}{8000}(0.02) = 15\,\text{mA}$$

[**b**] The current in an inductor is continuous. Therefore,

$$i_1(0^+) = i_1(0^-) = 5\,\text{mA}$$

$$i_2(0^+) = -i_1(0^+) = -5\,\text{mA} \qquad \text{(when switch is open)}$$

[c] $\tau = \dfrac{L}{R} = \dfrac{0.4 \times 10^{-3}}{8 \times 10^{3}} = 5 \times 10^{-5}\,\text{s}; \qquad \dfrac{1}{\tau} = 20{,}000$

$$i_1(t) = i_1(0^+)e^{-t/\tau} = 5e^{-20{,}000t}\,\text{mA}, \qquad t \geq 0$$

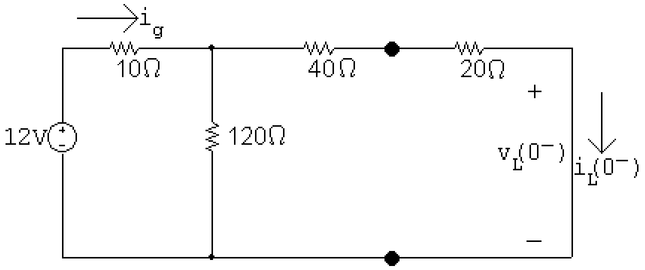
[d] $i_2(t) = -i_1(t) \qquad \text{when} \quad t \geq 0^+$

$$\therefore \quad i_2(t) = -5e^{-20{,}000t}\,\text{mA}, \qquad t \geq 0^+$$

[e] The current in a resistor can change instantaneously. The switching operation forces $i_2(0^-)$ to equal 15 mA and $i_2(0^+) = -5$ mA.

P 7.5 **[a]** $i_o(0^-) = 0$ since the switch is open for $t < 0$.

[b] For $t = 0^-$ the circuit is:

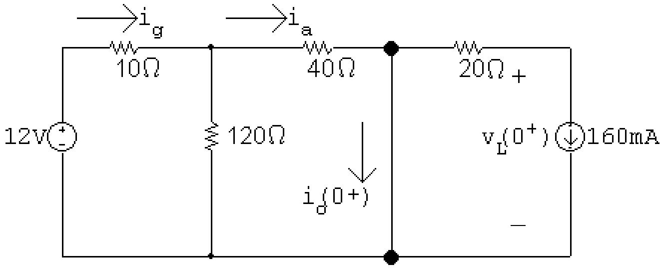


$$120\,\Omega \| 60\,\Omega = 40\,\Omega$$

$$\therefore \quad i_g = \frac{12}{10 + 40} = 0.24\,\text{A} = 240\,\text{mA}$$

$$i_L(0^-) = \left(\frac{120}{180}\right) i_g = 160\,\text{mA}$$

[c] For $t = 0^+$ the circuit is:



$$120\,\Omega \| 40\,\Omega = 30\,\Omega$$

$$\therefore \quad i_g = \frac{12}{10 + 30} = 0.30\,\text{A} = 300\,\text{mA}$$

$$i_a = \left(\frac{120}{160}\right) 300 = 225\,\text{mA}$$

$$\therefore \quad i_o(0^+) = 225 - 160 = 65\,\text{mA}$$

[d] $i_L(0^+) = i_L(0^-) = 160\,\text{mA}$

[e] $i_o(\infty) = i_{\text{a}} = 225\,\text{mA}$

[f] $i_L(\infty) = 0,$ since the switch short circuits the branch containing the 20 Ω resistor and the 100 mH inductor.

[g] $\tau = \dfrac{L}{R} = \dfrac{100 \times 10^{-3}}{20} = 5\,\text{ms}; \qquad \dfrac{1}{\tau} = 200$

$$\therefore \quad i_L = 0 + (160 - 0)e^{-200t} = 160e^{-200t}\,\text{mA}, \qquad t \geq 0$$

[h] $v_L(0^-) = 0$ since for $t < 0$ the current in the inductor is constant

[i] Refer to the circuit at $t = 0^+$ and note:

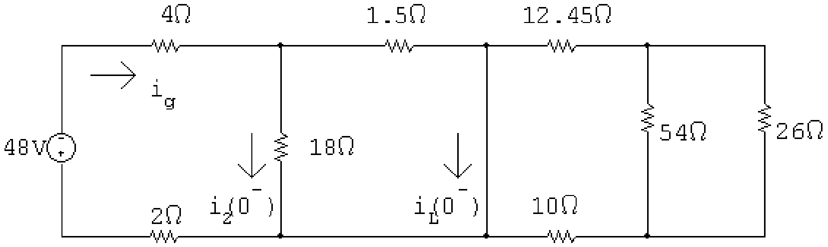
$$20(0.16) + v_L(0^+) = 0; \qquad \therefore \quad v_L(0^+) = -3.2\,\text{V}$$

[j] $v_L(\infty) = 0,$ since the current in the inductor is a constant at $t = \infty$.

[k] $v_L(t) = 0 + (-3.2 - 0)e^{-200t} = -3.2e^{-200t}\,\text{V}, \qquad t \geq 0^+$

[l] $i_o(t) = i_{\text{a}} - i_L = 225 - 160e^{-200t}\,\text{mA}, \qquad t \geq 0^+$

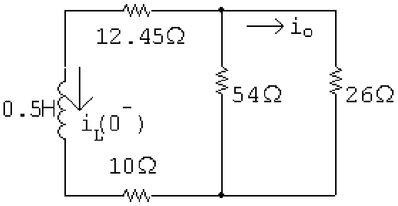
P 7.6 For $t < 0$



$$i_g = \frac{-48}{6 + (18\|1.5)} = -6.5\,\text{A}$$

$$i_L(0^-) = \frac{18}{18 + 1.5}(-6.5) = -6\,\text{A} = i_L(0^+)$$

For $t > 0$



$$i_L(t) = i_L(0^+)e^{-t/\tau}\,\text{A}, \qquad t \geq 0$$

$$\tau = \frac{L}{R} = \frac{0.5}{10 + 12.45 + (54\|26)} = 0.0125\,\text{s}; \qquad \frac{1}{\tau} = 80$$

$$i_L(t) = -6e^{-80t}\,\text{A}, \qquad t \geq 0$$

$$i_o(t) = \frac{54}{80}(-i_L(t)) = \frac{54}{80}(6e^{-80t}) = 4.05e^{-80t}\,\text{V}, \qquad t \geq 0^+$$

P 7.7 **[a]** $i(0) = \dfrac{24}{12} = 2\,\text{A}$

[b] $\tau = \dfrac{L}{R} = \dfrac{1.6}{80} = 20\,\text{ms}$

[c] $i = 2e^{-50t}\,\text{A}, \qquad t \geq 0$

$$v_1 = L\frac{d}{dt}(2e^{-50t}) = -160e^{-50t}\,\text{V} \qquad t \geq 0^+$$

$$v_2 = -72i = -144e^{-50t}\,\text{V} \qquad t \geq 0$$

[d] $w(0) = \dfrac{1}{2}(1.6)(2)^2 = 3.2\,\text{J}$

$$w_{72\Omega} = \int_0^t 72(4e^{-100x})\,dx = 288\frac{e^{-100x}}{-100}\Big|_0^t = 2.88(1 - e^{-100t})\,\text{J}$$

$$w_{72\Omega}(15\,\text{ms}) = 2.88(1 - e^{-1.5}) = 2.24\,\text{J}$$

$$\%\ \text{dissipated} = \frac{2.24}{3.2}(100) = 69.92\%$$

P 7.8 $w(0) = \dfrac{1}{2}(10 \times 10^{-3})(5)^2 = 125\,\text{mJ}$

$$0.9w(0) = 112.5\,\text{mJ}$$

$$w(t) = \frac{1}{2}(10 \times 10^{-3})i(t)^2, \qquad i(t) = 5e^{-t/\tau}\,\text{A}$$

$$\therefore\ w(t) = 0.005(25e^{-2t/\tau}) = 125e^{-2t/\tau})\,\text{mJ}$$

$$w(10\,\mu\text{s}) = 125e^{-20\times10^{-6}/\tau}\,\text{mJ}$$

$$\therefore\ 125e^{-20\times10^{-6}/\tau} = 112.5 \qquad \text{so} \qquad e^{20\times10^{-6}/\tau} = \frac{10}{9}$$

$$\tau = \frac{20 \times 10^{-6}}{\ln(10/9)} = \frac{L}{R}$$

$$R = \frac{10 \times 10^{-3}\ln(10/9)}{20 \times 10^{-6}} = 52.68\,\Omega$$

P 7.9 **[a]** $w(0) = \frac{1}{2}LI_g^2$

$$w_{\text{diss}} = \int_0^{t_o} I_g^2 Re^{-2t/\tau}\,dt = I_g^2 R \frac{e^{-2t/\tau}}{(-2/\tau)}\Big|_0^{t_o}$$

$$= \frac{1}{2}I_g^2 R\tau(1 - e^{-2t_o/\tau}) = \frac{1}{2}I_g^2 L(1 - e^{-2t_o/\tau})$$

$$w_{\text{diss}} = \sigma w(0)$$

$$\therefore \quad \frac{1}{2}LI_g^2(1 - e^{-2t_o/\tau}) = \sigma\left(\frac{1}{2}LI_g^2\right)$$

$$1 - e^{-2t_o/\tau} = \sigma; \qquad e^{2t_o/\tau} = \frac{1}{(1-\sigma)}$$

$$\frac{2t_o}{\tau} = \ln\left[\frac{1}{(1-\sigma)}\right]; \qquad \frac{R(2t_o)}{L} = \ln[1/(1-\sigma)]$$

$$R = \frac{L\,\ln[1/(1-\sigma)]}{2t_o}$$

[b] $R = \frac{(10 \times 10^{-3})\ln[1/0.9]}{20 \times 10^{-6}}$

$$R = 52.68\,\Omega$$

P 7.10 **[a]** $v_o(t) = v_o(0^+)e^{-t/\tau}$

$$\therefore \quad v_o(0^+)e^{-10^{-3}/\tau} = 0.5v_o(0^+)$$

$$\therefore \quad e^{10^{-3}/\tau} = 2$$

$$\therefore \quad \tau = \frac{L}{R} = \frac{10^{-3}}{\ln 2}$$

$$\therefore \quad L = \frac{10 \times 10^{-3}}{\ln 2} = 14.43\,\text{mH}$$

[b] $v_o(0^+) = -10i_L(0^+) = -10(1/10)(30 \times 10^{-3}) = -30\,\text{mV}$

$$v_o(t) = -0.03e^{-t/\tau}\,\text{V}$$

$$p_{10\Omega} = \frac{v_o^2}{10} = 9 \times 10^{-5}e^{-2t/\tau}$$

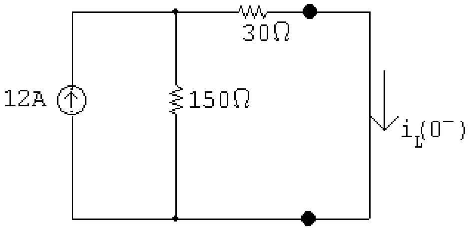
$$w_{10\Omega} = \int_0^{10^{-3}} 9 \times 10^{-5}e^{-2t/\tau}\,dt = 4.5\tau \times 10^{-5}(1 - e^{-2\times 10^{-3}/\tau})$$

$$\tau = \frac{1}{1000\ln 2} \qquad \therefore \qquad w_{10\Omega} = 48.69\,\text{nJ}$$

$$w_L(0) = \frac{1}{2}Li_L^2(0) = \frac{1}{2}(14.43\times10^{-3})(3\times10^{-3})^2 = 64.92\,\text{nJ}$$

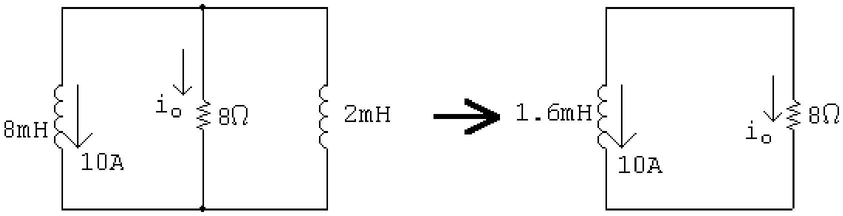
$$\%\text{ diss in 1 ms} = \frac{48.69}{64.92}\times 100 = 75\%$$

P 7.11 **[a]** $t < 0$



$$i_L(0^-) = \frac{150}{180}(12) = 10\,\text{A}$$

$t \geq 0$



$$\tau = \frac{1.6\times10^{-3}}{8} = 200\times10^{-6}; \qquad 1/\tau = 5000$$

$$i_o = -10e^{-5000t}\,\text{A} \quad t\geq 0$$

[b] $w_{\text{del}} = \frac{1}{2}(1.6\times10^{-3})(10)^2 = 80\,\text{mJ}$

[c] $0.95w_{\text{del}} = 76\,\text{mJ}$

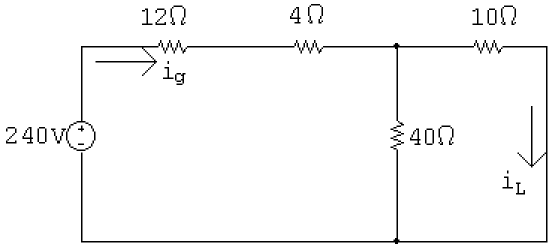
$$\therefore\quad 76\times10^{-3} = \int_0^{t_o} 8(100e^{-10{,}000t})\,dt$$

$$\therefore\quad 76\times10^{-3} = -80\times10^{-3}e^{-10{,}000t}\Big|_0^{t_o} = 80\times10^{-3}(1-e^{-10{,}000t_o})$$

$$\therefore\quad e^{-10{,}000t_o} = 0.05 \quad\text{so}\quad t_o = 299.57\,\mu\text{s}$$

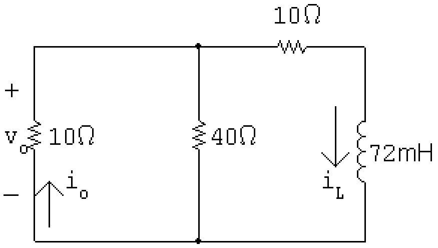
$$\therefore\quad \frac{t_o}{\tau} = \frac{299.57\times10^{-6}}{200\times10^{-6}} = 1.498 \qquad\text{so}\qquad t_o\approx 1.498\tau$$

P 7.12 $t < 0$:



$$i_L(0^+) = \frac{240}{16+8} = 10\,\text{A}; \qquad i_L(0^-) = 10\frac{40}{50} = 8\,\text{A}$$

$t > 0$:



$$R_e = \frac{(10)(40)}{50} + 10 = 18\,\Omega$$

$$\tau = \frac{L}{R_e} = \frac{72 \times 10^{-3}}{18} = 4\,\text{ms}; \qquad \frac{1}{\tau} = 250$$

$$\therefore \quad i_L = 8e^{-250t}\,\text{A}$$

$$v_o = 8i_o = 64e^{-250t}\,\text{V}, \quad t \geq 0^+$$

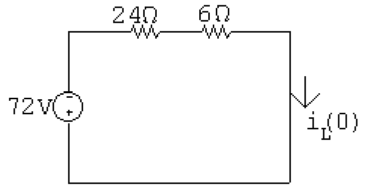
P 7.13 $$p_{40\Omega} = \frac{v_o^2}{40} = \frac{(64)^2}{40}e^{-500t} = 102.4e^{-500t}\,\text{W}$$

$$w_{40\Omega} = \int_0^\infty 102.4e^{-500t}\,dt = 102.4\frac{e^{-500t}}{-500}\bigg|_0^\infty = 204.8\,\text{mJ}$$

$$w(0) = \frac{1}{2}(72 \times 10^{-3})(8)^2 = 2304\,\text{mJ}$$

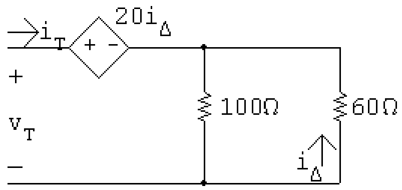
$$\%\text{ diss } = \frac{204.8}{2304}(100) = 8.89\%$$

P 7.14 **[a]** $t < 0:$



$$i_L(0) = -\frac{72}{24+6} = -2.4\,\text{A}$$

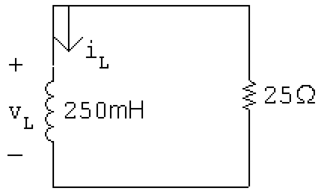
$t > 0$:



$$i_\Delta = -\frac{100}{160}i_T = -\frac{5}{8}i_T$$

$$v_T = 20i_\Delta + i_T\frac{(100)(60)}{160} = -12.5i_T + 37.5i_T$$

$$\frac{v_T}{i_T} = R_{\text{Th}} = -12.5 + 37.5 = 25\,\Omega$$



$$\tau = \frac{L}{R} = \frac{250\times10^{-3}}{25} \qquad \frac{1}{\tau} = 100$$

$$i_L = -2.4e^{-100t}\,\text{A}, \qquad t \geq 0$$

[b] $v_L = 250\times10^{-3}(240e^{-100t}) = 60e^{-100t}\,\text{V}, \quad t \geq 0^+$

[c] $i_\Delta = 0.625i_L = -1.5e^{-100t}\,\text{A} \qquad t \geq 0^+$

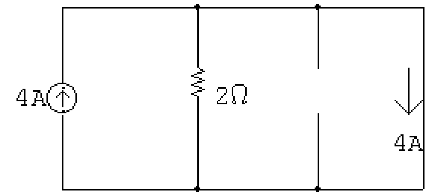
P 7.15 $w(0) = \frac{1}{2}(250\times10^{-3})(-2.4)^2 = 720\,\text{mJ}$

$$p_{60\Omega} = 60(-1.5e^{-100t})^2 = 135e^{-200t}\,\text{W}$$

$$w_{60\Omega} = \int_0^\infty 135e^{-200t}\,dt = 135\frac{e^{-200t}}{-200}\Big|_0^\infty = 675\,\text{mJ}$$

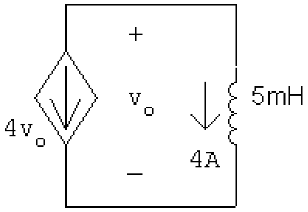
$$\%\text{ dissipated } = \frac{675}{720}(100) = 93.75\%$$

P 7.16 $t < 0$

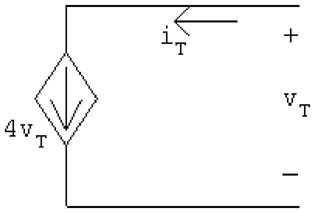


$$i_L(0^-) = i_L(0^+) = 4\,\mathrm{A}$$

$t > 0$

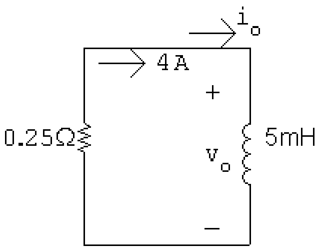


Find Thévenin resistance seen by inductor:



$$i_T = 4v_T; \qquad \frac{v_T}{i_T} = R_{\text{Th}} = \frac{1}{4} = 0.25\,\Omega$$

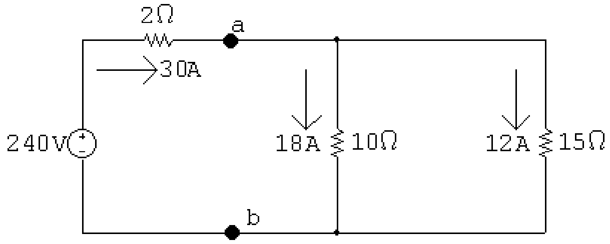
$$\tau = \frac{L}{R} = \frac{5 \times 10^{-3}}{0.25} = 20\,\mathrm{ms}; \qquad 1/\tau = 50$$



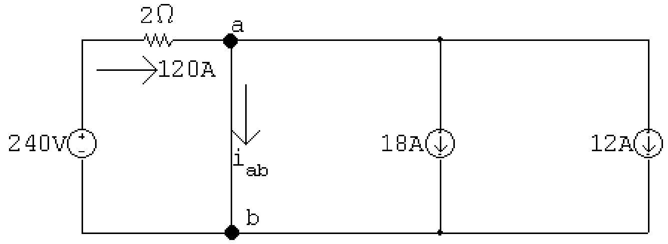
$$i_o = 4e^{-50t}\,\mathrm{A}, \qquad t \geq 0$$

$$v_o = L\frac{di_o}{dt} = (5 \times 10^{-3})(-200e^{-50t}) = -e^{-50t}\,\mathrm{V}, \quad t \geq 0^+$$

P 7.17 **[a]** $t < 0:$

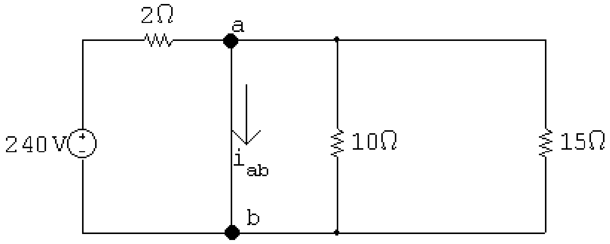


$t = 0^+$:

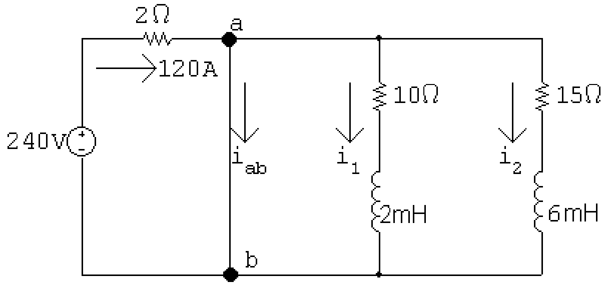


$$120 = i_{\text{ab}} + 18 + 12, \qquad i_{\text{ab}} = 90\,\text{A}, \quad t = 0^+$$

[b] At $t = \infty$:



$$i_{\text{ab}} = 240/2 = 120\,\text{A}, \quad t = \infty$$



[c] $i_1(0) = 18, \qquad \tau_1 = \dfrac{2 \times 10^{-3}}{10} = 0.2\,\text{ms}$

$$i_2(0) = 12, \qquad \tau_2 = \frac{6 \times 10^{-3}}{15} = 0.4\,\text{ms}$$

$$i_1(t) = 18e^{-5000t}\,\text{A}, \quad t \geq 0$$

$$i_2(t) = 12e^{-2500t}\,\text{A}, \quad t \geq 0$$

$$i_{ab} = 120 - 18e^{-5000t} - 12e^{-2500t}\,\text{A}, \quad t \geq 0$$

$$120 - 18e^{-5000t} - 12e^{-2500t} = 114$$

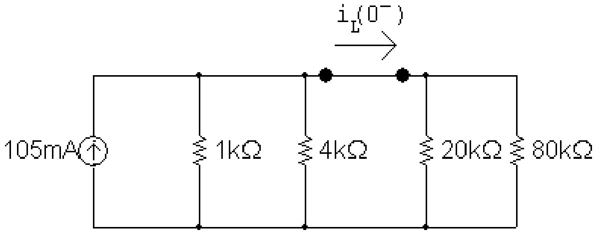
$$6 = 18e^{-5000t} + 12e^{-2500t}$$

Let $x = e^{-2500t}$ so $6 = 18x^2 + 12x$

Solving $x = \frac{1}{3} = e^{-2500t}$

$\therefore \quad e^{2500t} = 3$ and $t = \frac{\ln 3}{2500} = 439.44\,\mu\text{s}$

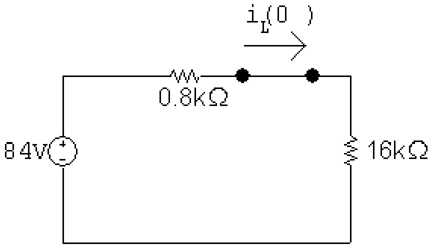
P 7.18 **[a]** $t < 0$



$$1\,\text{k}\Omega \| 4\,\text{k}\Omega = 0.8\,\text{k}\Omega$$

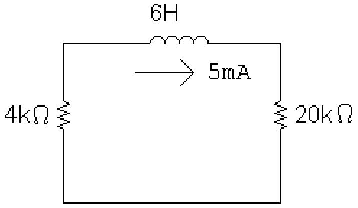
$$20\,\text{k}\Omega \| 80\,\text{k}\Omega = 16\,\text{k}\Omega$$

$$(105 \times 10^{-3})(0.8 \times 10^{3}) = 84\,\text{V}$$



$$i_L(0^-) = \frac{84}{16{,}800} = 5\,\text{mA}$$

$t > 0$



$$\tau = \frac{L}{R} = \frac{6}{24} \times 10^{-3} = 250\,\mu\text{s}; \qquad \frac{1}{\tau} = 4000$$

$$i_L(t) = 5e^{-4000t}\,\text{mA}, \qquad t \geq 0$$

$$p_{4k} = 25 \times 10^{-6} e^{-8000t}(4000) = 0.10e^{-8000t}\,\text{W}$$

$$w_{\text{diss}} = \int_0^t 0.10e^{-8000x}\,dx = 12.5 \times 10^{-6}[1 - e^{-8000t}]\,\text{J}$$

$$w(0) = \frac{1}{2}(6)(25 \times 10^{-6}) = 75\,\mu\text{J}$$

$$0.10w(0) = 7.5\,\mu\text{J}$$

$$12.5(1 - e^{-8000t}) = 7.5; \qquad \therefore \quad e^{8000t} = 2.5$$

$$t = \frac{\ln 2.5}{8000} = 114.54\,\mu\text{s}$$

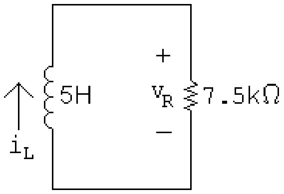
[b] $w_{\text{diss}}(\text{total}) = 75(1 - e^{-8000t})\,\mu\text{J}$

$$w_{\text{diss}}(114.54\,\mu\text{s}) = 45\,\mu\text{J}$$

$$\% = (45/75)(100) = 60\%$$

P 7.19 **[a]** $t > 0$:

$$L_{\text{eq}} = 1.25 + \frac{60}{16} = 5\,\text{H}$$



$$i_L(t) = i_L(0)e^{-t/\tau}\,\text{mA}; \qquad i_L(0) = 2\,\text{A}; \qquad \frac{1}{\tau} = \frac{R}{L} = \frac{7500}{5} = 1500$$

$$i_L(t) = 2e^{-1500t}\,\text{A}, \qquad t \geq 0$$

$$v_R(t) = Ri_L(t) = (7500)(2e^{-1500t}) = 15{,}000e^{-1500t}\,\text{V}, \qquad t \geq 0^+$$

$$v_o = -3.75\frac{di_L}{dt} = 11{,}250e^{-1500t}\,\text{V}, \qquad t \geq 0^+$$

[b] $i_o = \dfrac{-1}{6}\displaystyle\int_0^t 11{,}250e^{-1500x}\,dx + 0 = 1.25e^{-1500t} - 1.25\,\text{A}$

P 7.20 **[a]** From the solution to Problem 7.19,

$$w(0) = \frac{1}{2}L_{\text{eq}}[i_L(0)]^2 = \frac{1}{2}(5)(2)^2 = 10\,\text{J}$$

[b] $w_{\text{trapped}} = \dfrac{1}{2}(10)(1.25)^2 + \dfrac{1}{2}(6)(1.25)^2 = 12.5\,\text{J}$

P 7.21 **[a]** $R = \dfrac{v}{i} = 8\,\text{k}\Omega$

[b] $\dfrac{1}{\tau} = \dfrac{1}{RC} = 500; \qquad C = \dfrac{1}{(500)(8000)} = 0.25\,\mu\text{F}$

[c] $\tau = \dfrac{1}{500} = 2\,\text{ms}$

[d] $w(0) = \dfrac{1}{2}(0.25 \times 10^{-6})(72)^2 = 648\,\mu\text{J}$

[e] $w_{\text{diss}} = \displaystyle\int_0^{t_o} \frac{(72)^2 e^{-1000t}}{(800)} dt$

$$= 0.648 \frac{e^{-1000t}}{-1000} \Big|_0^{t_o} = 648(1 - e^{-1000t_o})\,\mu\text{J}$$

$\%\text{diss} = 100(1 - e^{-1000t_o}) = 68 \qquad \text{so} \qquad e^{1000t_o} = 3.125$

$\therefore \quad t = \dfrac{\ln 3.125}{1000} = 1139\,\mu\text{s}$

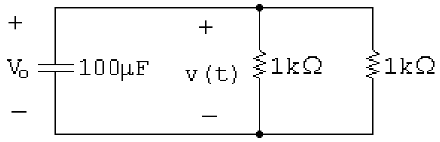
P 7.22 **[a]** Note that there are many different possible correct solutions to this problem.

$R = \dfrac{\tau}{C}$

Choose a $100\,\mu$F capacitor from Appendix H. Then,

$R = \dfrac{0.05}{100 \times 10^{-6}} = 500\,\Omega$

Construct a $500\,\Omega$ resistor by combining two $1\,\text{k}\Omega$ resistors in parallel:



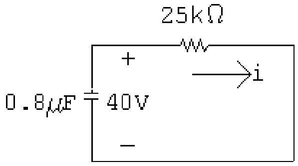
[b] $v(t) = V_o e^{-t/\tau} = 50e^{-20t}\,\text{V}, \qquad t \geq 0$

[c] $50e^{-20t} = 10 \qquad \text{so} \qquad e^{20t} = 5$

$\therefore \quad t = \dfrac{\ln 5}{20} = 80.47\,\text{ms}$

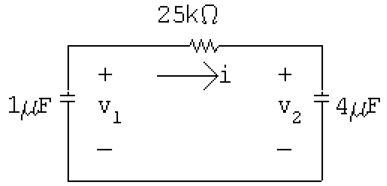
P 7.23 **[a]** $v_1(0^-) = v_1(0^+) = 40\,\text{V} \qquad\qquad v_2(0^+) = 0$

$C_{\text{eq}} = (1)(4)/5 = 0.8\,\mu\text{F}$



$\tau = (25 \times 10^3)(0.8 \times 10^{-6}) = 20\text{ms}; \qquad \dfrac{1}{\tau} = 50$

$$i = \frac{40}{25{,}000}e^{-50t} = 1.6e^{-50t}\,\text{mA}, \qquad t \geq 0^+$$



$$v_1 = \frac{-1}{10^{-6}}\int_0^t 1.6 \times 10^{-3}e^{-50x}\,dx + 40 = 32e^{-50t} + 8\,\text{V}, \qquad t \geq 0$$

$$v_2 = \frac{1}{4 \times 10^{-6}}\int_0^t 1.6 \times 10^{-3}e^{-50x}\,dx + 0 = -8e^{-50t} + 8\,\text{V}, \qquad t \geq 0$$

[b] $w(0) = \frac{1}{2}(10^{-6})(40)^2 = 800\,\mu\text{J}$

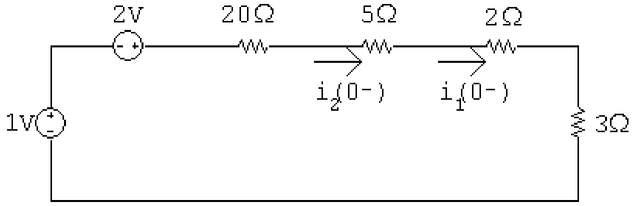
[c] $w_{\text{trapped}} = \frac{1}{2}(10^{-6})(8)^2 + \frac{1}{2}(4 \times 10^{-6})(8)^2 = 160\,\mu\text{J}.$

The energy dissipated by the 25 kΩ resistor is equal to the energy dissipated by the two capacitors; it is easier to calculate the energy dissipated by the capacitors:

$$w_{\text{diss}} = \frac{1}{2}(0.8 \times 10^{-6})(40)^2 = 640\,\mu\text{J}.$$

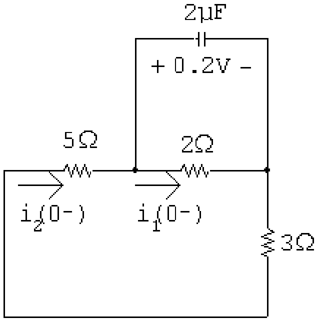
Check: $w_{\text{trapped}} + w_{\text{diss}} = 160 + 640 = 800\,\mu\text{J}; \qquad w(0) = 800\,\mu\text{J}.$

P 7.24 **[a]** $t < 0$:



$$i_1(0^-) = i_2(0^-) = \frac{3}{30} = 100\,\text{mA}$$

[b] $t > 0$:



$$i_1(0^+) = \frac{0.2}{2} = 100\,\text{mA}$$

$$i_2(0^+) = \frac{-0.2}{8} = -25\,\text{mA}$$

[c] Capacitor voltage cannot change instantaneously, therefore,

$$i_1(0^-) = i_1(0^+) = 100\,\text{mA}$$

[d] Switching can cause an instantaneous change in the current in a resistive branch. In this circuit

$$i_2(0^-) = 100\,\text{mA} \quad \text{and} \quad i_2(0^+) = 25\,\text{mA}$$

[e] $v_c = 0.2e^{-t/\tau}\,\text{V}, \qquad t \geq 0$

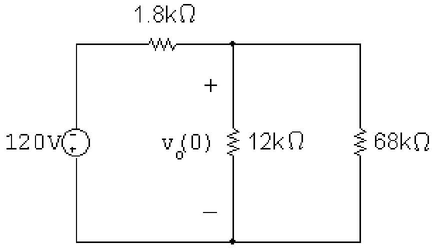
$$\tau = R_eC = 1.6(2 \times 10^{-6}) = 3.2\,\mu\text{s}; \qquad \frac{1}{\tau} = 312{,}500$$

$$v_c = 0.2e^{-312{,}000t}\,\text{V}, \qquad t \geq 0$$

$$i_1 = \frac{v_c}{2} = 0.1e^{-312{,}000t}\,\text{A}, \qquad t \geq 0$$

[f] $i_2 = \dfrac{-v_c}{8} = -25e^{-312{,}000t}\,\text{mA}, \qquad t \geq 0^+$

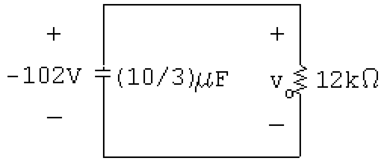
P 7.25 **[a]** $t < 0$:



$$R_{\text{eq}} = 12\,\text{k}\|8\,\text{k} = 10.2\,\text{k}\Omega$$

$$v_o(0) = \frac{10{,}200}{10{,}200 + 1800}(-120) = -102\,\text{V}$$

$t > 0$:



$$\tau = [(10/3) \times 10^{-6})(12{,}000) = 40\,\text{ms}; \qquad \frac{1}{\tau} = 25$$

$$v_o = -102e^{-25t}\,\text{V}, \quad t \geq 0$$

$$p = \frac{v_o^2}{12{,}000} = 867 \times 10^{-3}e^{-50t}\,\text{W}$$

$$w_{\text{diss}} = \int_0^{12\times10^{-3}} 867 \times 10^{-3} e^{-50t}\,dt$$
$$= 17.34 \times 10^{-3}(1 - e^{-50(12\times10^{-3})}) = 7824\,\mu\text{J}$$

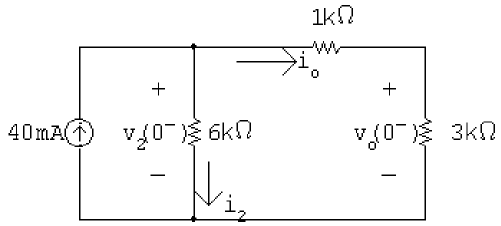
[b] $w(0) = \left(\frac{1}{2}\right)\left(\frac{10}{3}\right)(102)^2 \times 10^{-6} = 17.34\,\text{mJ}$

$0.75w(0) = 13\,\text{mJ}$

$$\int_0^{t_o} 867 \times 10^{-3} e^{-50x}\,dx = 13 \times 10^{-3}$$

$\therefore\ 1 - e^{-50t_o} = 0.75; \qquad e^{50t_o} = 4; \quad \text{so} \quad t_o = 27.73\,\text{ms}$

P 7.26 **[a]** $t < 0$:



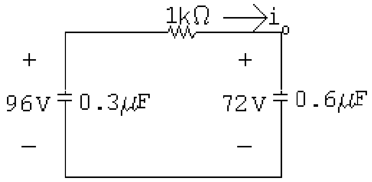
$$i_o(0^-) = \frac{6000}{6000 + 4000}(40\,\text{m}) = 24\,\text{mA}$$

$v_o(0^-) = (3000)(24\,\text{m}) = 72\,\text{V}$

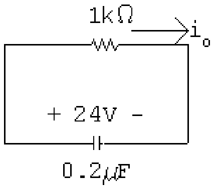
$i_2(0^-) = 40 - 24 = 16\,\text{mA}$

$v_2(0^-) = (6000)(16\,\text{m}) = 96\,\text{V}$

$t > 0$

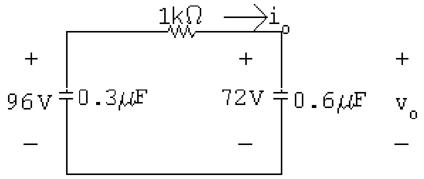


$\tau = RC = (1000)(0.2 \times 10^{-6}) = 200\,\mu\text{s}; \qquad \frac{1}{\tau} = 5000$



$$i_o(t) = \frac{24}{1 \times 10^3} e^{-t/\tau} = 24e^{-5000t}\,\text{mA}, \qquad t \geq 0^+$$

[b]



$$\begin{aligned} v_o &= \frac{1}{0.6\times 10^{-6}}\int_0^t 24\times 10^{-3}e^{-5000x}\,dx + 72 \\ &= (40{,}000)\frac{e^{-5000x}}{-5000}\Big|_0^t + 72 \\ &= -8e^{-5000t} + 8 + 72 \\ v_o &= [-8e^{-5000t} + 80]\,\mathrm{V}, \qquad t \geq 0 \end{aligned}$$

[c] $w_{\text{trapped}} = (1/2)(0.3\times 10^{-6})(80)^2 + (1/2)(0.6\times 10^{-6})(80)^2$

$w_{\text{trapped}} = 2880\,\mu\mathrm{J}.$

Check:

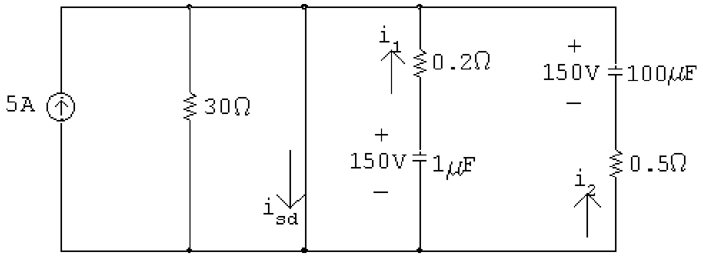
$$w_{\text{diss}} = \frac{1}{2}(0.2\times 10^{-6})(24)^2 = 57.6\,\mu\mathrm{J}$$

$$w(0) = \frac{1}{2}(0.3\times 10^{-6})(96)^2 + \frac{1}{2}(0.6\times 10^{-6})(72)^2 = 2937.6\,\mu\mathrm{J}.$$

$$w_{\text{trapped}} + w_{\text{diss}} = w(0)$$

$2880 + 57.6 = 2937.6$ OK.

P 7.27 [a] At $t = 0^-$ the voltage on each capacitor will be 150 V(5×30), positive at the upper terminal. Hence at $t \geq 0^+$ we have



$$\therefore \quad i_{sd}(0^+) = 5 + \frac{150}{0.2} + \frac{150}{0.5} = 1055\,\mathrm{A}$$

At $t = \infty$, both capacitors will have completely discharged.

$$\therefore \quad i_{sd}(\infty) = 5\,\mathrm{A}$$

[b] $i_{sd}(t) = 5 + i_1(t) + i_2(t)$

$$\tau_1 = 0.2(10^{-6}) = 0.2\,\mu\text{s}$$

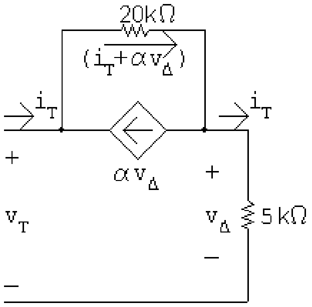
$$\tau_2 = 0.5(100 \times 10^{-6}) = 50\,\mu\text{s}$$

$$\therefore \quad i_1(t) = 750e^{-5\times10^6 t}\,\text{A}, \qquad t \geq 0^+$$

$$i_2(t) = 300e^{-20{,}000t}\,\text{A}, \qquad t \geq 0$$

$$\therefore \quad i_{sd} = 5 + 750e^{-5\times10^6 t} + 300e^{-20{,}000t}\,\text{A}, \qquad t \geq 0^+$$

P 7.28 **[a]**



$$v_T = 20 \times 10^3(i_T + \alpha v_\Delta) + 5 \times 10^3 i_T$$

$$v_\Delta = 5 \times 10^3 i_T$$

$$v_T = 25 \times 10^3 i_T + 20 \times 10^3 \alpha(5 \times 10^3 i_T)$$

$$R_{\text{Th}} = 25{,}000 + 100 \times 10^6 \alpha$$

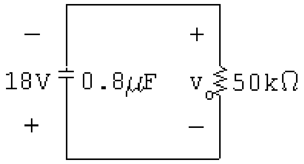
$$\tau = R_{\text{Th}}C = 40 \times 10^{-3} = R_{\text{Th}}(0.8 \times 10^{-6})$$

$$R_{\text{Th}} = 50\,\text{k}\Omega = 25{,}000 + 100 \times 10^6 \alpha$$

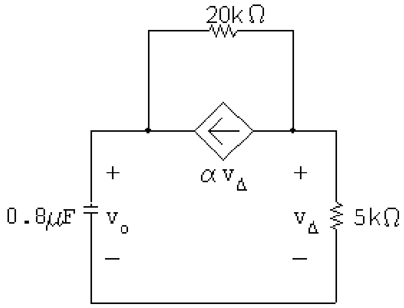
$$\alpha = \frac{25{,}000}{100 \times 10^6} = 2.5 \times 10^{-4}\,\text{A/V}$$

[b] $v_o(0) = (-5 \times 10^{-3})(3600) = -18\,\text{V} \qquad t < 0$

$t > 0$:



$$v_o = -18e^{-25t}\,\text{V}, \quad t \geq 0$$

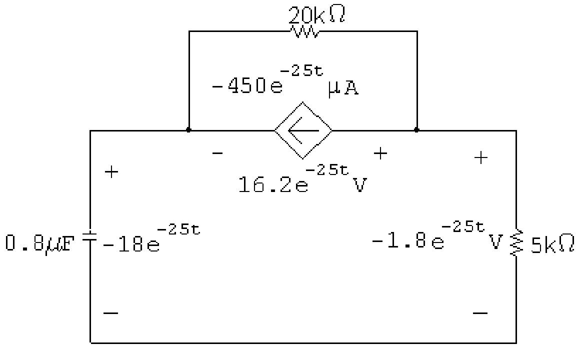


$$\frac{v_\Delta}{5000} + \frac{v_\Delta - v_o}{20{,}000} + 2.5 \times 10^{-4} v_\Delta = 0$$

$$4v_\Delta + v_\Delta - v_o + 5v_\Delta = 0$$

$$\therefore \quad v_\Delta = \frac{v_o}{10} = -1.8e^{-25t}\,\text{V}, \quad t \geq 0^+$$

P 7.29 **[a]**



$$p_{ds} = (16.2e^{-25t})(-450 \times 10^{-6}e^{-25t}) = -7290 \times 10^{-6}e^{-50t}\,\text{W}$$

$$w_{ds} = \int_0^\infty p_{ds}\,dt = -145.8\,\mu\text{J}.$$

$\therefore$ dependent source is delivering 145.8 μJ.

[b] $w_{5\text{k}} = \int_0^\infty (5000)(0.36 \times 10^{-3}e^{-25t})^2\,dt = 648 \times 10^{-6}\int_0^\infty e^{-50t}\,dt = 12.96\,\mu\text{J}$

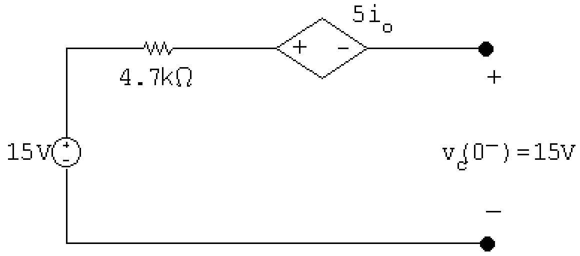
$$w_{20\text{k}} = \int_0^\infty \frac{(16.2e^{-25t})^2}{20{,}000}\,dt = 13{,}122 \times 10^{-6}\int_0^\infty e^{-50t}\,dt = 262.44\,\mu\text{J}$$

$$w_c(0) = \frac{1}{2}(0.8 \times 10^{-6})(18)^2 = 129.6\,\mu\text{J}$$

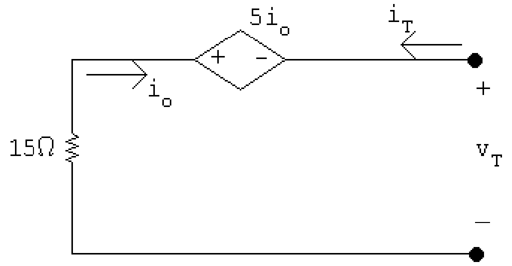
$$\sum w_{\text{diss}} = 12.96 + 262.44 = 275.4\,\mu\text{J}$$

$$\sum w_{\text{dev}} = 145.8 + 129.6 = 275.4\,\mu\text{J}.$$

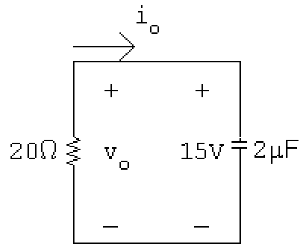
P 7.30 $t < 0$



$t > 0$



$$v_T = -5i_o - 15i_o = -20i_o = 20i_T \quad \therefore \quad R_{\text{Th}} = \frac{v_T}{i_T} = 20\,\Omega$$

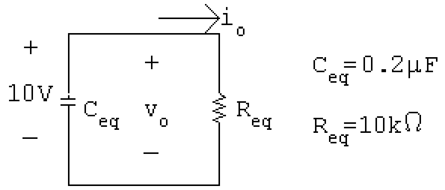


$$\tau = RC = 40\,\mu\text{s}; \qquad \frac{1}{\tau} = 25{,}000$$

$$v_o = 15e^{-25{,}000t}\,\text{V}, \qquad t \geq 0$$

$$i_o = -\frac{v_o}{20} = -0.75e^{-25{,}000t}\,\text{A}, \qquad t \geq 0^+$$

P 7.31 **[a]** The equivalent circuit for $t > 0$:



$$\tau = 2\,\text{ms}; \qquad 1/\tau = 500$$

$$v_o = 10e^{-500t}\,\text{V}, \qquad t \geq 0$$

$$i_o = e^{-500t}\,\text{mA}, \qquad t \geq 0^+$$

$$i_{24\text{k}\Omega} = e^{-500t}\left(\frac{16}{40}\right) = 0.4e^{-500t}\,\text{mA}, \qquad t \geq 0^+$$

$$p_{24\text{k}\Omega} = (0.16 \times 10^{-6}e^{-1000t})(24{,}000) = 3.84e^{-1000t}\,\text{mW}$$

$$w_{24\text{k}\Omega} = \int_0^\infty 3.84 \times 10^{-3}e^{-1000t}\,dt = -3.84 \times 10^{-6}(0-1) = 3.84\,\mu\text{J}$$

$$w(0) = \frac{1}{2}(0.25 \times 10^{-6})(40)^2 + \frac{1}{2}(1 \times 10^{-6})(50)^2 = 1.45\,\text{mJ}$$

$$\%\ \text{diss}\,(24\,\text{k}\Omega) = \frac{3.84 \times 10^{-6}}{1.45 \times 10^{-3}} \times 100 = 0.26\%$$

[b] $p_{400\Omega} = 400(1 \times 10^{-3}e^{-500t})^2 = 0.4 \times 10^{-3}e^{-1000t}$

$$w_{400\Omega} = \int_0^\infty p_{400}\,dt = 0.40\,\mu\text{J}$$

$$\%\ \text{diss}\,(400\,\Omega) = \frac{0.4 \times 10^{-6}}{1.45 \times 10^{-3}} \times 100 = 0.03\%$$

$$i_{16k\Omega} = e^{-500t}\left(\frac{24}{40}\right) = 0.6e^{-500t}\,\text{mA}, \quad t \geq 0^+$$

$$p_{16k\Omega} = (0.6 \times 10^{-3}e^{-500t})^2(16{,}000) = 5.76 \times 10^{-3}e^{-1000t}\,\text{W}$$

$$w_{16k\Omega} = \int_0^\infty 5.76 \times 10^{-3}e^{-1000t}\,dt = 5.76\,\mu\text{J}$$

$$\%\ \text{diss}\,(16\,\text{k}\Omega) = 0.4\%$$

[c] $\sum w_{\text{diss}} = 3.84 + 5.76 + 0.4 = 10\,\mu\text{J}$

$$w_{\text{trapped}} = w(0) - \sum w_{\text{diss}} = 1.45 \times 10^{-3} - 10 \times 10^{-6} = 1.44\,\text{mJ}$$

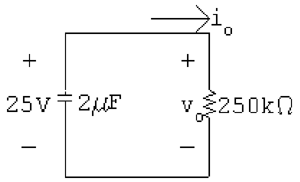
$$\%\ \text{trapped} = \frac{1.44}{1.45} \times 100 = 99.31\%$$

Check: $0.26 + 0.03 + 0.4 + 99.31 = 100\%$

P 7.32 **[a]** $C_e = \dfrac{(2+1)6}{2+1+6} = 2\,\mu\text{F}$

$$v_o(0) = -5 + 30 = 25\,\text{V}$$

$$\tau = (2 \times 10^{-6})(250 \times 10^3) = 0.5\,\text{s}; \qquad \frac{1}{\tau} = 2$$



$$v_o = 25e^{-2t}\,\text{V}, \qquad t \geq 0^+$$

[b] $w_o = \frac{1}{2}(3\times 10^{-6})(30)^2 + \frac{1}{2}(6\times 10^{-6})(5)^2 = 1425\,\mu\text{J}$

$$w_{\text{diss}} = \frac{1}{2}(2\times 10^{-6})(25)^2 = 625\,\mu\text{J}$$

$$\%\text{ diss } = \frac{625}{1425}\times 100 = 43.86\%$$

[c] $i_o = \frac{v_o}{250\times 10^{-3}} = 100e^{-2t}\,\mu\text{A}$

$$v_1 = -\frac{1}{6\times 10^{-6}}\int_0^t 100\times 10^{-6}e^{-2x}\,dx - 5 = -16.67\int_0^t e^{-2x}\,dx - 5$$

$$= -16.67\frac{e^{-2x}}{-2}\Big|_0^t -5 = 8.33e^{-2t} - 13.33\,\text{V} \qquad t\geq 0$$

[d] $v_1 + v_2 = v_o$

$$v_2 = v_o - v_1 = 25e^{-2t} - 8.33e^{-2t} + 13.33 = 16.67e^{-2t} + 13.33\,\text{V} \quad t\geq 0$$

[e] $w_{\text{trapped}} = \frac{1}{2}(6\times 10^{-6})(13.33)^2 + \frac{1}{2}(3\times 10^{-6})(13.33)^2 = 800\,\mu\text{J}$

$$w_{\text{diss}} + w_{\text{trapped}} = 625 + 800 = 1425\,\mu\text{J} \qquad \text{(check)}$$

P 7.33 **[a]** From Eqs. (7.35) and (7.42)

$$i = \frac{V_s}{R} + \left(I_o - \frac{V_s}{R}\right)e^{-(R/L)t}$$

$$v = (V_s - I_oR)e^{-(R/L)t}$$

$$\therefore \quad \frac{V_s}{R} = 4; \qquad I_o - \frac{V_s}{R} = 4$$

$$V_s - I_oR = -80; \qquad \frac{R}{L} = 40$$

$$\therefore \quad I_o = 4 + \frac{V_s}{R} = 8\,\text{A}$$

Now since $V_s = 4R$ we have

$$4R - 8R = -80; \qquad R = 20\,\Omega$$

$$V_s = 80\,\text{V}; \qquad L = \frac{R}{40} = 0.5\,\text{H}$$

[**b**] $i = 4 + 4e^{-40t}; \qquad i^2 = 16 + 32e^{-40t} + 16e^{-80t}$

$$w = \frac{1}{2}Li^2 = \frac{1}{2}(0.5)[16 + 32e^{-40t} + 16e^{-80t}] = 4 + 8e^{-40t} + 4e^{-80t}$$

$$\therefore \quad 4 + 8e^{-40t} + 4e^{-80t} = 9 \quad \text{or} \quad e^{-80t} + 2e^{-40t} - 1.25 = 0$$

Let $x = e^{-40t}$:

$x^2 + 2x - 1.25 = 0$; Solving, $x = 0.5$; $x = -2.5$

But $x \geq 0$ for all t. Thus,

$e^{-40t} = 0.5; \qquad e^{40t} = 2; \qquad t = 25 \ln 2 = 17.33\,\text{ms}$

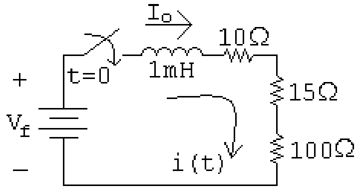
P 7.34 [**a**] Note that there are many different possible solutions to this problem.

$$R = \frac{L}{\tau}$$

Choose a 1 mH inductor from Appendix H. Then,

$$R = \frac{0.001}{8 \times 10^{-6}} = 125\,\Omega$$

Construct the resistance needed by combining $100\,\Omega$, $10\,\Omega$, and $15\,\Omega$ resistors in series:



[**b**] $i(t) = I_f + (I_o - I_f)e^{-t/\tau}$

$$I_o = 0\,\text{A}; \qquad I_f = \frac{V_f}{R} = \frac{25}{125} = 200\,\text{mA}$$

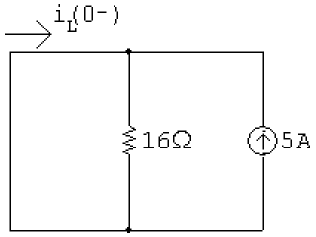
$$\therefore \quad i(t) = 200 + (0 - 200)e^{-125{,}000t}\,\text{mA} = 200 - 200e^{-125{,}000t}\,\text{mA}, \qquad t \geq 0$$

[**c**] $i(t) = 0.2 - 0.2e^{-125{,}000t} = (0.75)(0.2) = 0.15$

$$e^{-125{,}000t} = 0.25 \qquad \text{so} \qquad e^{125{,}000t} = 4$$

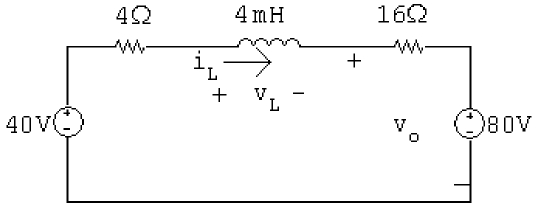
$$\therefore \quad t = \frac{\ln 4}{125{,}000} = 11.09\,\mu\text{s}$$

P 7.35 **[a]** $t < 0$



$$i_L(0^-) = -5\,\text{A}$$

$t > 0$



$$i_L(\infty) = \frac{40-80}{4+16} = -2\,\text{A}$$

$$\tau = \frac{L}{R} = \frac{4\times 10^{-3}}{4+16} = 200\,\mu\text{s}; \qquad \frac{1}{\tau} = 5000$$

$$i_L = i_L(\infty) + [i_L(0^+) - i_L(\infty)]e^{-t/\tau}$$

$$= -2 + (-5+2)e^{-5000t} = -2 - 3e^{-5000t}\,\text{A}, \qquad t \geq 0$$

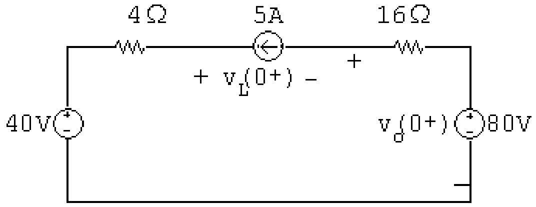
$$v_o = 16i_L + 80 = 16(-2 - 3e^{-5000t}) + 80 = 48 - 48e^{-5000t}\,\text{V}, \qquad t \geq 0$$

[b] $v_L = L\dfrac{di_L}{dt} = 4\times 10^{-3}(-5000)[-3e^{-5000t}] = 60e^{-5000t}\,\text{V}, \qquad t \geq 0^+$

$$v_L(0^+) = 60\,\text{V}$$

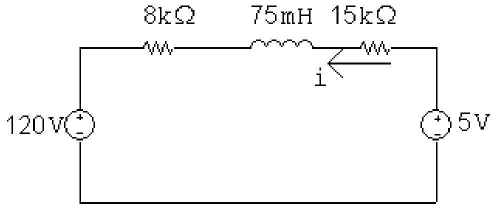
From part (a) $\quad v_o(0^+) = 0\,\text{V}$

Check: at $t = 0^+$ the circuit is:



$$v_L(0^+) = 40 + (5\,\text{A})(4\,\Omega) = 60\,\text{V}, \qquad v_o(0^+) = 80 - (16\,\Omega)(5\,\text{A}) = 0\,\text{V}$$

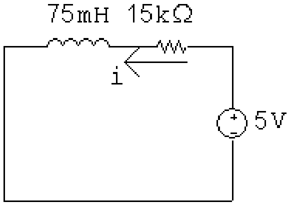
P 7.36 [**a**] For $t < 0$, calculate the Thévenin equivalent for the circuit to the left and right of the 75 mH inductor. We get



$$i(0^-) = \frac{5 - 120}{15\,\text{k} + 8\,\text{k}} = -5\,\text{mA}$$

$$i(0^-) = i(0^+) = -5\,\text{mA}$$

[**b**] For $t > 0$, the circuit reduces to



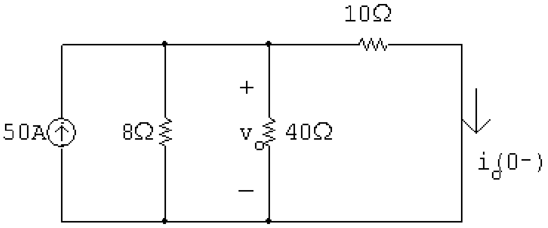
Therefore $i(\infty) = 5/15{,}000 = 0.333\,\text{mA}$

[**c**] $\tau = \dfrac{L}{R} = \dfrac{75 \times 10^{-3}}{15{,}000} = 5\,\mu\text{s}$

[**d**] $i(t) = i(\infty) + [i(0^+) - i(\infty)]e^{-t/\tau}$

$$= 0.333 + [-5 - 0.333]e^{-200{,}000t} = 0.333 - 5.333e^{-200{,}000t}\,\text{mA}, \qquad t \geq 0$$

P 7.37 [**a**] $t < 0$



KVL equation at the top node:

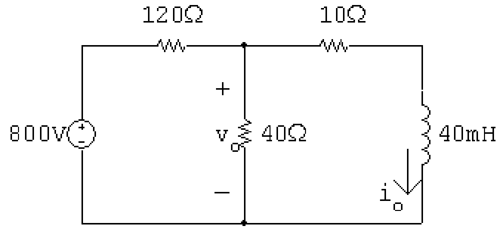
$$50 = \frac{v_o}{8} + \frac{v_o}{40} + \frac{v_o}{10}$$

Multiply by 40 and solve:

$$2000 = (5 + 1 + 4)v_o; \qquad v_o = 200\,\text{V}$$

$$\therefore \quad i_o(0^-) = \frac{v_o}{10} = 200/10 = 20\,\text{A}$$

$t > 0$



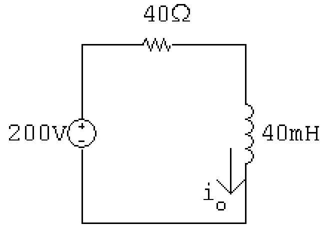
Use voltage division to find the Thévenin voltage:

$$V_{\mathrm{Th}} = v_o = \frac{40}{40+120}(800) = 200\,\mathrm{V}$$

Remove the voltage source and make series and parallel combinations of resistors to find the equivalent resistance:

$$R_{\mathrm{Th}} = 10 + 120\|40 = 10 + 30 = 40\,\Omega$$

The simplified circuit is:



$$\tau = \frac{L}{R} = \frac{40\times10^{-3}}{40} = 1\,\mathrm{ms}; \qquad \frac{1}{\tau} = 1000$$

$$i_o(\infty) = \frac{200}{40} = 5\,\mathrm{A}$$

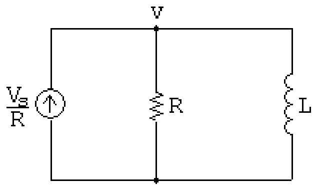
$$\therefore \quad i_o = i_o(\infty) + [i_o(0^+) - i_o(\infty)]e^{-t/\tau}$$

$$= 5 + (20-5)e^{-1000t} = 5 + 15e^{-1000t}\,\mathrm{A}, \qquad t \geq 0$$

[b]

$$\begin{aligned} v_o &= 10i_o + L\frac{di_o}{dt} \\ &= 10(5+15e^{-1000t}) + 0.04(-1000)(15e^{-1000t}) \\ &= 50 + 150e^{-1000t} - 600e^{-1000t} \\ v_o &= 50 - 450e^{-1000t}\,\mathrm{V}, \qquad t \geq 0^+ \end{aligned}$$

P 7.38 **[a]**



$$-\frac{V_s}{R} + \frac{v}{R} + \frac{1}{L}\int_0^t v\,dt + I_o = 0$$

Differentiating both sides,

$$\frac{1}{R}\frac{dv}{dt} + \frac{1}{L}v = 0$$

$$\therefore \qquad \frac{dv}{dt} + \frac{R}{L}v = 0$$

[b] $\frac{dv}{dt} = -\frac{R}{L}v$

$$\frac{dv}{dt}dt = -\frac{R}{L}v\,dt \qquad \text{so} \qquad dv = -\frac{R}{L}v\,dt$$

$$\frac{dv}{v} = -\frac{R}{L}dt$$

$$\int_{V_o}^{v(t)}\frac{dx}{x} = -\frac{R}{L}\int_0^t dy$$

$$\ln\frac{v(t)}{V_o} = -\frac{R}{L}t$$

$$\therefore \qquad v(t) = V_o e^{-(R/L)t} = (V_\mathrm{s} - RI_o)e^{-(R/L)t}$$

P 7.39 **[a]** $v_o(0^+) = -I_gR_2; \qquad \tau = \frac{L}{R_1 + R_2}$

$$v_o(\infty) = 0$$

$$v_o(t) = -I_gR_2e^{-[(R_1+R_2)/L]t}\,\mathrm{V}, \qquad t \geq 0^+$$

[b] $v_o(0^+) \to \infty$, and the duration of $v_o(t) \to$ zero

[c] $v_{sw} = R_2i_o; \qquad \tau = \frac{L}{R_1 + R_2}$

$$i_o(0^+) = I_g; \qquad i_o(\infty) = I_g\frac{R_1}{R_1 + R_2}$$

Therefore $\quad i_o(t) = \frac{I_gR_1}{R_1+R_2} + \left[I_g - \frac{I_gR_1}{R_1+R_2}\right]e^{-[(R_1+R_2)/L]t}$

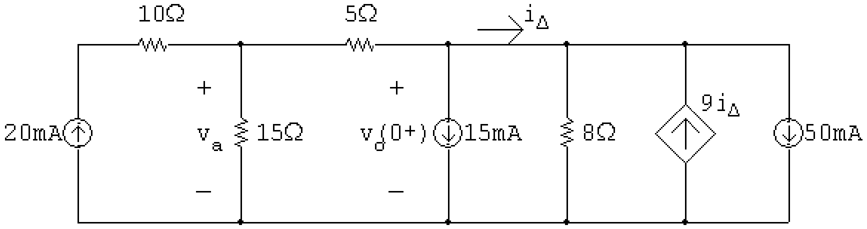
$$i_o(t) = \frac{R_1I_g}{(R_1+R_2)} + \frac{R_2I_g}{(R_1+R_2)}e^{-[(R_1+R_2)/L]t}$$

Therefore $\quad v_{sw} = \frac{R_1I_g}{(1+R_1/R_2)} + \frac{R_2I_g}{(1+R_1/R_2)}e^{-[(R_1+R_2)/L]t}, \qquad t \geq 0^+$

[d] $|v_{sw}(0^+)| \to \infty; \qquad$ duration $\to 0$

P 7.40 Opening the inductive circuit causes a very large voltage to be induced across the inductor L. This voltage also appears across the switch (part [d] of Problem 7.39), causing the switch to arc over. At the same time, the large voltage across L damages the meter movement.

P 7.41 $t > 0$; calculate $v_o(0^+)$



$$\frac{v_a}{15} + \frac{v_a - v_o(0^+)}{5} = 20 \times 10^{-3}$$

$$\therefore \quad v_a = 0.75v_o(0^+) + 75 \times 10^{-3}$$

$$15 \times 10^{-3} + \frac{v_o(0^+) - v_a}{5} + \frac{v_o(0^+)}{8} - 9i_\Delta + 50 \times 10^{-3} = 0$$

$$13v_o(0^+) - 8v_a - 360i_\Delta = -2600 \times 10^{-3}$$

$$i_\Delta = \frac{v_o(0^+)}{8} - 9i_\Delta + 50 \times 10^{-3}$$

$$\therefore \quad i_\Delta = \frac{v_o(0^+)}{80} + 5 \times 10^{-3}$$

$$\therefore \quad 360i_\Delta = 4.5v_o(0^+) + 1800 \times 10^{-3}$$

$$8v_a = 6v_o(0^+) + 600 \times 10^{-3}$$

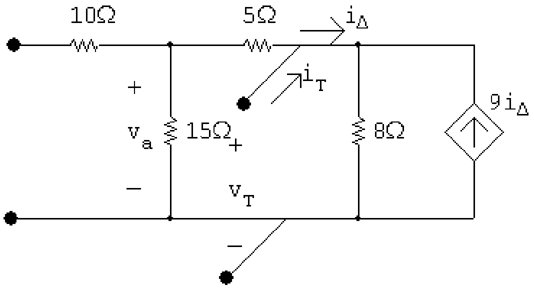
$$\therefore \quad 13v_o(0^+) - 6v_o(0^+) - 600 \times 10^{-3} - 4.5v_o(0^+) -$$

$$1800 \times 10^{-3} = -2600 \times 10^{-3}$$

$$2.5v_o(0^+) = -200 \times 10^{-3}; \qquad v_o(0^+) = -80\,\text{mV}$$

$$v_o(\infty) = 0$$

Find the Thévenin resistance seen by the 4 mH inductor:



$$i_T = \frac{v_T}{20} + \frac{v_T}{8} - 9i_\Delta$$

$$i_\Delta = \frac{v_T}{8} - 9i_\Delta \qquad \therefore \quad 10i_\Delta = \frac{v_T}{8}; \qquad i_\Delta = \frac{v_T}{80}$$

$$i_T = \frac{v_T}{20} + \frac{10v_T}{80} - \frac{9v_T}{80}$$

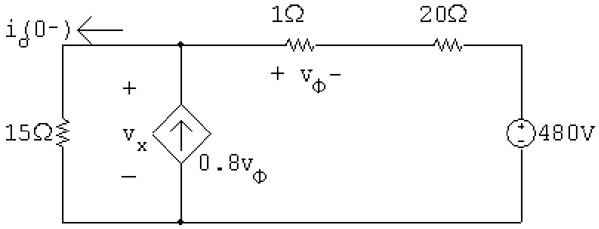
$$\frac{i_T}{v_T} = \frac{1}{20} + \frac{1}{80} = \frac{5}{80} = \frac{1}{16}\,\mathrm{S}$$

$$\therefore \quad R_{\mathrm{Th}} = 16\Omega$$

$$\tau = \frac{4 \times 10^{-3}}{16} = 0.25\,\mathrm{ms}; \qquad 1/\tau = 4000$$

$$\therefore \quad v_o = 0 + (-80 - 0)e^{-4000t} = -80e^{-4000t}\,\mathrm{mV}, \qquad t \geq 0^+$$

P 7.42 For $t < 0$



$$\frac{v_x}{15} - 0.8v_\phi + \frac{v_x - 480}{21} = 0$$

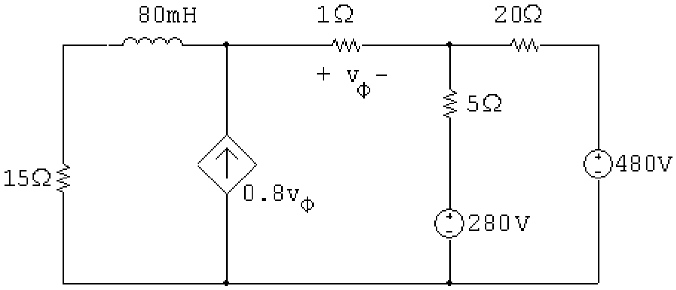
$$v_\phi = \frac{v_x - 480}{21}$$

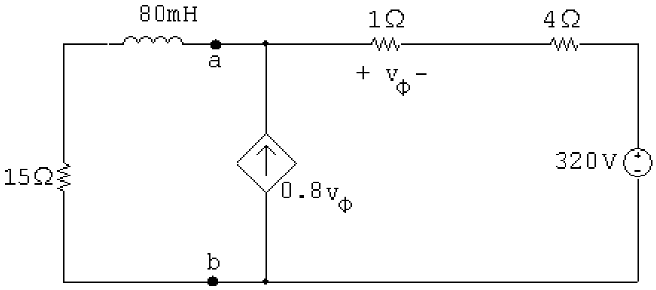
$$\frac{v_x}{15} - 0.8\left(\frac{v_x - 480}{21}\right) + \left(\frac{v_x - 480}{21}\right)$$

$$= \frac{v_x}{15} + 0.2\left(\frac{v_s - 480}{21}\right) = 21v_x + 3(v_x - 480) = 0$$

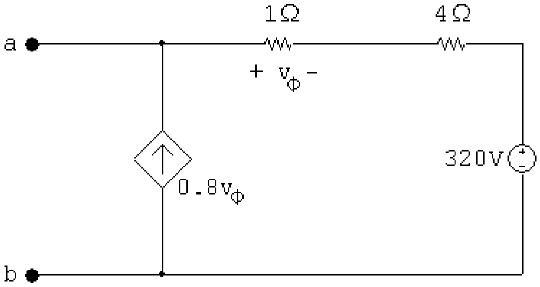
$$\therefore \qquad 24v_x = 1440 \quad \text{so} \quad v_x = 60\,\mathrm{V} \qquad i_o(0^-) = \frac{v_x}{15} = 4\,\mathrm{A}$$

$t > 0$

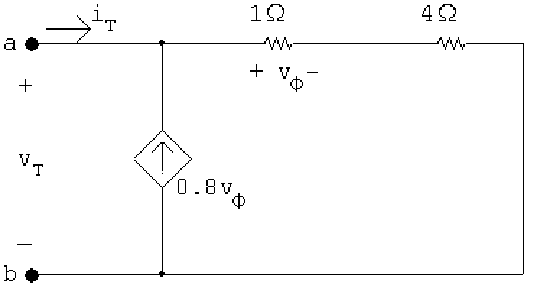




Find Thévenin equivalent with respect to a, b



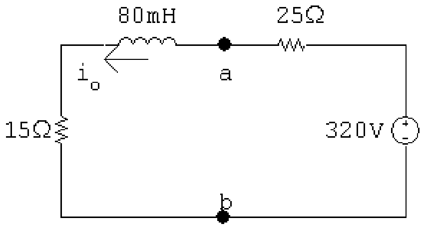
$$\frac{V_{\text{Th}} - 320}{5} - 0.8\left(\frac{V_{\text{Th}} - 320}{5}\right) = 0 \qquad V_{\text{Th}} = 320\,\text{V}$$



$$v_T = (i_T + 0.8v_\phi)(5) = \left(i_T + 0.8\frac{v_T}{5}\right)(5)$$

$$v_T = 5i_T + 0.8v_T \qquad \therefore \quad 0.2v_T = 5i_T$$

$$\frac{v_T}{i_T} = R_{\text{Th}} = 25\,\Omega$$



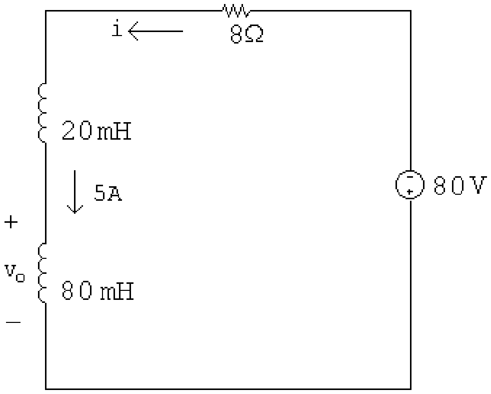
$$i_o(\infty) = 320/40 = 8\,\text{A}$$

$$\tau = \frac{80 \times 10^{-3}}{40} = 2\,\text{ms}; \qquad 1/\tau = 500$$

$$i_o = 8 + (4-8)e^{-500t} = 8 - 4e^{-500t}\,\text{A}, \qquad t \geq 0$$

P 7.43 For $t < 0$, $\quad i_{80\text{mH}}(0) = 50\,\text{V}/10\,\Omega = 5\,\text{A}$

For $t > 0$, after making a Thévenin equivalent we have



$$i = \frac{V_s}{R} + \left(I_o - \frac{V_s}{R}\right)e^{-t/\tau}$$

$$\frac{1}{\tau} = \frac{R}{L} = \frac{8}{100 \times 10^{-3}} = 80$$

$$I_o = 5\,\text{A}; \qquad I_f = \frac{V_s}{R} = \frac{-80}{8} = -10\,\text{A}$$

$$i = -10 + (5+10)e^{-80t} = -10 + 15e^{-80t}\,\text{A}, \qquad t \geq 0$$

$$v_o = 0.08\frac{di}{dt} = 0.08(-1200e^{-80t}) = -96e^{-80t}\,\text{V}, \qquad t \geq 0^+$$

P 7.44 **[a]** Let v be the voltage drop across the parallel branches, positive at the top node, then

$$-I_g + \frac{v}{R_g} + \frac{1}{L_1}\int_0^t v\,dx + \frac{1}{L_2}\int_0^t v\,dx = 0$$

$$\frac{v}{R_g} + \left(\frac{1}{L_1} + \frac{1}{L_2}\right)\int_0^t v\,dx = I_g$$

$$\frac{v}{R_g} + \frac{1}{L_e}\int_0^t v\,dx = I_g$$

$$\frac{1}{R_g}\frac{dv}{dt} + \frac{v}{L_e} = 0$$

$$\frac{dv}{dt} + \frac{R_g}{L_e}v = 0$$

Therefore $v = I_g R_g e^{-t/\tau}$; $\qquad \tau = L_e/R_g$

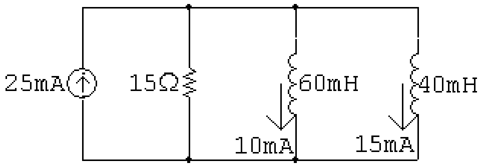
Thus

$$i_1 = \frac{1}{L_1}\int_0^t I_g R_g e^{-x/\tau}\,dx = \frac{I_g R_g}{L_1}\frac{e^{-x/\tau}}{(-1/\tau)}\Big|_0^t = \frac{I_g L_e}{L_1}(1 - e^{-t/\tau})$$

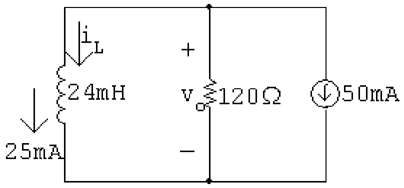
$$i_1 = \frac{I_g L_2}{L_1 + L_2}(1 - e^{-t/\tau}) \quad \text{and} \quad i_2 = \frac{I_g L_1}{L_1 + L_2}(1 - e^{-t/\tau})$$

[b] $i_1(\infty) = \dfrac{L_2}{L_1 + L_2}I_g; \qquad i_2(\infty) = \dfrac{L_1}{L_1 + L_2}I_g$

P 7.45 **[a]** $t < 0$



$t > 0$



$$i_L(0^-) = i_L(0^+) = 25\,\text{mA}; \qquad \tau = \frac{24 \times 10^{-3}}{120} = 0.2\,\text{ms}; \qquad \frac{1}{\tau} = 5000$$

$$i_L(\infty) = -50\,\text{mA}$$

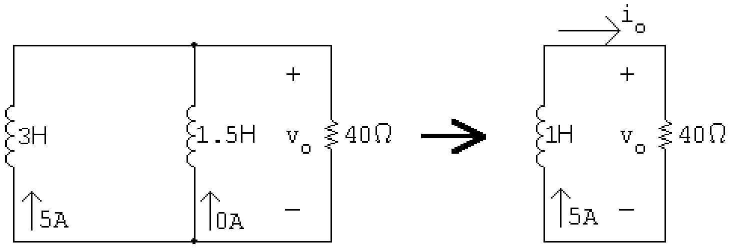
$$i_L = -50 + (25 + 50)e^{-5000t} = -50 + 75e^{-5000t}\,\text{mA}, \qquad t \geq 0$$

$$v_o = -120[75 \times 10^{-3}e^{-5000t}] = -9e^{-5000t}\,\text{V}, \qquad t \geq 0^+$$

[b] $i_1 = \dfrac{1}{60 \times 10^{-3}}\displaystyle\int_0^t -9e^{-5000x}\,dx + 10 \times 10^{-3} = (30e^{-5000t} - 20)\,\text{mA}, \qquad t \geq 0$

[c] $i_2 = \dfrac{1}{40 \times 10^{-3}}\displaystyle\int_0^t -9e^{-5000x}\,dx + 15 \times 10^{-3} = (45e^{-5000t} - 30)\,\text{mA}, \qquad t \geq 0$

P 7.46 $t > 0$



$$\tau = \frac{1}{40}$$

$$i_o = 5e^{-40t}\,\text{A}, \qquad t \geq 0$$

$$v_o = 40i_o = 200e^{-40t}\,\text{V}, \qquad t > 0^+$$

$$200e^{-40t} = 100; \qquad e^{40t} = 2$$

$$\therefore \quad t = \frac{1}{40}\ln 2 = 17.33\,\text{ms}$$

P 7.47 **[a]** $w_{\text{diss}} = \frac{1}{2}L_e i^2(0) = \frac{1}{2}(1)(5)^2 = 12.5\,\text{J}$

[b] $i_{3H} = \frac{1}{3}\int_0^t (200)e^{-40x}\,dx - 5$

$$= 1.67(1 - e^{-40t}) - 5 = -1.67e^{-40t} - 3.33\,\text{A}$$

$$i_{1.5H} = \frac{1}{1.5}\int_0^t (200)e^{-40x}\,dx + 0$$

$$= -3.33e^{-40t} + 3.33\,\text{A}$$

$$w_{\text{trapped}} = \frac{1}{2}(4.5)(3.33)^2 = 25\,\text{J}$$

[c] $w(0) = \frac{1}{2}(3)(5)^2 = 37.5\,\text{J}$

P 7.48 **[a]** $v = I_sR + (V_o - I_sR)e^{-t/RC} \qquad i = \left(I_s - \frac{V_o}{R}\right)e^{-t/RC}$

$$\therefore \quad I_sR = 40, \qquad V_o - I_sR = -24$$

$$\therefore \quad V_o = 16\,\text{V}$$

$$I_s - \frac{V_o}{R} = 3 \times 10^{-3}; \qquad I_s - \frac{16}{R} = 3 \times 10^{-3}; \qquad R = \frac{40}{I_s}$$

$$\therefore \quad I_s - 0.4I_s = 3 \times 10^{-3}; \qquad I_s = 5\,\text{mA}$$

$$R = \frac{40}{5} \times 10^3 = 8\,\text{k}\Omega$$

$$\frac{1}{RC} = 2500; \quad C = \frac{1}{2500R} = \frac{10^{-3}}{20 \times 10^3} = 50\,\text{nF}; \quad \tau = RC = \frac{1}{2500} = 400\,\mu\text{s}$$

[b] $v(\infty) = 40\,\text{V}$

$$w(\infty) = \frac{1}{2}(50 \times 10^{-9})(1600) = 40\,\mu\text{J}$$

$$0.81w(\infty) = 32.4\,\mu\text{J}$$

$$v^2(t_o) = \frac{32.4 \times 10^{-6}}{25 \times 10^{-9}} = 1296; \qquad v(t_o) = 36\,\text{V}$$

$$40 - 24e^{-2500t_o} = 36; \qquad e^{2500t_o} = 6; \qquad \therefore \quad t_o = 716.70\,\mu\text{s}$$

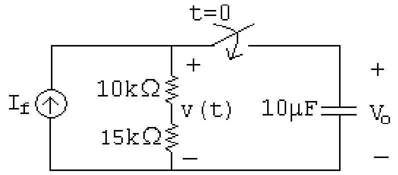
P 7.49 **[a]** Note that there are many different possible solutions to this problem.

$$R = \frac{\tau}{C}$$

Choose a 10 μH capacitor from Appendix H. Then,

$$R = \frac{0.25}{10 \times 10^{-6}} = 25\,\text{k}\Omega$$

Construct the resistance needed by combining 10 kΩ and 15 kΩ resistors in series:



[b] $v(t) = V_f + (V_o - V_f)e^{-t/\tau}$

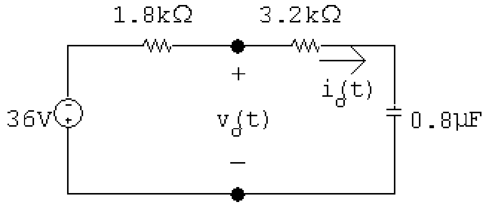
$$V_o = 100\,\text{V}; \qquad V_f = (I_f)(R) = (1 \times 10^{-3})(25 \times 10^3) = 25\,\text{V}$$

$$\therefore \quad v(t) = 25 + (100 - 25)e^{-4t}\,\text{V} = 25 + 75e^{-4t}\,\text{V}, \qquad t \geq 0$$

[c] $v(t) = 25 + 75e^{-4t} = 50 \qquad \text{so} \qquad e^{-4t} = \frac{1}{3}$

$$\therefore \quad t = \frac{\ln 3}{4} = 274.65\,\text{ms}$$

P 7.50 **[a]**



$$i_o(0^+) = \frac{-36}{5000} = -7.2\,\text{mA}$$

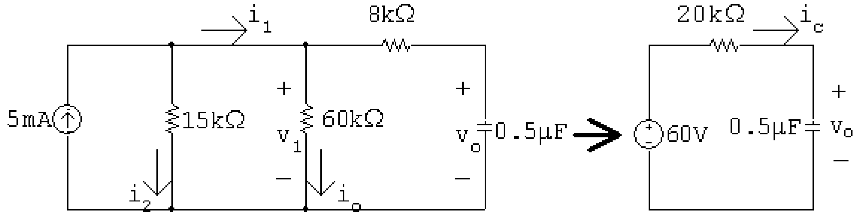
[b] $i_o(\infty) = 0$

[c] $\tau = RC = (5000)(0.8 \times 10^{-6}) = 4\,\text{ms}$

[d] $i_o = 0 + (-7.2)e^{-250t} = -7.2e^{-250t}\,\text{mA}, \qquad t \geq 0^+$

[e] $v_o = -[36 + 1800(-7.2 \times 10^{-3}e^{-250t})] = -36 + 12.96e^{-250t}\,\text{V}, \qquad t \geq 0^+$

P 7.51 **[a]** Simplify the circuit for $t > 0$ using source transformation:



Since there is no source connected to the capacitor for $t < 0$

$v_o(0^-) = v_o(0^+) = 0\,\text{V}$

From the simplified circuit,

$v_o(\infty) = 60\,\text{V}$

$\tau = RC = (20 \times 10^3)(0.5 \times 10^{-6}) = 10\,\text{ms} \qquad 1/\tau = 100$

$v_o = v_o(\infty) + [v_o(0^+) - v_o(\infty)]e^{-t/\tau} = (60 - 60e^{-100t})\,\text{V}, \qquad t \geq 0$

[b] $i_c = C\dfrac{dv_o}{dt}$

$i_c = 0.5 \times 10^{-6}(-100)(-60e^{-100t}) = 3e^{-100t}\,\text{mA}$

$v_1 = 8000i_c + v_o = (8000)(3 \times 10^{-3})e^{-100t} + (60 - 60e^{-100t}) = 60 - 36e^{-100t}\,\text{V}$

$i_o = \dfrac{v_1}{60 \times 10^3} = 1 - 0.6e^{-100t}\,\text{mA}, \quad t \geq 0^+$

[c] $i_1(t) = i_o + i_c = 1 + 2.4e^{-100t}\,\text{mA}, \quad t \geq 0^+$

[d] $i_2(t) = \dfrac{v_1}{15 \times 10^3} = 4 - 2.4e^{-100t}\,\text{mA}, \quad t \geq 0^+$

[e] $i_1(0^+) = 1 + 2.4 = 3.4\,\text{mA}$

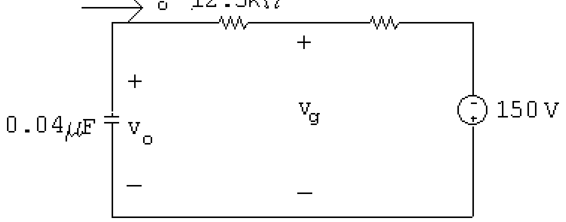
At $t = 0^+$:

$R_e = 15\,\text{k}\|60\,\text{k}\|8\,\text{k} = 4800\,\Omega$

$v_1(0^+) = (5 \times 10^{-3})(4800) = 24\,\text{V}$

$i_1(0^+) = \dfrac{v_1(0^+)}{60{,}000} + \dfrac{v_1(0^+)}{8000} = 0.4\,\text{m} + 3\,\text{m} = 3.4\,\text{mA} \quad \text{(checks)}$

P 7.52 **[a]** $v_o(0^-) = v_o(0^+) = 120\,\text{V}$



$v_o(\infty) = -150\,\text{V}; \qquad \tau = 2\,\text{ms}; \qquad \dfrac{1}{\tau} = 500$

$$v_o = -150 + (120 - (-150))e^{-500t}$$

$$v_o = -150 + 270e^{-500t}\,\text{V}, \qquad t \geq 0$$

[b] $i_o = -0.04 \times 10^{-6}(-500)[270e^{-500t}] = 5.4e^{-500t}\,\text{mA}, \qquad t \geq 0^+$

[c] $v_g = v_o - 12.5 \times 10^3 i_o = -150 + 202.5e^{-500t}\,\text{V}$

[d] $v_g(0^+) = -150 + 202.5 = 52.5\,\text{V}$

Checks:

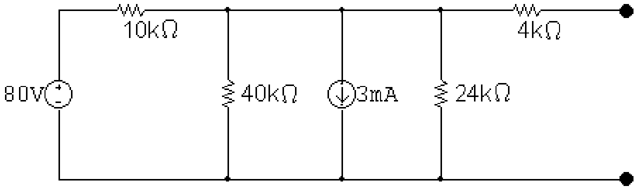
$$v_g(0^+) = i_o(0^+)[37.5 \times 10^3] - 150 = 202.5 - 150 = 52.5\,\text{V}$$

$$i_{50k} = \frac{v_g}{50\text{k}} = -3 + 4.05e^{-500t}\,\text{mA}$$

$$i_{150k} = \frac{v_g}{150\text{k}} = -1 + 1.35e^{-500t}\,\text{mA}$$

$$\text{-}i_o + i_{50k} + i_{150k} + 4 = 0 \qquad \text{(ok)}$$

P 7.53 For $t < 0$



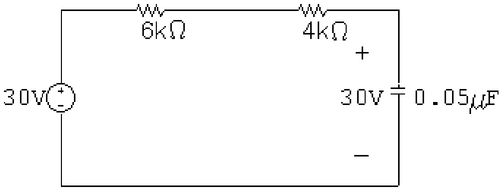
Simplify the circuit:

$80/10{,}000 = 8\,\text{mA}, \qquad 10\,\text{k}\Omega \| 40\,\text{k}\Omega \| 24\,\text{k}\Omega = 6\,\text{k}\Omega$

$8\,\text{mA} - 3\,\text{mA} = 5\,\text{mA}$

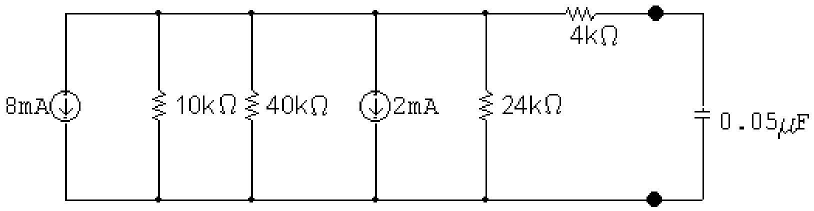
$5\,\text{mA} \times 6\,\text{k}\Omega = 30\,\text{V}$

Thus, for $t < 0$



$\therefore \quad v_o(0^-) = v_o(0^+) = 30\,\text{V}$

$t > 0$



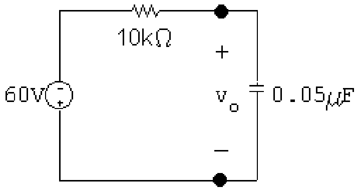
Simplify the circuit:

$8\,\mathrm{mA} + 2\,\mathrm{mA} = 10\,\mathrm{mA}$

$10\,\mathrm{k}\|40\,\mathrm{k}\|24\,\mathrm{k} = 6\,\mathrm{k}\Omega$

$(10\,\mathrm{mA})(6\,\mathrm{k}\Omega) = 60\,\mathrm{V}$

Thus, for $t > 0$



$$v_o(\infty) = -10 \times 10^{-3}(6 \times 10^3) = -60\,\mathrm{V}$$

$$\tau = RC = (10\,\mathrm{k})(0.05\,\mu) = 0.5\,\mathrm{ms}; \qquad \frac{1}{\tau} = 2000$$

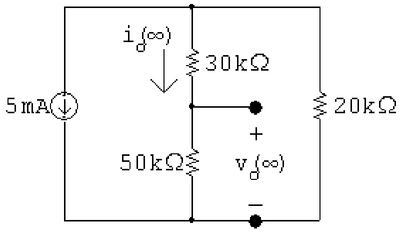
$$v_o = v_o(\infty) + [v_o(0^+) - v_o(\infty)]e^{-t/\tau} = -60 + [30 - (-60)]e^{-2000t}$$

$$= -60 + 90e^{-2000t}\,\mathrm{V} \qquad t \geq 0$$

P 7.54 $t < 0$:

$$i_o(0^-) = \frac{20}{100}(10 \times 10^{-3}) = 2\,\mathrm{mA}; \qquad v_o(0^-) = (2 \times 10^{-3})(50{,}000) = 100\,\mathrm{V}$$

$t = \infty$:

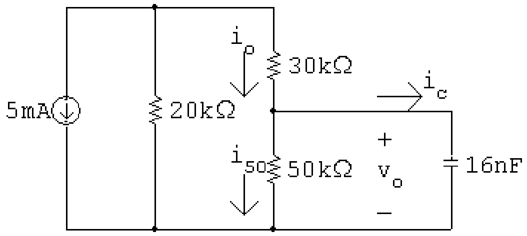


$$i_o(\infty) = -5 \times 10^{-3}\left(\frac{20}{100}\right) = -1\,\mathrm{mA}; \qquad v_o(\infty) = i_o(\infty)(50{,}000) = -50\,\mathrm{V}$$

$R_{\text{Th}} = 50\,\text{k}\Omega \| 50\,\text{k}\Omega = 25\,\text{k}\Omega; \qquad C = 16\,\text{nF}$

$\tau = (25{,}000)(16 \times 10^{-9}) = 0.4\,\text{ms}; \qquad \frac{1}{\tau} = 2500$

$\therefore \quad v_o(t) = -50 + 150e^{-2500t}\,\text{V}, \qquad t \geq 0$



$$i_c = C\frac{dv_o}{dt} = -6e^{-2500t}\,\text{mA}, \qquad t \geq 0^+$$

$$i_{50k} = \frac{v_o}{50{,}000} = -1 + 3e^{-2500t}\,\text{mA}, \qquad t \geq 0^+$$

$$i_o = i_c + i_{50k} = -(1 + 3e^{-2500t})\,\text{mA}, \qquad t \geq 0^+$$

P 7.55 **[a]** $v_c(0^+) = 50\,\text{V}$

[b] Use voltage division to find the final value of voltage:

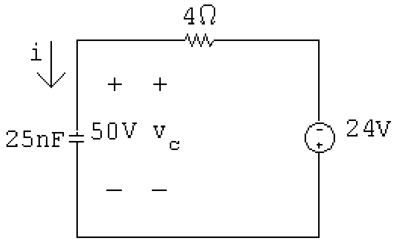
$$v_c(\infty) = \frac{20}{20+5}(-30) = -24\,\text{V}$$

[c] Find the Thévenin equivalent with respect to the terminals of the capacitor:

$V_{\text{Th}} = -24\,\text{V}, \qquad R_{\text{Th}} = 20\|5 = 4\,\Omega,$

Therefore $\tau = R_{\text{eq}}C = 4(25 \times 10^{-9}) = 0.1\,\mu\text{s}$

The simplified circuit for $t > 0$ is:



[d] $i(0^+) = \frac{-24 - 50}{4} = -18.5\,\text{A}$

[e] $v_c = v_c(\infty) + [v_c(0^+) - v_c(\infty)]e^{-t/\tau}$

$$= -24 + [50 - (-24)]e^{-t/\tau} = -24 + 74e^{-10^7 t}\,\text{V}, \qquad t \geq 0$$

[f] $i = C\dfrac{dv_c}{dt} = (25 \times 10^{-9})(-10^7)(74e^{-10^7 t}) = -18.5e^{-10^7 t}\,\text{A}, \qquad t \geq 0^+$

P 7.56 **[a]** Use voltage division to find the initial value of the voltage:

$$v_c(0^+) = v_{9k} = \frac{9\,\text{k}}{9\,\text{k} + 3\,\text{k}}(120) = 90\,\text{V}$$

[b] Use Ohm's law to find the final value of voltage:

$$v_c(\infty) = v_{40k} = -(1.5 \times 10^{-3})(40 \times 10^3) = -60\,\text{V}$$

[c] Find the Thévenin equivalent with respect to the terminals of the capacitor:

$$V_{\text{Th}} = -60\,\text{V}, \qquad R_{\text{Th}} = 10\,\text{k} + 40\,\text{k} = 50\,\text{k}\Omega$$

$$\tau = R_{\text{Th}}C = 1\,\text{ms} \;= 1000\,\mu\text{s}$$

[d] $v_c = v_c(\infty) + [v_c(0^+) - v_c(\infty)]e^{-t/\tau}$

$$= -60 + (90 + 60)e^{-1000t} = -60 + 150e^{-1000t}\,\text{V}, \quad t \geq 0$$

We want $v_c = -60 + 150e^{-1000t} = 0$:

Therefore $t = \dfrac{\ln(150/60)}{1000} = 916.3\,\mu\text{s}$

P 7.57 Use voltage division to find the initial voltage:

$$v_o(0) = \frac{60}{40 + 60}(50) = 30\,\text{V}$$

Use Ohm's law to find the final value of voltage:

$$v_o(\infty) = (-5\,\text{mA})(20\,\text{k}\Omega) = -100\,\text{V}$$

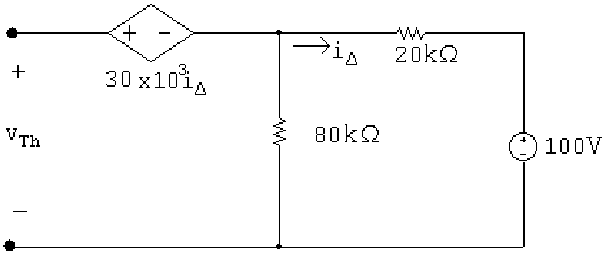
$$\tau = RC = (20 \times 10^3)(250 \times 10^{-9}) = 5\,\text{ms}; \qquad \frac{1}{\tau} = 200$$

$$v_o = v_o(\infty) + [v_o(0^+) - v_o(\infty)]e^{-t/\tau}$$

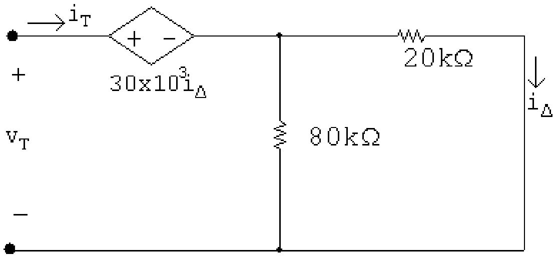
$$= -100 + (30 + 100)e^{-200t} = -100 + 130e^{-200t}\,\text{V}, \qquad t \geq 0$$

P 7.58 For $t < 0$, $v_o(0) = 80\,\text{V}$

$t > 0$:



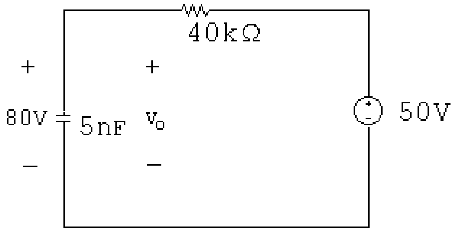
$$v_{\text{Th}} = 30 \times 10^3 i_\Delta + 0.8(100) = 30 \times 10^3 \left(\frac{-100}{100 \times 10^3}\right) + 80 = 50\,\text{V}$$



$$v_T = 30 \times 10^3 i_\Delta + 16 \times 10^3 i_T = 30 \times 10^3 (0.8) i_T + 16 \times 10^3 i_T = 40 \times 10^3 i_T$$

$$R_{\text{Th}} = \frac{v_T}{i_T} = 40\,\text{k}\Omega$$

$t > 0$



$$v_o = 50 + (80 - 50)e^{-t/\tau}$$

$$\tau = RC = (40 \times 10^3)(5 \times 10^{-9}) = 200 \times 10^{-6}; \qquad \frac{1}{\tau} = 5000$$

$$v_o = 50 + 30e^{-5000t}\,\text{V}, \quad t \geq 0$$

P 7.59 $v_o(0) = 50\,\text{V}$; $v_o(\infty) = 80\,\text{V}$

$$R_{\text{Th}} = 16\,\text{k}\Omega$$

$$\tau = (16)(5 \times 10^{-6}) = 80 \times 10^{-6}; \qquad \frac{1}{\tau} = 12{,}500$$

$$v = 80 + (50 - 80)e^{-12{,}500t} = 80 - 30e^{-12{,}500t}\,\text{V}, \quad t \geq 0$$

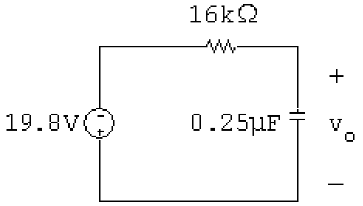
P 7.60 For $t > 0$

$$V_{\mathrm{Th}} = (-25)(16{,}000)i_{\mathrm{b}} = -400 \times 10^3 i_{\mathrm{b}}$$

$$i_{\mathrm{b}} = \frac{33{,}000}{80{,}000}(120 \times 10^{-6}) = 49.5\,\mu\mathrm{A}$$

$$V_{\mathrm{Th}} = -400 \times 10^3(49.5 \times 10^{-6}) = -19.8\,\mathrm{V}$$

$$R_{\mathrm{Th}} = 16\,\mathrm{k}\Omega$$



$$v_o(\infty) = -19.8\,\mathrm{V}; \qquad v_o(0^+) = 0$$

$$\tau = (16,000)(0.25 \times 10^{-6}) = 4\,\mathrm{ms}; \qquad 1/\tau = 250$$

$$v_o = -19.8 + 19.8e^{-250t}\,\mathrm{V}, \qquad t \geq 0$$

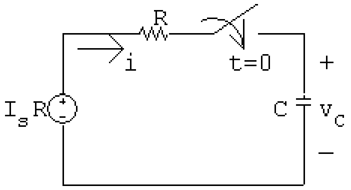
$$w(t) = \frac{1}{2}(0.25 \times 10^{-6})v_o^2 = w(\infty)(1 - e^{-250t})^2\,\mathrm{J}$$

$$(1 - e^{-250t})^2 = \frac{0.36w(\infty)}{w(\infty)} = 0.36$$

$$1 - e^{-250t} = 0.6$$

$$e^{-250t} = 0.4 \quad \therefore \qquad t = 3.67\,\mathrm{ms}$$

P 7.61 [**a**]



$$I_sR = Ri + \frac{1}{C}\int_{0^+}^{t} i\,dx + V_o$$

$$0 = R\frac{di}{dt} + \frac{i}{C} + 0$$

$$\therefore \quad \frac{di}{dt} + \frac{i}{RC} = 0$$

[b] $\frac{di}{dt} = -\frac{i}{RC}; \qquad \frac{di}{i} = -\frac{dt}{RC}$

$$\int_{i(0^+)}^{i(t)} \frac{dy}{y} = -\frac{1}{RC}\int_{0^+}^{t} dx$$

$$\ln\frac{i(t)}{i(0^+)} = \frac{-t}{RC}$$

$$i(t) = i(0^+)e^{-t/RC}; \qquad i(0^+) = \frac{I_sR - V_o}{R} = \left(I_s - \frac{V_o}{R}\right)$$

$$\therefore \; i(t) = \left(I_s - \frac{V_o}{R}\right)e^{-t/RC}$$

P 7.62 **[a]** Let i be the current in the clockwise direction around the circuit. Then

$$\begin{aligned} V_g &= iR_g + \frac{1}{C_1}\int_0^t i\,dx + \frac{1}{C_2}\int_0^t i\,dx \\ &= iR_g + \left(\frac{1}{C_1} + \frac{1}{C_2}\right)\int_0^t i\,dx = iR_g + \frac{1}{C_e}\int_0^t i\,dx \end{aligned}$$

Now differentiate the equation

$$0 = R_g\frac{di}{dt} + \frac{i}{C_e} \quad \text{or} \quad \frac{di}{dt} + \frac{1}{R_gC_e}i = 0$$

Therefore $i = \frac{V_g}{R_g}e^{-t/R_gC_e} = \frac{V_g}{R_g}e^{-t/\tau}; \qquad \tau = R_gC_e$

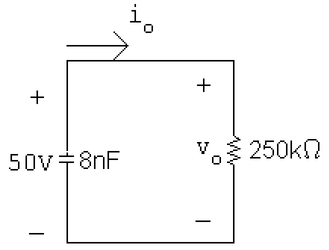
$$v_1(t) = \frac{1}{C_1}\int_0^t \frac{V_g}{R_g}e^{-x/\tau}\,dx = \frac{V_g}{R_gC_1}\,\frac{e^{-x/\tau}}{-1/\tau}\Big|_0^t = -\frac{V_gC_e}{C_1}(e^{-t/\tau} - 1)$$

$$v_1(t) = \frac{V_gC_2}{C_1 + C_2}(1 - e^{-t/\tau}); \qquad \tau = R_gC_e$$

$$v_2(t) = \frac{V_gC_1}{C_1 + C_2}(1 - e^{-t/\tau}); \qquad \tau = R_gC_e$$

[b] $v_1(\infty) = \frac{C_2}{C_1 + C_2}V_g; \qquad v_2(\infty) = \frac{C_1}{C_1 + C_2}V_g$

P 7.63 **[a]** For $t > 0$:



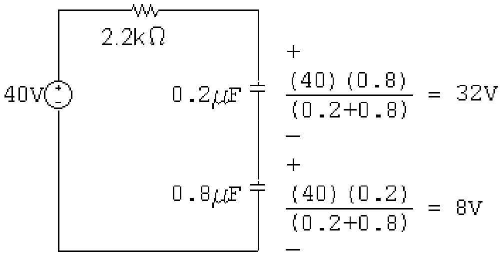
$$\tau = RC = 250 \times 10^3 \times 8 \times 10^{-9} = 2\,\text{ms}; \qquad \frac{1}{\tau} = 500$$

$$v_o = 50e^{-500t}\,\text{V}, \qquad t \geq 0^+$$

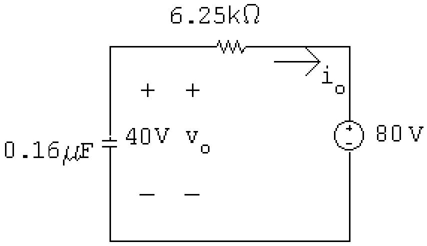
[b] $i_o = \dfrac{v_o}{250{,}000} = \dfrac{50e^{-500t}}{250{,}000} = 200e^{-500t}\,\mu\text{A}$

$$v_1 = \frac{-1}{40\times 10^{-9}} \times 200 \times 10^{-6} \int_0^t e^{-500x}\,dx + 50 = 10e^{-500t} + 40\,\text{V}, \quad t \geq 0$$

P 7.64 **[a]** $t < 0$



$t > 0$



$$v_o(0^-) = v_o(0^+) = 40\,\text{V}$$

$$v_o(\infty) = 80\,\text{V}$$

$$\tau = (0.16\times 10^{-6})(6.25\times 10^3) = 1\,\text{ms}; \qquad 1/\tau = 1000$$

$$v_o = 80 - 40e^{-1000t}\,\text{V}, \qquad t \geq 0$$

[b] $i_o = -C\dfrac{dv_o}{dt} = -0.16\times 10^{-6}[40{,}000e^{-1000t}]$

$$= -6.4e^{-1000t}\,\text{mA}; \qquad t \geq 0^+$$

[c] $v_1 = \dfrac{-1}{0.2\times 10^{-6}} \displaystyle\int_0^t -6.4\times 10^{-3}e^{-1000x}\,dx + 32$

$$= 64 - 32e^{-1000t}\,\text{V}, \qquad t \geq 0$$

[d] $v_2 = \dfrac{-1}{0.8\times 10^{-6}} \displaystyle\int_0^t -6.4\times 10^{-3}e^{-1000x}\,dx + 8$

$$= 16 - 8e^{-1000t}\,\text{V}, \qquad t \geq 0$$

[e] $w_{\text{trapped}} = \dfrac{1}{2}(0.2\times 10^{-6})(64)^2 + \dfrac{1}{2}(0.8\times 10^{-6})(16)^2 = 512\,\mu\text{J}.$

P 7.65 **[a]** $L_{\text{eq}} = \dfrac{(3)(15)}{3+15} = 2.5\,\text{H}$

$$\tau = \frac{L_{\text{eq}}}{R} = \frac{2.5}{7.5} = \frac{1}{3}\,\text{s}$$

$$i_o(0) = 0; \qquad i_o(\infty) = \frac{120}{7.5} = 16\,\text{A}$$

$$\therefore\ i_o = 16 - 16e^{-3t}\,\text{A}, \qquad t \geq 0$$

$$v_o = 120 - 7.5i_o = 120e^{-3t}\,\text{V}, \qquad t \geq 0^+$$

$$i_1 = \frac{1}{3}\int_0^t 120e^{-3x}\,dx = \frac{40}{3} - \frac{40}{3}e^{-3t}\,\text{A}, \qquad t \geq 0$$

$$i_2 = i_o - i_1 = \frac{8}{3} - \frac{8}{3}e^{-3t}\,\text{A}, \qquad t \geq 0$$

[b] $i_o(0) = i_1(0) = i_2(0) = 0$, consistent with initial conditions.
$v_o(0^+) = 120$ V, consistent with $i_o(0) = 0$.

$$v_o = 3\frac{di_1}{dt} = 120e^{-3t}\,\text{V}, \qquad t \geq 0^+$$

or

$$v_o = 15\frac{di_2}{dt} = 120e^{-3t}\,\text{V}, \qquad t \geq 0^+$$

The voltage solution is consistent with the current solutions.

$$\lambda_1 = 3i_1 = 40 - 40e^{-3t}\ \text{Wb-turns}$$

$$\lambda_2 = 15i_2 = 40 - 40e^{-3t}\ \text{Wb-turns}$$

$\therefore\ \lambda_1 = \lambda_2$ as it must, since

$$v_o = \frac{d\lambda_1}{dt} = \frac{d\lambda_2}{dt}$$

$$\lambda_1(\infty) = \lambda_2(\infty) = 40\ \text{Wb-turns}$$

$$\lambda_1(\infty) = 3i_1(\infty) = 3(40/3) = 40\ \text{Wb-turns}$$

$$\lambda_2(\infty) = 15i_2(\infty) = 15(8/3) = 40\ \text{Wb-turns}$$

$\therefore\ i_1(\infty)$ and $i_2(\infty)$ are consistent with $\lambda_1(\infty)$ and $\lambda_2(\infty)$.

P 7.66 **[a]** $L_{\text{eq}} = 5 + 10 - 2.5(2) = 10\,\text{H}$

$$\tau = \frac{L}{R} = \frac{10}{40} = \frac{1}{4}; \qquad \frac{1}{\tau} = 4$$

$$i = 2 - 2e^{-4t}\,\text{A}, \quad t \geq 0$$

[b] $v_1(t) = 5\frac{di_1}{dt} - 2.5\frac{di}{dt} = 2.5\frac{di}{dt} = 2.5(8e^{-4t}) = 20e^{-4t}\,\text{V}, \quad t \geq 0^+$

[c] $v_2(t) = 10\frac{di_1}{dt} - 2.5\frac{di}{dt} = 7.5\frac{di}{dt} = 7.5(8e^{-4t}) = 60e^{-4t}\,\text{V}, \quad t \geq 0^+$

[d] $i(0) = 2 - 2 = 0,$ which agrees with initial conditions.

$$80 = 40i_1 + v_1 + v_2 = 40(2 - 2e^{-4t}) + 20e^{-4t} + 60e^{-4t} = 80\,\text{V}$$

Therefore, Kirchhoff's voltage law is satisfied for all values of $t \geq 0$. Thus, the answers make sense in terms of known circuit behavior.

P 7.67 **[a]** $L_{\text{eq}} = 5 + 10 + 2.5(2) = 20\,\text{H}$

$$\tau = \frac{L}{R} = \frac{20}{40} = \frac{1}{2}; \qquad \frac{1}{\tau} = 2$$

$$i = 2 - 2e^{-2t}\,\text{A}, \quad t \geq 0$$

[b] $v_1(t) = 5\frac{di_1}{dt} + 2.5\frac{di}{dt} = 7.5\frac{di}{dt} = 7.5(4e^{-2t}) = 30e^{-2t}\,\text{V}, \quad t \geq 0^+$

[c] $v_2(t) = 10\frac{di_1}{dt} + 2.5\frac{di}{dt} = 12.5\frac{di}{dt} = 12.5(4e^{-2t}) = 50e^{-2t}\,\text{V}, \quad t \geq 0^+$

[d] $i(0) = 0,$ which agrees with initial conditions.

$$80 = 40i_1 + v_1 + v_2 = 40(2 - 2e^{-2t}) + 30e^{-2t} + 50e^{-2t} = 80\,\text{V}$$

Therefore, Kirchhoff's voltage law is satisfied for all values of $t \geq 0$. Thus, the answers make sense in terms of known circuit behavior.

P 7.68 **[a]** From Example 7.10,

$$L_{\text{eq}} = \frac{L_1L_2 - M^2}{L_1 + L_2 + 2M} = \frac{50 - 25}{15 + 10} = 1\,\text{H}$$

$$\tau = \frac{L}{R} = \frac{1}{20}; \qquad \frac{1}{\tau} = 20$$

$$\therefore \quad i_o(t) = 4 - 4e^{-20t}\,\text{A}, \qquad t \geq 0$$

[b] $v_o = 80 - 20i_o = 80 - 80 + 80e^{-20t} = 80e^{-20t}\,\text{V}, \qquad t \geq 0^+$

[c] $v_o = 5\frac{di_1}{dt} - 5\frac{di_2}{dt} = 80e^{-20t}\,\text{V}$

$$i_o = i_1 + i_2$$

$$\frac{di_o}{dt} = \frac{di_1}{dt} + \frac{di_2}{dt} = 80e^{-20t}\ \text{A/s}$$

$$\therefore \quad \frac{di_2}{dt} = 80e^{-20t} - \frac{di_1}{dt}$$

$$\therefore \quad 80e^{-20t} = 5\frac{di_1}{dt} - 400e^{-20t} + 5\frac{di_1}{dt}$$

$$\therefore \quad 10\frac{di_1}{dt} = 480e^{-20t}; \qquad di_1 = 48e^{-20t}\,dt$$

$$\int_0^{t_1} dx = \int_0^t 48e^{-20y}\,dy$$

$$i_1 = \frac{48}{-20}e^{-20y}\Big|_0^t = 2.4 - 2.4e^{-20t}\,\text{A}, \qquad t \geq 0$$

[d] $i_2 = i_o - i_1 = 4 - 4e^{-20t} - 2.4 + 2.4e^{-20t}$

$$= 1.6 - 1.6e^{-20t}\,\text{A}, \qquad t \geq 0$$

[e] $i_o(0) = i_1(0) = i_2(0) = 0$, consistent with zero initial stored energy.

$$v_o = L_{\text{eq}}\frac{di_o}{dt} = 1(80)e^{-20t} = 80e^{-20t}\,\text{V}, \qquad t \geq 0^+ \text{ (checks)}$$

Also,

$$v_o = 5\frac{di_1}{dt} - 5\frac{di_2}{dt} = 80e^{-20t}\,\text{V}, \qquad t \geq 0^+ \text{ (checks)}$$

$$v_o = 10\frac{di_2}{dt} - 5\frac{di_1}{dt} = 80e^{-20t}\,\text{V}, \qquad t \geq 0^+ \text{ (checks)}$$

$v_o(0^+) = 80\,\text{V}$, which agrees with $i_o(0^+) = 0\,\text{A}$

$i_o(\infty) = 4\,\text{A}; \qquad i_o(\infty)L_{\text{eq}} = (4)(1) = 4$ Wb-turns

$i_1(\infty)L_1 + i_2(\infty)M = (2.4)(5) + (1.6)(-5) = 4$ Wb-turns (ok)

$i_2(\infty)L_2 + i_1(\infty)M = (1.6)(10) + (2.4)(-5) = 4$ Wb-turns (ok)

Therefore, the final values of i_o, i_1, and i_2 are consistent with conservation of flux linkage. Hence, the answers make sense in terms of known circuit behavior.

P 7.69 **[a]** From Example 7.10,

$$L_{\text{eq}} = \frac{L_1L_2 - M^2}{L_1 + L_2 + 2M} = \frac{0.125 - 0.0625}{0.75 + 0.5} = 50\,\text{mH}$$

$$\tau = \frac{L}{R} = \frac{1}{5000}; \qquad \frac{1}{\tau} = 5000$$

$$\therefore \quad i_o(t) = 40 - 40e^{-5000t}\,\text{mA}, \qquad t \geq 0$$

[b] $v_o = 10 - 250i_o = 10 - 250(0.04 + 0.04e^{-5000t} = 10e^{-5000t}\,\text{V}, \qquad t \geq 0^+$

[c] $v_o = 0.5\dfrac{di_1}{dt} - 0.25\dfrac{di_2}{dt} = 10e^{-5000t}\,\text{V}$

$$i_o = i_1 + i_2$$

$$\frac{di_o}{dt} = \frac{di_1}{dt} + \frac{di_2}{dt} = 200e^{-5000t}\ \text{A/s}$$

$$\therefore\quad \frac{di_2}{dt} = 200e^{-5000t} - \frac{di_1}{dt}$$

$$\therefore\quad 10e^{-5000t} = 0.5\frac{di_1}{dt} - 50e^{-5000t} + 0.25\frac{di_1}{dt}$$

$$\therefore\quad 0.75\frac{di_1}{dt} = 60e^{-5000t}; \qquad di_1 = 80e^{-5000t}\,dt$$

$$\int_0^{t_1} dx = \int_0^t 80e^{-5000y}\,dy$$

$$i_1 = \frac{80}{-5000}e^{-5000y}\Big|_0^t = 16 - 16e^{-5000t}\,\text{mA}, \qquad t \geq 0$$

[d] $i_2 = i_o - i_1 = 40 - 40e^{-5000t} - 16 + 16e^{-5000t}$

$$= 24 - 24e^{-5000t}\,\text{mA}, \qquad t \geq 0$$

[e] $i_o(0) = i_1(0) = i_2(0) = 0$, consistent with zero initial stored energy.

$$v_o = L_{\text{eq}}\frac{di_o}{dt} = (0.05)(200)e^{-5000t} = 10e^{-5000t}\,\text{V}, \qquad t \geq 0^+ \text{ (checks)}$$

Also,

$$v_o = 0.5\frac{di_1}{dt} - 0.25\frac{di_2}{dt} = 10e^{-5000t}\,\text{V}, \qquad t \geq 0^+ \text{ (checks)}$$

$$v_o = 0.25\frac{di_2}{dt} - 0.25\frac{di_1}{dt} = 10e^{-5000t}\,\text{V}, \qquad t \geq 0^+ \text{ (checks)}$$

$v_o(0^+) = 10\,\text{V}$, which agrees with $i_o(0^+) = 0\,\text{A}$

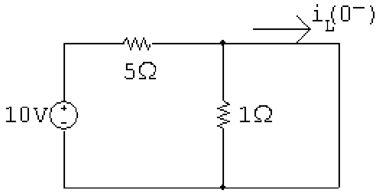
$i_o(\infty) = 40\,\text{mA}; \qquad i_o(\infty)L_{\text{eq}} = (0.04)(0.05) = 2$ mWb-turns

$i_1(\infty)L_1 + i_2(\infty)M = (16\,\text{m})(500) + (24\,\text{m})(-250) = 2$ mWb-turns (ok)

$i_2(\infty)L_2 + i_1(\infty)M = (24\,\text{m})(250) + (16\,\text{m})(-250) = 2$ mWb-turns (ok)

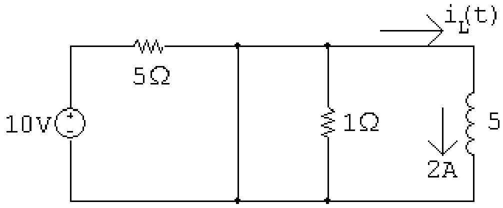
Therefore, the final values of i_o, i_1, and i_2 are consistent with conservation of flux linkage. Hence, the answers make sense in terms of known circuit behavior.

P 7.70 $t < 0$:



$$i_L(0^-) = 10\,\text{V}/5\,\Omega = 2\,\text{A} = i_L(0^+)$$

$0 \le t \le 5$:

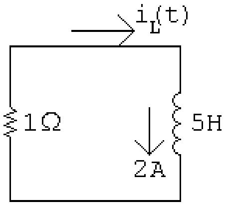


$$\tau = 5/0 = \infty$$

$$i_L(t) = 2e^{-t/\infty} = 2e^{-0} = 2$$

$$i_L(t) = 2\,\text{A} \quad 0 \le t \le 5\,\text{s}$$

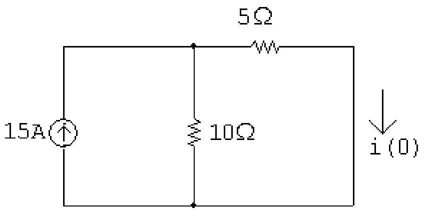
$5 \le t < \infty$:



$$\tau = \frac{5}{1} = 5\,\text{s}; \qquad 1/\tau = 0.2$$

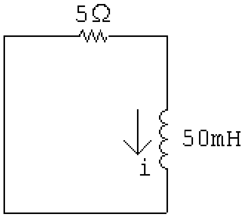
$$i_L(t) = 2e^{-0.2(t-5)}\,\text{A}, \quad t \ge 5\,\text{s}$$

P 7.71 For $t < 0$:



$$i(0) = \frac{10}{15}(15) = 10\,\text{A}$$

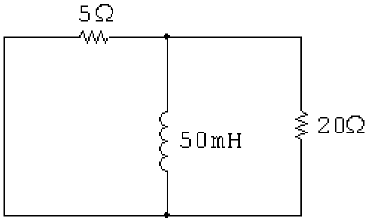
$0 \le t \le 10\,\text{ms}$:



$$i = 10e^{-100t}\,\text{A}$$

$$i(10\,\text{ms}) = 10e^{-1} = 3.68\,\text{A}$$

$10\,\text{ms} \le t \le 20\,\text{ms}$:



$$R_{\text{eq}} = \frac{(5)(20)}{25} = 4\,\Omega$$

$$\frac{1}{\tau} = \frac{R}{L} = \frac{4}{50 \times 10^{-3}} = 80$$

$$i = 3.68e^{-80(t-0.01)}\,\text{A}$$

$20\,\text{ms} \le t < \infty$:

$$i(20\,\text{ms}) = 3.68e^{-80(0.02-0.01)} = 1.65\,\text{A}$$

$$i = 1.65e^{-100(t-0.02)}\,\text{A}$$

$$v_o = L\frac{di}{dt}; \qquad L = 50\,\text{mH}$$

$$\frac{di}{dt} = 1.65(-100)e^{-100(t-0.02)} = -165e^{-100(t-0.02)}$$

$$v_o = (50 \times 10^{-3})(-165)e^{-100(t-0.02)}$$

$$= -8.26e^{-100(t-0.02)}\,\text{V}, \qquad t > 20^{+}\,\text{ms}$$

$$v_o(25\,\text{ms}) = -8.26e^{-100(0.025-0.02)} = -5.013\,\text{V}$$

P 7.72 From the solution to Problem 7.71, the initial energy is

$$w(0) = \frac{1}{2}(50\,\text{mH})(10\,\text{A})^2 = 2.5\,\text{J}$$

$$0.04w(0) = 0.1\,\text{J}$$

$$\therefore \quad \frac{1}{2}(50 \times 10^{-3})i_L^2 = 0.1 \quad \text{so} \quad i_L = 2\,\text{A}$$

Again, from the solution to Problem 7.73, t must be between 10 ms and 20 ms since

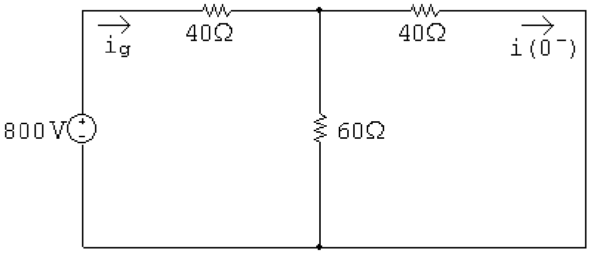
$$i(10\,\text{ms}) = 3.68\,\text{A} \quad \text{and} \quad i(20\,\text{ms}) = 1.65\,\text{A}$$

For $10\,\text{ms} \leq t \leq 20\,\text{ms}$:

$$i = 3.68e^{-80(t-0.01)} = 2$$

$$e^{80(t-0.01)} = \frac{3.68}{2} \quad \text{so} \quad t - 0.01 = 0.0076 \quad \therefore \quad t = 17.6\,\text{ms}$$

P 7.73 **[a]** $t < 0$:



Using Ohm's law,

$$i_g = \frac{800}{40 + 60\|40} = 12.5\,\text{A}$$

Using current division,

$$i(0^-) = \frac{60}{60 + 40}(12.5) = 7.5\,\text{A} = i(0^+)$$

[b] $0 \leq t \leq 1\,\text{ms}$:

$$i = i(0^+)e^{-t/\tau} = 7.5e^{-t/\tau}$$

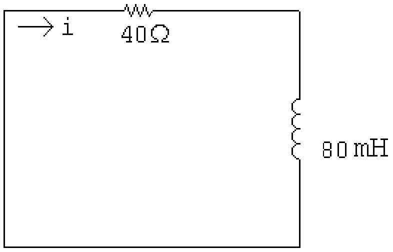
$$\frac{1}{\tau} = \frac{R}{L} = \frac{40 + 120\|60}{80 \times 10^{-3}} = 1000$$

$$i = 7.5e^{-1000t}$$

$$i(200\mu\text{s}) = 7.5e^{-10^3(200\times10^{-6})} = 7.5e^{-0.2} = 6.14\,\text{A}$$

[c] $i(1\,\text{ms}) = 7.5e^{-1} = 2.7591\,\text{A}$

$1\,\text{ms} \leq t < \infty$:



$$\frac{1}{\tau} = \frac{R}{L} = \frac{40}{80 \times 10^{-3}} = 500$$

$$i = i(1\,\text{ms})e^{-(t-1\,ms)/\tau} = 2.7591e^{-500(t-0.001)}\,\text{A}$$

$$i(6\text{ms}) = 2.7591e^{-500(0.005)} = 2.7591e^{-2.5} = 226.48\,\text{mA}$$

[**d**] $0 \leq t \leq 1\,\text{ms}$:

$$i = 7.5e^{-1000t}$$

$$v = L\frac{di}{dt} = (80 \times 10^{-3})(-1000)(7.5e^{-1000t}) = -600e^{-1000t}\,\text{V}$$

$$v(1^{-}\text{ms}) = -600e^{-1} = -220.73\,\text{V}$$

[**e**] $1\,\text{ms} \leq t \leq \infty$:

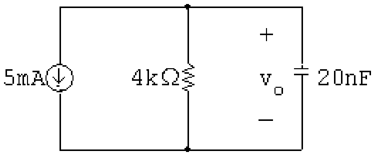
$$i = 2.759e^{-500(t-0.001)}$$

$$v = L\frac{di}{dt} = (80 \times 10^{-3})(-500)(2.759e^{-500(t-0.001)})$$

$$= -110.4e^{-500(t-0.001)}\,\text{V}$$

$$v(1^{+}\text{ms}) = -110.4\,\text{V}$$

P 7.74 $0 \leq t \leq 10\,\mu\text{s}$:

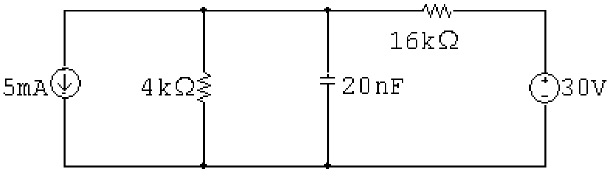


$$\tau = RC = (4 \times 10^{3})(20 \times 10^{-9}) = 80\,\mu\text{s}; \qquad 1/\tau = 12{,}500$$

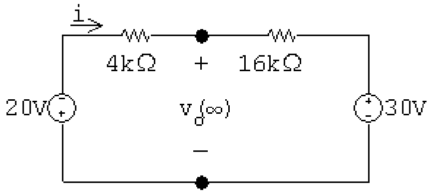
$$v_o(0) = 0\,\text{V}; \qquad v_o(\infty) = -20\,\text{V}$$

$$v_o = -20 + 20e^{-12{,}500t}\,\text{V} \qquad 0 \leq t \leq 10\,\mu\text{s}$$

$10\,\mu\text{s} \le t < \infty$:



$t \to \infty$:



$$i = \frac{-50\,\text{V}}{20\,\text{k}\Omega} = -2.5\,\text{mA}$$

$$v_o(\infty) = (-2.5\times10^{-3})(16{,}000) + 30 = -10\,\text{V}$$

$$v_o(10\,\mu\text{s}) = -20 + 20^{-0.125} = -2.35\,\text{V}$$

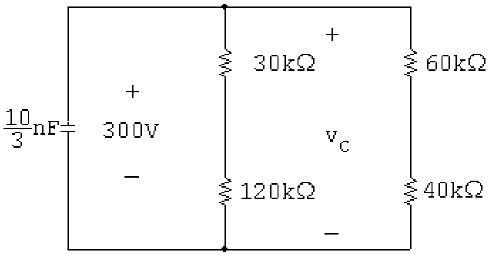
$$v_o = -10 + (-2.35+10)e^{-(t\,-\,10\times10^{-6})/\tau}$$

$$R_{\text{Th}} = 4\,\text{k}\Omega\|16\,\text{k}\Omega = 3.2\,\text{k}\Omega$$

$$\tau = (3200)(20\times10^{-9}) = 64\,\mu\text{s}; \qquad 1/\tau = 15{,}625$$

$$v_o = -10 + 7.65e^{-15{,}625(t\,-\,10\times10^{-6})} \qquad 10\,\mu\text{s} \le t < \infty$$

P 7.75 $0 \le t \le 200\,\mu\text{s}$;

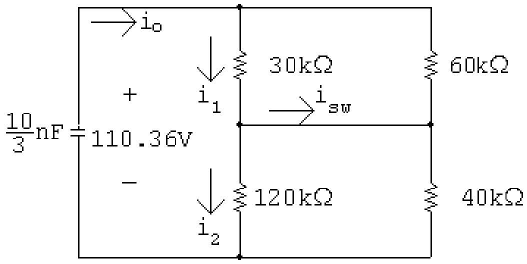


$$R_e = 150\|100 = 60\,\text{k}\Omega; \qquad \tau = \left(\frac{10}{3}\times10^{-9}\right)(60{,}000) = 200\,\mu\text{s}$$

$$v_c = 300e^{-5000t}\,\text{V}$$

$$v_c(200\,\mu\text{s}) = 300e^{-1} = 110.36\,\text{V}$$

$200\,\mu\text{s} \leq t < \infty$:



$$R_e = 30\|60 + 120\|40 = 20 + 30 = 50\,\text{k}\Omega$$

$$\tau = \left(\frac{10}{3} \times 10^{-9}\right)(50{,}000) = 166.67\,\mu\text{s}; \qquad \frac{1}{\tau} = 6000$$

$$v_c = 110.36e^{-6000(t - 200\,\mu s)}\,\text{V}$$

$$v_c(300\,\mu\text{s}) = 110.36e^{-6000(100\,\mu\text{S})} = 60.57\,\text{V}$$

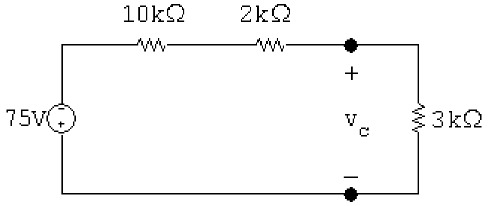
$$i_o(300\,\mu\text{s}) = \frac{60.57}{50{,}000} = 1.21\,\text{mA}$$

$$i_1 = \frac{60}{90}i_o = \frac{2}{3}i_o; \qquad i_2 = \frac{40}{160}i_o = \frac{1}{4}i_o$$

$$i_{\text{sw}} = i_1 - i_2 = \frac{2}{3}i_o - \frac{1}{4}i_o = \frac{5}{12}i_o = \frac{5}{12}(1.21 \times 10^{-3}) = 0.50\,\text{mA}$$

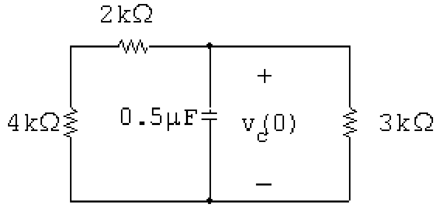
P 7.76 Note that for $t > 0$, $v_o = (4/6)v_c$, where v_c is the voltage across the $0.5\,\mu$F capacitor. Thus we will find v_c first.

$t < 0$



$$v_c(0) = \frac{3}{15}(-75) = -15\,\text{V}$$

$0 \le t \le 800\,\mu\text{s}$:



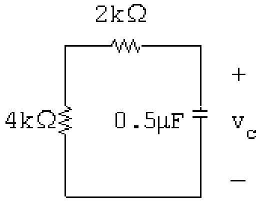
$$\tau = R_eC, \qquad R_e = \frac{(6000)(3000)}{9000} = 2\,\text{k}\Omega$$

$$\tau = (2\times10^3)(0.5\times10^{-6}) = 1\,\text{ms}, \qquad \frac{1}{\tau} = 1000$$

$$v_\text{c} = -15e^{-1000t}\,\text{V}, \qquad t \ge 0$$

$$v_\text{c}(800\,\mu\text{s}) = -15e^{-0.8} = -6.74\,\text{V}$$

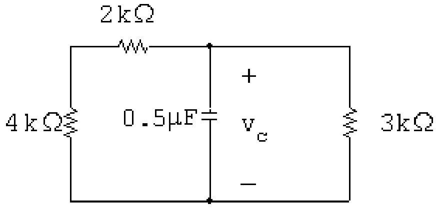
$800\,\mu\text{s} \le t \le 1.1\,\text{ms}$:



$$\tau = (6\times10^3)(0.5\times10^{-6}) = 3\,\text{ms}, \qquad \frac{1}{\tau} = 333.33$$

$$v_\text{c} = -6.74e^{-333.33(t-800\times10^{-6})}\,\text{V}$$

$1.1\,\text{ms} \le t < \infty$:



$$\tau = 1\,\text{ms}, \qquad \frac{1}{\tau} = 1000$$

$$v_\text{c}(1.1\text{ms}) = -6.74e^{-333.33(1100-800)10^{-6}} = -6.74e^{-0.1} = -6.1\,\text{V}$$

$$v_\text{c} = -6.1e^{-1000(t-1.1\times10^{-3})}\,\text{V}$$

$$v_\text{c}(1.5\text{ms}) = -6.1e^{-1000(1.5-1.1)10^{-3}} = -6.1e^{-0.4} = -4.09\,\text{V}$$

$$v_o = (4/6)(-4.09) = -2.73\,\text{V}$$

P 7.77 $w(0) = \frac{1}{2}(0.5 \times 10^{-6})(-15)^2 = 56.25\,\mu\text{J}$

$0 \leq t \leq 800\,\mu\text{s}$:

$v_c = -15e^{-1000t}; \qquad v_c^2 = 225e^{-2000t}$

$p_{3k} = 75e^{-2000t}\,\text{mW}$

$$\begin{aligned} w_{3k} &= \int_0^{800\times10^{-6}} 75 \times 10^{-3} e^{-2000t}\,dt \\ &= 75 \times 10^{-3} \frac{e^{-2000t}}{-2000}\Big|_0^{800\times10^{-6}} \\ &= -37.5 \times 10^{-6}(e^{-1.6} - 1) = 29.93\,\mu\text{J} \end{aligned}$$

$1.1\,\text{ms} \leq t \leq \infty$:

$v_c = -6.1e^{-1000(t-1.1\times10^{-3})}\,\text{V}; \qquad v_c^2 = 37.19e^{-2000(t-1.1\times10^{-3})}$

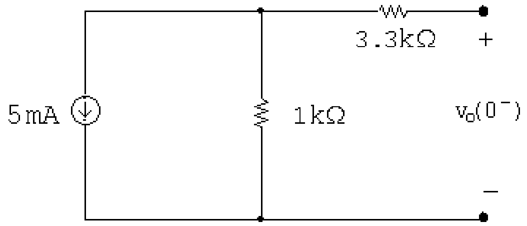
$p_{3k} = 12.4e^{-2000(t-1.1\times10^{-3})}\,\text{mW}$

$$\begin{aligned} w_{3k} &= \int_{1.1\times10^{-3}}^{\infty} 12.4 \times 10^{-3} e^{-2000(t-1.1\times10^{-3})}\,dt \\ &= 12.4 \times 10^{-3} \frac{e^{-2000(t-1.1\times10^{-3})}}{-2000}\Big|_{1.1\times10^{-3}}^{\infty} \\ &= -6.2 \times 10^{-6}(0 - 1) = 6.2\,\mu\text{J} \end{aligned}$$

$w_{3k} = 29.93 + 6.2 = 36.13\,\mu\text{J}$

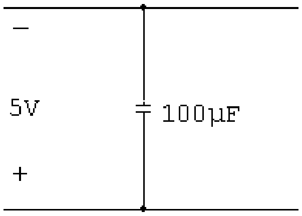
$\% = \frac{36.13}{56.25}(100) = 64.23\%$

P 7.78 $t < 0$:



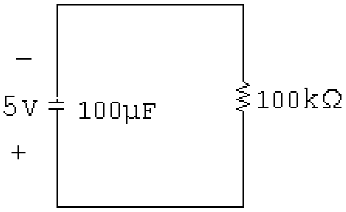
$v_c(0^-) = -(5)(1000) \times 10^{-3} = -5\,\text{V} = v_c(0^+)$

$0 \leq t \leq 5\,\text{s}$:



$$\tau = \infty; \qquad 1/\tau = 0; \qquad v_o = -5e^{-0} = -5\,\text{V}$$

$5\,\text{s} \leq t < \infty$:



$$\tau = (100)(0.1) = 10\,\text{s}; \qquad 1/\tau = 0.1; \qquad v_o = -5e^{-0.1(t-5)}\,\text{V}$$

Summary:

$$v_o = -5\,\text{V}, \qquad 0 \leq t \leq 5\,\text{s}$$

$$v_o = -5e^{-0.1(t-5)}\,\text{V}, \qquad 5\,\text{s} \leq t < \infty$$

P 7.79 **[a]** $0 \leq t \leq 2.5\,\text{ms}$

$$v_o(0^+) = 80\,\text{V}; \qquad v_o(\infty) = 0$$

$$\tau = \frac{L}{R} = 2\,\text{ms}; \qquad 1/\tau = 500$$

$$v_o(t) = 80e^{-500t}\,\text{V}, \qquad 0^+ \leq t \leq 2.5^-\,\text{ms}$$

$$v_o(2.5^-\,\text{ms}) = 80e^{-1.25} = 22.92\,\text{V}$$

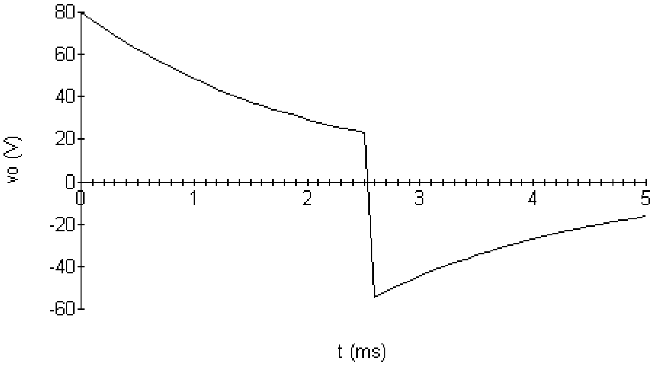
$$i_o(2.5^-\,\text{ms}) = \frac{(80 - 22.92)}{20} = 2.85\,\text{A}$$

$$v_o(2.5^+\,\text{ms}) = -20(2.85) = -57.08\,\text{V}$$

$$v_o(\infty) = 0; \qquad \tau = 2\,\text{ms}; \qquad 1/\tau = 500$$

$$v_o = -57.08e^{-500(t-0.0025)}\,\text{V} \qquad t \geq 2.5^+\,\text{ms}$$

[b]



[c] $v_o(5\,\text{ms}) = -16.35\,\text{V}$

$$i_o = \frac{+16.35}{20} = 817.68\,\text{mA}$$

P 7.80 [a] $i_o(0) = 0; \qquad i_o(\infty) = 25\,\text{mA}$

$$\frac{1}{\tau} = \frac{R}{L} = \frac{2000}{250} \times 10^3 = 8000$$

$$i_o = (25 - 25e^{-8000t})\,\text{mA}, \qquad 0 \le t \le 75\,\mu\text{s}$$

$$v_o = 0.25\frac{di_o}{dt} = 50e^{-8000t}\,\text{V}, \qquad 0 \le t \le 75\,\mu s$$

$75\,\mu\text{s} \le t < \infty$:

$$i_o(75\mu\text{s}) = 25 - 25e^{-0.6} = 11.28\,\text{mA}; \qquad i_o(\infty) = 0$$

$$i_o = 11.28e^{-8000(t-75\times10^{-6})}\,\text{mA}$$

$$v_o = (0.25)\frac{di_o}{dt} = -22.56e^{-8000(t-75\mu s)}$$

$$\begin{aligned} \therefore\quad t < 0: &\qquad v_o = 0 \\ 0 \le t \le 75\,\mu\text{s}: &\qquad v_o = 50e^{-8000t}\,\text{V} \\ 75\,\mu\text{s} \le t < \infty: &\qquad v_o = -22.56e^{-8000(t-75\mu s)} \end{aligned}$$

[b] $v_o(75^-\mu\text{s}) = 50e^{-0.6} = 27.44\,\text{V}$

$$v_o(75^+\mu\text{s}) = -22.56\,\text{V}$$

[c] $i_o(75^-\mu\text{s}) = i_o(75^+\mu\text{s}) = 11.28\,\text{mA}$

P 7.81 **[a]** $0 \le t \le 1\,\text{ms}$:

$$v_c(0^+) = 0; \qquad v_c(\infty) = 50\,\text{V};$$

$$RC = 400 \times 10^3(0.01 \times 10^{-6}) = 4\,\text{ms}; \qquad 1/RC = 250$$

$$v_c = 50 - 50e^{-250t}$$

$$v_o = 50 - 50 + 50e^{-250t} = 50e^{-250t}\,\text{V}, \qquad 0 \le t \le 1\,\text{ms}$$

$1\,\text{ms} \le t < \infty$:

$$v_c(1\,\text{ms}) = 50 - 50e^{-0.25} = 11.06\,\text{V}$$

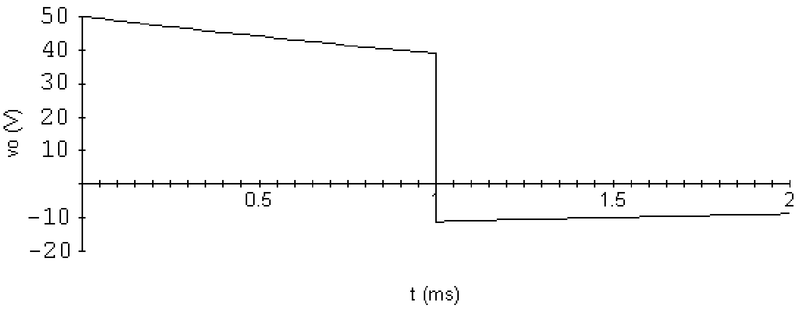
$$v_c(\infty) = 0\,\text{V}$$

$$\tau = 4\,\text{ms}; \qquad 1/\tau = 250$$

$$v_c = 11.06e^{-250(t-0.001)}\,\text{V}$$

$$v_o = -v_c = -11.06e^{-250(t-0.001)}\,\text{V}, \qquad t \ge 1\,\text{ms}$$

[b]



P 7.82 **[a]** $t < 0; \qquad v_o = 0$

$0 \le t \le 4\,\text{ms}$:

$$\tau = (200 \times 10^3)(0.025 \times 10^{-6}) = 5\,\text{ms}; \qquad 1/\tau = 200$$

$$v_o = 100 - 100e^{-200t}\,\text{V}, \qquad 0 \le t \le 4\,\text{ms}$$

$$v_o(4\,\text{ms}) = 100(1 - e^{-0.8}) = 55.07\,\text{V}$$

$4\,\text{ms} \le t \le 8\,\text{ms}$:

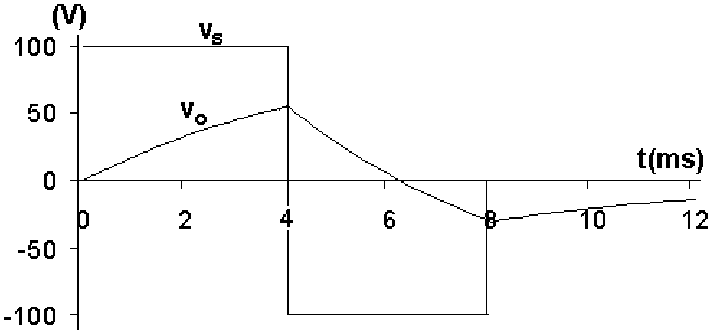
$$v_o = -100 + 155.07e^{-200(t-0.004)}\,\text{V}, \quad 4\,\text{ms} \le t \le 8\,\text{ms}$$

$$v_o(8\,\text{ms}) = -100 + 155.07e^{-0.8} = -30.32\,\text{V}$$

$t \ge 8\,\text{ms}$:

$$v_o = -30.32e^{-200(t-0.008)}\,\text{V}, \quad t \ge 8\,\text{ms}$$

[b]



[c] $t \leq 0: \qquad v_o = 0$

$0 \leq t \leq 4\,\text{ms}$:

$$\tau = (50 \times 10^3)(0.025 \times 10^{-6}) = 1.25\,\text{ms} \quad 1/\tau = 800$$

$$v_o = 100 - 100e^{-800t}\,\text{V}, \qquad 0 \leq t \leq 4\,\text{ms}$$

$$v_o(4\,\text{ms}) = 100 - 100e^{-3.2} = 95.92\,\text{V}$$

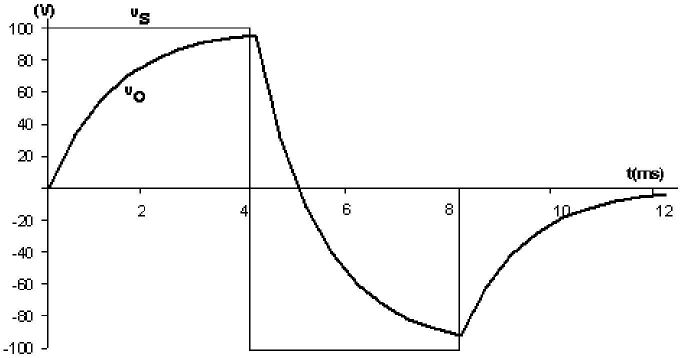
$4\,\text{ms} \leq t \leq 8\,\text{ms}$:

$$v_o = -100 + 195.92e^{-800(t-0.004)}\,\text{V}, \qquad 4\,\text{ms} \leq t \leq 8\,\text{ms}$$

$$v_o(8\,\text{ms}) = -100 + 195.92e^{-3.2} = -92.01\,\text{V}$$

$t \geq 8\,\text{ms}$:

$$v_o = -92.01e^{-800(t-0.008)}\,\text{V}, \quad t \geq 8\,\text{ms}$$



P 7.83 [a] $\tau = RC = (20{,}000)(0.2 \times 10^{-6}) = 4\,\text{ms}; \qquad 1/\tau = 250$

$$i_o = v_o = 0 \qquad t < 0$$

$$i_o(0^+) = 20\left(\frac{16}{20}\right) = 16\,\text{mA}, \qquad i_o(\infty) = 0$$

$$\therefore \quad i_o = 16e^{-250t}\,\text{mA} \qquad 0^+ \leq t \leq 2^-\,\text{ms}$$

$$i_{16k\Omega} = 20 - 16e^{-250t}\,\text{mA}$$

$$\therefore\quad v_o = 320 - 256e^{-250t}\,\text{V} \qquad 0^+ \leq t \leq 2^-\,\text{ms}$$

$$v_c = v_o - 4\times 10^3 i_o = 320 - 320e^{-250t}\,\text{V} \qquad 0 \leq t \leq 2\,\text{ms}$$

$$v_c(2\,\text{ms}) = 320 - 320e^{-0.5} = 125.91\,\text{V}$$

$$\therefore\quad i_o(2^+\,\text{ms}) = 16e^{-0.5} = 9.7\,\text{mA}$$

$$i_o(\infty) = 0$$

$$v_c = 125.91e^{-250(t-0.002)}, \quad t \geq 2\,\text{ms}$$

$$i_o = C\frac{dv_c}{dt} = (0.2\times 10^{-6})(-250)(125.91)e^{-250(t-0.002)}$$

$$= -6.3e^{-250(t-0.002)}\,\text{mA}, \quad t \geq 2^+\,\text{ms}$$

$$v_o = 4000i_o + v_c = 100.73e^{-250(t-0.002)}\,\text{V} \qquad t \geq 2^+\,\text{ms}$$

Summary part (a)

$$i_o = 0 \qquad t < 0$$

$$i_o = 16e^{-250t}\,\text{mA} \qquad (0^+ \leq t \leq 2^-\,\text{ms})$$

$$i_o = -6.3e^{-250(t-0.002)}\,\text{mA} \qquad t \geq 2^+\,\text{ms}$$

$$v_o = 0 \qquad t < 0$$

$$v_o = 320 - 256e^{-250t}\,\text{V}, \qquad 0^+ \leq t \leq 2^-\,\text{ms}$$

$$v_o = 100.73e^{-250(t-0.002)}\,\text{V}, \qquad t \geq 2^+\,\text{ms}$$

[b] $i_o(0^-) = 0$

$$i_o(0^+) = 16\,\text{mA}$$

$$i_o(2^-\,\text{ms}) = 16e^{-0.5} = 9.7\,\text{mA}$$

$$i_o(2^+\,\text{ms}) = -6.3\,\text{mA}$$

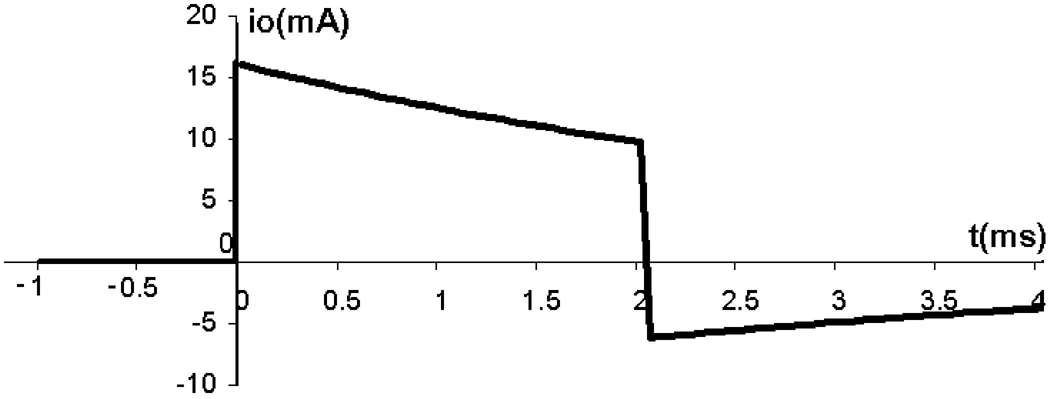
[c] $v_o(0^-) = 0$

$$v_o(0^+) = 64\,\text{V}$$

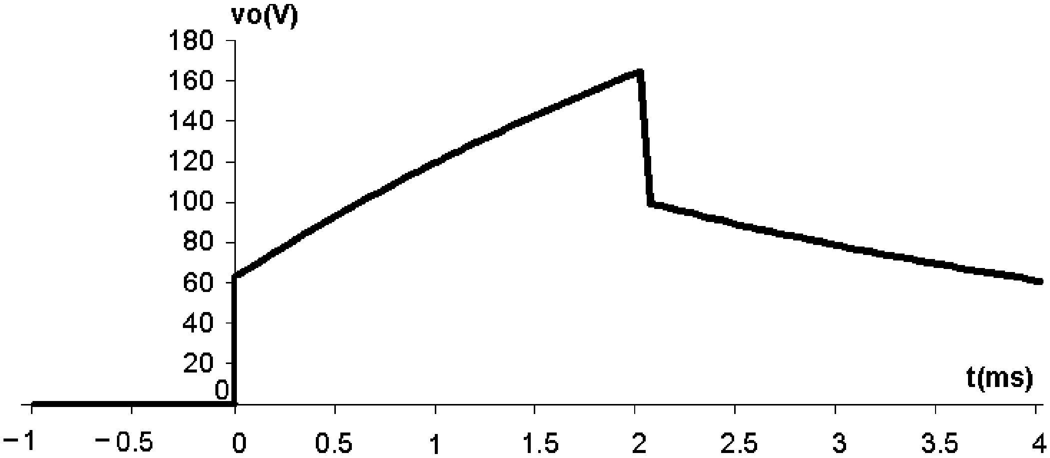
$$v_o(2^-\,\text{ms}) = 320 - 256e^{-0.5} = 164.73\,\text{V}$$

$$v_o(2^+\,\text{ms}) = 100.73$$

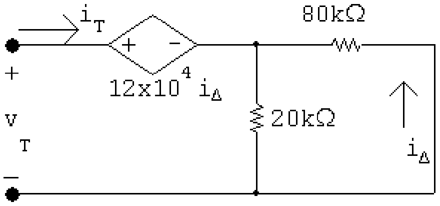
[d]



[e]



P 7.84 $t > 0$:



$$v_T = 12 \times 10^4 i_\Delta + 16 \times 10^3 i_T$$

$$i_\Delta = -\frac{20}{100} i_T = -0.2 i_T$$

$$\therefore \quad v_T = -24 \times 10^3 i_T + 16 \times 10^3 i_T$$

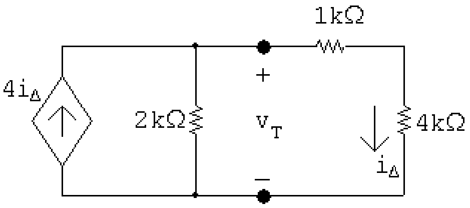
$$R_{\text{Th}} = \frac{v_T}{i_T} = -8\,\text{k}\Omega$$

$$\tau = RC = (-8 \times 10^3)(2.5 \times 10^{-6}) = -0.02 \quad 1/\tau = -50$$

$v_c = 20e^{50t}\,\mathrm{V}; \qquad 20e^{50t} = 20{,}000$

$50t = \ln 1000 \quad \therefore \quad t = 138.16\,\mathrm{ms}$

P 7.85 Find the Thévenin equivalent with respect to the terminals of the capacitor. R_{Th} calculation:

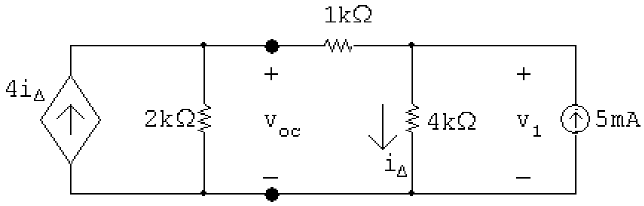


$$i_T = \frac{v_T}{2000} + \frac{v_T}{5000} - 4\frac{v_T}{5000}$$

$$\therefore \quad \frac{i_T}{v_T} = \frac{5+2-8}{10{,}000} = -\frac{1}{10{,}000}$$

$$\frac{v_T}{i_T} = -\frac{10{,}000}{1} = -10\,\mathrm{k}\Omega$$

Open circuit voltage calculation:



The node voltage equations:

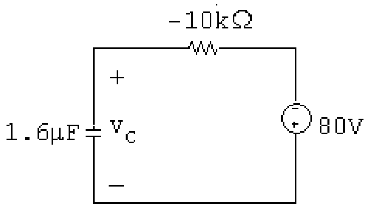
$$\frac{v_{\mathrm{oc}}}{2000} + \frac{v_{\mathrm{oc}} - v_1}{1000} - 4i_\Delta = 0$$

$$\frac{v_1 - v_{\mathrm{oc}}}{1000} + \frac{v_1}{4000} - 5 \times 10^{-3} = 0$$

The constraint equation:

$$i_\Delta = \frac{v_1}{4000}$$

Solving, $v_{\mathrm{oc}} = -80\,\mathrm{V}, \quad v_1 = -60\,\mathrm{V}$



$v_c(0) = 0; \qquad v_c(\infty) = -80\,\text{V}$

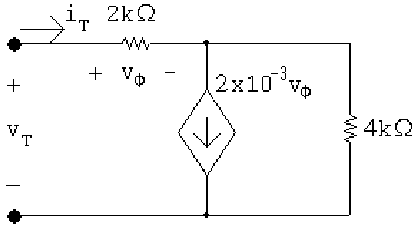
$\tau = RC = (-10{,}000)(1.6 \times 10^{-6}) = -16\,\text{ms}; \qquad \frac{1}{\tau} = -62.5$

$v_c = v_c(\infty) + [v_c(0^+) - v_c(\infty)]e^{-t/\tau} = -80 + 80e^{62.5t} = 14{,}400$

Solve for the time of the maximum voltage rating:

$e^{62.5t} = 181; \qquad 62.5t = \ln 181; \qquad t = 83.09\,\text{ms}$

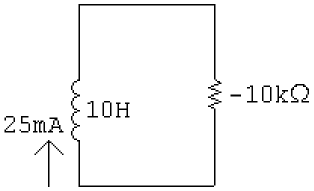
P 7.86



$v_T = 2000i_T + 4000(i_T - 2 \times 10^{-3}v_\phi) = 6000i_T - 8v_\phi$

$= 6000i_T - 8(2000i_T)$

$\frac{v_T}{i_T} = -10{,}000$

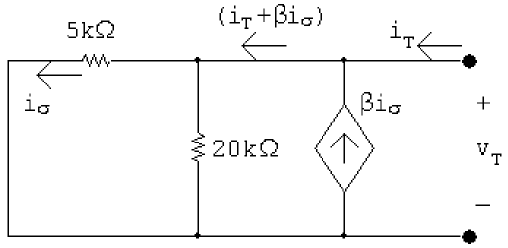


$\tau = \frac{10}{-10{,}000} = -1\,\text{ms}; \qquad 1/\tau = -1000$

$i = 25e^{1000t}\,\text{mA}$

$\therefore \quad 25e^{1000t} \times 10^{-3} = 5; \qquad t = \frac{\ln 200}{1000} = 5.3\,\text{ms}$

P 7.87 [**a**]



Using Ohm's law,

$$v_T = 5000 i_\sigma$$

Using current division,

$$i_\sigma = \frac{20{,}000}{20{,}000 + 5000}(i_T + \beta i_\sigma) = 0.8 i_T + 0.8\beta i_\sigma$$

Solve for i_σ:

$$i_\sigma(1 - 0.8\beta) = 0.8 i_T$$

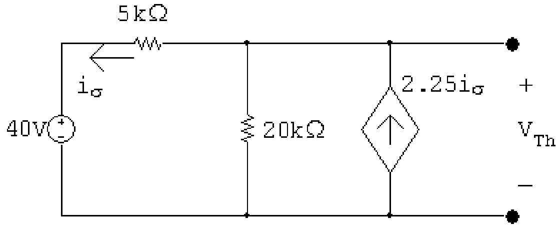
$$i_\sigma = \frac{0.8 i_T}{1 - 0.8\beta}; \qquad v_T = 5000 i_\sigma = \frac{4000 i_T}{(1 - 0.8\beta)}$$

Find β such that $R_{\rm Th} = -5\,{\rm k}\Omega$:

$$R_{\rm Th} = \frac{v_T}{i_T} = \frac{4000}{1 - 0.8\beta} = -5000$$

$$1 - 0.8\beta = -0.8 \qquad \therefore \quad \beta = 2.25$$

[**b**] Find $V_{\rm Th}$;



Write a KCL equation at the top node:

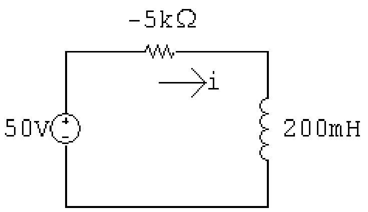
$$\frac{V_{\rm Th} - 40}{5000} + \frac{V_{\rm Th}}{20{,}000} - 2.25 i_\sigma = 0$$

The constraint equation is:

$$i_\sigma = \frac{(V_{\rm Th} - 40)}{5000} = 0$$

Solving,

$$V_{\rm Th} = 50\,{\rm V}$$



Write a KVL equation around the loop:

$$50 = -5000i + 0.2\frac{di}{dt}$$

Rearranging:

$$\frac{di}{dt} = 250 + 25{,}000i = 25{,}000(i + 0.01)$$

Separate the variables and integrate to find i;

$$\frac{di}{i + 0.01} = 25{,}000\,dt$$

$$\int_0^i \frac{dx}{x + 0.01} = \int_0^t 25{,}000\,dx$$

$$\therefore \quad i = -10 + 10e^{25{,}000t}\,\text{mA}$$

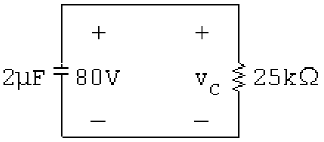
$$\frac{di}{dt} = (10 \times 10^{-3})(25{,}000)e^{25{,}000t} = 250e^{25{,}000t}$$

Solve for the arc time:

$$v = 0.2\frac{di}{dt} = 50e^{25{,}000t} = 45{,}000; \qquad e^{25{,}000t} = 900$$

$$\therefore \quad t = \frac{\ln 900}{25{,}000} = 272.1\,\mu\text{s}$$

P 7.88 **[a]**



$$\tau = (25)(2) \times 10^{-3} = 50\,\text{ms}; \qquad 1/\tau = 20$$

$$v_c(0^+) = 80\,\text{V}; \qquad v_c(\infty) = 0$$

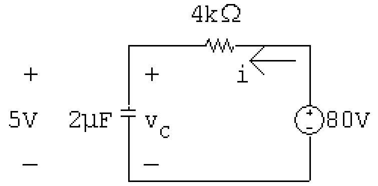
$$v_c = 80e^{-20t}\,\text{V}$$

$$\therefore \quad 80e^{-20t} = 5; \qquad e^{20t} = 16; \qquad t = \frac{\ln 16}{20} = 138.63\,\text{ms}$$

[b] $0^+ \leq t \leq 138.63^-$ ms:

$$i = (2 \times 10^{-6})(-1600e^{-20t}) = -3.2e^{-20t}\,\text{mA}$$

$t \geq 138.63^+$ ms:



$$\tau = (2)(4) \times 10^{-3} = 8\,\text{ms}; \qquad 1/\tau = 125$$

$$v_c(138.63^+\,\text{ms}) = 5\,\text{V}; \qquad v_c(\infty) = 80\,\text{V}$$

$$v_c = 80 - 75e^{-125(t-0.13863)}\,\text{V}, \qquad t \geq 138.63\,\text{ms}$$

$$i = 2 \times 10^{-6}(9375)e^{-125(t-0.13863)}$$
$$= 18.75e^{-125(t-0.13863)}\,\text{mA}, \qquad t \geq 138.63^+\,\text{ms}$$

[c] $80 - 75e^{-125\Delta t} = 0.85(80) = 68$

$$80 - 68 = 75e^{-125\Delta t} = 12$$

$$e^{125\Delta t} = 6.25; \qquad \Delta t = \frac{\ln 6.25}{125} \cong 14.66\,\text{ms}$$

P 7.89 **[a]** $RC = (25 \times 10^3)(0.4 \times 10^{-6}) = 10\,\text{ms}; \qquad \frac{1}{RC} = 100$

$$v_o = 0, \quad t < 0$$

[b] $0 \leq t \leq 250\,\text{ms}:$

$$v_o = -100\int_0^t -0.20\,dx = 20t\,\text{V}$$

[c] $250\,\text{ms} \leq t \leq 500\,\text{ms};$

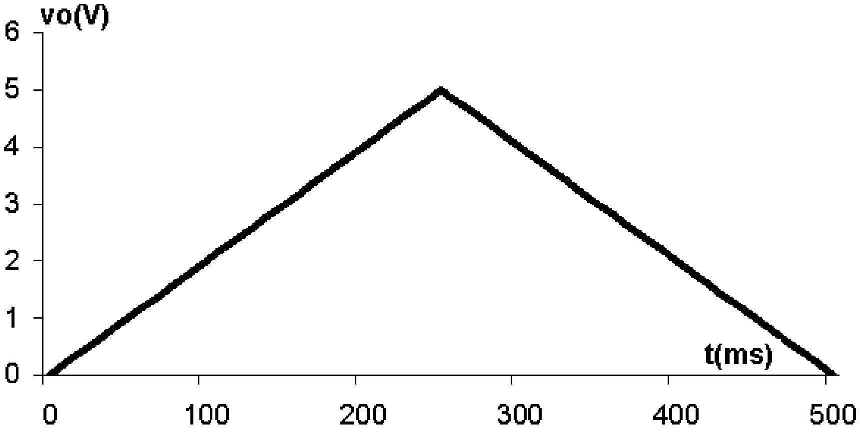
$$v_o(0.25) = 20(0.25) = 5\,\text{V}$$

$$v_o(t) = -100\int_{0.25}^t 0.20\,dx + 5 = -20(t - 0.25) + 5 = -20t + 10\,\text{V}$$

[d] $t \geq 500\,\text{ms}:$

$$v_o(0.5) = -10 + 10 = 0\,\text{V}$$

$$v_o(t) = 0\,\text{V}$$



P 7.90 [a] $v_o = 0, \qquad t < 0$

$$RC = (25\times10^3)(0.4\times10^{-6}) = 10\,\text{ms} \qquad \frac{1}{RC} = 100$$

[b] $R_fC_f = (5\times10^6)(0.4\times10^{-6}) = 2; \qquad \dfrac{1}{R_fC_f} = 0.5$

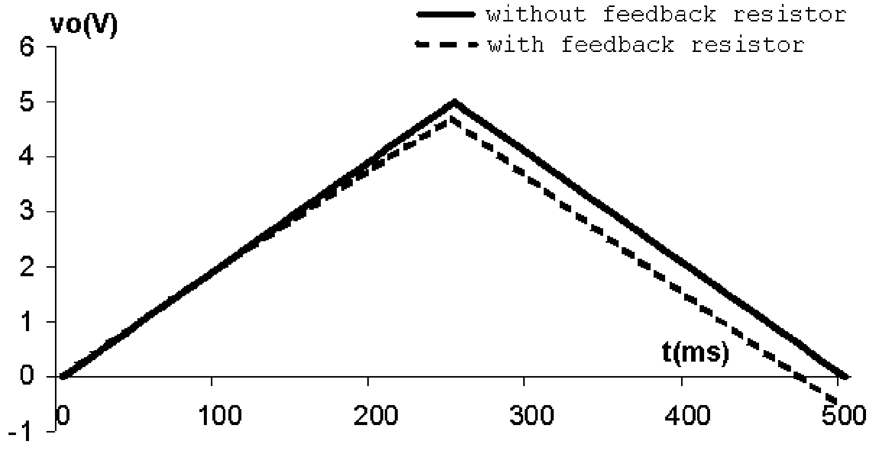
$$v_o = \frac{-5\times10^6}{25\times10^3}(-0.2)[1-e^{-0.5t}] = 40(1-e^{-0.5t})\,\text{V}, \qquad 0 \le t \le 250\,\text{ms}$$

[c] $v_o(0.25) = 40(1-e^{-0.125}) \cong 4.70\,\text{V}$

$$\begin{aligned} v_o &= \frac{-V_mR_f}{R_s} + \frac{V_mR_f}{R_s}(2-e^{-0.125})e^{-0.5(t-0.25)} \\ &= -40 + 40(2-e^{-0.125})e^{-0.5(t-0.25)} \\ &= -40 + 44.70e^{-0.5(t-0.25)}\,\text{V}, \qquad 250\,\text{ms} \le t \le 500\,\text{ms} \end{aligned}$$

[d] $v_o(0.5) = -40 + 44.70e^{-0.125} \cong -0.55\,\text{V}$

$$v_o = -0.55e^{-0.5(t-0.5)}\,\text{V}, \qquad t \ge 500\,\text{ms}$$



P 7.91 $$v_o = -\frac{1}{R(0.5\times 10^{-6})}\int_0^t 4\,dx + 0 = \frac{-4t}{R(0.5\times 10^{-6})}$$

$$\frac{-4(15\times 10^{-3})}{R(0.5\times 10^{-6})} = -10$$

$$\therefore \qquad R = \frac{-4(15\times 10^{-3})}{-10(0.5\times 10^{-6})} = 12\,\text{k}\Omega$$

P 7.92 $$v_o = \frac{-4t}{R(0.5\times 10^{-6})} + 6 = \frac{-4(40\times 10^{-3})}{R(0.5\times 10^{-6})} + 6 = -10$$

$$\therefore \qquad R = \frac{-4(40\times 10^{-3})}{-16(0.5\times 10^{-6})} = 20\,\text{k}\Omega$$

P 7.93 **[a]** $RC = (1000)(800\times 10^{-12}) = 800\times 10^{-9};\qquad \frac{1}{RC} = 1{,}250{,}000$

$0 \le t \le 1\,\mu$s:

$$v_g = 2\times 10^6 t$$

$$v_o = -1.25\times 10^6\int_0^t 2\times 10^6 x\,dx + 0$$

$$= -2.5\times 10^{12}\frac{x^2}{2}\Big|_0^t = -125\times 10^{10}t^2\,\text{V},\quad 0\le t\le 1\,\mu\text{s}$$

$$v_o(1\,\mu\text{s}) = -125\times 10^{10}(1\times 10^{-6})^2 = -1.25\,\text{V}$$

$1\,\mu\text{s} \le t \le 3\,\mu$s:

$$v_g = 4 - 2\times 10^6 t$$

$$v_o = -125\times 10^4\int_{1\times 10^{-6}}^t (4 - 2\times 10^6 x)\,dx - 1.25$$

$$= -125\times 10^4\left[4x\Big|_{1\times 10^{-6}}^t - 2\times 10^6\frac{x^2}{2}\Big|_{1\times 10^{-6}}^t\right] - 1.25$$

$$= -5\times 10^6 t + 5 + 125\times 10^{10}t^2 - 1.25 - 1.25$$

$$= 125\times 10^{10}t^2 - 5\times 10^6 t + 2.5\,\text{V},\quad 1\,\mu\text{s}\le t\le 3\,\mu\text{s}$$

$$v_o(3\,\mu\text{s}) = 125\times 10^{10}(3\times 10^{-6})^2 - 5\times 10^6(3\times 10^{-6}) + 2.5$$

$$= -1.25$$

$3\,\mu\text{s} \le t \le 4\,\mu$s:

$$v_g = -8 + 2\times 10^6 t$$

$$v_o = -125\times 10^4\int_{3\times 10^{-6}}^t (-8 + 2\times 10^6 x)\,dx - 1.25$$

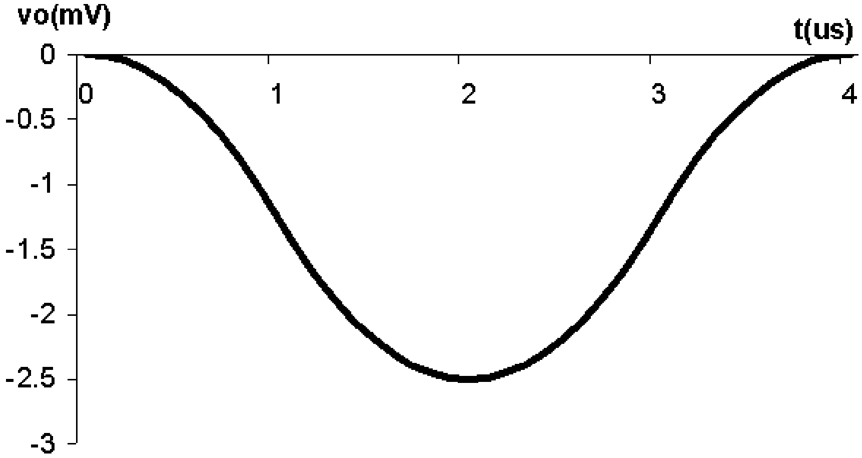
$$= -125\times 10^4\left[-8x\Big|_{3\times 10^{-6}}^t + 2\times 10^6\frac{x^2}{2}\Big|_{3\times 10^{-6}}^t\right] - 1.25$$

$$= 10^7 t - 30 - 125\times 10^{10}t^2 + 11.25 - 1.25$$

$$= -125\times 10^{10}t^2 + 10^7 t - 20\,\text{V},\quad 3\,\mu\text{s}\le t\le 4\,\mu\text{s}$$

$$v_o(4\,\mu\text{s}) = -125\times10^{10}(4\times10^{-6})^2 + 10^7(4\times10^{-6}) - 20 = 0$$

[b]



[c] The output voltage will also repeat. This follows from the observation that at $t = 4\,\mu$s the output voltage is zero, hence there is no energy stored in the capacitor. This means the circuit is in the same state at $t = 4\,\mu$s as it was at $t = 0$, thus as v_g repeats itself, so will v_o.

P 7.94 [a] $\dfrac{C dv_p}{dt} + \dfrac{v_p - v_\text{b}}{R} = 0;$ therefore $\dfrac{dv_p}{dt} + \dfrac{1}{RC}v_p = \dfrac{v_\text{b}}{RC}$

$$\frac{v_n - v_\text{a}}{R} + C\frac{d(v_n - v_o)}{dt} = 0;$$

$$\text{therefore}\quad \frac{dv_o}{dt} = \frac{dv_n}{dt} + \frac{v_n}{RC} - \frac{v_\text{a}}{RC}$$

But $v_n = v_p$

$$\text{Therefore}\quad \frac{dv_n}{dt} + \frac{v_n}{RC} = \frac{dv_p}{dt} + \frac{v_p}{RC} = \frac{v_\text{b}}{RC}$$

$$\text{Therefore}\quad \frac{dv_o}{dt} = \frac{1}{RC}(v_\text{b} - v_\text{a});\qquad v_o = \frac{1}{RC}\int_0^t (v_\text{b} - v_\text{a})\,dy$$

[b] The output is the integral of the difference between v_b and v_a and then scaled by a factor of $1/RC$.

[c] $v_o = \dfrac{1}{RC}\displaystyle\int_0^t (v_\text{b} - v_\text{a})\,dx$

$$RC = (50\times10^3)(10\times10^{-9}) = 0.5\,\text{ms}$$

$$v_\text{b} - v_\text{a} = -25\,\text{mV}$$

$$v_o = \frac{1}{0.0005}\int_0^t -25\times10^{-3}dx = -50t$$

$$-50t_\text{sat} = -6;\qquad t_\text{sat} = 120\,\text{ms}$$

P 7.95 The equation for an integrating amplifier:

$$v_o = \frac{1}{RC}\int_0^t (v_{\rm b} - v_{\rm a})\,dy + v_o(0)$$

Find the values and substitute them into the equation:

$$RC = (100 \times 10^3)(0.05 \times 10^{-6}) = 5\,\text{ms}$$

$$\frac{1}{RC} = 200; \qquad v_{\rm b} - v_{\rm a} = -15 - (-7) = -8\,\text{V}$$

$$v_o(0) = -4 + 12 = 8\,\text{V}$$

$$v_o = 200\int_0^t -8\,dx + 8 = (-1600t + 8)\,\text{V}, \quad 0 \le t \le t_{\rm sat}$$

RC circuit analysis for v_2:

$$v_2(0^+) = -4\,\text{V}; \qquad v_2(\infty) = -15\,\text{V}; \qquad \tau = RC = (100\,\text{k})(0.05\,\mu) = 5\,\text{ms}$$

$$v_2 = v_2(\infty) + [v_2(0^+) - v_2(\infty)]e^{-t/\tau}$$

$$= -15 + (-4 + 15)e^{-200t} = -15 + 11e^{-200t}\,\text{V}, \quad 0 \le t \le t_{\rm sat}$$

$$v_f + v_2 = v_o \quad \therefore \quad v_f = v_o - v_2 = 23 - 1600t - 11e^{-200t}\,\text{V}, \quad 0 \le t \le t_{\rm sat}$$

Note that

$$-1600t_{\rm sat} + 8 = -20 \quad \therefore \quad t_{\rm sat} = \frac{-28}{-1600} = 17.5\,\text{ms}$$

so the op amp operates in its linear region until it saturates at 17.5 ms.

P 7.96 Use voltage division to find the voltage at the non-inverting terminal:

$$v_p = \frac{80}{100}(-45) = -36\,\text{V} = v_n$$

Write a KCL equation at the inverting terminal:

$$\frac{-36 - 14}{80{,}000} + 2.5 \times 10^{-6}\frac{d}{dt}(-36 - v_o) = 0$$

$$\therefore \quad 2.5 \times 10^{-6}\frac{dv_o}{dt} = \frac{-50}{80{,}000}$$

Separate the variables and integrate:

$$\frac{dv_o}{dt} = -250 \quad \therefore \quad dv_o = -250dt$$

$$\int_{v_o(0)}^{v_o(t)} dx = -250\int_0^t dy \quad \therefore \quad v_o(t) - v_o(0) = -250t$$

$$v_o(0) = -36 + 56 = 20\,\text{V}$$

$$v_o(t) = -250t + 20$$

Find the time when the voltage reaches 0:

$$0 = -250t + 20 \quad \therefore \quad t = \frac{20}{250} = 80\,\text{ms}$$

P 7.97 [**a**] T_2 is normally ON since its base current i_{b2} is greater than zero, i.e., $i_{b2} = V_{CC}/R$ when T_2 is ON. When T_2 is ON, $v_{ce2} = 0$, therefore $i_{b1} = 0$. When $i_{b1} = 0$, T_1 is OFF. When T_1 is OFF and T_2 is ON, the capacitor C is charged to V_{CC}, positive at the left terminal. This is a stable state; there is nothing to disturb this condition if the circuit is left to itself.

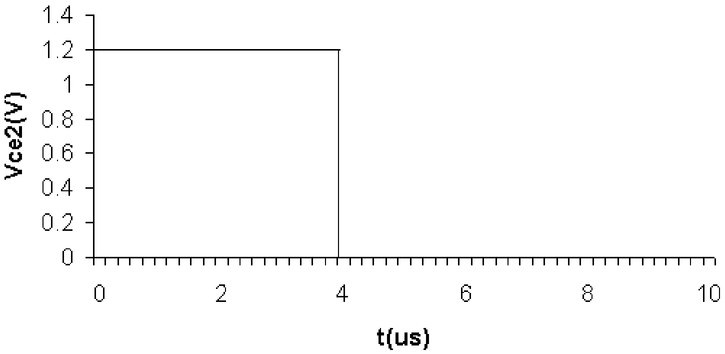
[**b**] When S is closed momentarily, v_{be2} is changed to $-V_{CC}$ and T_2 snaps OFF. The instant T_2 turns OFF, v_{ce2} jumps to $V_{CC}R_1/(R_1 + R_L)$ and i_{b1} jumps to $V_{CC}/(R_1 + R_L)$, which turns T_1 ON.

[**c**] As soon as T_1 turns ON, the charge on C starts to reverse polarity. Since v_{be2} is the same as the voltage across C, it starts to increase from $-V_{CC}$ toward $+V_{CC}$. However, T_2 turns ON as soon as $v_{be2} = 0$. The equation for v_{be2} is $v_{be2} = V_{CC} - 2V_{CC}e^{-t/RC}$. $v_{be2} = 0$ when $t = RC \ln 2$, therefore T_2 stays OFF for $RC \ln 2$ seconds.

P 7.98 [**a**] For $t < 0$, $v_{ce2} = 0$. When the switch is momentarily closed, v_{ce2} jumps to

$$v_{ce2} = \left(\frac{V_{CC}}{R_1 + R_L}\right)R_1 = \frac{6(5)}{25} = 1.2\,\text{V}$$

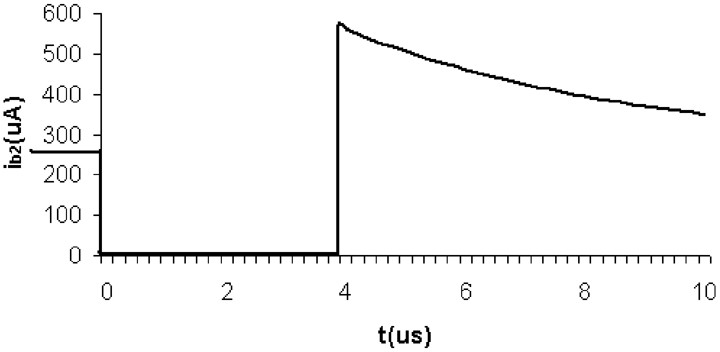
T_2 remains open for (23,083)(250) $\times 10^{-12} \ln 2 \cong 4\,\mu$s.



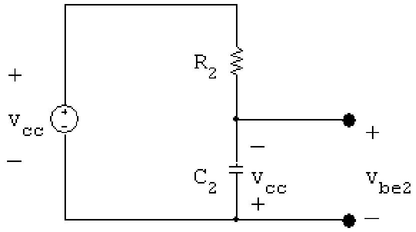
[b] $i_{\rm b2} = \dfrac{V_{CC}}{R} = 259.93\,\mu\text{A}, \qquad -5 \le t \le 0\,\mu\text{s}$

$$i_{\rm b2} = 0, \qquad 0 < t < RC\ln 2$$

$$\begin{aligned} i_{\rm b2} &= \frac{V_{CC}}{R} + \frac{V_{CC}}{R_{\rm L}} e^{-(t-RC\ln 2)/R_{\rm L}C} \\ &= 259.93 + 300e^{-0.2\times 10^{6}(t-4\times 10^{-6})}\,\mu\text{A}, \qquad RC\ln 2 < t \end{aligned}$$

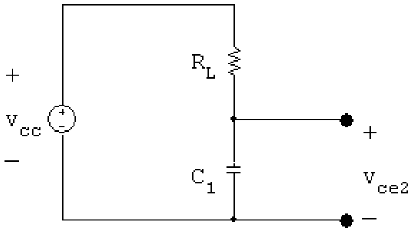


P 7.99 [a] While T_2 has been ON, C_2 is charged to V_{CC}, positive on the left terminal. At the instant T_1 turns ON the capacitor C_2 is connected across $b_2 - e_2$, thus $v_{\rm be2} = -V_{CC}$. This negative voltage snaps T_2 OFF. Now the polarity of the voltage on C_2 starts to reverse, that is, the right-hand terminal of C_2 starts to charge toward $+V_{CC}$. At the same time, C_1 is charging toward V_{CC}, positive on the right. At the instant the charge on C_2 reaches zero, $v_{\rm be2}$ is zero, T_2 turns ON. This makes $v_{\rm be1} = -V_{CC}$ and T_1 snaps OFF. Now the capacitors C_1 and C_2 start to charge with the polarities to turn T_1 ON and T_2 OFF. This switching action repeats itself over and over as long as the circuit is energized. At the instant T_1 turns ON, the voltage controlling the state of T_2 is governed by the following circuit:



It follows that $v_{\rm be2} = V_{CC} - 2V_{CC}e^{-t/R_2C_2}$.

[b] While T_2 is OFF and T_1 is ON, the output voltage $v_{\rm ce2}$ is the same as the voltage across C_1, thus



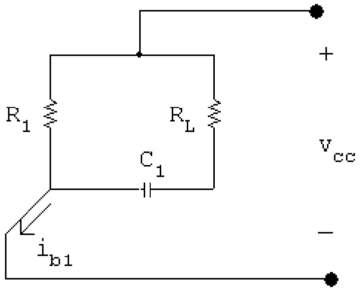
It follows that $v_{\rm ce2} = V_{CC} - V_{CC}e^{-t/R_{\rm L}C_1}$.

[c] T_2 will be OFF until $v_{\rm be2}$ reaches zero. As soon as $v_{\rm be2}$ is zero, $i_{\rm b2}$ will become positive and turn T_2 ON. $v_{\rm be2} = 0$ when $V_{CC} - 2V_{CC}e^{-t/R_2C_2} = 0$, or when $t = R_2C_2 \ln 2$.

[d] When $t = R_2C_2 \ln 2$, we have

$$v_{\rm ce2} = V_{CC} - V_{CC}e^{-[(R_2C_2 \ln 2)/(R_{\rm L}C_1)]} = V_{CC} - V_{CC}e^{-10 \ln 2} \cong V_{CC}$$

[e] Before T_1 turns ON, $i_{\rm b1}$ is zero. At the instant T_1 turns ON, we have

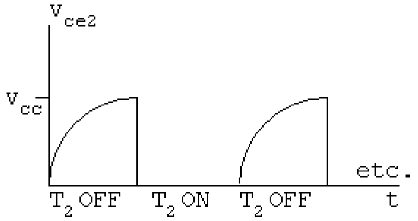


$$i_{b1} = \frac{V_{CC}}{R_1} + \frac{V_{CC}}{R_{\rm L}}e^{-t/R_{\rm L}C_1}$$

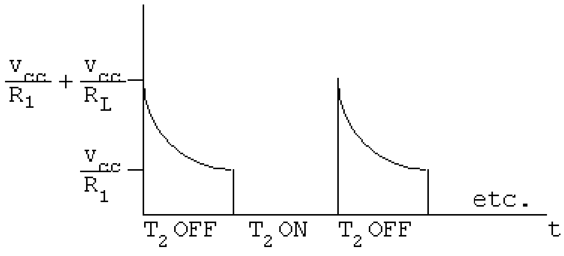
[f] At the instant T_2 turns back ON, $t = R_2C_2 \ln 2$; therefore

$$i_{\rm b1} = \frac{V_{CC}}{R_1} + \frac{V_{CC}}{R_{\rm L}}e^{-10 \ln 2} \cong \frac{V_{CC}}{R_1}$$

[g]



[h]



P 7.100 [a] $t_{\rm OFF2} = R_2C_2 \ln 2 = 18 \times 10^3(2 \times 10^{-9}) \ln 2 \cong 25\,\mu$s

[b] $t_{\rm ON2} = R_1C_1 \ln 2 \cong 25\,\mu$s

[c] $t_{\rm OFF1} = R_1C_1 \ln 2 \cong 25\,\mu$s

[d] $t_{\rm ON1} = R_2C_2 \ln 2 \cong 25\,\mu$s

[e] $i_{\rm b1} = \frac{9}{3} + \frac{9}{18} = 3.5\,$mA

[f] $i_{\rm b1} = \frac{9}{18} + \frac{9}{3}e^{-6\ln 2} \cong 0.5469\,$mA

[g] $v_{\rm ce2} = 9 - 9e^{-6\ln 2} \cong 8.86\,$V

P 7.101 [a] $t_{\rm OFF2} = R_2C_2 \ln 2 = (18 \times 10^3)(2.8 \times 10^{-9}) \ln 2 \cong 35\,\mu$s

[b] $t_{\rm ON2} = R_1C_1 \ln 2 \cong 37.4\,\mu$s

[c] $t_{\rm OFF1} = R_1C_1 \ln 2 \cong 37.4\,\mu$s

[d] $t_{\rm ON1} = R_2C_2 \ln 2 = 35\,\mu$s

[e] $i_{\rm b1} = 3.5\,$mA

[f] $i_{\rm b1} = \frac{9}{18} + 3e^{-5.6\ln 2} \cong 0.562\,$mA

[g] $v_{\rm ce2} = 9 - 9e^{-5.6\ln 2} \cong 8.81\,$V

Note in this circuit T_2 is OFF 35 μs and ON 37.4 μs of every cycle, whereas T_1 is ON 35 μs and OFF 37.4 μs every cycle.

P 7.102 If $R_1 = R_2 = 50R_{\rm L} = 100\,\text{k}\Omega$, then

$$C_1 = \frac{48 \times 10^{-6}}{100 \times 10^3 \ln 2} = 692.49\,\text{pF}; \qquad C_2 = \frac{36 \times 10^{-6}}{100 \times 10^3 \ln 2} = 519.37\,\text{pF}$$

If $R_1 = R_2 = 6R_{\rm L} = 12\,\text{k}\Omega$, then

$$C_1 = \frac{48 \times 10^{-6}}{12 \times 10^3 \ln 2} = 5.77\,\text{nF}; \qquad C_2 = \frac{36 \times 10^{-6}}{12 \times 10^3 \ln 2} = 4.33\,\text{nF}$$

Therefore $692.49\,\text{pF} \le C_1 \le 5.77\,\text{nF}$ and $519.37\,\text{pF} \le C_2 \le 4.33\,\text{nF}$

P 7.103 [a] We want the lamp to be in its nonconducting state for no more than 10 s, the value of t_o:

$$10 = R(10 \times 10^{-6}) \ln \frac{1-6}{4-6} \quad \text{and} \quad R = 1.091\,\text{M}\Omega$$

[b] When the lamp is conducting

$$V_{\rm Th} = \frac{20 \times 10^3}{20 \times 10^3 + 1.091 \times 10^6}(6) = 0.108\,\text{V}$$

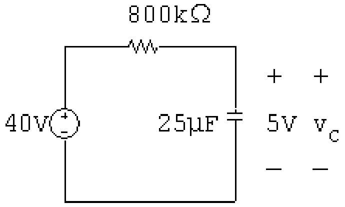
$$R_{\rm Th} = 20\,\text{k}\|1.091\ \text{M} = 19{,}640\,\Omega$$

So,

$$(t_c - t_o) = (19{,}640)(10 \times 10^{-6}) \ln \frac{4 - 0.108}{1 - 0.108} = 0.289\,\text{s}$$

The flash lasts for 0.289 s.

P 7.104 **[a]** At $t = 0$ we have



$$\tau = (800)(25) \times 10^{-3} = 20\text{ sec}; \qquad 1/\tau = 0.05$$

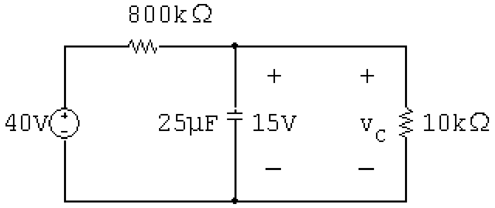
$$v_c(\infty) = 40\,\text{V}; \qquad v_c(0) = 5\,\text{V}$$

$$v_c = 40 - 35e^{-0.05t}\,\text{V}, \qquad 0 \le t \le t_o$$

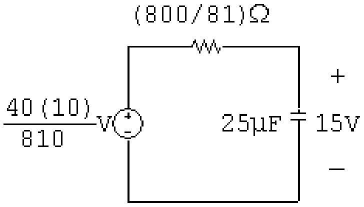
$$40 - 35e^{-0.05t_o} = 15; \qquad \therefore \quad e^{0.05t_o} = 1.4$$

$$t_o = 20\ \ln 1.4\,\text{s} = 6.73\,\text{s}$$

At $t = t_o$ we have



The Thévenin equivalent with respect to the capacitor is



$$\tau = \left(\frac{800}{81}\right)(25) \times 10^{-3} = \frac{20}{81}\,\text{s}; \qquad \frac{1}{\tau} = \frac{81}{20} = 4.05$$

$$v_c(t_o) = 15\,\text{V}; \qquad v_c(\infty) = \frac{40}{81}\,\text{V}$$

$$v_c(t) = \frac{40}{81} + \left(15 - \frac{40}{81}\right) e^{-4.05(t-t_o)}\,\text{V} = \frac{40}{81} + \frac{1175}{81} e^{-4.05(t-t_o)}$$

$$\therefore \quad \frac{40}{81} + \frac{1175}{81} e^{-4.05(t-t_o)} = 5$$

$$\frac{1175}{81}e^{-4.05(t-t_o)} = \frac{365}{81}$$

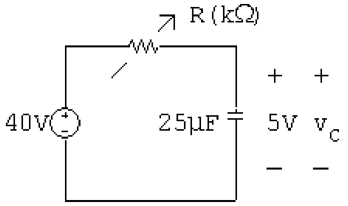
$$e^{4.05(t-t_o)} = \frac{1175}{365} = 3.22$$

$$t - t_o = \frac{1}{4.05}\ln 3.22 \cong 0.29\,\text{s}$$

One cycle = 7.02 seconds.

$N = 60/7.02 = 8.55$ flashes per minute

[b] At $t = 0$ we have



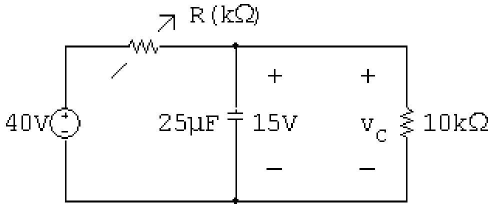
$$\tau = 25R \times 10^{-3}; \qquad 1/\tau = 40/R$$

$$v_c = 40 - 35e^{-(40/R)t}$$

$$40 - 35e^{-(40/R)t_o} = 15$$

$$\therefore\quad t_o = \frac{R}{40}\ln 1.4, \qquad R \quad \text{in} \quad \text{k}\Omega$$

At $t = t_o$:



$$v_{\text{Th}} = \frac{10}{R+10}(40) = \frac{400}{R+10}; \qquad R_{\text{Th}} = \frac{10R}{R+10}\,\text{k}\Omega$$

$$\tau = \frac{(25)(10R)\times 10^{-3}}{R+10} = \frac{0.25R}{R+10}; \qquad \frac{1}{\tau} = \frac{4(R+10)}{R}$$

$$v_c = \frac{400}{R+10} + \left(15 - \frac{400}{R+10}\right)e^{-\frac{4(R+10)}{R}(t-t_o)}$$

$$\therefore\quad \frac{400}{R+10} + \left[\frac{15R-250}{R+10}\right]e^{-\frac{4(R+10)}{R}(t-t_o)} = 5$$

$$\text{or}\quad \left(\frac{15R-250}{R+10}\right)e^{-\frac{4(R+10)}{R}(t-t_o)} = \frac{5R-350}{(R+10)}$$

$$\therefore \quad e^{\frac{4(R+10)}{R}(t-t_o)} = \frac{3R-50}{R-70}$$

$$\therefore \quad t - t_o = \frac{R}{4(R+10)} \ln\left(\frac{3R-50}{R-70}\right)$$

At 12 flashes per minute $t_o + (t - t_o) = 5\,\text{s}$

$$\therefore \quad \underbrace{\frac{R}{40} \ln 1.4}_{\substack{\text{dominant} \\ \text{term}}} + \frac{R}{4(R+10)} \ln\left(\frac{3R-50}{R-70}\right) = 5$$

Start the trial-and-error procedure by setting $(R/40)\ \ln 1.4 = 5$, then $R = 200/(\ln 1.4)$ or $594.40\,\text{k}\Omega$. If $R = 594.40\,\text{k}\Omega$ then $t - t_o \cong 0.29\,\text{s}$. Second trial set $(R/40)\ \ln 1.4 = 4.7\,\text{s}$ or $R = 558.74\,\text{k}\Omega$.

With $R = 558.74\,\text{k}\Omega, \quad t - t_o \cong 0.30\,\text{s}$

This procedure converges to $R = 559.3\,\text{k}\Omega$.

P 7.105 **[a]** $t_o = RC \ln\left(\frac{V_{\text{min}} - V_s}{V_{\text{max}} - V_s}\right) = (3700)(250 \times 10^{-6}) \ln\left(\frac{-700}{-100}\right)$

$$= 1.80\,\text{s}$$

$$t_c - t_o = \frac{RCR_{\text{L}}}{R + R_{\text{L}}} \ln\left(\frac{V_{\text{max}} - V_{\text{Th}}}{V_{\text{min}} - V_{\text{Th}}}\right)$$

$$\frac{R_{\text{L}}}{R + R_{\text{L}}} = \frac{1.3}{1.3 + 3.7} = 0.26; \qquad RC = (3700)(250\text{\c{1}}0^{-6}) = 0.925\,\text{s}$$

$$V_{\text{Th}} = \frac{1000(1.3)}{1.3 + 3.7} = 260\,\text{V}; \qquad R_{\text{Th}} = 3.7\,\text{k}\|1.3\,\text{k} = 962\,\Omega$$

$$\therefore \quad t_c - t_o = (0.925)(0.26)\ln(640/40) = 0.67\,\text{s}$$

$$\therefore \quad t_c = 1.8 + 0.67 = 2.47\,\text{s}$$

$$\text{flashes/min} \ = \frac{60}{2.47} = 24.32$$

[b] $0 \leq t \leq t_o$:

$$v_L = 1000 - 700e^{-t/\tau_1}$$

$$\tau_1 = RC = 0.925\,\text{s}$$

$t_o \leq t \leq t_c$:

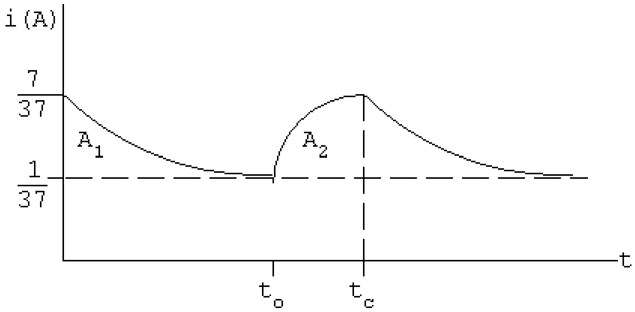
$$v_L = 260 + 640e^{-(t-t_o)/\tau_2}$$

$$\tau_2 = R_{\text{Th}}C = 962(250) \times 10^{-6} = 0.2405\,\text{s}$$

$$0 \leq t \leq t_o: \qquad i = \frac{1000 - v_L}{3700} = \frac{7}{37}e^{-t/0.925}\,\text{A}$$

$$t_o \leq t \leq t_c: \qquad i = \frac{1000 - v_L}{3700} = \frac{74}{370} - \frac{64}{370}e^{-(t-t_o)/0.2405}$$

Graphically, i versus t is



The average value of i will equal the areas $(A_1 + A_2)$ divided by t_c.

$$\therefore \quad i_{\text{avg}} = \frac{A_1 + A_2}{t_c}$$

$$\begin{aligned} A_1 &= \frac{7}{37}\int_0^{t_o} e^{-t/0.925}\,dt \\ &= \frac{6.475}{37}(1 - e^{-\ln 7}) = 0.15 \text{ A–s} \\ A_2 &= \int_{t_o}^{t_c} \frac{74 - 64e^{-(t-t_o)/0.2405}}{370}\,dt \\ &= \frac{74}{370}(t_c - t_o) + \frac{15.392}{370}(e^{-\ln 16} - 1) \\ &= \frac{17.797}{370}\ln 16 - \frac{15.392}{370}(1 - e^{-\ln 16}) \\ &= 0.09436 \text{ A–s} \end{aligned}$$

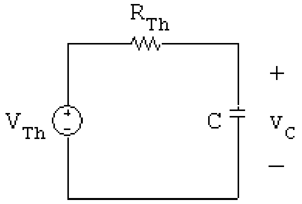
$$i_{\text{avg}} = \frac{(0.15 + 0.09436)}{0.925\ln 7 + 0.2405\ln 16}(1000) = 99.06\,\text{mA}$$

[c] $P_{\text{avg}} = (1000)(99.06 \times 10^{-3}) = 99.06\,\text{W}$

$$\text{No. of kw hrs/yr} \;= \frac{(99.06)(24)(365)}{1000} = 867.77$$

$$\text{Cost/year} \;= (867.77)(0.05) = 43.39 \text{ dollars/year}$$

P 7.106 **[a]** Replace the circuit attached to the capacitor with its Thévenin equivalent, where the equivalent resistance is the parallel combination of the two resistors, and the open-circuit voltage is obtained by voltage division across the lamp resistance. The resulting circuit is



$$R_{\text{Th}} = R\|R_{\text{L}} = \frac{RR_{\text{L}}}{R+R_{\text{L}}}; \qquad V_{\text{Th}} = \frac{R_{\text{L}}}{R+R_{\text{L}}}V_s$$

From this circuit,

$$v_{\text{C}}(\infty) = V_{\text{Th}}; \qquad v_{\text{C}}(0) = V_{\text{max}}; \qquad \tau = R_{\text{Th}}C$$

Thus,

$$v_{\text{C}}(t) = V_{\text{Th}} + (V_{\text{max}} - V_{\text{Th}})e^{-(t-t_o)/\tau}$$

where

$$\tau = \frac{RR_{\text{L}}C}{R+R_{\text{L}}}$$

[b] Now, set $v_{\text{C}}(t_c) = V_{\text{min}}$ and solve for $(t_c - t_o)$:

$$V_{\text{Th}} + (V_{\text{max}} - V_{\text{Th}})e^{-(t_c-t_o)/\tau} = V_{\text{min}}$$

$$e^{-(t_c-t_o)/\tau} = \frac{V_{\text{min}} - V_{\text{Th}}}{V_{\text{max}} - V_{\text{Th}}}$$

$$\frac{-(t_c-t_o)}{\tau} = \ln\frac{V_{\text{min}} - V_{\text{Th}}}{V_{\text{max}} - V_{\text{Th}}}$$

$$(t_c - t_o) = -\frac{RR_{\text{L}}C}{R+R_{\text{L}}}\ln\frac{V_{\text{min}} - V_{\text{Th}}}{V_{\text{max}} - V_{\text{Th}}} = \frac{RR_{\text{L}}C}{R+R_{\text{L}}}\ln\frac{V_{\text{max}} - V_{\text{Th}}}{V_{\text{min}} - V_{\text{Th}}}$$

P 7.107 **[a]** $0 \le t \le 0.5$:

$$i = \frac{21}{60} + \left(\frac{30}{60} - \frac{21}{60}\right)e^{-t/\tau} \qquad \text{where } \tau = L/R.$$

$$i = 0.35 + 0.15e^{-60t/L}$$

$$i(0.5) = 0.35 + 0.15e^{-30/L} = 0.40$$

$$\therefore \quad e^{30/L} = 3; \qquad L = \frac{30}{\ln 3} = 27.31\,\text{H}$$

[b] $0 \le t \le t_r$, where t_r is the time the relay releases:

$$i = 0 + \left(\frac{30}{60} - 0\right)e^{-60t/L} = 0.5e^{-60t/L}$$

$$\therefore \quad 0.4 = 0.5e^{-60t_r/L}; \qquad e^{60t_r/L} = 1.25$$

$$t_r = \frac{27.31\ln 1.25}{60} \cong 0.10\,\text{s}$$

8

Natural and Step Responses of *RLC* Circuits

Assessment Problems

AP 8.1 [**a**] $\frac{1}{(2RC)^2} = \frac{1}{LC}$, therefore $C = 500\,\text{nF}$

[**b**] $\alpha = 5000 = \frac{1}{2RC}$, therefore $C = 1\,\mu\text{F}$

$$s_{1,2} = -5000 \pm \sqrt{25 \times 10^6 - \frac{(10^3)(10^6)}{20}} = (-5000 \pm j5000)\ \text{rad/s}$$

[**c**] $\frac{1}{\sqrt{LC}} = 20{,}000$, therefore $C = 125\,\text{nF}$

$$s_{1,2} = \left[-40 \pm \sqrt{(40)^2 - 20^2}\right] 10^3,$$

$$s_1 = -5.36\,\text{krad/s}, \qquad s_2 = -74.64\,\text{krad/s}$$

AP 8.2

$$\begin{aligned} i_{\text{L}} &= \frac{1}{50 \times 10^{-3}} \int_0^t [-14e^{-5000x} + 26e^{-20{,}000x}]\,dx + 30 \times 10^{-3} \\ &= 20\left\{ \frac{-14e^{-5000x}}{-5000}\Bigg|_0^t + \frac{26e^{-20{,}000t}}{-20{,}000}\Bigg|_0^t \right\} + 30 \times 10^{-3} \\ &= 56 \times 10^{-3}(e^{-5000t} - 1) - 26 \times 10^{-3}(e^{-20{,}000t} - 1) + 30 \times 10^{-3} \\ &= [56e^{-5000t} - 56 - 26e^{-20{,}000t} + 26 + 30]\,\text{mA} \\ &= 56e^{-5000t} - 26e^{-20{,}000t}\,\text{mA}, \qquad t \geq 0 \end{aligned}$$

AP 8.3 From the given values of R, L, and C, $s_1 = -10\,\text{krad/s}$ and $s_2 = -40$ krad/s.

[**a**] $v(0^-) = v(0^+) = 0$, therefore $i_{\text{R}}(0^+) = 0$

[b] $i_C(0^+) = -(i_L(0^+) + i_R(0^+)) = -(-4+0) = 4\,\text{A}$

[c] $C\dfrac{dv_c(0^+)}{dt} = i_c(0^+) = 4, \qquad \text{therefore} \quad \dfrac{dv_c(0^+)}{dt} = \dfrac{4}{C} = 4\times 10^8\,\text{V/s}$

[d] $v = [A_1e^{-10{,}000t} + A_2e^{-40{,}000t}]\,\text{V}, \qquad t \geq 0^+$

$$v(0^+) = A_1 + A_2, \qquad \frac{dv(0^+)}{dt} = -10{,}000A_1 - 40{,}000A_2$$

Therefore $A_1 + A_2 = 0, \qquad -A_1 - 4A_2 = 40{,}000; \qquad A_1 = 40{,}000/3\,\text{V}$

[e] $A_2 = -40{,}000/3\,\text{V}$

[f] $v = [40{,}000/3][e^{-10{,}000t} - e^{-40{,}000t}]\,\text{V}, \qquad t \geq 0$

AP 8.4 **[a]** $\dfrac{1}{2RC} = 8000, \qquad \text{therefore} \quad R = 62.5\,\Omega$

[b] $i_\text{R}(0^+) = \dfrac{10\,\text{V}}{62.5\,\Omega} = 160\,\text{mA}$

$$i_\text{C}(0^+) = -(i_L(0^+) + i_R(0^+)) = -80 - 160 = -240\,\text{mA} = C\frac{dv(0^+)}{dt}$$

Therefore $\dfrac{dv(0^+)}{dt} = \dfrac{-240\,\text{m}}{C} = -240\,\text{kV/s}$

[c] $B_1 = v(0^+) = 10\,\text{V}, \qquad \dfrac{dv_c(0^+)}{dt} = \omega_d B_2 - \alpha B_1$

Therefore $6000B_2 - 8000B_1 = -240{,}000, \qquad B_2 = (-80/3)\,\text{V}$

[d] $i_\text{L} = -(i_\text{R} + i_\text{C}); \qquad i_\text{R} = v/R; \qquad i_\text{C} = C\dfrac{dv}{dt}$

$$v = e^{-8000t}[10\cos 6000t - \frac{80}{3}\sin 6000t]\,\text{V}$$

Therefore $i_\text{R} = e^{-8000t}[160\cos 6000t - \dfrac{1280}{3}\sin 6000t]\,\text{mA}$

$$i_\text{C} = e^{-8000t}[-240\cos 6000t + \frac{460}{3}\sin 6000t]\,\text{mA}$$

$$i_\text{L} = 10e^{-8000t}[8\cos 6000t + \frac{82}{3}\sin 6000t]\,\text{mA}, \qquad t \geq 0$$

AP 8.5 **[a]** $\left(\dfrac{1}{2RC}\right)^2 = \dfrac{1}{LC} = \dfrac{10^6}{4}, \qquad \text{therefore} \quad \dfrac{1}{2RC} = 500, \quad R = 100\,\Omega$

[b] $0.5CV_0^2 = 12.5\times 10^{-3}, \qquad \text{therefore} \quad V_0 = 50\,\text{V}$

[c] $0.5LI_0^2 = 12.5\times 10^{-3}, \qquad I_0 = 250\,\text{mA}$

[d] $D_2 = v(0^+) = 50, \qquad \dfrac{dv(0^+)}{dt} = D_1 - \alpha D_2$

$$i_R(0^+) = \frac{50}{100} = 500\,\text{mA}$$

Therefore $i_C(0^+) = -(500 + 250) = -750\,\text{mA}$

Therefore $\dfrac{dv(0^+)}{dt} = -750 \times \dfrac{10^{-3}}{C} = -75{,}000\,\text{V/s}$

Therefore $D_1 - \alpha D_2 = -75{,}000; \qquad \alpha = \dfrac{1}{2RC} = 500, \quad D_1 = -50{,}000$ V/s

[e] $v = [50e^{-500t} - 50{,}000te^{-500t}]\,\text{V}$

$i_R = \dfrac{v}{R} = [0.5e^{-500t} - 500te^{-500t}]\,\text{A}, \qquad t \geq 0^+$

AP 8.6 **[a]** $i_R(0^+) = \dfrac{V_0}{R} = \dfrac{40}{500} = 0.08\,\text{A}$

[b] $i_C(0^+) = I - i_R(0^+) - i_L(0^+) = -1 - 0.08 - 0.5 = -1.58\,\text{A}$

[c] $\dfrac{di_L(0^+)}{dt} = \dfrac{V_o}{L} = \dfrac{40}{0.64} = 62.5\,\text{A/s}$

[d] $\alpha = \dfrac{1}{2RC} = 1000; \qquad \dfrac{1}{LC} = 1{,}562{,}500; \qquad s_{1,2} = -1000 \pm j750$ rad/s

[e] $i_L = i_f + B_1'e^{-\alpha t}\cos\omega_d t + B_2'e^{-\alpha t}\sin\omega_d t, \qquad i_f = I = -1\,\text{A}$

$i_L(0^+) = 0.5 = i_f + B_1', \qquad$ therefore $B_1' = 1.5\,\text{A}$

$\dfrac{di_L(0^+)}{dt} = 62.5 = -\alpha B_1' + \omega_d B_2', \qquad$ therefore $B_2' = (25/12)\,\text{A}$

Therefore $i_L(t) = -1 + e^{-1000t}[1.5\cos 750t + (25/12)\sin 750t]\,\text{A}, \qquad t \geq 0$

[f] $v(t) = \dfrac{Ldi_L}{dt} = 40e^{-1000t}[\cos 750t - (154/3)\sin 750t]V \qquad t \geq 0$

AP 8.7 **[a]** $i(0^+) = 0$, since there is no source connected to L for $t < 0$.

[b] $v_c(0^+) = v_C(0^-) = \left(\dfrac{15\,\text{k}}{15\,\text{k} + 9\,\text{k}}\right)(80) = 50\,\text{V}$

[c] $50 + 80i(0^+) + L\dfrac{di(0^+)}{dt} = 100, \qquad \dfrac{di(0^+)}{dt} = 10{,}000\,\text{A/s}$

[d] $\alpha = 8000; \qquad \dfrac{1}{LC} = 100 \times 10^6; \qquad s_{1,2} = -8000 \pm j6000$ rad/s

[e] $i = i_f + e^{-\alpha t}[B_1'\cos\omega_d t + B_2'\sin\omega_d t]; \qquad i_f = 0, \quad i(0^+) = 0$

Therefore $B_1' = 0; \qquad \dfrac{di(0^+)}{dt} = 10{,}000 = -\alpha B_1' + \omega_d B_2'$

Therefore $B_2' = 1.67\,\text{A}; \qquad i = 1.67e^{-8000t}\sin 6000t\,\text{A}, \qquad t \geq 0$

AP 8.8 $v_c(t) = v_f + e^{-\alpha t}[B_1' \cos\omega_d t + B_2' \sin\omega_d t], \qquad v_f = 100\,\text{V}$

$$v_c(0^+) = 50\,\text{V}; \qquad \frac{dv_c(0^+)}{dt} = 0; \qquad \text{therefore} \quad 50 = 100 + B_1'$$

$$B_1' = -50\,\text{V}; \qquad 0 = -\alpha B_1' + \omega_d B_2'$$

$$\text{Therefore} \quad B_2' = \frac{\alpha}{\omega_d} B_1' = \left(\frac{8000}{6000}\right)(-50) = -66.67\,\text{V}$$

$$\text{Therefore} \quad v_c(t) = 100 - e^{-8000t}[50\cos 6000t + 66.67\sin 6000t]\,\text{V}, \qquad t \geq 0$$

Problems

P 8.1 **[a]** $\alpha = \frac{1}{2RC} = \frac{10^{12}}{(4000)(10)} = 25{,}000$

$$\omega_o^2 = \frac{1}{LC} = \frac{10^{12}}{(250)(10)} = 4 \times 10^8$$

$$s_{1,2} = -25{,}000 \pm \sqrt{625 \times 10^6 - 400 \times 10^6} = -25{,}000 \pm 15{,}000$$

$$s_1 = -10{,}000 \text{ rad/s}$$

$$s_2 = -40{,}000 \text{ rad/s}$$

[b] overdamped

[c] $\omega_d = \sqrt{\omega_o^2 - \alpha^2}$

$$\therefore \quad \alpha^2 = \omega_o^2 - \omega_d^2 = 4 \times 10^8 - 144 \times 10^6 = 256 \times 10^6$$

$$\alpha = 16 \times 10^3 = 16{,}000$$

$$\frac{1}{2RC} = 16{,}000; \qquad \therefore \quad R = \frac{10^9}{(32{,}000)(10)} = 3125\,\Omega$$

[d] $s_1 = -16{,}000 + j12{,}000$ rad/s; $\qquad s_2 = -16{,}000 - j12{,}000$ rad/s

[e] $\alpha = 4 \times 10^4 = \frac{1}{2RC}; \qquad \therefore \quad R = \frac{1}{2C(4 \times 10^4)} = 2500\,\Omega$

P 8.2 **[a]** $i_R(0) = \dfrac{15}{200} = 75\,\text{mA}$

$i_L(0) = -45\,\text{mA}$

$i_C(0) = -i_L(0) - i_R(0) = 45 - 75 = -30\,\text{mA}$

[b] $\alpha = \dfrac{1}{2RC} = \dfrac{1}{2(200)(0.2\times10^{-6})} = 12{,}500$

$\omega_o^2 = \dfrac{1}{LC} = \dfrac{1}{(50\times10^{-3})(0.2\times10^{-6})} = 10^8$

$s_{1,2} = -12{,}500 \pm \sqrt{1.5625\times10^8 - 10^8} = -12{,}500 \pm 7500$

$s_1 = -5000$ rad/s; $\qquad s_2 = -20{,}000$ rad/s

$v = A_1e^{-5000t} + A_2e^{-20{,}000t}$

$v(0) = A_1 + A_2 = 15$

$\dfrac{dv}{dt}(0) = -5000A_1 - 20{,}000A_2 = \dfrac{-30\times10^{-3}}{0.2\times10^{-6}} = -15\times10^4\text{V/s}$

Solving, $\quad A_1 = 10; \quad A_2 = 5$

$v = 10e^{-5000t} + 5e^{-20{,}000t}\,\text{V}, \qquad t\ge0$

[c]
$$\begin{aligned} i_C &= C\frac{dv}{dt} \\ &= 0.2\times10^{-6}[-50{,}000e^{-5000t} - 100{,}000e^{-20{,}000t}] \\ &= -10e^{-5000t} - 20e^{-20{,}000t}\,\text{mA} \end{aligned}$$

$i_R = 50e^{-5000t} + 25e^{-20{,}000t}\,\text{mA}$

$i_L = -i_C - i_R = -40e^{-5000t} - 5e^{-20{,}000t}\,\text{mA}, \quad t\ge0$

P 8.3 $\dfrac{1}{2RC} = \dfrac{1}{2(312.5)(0.2\times10^{-6})} = 8000$

$\dfrac{1}{LC} = \dfrac{1}{(50\times10^{-3})(0.2\times10^{-6})} = 10^8$

$s_{1,2} = -8000 \pm \sqrt{8000^2 - 10^8} = -8000 \pm j6000\,\text{rad/s}$

$\therefore$ response is underdamped

$v(t) = B_1e^{-8000t}\cos 6000t + B_2e^{-8000t}\sin 6000t$

$$v(0^+) = 15\,\text{V} = B_1; \qquad i_{\text{R}}(0^+) = \frac{15}{312.5} = 48\,\text{mA}$$

$$i_{\text{C}}(0^+) = [-i_{\text{L}}(0^+) + i_{\text{R}}(0^+)] = -[-45 + 48] = -3\,\text{mA}$$

$$\frac{dv(0^+)}{dt} = \frac{-3 \times 10^{-3}}{0.2 \times 10^{-6}} = -15{,}000\,\text{V/s}$$

$$\frac{dv(0)}{dt} = -8000B_1 + 6000B_2 = -15{,}000$$

$$6000B_2 = 8000(15) - 15{,}000; \qquad \therefore \quad B_2 = 17.5\,\text{V}$$

$$v(t) = 15e^{-8000t}\cos 6000t + 17.5e^{-8000t}\sin 6000t\,\text{V}, \qquad t \geq 0$$

P 8.4 $$\alpha = \frac{1}{2RC} = \frac{1}{2(250)(0.2 \times 10^{-6})} = 10^4$$

$$\alpha^2 = 10^8; \qquad \therefore \quad \alpha^2 = \omega_o^2$$

Critical damping:

$$v = D_1te^{-\alpha t} + D_2e^{-\alpha t}$$

$$i_R(0^+) = \frac{15}{250} = 60\,\text{mA}$$

$$i_C(0^+) = -[i_L(0^+) + i_R(0^+)] = -[-45 + 60] = -15\,\text{mA}$$

$$v(0) = D_2 = 15$$

$$\frac{dv}{dt} = D_1[t(-\alpha e^{-\alpha t}) + e^{-\alpha t}] - \alpha D_2e^{-\alpha t}$$

$$\frac{dv}{dt}(0) = D_1 - \alpha D_2 = \frac{i_{\text{C}}(0)}{C} = \frac{-15 \times 10^{-3}}{0.2 \times 10^{-6}} = -75{,}000$$

$$D_1 = \alpha D_2 - 75{,}000 = (10^4)(15) - 75{,}000 = 75{,}000$$

$$v = (75{,}000t + 15)e^{-10{,}000t}\,\text{V}, \qquad t \geq 0$$

P 8.5 [a] $\frac{1}{LC} = 5000^2$

There are many possible solutions. This one begins by choosing $L = 10\,\text{mH}$. Then,

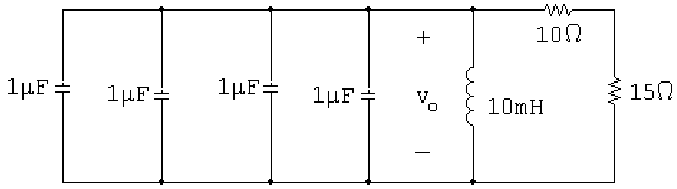
$$C = \frac{1}{(10 \times 10^{-3})(5000)^2} = 4\,\mu\text{F}$$

We can achieve this capacitor value using components from Appendix H by combining four $1\,\mu$F capacitors in parallel.

Critically damped: $\alpha = \omega_0 = 5000$ so $\frac{1}{2RC} = 5000$

$$\therefore \quad R = \frac{1}{2(4 \times 10^{-6})(5000)} = 25\,\Omega$$

We can create this resistor value using components from Appendix H by combining a $10\,\Omega$ resistor and a $15\,\Omega$ resistor in series. The final circuit:



[b] $s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2} = -5000 \pm 0$

Therefore there are two repeated real roots at -5000 rad/s.

P 8.6 [a] Underdamped response:

$\alpha < \omega_0$ so $\alpha < 5000$

Therefore we choose a larger resistor value than the one used in Problem 8.5. Choose $R = 100\,\Omega$:

$$\alpha = \frac{1}{2(100)(4 \times 10^{-6})} = 1250$$

$$s_{1,2} = -1250 \pm \sqrt{1250^2 - 5000^2} = -1250 \pm j4841.23 \text{ rad/s}$$

[b] Overdamped response:

$\alpha > \omega_0$ so $\alpha > 5000$

Therefore we choose a smaller resistor value than the one used in Problem 8.5. Choose $R = 20\,\Omega$:

$$\alpha = \frac{1}{2(20)(4 \times 10^{-6})} = 6250$$

$$s_{1,2} = -1250 \pm \sqrt{6250^2 - 5000^2} = -1250 \pm 3750$$

$$= -2500 \text{ rad/s}; \qquad \text{and} \qquad -10{,}000 \text{ rad/s}$$

P 8.7 **[a]** $\alpha = 8000; \qquad \omega_d = 6000$

$$\omega_d = \sqrt{\omega_o^2 - \alpha^2}$$

$$\therefore \quad \omega_o^2 = \omega_d^2 + \alpha^2 = 36 \times 10^6 + 64 \times 10^6 = 100 \times 10^6$$

$$\frac{1}{LC} = 100 \times 10^6$$

$$C = \frac{1}{(100 \times 10^6)(0.4)} = 25\,\text{nF}$$

[b] $\alpha = \dfrac{1}{2RC}$

$$\therefore \quad R = \frac{1}{2\alpha C} = \frac{1}{(16{,}000)(25 \times 10^{-9})} = 2500\,\Omega$$

[c] $V_o = v(0) = 75\,\text{V}$

[d] $I_o = i_{\text{L}}(0) = -i_{\text{R}}(0) - i_{\text{C}}(0)$

$$i_{\text{R}}(0) = \frac{75}{2500} = 30\,\text{mA}$$

$$i_{\text{C}}(0) = C\frac{dv}{dt}(0) = 25 \times 10^{-9}[6000(-300) - 8000(75)] = -60\,\text{mA}$$

$$\therefore \quad I_o = -30 + 60 = 30\,\text{mA}$$

[e] $i_{\text{C}}(t) = 25 \times 10^{-9}\dfrac{dv(t)}{dt} = e^{-8000t}(48.75\sin 6000t - 60\cos 6000t)\,\text{mA}$

$$i_{\text{R}}(t) = \frac{v(t)}{2500} = e^{-8000t}(30\cos 6000t - 120\sin 6000t)\,\text{mA}$$

$$i_{\text{L}}(t) = -i_{\text{R}}(t) - i_{\text{C}}(t)$$

$$= e^{-8000t}(30\cos 6000t + 71.25\sin 6000t)\,\text{mA}, \quad t \geq 0$$

Check:

$$L\frac{di_{\text{L}}}{dt} = 0.4 \times 10^{-3}e^{-8000t}[187{,}000\cos 6000t - 750{,}000\sin 6000t]$$

$$v(t) = e^{-8000t}[75\cos 6000t - 300\sin 6000t]\,\text{V}$$

P 8.8 **[a]** $-\alpha + \sqrt{\alpha^2 - \omega_o^2} = -250$

$$-\alpha - \sqrt{\alpha^2 - \omega_o^2} = -1000$$

Adding the above equations, $\qquad -2\alpha = -1250$

$$\alpha = 625\,\text{rad/s}$$

$$\frac{1}{2RC} = \frac{1}{2R(0.1 \times 10^{-6})} = 625$$

$$R = 8\,\text{k}\Omega$$

$$2\sqrt{\alpha^2 - \omega_o^2} = 750$$

$$4(\alpha^2 - \omega_o^2) = 562{,}500$$

$$\therefore \quad \omega_o = 500\,\text{rad/s}$$

$$\omega_o^2 = 25 \times 10^4 = \frac{1}{LC}$$

$$\therefore \quad L = \frac{1}{(25 \times 10^4)(0.1 \times 10^{-6})} = 40\,\text{H}$$

[b] $i_\text{R} = \dfrac{v(t)}{R} = -1e^{-250t} + 4e^{-1000t}\,\text{mA}, \qquad t \geq 0^+$

$$i_\text{C} = C\frac{dv(t)}{dt} = 0.2e^{-250t} - 3.2e^{-1000t}\,\text{mA}, \qquad t \geq 0^+$$

$$i_\text{L} = -(i_\text{R} + i_\text{C}) = 0.8e^{-250t} - 0.8e^{-1000t}\,\text{mA}, \qquad t \geq 0$$

P 8.9 **[a]** $\left(\dfrac{1}{2RC}\right)^2 = \dfrac{1}{LC} = (500)^2$

$$\therefore \quad C = \frac{1}{(500)^2(4)} = 1\,\mu\text{F}$$

$$\frac{1}{2RC} = 500$$

$$\therefore \quad R = \frac{1}{2(500)(10^{-6})} = 1\,\text{k}\Omega$$

$$v(0) = D_2 = 8\,\text{V}$$

$$i_\text{R}(0) = \frac{8}{1000} = 8\text{mA}$$

$$i_\text{C}(0) = -8 + 10 = 2\,\text{mA}$$

$$\frac{dv}{dt}(0) = D_1 - 500D_2 = \frac{2 \times 10^{-3}}{10^{-6}} = 2000\,\text{V/s}$$

$$\therefore \quad D_1 = 2000 + 500(8) = 6000\ \text{V/s}$$

[b] $v = 6000te^{-500t} + 8e^{-500t}\,\text{V}, \quad t \geq 0$

$$\frac{dv}{dt} = [-3 \times 10^6 t + 2000]e^{-500t}$$

$$i_\text{C} = C\frac{dv}{dt} = (-3000t + 2)e^{-500t}\,\text{mA}, \quad t \geq 0^+$$

P 8.10 $\alpha = 500/2 = 250$

$$R = \frac{1}{2\alpha C} = \frac{10^6}{(500)(18)} = 1000\,\Omega$$

$$v(0^+) = -11 + 20 = 9\,\mathrm{V}$$

$$i_\mathrm{R}(0^+) = \frac{9}{1000} = 9\,\mathrm{mA}$$

$$\frac{dv}{dt} = 1100e^{-100t} - 8000e^{-400t}$$

$$\frac{dv(0^+)}{dt} = 1100 - 8000 = -6900\,\mathrm{V/s}$$

$$i_\mathrm{C}(0^+) = 2\times 10^{-6}(-6900) = -13.8\,\mathrm{mA}$$

$$i_\mathrm{L}(0^+) = -[i_\mathrm{R}(0^+) + i_\mathrm{C}(0^+)] = -[9 - 13.8] = 4.8\,\mathrm{mA}$$

P 8.11 **[a]** $2\alpha = 1000; \qquad \alpha = 500\,\mathrm{rad/s}$

$$2\sqrt{\alpha^2 - \omega_o^2} = 600; \qquad \omega_o = 400\,\mathrm{rad/s}$$

$$C = \frac{1}{2\alpha R} = \frac{1}{2(500)(250)} = 4\,\mu F$$

$$L = \frac{1}{\omega_o^2 C} = \frac{1}{(400)^2(4\times 10^{-6})} = 1.5625\,\mathrm{H}$$

$$i_\mathrm{C}(0^+) = A_1 + A_2 = 45\,\mathrm{mA}$$

$$\frac{di_\mathrm{C}}{dt} + \frac{di_\mathrm{L}}{dt} + \frac{di_\mathrm{R}}{dt} = 0$$

$$\frac{di_\mathrm{C}(0)}{dt} = -\frac{di_\mathrm{L}(0)}{dt} - \frac{di_\mathrm{R}(0)}{dt}$$

$$\frac{di_\mathrm{L}(0)}{dt} = \frac{0}{1.5625} = 0\,\mathrm{A/s}$$

$$\frac{di_\mathrm{R}(0)}{dt} = \frac{1}{R}\frac{dv(0)}{dt} = \frac{1}{R}\frac{i_\mathrm{C}(0)}{C} = \frac{45\times 10^{-3}}{(250)(4\times 10^{-6})} = 45\,\mathrm{A/s}$$

$$\therefore \quad \frac{di_\mathrm{C}(0)}{dt} = 0 - 45 = -45\,\mathrm{A/s}$$

$$\therefore \quad 200A_1 + 800A_2 = 45; \qquad A_1 + A_2 = 0.045$$

Solving, $A_1 = -15\,\mathrm{mA}; \qquad A_2 = 60\,\mathrm{mA}$

$$\therefore \quad i_\mathrm{C} = -15e^{-200t} + 60e^{-800t}\,\mathrm{mA}, \qquad t \geq 0^+$$

[b] By hypothesis

$$v = A_3e^{-200t} + A_4e^{-800t}, \qquad t \geq 0$$

$$v(0) = A_3 + A_4 = 0$$

$$\frac{dv(0)}{dt} = \frac{45 \times 10^{-3}}{4 \times 10^{-6}} = 11{,}250\,\text{V/s}$$

$$-200A_3 - 800A_4 = 11{,}250; \qquad \therefore \;\; A_3 = 18.75\,\text{V}; \qquad A_4 = -18.75\,\text{V}$$

$$v = 18.75e^{-200t} - 18.75e^{-800t}\,\text{V}, \qquad t \geq 0$$

[c] $i_{\text{R}}(t) = \dfrac{v}{250} = 75e^{-200t} - 75e^{-800t}\,\text{mA}, \qquad t \geq 0^+$

[d] $i_{\text{L}} = -i_{\text{R}} - i_{\text{C}}$

$$i_{\text{L}} = -60e^{-200t} + 15e^{-800t}\,\text{mA}, \qquad t \geq 0$$

P 8.12 From the form of the solution we have

$$v(0) = A_1 + A_2$$

$$\frac{dv(0^+)}{dt} = -\alpha(A_1 + A_2) + j\omega_d(A_1 - A_2)$$

We know both $v(0)$ and $dv(0^+)/dt$ will be real numbers. To facilitate the algebra we let these numbers be K_1 and K_2, respectively. Then our two simultaneous equations are

$$K_1 = A_1 + A_2$$

$$K_2 = (-\alpha + j\omega_d)A_1 + (-\alpha - j\omega_d)A_2$$

The characteristic determinant is

$$\Delta = \begin{vmatrix} 1 & 1 \\ (-\alpha + j\omega_d) & (-\alpha - j\omega_d) \end{vmatrix} = -j2\omega_d$$

The numerator determinants are

$$N_1 = \begin{vmatrix} K_1 & 1 \\ K_2 & (-\alpha - j\omega_d) \end{vmatrix} = -(\alpha + j\omega_d)K_1 - K_2$$

and $N_2 = \begin{vmatrix} 1 & K_1 \\ (-\alpha + j\omega_d) & K_2 \end{vmatrix} = K_2 + (\alpha - j\omega_d)K_1$

It follows that $A_1 = \dfrac{N_1}{\Delta} = \dfrac{\omega_d K_1 - j(\alpha K_1 + K_2)}{2\omega_d}$

and $A_2 = \dfrac{N_2}{\Delta} = \dfrac{\omega_d K_1 + j(\alpha K_1 + K_2)}{2\omega_d}$

We see from these expressions that $A_1 = A_2^*$.

P 8.13 By definition, $B_1 = A_1 + A_2$. From the solution to Problem 8.12 we have

$$A_1 + A_2 = \frac{2\omega_d K_1}{2\omega_d} = K_1$$

But K_1 is $v(0)$, therefore, $B_1 = v(0)$, which is identical to Eq. (8.30).
By definition, $B_2 = j(A_1 - A_2)$. From Problem 8.12 we have

$$B_2 = j(A_1 - A_2) = \frac{j[-2j(\alpha K_1 + K_2)]}{2\omega_d} = \frac{\alpha K_1 + K_2}{\omega_d}$$

It follows that

$$K_2 = -\alpha K_1 + \omega_d B_2, \quad \text{but} \quad K_2 = \frac{dv(0^+)}{dt} \quad \text{and} \quad K_1 = B_1.$$

Thus we have

$$\frac{dv}{dt}(0^+) = -\alpha B_1 + \omega_d B_2,$$

which is identical to Eq. (8.31).

P 8.14 **[a]** $\alpha = \dfrac{1}{2RC} = 800$ rad/s

$$\omega_o^2 = \frac{1}{LC} = 10^6$$

$$\omega_d = \sqrt{10^6 - 800^2} = 600 \text{ rad/s}$$

$$\therefore \quad v = B_1 e^{-800t}\cos 600t + B_2 e^{-800t}\sin 600t$$

$$v(0) = B_1 = 30$$

$$i_R(0^+) = \frac{30}{5000} = 6\,\text{mA}; \qquad i_C(0^+) = -12\,\text{mA}$$

$$\therefore \quad \frac{dv}{dt}(0^+) = \frac{-0.012}{125 \times 10^{-9}} = -96{,}000 \text{ V/s}$$

$$-96{,}000 = -\alpha B_1 + \omega_d B_2 = -(800)(30) + 600B_2$$

$$\therefore \quad B_2 = -120$$

$$\therefore \quad v = 30e^{-800t}\cos 600t - 120e^{-800t}\sin 600t\,\text{V}, \qquad t \ge 0$$

[b] $\dfrac{dv}{dt} = 6000e^{-800t}(13\sin 600t - 16\cos 600t)$

$$\frac{dv}{dt} = 0 \quad \text{when} \quad 16\cos 600t = 13\sin 600t \quad \text{or} \quad \tan 600t = \frac{16}{13}$$

$$\therefore \quad 600t_1 = 0.8885, \qquad t_1 = 1.48\,\text{ms}$$

$$600t_2 = 0.8885 + \pi, \qquad t_2 = 6.72\,\text{ms}$$

$$600t_3 = 0.8885 + 2\pi, \qquad t_3 = 11.95\,\text{ms}$$

[c] $t_3 - t_1 = 10.47\,\text{ms}; \qquad T_d = \dfrac{2\pi}{\omega_d} = \dfrac{2\pi}{600} = 10.47\,\text{ms}$

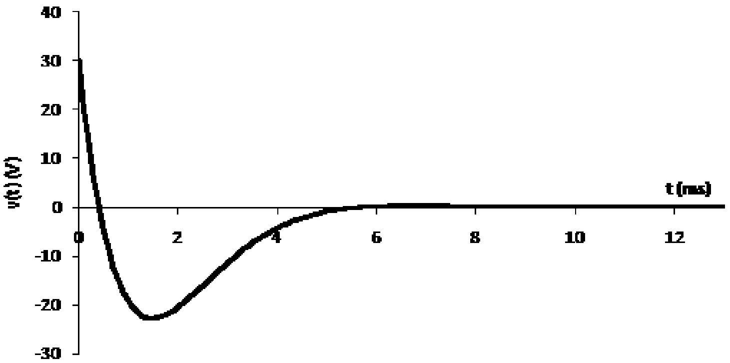
[d] $t_2 - t_1 = 5.24\,\text{ms}; \qquad \dfrac{T_d}{2} = \dfrac{10.48}{2} = 5.24\,\text{ms}$

[e] $v(t_1) = 30e^{-(1.184)}(\cos 0.8885 - 4\sin 0.8885) = -22.7\,\text{V}$

$$v(t_2) = 30e^{-(5.376)}(\cos 4.032 - 4\sin 4.032) = 0.334\,\text{V}$$

$$v(t_3) = 30e^{-(9.56)}(\cos 7.17 - 4\sin 7.17) = -5.22\,\text{mV}$$

[f]



P 8.15 [a] $\alpha = 0; \qquad \omega_d = \omega_o = \sqrt{10^6} = 1000\,\text{rad/s}$

$$v = B_1\cos\omega_o t + B_2\sin\omega_o t; \qquad v(0) = B_1 = 30$$

$$C\frac{dv}{dt}(0) = -i_{\text{L}}(0) = -0.006$$

$$-48{,}000 = -\alpha B_1 + \omega_d B_2 = -0 + 1000B_2$$

$$\therefore \quad B_2 = \frac{-48{,}000}{1000} = -48\,\text{V}$$

$$v = 30\cos 1000t - 48\sin 1000t\,\text{V}, \qquad t \geq 0$$

[b] $2\pi f = 1000; \qquad f = \dfrac{1000}{2\pi} \cong 159.15\,\text{Hz}$

[c] $\sqrt{30^2 + 48^2} = 56.6\,\text{V}$

P 8.16 **[a]** $\omega_o^2 = \dfrac{1}{LC} = \dfrac{10^9}{(2.5)(100)} = 4 \times 10^6$

$$\omega_o = 2000 \text{ rad/s}$$

$$\frac{1}{2RC} = 2000; \qquad R = \frac{1}{4000C} = 2500\,\Omega$$

[b] $v(t) = D_1te^{-5000t} + D_2e^{-5000t}$

$$v(0) = -15\,\text{V} = D_2$$

$$i_{\text{C}}(0) = 5 + \frac{15}{2.5} = 11\,\text{mA}$$

$$\frac{dv}{dt}(0) = \frac{i_{\text{C}}(0)}{C} = \frac{11 \times 10^{-3}}{100 \times 10^{-9}} = 110{,}000$$

$$D_1 - 2000(-15) = 110{,}000 \qquad \text{so} \qquad D_1 = 80{,}000 \text{ V/s}$$

$$\therefore \quad v(t) = (80{,}000t - 15)e^{-2000t}\,\text{V}, \qquad t \geq 0$$

[c] $i_{\text{C}}(t) = 0$ when $\dfrac{dv}{dt}(t) = 0$

$$\frac{dv}{dt} = (110{,}000 - 160 \times 10^6t))e^{-2000t}$$

$$\frac{dv}{dt} = 0 \text{ when } 160 \times 10^6t_1 = 110{,}000; \qquad \therefore \quad t_1 = 687.5\,\mu\text{s}$$

$$v(687.5\mu\text{s}) = (55 - 15)e^{-1.375} = 10.1136\,\text{V}$$

[d] $w(0) = \frac{1}{2}(100 \times 10^{-9})(15)^2 + \frac{1}{2}(2.5)(0.005)^2 = 42.5\,\mu\text{J}$

$$w(687.5\,\mu\text{s}) = \frac{1}{2}(100 \times 10^{-9})(10.1136)^2 + \frac{1}{2}(2.5)\left(\frac{10.1136}{2500}\right)^2 = 25.571\,\mu\text{J}$$

$$\% \text{ remaining } = \frac{25.571}{42.5}(100) = 60.17\%$$

P 8.17 **[a]** $\alpha = \dfrac{1}{2RC} = 1250, \qquad \omega_o = 10^3, \qquad$ therefore overdamped

$$s_1 = -500, \qquad s_2 = -2000$$

$$\text{therefore} \quad v = A_1e^{-500t} + A_2e^{-2000t}$$

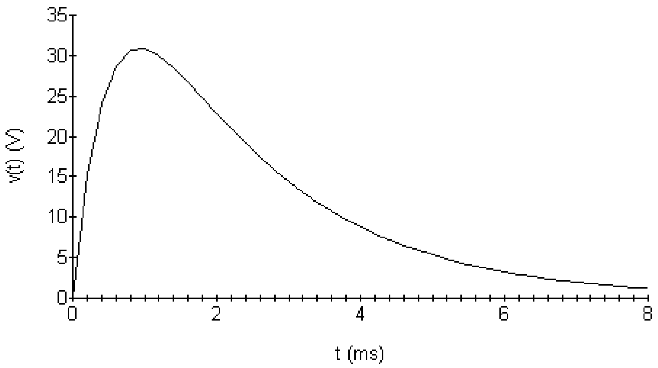
$$v(0^+) = 0 = A_1 + A_2; \qquad \left[\frac{dv(0^+)}{dt}\right] = \frac{i_{\text{C}}(0^+)}{C} = 98{,}000\,\text{V/s}$$

$$\text{Therefore} \quad -500A_1 - 2000A_2 = 98{,}000$$

$$A_1 = \frac{+980}{15}, \quad A_2 = \frac{-980}{15}$$

$$v(t) = \left[\frac{980}{15}\right][e^{-500t} - e^{-2000t}]\,\mathrm{V}, \qquad t \geq 0$$

[b]

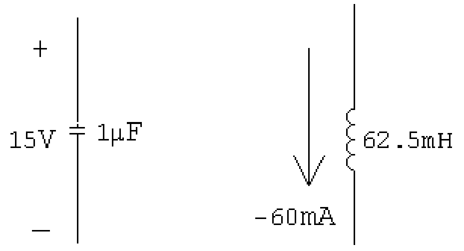


Example 8.4: $v_{\max} \cong 74.1\,\mathrm{V}$ at 1.4 ms

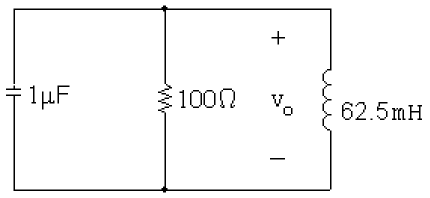
Example 8.5: $v_{\max} \cong 36.1\,\mathrm{V}$ at 1.0 ms

Problem 8.17: $v_{\max} \cong 30.9$ at 0.92 ms

P 8.18 $t < 0: \qquad V_o = 15\,\mathrm{V}, \qquad I_o = -60\,\mathrm{mA}$



$t > 0$:



$$i_R(0) = \frac{15}{100} = 150\,\mathrm{mA}; \qquad i_{\mathrm{L}}(0) = -60\,\mathrm{mA}$$

$$i_{\mathrm{C}}(0) = -150 - (-60) = -90\,\mathrm{mA}$$

$$\alpha = \frac{1}{2RC} = \frac{1}{2(100)(10^{-6})} = 5000 \text{ rad/s}$$

$$\omega_o^2 = \frac{1}{LC} = \frac{1}{(62.5 \times 10^{-3})(10^{-6})} = 16 \times 10^6$$

$$s_{1,2} = -5000 \pm \sqrt{25 \times 10^6 - 16 \times 10^6} = -5000 \pm 3000$$

$$s_1 = -2000 \text{ rad/s}; \qquad s_2 = -8000 \text{ rad/s}$$

$$\therefore \quad v_o = A_1e^{-2000t} + A_2e^{-8000t}$$

$$A_1 + A_2 = v_o(0) = 15$$

$$\frac{dv_o}{dt}(0) = -2000A_1 - 8000A_2 = \frac{-90 \times 10^{-3}}{10^{-6}} = -90{,}000$$

Solving, $\quad A_1 = 5\,\text{V}, \qquad A_2 = 10\,\text{V}$

$$\therefore \quad v_o = 5e^{-2000t} + 10e^{-8000t}\,\text{V}, \qquad t \geq 0$$

P 8.19 $\omega_o^2 = \dfrac{1}{LC} = \dfrac{1}{(62.5 \times 10^{-3})(10^{-6})} = 16 \times 10^6$

$$\alpha = \frac{1}{2RC} = \frac{1}{2(200)(10^{-6})} = 2500$$

$$s_{1,2} = -2500 \pm \sqrt{2500^2 - 16 \times 10^6} = -2500 \pm j3122.5\text{rad/s}$$

$$v_o(t) = B_1e^{-2500t}\cos 3122.5t + B_2e^{-2500t}\sin 3122.5t$$

$$v_o(0) = B_1 = 15\,\text{V}$$

$$i_R(0) = \frac{15}{200} = 75\,\text{mA}$$

$$i_\text{L}(0) = -60\,\text{mA}$$

$$i_\text{C}(0) = -i_R(0) - i_\text{L}(0) = -15\,\text{mA} \quad \therefore \quad \frac{i_\text{C}(0)}{C} = -15{,}000$$

$$\frac{dv_o}{dt}(0) = -2500B_1 + 3122.5B_2 = -15{,}000$$

$$\therefore \qquad B_2 = 7.21$$

$$v_o(t) = 15e^{-2500t}\cos 3122.5t + 7.21e^{-2500t}\sin 3122.5t\,\text{V}, \qquad t \geq 0$$

P 8.20 $$\omega_o^2 = \frac{1}{LC} = \frac{1}{(62.5 \times 10^{-3})(10^{-6})} = 16 \times 10^6$$

$$\alpha = \frac{1}{2RC} = \frac{1}{2(125)(10^{-6})} = 4000$$

$$\therefore \quad \alpha^2 = \omega_o^2 \text{ (critical damping)}$$

$$v_o(t) = D_1te^{-4000t} + D_2e^{-4000t}$$

$$v_o(0) = D_2 = 15\,\text{V}$$

$$i_R(0) = \frac{15}{125} = 120\,\text{mA}$$

$$i_{\text{L}}(0) = -60\,\text{mA}$$

$$i_{\text{C}}(0) = -60\,\text{mA}$$

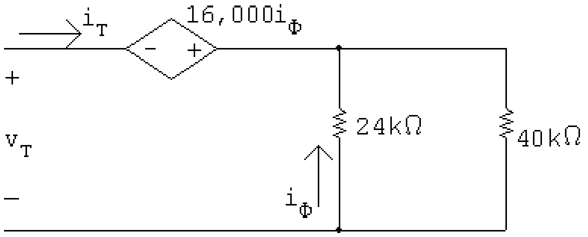
$$\frac{dv_o}{dt}(0) = -4000D_2 + D_1$$

$$\frac{i_{\text{C}}(0)}{C} = \frac{-60 \times 10^{-3}}{10^{-6}} = -60{,}000$$

$$D_1 - 4000D_2 = -60{,}000; \qquad D_1 = 0$$

$$v_o(t) = 15e^{-4000t}\,\text{V}, \qquad t \geq 0$$

P 8.21



$$v_T = -16{,}000i_\phi + i_T(15{,}000) = -16{,}000\frac{-i_T(40)}{64} + i_t(15{,}000)$$

$$\frac{v_T}{i_T} = 10{,}000 + 15{,}000 = 25\,\text{k}\Omega$$

$$V_o = \frac{4000}{5000}(7.5) = 6\,\text{V}; \qquad I_o = 0$$

$$i_{\text{C}}(0) = -i_R(0) - i_{\text{L}}(0) = -\frac{6}{25{,}000} = -240\,\mu\text{A}$$

$$\frac{i_{\text{C}}(0)}{C} = \frac{-240 \times 10^{-6}}{4 \times 10^{-9}} = -60{,}000$$

$$\omega_o^2 = \frac{1}{LC} = \frac{10^9}{(4)(15.625)} = 16 \times 10^6; \qquad \omega_o = 4000 \text{ rad/s}$$

$$\alpha = \frac{1}{2RC} = \frac{10^9}{(2)(4)(25 \times 10^3)} = 5000 \text{ rad/s}$$

$\alpha^2 > \omega_0^2$ so the response is overdamped

$$s_{1,2} = -5000 \pm \sqrt{5000^2 - 4000^2} = -5000 \pm 3000 \text{ rad/s}$$

$$v_o = A_1 e^{-2000t} + A_2 e^{-8000t}$$

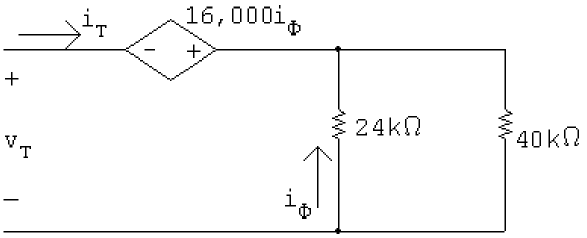
$$v_o(0) = A_1 + A_2 = 6\,\text{V}$$

$$\frac{dv_o}{dt}(0) = -2000A_1 - 8000A_2 = -60{,}000$$

$$\therefore \quad A_1 = -2\,\text{V}; \qquad A_2 = 8\,\text{V}$$

$$v_o = 8e^{-8000t} - 2e^{-2000t}\,\text{V}, \qquad t \geq 0$$

P 8.22



$$v_T = -16{,}000 i_\phi + i_T(15{,}000) = -16{,}000\frac{-i_T(40)}{64} + i_t(15{,}000$$

$$\frac{v_T}{i_T} = 10{,}000 + 15{,}000 = 25\,\text{k}\Omega$$

$$V_o = \frac{4000}{5000}(7.5) = 6\,\text{V}; \qquad I_o = 0$$

$$i_{\text{C}}(0) = -i_R(0) - i_{\text{L}}(0) = -\frac{6}{25{,}000} = -240\,\mu\text{A}$$

$$\frac{i_{\text{C}}(0)}{C} = \frac{-240 \times 10^{-6}}{4 \times 10^{-9}} = -60{,}000$$

$$\omega_o^2 = \frac{1}{LC} = \frac{10^9}{(4)(10)} = 25 \times 10^6; \qquad \omega_o = 5000 \text{ rad/s}$$

$$\alpha = \frac{1}{2RC} = \frac{10^9}{(2)(4)(25 \times 10^3)} = 5000 \text{ rad/s}$$

$\alpha^2 = \omega_0^2$ so the response is critically damped

$$v_o = D_1 te^{-5000t} + D_2 e^{-5000t}$$

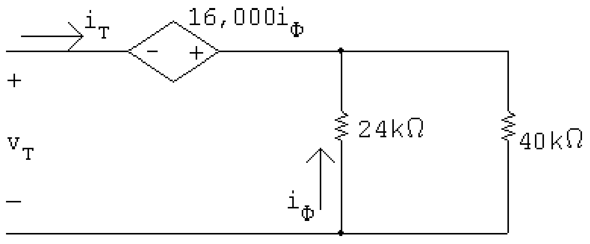
$$v_o(0) = D_2 = 6\,\text{V}$$

$$\frac{dv_o}{dt}(0) = D_1 - \alpha D_2 = -60{,}000$$

$$\therefore \quad D_1 = -60{,}000 + (5000)(6) = -30{,}000 \text{ V/s}$$

$$v_o = -30{,}000te^{-5000t} + 6e^{-5000t}\,\text{V}, \qquad t \geq 0$$

P 8.23



$$v_T = -16{,}000 i_\phi + i_T(15{,}000) = -16{,}000\frac{-i_T(40)}{64} + i_t(15{,}000$$

$$\frac{v_T}{i_T} = 10{,}000 + 15{,}000 = 25\,\text{k}\Omega$$

$$V_o = \frac{4000}{5000}(7.5) = 6\,\text{V}; \qquad I_o = 0$$

$$i_{\rm C}(0) = -i_R(0) - i_{\rm L}(0) = -\frac{6}{25{,}000} = -240\,\mu\text{A}$$

$$\frac{i_{\rm C}(0)}{C} = \frac{-240 \times 10^{-6}}{4 \times 10^{-9}} = -60{,}000$$

$$\omega_o^2 = \frac{1}{LC} = \frac{10^9}{(4)(6.4)} = 6250^2; \qquad \omega_o = 6250 \text{ rad/s}$$

$$\alpha = \frac{1}{2RC} = \frac{10^9}{(2)(4)(25 \times 10^3)} = 5000 \text{ rad/s}$$

$\alpha^2 < \omega_0^2$ so the response is underdamped

$$\omega_d = \sqrt{6250^2 - 5000^2} = 3750 \text{ rad/s}$$

$$v_o = B_1 e^{-5000t} \cos 3750t + B_2 e^{-5000t} \sin 3750t$$

$$v_o(0) = B_1 = 6\,\text{V}$$

$$\frac{dv_o}{dt}(0) = -5000B_1 + 3750B_2 = -60{,}000$$

$$\therefore \quad B_2 = -8\,\text{V}$$

$$v_o = e^{-5000t}(6 \cos 3750t - 8 \sin 3750t)\,\text{V}, \qquad t \geq 0$$

P 8.24 **[a]** $v = L\left(\frac{di_{\rm L}}{dt}\right) = 16[e^{-20{,}000t} - e^{-80{,}000t}]\,\text{V}, \qquad t \geq 0$

[b] $i_{\rm R} = \frac{v}{R} = 40[e^{-20{,}000t} - e^{-80{,}000t}]\,\text{mA}, \qquad t \geq 0^+$

[c] $i_{\rm C} = I - i_{\rm L} - i_{\rm R} = [-8e^{-20{,}000t} + 32e^{-80{,}000t}]\,\text{mA}, \qquad t \geq 0^+$

P 8.25 **[a]** $v = L\left(\frac{di_{\rm L}}{dt}\right) = 40e^{-32{,}000t} \sin 24{,}000t\,\text{V}, \qquad t \geq 0$

[b] $i_{\rm C}(t) = I - i_{\rm R} - i_{\rm L} = 24 \times 10^{-3} - \frac{v}{625} - i_{\rm L}$

$$= [24e^{-32{,}000t} \cos 24{,}000t - 32e^{-32{,}000t} \sin 24{,}000t]\,\text{mA}, \qquad t \geq 0^+$$

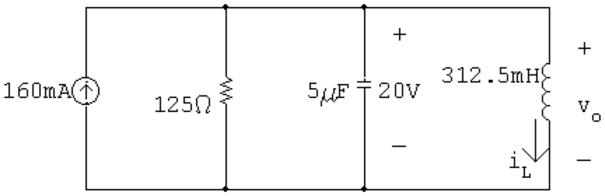
P 8.26 $v = L\left(\frac{di_{\rm L}}{dt}\right) = 960{,}000te^{-40{,}000t}\,\text{V}, \qquad t \geq 0$

P 8.27 $t < 0$:

$$v_o(0^-) = v_o(0^+) = \frac{625}{781.25}(25) = 20\,\text{V}$$

$$i_\text{L}(0^-) = i_\text{L}(0^+) = 0$$

$t > 0$:



$$-160 \times 10^{-3} + \frac{20}{125} + i_\text{C}(0^+) + 0 = 0; \qquad \therefore \quad i_\text{C}(0^+) = 0$$

$$\frac{1}{2RC} = \frac{1}{2(125)(5 \times 10^{-6})} = 800\,\text{rad/s}$$

$$\omega_o^2 = \frac{1}{LC} = \frac{1}{(312.5 \times 10^{-3})(5 \times 10^{-6})} = 64 \times 10^4$$

$$\therefore \quad \alpha^2 = \omega_o^2 \quad \text{critically damped}$$

[**a**] $v_o = V_f + D_1'te^{-800t} + D_2'e^{-800t}$

$$V_f = 0$$

$$\frac{dv_o(0)}{dt} = -800D_2' + D_1' = 0$$

$$v_o(0^+) = 20 = D_2'$$

$$D_1' = 800D_2' = 16{,}000\,\text{V/s}$$

$$\therefore \quad v_o = 16{,}000te^{-800t} + 20e^{-800t}\,\text{V}, \quad t \geq 0^+$$

[**b**] $i_\text{L} = I_f + D_3'te^{-800t} + D_4'e^{-800t}$

$$i_\text{L}(0^+) = 0; \qquad I_f = 160\,\text{mA}; \qquad \frac{di_\text{L}(0^+)}{dt} = \frac{20}{312.5 \times 10^{-3}} = 64\ \text{A/s}$$

$$\therefore \quad 0 = 160 + D_4'; \qquad D_4' = -160\,\text{mA};$$

$$-800D_4' + D_3' = 64; \qquad D_3' = -64\,\text{A/s}$$

$$\therefore \quad i_\text{L} = 160 - 64{,}000te^{-800t} - 160e^{-800t}\,\text{mA} \qquad t \geq 0$$

P 8.28 **[a]** $w_{\rm L} = \int_0^\infty p dt = \int_0^\infty v_o i_{\rm L}\, dt$

$$v_o = 16{,}000te^{-800t} + 20e^{-800t}\,\text{V}$$

$$i_{\rm L} = 0.16 - 64te^{-800t} - 0.16e^{-800t}\,\text{A}$$

$$p = 3.2e^{-800t} + 2560te^{-800t} - 3840te^{-1600t}$$
$$-1{,}024{,}000t^2e^{-1600t} - 3.2e^{-1600t}\,\text{W}$$

$$w_{\rm L} = 3.2\int_0^\infty e^{-800t}\,dt + 2560\int_0^\infty te^{-800t}\,dt - 3480\int_0^\infty te^{-1600t}\,dt$$
$$- 1{,}024{,}000\int_0^\infty t^2e^{-1600t}\,dt - 3.2\int_0^\infty e^{-1600t}\,dt$$

$$= 3.2\frac{e^{-800t}}{-800}\Big|_0^\infty + \frac{2560}{(800)^2}e^{-800t}(-2560t-1)\Big|_0^\infty$$
$$- \frac{3840}{(1600)^2}e^{-1600t}(-1600t-1)\Big|_0^\infty$$
$$- \frac{1{,}024{,}000}{(-1600)^3}e^{-1600t}(1600^2t^2+3200t+2)\Big|_0^\infty$$
$$- 3.2\frac{e^{-1600t}}{(-1600)}\Big|_0^\infty$$

All the upper limits evaluate to zero hence

$$w_{\rm L} = \frac{3.2}{800} + \frac{2560}{800^2} - \frac{3840}{1600^2} - \frac{(1{,}024{,}000)(2)}{1600^3} - \frac{3.2}{1600} = 4\,\text{mJ}$$

Note this value corresponds to the final energy stored in the inductor, i.e.

$$w_{\rm L}(\infty) = \frac{1}{2}(312.5\times10^{-3})(0.16)^2 = 4\,\text{mJ}.$$

[b] $v = 16{,}000te^{-800t} + 20e^{-800t}\,\text{V}$

$$i_{\rm R} = \frac{v}{125} = 128te^{-800t} + 0.16e^{-800t}\,\text{A}$$

$$p_{\rm R} = vi_{\rm R} = 2{,}048{,}000t^2e^{-1600t} + 5120te^{-1600t} + 3.2e^{-1600t}$$

$$w_{\rm R} = \int_0^\infty p_{\rm R}\,dt$$

$$= 2{,}048{,}000\int_0^\infty t^2e^{-1600t}\,dt + 5120\int_0^\infty te^{-1600t}\,dt + 3.2\int_0^\infty e^{-1600t}\,dt$$

$$= \frac{2{,}048{,}000e^{-1600t}}{-1600^3}[1600^2t^2+3200t+2]\Big|_0^\infty +$$

$$\frac{5120e^{-1600t}}{1600^2}(-1600t-1)\Big|_0^\infty + \frac{3.2e^{-1600t}}{(-1600)}\Big|_0^\infty$$

Since all the upper limits evaluate to zero we have

$$w_\mathrm{R} = \frac{2{,}048{,}000(2)}{1600^3} + \frac{5120}{1600^2} + \frac{3.2}{1600} = 5\,\mathrm{mJ}$$

[c] $160 = i_\mathrm{R} + i_\mathrm{C} + i_\mathrm{L} \quad (\mathrm{mA})$

$$i_\mathrm{R} + i_\mathrm{L} = 160 + 64{,}000te^{-800t}\,\mathrm{mA}$$

$$\therefore \quad i_\mathrm{C} = 160 - (i_\mathrm{R} + i_\mathrm{L}) = -64{,}000te^{-800t}\,\mathrm{mA} = -64te^{-800t}\,\mathrm{A}$$

$$p_\mathrm{C} = vi_\mathrm{C} = [16{,}000te^{-800t} + 20e^{-800t}][-64te^{-800t}]$$

$$= -1{,}024{,}000t^2e^{-1600t} - 1280e^{-1600t}$$

$$w_\mathrm{C} = -1{,}024{,}000\int_0^\infty t^2e^{-1600t}\,dt - 1280\int_0^\infty te^{-1600t}\,dt$$

$$w_\mathrm{C} = \frac{-1{,}024{,}000e^{-1600t}}{-1600^3}[1600^2t^2 + 3200t + 2]\Big|_0^\infty - \frac{1280e^{-1600t}}{1600^2}(-1600t - 1)\Big|_0^\infty$$

Since all upper limits evaluate to zero we have

$$w_\mathrm{C} = \frac{-1{,}024{,}000(2)}{1600^3} - \frac{1280(1)}{1600^2} = -1\,\mathrm{mJ}$$

Note this 1 mJ corresponds to the initial energy stored in the capacitor, i.e.,

$$w_\mathrm{C}(0) = \frac{1}{2}(5 \times 10^{-6})(20)^2 = 1\,\mathrm{mJ}.$$

Thus $w_\mathrm{C}(\infty) = 0\,\mathrm{mJ}$ which agrees with the final value of $v = 0$.

[d] $i_s = 160\,\mathrm{mA}$

$$p_s(\mathrm{del}) = 160v\,\mathrm{mW}$$

$$= 0.16[16{,}000te^{-800t} + 20e^{-800t}]$$

$$= 3.2e^{-800t} + 2560te^{-800t}\,\mathrm{W}$$

$$w_s = 3.2\int_0^\infty e^{-800t}\,dt + \int_0^\infty 2560te^{-800t}\,dt$$

$$= \frac{3.2e^{-800t}}{-800}\Big|_0^\infty + \frac{2560e^{-800t}}{800^2}(-800t - 1)\Big|_0^\infty$$

$$= \frac{3.2}{800} + \frac{2560}{800} = 8\,\mathrm{mJ}$$

[e] $w_\mathrm{L} = 4\,\mathrm{mJ} \quad$ (absorbed)

$w_\mathrm{R} = 5\,\mathrm{mJ} \quad$ (absorbed)

$w_\mathrm{C} = 1\,\mathrm{mJ} \quad$ (delivered)

$$w_S = 8\,\text{mJ} \quad \text{(delivered)}$$

$$\sum w_{\text{del}} = w_{\text{abs}} = 9\,\text{mJ}.$$

P 8.29 $$\omega_o^2 = \frac{1}{LC} = \frac{1}{(50\times 10^{-3})(0.2\times 10^{-6})} = 10^8; \qquad \omega_o = 10^4\ \text{rad/s}$$

$$\alpha = \frac{1}{2RC} = \frac{1}{2(200)(0.2\times 10^{-6})} = 12{,}500\ \text{rad/s} \qquad \therefore\ \ \text{overdamped}$$

$$s_{1,2} = -12{,}500 \pm \sqrt{(12{,}500)^2 - 10^8} = -12{,}500 \pm 7500\ \text{rad/s}$$

$$s_1 = -5000\ \text{rad/s}; \qquad s_2 = -20{,}000\ \text{rad/s}$$

$$I_f = 60\,\text{mA}$$

$$i_{\text{L}} = 60\times 10^{-3} + A_1'e^{-5000t} + A_2'e^{-20{,}000t}$$

$$\therefore\ \ -45\times 10^{-3} = 60\times 10^{-3} + A_1' + A_2'; \qquad A_1' + A_2' = -105\times 10^{-3}$$

$$\frac{di_{\text{L}}}{dt} = -5000A_1' - 20{,}000A_2' = \frac{15}{0.05} = 300$$

$$\text{Solving,} \qquad A_1' = -120\,\text{mA}; \qquad A_2' = 15\,\text{mA}$$

$$i_{\text{L}} = 60 - 120e^{-5000t} + 15e^{-20{,}000t}\,\text{mA}, \quad t \geq 0$$

P 8.30 $$\alpha = \frac{1}{2RC} = \frac{1}{2(312.5)(0.2\times 10^{-6})} = 8000; \qquad \alpha^2 = 64\times 10^6$$

$$\omega_o = 10^4 \qquad \text{underdamped}$$

$$s_{1,2} = -8000 \pm j\sqrt{8000^2 - 10^8} = -8000 \pm j6000\ \text{rad/s}$$

$$i_{\text{L}} = 60\times 10^{-3} + B_1'e^{-8000t}\cos 6000t + B_2'e^{-8000t}\sin 6000t$$

$$-45\times 10^{-3} = 60\times 10^{-3} + B_1' \qquad \therefore\ \ B_1' = -105\,\text{mA}$$

$$\frac{di_{\text{L}}}{dt}(0) = -8000B_1' + 6000B_2' = 300$$

$$\therefore\ \ B_2' = -90\,\text{mA}$$

$$i_{\text{L}} = 60 - 105e^{-8000t}\cos 6000t - 90e^{-8000t}\sin 6000t\,\text{mA}, \quad t \geq 0$$

P 8.31 $\alpha = \dfrac{1}{2RC} = \dfrac{1}{2(250)(0.2 \times 10^{-6})} = 10^4$

$\alpha^2 = 10^4 = \omega_o^2$ critical damping

$i_{\rm L} = I_f + D_1' te^{-10^4 t} + D_2' e^{-10^4 t} = 60 \times 10^{-3} + D_1' te^{-10^4 t} + D_2' e^{-10^4 t}$

$i_{\rm L}(0) = -45 \times 10^{-3} = 60 \times 10^{-3} + D_2'; \qquad \therefore \; D_2' = -105\,\text{mA}$

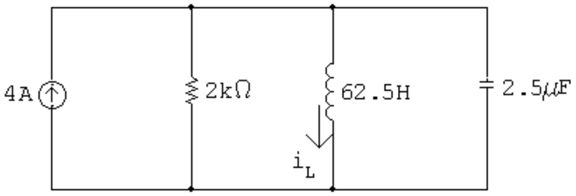
$\dfrac{di_{\rm L}}{dt}(0) = -10^4 D_2' + D_1' = 300\,\text{A/s}$

$\therefore \; D_1' = 300 + 10^4(-105 \times 10^{-3}) = -750\,\text{A/s}$

$i_{\rm L} = 60 - 750{,}000te^{-10^4 t} - 105e^{-10^4 t}\,\text{mA}, \quad t \geq 0$

P 8.32 $t < 0: \qquad i_{\rm L}(0^-) = \dfrac{-15}{3000} = -5\,\text{mA}; \qquad v_{\rm C}(0^-) = 0\,\text{V}$

The circuit reduces to:



$i_{\rm L}(\infty) = 4\,\text{mA}$

$\omega_o^2 = \dfrac{1}{LC} = \dfrac{10^6}{(62.5)(2.5)} = 6400; \qquad \omega_o = 80\text{ rad/s}$

$\alpha = \dfrac{1}{2RC} = \dfrac{10^6}{(4000)(2.5)} = 100$

$s_{1,2} = -100 \pm \sqrt{100^2 - 80^2} = -100 \pm 60$

$s_1 = -40\text{ rad/s}; \qquad s_2 = -160\text{ rad/s}$

$i_{\rm L} = I_f + A_1' e^{-40t} + A_2' e^{-160t}$

$i_{\rm L}(\infty) = I_f = 4\text{mA}$

$i_{\rm L}(0) = A_1' + A_2' + I_f = -5\,\text{mA}$

$\therefore \quad A_1' + A_2' + 4 = -5 \quad \text{so} \quad A_1' + A_2' = -9\,\text{mA}$

$\frac{di_\text{L}}{dt}(0) = 0 = -40A_1 - 160A_2'$

Solving, $\quad A_1' = -12\,\text{mA}, \qquad A_2' = 3\,\text{mA}$

$i_\text{L} = 4 - 12e^{-40t} + 3e^{-160t}\,\text{mA}, \qquad t \geq 0$

P 8.33 $v_\text{C}(0^+) = \frac{1}{2}(240) = 120\,\text{V}$

$i_\text{L}(0^+) = 60\,\text{mA}; \qquad i_\text{L}(\infty) = \frac{240}{5} \times 10^{-3} = 48\,\text{mA}$

$\alpha = \frac{1}{2RC} = \frac{10^6}{2(2500)(5)} = 40$

$\omega_o^2 = \frac{1}{LC} = \frac{10^6}{400} = 2500$

$\alpha^2 = 1600; \qquad \alpha^2 < \omega_o^2; \qquad \therefore \quad \text{underdamped}$

$s_{1,2} = -40 \pm j\sqrt{2500 - 1600} = -40 \pm j30 \text{ rad/s}$

$$\begin{aligned} i_\text{L} &= I_f + B_1'e^{-\alpha t}\cos\omega_d t + B_2'e^{-\alpha t}\sin\omega_d t \\ &= 48 + B_1'e^{-40t}\cos 30t + B_2'e^{-40t}\sin 30t \end{aligned}$$

$i_\text{L}(0) = 48 + B_1'; \qquad B_1' = 60 - 48 = 12\,\text{mA}$

$\frac{di_\text{L}}{dt}(0) = 30B_2' - 40B_1' = \frac{120}{80} = 1.5 = 1500 \times 10^{-3}$

$\therefore \quad 30B_2' = 40(12) \times 10^{-3} + 1500 \times 10^{-3}; \qquad B_2' = 66\,\text{mA}$

$\therefore \quad i_\text{L} = 48 + 12e^{-40t}\cos 30t + 66e^{-40t}\sin 30t\,\text{mA}, \qquad t \geq 0$

P 8.34 $\alpha = \dfrac{1}{2RC} = \dfrac{1}{2(400)(1.25 \times 10^{-6})} = 1000$

$$\omega_o^2 = \frac{1}{LC} = \frac{1}{(1.25 \times 10^{-6})(1.25)} = 64 \times 10^4$$

$$s_{1,2} = -1000 \pm \sqrt{1000^2 - 64 \times 10^4} = -1000 \pm 600 \text{ rad/s}$$

$$s_1 = -400 \text{ rad/s}; \qquad s_2 = -1600 \text{ rad/s}$$

$$v_o(\infty) = 0 = V_f$$

$$\therefore \quad v_o = A_1' e^{-400t} + A_2' e^{-1600t}$$

$$v_o(0) = 12 = A_1' + A_2'$$

Note: $i_{\mathrm{C}}(0^+) = 0$

$$\therefore \quad \frac{dv_o}{dt}(0) = 0 = -400A_1' - 1600A_2'$$

Solving, $A_1' = 16\,\text{V}$, $A_2' = -4\,\text{V}$

$$v_o(t) = 16e^{-400t} - 4e^{-1600t}\,\text{V}, \qquad t \geq 0$$

P 8.35 **[a]** $i_o = I_f + A_1' e^{-400t} + A_2' e^{-1600t}$

$$I_f = \frac{12}{400} = 30\text{mA}; \qquad i_o(0) = 0$$

$$0 = 30 \times 10^{-3} + A_1' + A_2', \qquad \therefore \quad A_1' + A_2' = -30 \times 10^{-3}$$

$$\frac{di_o}{dt}(0) = \frac{12}{1.25} = -400A_1' - 1600A_2'$$

Solving, $A_1' = -32\,\text{mA}$; $A_2' = 2\,\text{mA}$

$$i_o = 30 - 32e^{-400t} + 2e^{-1600t}\,\text{mA}, \quad t \geq 0$$

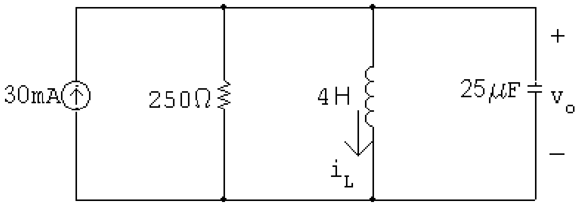
[b] $\dfrac{di_o}{dt} = [12.8e^{-400t} - 3.2e^{-1600t}]$

$$v_o = L\frac{di_o}{dt} = 16e^{-400t} - 4e^{-1600t}\,\text{V}, \quad t \geq 0$$

This agrees with the solution to Problem 8.34.

P 8.36 $i_L(0^-) = i_L(0^+) = \dfrac{7.5}{250} = 30\,\text{mA}$

For $t > 0$



$$i_L(0^-) = i_L(0^+) = 30\,\text{mA}$$

$$\alpha = \frac{1}{2RC} = 80\,\text{rad/s}; \qquad \omega_o^2 = \frac{1}{LC} = 10^4 \quad \text{so} \quad \omega_o = 100 \text{ rad/s}$$

$$\omega_d = \sqrt{100^2 - 80^2} = 60\,\text{rad/s}$$

$$v_o(\infty) = 0 = V_f; \qquad B_1' = v(0) = 0$$

$$v_o = e^{-80t} B_2' \sin 60t$$

$$i_C(0^+) = -30 + 30 + 0 = 0$$

$$\therefore \quad \frac{dv_o}{dt} = 0$$

$$\frac{dv_o}{dt}(0) = -\alpha B_1' + \omega_d B_2' = 0 + 60B_2' = 0$$

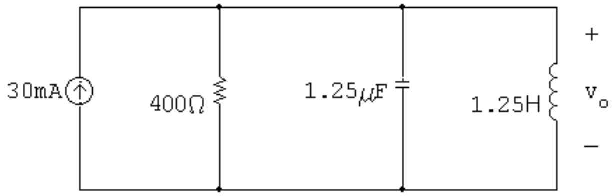
$$\therefore \quad B_1' = 0; \qquad B_2' = 0$$

$$\therefore \quad v_o = 0 \text{ for } t \geq 0$$

Note: $\quad v_o(0) = 0; \qquad v_o(\infty) = 0; \qquad \dfrac{dv_o(0)}{dt} = 0$

Hence, the 30 mA current circulates between the current source and the ideal inductor in the equivalent circuit. In the original circuit, the 7.5 V source sustains a current of 30 mA in the inductor. This is an example of a circuit going directly into steady state when the switch is closed. There is no transient period, or interval.

P 8.37 For $t > 0$



$$\alpha = \frac{1}{2RC} = 1000; \qquad \frac{1}{LC} = 64 \times 10^4$$

$$s_{1,2} = -1000 \pm 600 \text{ rad/s}$$

$$s_1 = -400\,\text{rad/s}; \qquad s_2 = -1600\,\text{rad/s}$$

$$v_o = V_f + A_1' e^{-400t} + A_2' e^{-1600t}$$

$$V_f = 0; \qquad v_o(0^+) = 0; \qquad i_{\text{C}}(0^+) = 30\,\text{mA}$$

$$\therefore \quad A_1' + A_2' = 0$$

$$\frac{dv_o(0^+)}{dt} = \frac{i_{\text{C}}(0^+)}{1.25 \times 10^{-6}} = 24{,}000\,\text{V/s}$$

$$\frac{dv_o(0^+)}{dt} = -400A_1' - 1600A_2' = 24{,}000$$

Solving,

$$A_1' = 20\,\text{V}; \qquad A_2' = -20\,\text{V}$$

$$v_o = 20e^{-400t} - 20e^{-1600t}\,\text{V}, \qquad t \geq 0$$

P 8.38 **[a]** From the solution to Prob. 8.37 $s_1 = -400\,\text{rad/s}$ and $s_2 = -1600\,\text{rad/s}$, therefore

$$i_o = I_f + A_1' e^{-400t} + A_2' e^{-1600t}$$

$$I_f = 30\,\text{mA}; \qquad i_o(0^+) = 0; \qquad \frac{di_o(0^+)}{dt} = 0$$

$$\therefore \quad 0 = 30 \times 10^{-3} + A_1' + A_2'; \qquad -400A_1' - 1600A_2' = 0$$

Solving

$$A_1' = -40\,\text{mA}; \qquad A_2' = 10\,\text{mA}$$

$$\therefore \quad i_o = 30 - 40e^{-400t} + 10e^{-1600t}\,\text{mA}, \qquad t \geq 0$$

[b] $\dfrac{di_o}{dt} = 16e^{-400t} - 16e^{-1600t}$

$$v_o = L\frac{di_o}{dt} = 20e^{-400t} - 20e^{-1600t}\,\text{V}, \qquad t \geq 0$$

This agrees with the solution to Problem 8.27.

P 8.39 **[a]** $-\alpha + \sqrt{\alpha^2 - \omega_0^2} = -4000; \qquad -\alpha - \sqrt{\alpha^2 - \omega_0^2} = -16{,}000$

$$\therefore \quad \alpha = 10{,}000 \text{ rad/s}, \qquad \omega_0^2 = 64 \times 10^6$$

$$\alpha = \frac{R}{2L} = 10{,}000; \qquad R = 20{,}000L$$

$$\omega_o^2 = \frac{1}{LC} = 64 \times 10^6; \qquad L = \frac{10^9}{64 \times 10^6(31.25)} = 0.5\,\text{H}$$

$$R = 10{,}000\ \Omega$$

[b] $i(0) = 0$

$$L\frac{di(0)}{dt} = v_c(0); \qquad \frac{1}{2}(31.25) \times 10^{-9}v_c^2(0) = 9 \times 10^{-6}$$

$$\therefore \quad v_c^2(0) = 576; \qquad v_c(0) = 24\,\text{V}$$

$$\frac{di(0)}{dt} = \frac{24}{0.5} = 48\,\text{A/s}$$

[c] $i(t) = A_1e^{-4000t} + A_2e^{-16{,}000t}$

$$i(0) = A_1 + A_2 = 0$$

$$\frac{di(0)}{dt} = -4000A_1 - 16{,}000A_2 = 48$$

Solving,

$$\therefore \quad A_1 = 4\,\text{mA}; \qquad A_2 = -4\,\text{mA}$$

$$i(t) = 4e^{-4000t} - 4e^{-16{,}000t}\,\text{mA}, \qquad t \geq 0$$

[d] $\dfrac{di(t)}{dt} = -16e^{-4000t} + 64e^{-16{,}000t}$

$$\frac{di}{dt} = 0 \text{ when } 64e^{-16{,}000t} = 16e^{-4000t}$$

$$\text{or } e^{12{,}000t} = 4$$

$$\therefore \quad t = \frac{\ln 4}{12{,}000} = 115.52\,\mu\text{s}$$

[e] $i_{\max} = 4e^{-0.4621} - 4e^{-1.8484} = 1.89\,\text{mA}$

[f] $v_L(t) = 0.5\dfrac{di}{dt} = [-8e^{-1000t} + 32e^{-4000t}]\,\text{V}, \quad t \geq 0^+$

P 8.40 **[a]** $\dfrac{1}{LC} = 20{,}000^2$

There are many possible solutions. This one begins by choosing $L = 1\,\text{mH}$. Then,

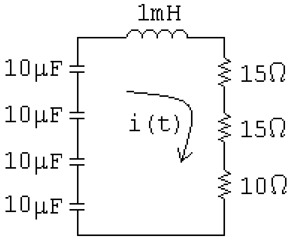
$$C = \frac{1}{(1 \times 10^{-3})(20{,}000)^2} = 2.5\,\mu\text{F}$$

We can achieve this capacitor value using components from Appendix H by combining four $10\,\mu$F capacitors in series.

Critically damped: $\qquad \alpha = \omega_0 = 20{,}000 \qquad$ so $\qquad \dfrac{R}{2L} = 20{,}000$

$$\therefore \quad R = 2(10^{-3})(20{,}000) = 40\,\Omega$$

We can create this resistor value using components from Appendix H by combining a $10\,\Omega$ resistor and two $15\,\Omega$ resistors in series. The final circuit:



[b] $s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2} = -20{,}000 \pm 0$

Therefore there are two repeated real roots at $-20{,}000$ rad/s.

P 8.41 **[a]** Underdamped response:

$\alpha < \omega_0 \qquad$ so $\qquad \alpha < 20{,}000$

Therefore we choose a larger resistor value than the one used in Problem 8.40 to give a smaller value of α. For convenience, pick $\alpha = 16{,}000$ rad/s:

$$\alpha = \frac{R}{2L} = 16{,}000 \qquad \text{so} \qquad R = 2(16{,}000)(10^{-3}) = 32\,\Omega$$

We can create a $32\,\Omega$ resistance by combining a $10\,\Omega$ resistor and a $22\,\Omega$ resistor in series.

$$s_{1,2} = -16{,}000 \pm \sqrt{16{,}000^2 - 20{,}000^2} = -16{,}000 \pm j12{,}000 \text{ rad/s}$$

[b] Overdamped response:

$\alpha > \omega_0 \qquad \text{so} \qquad \alpha > 20{,}000$

Therefore we choose a smaller resistor value than the one used in Problem 8.40. Choose $R = 50\,\Omega$, which can be created by combining two $100\,\Omega$ resistors in parallel:

$$\alpha = \frac{R}{2L} = 25{,}000$$

$$s_{1,2} = -25{,}000 \pm \sqrt{25{,}000^2 - 20{,}000^2} = -25{,}000 \pm 15{,}000$$

$$= -10{,}000 \text{ rad/s}; \qquad \text{and} \qquad -40{,}000 \text{ rad/s}$$

P 8.42 $\alpha = 2000\,\text{rad/s}; \qquad \omega_d = 1500\,\text{rad/s}$

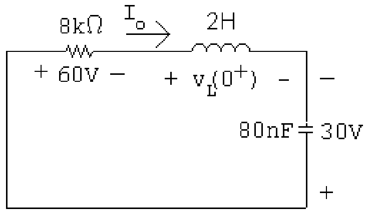
$$\omega_o^2 - \alpha^2 = 225 \times 10^4; \qquad \omega_o^2 = 625 \times 10^4; \quad w_o = 25{,}000\,\text{rad/s}$$

$$\alpha = \frac{R}{2L} = 2000; \qquad R = 4000L$$

$$\frac{1}{LC} = 625 \times 10^4; \qquad L = \frac{1}{(625 \times 10^4)(80 \times 10^{-9})} = 2\,\text{H}$$

$$\therefore \quad R = 8\,\text{k}\Omega$$

$$i(0^+) = B_1 = 7.5\,\text{mA}; \qquad \text{at } t = 0^+$$



$$60 + v_{\text{L}}(0^+) - 30 = 0; \qquad \therefore \qquad v_{\text{L}}(0^+) = -30\,\text{V}$$

$$\frac{di(0^+)}{dt} = \frac{-30}{2} = -15\,\text{A/s}$$

$$\therefore \quad \frac{di(0^+)}{dt} = 1500B_2 - 2000B_1 = -15$$

$$\therefore \quad 1500B_2 = 2000(7.5 \times 10^{-3}) - 15; \qquad \therefore \quad B_2 = 0\,\text{A}$$

$$\therefore \quad i = 7.5e^{-2000t} \sin 1500t\,\text{mA}, \quad t \geq 0$$

P 8.43 From Prob. 8.42 we know v_c will be of the form

$$v_c = B_3e^{-2000t}\cos 1500t + B_4e^{-2000t}\sin 1500t$$

From Prob. 8.42 we have

$$v_c(0) = -30\,\text{V} = B_3$$

and

$$\frac{dv_c(0)}{dt} = \frac{i_\text{C}(0)}{C} = \frac{7.5\times 10^{-3}}{80\times 10^{-9}} = 93.75\times 10^3$$

$$\frac{dv_c(0)}{dt} = 1500B_4 - 2000B_3 = 93{,}750$$

$$\therefore\quad 1500B_4 = 2000(-30) + 93{,}750; \qquad B_4 = 22.5\,\text{V}$$

$$v_c(t) = -30e^{-2000t}\cos 1500t + 22.5e^{-2000t}\sin 1500t\,\text{V} \qquad t\geq 0$$

P 8.44 **[a]** $\omega_o^2 = \dfrac{1}{LC} = \dfrac{10^9}{(125)(0.32)} = 25\times 10^6$

$$\alpha = \frac{R}{2L} = \omega_o = 5000\,\text{rad/s}$$

$$\therefore\quad R = (5000)(2)L = 1250\,\Omega$$

[b] $i(0) = i_\text{L}(0) = 6\,\text{mA}$

$$v_\text{L}(0) = 15 - (0.006)(1250) = 7.5\,\text{V}$$

$$\frac{di}{dt}(0) = \frac{7.5}{0.125} = 60\text{ A/s}$$

[c] $v_\text{C} = D_1te^{-5000t} + D_2e^{-5000t}$

$$v_\text{C}(0) = D_2 = 15\,\text{V}$$

$$\frac{dv_\text{C}}{dt}(0) = D_1 - 5000D_2 = \frac{i_\text{C}(0)}{C} = \frac{-i_\text{L}(0)}{C} = -18{,}750$$

$$\therefore\quad D_1 = 56{,}250\text{ V/s}$$

$$v_\text{C} = 56{,}250te^{-5000t} + 15e^{-5000t}\,\text{V}, \qquad t\geq 0$$

P 8.45 $\omega_o^2 = \dfrac{1}{LC} = \dfrac{1}{(10)(4\times 10^{-3})} = 25$

$$\alpha = \frac{R}{2L} = \frac{80}{2(10)} = 4; \qquad \alpha^2 = 16$$

$$\alpha^2 < \omega_o^2 \quad \therefore \qquad \text{underdamped}$$

$$s_{1,2} = -4 \pm j\sqrt{9} = -4 \pm j3 \text{ rad/s}$$

$$i = B_1e^{-4t}\cos 3t + B_2e^{-4t}\sin 3t$$

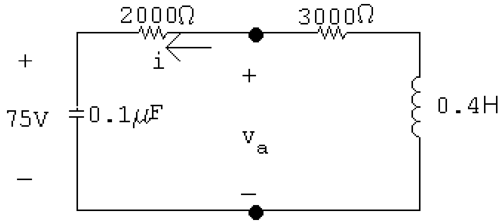
$$i(0) = B_1 = -240/100 = -2.4\,\text{A}$$

$$\frac{di}{dt}(0) = 3B_2 - 4B_1 = 0$$

$$\therefore \quad B_2 = -3.2\,\text{A}$$

$$i = -2.4e^{-4t}\cos 3t - 3.2\sin 3t\,\text{A}, \qquad t \geq 0$$

P 8.46 **[a]** For $t > 0$:



Since $i(0^-) = i(0^+) = 0$

$v_a(0^+) = 75\,\text{V}$

[b] $v_a = 2000i + 10^7\displaystyle\int_0^t i\,dx + 75$

$$\frac{dv_a}{dt} = 2000\frac{di}{dt} + 10^7 i$$

$$\frac{dv_a(0^+)}{dt} = 2000\frac{di(0^+)}{dt} + 10^7 i(0^+) = 2000\frac{di(0^+)}{dt}$$

$$-L\frac{di(0^+)}{dt} = 75$$

$$\frac{di(0^+)}{dt} = -2.5(75) = -187.5\,\text{A/s}$$

$$\therefore \quad \frac{dv_a(0^+)}{dt} = -375{,}000\,\text{V/s}$$

[c] $\alpha = \frac{R}{2L} = \frac{5000}{0.8} = 6250\,\text{rad/s}$

$$\omega_o^2 = \frac{1}{LC} = \frac{10^6}{(0.4)(0.1)} = 25 \times 10^6$$

$$s_{1,2} = -6250 \pm \sqrt{6250^2 - 25 \times 10^6} = -6250 \pm 3750\,\text{rad/s}$$

$\therefore \quad s_1 = -2500\,\text{rad/s}; \qquad s_2 = -10{,}000\,\text{rad/s}$

Overdamped:

$$v_a = A_1e^{-2500t} + A_2e^{-10{,}000t}$$

$$v_a(0) = A_1 + A_2 = 75\,\text{V}$$

$$\frac{dv_a(0)}{dt} = -2500A_1 - 10{,}000A_2 = -375{,}000; \qquad \therefore \quad A_1 = 50\,\text{V}, \qquad A_2 = 25\,\text{V}$$

$$v_a = 50e^{-2500t} + 25e^{-10{,}000t}\,\text{V}, \quad t \geq 0^+$$

P 8.47 **[a]** $t < 0$:

$$i_o = \frac{80}{800} = 100\,\text{mA}; \qquad v_o = 500i_o = (500)(0.01) = 50\,\text{V}$$

$t > 0$:

$$\alpha = \frac{R}{2L} = \frac{500}{2(2.5 \times 10^{-3})} = 10^5 \text{ rad/s}$$

$$\omega_o^2 = \frac{1}{LC} = \frac{1}{(2.5 \times 10^{-3})(40 \times 10^{-9})} = 100 \times 10^8$$

$\alpha^2 = \omega_o^2 \quad \therefore \quad$ critically damped

$$\therefore \quad i_o(t) = D_1te^{-10^5t} + D_2e^{-10^5t}$$

$$i_o(0) = D_2 = 100\,\text{mA}$$

$$\frac{di_o}{dt}(0) = -\alpha D_2 + D_1 = 0$$

$$\therefore \quad D_1 = 10^5(100 \times 10^{-3}) = 10{,}000$$

$$i_o(t) = 10{,}000te^{-10^5t} + 0.1e^{-10^5t}\,\text{A}, \qquad t \geq 0^+$$

[b] $v_o(t) = D_3te^{-10^5t} + D_4e^{-10^5t}$

$$v_o(0) = D_4 = 50$$

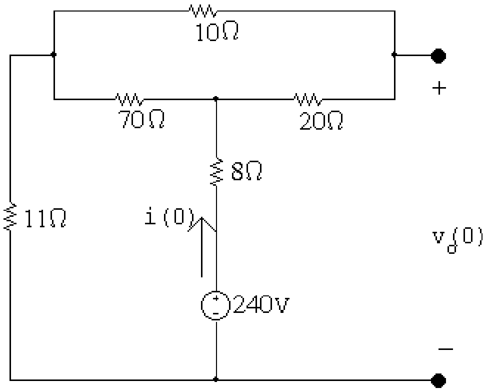
$$C\frac{dv_o}{dt}(0) = -0.1$$

$$\frac{dv_o}{dt}(0) = \frac{-0.1}{40 \times 10^{-9}} = -25 \times 10^5\,\text{V/s} = -\alpha D_4 + D_3$$

$$\therefore \qquad D_3 = 10^5(50) - 25 \times 10^5 = 25 \times 10^5$$

$$v_o(t) = 25 \times 10^5 t e^{-10^5 t} + 50 e^{-10^5 t}\,\text{V}, \quad t \geq 0^+$$

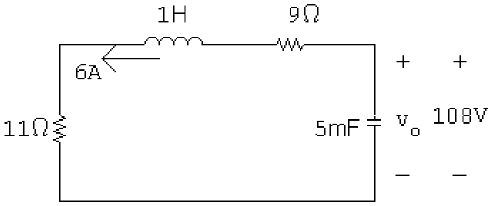
P 8.48 $t < 0$:



$$i(0) = \frac{240}{8 + 30\|70 + 11} = \frac{240}{40} = 6\,\text{A}$$

$$v_o(0) = 240 - 8(6) - \frac{70}{100}(6)(20) = 108\,\text{V}$$

$t > 0$:



$$\alpha = \frac{R}{2L} = \frac{20}{2(1)} = 10, \qquad \alpha^2 = 100$$

$$\omega_o^2 = \frac{1}{LC} = \frac{1}{(1)(5 \times 10^{-3})} = 200$$

$$\omega_o^2 > \alpha^2 \quad \text{underdamped}$$

$$s_{1,2} = -100 \pm \sqrt{100 - 200} = -10 \pm j10 \text{ rad/s}$$

$$v_o = B_1 e^{-10t} \cos 10t + B_2 e^{-10t} \sin 10t$$

$$v_o(0) = B_1 = 108\,\text{V}$$

$$C\frac{dv_o}{dt}(0) = -6, \qquad \frac{dv_o}{dt} = \frac{-6}{5 \times 10^{-3}} = -1200\,\text{V/s}$$

$$\frac{dv_o}{dt}(0) = -10B_1 + 10B_2 = -1200$$

$$10B_2 = -1200 + 10B_1 = -1200 + 1080; \qquad B_2 = -120/10 = -12\,\text{V}$$

$$\therefore \quad v_o = 108e^{-10t}\cos 10t - 12e^{-10t}\sin 10t\,\text{V}, \qquad t \geq 0$$

P 8.49 $i_{\text{C}}(0) = 0; \qquad v_o(0) = 50\,\text{V}$

$$\alpha = \frac{R}{2L} = \frac{8000}{2(160 \times 10^{-3})} = 25{,}000\,\text{rad/s}$$

$$\omega_o^2 = \frac{1}{LC} = \frac{1}{(160 \times 10^{-3})(10 \times 10^{-9})} = 625 \times 10^6$$

$$\therefore \quad \alpha^2 = \omega_o^2; \qquad \text{critical damping}$$

$$v_o(t) = V_f + D_1'te^{-25{,}000t} + D_2'e^{-25{,}000t}$$

$$V_f = 250\,\text{V}$$

$$v_o(0) = 250 + D_2' = 50; \qquad D_2' = -200\,V$$

$$\frac{dv_o}{dt}(0) = -25{,}000D_2' + D_1' = 0$$

$$D_1' = 25{,}000D_2' = -5 \times 10^6\ \text{V/s}$$

$$v_o = 250 - 5 \times 10^6 te^{-25{,}000t} - 200e^{-25{,}000t}\,\text{V}, \quad t \geq 0$$

P 8.50 $\alpha = \dfrac{R}{2L} = 2000\,\text{rad/s}$

$$\omega_o^2 = \frac{1}{LC} = \frac{1}{(62.5 \times 10^{-3})(6.25 \times 10^{-6})} = 256 \times 10^4$$

$$s_{1,2} = -2000 \pm \sqrt{4 \times 10^6 - 256 \times 10^4} = -2000 \pm j1200\,\text{rad/s}$$

$$v_o = V_f + A_1' e^{-800t} + A_2' e^{-3200t}$$

$$v_o(0) = 0 = V_f + A_1' + A_2'$$

$$v_o(\infty) = 60\,\text{V}; \qquad \therefore \quad A_1' + A_2' = -60$$

$$\frac{dv_o(0)}{dt} = 0 = -800A_1' - 3200A_2'$$

$$\therefore \quad A_1' = -80\,\text{V}; \qquad\qquad A_2' = 20\,\text{V}$$

$$v_o = 60 - 80e^{-800t} + 20e^{-3200t}\,\text{V}, \quad t \geq 0$$

P 8.51 $\alpha = \dfrac{R}{2L} = 2000\,\text{rad/s}$

$$\omega_o^2 = \frac{1}{LC} = \frac{1}{(62.5 \times 10^{-3})(4 \times 10^{-6})} = 4 \times 10^6 \qquad \therefore \quad \omega_o = 2000 \text{ rad/s}$$

The response is therefore critically damped

$$v_o = V_f + D_1' te^{-2000t} + D_2' e^{-2000t}$$

$$v_o(0) = 0 = V_f + D_2'$$

$$v_o(\infty) = 60\,\text{V}; \qquad \therefore \quad D_2' = -60\,\text{V}$$

$$\frac{dv_o(0)}{dt} = 0 = D_1' - \alpha D_2'$$

$$\therefore \quad D_1' = (2000)(-60) = -120{,}000 \text{ V/s}$$

$$v_o = 60 - 120{,}000te^{-2000t} - 60e^{-2000t}\,\text{V}, \quad t \geq 0$$

P 8.52 $\alpha = \dfrac{R}{2L} = 2000\,\text{rad/s}$

$$\omega_o^2 = \frac{1}{LC} = \frac{1}{(62.5 \times 10^{-3})(2.56 \times 10^{-6})} = 625 \times 10^4 \qquad \therefore \quad \omega_o = 2500 \text{ rad/s}$$

The response is therefore underdamped.

$$\omega_d = \sqrt{2500^2 - 2000^2} = 1500 \text{ rad/s}$$

$$v_o = V_f + B_1' e^{-2000t} \cos 1500t + B_2' e^{-2000t} \sin 1500t$$

$$v_o(0) = 0 = V_f + B_1'$$

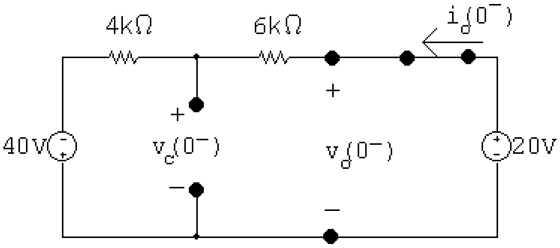
$$v_o(\infty) = 60\,\text{V}; \qquad \therefore \quad B_1' = -60\,\text{V}$$

$$\frac{dv_o(0)}{dt} = 0 = -2000B_1' + 1500B_2'$$

$$\therefore \quad B_2' = -80\,\text{V}$$

$$v_o = \ \text{V}, \quad t \geq 0$$

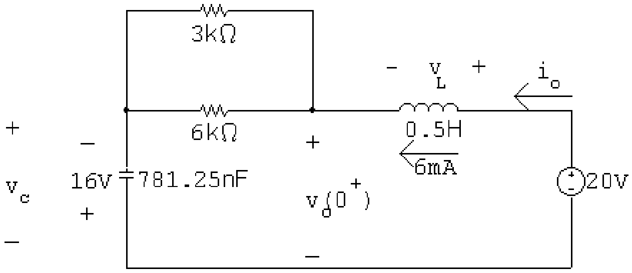
P 8.53 **[a]** $t < 0$:



$$i_o(0^-) = \frac{60}{10{,}000} = 6\,\text{mA}$$

$$v_C(0^-) = 20 - (6000)(0.006) = -16\,\text{V}$$

$t = 0^+$:



$$3\,\text{k}\Omega \| 6\,\text{k}\Omega = 2\,\text{k}\Omega$$

$$\therefore \quad v_o(0^+) = (0.006)(2000) - 16 = 12 - 16 = -4\,\text{V}$$

and $v_L(0^+) = 20 - (-4) = 24\,\text{V}$

[b] $v_o(t) = 2000 i_o + v_{\rm C}$

$$\frac{dv_o}{dt}(t) = 2000\frac{di_o}{dt} + \frac{dv_{\rm C}}{dt}$$

$$\frac{dv_o}{dt}(0^+) = 2000\frac{di_o}{dt}(0^+) + \frac{dv_{\rm C}}{dt}(0^+)$$

$$v_L(0^+) = L\frac{di_o}{dt}(0^+)$$

$$\frac{di_o}{dt}(0^+) = \frac{v_L(0^+)}{L} = \frac{24}{0.5} = 48 \text{ A/s}$$

$$C\frac{dv_c}{dt}(0^+) = i_o(0^+)$$

$$\therefore \quad \frac{dv_c}{dt}(0^+) = \frac{6\times 10^{-3}}{781.25\times 10^{-9}} = 7680$$

$$\therefore \quad \frac{dv_o}{dt}(0^+) = 2000(48) + 7680 = 103{,}680 \text{ V/s}$$

[c] $\omega_o^2 = \dfrac{1}{LC} = 2.56\times 10^6; \qquad \omega_o = 1600 \text{ rad/s}$

$$\alpha = \frac{R}{2L} = 2000 \text{ rad/s}$$

$\alpha^2 > \omega_o^2$ overdamped

$$s_{1,2} = -2000 \pm j1200 \text{ rad/s}$$

$$v_o(t) = V_f + A_1' e^{-800t} + A_2' e^{-3200t}$$

$$V_f = v_o(\infty) = 20\,\text{V}$$

$$20 + A_1' + A_2' = -4; \qquad -800A_1' - 3200A_2' = 103{,}680$$

Solving $A_1' = 11.2; \qquad A_2' = -35.2$

$$\therefore \quad v_o(t) = 20 + 11.2e^{-800t} - 35.2e^{-3200t}\,\text{V}, \qquad t \geq 0^+$$

P 8.54 **[a]** Let i be the current in the direction of the voltage drop $v_o(t)$. Then by hypothesis

$$i = i_f + B_1' e^{-\alpha t}\cos\omega_d t + B_2' e^{-\alpha t}\sin\omega_d t$$

$$i_f = i(\infty) = 0, \qquad i(0) = \frac{V_g}{R} = B_1'$$

Therefore $i = B_1' e^{-\alpha t}\cos\omega_d t + B_2' e^{-\alpha t}\sin\omega_d t$

$$L\frac{di(0)}{dt} = 0, \qquad \text{therefore} \quad \frac{di(0)}{dt} = 0$$

$$\frac{di}{dt} = [(\omega_d B_2' - \alpha B_1')\cos\omega_d t - (\alpha B_2' + \omega_d B_1')\sin\omega_d t]\, e^{-\alpha t}$$

Therefore $\omega_d B_2' - \alpha B_1' = 0; \qquad B_2' = \frac{\alpha}{\omega_d}B_1' = \frac{\alpha}{\omega_d}\frac{V_g}{R}$

Therefore

$$v_o = L\frac{di}{dt} = -\left\{L\left(\frac{\alpha^2 V_g}{\omega_d R} + \frac{\omega_d V_g}{R}\right)\sin\omega_d t\right\} e^{-\alpha t}$$

$$= -\left\{\frac{LV_g}{R}\left(\frac{\alpha^2}{\omega_d} + \omega_d\right)\sin\omega_d t\right\} e^{-\alpha t}$$

$$= -\frac{V_g L}{R}\left(\frac{\alpha^2 + \omega_d^2}{\omega_d}\right) e^{-\alpha t}\sin\omega_d t$$

$$= -\frac{V_g L}{R}\left(\frac{\omega_o^2}{\omega_d}\right) e^{-\alpha t}\sin\omega_d t$$

$$= -\frac{V_g L}{R\omega_d}\left(\frac{1}{LC}\right) e^{-\alpha t}\sin\omega_d t$$

$$v_o = -\frac{V_g}{RC\omega_d}e^{-\alpha t}\sin\omega_d t\,\text{V}, \quad t \geq 0$$

[b] $$\frac{dv_o}{dt} = -\frac{V_g}{\omega_d RC}\{\omega_d\cos\omega_d t - \alpha\sin\omega_d t\}e^{-\alpha t}$$

$$\frac{dv_o}{dt} = 0 \quad \text{when} \quad \tan\omega_d t = \frac{\omega_d}{\alpha}$$

Therefore $\omega_d t = \tan^{-1}(\omega_d/\alpha)$ (smallest t)

$$t = \frac{1}{\omega_d}\tan^{-1}\left(\frac{\omega_d}{\alpha}\right)$$

P 8.55 **[a]** From Problem 8.54 we have

$$v_o = \frac{-V_g}{RC\omega_d}e^{-\alpha t}\sin\omega_d t$$

$$\alpha = \frac{R}{2L} = \frac{4800}{2(64\times 10^{-3})} = 37{,}500\,\text{rad/s}$$

$$\omega_o^2 = \frac{1}{LC} = \frac{1}{(64\times 10^{-3})(4\times 10^{-9})} = 3906.25\times 10^6$$

$$\omega_d = \sqrt{\omega_o^2 - \alpha^2} = 50\,\text{krad/s}$$

$$\frac{-V_g}{RC\omega_d} = \frac{-(-72)}{(4800)(4\times10^{-9})(50\times10^3)} = 75$$

$$\therefore\quad v_o = 75e^{-37{,}500t}\sin 50{,}000t\,\text{V}$$

[b] From Problem 8.54

$$t_d = \frac{1}{\omega_d}\tan^{-1}\left(\frac{\omega_d}{\alpha}\right) = \frac{1}{50{,}000}\tan^{-1}\left(\frac{50{,}000}{37{,}500}\right)$$

$$t_d = 18.55\,\mu\text{s}$$

[c] $v_{\max} = 75e^{-0.0375(18.55)}\sin[(0.05)(18.55)] = 29.93\,\text{V}$

[d] $R = 480\,\Omega;\qquad \alpha = 3750\,\text{rad/s}$

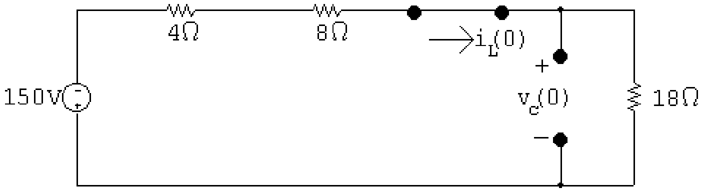
$$\omega_d = 62{,}387.4\,\text{rad/s}$$

$$v_o = 601.08e^{-3750t}\sin 62{,}387.4t\,\text{V},\quad t \geq 0$$

$$t_d = 24.22\,\mu\text{s}$$

$$v_{\max} = 547.92\,\text{V}$$

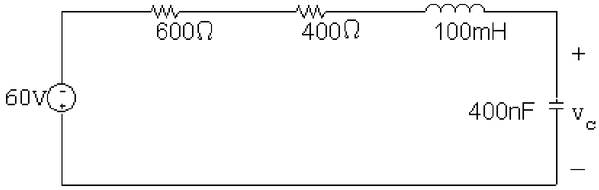
P 8.56 $t < 0$:



$$i_L(0) = \frac{-150}{30} = -5\,\text{A}$$

$$v_C(0) = 18i_L(0) = -90\,\text{V}$$

$t > 0$:



$$\alpha = \frac{R}{2L} = \frac{10}{2(0.1)} = 50\text{ rad/s}$$

$$\omega_o^2 = \frac{1}{LC} = \frac{1}{(0.1)(2\times10^{-3})} = 5000$$

$\omega_o > \alpha^2 \quad \therefore \quad$ underdamped

$$s_{1,2} = -50 \pm \sqrt{50^2 - 5000} = -50 \pm j50$$

$$v_c = 60 + B_1' e^{-50t} \cos 50t + B_2' e^{-50t} \sin 50t$$

$$v_c(0) = -90 = 60 + B_1' \quad \therefore \quad B_1' = -150$$

$$C\frac{dv_c}{dt}(0) = -5; \qquad \frac{dv_c}{dt}(0) = \frac{-5}{2 \times 10^{-3}} = -2500$$

$$\frac{dv_c}{dt}(0) = -50B_1' + 50B_2 = -2500 \quad \therefore \quad B_2' = -200$$

$$v_c = 60 - 150e^{-50t} \cos 50t - 200e^{-50t} \sin 50t \,\text{V}, \quad t \geq 0$$

P 8.57 **[a]** $v_c = V_f + [B_1' \cos \omega_d t + B_2' \sin \omega_d t]\, e^{-\alpha t}$

$$\frac{dv_c}{dt} = [(\omega_d B_2' - \alpha B_1') \cos \omega_d t - (\alpha B_2' + \omega_d B_1') \sin \omega_d t] e^{-\alpha t}$$

Since the initial stored energy is zero,

$$v_c(0^+) = 0 \quad \text{and} \quad \frac{dv_c(0^+)}{dt} = 0$$

It follows that $B_1' = -V_f$ and $B_2' = \dfrac{\alpha B_1'}{\omega_d}$

When these values are substituted into the expression for $[dv_c/dt]$, we get

$$\frac{dv_c}{dt} = \left(\frac{\alpha^2}{\omega_d} + \omega_d\right) V_f e^{-\alpha t} \sin \omega_d t$$

But $V_f = V$ and $\dfrac{\alpha^2}{\omega_d} + \omega_d = \dfrac{\alpha^2 + \omega_d^2}{\omega_d} = \dfrac{\omega_o^2}{\omega_d}$

Therefore $\dfrac{dv_c}{dt} = \left(\dfrac{\omega_o^2}{\omega_d}\right) V e^{-\alpha t} \sin \omega_d t$

[b] $\dfrac{dv_c}{dt} = 0$ when $\sin \omega_d t = 0$, or $\omega_d t = n\pi$

where $n = 0, 1, 2, 3, \ldots$

Therefore $t = \dfrac{n\pi}{\omega_d}$

[c] When $t_n = \dfrac{n\pi}{\omega_d}$, $\cos\omega_d t_n = \cos n\pi = (-1)^n$

and $\sin\omega_d t_n = \sin n\pi = 0$

Therefore $v_c(t_n) = V[1 - (-1)^n e^{-\alpha n\pi/\omega_d}]$

[d] It follows from [c] that

$$v(t_1) = V + Ve^{-(\alpha\pi/\omega_d)} \quad \text{and} \quad v_c(t_3) = V + Ve^{-(3\alpha\pi/\omega_d)}$$

$$\text{Therefore} \quad \frac{v_c(t_1) - V}{v_c(t_3) - V} = \frac{e^{-(\alpha\pi/\omega_d)}}{e^{-(3\alpha\pi/\omega_d)}} = e^{(2\alpha\pi/\omega_d)}$$

$$\text{But} \quad \frac{2\pi}{\omega_d} = t_3 - t_1 = T_d, \quad \text{thus} \quad \alpha = \frac{1}{T_d}\ln\frac{[v_c(t_1) - V]}{[v_c(t_3) - V]}$$

P 8.58 $\dfrac{1}{T_d}\ln\left\{\dfrac{v_c(t_1) - V}{v_c(t_3) - V}\right\}$; $\quad T_d = t_3 - t_1 = \dfrac{3\pi}{7} - \dfrac{\pi}{7} = \dfrac{2\pi}{7}$ ms

$$\alpha = \frac{7000}{2\pi}\ln\left[\frac{63.84}{26.02}\right] = 1000; \qquad \omega_d = \frac{2\pi}{T_d} = 7000\,\text{rad/s}$$

$$\omega_o^2 = \omega_d^2 + \alpha^2 = 49 \times 10^6 + 10^6 = 50 \times 10^6$$

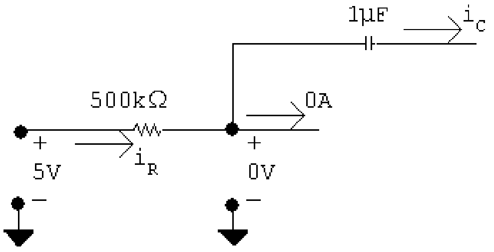
$$L = \frac{1}{(50 \times 10^6)(0.1 \times 10^{-6})} = 200\,\text{mH}; \qquad R = 2\alpha L = 400\,\Omega$$

P 8.59 At $t = 0$ the voltage across each capacitor is zero. It follows that since the operational amplifiers are ideal, the current in the 500 kΩ is zero. Therefore there cannot be an instantaneous change in the current in the 1 μF capacitor. Since the capacitor current equals $C(dv_o/dt)$, the derivative must be zero.

P 8.60 **[a]** From Example 8.13 $\dfrac{d^2v_o}{dt^2} = 2$

$$\text{therefore} \quad \frac{dg(t)}{dt} = 2, \quad g(t) = \frac{dv_o}{dt}$$

$$g(t) - g(0) = 2t; \qquad g(t) = 2t + g(0); \qquad g(0) = \frac{dv_o(0)}{dt}$$



$$i_R = \frac{5}{500} \times 10^{-3} = 10\,\mu\text{A} = i_C = -C\frac{dv_o(0)}{dt}$$

$$\frac{dv_o(0)}{dt} = \frac{-10 \times 10^{-6}}{1 \times 10^{-6}} = -10 = g(0)$$

$$\frac{dv_o}{dt} = 2t - 10$$

$$dv_o = 2t\,dt - 10\,dt$$

$$v_o - v_o(0) = t^2 - 10t; \qquad v_o(0) = 8\,\mathrm{V}$$

$$v_o = t^2 - 10t + 8, \qquad 0 \le t \le t_{\mathrm{sat}}$$

[**b**] $t^2 - 10t + 8 = -9$

$$t^2 - 10t + 17 = 0$$

$$t \cong 2.17\,\mathrm{s}$$

P 8.61 Part (1) — Example 8.14, with R_1 and R_2 removed:

[**a**] $R_\mathrm{a} = 100\,\mathrm{k}\Omega; \qquad C_1 = 0.1\,\mu\mathrm{F}; \qquad R_\mathrm{b} = 25\,\mathrm{k}\Omega; \qquad C_2 = 1\,\mu\mathrm{F}$

$$\frac{d^2v_o}{dt^2} = \left(\frac{1}{R_\mathrm{a}C_1}\right)\left(\frac{1}{R_\mathrm{b}C_2}\right)v_g; \qquad \frac{1}{R_\mathrm{a}C_1} = 100 \quad \frac{1}{R_\mathrm{b}C_2} = 40$$

$$v_g = 250 \times 10^{-3}; \qquad \text{therefore} \quad \frac{d^2v_o}{dt^2} = 1000$$

[**b**] Since $v_o(0) = 0 = \dfrac{dv_o(0)}{dt}$, our solution is $v_o = 500t^2$

The second op-amp will saturate when

$$v_o = 6\,\mathrm{V}, \quad \text{or} \quad t_{\mathrm{sat}} = \sqrt{6/500} \cong 0.1095\,\mathrm{s}$$

[**c**] $\dfrac{dv_{o1}}{dt} = -\dfrac{1}{R_\mathrm{a}C_1}v_g = -25$

[**d**] Since $v_{o1}(0) = 0, \quad v_{o1} = -25t\,\mathrm{V}$

At $t = 0.1095\,\mathrm{s}, \quad v_{o1} \cong -2.74\,\mathrm{V}$

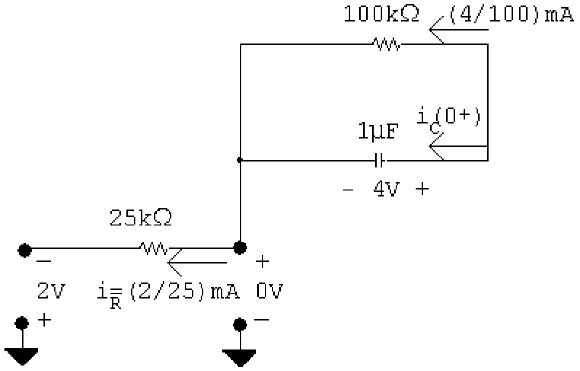
Therefore the second amplifier saturates before the first amplifier saturates. Our expressions are valid for $0 \le t \le 0.1095\,\mathrm{s}$. Once the second op-amp saturates, our linear model is no longer valid.

Part (2) — Example 8.14 with $v_{o1}(0) = -2\,\mathrm{V}$ and $v_o(0) = 4\,\mathrm{V}$:

[**a**] Initial conditions will not change the differential equation; hence the equation is the same as Example 8.14.

[**b**] $v_o = 5 + A_1' e^{-10t} + A_2' e^{-20t}$ (from Example 8.14)

$$v_o(0) = 4 = 5 + A_1' + A_2'$$



$$\frac{4}{100} + i_{\text{C}}(0^+) - \frac{2}{25} = 0$$

$$i_{\text{C}}(0^+) = \frac{4}{100}\,\text{mA} = C\frac{dv_o(0^+)}{dt}$$

$$\frac{dv_o(0^+)}{dt} = \frac{0.04 \times 10^{-3}}{10^{-6}} = 40\,\text{V/s}$$

$$\frac{dv_o}{dt} = -10A_1' e^{-10t} - 20A_2' e^{-20t}$$

$$\frac{dv_o}{dt}(0^+) = -10A_1' - 20A_2' = 40$$

Therefore $-A_1' - 2A_2' = 4$ and $A_1' + A_2' = -1$
Thus, $A_1' = 2$ and $A_2' = -3$

$$v_o = 5 + 2e^{-10t} - 3e^{-20t}\,\text{V}$$

[**c**] Same as Example 8.14:

$$\frac{dv_{o1}}{dt} + 20v_{o1} = -25$$

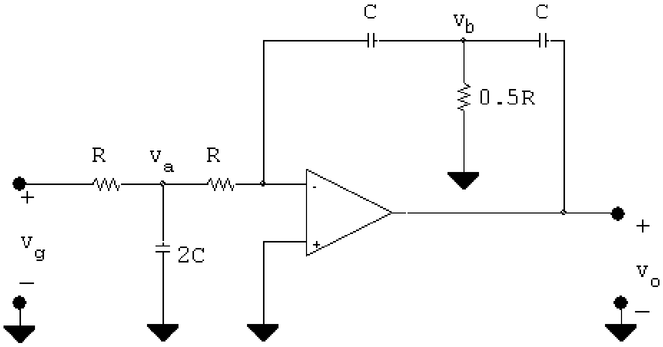
[**d**] From Example 8.14:

$v_{o1}(\infty) = -1.25\,\text{V}$; $v_1(0) = -2\,\text{V}$ (given)

Therefore

$$v_{o1} = -1.25 + (-2 + 1.25)e^{-20t} = -1.25 - 0.75e^{-20t}\,\text{V}$$

P 8.62 [**a**]



$$2C\frac{dv_{\rm a}}{dt} + \frac{v_{\rm a} - v_g}{R} + \frac{v_{\rm a}}{R} = 0$$

$$\text{(1) Therefore} \quad \frac{dv_{\rm a}}{dt} + \frac{v_{\rm a}}{RC} = \frac{v_g}{2RC}$$

$$\frac{0 - v_{\rm a}}{R} + C\frac{d(0 - v_{\rm b})}{dt} = 0$$

$$\text{(2) Therefore} \quad \frac{dv_{\rm b}}{dt} + \frac{v_{\rm a}}{RC} = 0, \quad v_{\rm a} = -RC\frac{dv_{\rm b}}{dt}$$

$$\frac{2v_{\rm b}}{R} + C\frac{dv_{\rm b}}{dt} + C\frac{d(v_{\rm b} - v_o)}{dt} = 0$$

$$\text{(3) Therefore} \quad \frac{dv_{\rm b}}{dt} + \frac{v_{\rm b}}{RC} = \frac{1}{2}\frac{dv_o}{dt}$$

From (2) we have $\frac{dv_{\rm a}}{dt} = -RC\frac{d^2v_{\rm b}}{dt^2}$ and $v_{\rm a} = -RC\frac{dv_{\rm b}}{dt}$

When these are substituted into (1) we get

$$\text{(4)} \quad -RC\frac{d^2v_{\rm b}}{dt^2} - \frac{dv_{\rm b}}{dt} = \frac{v_g}{2RC}$$

Now differentiate (3) to get

$$\text{(5)} \quad \frac{d^2v_{\rm b}}{dt^2} + \frac{1}{RC}\frac{dv_{\rm b}}{dt} = \frac{1}{2}\frac{d^2v_o}{dt^2}$$

But from (4) we have

$$\text{(6)} \quad \frac{d^2v_{\rm b}}{dt^2} + \frac{1}{RC}\frac{dv_{\rm b}}{dt} = -\frac{v_g}{2R^2C^2}$$

Now substitute (6) into (5)

$$\frac{d^2v_o}{dt^2} = -\frac{v_g}{R^2C^2}$$

[**b**] When $R_1C_1 = R_2C_2 = RC$: $\dfrac{d^2v_o}{dt^2} = \dfrac{v_g}{R^2C^2}$

The two equations are the same except for a reversal in algebraic sign.

[**c**] Two integrations of the input signal with one operational amplifier.

P 8.63 [**a**] $\dfrac{d^2v_o}{dt^2} = \dfrac{1}{R_1C_1R_2C_2}v_g$

$$\frac{1}{R_1C_1R_2C_2} = \frac{10^{-6}}{(100)(400)(0.5)(0.2)\times 10^{-6}\times 10^{-6}} = 250$$

$$\therefore \quad \frac{d^2v_o}{dt^2} = 250v_g$$

$0 \le t \le 0.5^-$:

$v_g = 80\,\text{mV}$

$$\frac{d^2v_o}{dt^2} = 20$$

Let $g(t) = \dfrac{dv_o}{dt}$, then $\dfrac{dg}{dt} = 20$ or $dg = 20\,dt$

$$\int_{g(0)}^{g(t)} dx = 20\int_0^t dy$$

$$g(t) - g(0) = 20t, \quad g(0) = \frac{dv_o}{dt}(0) = 0$$

$$g(t) = \frac{dv_o}{dt} = 20t$$

$$dv_o = 20t\,dt$$

$$\int_{v_o(0)}^{v_o(t)} dx = 20\int_0^t x\,dx; \qquad v_o(t) - v_o(0) = 10t^2, \quad v_o(0) = 0$$

$$v_o(t) = 10t^2\,\text{V}, \quad 0 \le t \le 0.5^-$$

$$\frac{dv_{o1}}{dt} = -\frac{1}{R_1C_1}v_g = -20v_g = -1.6$$

$$dv_{o1} = -1.6\,dt$$

$$\int_{v_{o1}(0)}^{v_{o1}(t)} dx = -1.6\int_0^t dy$$

$$v_{o1}(t) - v_{o1}(0) = -1.6t, \qquad v_{o1}(0) = 0$$

$$v_{o1}(t) = -1.6t\,\text{V}, \qquad 0 \le t \le 0.5^-$$

$0.5^+ \leq t \leq t_{\text{sat}}$:

$$\frac{d^2v_o}{dt^2} = -10, \qquad \text{let} \quad g(t) = \frac{dv_o}{dt}$$

$$\frac{dg(t)}{dt} = -10; \qquad dg(t) = -10\,dt$$

$$\int_{g(0.5^+)}^{g(t)} dx = -10\int_{0.5}^{t} dy$$

$$g(t) - g(0.5^+) = -10(t - 0.5) = -10t + 5$$

$$g(0.5^+) = \frac{dv_o(0.5^+)}{dt}$$

$$C\frac{dv_o}{dt}(0.5^+) = \frac{0 - v_{o1}(0.5^+)}{400 \times 10^3}$$

$$v_{o1}(0.5^+) = v_o(0.5^-) = -1.6(0.5) = -0.80\,\text{V}$$

$$\therefore \quad C\frac{dv_{o1}(0.5^+)}{dt} = \frac{0.80}{0.4 \times 10^3} = 2\,\mu\text{A}$$

$$\frac{dv_{o1}}{dt}(0.5^+) = \frac{2 \times 10^{-6}}{0.2 \times 10^{-6}} = 10\,\text{V/s}$$

$$\therefore \quad g(t) = -10t + 5 + 10 = -10t + 15 = \frac{dv_o}{dt}$$

$$\therefore \quad dv_o = -10t\,dt + 15\,dt$$

$$\int_{v_o(0.5^+)}^{v_o(t)} dx = \int_{0.5^+}^{t} -10y\,dy + \int_{0.5^+}^{t} 15\,dy$$

$$v_o(t) - v_o(0.5^+) = -5y^2 \Big|_{0.5}^{t} + 15y \Big|_{0.5}^{t}$$

$$v_o(t) = v_o(0.5^+) - 5t^2 + 1.25 + 15t - 7.5$$

$$v_o(0.5^+) = v_o(0.5^-) = 2.5\,\text{V}$$

$$\therefore \quad v_o(t) = -5t^2 + 15t - 3.75\,\text{V}, \qquad 0.5^+ \leq t \leq t_{\text{sat}}$$

$$\frac{dv_{o1}}{dt} = -20(-0.04) = 0.8, \qquad 0.5^+ \leq t \leq t_{\text{sat}}$$

$$dv_{o1} = 0.8\,dt; \qquad \int_{v_{o1}(0.5^+)}^{v_{o1}(t)} dx = 0.8\int_{0.5^+}^{t} dy$$

$$v_{o1}(t) - v_{o1}(0.5^+) = 0.8t - 0.4; \qquad v_{o1}(0.5^+) = v_{o1}(0.5^-) = -0.8\,\text{V}$$

$$\therefore \quad v_{o1}(t) = 0.8t - 1.2\,\text{V}, \qquad 0.5^+ \leq t \leq t_{\text{sat}}$$

Summary:

$0 \le t \le 0.5^{-}\text{s}: \qquad v_{o1} = -1.6t\,\text{V}, \quad v_o = 10t^2\,\text{V}$

$0.5^{+}\text{s} \le t \le t_{\text{sat}}: \qquad v_{o1} = 0.8t - 1.2\,\text{V}, \quad v_o = -5t^2 + 15t - 3.75\,\text{V}$

[b] $-12.5 = -5t_{\text{sat}}^2 + 15t_{\text{sat}} - 3.75$

$\therefore \; 5t_{\text{sat}}^2 - 15t_{\text{sat}} - 8.75 = 0$

Solving, $\qquad t_{\text{sat}} = 3.5\,\text{sec}$

$v_{o1}(t_{\text{sat}}) = 0.8(3.5) - 1.2 = 1.6\,\text{V}$

P 8.64 $\tau_1 = (10^6)(0.5 \times 10^{-6}) = 0.50\,\text{s}$

$$\frac{1}{\tau_1} = 2; \qquad \tau_2 = (5 \times 10^6)(0.2 \times 10^{-6}) = 1\,\text{s}; \qquad \therefore \; \frac{1}{\tau_2} = 1$$

$$\therefore \; \frac{d^2v_o}{dt^2} + 3\frac{dv_o}{dt} + 2v_o = 20$$

$$s^2 + 3s + 2 = 0$$

$$(s+1)(s+2) = 0; \qquad s_1 = -1, \quad s_2 = -2$$

$$v_o = V_f + A_1'e^{-t} + A_2'e^{-2t}; \qquad V_f = \frac{20}{2} = 10\,\text{V}$$

$$v_o = 10 + A_1'e^{-t} + A_2'e^{-2t}$$

$$v_o(0) = 0 = 10 + A_1' + A_2'; \qquad \frac{dv_o}{dt}(0) = 0 = -A_1' - 2A_2'$$

$$\therefore \; A_1' = -20, \qquad A_2' = 10\,\text{V}$$

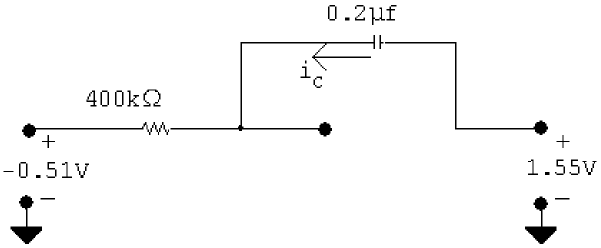
$$v_o(t) = 10 - 20e^{-t} + 10e^{-2t}\,\text{V}, \qquad 0 \le t \le 0.5\,\text{s}$$

$$\frac{dv_{o1}}{dt} + 2v_{o1} = -1.6; \qquad \therefore \; v_{o1} = -0.8 + 0.8e^{-2t}\,\text{V}, \quad 0 \le t \le 0.5\,\text{s}$$

$$v_o(0.5) = 10 - 20e^{-0.5} + 10e^{-1} = 1.55\,\text{V}$$

$$v_{o1}(0.5) = -0.8 + 0.8e^{-1} = -0.51\,\text{V}$$

At $t = 0.5\,\text{s}$



$$i_\text{C} = \frac{0 + 0.51}{400 \times 10^3} = 1.26\,\mu\text{A}$$

$$C\frac{dv_o}{dt} = 1.26\,\mu\text{A}; \qquad \frac{dv_o}{dt} = \frac{1.26}{0.2} = 6.32\,\text{V/s}$$

$0.5\,\text{s} \leq t \leq \infty$:

$$\frac{d^2v_o}{dt^2} + 3\frac{dv_o}{dt} + 2 = -10$$

$$v_o(\infty) = -5$$

$$\therefore \quad v_o = -5 + A_1'e^{-(t-0.5)} + A_2'e^{-2(t-0.5)}$$

$$1.55 = -5 + A_1' + A_2'$$

$$\frac{dv_o}{dt}(0.5) = 6.32 = -A_1' - 2A_2'$$

$$\therefore \quad A_1' + A_2' = 6.55; \qquad -A_1' - 2A_2' = 6.32$$

Solving,

$$A_1' = 19.42; \qquad A_2' = -12.87$$

$$\therefore \quad v_o = -5 + 19.42e^{-(t-0.5)} - 12.87e^{-2(t-0.5)}\,\text{V}, \qquad 0.5 \leq t \leq \infty$$

$$\frac{dv_{o1}}{dt} + 2v_{o1} = 0.8$$

$$\therefore \quad v_{o1} = 0.4 + (-0.51 - 0.4)e^{-2(t-0.5)} = 0.4 - 0.91e^{-2(t-0.5)}\,\text{V}, \qquad 0.5 \leq t \leq \infty$$

P 8.65 **[a]** $f(t) =$ inertial force + frictional force + spring force

$$= M[d^2x/dt^2] + D[dx/dt] + Kx$$

[b] $$\frac{d^2x}{dt^2} = \frac{f}{M} - \left(\frac{D}{M}\right)\left(\frac{dx}{dt}\right) - \left(\frac{K}{M}\right)x$$

Given $v_A = \dfrac{d^2x}{dt^2}$, then

$$v_B = -\frac{1}{R_1C_1}\int_0^t \left(\frac{d^2x}{dy^2}\right)dy = -\frac{1}{R_1C_1}\frac{dx}{dt}$$

$$v_{\text{C}} = -\frac{1}{R_2C_2}\int_0^t v_B\,dy = \frac{1}{R_1R_2C_1C_2}x$$

$$v_D = -\frac{R_3}{R_4}\cdot v_B = \frac{R_3}{R_4R_1C_1}\frac{dx}{dt}$$

$$v_E = \left[\frac{R_5+R_6}{R_6}\right]v_{\text{C}} = \left[\frac{R_5+R_6}{R_6}\right]\cdot\frac{1}{R_1R_2C_1C_2}\cdot x$$

$$v_F = \left[\frac{-R_8}{R_7}\right]f(t), \qquad v_A = -(v_D + v_E + v_F)$$

Therefore $$\frac{d^2x}{dt^2} = \left[\frac{R_8}{R_7}\right]f(t) - \left[\frac{R_3}{R_4R_1C_1}\right]\frac{dx}{dt} - \left[\frac{R_5+R_6}{R_6R_1R_2C_1C_2}\right]x$$

Therefore $M = \dfrac{R_7}{R_8}$, $\quad D = \dfrac{R_3R_7}{R_8R_4R_1C_1}$ and $K = \dfrac{R_7(R_5+R_6)}{R_8R_6R_1R_2C_1C_2}$

Box Number	Function
1	inverting and scaling
2	summing and inverting
3	integrating and scaling
4	integrating and scaling
5	inverting and scaling
6	noninverting and scaling

P 8.66 **[a]** Given that the current response is underdamped, we know i will be of the form

$$i = I_f + [B_1' \cos\omega_d t + B_2' \sin\omega_d t]e^{-\alpha t}$$

where $$\alpha = \frac{R}{2L}$$

and $$\omega_d = \sqrt{\omega_o^2 - \alpha^2} = \sqrt{\frac{1}{LC} - \alpha^2}$$

The capacitor will force the final value of i to be zero, therefore $I_f = 0$.
By hypothesis $i(0^+) = V_{dc}/R$; therefore $B_1' = V_{dc}/R$.
At $t = 0^+$ the voltage across the primary winding is approximately zero; hence $di(0^+)/dt = 0$.
From our equation for i we have

$$\frac{di}{dt} = [(\omega_d B_2' - \alpha B_1')\cos\omega_d t - (\omega_d B_1' + \alpha B_2')\sin\omega_d t]e^{-\alpha t}$$

Hence

$$\frac{di(0^+)}{dt} = \omega_d B_2' - \alpha B_1' = 0$$

Thus

$$B_2' = \frac{\alpha}{\omega_d}B_1' = \frac{\alpha V_{dc}}{\omega_d R}$$

It follows directly that

$$i = \frac{V_{dc}}{R}\left[\cos\omega_d t + \frac{\alpha}{\omega_d}\sin\omega_d t\right]e^{-\alpha t}$$

[b] Since $\omega_d B_2' - \alpha B_1' = 0$, it follows that

$$\frac{di}{dt} = -(\omega_d B_1' + \alpha B_2')e^{-\alpha t}\sin\omega_d t$$

But $\alpha B_2' = \dfrac{\alpha^2 V_{dc}}{\omega_d R}$ and $\omega_d B_1' = \dfrac{\omega_d V_{dc}}{R}$

Therefore

$$\omega_d B_1' + \alpha B_2' = \frac{\omega_d V_{dc}}{R} + \frac{\alpha^2 V_{dc}}{\omega_d R} = \frac{V_{dc}}{R}\left[\frac{\omega_d^2 + \alpha^2}{\omega_d}\right]$$

But $\omega_d^2 + \alpha^2 = \omega_o^2 = \dfrac{1}{LC}$

Hence

$$\omega_d B_1' + \alpha B_2' = \frac{V_{dc}}{\omega_d RLC}$$

Now since $v_1 = L\dfrac{di}{dt}$ we get

$$v_1 = -L\frac{V_{dc}}{\omega_d RLC}e^{-\alpha t}\sin\omega_d t = -\frac{V_{dc}}{\omega_d RC}e^{-\alpha t}\sin\omega_d t$$

[c] $v_c = V_{dc} - iR - L\dfrac{di}{dt}$

$$iR = V_{dc}\left(\cos\omega_d t + \frac{\alpha}{\omega_d}\sin\omega_d t\right)e^{-\alpha t}$$

$$\begin{aligned} v_c &= V_{dc} - V_{dc}\left(\cos\omega_d t + \frac{\alpha}{\omega_d}\,\sin\omega_d t\right)e^{-\alpha t} + \frac{V_{dc}}{\omega_d RC}e^{-\alpha t}\sin\omega_d t \\ &= V_{dc} - V_{dc}e^{-\alpha t}\cos\omega_d t + \left(\frac{V_{dc}}{\omega_d RC} - \frac{\alpha V_{dc}}{\omega_d}\right)e^{-\alpha t}\sin\omega_d t \\ &= V_{dc}\left[1 - e^{-\alpha t}\cos\omega_d t + \frac{1}{\omega_d}\left(\frac{1}{RC} - \alpha\right)e^{-\alpha t}\sin\omega_d t\right] \\ &= V_{dc}\left[1 - e^{-\alpha t}\cos\omega_d t + Ke^{-\alpha t}\sin\omega_d t\right] \end{aligned}$$

P 8.67 $v_{sp} = V_{dc}\left[1 - \frac{a}{\omega_d RC}e^{-\alpha t}\,\sin\omega_d t\right]$

$$\begin{aligned} \frac{dv_{sp}}{dt} &= \frac{-aV_{dc}}{\omega_d RC}\,\frac{d}{dt}[e^{-\alpha t}\sin\omega_d t] \\ &= \frac{-aV_{dc}}{\omega_d RC}[-\alpha e^{-\alpha t}\sin\omega_d t + \omega_d e^{-\alpha t}\cos\omega_d t] \\ &= \frac{aV_{dc}e^{-\alpha t}}{\omega_d RC}[\alpha\,\sin\omega_d t - \omega_d\,\cos\omega_d t] \end{aligned}$$

$$\frac{dv_{sp}}{dt} = 0 \quad \text{when} \quad \alpha\,\sin\omega_d t = \omega_d\,\cos\omega_d t$$

$$\text{or} \quad \tan\omega_d t = \frac{\omega_d}{\alpha}; \qquad \omega_d t = \tan^{-1}\left(\frac{\omega_d}{\alpha}\right)$$

$$\therefore\quad t_{\max} = \frac{1}{\omega_d}\,\tan^{-1}\left(\frac{\omega_d}{\alpha}\right)$$

Note that because $\tan\theta$ is periodic, i.e., $\tan\theta = \tan(\theta \pm n\pi)$, where n is an integer, there are an infinite number of solutions for t where $dv_{sp}/dt = 0$, that is

$$t = \frac{\tan^{-1}(\omega_d/\alpha) \pm n\pi}{\omega_d}$$

Because of $e^{-\alpha t}$ in the expression for v_{sp} and knowing $t \geq 0$ we know v_{sp} will be maximum when t has its smallest positive value. Hence

$$t_{\max} = \frac{\tan^{-1}(\omega_d/\alpha)}{\omega_d}.$$

P 8.68 [**a**] $v_c = V_{dc}[1 - e^{-\alpha t}\cos\omega_d t + Ke^{-\alpha t}\sin\omega_d t]$

$$\frac{dv_c}{dt} = V_{dc}\frac{d}{dt}[1 + e^{-\alpha t}(K\ \sin\omega_d t - \cos\omega_d t)]$$

$$= V_{dc}\{(-\alpha e^{-\alpha t})(K\ \sin\omega_d t - \cos\omega_d t) +$$

$$e^{-\alpha t}[\omega_d K\ \cos\omega_d t + \omega_d\ \sin\omega_d t]\}$$

$$= V_{dc}e^{-\alpha t}[(\omega_d - \alpha K)\ \sin\omega_d t + (\alpha + \omega_d K)\ \cos\omega_d t]$$

$$\frac{dv_c}{dt} = 0 \quad \text{when} \quad (\omega_d - \alpha K)\ \sin\omega_d t = -(\alpha + \omega_d K)\ \cos\omega_d t$$

$$\text{or} \quad \tan\omega_d t = \left[\frac{\alpha + \omega_d K}{\alpha K - \omega_d}\right]$$

$$\therefore \quad \omega_d t \pm n\pi = \tan^{-1}\left[\frac{\alpha + \omega_d K}{\alpha K - \omega_d}\right]$$

$$t_c = \frac{1}{\omega_d}\left\{\tan^{-1}\left(\frac{\alpha + \omega_d K}{\alpha K - \omega_d}\right) \pm n\pi\right\}$$

$$\alpha = \frac{R}{2L} = \frac{4 \times 10^3}{6} = 666.67\,\text{rad/s}$$

$$\omega_d = \sqrt{\frac{10^9}{1.2} - (666.67)^2} = 28{,}859.81\,\text{rad/s}$$

$$K = \frac{1}{\omega_d}\left(\frac{1}{RC} - \alpha\right) = 21.63$$

$$t_c = \frac{1}{\omega_d}\left\{\tan^{-1}(-43.29) + n\pi\right\} = \frac{1}{\omega_d}\{-1.55 + n\pi\}$$

The smallest positive value of t occurs when $n = 1$, therefore

$t_{c\,\max} = 55.23\,\mu\text{s}$

[**b**] $v_c(t_{c\max}) = 12[1 - e^{-\alpha t_{c\max}}\cos\omega_d t_{c\max} + Ke^{-\alpha t_{c\max}}\sin\omega_d t_{c\max}]$

$$= 262.42\,\text{V}$$

[**c**] From the text example the voltage across the spark plug reaches its maximum value in 53.63 μs. If the spark plug does not fire the capacitor voltage peaks in 55.23 μs. When v_{sp} is maximum the voltage across the capacitor is 262.15 V. If the spark plug does not fire the capacitor voltage reaches 262.42 V.

P 8.69 [**a**] $w = \frac{1}{2}L[i(0^+)]^2 = \frac{1}{2}(5)(16) \times 10^{-3} = 40\,\text{mJ}$

[b] $\alpha = \dfrac{R}{2L} = \dfrac{3 \times 10^3}{10} = 300\,\text{rad/s}$

$$\omega_d = \sqrt{\frac{10^9}{1.25} - (300)^2} = 28{,}282.68\,\text{rad/s}$$

$$\frac{1}{RC} = \frac{10^6}{0.75} = \frac{4 \times 10^6}{3}$$

$$t_{\max} = \frac{1}{\omega_d}\,\tan^{-1}\left(\frac{\omega_d}{\alpha}\right) = 55.16\,\mu\text{s}$$

$$v_{sp}\ (t_{\max}) = 12 - \frac{12(50)(4 \times 10^6)}{3(28{,}282.68)} e^{-\alpha t_{\max}} \sin\omega_d t_{\max} = -27{,}808.04\,\text{V}$$

[c] $v_c\ (t_{\max}) = 12[1 - e^{-\alpha t_{\max}} \cos\omega_d t_{\max} + K e^{-\alpha t_{\max}} \sin\omega_d t_{\max}]$

$$K = \frac{1}{\omega_d}\left[\frac{1}{RC} - \alpha\right] = 47.13$$

$$v_c\ (t_{\max}) = 568.15\,\text{V}$$

P 8.70 **[a]** $v_c = V_{dc}[1 - e^{-\alpha t}\cos\omega_d t + K e^{-\alpha t}\sin\omega_d t]$

$$\frac{dv_c}{dt} = V_{dc}\frac{d}{dt}[1 + e^{-\alpha t}(K\ \sin\omega_d t - \cos\omega_d t)]$$

$$= V_{dc}\{(-\alpha e^{-\alpha t})(K\ \sin\omega_d t - \cos\omega_d t) +$$
$$e^{-\alpha t}[\omega_d K\ \cos\omega_d t + \omega_d\ \sin\omega_d t]\}$$
$$= V_{dc} e^{-\alpha t}[(\omega_d - \alpha K)\ \sin\omega_d t + (\alpha + \omega_d K)\ \cos\omega_d t]$$

$$\frac{dv_c}{dt} = 0 \quad \text{when} \quad (\omega_d - \alpha K)\ \sin\omega_d t = -(\alpha + \omega_d K)\ \cos\omega_d t$$

or $\quad \tan\omega_d t = \left[\dfrac{\alpha + \omega_d K}{\alpha K - \omega_d}\right]$

$$\therefore\quad \omega_d t \pm n\pi = \tan^{-1}\left[\frac{\alpha + \omega_d K}{\alpha K - \omega_d}\right]$$

$$t_c = \frac{1}{\omega_d}\left\{\tan^{-1}\left(\frac{\alpha + \omega_d K}{\alpha K - \omega_d}\right) \pm n\pi\right\}$$

$$\alpha = \frac{R}{2L} = \frac{3}{2(5 \times 10^{-3})} = 300\,\text{rad/s}$$

$$\omega_d = \sqrt{\frac{10^9}{1.25} - (300)^2} = 28{,}282.68\,\text{rad/s}$$

$$K = \frac{1}{\omega_d}\left(\frac{1}{RC} - \alpha\right) = 47.13$$

$$t_c = \frac{1}{\omega_d}\{-1.56 + n\pi\}$$

The smallest positive value of t occurs when $n = 1$, therefore

$$t_{c\,\max} = 55.91\,\mu\text{s}$$

[b] $v_c(t_{c\,\max}) = 12[1 - e^{-\alpha t_{c\,\max}} \cos\omega_d t_{c\,\max} + Ke^{-\alpha t_{c\,\max}} \sin\omega_d t_{c\,\max}] = 568.28\,\text{V}$

[c] From Problem 8.69, the voltage across the spark plug reaches its maximum value in $55.16\,\mu$s. If the spark plug does not fire the capacitor voltage peaks in $55.91\,\mu$s. When v_{sp} is maximum the voltage across the capacitor is 568.15 V. If the spark plug does not fire the capacitor voltage reaches 568.28 V.

9

Sinusoidal Steady State Analysis

Assessment Problems

AP 9.1 **[a]** $\mathbf{V} = 170\underline{/-40^\circ}\,\text{V}$

[b] $10\sin(1000t + 20^\circ) = 10\cos(1000t - 70^\circ)$

$$\therefore \qquad \mathbf{I} = 10\underline{/-70^\circ}\,\text{A}$$

[c] $\mathbf{I} = 5\underline{/36.87^\circ} + 10\underline{/-53.13^\circ}$

$$= 4 + j3 + 6 - j8 = 10 - j5 = 11.18\underline{/-26.57^\circ}\,\text{A}$$

[d] $\sin(20{,}000\pi t + 30^\circ) = \cos(20{,}000\pi t - 60^\circ)$

Thus,

$$\mathbf{V} = 300\underline{/45^\circ} - 100\underline{/-60^\circ} = 212.13 + j212.13 - (50 - j86.60)$$

$$= 162.13 + j298.73 = 339.90\underline{/61.51^\circ}\,\text{mV}$$

AP 9.2 **[a]** $v = 18.6\cos(\omega t - 54^\circ)\,\text{V}$

[b] $\mathbf{I} = 20\underline{/45^\circ} - 50\underline{/-30^\circ} = 14.14 + j14.14 - 43.3 + j25$

$$= -29.16 + j39.14 = 48.81\underline{/126.68^\circ}$$

Therefore $\quad i = 48.81\cos(\omega t + 126.68^\circ)\,\text{mA}$

[c] $\mathbf{V} = 20 + j80 - 30\underline{/15^\circ} = 20 + j80 - 28.98 - j7.76$

$$= -8.98 + j72.24 = 72.79\underline{/97.08^\circ}$$

$$v = 72.79\cos(\omega t + 97.08^\circ)\,\text{V}$$

AP 9.3 **[a]** $\omega L = (10^4)(20 \times 10^{-3}) = 200\,\Omega$

[b] $Z_L = j\omega L = j200\,\Omega$

[c] $\mathbf{V}_L = \mathbf{I}Z_L = (10\angle 30^\circ)(200\angle 90^\circ) \times 10^{-3} = 2\angle 120^\circ\,\text{V}$

[d] $v_L = 2\cos(10{,}000t + 120^\circ)\,\text{V}$

AP 9.4 [a] $X_C = \dfrac{-1}{\omega C} = \dfrac{-1}{4000(5 \times 10^{-6})} = -50\,\Omega$

[b] $Z_C = jX_C = -j50\,\Omega$

[c] $\mathbf{I} = \dfrac{\mathbf{V}}{Z_C} = \dfrac{30\angle 25^\circ}{50\angle -90^\circ} = 0.6\angle 115^\circ\,\text{A}$

[d] $i = 0.6\cos(4000t + 115^\circ)\,\text{A}$

AP 9.5 $\mathbf{I}_1 = 100\angle 25^\circ = 90.63 + j42.26$

$\mathbf{I}_2 = 100\angle 145^\circ = -81.92 + j57.36$

$\mathbf{I}_3 = 100\angle -95^\circ = -8.72 - j99.62$

$\mathbf{I}_4 = -(\mathbf{I}_1 + \mathbf{I}_2 + \mathbf{I}_3) = (0 + j0)\,\text{A}, \qquad \text{therefore} \quad i_4 = 0\,\text{A}$

AP 9.6 [a] $\mathbf{I} = \dfrac{125\angle -60^\circ}{|Z|\angle \theta_z} = \dfrac{125}{|Z|}\angle (-60 - \theta_Z)^\circ$

But $-60 - \theta_Z = -105^\circ \qquad \therefore \ \theta_Z = 45^\circ$

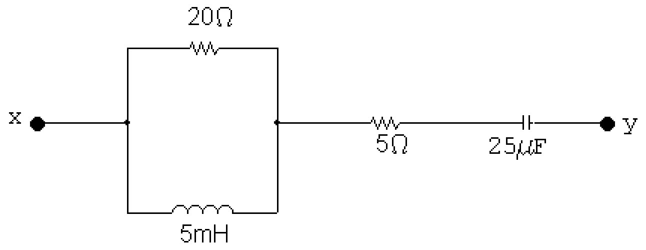
$Z = 90 + j160 + jX_C$

$\therefore \ X_C = -70\,\Omega; \qquad X_C = -\dfrac{1}{\omega C} = -70$

$\therefore \ C = \dfrac{1}{(70)(5000)} = 2.86\,\mu\text{F}$

[b] $\mathbf{I} = \dfrac{\mathbf{V}_s}{Z} = \dfrac{125\angle -60^\circ}{(90 + j90)} = 0.982\angle -105^\circ A; \qquad \therefore \ |\mathbf{I}| = 0.982\,\text{A}$

AP 9.7 [a]



$\omega = 2000\,\text{rad/s}$

$\omega L = 10\,\Omega, \qquad \dfrac{-1}{\omega C} = -20\,\Omega$

$$Z_{xy} = 20\|j10 + 5 + j20 = \frac{20(j10)}{(20 + j10)} + 5 - j20$$

$$= 4 + j8 + 5 - j20 = (9 - j12)\,\Omega$$

[b] $\omega L = 40\,\Omega, \qquad \frac{-1}{\omega C} = -5\,\Omega$

$$Z_{xy} = 5 - j5 + 20\|j40 = 5 - j5 + \left[\frac{(20)(j40)}{20 + j40}\right]$$

$$= 5 - j5 + 16 + j8 = (21 + j3)\,\Omega$$

[c] $Z_{xy} = \left[\frac{20(j\omega L)}{20 + j\omega L}\right] + \left(5 - \frac{j10^6}{25\omega}\right)$

$$= \frac{20\omega^2 L^2}{400 + \omega^2 L^2} + \frac{j400\omega L}{400 + \omega^2 L^2} + 5 - \frac{j10^6}{25\omega}$$

The impedance will be purely resistive when the j terms cancel, i.e.,

$$\frac{400\omega L}{400 + \omega^2 L^2} = \frac{10^6}{25\omega}$$

Solving for ω yields $\omega = 4000\,\text{rad/s}$.

[d] $Z_{xy} = \frac{20\omega^2 L^2}{400 + \omega^2 L^2} + 5 = 10 + 5 = 15\,\Omega$

AP 9.8 The frequency 4000 rad/s was found to give $Z_{\text{xy}} = 15\,\Omega$ in Assessment Problem 9.7. Thus,

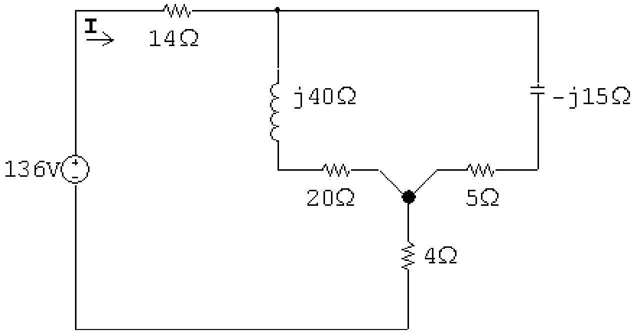
$$\mathbf{V} = 150\underline{/0^\circ}, \qquad \mathbf{I}_s = \frac{\mathbf{V}}{Z_{\text{xy}}} = \frac{150\underline{/0^\circ}}{15} = 10\underline{/0^\circ}\,\text{A}$$

Using current division,

$$\mathbf{I}_L = \frac{20}{20 + j20}(10) = 5 - j5 = 7.07\underline{/-45^\circ}\,\text{A}$$

$$i_L = 7.07\cos(4000t - 45^\circ)\,\text{A}, \qquad I_m = 7.07\,\text{A}$$

AP 9.9 After replacing the delta made up of the $50\,\Omega$, $40\,\Omega$, and $10\,\Omega$ resistors with its equivalent wye, the circuit becomes



The circuit is further simplified by combining the parallel branches,

$$(20 + j40)\|(5 - j15) = (12 - j16)\,\Omega$$

Therefore $\mathbf{I} = \dfrac{136\underline{/0^\circ}}{14 + 12 - j16 + 4} = 4\underline{/28.07^\circ}\,\text{A}$

AP 9.10

$$\mathbf{V}_1 = 240\underline{/53.13^\circ} = 144 + j192\,\text{V}$$

$$\mathbf{V}_2 = 96\underline{/-90^\circ} = -j96\,\text{V}$$

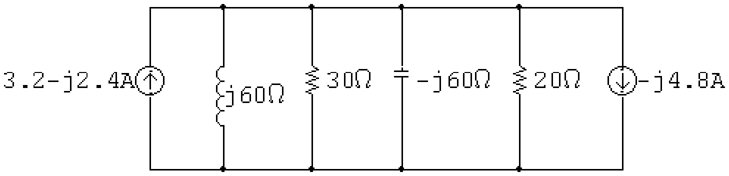
$$j\omega L = j(4000)(15 \times 10^{-3}) = j60\,\Omega$$

$$\frac{1}{j\omega C} = -j\frac{6 \times 10^6}{(4000)(25)} = -j60\,\Omega$$

Perform a source transformation:

$$\frac{\mathbf{V}_1}{j60} = \frac{144 + j192}{j60} = 3.2 - j2.4\,\text{A}$$

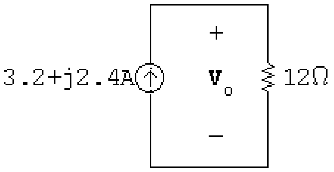
$$\frac{\mathbf{V}_2}{20} = -j\frac{96}{20} = -j4.8\,\text{A}$$



Combine the parallel impedances:

$$Y = \frac{1}{j60} + \frac{1}{30} + \frac{1}{-j60} + \frac{1}{20} = \frac{j5}{j60} = \frac{1}{12}$$

$$Z = \frac{1}{Y} = 12\,\Omega$$



$$\mathbf{V}_o = 12(3.2 + j2.4) = 38.4 + j28.8\,\text{V} = 48\underline{/36.87^\circ}\,\text{V}$$

$$v_o = 48\cos(4000t + 36.87^\circ)\,\text{V}$$

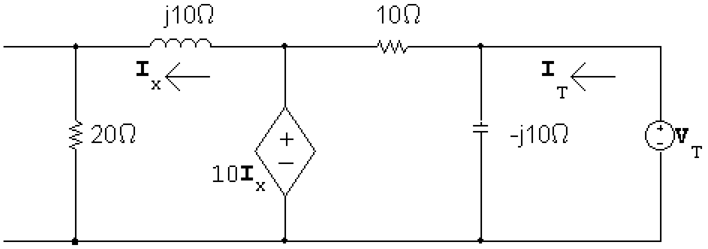
AP 9.11 Use the lower node as the reference node. Let $\mathbf{V}_1$ = node voltage across the $20\,\Omega$ resistor and $\mathbf{V}_{\rm Th}$ = node voltage across the capacitor. Writing the node voltage equations gives us

$$\frac{\mathbf{V}_1}{20} - 2\underline{/45^\circ} + \frac{\mathbf{V}_1 - 10\mathbf{I}_x}{j10} = 0 \quad \text{and} \quad \mathbf{V}_{\rm Th} = \frac{-j10}{10 - j10}(10\mathbf{I}_x)$$

We also have

$$\mathbf{I}_x = \frac{\mathbf{V}_1}{20}$$

Solving these equations for $\mathbf{V}_{\rm Th}$ gives $\mathbf{V}_{\rm Th} = 10\underline{/45^\circ}V$. To find the Thévenin impedance, we remove the independent current source and apply a test voltage source at the terminals a, b. Thus



It follows from the circuit that

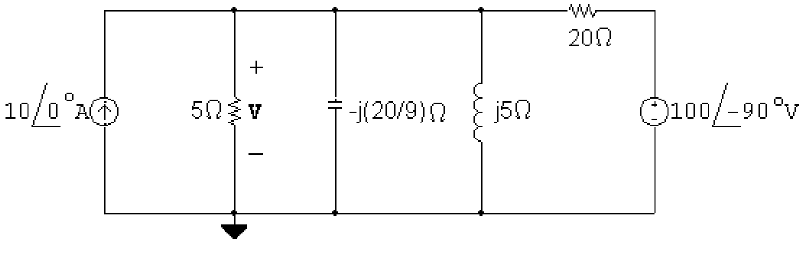
$$10\mathbf{I}_x = (20 + j10)\mathbf{I}_x$$

Therefore

$$\mathbf{I}_x = 0 \quad \text{and} \quad \mathbf{I}_T = \frac{\mathbf{V}_T}{-j10} + \frac{\mathbf{V}_T}{10}$$

$$Z_{\rm Th} = \frac{\mathbf{V}_T}{\mathbf{I}_T}, \quad \text{therefore} \quad Z_{\rm Th} = (5 - j5)\,\Omega$$

AP 9.12 The phasor domain circuit is as shown in the following diagram:



The node voltage equation is

$$-10 + \frac{\mathbf{V}}{5} + \frac{\mathbf{V}}{-j(20/9)} + \frac{\mathbf{V}}{j5} + \frac{\mathbf{V} - 100\underline{/-90^\circ}}{20} = 0$$

Therefore $\mathbf{V} = 10 - j30 = 31.62\underline{/-71.57^\circ}$

Therefore $v = 31.62\cos(50{,}000t - 71.57^\circ)\,\mathrm{V}$

AP 9.13 Let $\mathbf{I}_\mathrm{a}$, $\mathbf{I}_\mathrm{b}$, and $\mathbf{I}_\mathrm{c}$ be the three clockwise mesh currents going from left to right. Summing the voltages around meshes a and b gives

$$33.8 = (1 + j2)\mathbf{I}_\mathrm{a} + (3 - j5)(\mathbf{I}_\mathrm{a} - \mathbf{I}_\mathrm{b})$$

and

$$0 = (3 - j5)(\mathbf{I}_\mathrm{b} - \mathbf{I}_\mathrm{a}) + 2(\mathbf{I}_\mathrm{b} - \mathbf{I}_\mathrm{c}).$$

But

$$\mathbf{V}_x = -j5(\mathbf{I}_\mathrm{a} - \mathbf{I}_\mathrm{b}),$$

therefore

$$\mathbf{I}_\mathrm{c} = -0.75[-j5(\mathbf{I}_\mathrm{a} - \mathbf{I}_\mathrm{b})].$$

Solving for $\mathbf{I} = \mathbf{I}_\mathrm{a} = 29 + j2 = 29.07\underline{/3.95^\circ}\,\mathrm{A}$.

AP 9.14 **[a]** $M = 0.4\sqrt{0.0625} = 0.1\,\mathrm{H}, \qquad \omega M = 80\,\Omega$

$$Z_{22} = 40 + j800(0.125) + 360 + j800(0.25) = (400 + j300)\,\Omega$$

Therefore $|Z_{22}| = 500\,\Omega, \qquad Z_{22}^* = (400 - j300)\,\Omega$

$$Z_\tau = \left(\frac{80}{500}\right)^2(400 - j300) = (10.24 - j7.68)\,\Omega$$

[b] $$\mathbf{I}_1 = \frac{245.20}{184 + 100 + j400 + Z_\tau} = 0.50\underline{/-53.13^\circ}\,\mathrm{A}$$

$$i_1 = 0.5\cos(800t - 53.13^\circ)\,\mathrm{A}$$

[c] $$\mathbf{I}_2 = \left(\frac{j\omega M}{Z_{22}}\right)\mathbf{I}_1 = \frac{j80}{500\underline{/36.87^\circ}}(0.5\underline{/-53.13^\circ}) = 0.08\underline{/0^\circ}\,\mathrm{A}$$

$$i_2 = 80\cos 800t\,\mathrm{mA}$$

AP 9.15

$$\mathbf{I}_1 = \frac{\mathbf{V}_s}{Z_1 + 2s^2 Z_2} = \frac{25 \times 10^3\underline{/0^\circ}}{1500 + j6000 + (25)^2(4 - j14.4)}$$

$$= 4 + j3 = 5\underline{/36.87^\circ}\text{ A}$$

$$\mathbf{V}_1 = \mathbf{V}_s - Z_1\mathbf{I}_1 = 25{,}000\underline{/0^\circ} - (4 + j3)(1500 + j6000)$$

$$= 37{,}000 - j28{,}500$$

$$\mathbf{V}_2 = -\frac{1}{25}\mathbf{V}_1 = -1480 + j1140 = 1868.15\underline{/142.39^\circ}\text{ V}$$

$$\mathbf{I}_2 = \frac{\mathbf{V}_2}{Z_2} = \frac{1868.15\underline{/142.39^\circ}}{4 - j14.4} = 125\underline{/216.87^\circ}\text{ A}$$

Problems

P 9.1 **[a]** 80 V

[b] $2\pi f = 1000\pi; \qquad f = 500\,\text{Hz}$

[c] $\omega = 1000\pi = 3141.59$ rad/s

[d] $\theta(\text{rad}) = \dfrac{-\pi}{6} = -0.5236$ rad

[e] $\theta = -30^\circ$

[f] $T = \dfrac{1}{f} = \dfrac{1}{500} = 2\,\text{ms}$

[g] $1000\pi t - \dfrac{\pi}{6} = 0; \qquad \therefore \quad t = \dfrac{1}{6000} = 166.67\,\mu\text{s}$

[h]
$$v = 80\cos\left[1000\pi\left(t + \frac{0.002}{3}\right) - \frac{\pi}{6}\right]$$
$$= 80\cos[1000\pi t + (2\pi/3) - (\pi/6)]$$
$$= 80\cos[1000\pi t + (\pi/2)]$$
$$= -80\sin 1000\pi t\,\text{V}$$

[i] $1000\pi(t - t_o) - (\pi/6) = 1000\pi t - (\pi/2)$

$$\therefore \quad 1000\pi t_o = \frac{\pi}{3}; \qquad t_o = \frac{1}{3000} = 333.33\,\mu\text{s}$$

[j] $1000\pi(t + t_o) - (\pi/6) = 1000\pi t$

$$\therefore \quad 1000\pi t_o = \frac{\pi}{6}; \qquad t_o = \frac{1}{6000} = 166.67\,\mu\text{s}$$

P 9.2 **[a]** $\dfrac{T}{2} = 8 + 2 = 10\,\text{ms}; \qquad T = 20\,\text{ms}$

$$f = \frac{1}{T} = \frac{1}{20\times10^{-3}} = 50\,\text{Hz}$$

[b] $v = V_m\sin(\omega t + \theta)$

$$\omega = 2\pi f = 100\pi \text{ rad/s}$$

$$100\pi(-2\times10^{-3}) + \theta = 0; \qquad \therefore \quad \theta = \frac{\pi}{5}\text{ rad} = 36^\circ$$

$$v = V_m\sin[100\pi t + 36^\circ]$$

$$80.9 = V_m\sin 36^\circ; \qquad V_m = 137.64\,\text{V}$$

$$v = 137.64\sin[100\pi t + 36^\circ] = 137.64\cos[100\pi t - 54^\circ]\,\text{V}$$

P 9.3 [a] By hypothesis

$$i = 20\cos(\omega t + \theta)$$

$$\frac{di}{dt} = -20\omega\sin(\omega t + \theta)$$

$$\therefore \quad 20\omega = 8000\pi; \qquad \omega = 400\pi\ \text{rad/s}$$

[b] $f = \dfrac{\omega}{2\pi} = 200$ Hz; $\quad T = \dfrac{1}{f} = 5$ ms $= 5000\,\mu$s

$$\frac{625}{5000} = \frac{1}{8}, \qquad \therefore \quad \theta = -\frac{1}{8}(360) = -45^\circ$$

$$\therefore \quad i = 20\cos(400\pi t - 45^\circ)\ \text{A}$$

P 9.4 [a] $\omega = 2\pi f = 3769.91\,\text{rad/s}, \qquad f = \dfrac{\omega}{2\pi} = 600\,\text{Hz}$

[b] $T = 1/f = 1.67\,\text{ms}$

[c] $V_m = 10\,\text{V}$

[d] $v(0) = 10\cos(-53.13^\circ) = 6\,\text{V}$

[e] $\phi = -53.13^\circ; \qquad \phi = \dfrac{-53.13^\circ(2\pi)}{360^\circ} = -0.9273$ rad

[f] $V = 0$ when $3769.91t - 53.13^\circ = 90^\circ$. Now resolve the units:

$$(3769.91\ \text{rad/s})t = \frac{143.13^\circ}{57.3^\circ/\text{rad}} = 2.498\,\text{rad}, \qquad t = 662.64\,\mu\text{s}$$

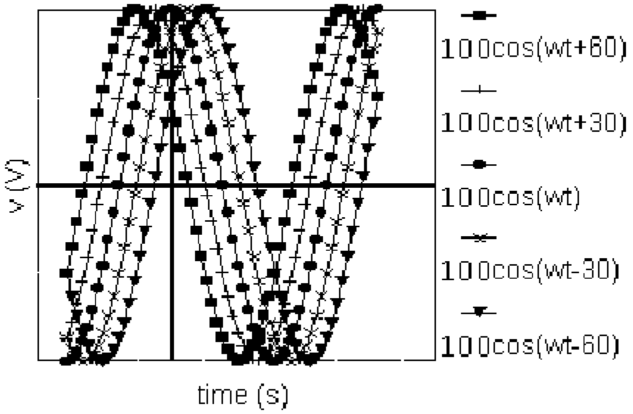
[g] $(dv/dt) = (-10)3769.91\sin(3769.91t - 53.13^\circ)$

$$(dv/dt) = 0 \quad \text{when} \quad 3769.91t - 53.13^\circ = 0^\circ$$

$$\text{or} \qquad 3769.91t = \frac{53.13^\circ}{57.3^\circ/\text{rad}} = 0.9273\,\text{rad}$$

Therefore $t = 245.97\,\mu$s

P 9.5



[a] Left as ϕ becomes more positive

[b] Left

P 9.6

$$\begin{aligned}\int_{t_o}^{t_o+T} V_m^2\cos^2(\omega t+\phi)\,dt &= V_m^2\int_{t_o}^{t_o+T}\frac{1}{2}+\frac{1}{2}\cos(2\omega t+2\phi)\,dt\\ &= \frac{V_m^2}{2}\left\{\int_{t_o}^{t_o+T} dt + \int_{t_o}^{t_o+T}\cos(2\omega t+2\phi)\,dt\right\}\\ &= \frac{V_m^2}{2}\left\{T+\frac{1}{2\omega}\left[\sin(2\omega t+2\phi)\,\Big|_{t_o}^{t_o+T}\right]\right\}\\ &= \frac{V_m^2}{2}\left\{T+\frac{1}{2\omega}\left[\sin(2\omega t_o+4\pi+2\phi)-\sin(2\omega t_o+2\phi)\right]\right\}\\ &= V_m^2\left(\frac{T}{2}\right)+\frac{1}{2\omega}(0)=V_m^2\left(\frac{T}{2}\right)\end{aligned}$$

P 9.7 $V_m=\sqrt{2}V_{\text{rms}}=\sqrt{2}(240)=339.41\,\text{V}$

P 9.8 $V_{\text{rms}}=\sqrt{\frac{1}{T}\int_0^{T/2}V_m^2\sin^2\frac{2\pi}{T}t\,dt}$

$$\int_0^{T/2}V_m^2\sin^2\left(\frac{2\pi}{T}t\right)\,dt=\frac{V_m^2}{2}\int_0^{T/2}\left(1-\cos\frac{4\pi}{T}t\right)\,dt=\frac{V_m^2T}{4}$$

Therefore $V_{\text{rms}}=\sqrt{\frac{1}{T}\frac{V_m^2T}{4}}=\frac{V_m}{2}$

P 9.9 **[a]** The numerical values of the terms in Eq. 9.8 are

$V_m=20,\qquad R/L=1066.67,\qquad \omega L=60$

$\sqrt{R^2+\omega^2L^2}=100$

$\phi=25^\circ,\qquad \theta=\tan^{-1}60/80,\qquad \theta=36.87^\circ$

Substitute these values into Equation 9.9:

$i=\left[-195.72e^{-1066.67t}+200\cos(800t-11.87^\circ)\right]\text{ mA},\qquad t\geq 0$

[b] Transient component $=-195.72e^{-1066.67t}\,\text{mA}$
Steady-state component $=200\cos(800t-11.87^\circ)\,\text{mA}$

[c] By direct substitution into Eq 9.9 in part (a), $i(1.875\,\text{ms})=28.39\,\text{mA}$

[d] $200\,\text{mA},\quad 800\,\text{rad/s},\quad -11.87^\circ$

[e] The current lags the voltage by 36.87°.

P 9.10 [a] From Eq. 9.9 we have

$$L\frac{di}{dt} = \frac{V_m R\cos(\phi - \theta)}{\sqrt{R^2 + \omega^2 L^2}}e^{-(R/L)t} - \frac{\omega L V_m \sin(\omega t + \phi - \theta)}{\sqrt{R^2 + \omega^2 L^2}}$$

$$Ri = \frac{-V_m R\cos(\phi - \theta)e^{-(R/L)t}}{\sqrt{R^2 + \omega^2 L^2}} + \frac{V_m R\cos(\omega t + \phi - \theta)}{\sqrt{R^2 + \omega^2 L^2}}$$

$$L\frac{di}{dt} + Ri = V_m\left[\frac{R\cos(\omega t + \phi - \theta) - \omega L\sin(\omega t + \phi - \theta)}{\sqrt{R^2 + \omega^2 L^2}}\right]$$

But

$$\frac{R}{\sqrt{R^2 + \omega^2 L^2}} = \cos\theta \quad \text{and} \quad \frac{\omega L}{\sqrt{R^2 + \omega^2 L^2}} = \sin\theta$$

Therefore the right-hand side reduces to

$$V_m\cos(\omega t + \phi)$$

At $t = 0$, Eq. 9.9 reduces to

$$i(0) = \frac{-V_m\cos(\phi - \theta)}{\sqrt{R^2 + \omega^2 L^2}} + \frac{V_m\cos(\phi - \theta)}{\sqrt{R^2 + \omega^2 L^2}} = 0$$

[b] $$i_{ss} = \frac{V_m}{\sqrt{R^2 + \omega^2 L^2}}\cos(\omega t + \phi - \theta)$$

Therefore

$$L\frac{di_{ss}}{dt} = \frac{-\omega L V_m}{\sqrt{R^2 + \omega^2 L^2}}\sin(\omega t + \phi - \theta)$$

and

$$Ri_{ss} = \frac{V_m R}{\sqrt{R^2 + \omega^2 L^2}}\cos(\omega t + \phi - \theta)$$

$$L\frac{di_{ss}}{dt} + Ri_{ss} = V_m\left[\frac{R\cos(\omega t + \phi - \theta) - \omega L\sin(\omega t + \phi - \theta)}{\sqrt{R^2 + \omega^2 L^2}}\right]$$

$$= V_m\cos(\omega t + \phi)$$

P 9.11 [a] $\mathbf{Y} = 50\underline{/60^\circ} + 100\underline{/-30^\circ} = 111.8\underline{/-3.43^\circ}$

$$y = 111.8\cos(500t - 3.43^\circ)$$

[b] $\mathbf{Y} = 200\underline{/50^\circ} - 100\underline{/60^\circ} = 102.99\underline{/40.29^\circ}$

$$y = 102.99\cos(377t + 40.29^\circ)$$

[c] $\mathbf{Y} = 80\underline{/30^\circ} - 100\underline{/-225^\circ} + 50\underline{/-90^\circ} = 161.59\underline{/-29.96^\circ}$

$$y = 161.59\cos(100t - 29.96^\circ)$$

[d] $\mathbf{Y} = 250\underline{/0^\circ} + 250\underline{/120^\circ} + 250\underline{/-120^\circ} = 0$

$y = 0$

P 9.12 **[a]** $\mathbf{V}_g = 300\underline{/78^\circ}$; $\mathbf{I}_g = 6\underline{/33^\circ}$

$$\therefore \quad Z = \frac{\mathbf{V}_g}{\mathbf{I}_g} = \frac{300\underline{/78^\circ}}{6\underline{/33^\circ}} = 50\underline{/45^\circ}\,\Omega$$

[b] i_g lags v_g by 45°:

$2\pi f = 5000\pi$; $\quad f = 2500\,\text{Hz}$; $\quad T = 1/f = 400\,\mu\text{s}$

$$\therefore \quad i_g \text{ lags } v_g \text{ by } \frac{45^\circ}{360^\circ}(400\,\mu\text{s}) = 50\,\mu\text{s}$$

P 9.13 **[a]** $\omega = 2\pi f = 160\pi \times 10^3 = 502.65\,\text{krad/s} = 502{,}654.82\,\text{rad/s}$

[b] $\mathbf{I} = \dfrac{25 \times 10^{-3}\underline{/0^\circ}}{1/j\omega C} = j\omega C(25 \times 10^{-3})\underline{/0^\circ} = 25 \times 10^{-3}\omega C\underline{/90^\circ}$

$\therefore \quad \theta_i = 90^\circ$

[c] $628.32 \times 10^{-6} = 25 \times 10^{-3}\,\omega C$

$$\frac{1}{\omega C} = \frac{25 \times 10^{-3}}{628.32 \times 10^{-6}} = 39.79\,\Omega, \quad \therefore \quad X_{\text{C}} = -39.79\,\Omega$$

[d] $C = \dfrac{1}{39.79(\omega)} = \dfrac{1}{(39.79)(160\pi \times 10^3)}$

$C = 0.05 \times 10^{-6} = 0.05\,\mu\text{F}$

[e] $Z_c = j\left(\dfrac{-1}{\omega C}\right) = -j39.79\,\Omega$

P 9.14 **[a]** 400 Hz

[b] $\theta_v = 0^\circ$

$$\mathbf{I} = \frac{100\underline{/0^\circ}}{j\omega L} = \frac{100}{\omega L}\underline{/-90^\circ}; \qquad \theta_i = -90^\circ$$

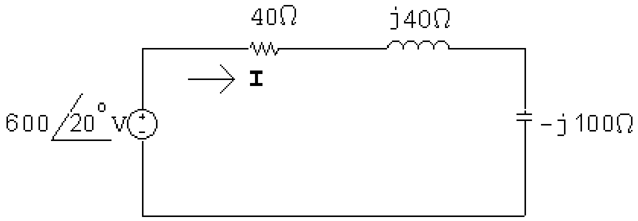
[c] $\dfrac{100}{\omega L} = 20$; $\quad \omega L = 5\,\Omega$

[d] $L = \dfrac{5}{800\pi} = 1.99\,\text{mH}$

[e] $Z_L = j\omega L = j5\,\Omega$

P 9.15 [a] $Z_L = j(8000)(5 \times 10^{-3}) = j40\,\Omega$

$$Z_C = \frac{-j}{(8000)(1.25 \times 10^{-6})} = -j100\,\Omega$$

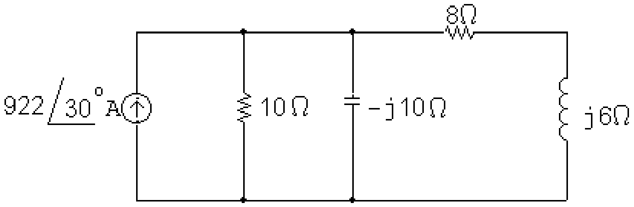


[b] $\mathbf{I} = \dfrac{600\underline{/20^\circ}}{40 + j40 - j100} = 8.32\underline{/76.31^\circ}\,\mathrm{A}$

[c] $i = 8.32\cos(8000t + 76.31^\circ)\,\mathrm{A}$

P 9.16 [a] $j\omega L = j(2 \times 10^4)(300 \times 10^{-6}) = j6\,\Omega$

$$\frac{1}{j\omega C} = -j\frac{1}{(2 \times 10^4)(5 \times 10^{-6})} = -j10\,\Omega; \qquad \mathbf{I}_g = 922\underline{/30^\circ}\,\mathrm{A}$$



[b] $\mathbf{V}_o = 922\underline{/30^\circ}Z_e$

$$Z_e = \frac{1}{Y_e}; \qquad Y_e = \frac{1}{10} + j\frac{1}{10} + \frac{1}{8 + j6}$$

$$Y_e = 0.18 + j0.04\,\mathrm{S}$$

$$Z_e = \frac{1}{0.18 + j0.04} = 5.42\underline{/-12.53^\circ}\,\Omega$$

$$\mathbf{V}_o = (922\underline{/30^\circ})(5.42\underline{/-12.53^\circ}) = 5000.25\underline{/17.47^\circ}\,\mathrm{V}$$

[c] $v_o = 5000.25\cos(2 \times 10^4 t + 17.47^\circ)\,\mathrm{V}$

P 9.17 [a] $Z_1 = R_1 + j\omega L_1$

$$Z_2 = \frac{R_2(j\omega L_2)}{R_2 + j\omega L_2} = \frac{\omega^2 L_2^2 R_2 + j\omega L_2 R_2^2}{R_2^2 + \omega^2 L_2^2}$$

$$Z_1 = Z_2 \quad \text{when} \quad R_1 = \frac{\omega^2 L_2^2 R_2}{R_2^2 + \omega^2 L_2^2} \quad \text{and} \quad L_1 = \frac{R_2^2 L_2}{R_2^2 + \omega^2 L_2^2}$$

[b] $$R_1 = \frac{(4000)^2(1.25)^2(5000)}{5000^2 + 4000^2(1.25)^2} = 2500\,\Omega$$

$$L_1 = \frac{(5000)^2(1.25)}{5000^2 + 4000^2(1.25)^2} = 625\,\mathrm{mH}$$

P 9.18 **[a]** $$Y_2 = \frac{1}{R_2} - \frac{j}{\omega L_2}$$

$$Y_1 = \frac{1}{R_1 + j\omega L_1} = \frac{R_1 - j\omega L_1}{R_1^2 + \omega^2 L_1^2}$$

Therefore $Y_2 = Y_1$ when

$$R_2 = \frac{R_1^2 + \omega^2 L_1^2}{R_1} \quad \text{and} \quad L_2 = \frac{R_1^2 + \omega^2 L_1^2}{\omega^2 L_1}$$

[b] $$R_2 = \frac{8000^2 + 1000^2(4)^2}{8000} = 10\,\mathrm{k}\Omega$$

$$L_2 = \frac{8000^2 + 1000^2(4)^2}{1000^2(4)} = 20\,\mathrm{H}$$

P 9.19 **[a]** $$Z_1 = R_1 - j\frac{1}{\omega C_1}$$

$$Z_2 = \frac{R_2/j\omega C_2}{R_2 + (1/j\omega C_2)} = \frac{R_2}{1 + j\omega R_2 C_2} = \frac{R_2 - j\omega R_2^2 C_2}{1 + \omega^2 R_2^2 C_2^2}$$

$$Z_1 = Z_2 \quad \text{when} \quad R_1 = \frac{R_2}{1 + \omega^2 R_2^2 C_2^2} \quad \text{and}$$

$$\frac{1}{\omega C_1} = \frac{\omega R_2^2 C_2}{1 + \omega^2 R_2^2 C_2^2} \quad \text{or} \quad C_1 = \frac{1 + \omega^2 R_2^2 C_2^2}{\omega^2 R_2^2 C_2}$$

[b] $$R_1 = \frac{1000}{1 + (40 \times 10^3)^2(1000)^2(50 \times 10^{-4})^2} = 200\,\Omega$$

$$C_1 = \frac{1 + (40 \times 10^3)^2(1000)^2(50 \times 10^{-9})^2}{(40 \times 10^3)^2(1000)^2(50 \times 10^{-9})} = 62.5\,\mathrm{nF}$$

P 9.20 **[a]** $$Y_2 = \frac{1}{R_2} + j\omega C_2$$

$$Y_1 = \frac{1}{R_1 + (1/j\omega C_1)} = \frac{j\omega C_1}{1 + j\omega R_1 C_1} = \frac{\omega^2 R_1 C_1^2 + j\omega C_1}{1 + \omega^2 R_1^2 C_1^2}$$

Therefore $Y_1 = Y_2$ when

$$R_2 = \frac{1 + \omega^2 R_1^2 C_1^2}{\omega^2 R_1 C_1^2} \quad \text{and} \quad C_2 = \frac{C_1}{1 + \omega^2 R_1^2 C_1^2}$$

[**b**] $$R_2 = \frac{1 + (50 \times 10^3)^2(1000)^2(40 \times 10^{-9})^2}{(50 \times 10^3)^2(1000)(40 \times 10^{-9})^2} = 1250\,\Omega$$

$$C_2 = \frac{40 \times 10^{-9}}{1 + (50 \times 10^3)^2(1000)^2(40 \times 10^{-9})^2} = 8\,\text{nF}$$

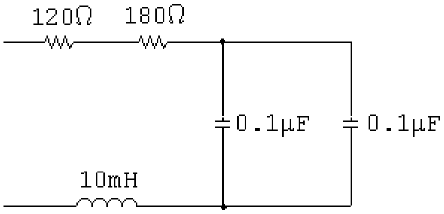
P 9.21 [**a**] $R = 300\,\Omega = 120\,\Omega + 180\,\Omega$

$$\omega L - \frac{1}{\omega C} = -400 \quad \text{so} \quad 10{,}000L - \frac{1}{10{,}000C} = -400$$

Choose L = 10 mH. Then,

$$\frac{1}{10{,}000C} = 100 + 400 \quad \text{so} \quad C = \frac{1}{10{,}000(500)} = 0.2\,\mu\text{F}$$

We can achieve the desired capacitance by combining two $0.1\,\mu$F capacitors in parallel. The final circuit is shown here:



[**b**] $$0.01\omega = \frac{1}{\omega(0.2 \times 10^{-6})} \quad \text{so} \quad \omega^2 = \frac{1}{0.01(0.2 \times 10^{-6})} = 5 \times 10^8$$

$$\therefore \quad \omega = 22{,}360.7 \text{ rad/s}$$

P 9.22 [**a**] Using the notation and results from Problem 9.18:

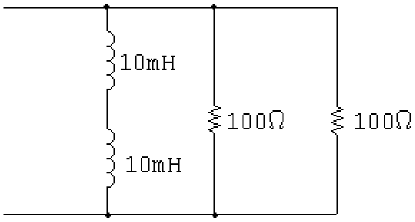
$$R\|L = 40 + j20 \quad \text{so} \quad R_1 = 40, \quad L_1 = \frac{20}{5000} = 4\,\text{mH}$$

$$R_2 = \frac{40^2 + 5000^2(0.004)^2}{40} = 50\,\Omega$$

$$L_2 = \frac{40^2 + 5000^2(0.004)^2}{5000^2(0.004)} = 20\,\text{mH}$$

$$R_2\|j\omega L_2 = 50\|j100 = 40 + j20\,\Omega \quad \text{(checks)}$$

The circuit, using combinations of components from Appendix H, is shown here:



[b] Using the notation and results from Problem 9.22:

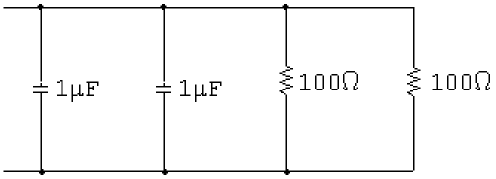
$$R\|C = 40 - j20 \quad \text{so} \quad R_1 = 40, \quad C_1 = 10\,\mu\text{F}$$

$$R_2 = \frac{1 + 5000^2(40)^2(10\,\mu)^2}{5000^2(40)(10\,\mu)^2} = 50\,\Omega$$

$$C_2 = \frac{10\,\mu}{1 + 5000^2(40)^2(10\,\mu)^2} = 2\,\mu\text{F}$$

$$R_2\|(-j/\omega C_2) = 50\|(-j100) = 40 - j20\,\Omega \quad \text{(checks)}$$

The circuit, using combinations of components from Appendix H, is shown here:



P 9.23 **[a]** $(40 + j20)\|(-j/\omega C) = 50\|j100\|(-j/\omega C)$

To cancel out the $j100\,\Omega$ impedance, the capacitive impedance must be $-j100\,\Omega$:

$$\frac{-j}{5000C} = -j100 \quad \text{so} \quad C = \frac{1}{(100)(5000)} = 2\,\mu\text{F}$$

Check:

$$R\|j\omega L\|(-j/\omega C) = 50\|j100\|(-j100) = 50\,\Omega$$

Create the equivalent of a $2\,\mu$F capacitor from components in Appendix H by combining two $1\,\mu$F capacitors in parallel.

[b] $(40 - j20)\|(j\omega L) = 50\|(-j100)\|(j\omega L)$

To cancel out the $-j100\,\Omega$ impedance, the inductive impedance must be $j100\,\Omega$:

$$j5000L = j100 \quad \text{so} \quad L = \frac{100}{5000} = 20\,\text{mH}$$

Check:

$$R\|j\omega L\|(-j/\omega C) = 50\|j100\|(-j100) = 50\,\Omega$$

Create the equivalent of a 20 mH inductor from components in Appendix H by combining two 10 mH inductors in series.

P 9.24 **[a]** $Y = \dfrac{1}{3 + j4} + \dfrac{1}{16 - j12} + \dfrac{1}{-j4}$

$$= 0.12 - j0.16 + 0.04 + j0.03 + j0.25$$

$$= 0.16 + j0.12 = 200\underline{/36.87^\circ}\,\text{mS}$$

[**b**] $G = 160\,\text{mS}$

[**c**] $B = 120\,\text{mS}$

[**d**] $\mathbf{I} = 8\underline{/0^\circ}\,\text{A}, \qquad \mathbf{V} = \dfrac{\mathbf{I}}{Y} = \dfrac{8}{0.2\underline{/36.87^\circ}} = 40\underline{/-36.87^\circ}\,\text{V}$

$$\mathbf{I}_C = \frac{\mathbf{V}}{Z_C} = \frac{40\underline{/-36.87^\circ}}{4\underline{/-90^\circ}} = 10\underline{/53.13^\circ}\,\text{A}$$

$$i_C = 10\cos(\omega t + 53.13^\circ)\,\text{A}, \qquad I_m = 10\,\text{A}$$

P 9.25 [**a**] $j\omega L = R\|(-j/\omega C) = j\omega L + \dfrac{-jR/\omega C}{R - j/\omega C}$

$$j\omega L + \frac{-jR}{\omega CR - j}$$

$$j\omega L + \frac{-jR(\omega CR + j)}{\omega^2 C^2 R^2 + 1}$$

$$\mathbf{Im}(Z_{\text{ab}}) = \omega L - \frac{\omega CR^2}{\omega^2 C^2 R^2 + 1} = 0$$

$$\therefore \quad L = \frac{CR^2}{\omega^2 C^2 R^2 + 1}$$

$$\therefore \quad \omega^2 C^2 R^2 + 1 = \frac{CR^2}{L}$$

$$\therefore \quad \omega^2 = \frac{(CR^2/L) - 1}{C^2 R^2} = \frac{\frac{(25\times 10^{-9})(100)^2}{160\times 10^{-6}} - 1}{(25 \times 10^{-9})^2 (100)^2} = 900 \times 10^8$$

$$\omega = 300\,\text{krad/s}$$

[**b**] $Z_{\text{ab}}(300 \times 10^3) = j48 + \dfrac{(100)(-j133.33)}{100 - j133.33} = 64\,\Omega$

P 9.26 First find the admittance of the parallel branches

$$Y_p = \frac{1}{6 - j2} + \frac{1}{4 + j12} + \frac{1}{5} + \frac{1}{j10} = 0.375 - j0.125\,\text{S}$$

$$Z_p = \frac{1}{Y_p} = \frac{1}{0.375 - j0.125} = 2.4 + j0.8\,\Omega$$

$$Z_{\text{ab}} = -j12.8 + 2.4 + j0.8 + 13.6 = 16 - j12\,\Omega$$

$$Y_{\text{ab}} = \frac{1}{Z_{\text{ab}}} = \frac{1}{16 - j12} = 0.04 + j0.03\,\text{S}$$

$$= 40 + j30\,\text{mS} = 50\underline{/36.87^\circ}\,\text{mS}$$

P 9.27 $Z_{\rm ab} = 1 - j8 + (2 + j4)\|(10 - j20) + (40\|j20)$

$$= 1 - j8 + 3 + j4 + 8 + j16 = 12 + j12\,\Omega = 16.97\underline{/45^\circ}\,\Omega$$

P 9.28 $\mathbf{V}_g = 40\underline{/-15^\circ}\,\mathrm{V};\qquad \mathbf{I}_g = 40\underline{/-68.13^\circ}\,\mathrm{mA}$

$$Z = \frac{\mathbf{V}_g}{\mathbf{I}_g} = 1000\underline{/53.13^\circ}\,\Omega = 600 + j800\,\Omega$$

$$Z = 600 + j\left(3.2\omega - \frac{0.4\times10^6}{\omega}\right)$$

$$\therefore\quad 3.2\omega - \frac{0.4\times10^6}{\omega} = 800$$

$$\therefore\quad \omega^2 - 250\omega - 125{,}000 = 0$$

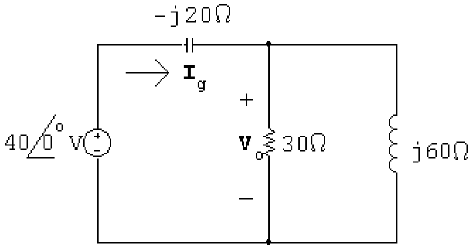
Solving,

$\omega = 500\,\mathrm{rad/s}$

P 9.29 $\dfrac{1}{j\omega C} = \dfrac{1}{(1\times10^{-6})(50\times10^3)} = -j20\,\Omega$

$$j\omega L = j50\times10^3(1.2\times10^{-3}) = j60\,\Omega$$

$$\mathbf{V}_g = 40\underline{/0^\circ}\,\mathrm{V}$$



$$Z_e = -j20 + 30\|j60 = 24 - j8\,\Omega$$

$$\mathbf{I}_g = \frac{40\underline{/0^\circ}}{24 - j8} = 1.5 + j0.5\,\mathrm{mA}$$

$$\mathbf{V}_o = (30\|j60)\mathbf{I}_g = \frac{30(j60)}{30 + j60}(1.5 + j0.5) = 30 + j30 = 42.43\underline{/45^\circ}\,\mathrm{V}$$

$$v_o = 42.43\cos(50{,}000t + 45^\circ)\,\mathrm{V}$$

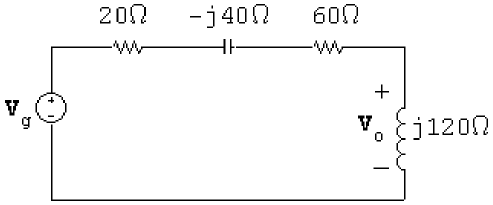
P 9.30 **[a]** $\frac{1}{j\omega C} = -j50\,\Omega$

$$j\omega L = j120\,\Omega$$

$$Z_e = 100\| - j50 = 20 - j40\,\Omega$$

$$\mathbf{I}_g = 2\underline{/0^\circ}$$

$$\mathbf{V}_g = \mathbf{I}_g Z_e = 2(20 - j40) = 40 - j80\,\text{V}$$



$$\mathbf{V}_o = \frac{j120}{80 + j80}(40 - j80) = 90 - j30 = 94.87\underline{/-18.43^\circ}\,\text{V}$$

$$v_o = 94.87\cos(16 \times 10^5 t - 18.435^\circ)\,\text{V}$$

[b] $\omega = 2\pi f = 16 \times 10^5; \qquad f = \frac{8 \times 10^5}{\pi}$

$$T = \frac{1}{f} = \frac{\pi}{8 \times 10^5} = 1.25\pi\,\mu\text{s}$$

$$\therefore \quad \frac{18.435}{360}(1.25\pi\,\mu\text{s}) = 201.09\,\text{ns}$$

$\therefore \quad v_o$ lags i_g by 201.09 ns.

P 9.31 $Z = 4 + j(50)(0.24) - j\frac{1}{(50)(0.0025)} = 5.66\underline{/45^\circ}\,\Omega$

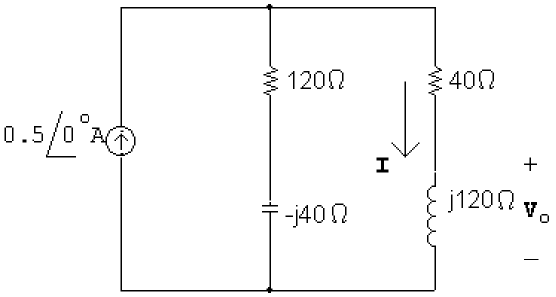
$$\mathbf{I}_o = \frac{\mathbf{V}}{Z} = \frac{0.1\underline{/-90^\circ}}{5.66\underline{/45^\circ}} = 17.67\underline{/-135^\circ}\,\text{mA}$$

$$i_o(t) = 17.67\cos(50t - 135^\circ)\,\text{mA}$$

P 9.32 $Z_L = j(2000)(60 \times 10^{-3}) = j120\,\Omega$

$$Z_C = \frac{-j}{(2000)(12.5 \times 10^{-6})} = -j40\,\Omega$$

Construct the phasor domain equivalent circuit:



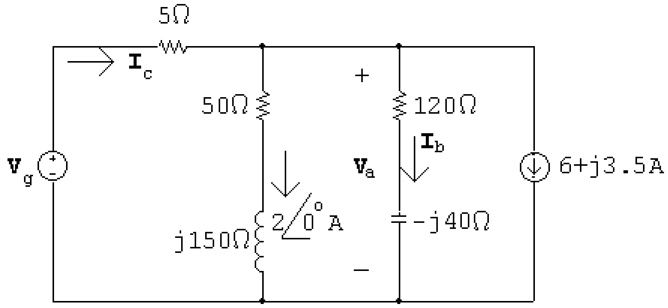
Using current division:

$$\mathbf{I} = \frac{(120 - j40)}{120 - j40 + 40 + j120}(0.5) = 0.25 - j0.25\,\text{A}$$

$$\mathbf{V}_o = j120\mathbf{I} = 30 + j30 = 42.43\underline{/45^\circ}$$

$$v_o = 42.43\cos(2000t + 45^\circ)\,\text{V}$$

P 9.33 **[a]**



$$\mathbf{V}_\text{a} = (50 + j150)(2\underline{/0^\circ}) = 100 + j300\,\text{V}$$

$$\mathbf{I}_\text{b} = \frac{100 + j300}{120 - j40} = j2.5\,\text{A}$$

$$\mathbf{I}_\text{c} = 2\underline{/0^\circ} + j2.5 + 6 + j3.5 = 8 + j6\,\text{A}$$

$$\mathbf{V}_g = 5\mathbf{I}_\text{c} + \mathbf{V}_\text{a} = 5(8 + j6) + 100 + j300 = 140 + j330\,\text{V}$$

[b] $i_\text{b} = 2.5\cos(800t + 90^\circ)\,\text{A}$

$$i_\text{c} = 10\cos(800t + 36.87^\circ)\,\text{A}$$

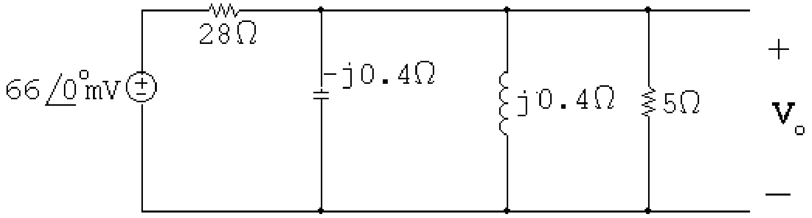
$$v_g = 358.47\cos(800t + 67.01^\circ)\,\text{V}$$

P 9.34 $\mathbf{I}_s = 3\underline{/0^\circ}\,\text{mA}$

$$\frac{1}{j\omega C} = -j0.4\,\Omega$$

$$j\omega L = j0.4\,\Omega$$

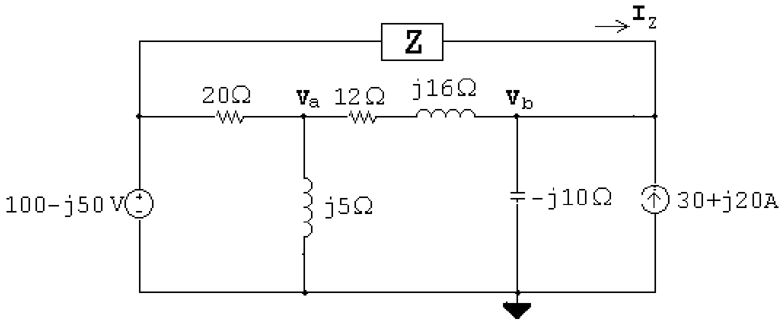
After source transformation we have



$$\mathbf{V}_o = \frac{-j0.4\|j0.4\|5}{28 + -j0.4\|j0.4\|5}(66 \times 10^{-3}) = 10\,\text{mV}$$

$$v_o = 10\cos 200t\,\text{mV}$$

P 9.35



$$\frac{\mathbf{V}_\text{a} - (100 - j50)}{20} + \frac{\mathbf{V}_\text{a}}{j5} + \frac{\mathbf{V}_\text{a} - (140 + j30)}{12 + j16} = 0$$

Solving,

$$\mathbf{V}_\text{a} = 40 + j30\,\text{V}$$

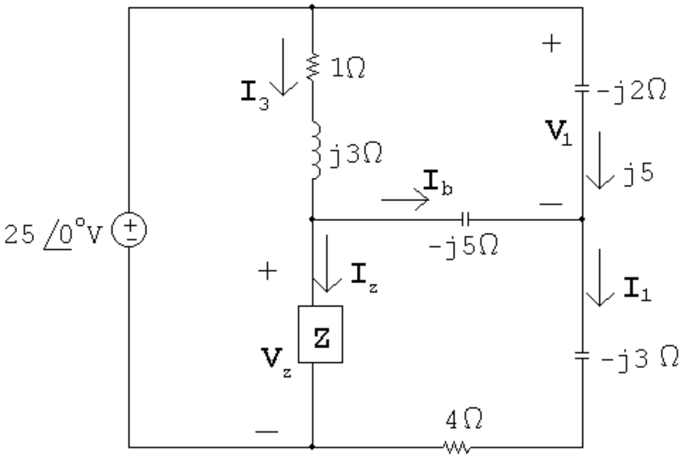
$$\mathbf{I}_Z + (30 + j20) - \frac{140 + j30}{-j10} + \frac{(40 + j30) - (140 + j30)}{12 + j16} = 0$$

Solving,

$$\mathbf{I}_Z = -30 - j10\,\text{A}$$

$$Z = \frac{(100 - j50) - (140 + j30)}{-30 - j10} = 2 + j2\,\Omega$$

P 9.36



$$\mathbf{V}_1 = j5(-j2) = 10\,\text{V}$$

$$-25 + 10 + (4 - j3)\mathbf{I}_1 = 0 \quad \therefore \quad \mathbf{I}_1 = \frac{15}{4 - j3} = 2.4 + j1.8\,\text{A}$$

$$\mathbf{I}_\text{b} = \mathbf{I}_1 - j5 = (2.4 + j1.8) - j5 = 2.4 - j3.2\,\text{A}$$

$$\mathbf{V}_Z = -j5\mathbf{I}_2 + (4 - j3)\mathbf{I}_1 = -j5(2.4 - j3.2) + (4 - j3)(2.4 + j1.8) = -1 - j12\,\text{V}$$

$$-25 + (1 + j3)\mathbf{I}_3 + (-1 - j12) = 0 \quad \therefore \quad \mathbf{I}_3 = 6.2 - j6.6\,\text{A}$$

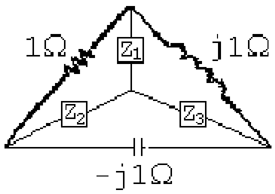
$$\mathbf{I}_Z = \mathbf{I}_3 - \mathbf{I}_2 = (6.2 - j6.6) - (2.4 - j3.2) = 3.8 - j3.4\,\text{A}$$

$$Z = \frac{\mathbf{V}_Z}{\mathbf{I}_Z} = \frac{-1 - j12}{3.8 - j3.4} = 1.42 - j1.88\,\Omega$$

P 9.37 Simplify the top triangle using series and parallel combinations:

$$(1 + j1)\|(1 - j1) = 1\,\Omega$$

Convert the lower left delta to a wye:

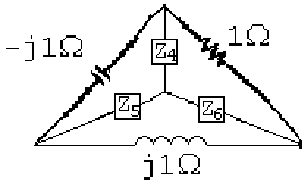


$$Z_1 = \frac{(j1)(1)}{1 + j1 - j1} = j1\,\Omega$$

$$Z_2 = \frac{(-j1)(1)}{1 + j1 - j1} = -j1\,\Omega$$

$$Z_3 = \frac{(j1)(-j1)}{1 + j1 - j1} = 1\,\Omega$$

Convert the lower right delta to a wye:

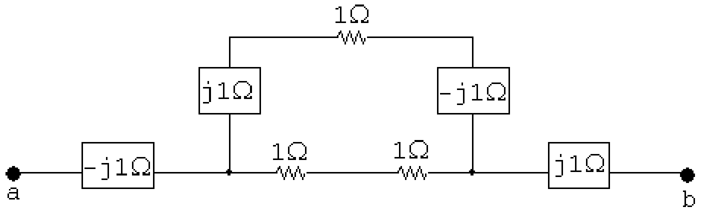


$$Z_4 = \frac{(-j1)(1)}{1 + j1 - j1} = -j1\,\Omega$$

$$Z_5 = \frac{(-j1)(j1)}{1 + j1 - j1} = 1\,\Omega$$

$$Z_6 = \frac{(j1)(1)}{1 + j1 - j1} = j1\,\Omega$$

The resulting circuit is shown below:



Simplify the middle portion of the circuit by making series and parallel combinations:

$$(1 + j1 - j1)\|(1 + 1) = 1\|2 = 2/3\,\Omega$$

$$Z_{\rm ab} = -j1 + 2/3 + j1 = 2/3\,\Omega$$

P 9.38 **[a]** $Z_g = 500 - j\dfrac{10^6}{\omega} + \dfrac{10^3(j0.5\omega)}{10^3 + j0.5\omega}$

$$= 500 - j\frac{10^6}{\omega} + \frac{500j\omega(1000 - j0.5\omega)}{10^6 + 0.25\omega^2}$$

$$= 500 - j\frac{10^6}{\omega} + \frac{250\omega^2}{10^6 + 0.25\omega^2} + j\frac{5 \times 10^5\omega}{10^6 + 0.25\omega^2}$$

$\therefore$ If Z_g is purely real, $\dfrac{10^6}{\omega} = \dfrac{5 \times 10^5\omega}{10^6 + 0.25\omega^2}$

$$2(10^6 + 0.25\omega^2) = \omega^2 \qquad \therefore \qquad 4 \times 10^6 = \omega^2$$

$$\therefore \qquad \omega = 2000 \text{ rad/s}$$

[b] When $\omega = 2000\,\text{rad/s}$

$$Z_g = 500 - j500 + (j1000\|1000) = 1000\,\Omega$$

$$\therefore \quad \mathbf{I}_g = \frac{20\underline{/0^\circ}}{1000} = 20\underline{/0^\circ}\,\text{mA}$$

$$\mathbf{V}_o = \mathbf{V}_g - \mathbf{I}_g Z_1$$

$$Z_1 = 500 - j500\,\Omega$$

$$\mathbf{V}_o = 20\underline{/0^\circ} - (0.02\underline{/0^\circ})(500 - j500) = 10 + j10 = 14.14\underline{/45^\circ}\,\text{V}$$

$$v_o = 14.14\cos(2000t + 45^\circ)\,\text{V}$$

P 9.39 **[a]** $Z_{\text{eq}} = \dfrac{50{,}000}{3} + \dfrac{-j20 \times 10^6}{\omega}\|(1200 + j0.2\omega)$

$$= \frac{50{,}000}{3} + \frac{-j20 \times 10^6}{\omega}\frac{(1200 + j0.2\omega)}{1200 + j[0.2\omega - \frac{20\times10^6}{\omega}]}$$

$$= \frac{50{,}000}{3} + \frac{\frac{-j20\times10^6}{\omega}(1200 + j0.2\omega)\left[1200 - j\left(0.2\omega - \frac{20\times10^6}{\omega}\right)\right]}{1200^2 + \left(0.2\omega - \frac{20\times10^6}{\omega}\right)^2}$$

$$\mathbf{Im}(Z_{\text{eq}}) = -\frac{20 \times 10^6}{\omega}(1200)^2 - \frac{20 \times 10^6}{\omega}\left[0.2\omega\left(0.2\omega - \frac{20 \times 10^6}{\omega}\right)\right] = 0$$

$$-20 \times 10^6(1200)^2 - 20 \times 10^6\left[0.2\omega\left(0.2\omega - \frac{20 \times 10^6}{\omega}\right)\right] = 0$$

$$-(1200)^2 = 0.2\omega\left(0.2\omega - \frac{20 \times 10^6}{\omega}\right)$$

$$0.2^2\omega^2 - 0.2(20 \times 10^6) - 1200^2 = 0$$

$$\omega^2 = 64 \times 10^6 \qquad \therefore \qquad \omega = 8000\,\text{rad/s}$$

$$\therefore \qquad f = 1273.24 \text{ Hz}$$

[b] $Z_{\text{eq}} = \dfrac{50{,}000}{3} + -j2500\|(1200 + j1600)$

$$= \frac{50{,}000}{3} + \frac{(-j2500)(1200 + j1600)}{1200 - j900} = 20{,}000\,\Omega$$

$$\mathbf{I}_g = \frac{30\underline{/0^\circ}}{20{,}000} = 1.5\underline{/0^\circ}\,\text{mA}$$

$$i_g(t) = 1.5\cos 8000t\,\text{mA}$$

P 9.40 **[a]** $$Z_p = \frac{\frac{R}{j\omega C}}{R + (1/j\omega C)} = \frac{R}{1 + j\omega RC}$$

$$= \frac{10{,}000}{1 + j(5000)(10{,}000)C} = \frac{10{,}000}{1 + j50 \times 10^6 C}$$

$$= \frac{10{,}000(1 - j50 \times 10^6 C)}{1 + 25 \times 10^{14} C^2}$$

$$= \frac{10{,}000}{1 + 25 \times 10^{14} C^2} - j\frac{5 \times 10^{11} C}{1 + 25 \times 10^{14} C^2}$$

$$j\omega L = j5000(0.8) = j4000$$

$$\therefore \quad 4000 = \frac{5 \times 10^{11} C}{1 + 25 \times 10^{14} C^2}$$

$$\therefore \quad 10^{14} C^2 - 125 \times 10^6 C + 1 = 0$$

$$\therefore \quad C^2 - 5 \times 10^{-8} C + 4 \times 10^{-16} = 0$$

Solving,

$C_1 = 40\,\text{nF} \qquad C_2 = 10\,\text{nF}$

[b] $$R_e = \frac{10{,}000}{1 + 25 \times 10^{14} C^2}$$

When $C = 40\,\text{nF} \qquad R_e = 2000\,\Omega$;

$$\mathbf{I}_g = \frac{80\underline{/0^\circ}}{2000} = 40\underline{/0^\circ}\,\text{mA}; \qquad i_g = 40\cos 5000t\,\text{mA}$$

When $C = 10\,\text{nF} \qquad R_e = 8000\,\Omega$;

$$\mathbf{I}_g = \frac{80\underline{/0^\circ}}{8000} = 10\underline{/0^\circ}\,\text{mA}; \qquad i_g = 10\cos 5000t\,\text{mA}$$

P 9.41 **[a]** $$Z_C = \frac{10^9}{j(50{,}000)(5)} = -j4000\,\Omega$$

$$Z_1 = 10{,}000 \| j50{,}000L = \frac{10{,}000(j50{,}000L)}{10{,}000 + j50{,}000L} = \frac{250{,}000L^2 + j50{,}000L}{1 + 25L^2}$$

$$Z_T = Z_1 + Z_R + Z_C = \frac{250{,}000L^2 + j50{,}000L}{1 + 25L^2} - j4000 + 2000$$

Z_T is resistive when

$$\frac{50{,}000L}{1 + 25L^2} = 4000 \qquad \text{or}$$

$$L^2 - 0.5L + 0.04 = 0$$

Solving, $L_1 = 0.4$ H and $L_2 = 0.1$ H.

[b] When $L = 0.4$ H:

$$Z_T = 2000 + \frac{250{,}000(0.16)}{1 + 25(0.16)} = 10{,}000\,\Omega$$

$$\mathbf{I}_g = \frac{50\underline{/0^\circ}}{10{,}000} = 5\underline{/0^\circ}\,\text{mA}$$

$$i_g = 5\cos 50{,}000t\,\text{mA}$$

When $L = 0.1$ H:

$$Z_T = 2000 + \frac{250{,}000(0.01)}{1 + 25(0.01)} = 4000\,\Omega$$

$$\mathbf{I}_g = \frac{50\underline{/0^\circ}}{4000} = 12.5\underline{/0^\circ}\,\text{mA}$$

$$i_g = 12.5\cos 50{,}000t\,\text{mA}$$

P 9.42 **[a]** $Y_1 = \dfrac{1}{5000} = 0.2 \times 10^{-3}\,\text{S}$

$$Y_2 = \frac{1}{1200 + j0.2\omega}$$

$$= \frac{1200}{1.44 \times 10^6 + 0.04\omega^2} - j\frac{0.2\omega}{1.44 \times 10^6 + 0.04\omega^2}$$

$$Y_3 = j\omega 50 \times 10^{-9}$$

$$Y_T = Y_1 + Y_2 + Y_3$$

For i_g and v_o to be in phase the j component of Y_T must be zero; thus,

$$\omega 50 \times 10^{-9} = \frac{0.2\omega}{1.44 \times 10^6 + 0.04\omega^2}$$

or

$$0.04\omega^2 + 1.44 \times 10^6 = \frac{0.2 \times 10^9}{50} = 4 \times 10^6$$

$$\therefore\ \ 0.04\omega^2 = 2.56 \times 10^6 \qquad \therefore\ \ \omega = 8000\,\text{rad/s} = 8\,\text{krad/s}$$

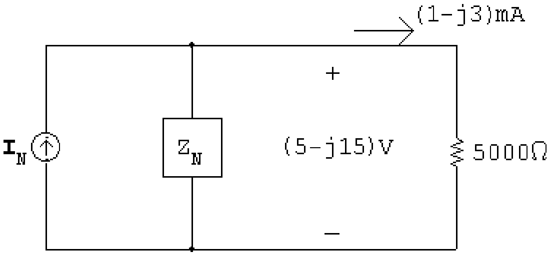
[b] $Y_T = 0.2 \times 10^{-3} + \dfrac{1200}{1.44 \times 10^6 + 0.04(64) \times 10^6} = 0.5 \times 10^{-3}\,\text{S}$

$$\therefore\ \ Z_T = 2000\,\Omega$$

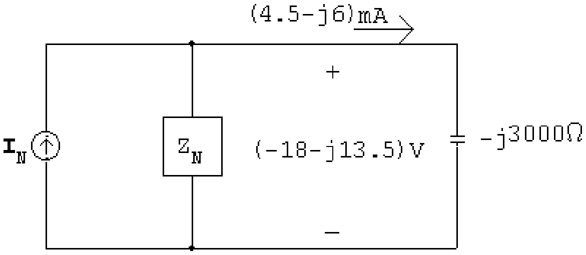
$$\mathbf{V}_o = (2.5 \times 10^{-3}\underline{/0^\circ})(2000) = 5\underline{/0^\circ}$$

$$v_o = 5\cos 8000t\,\text{V}$$

P 9.43



$$\mathbf{I}_N = \frac{5 - j15}{Z_{\rm N}} + (1 - j3)\,\text{mA}, \quad Z_N \text{ in k}\Omega$$

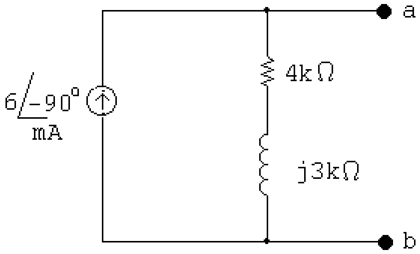


$$\mathbf{I}_N = \frac{-18 - j13.5}{Z_N} + 4.5 - j6\,\text{mA}, \quad Z_N \text{ in k}\Omega$$

$$\frac{5 - j15}{Z_N} + 1 - j3 = \frac{-18 - j13.5}{Z_N} + (4.5 - j6)$$

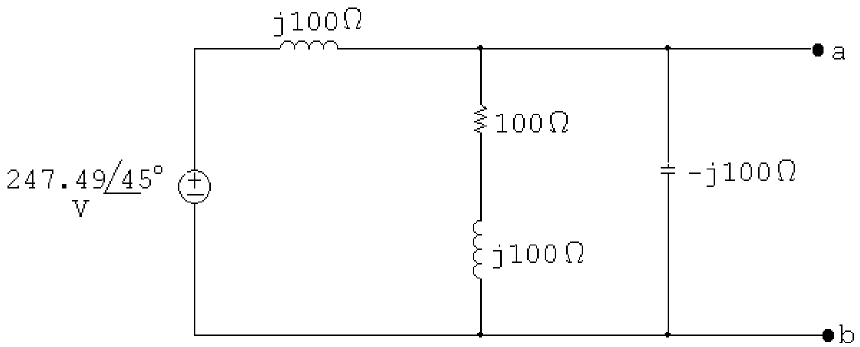
$$\frac{23 - j15}{Z_N} = 3.5 - j3 \quad \therefore \quad Z_N = 4 + j3\,\text{k}\Omega$$

$$\mathbf{I}_N = \frac{5 - j15}{4 + j3} + 1 - j3 = -j6\,\text{mA}$$



P 9.44 [**a**] $j\omega L = j(1000)(100) \times 10^{-3} = j100\,\Omega$

$$\frac{1}{j\omega C} = -j\frac{10^6}{(1000)(10)} = -j100\,\Omega$$



Using voltage division,

$$\mathbf{V}_{\rm ab} = \frac{(100+j100)\|(-j100)}{j100+(100+j100)\|(-j100)}(247.49\underline{/45^\circ}) = 350\underline{/0^\circ}$$

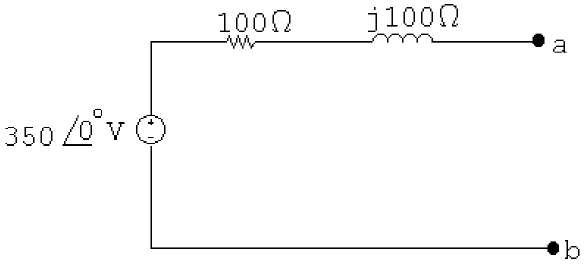
$$\mathbf{V}_{\rm Th} = \mathbf{V}_{\rm ab} = 350\underline{/0^\circ}\,\text{V}$$

[**b**] Remove the voltage source and combine impedances in parallel to find $Z_{\rm Th} = Z_{\rm ab}$:

$$Y_{\rm ab} = \frac{1}{j100} + \frac{1}{100+j100} + \frac{1}{-j100} = 5 - j5\text{ mS}$$

$$Z_{\rm Th} = Z_{\rm ab} = \frac{1}{Y_{\rm ab}} = 100 + j100\,\Omega$$

[**c**]



P 9.45 Step 1 to Step 2:

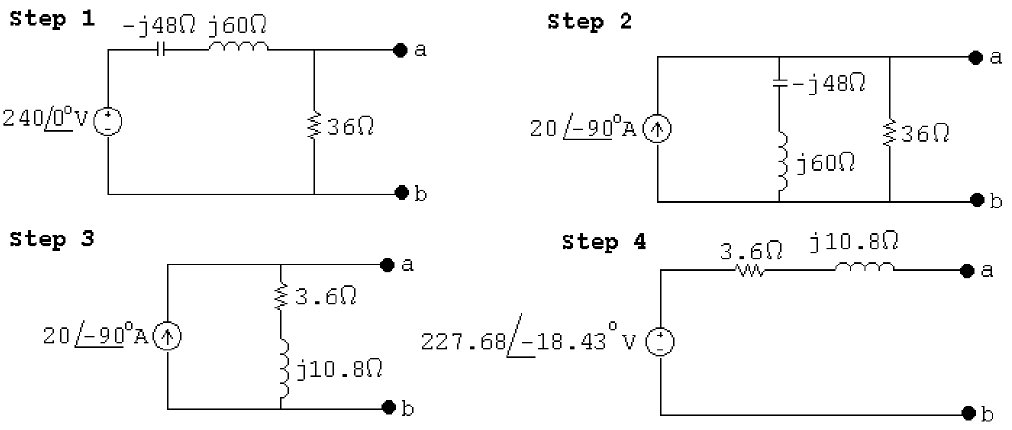
$$\frac{240\underline{/0^\circ}}{j12} = -j20 = 20\underline{/-90^\circ}\,\text{A}$$

Step 2 to Step 3:

$$(j12)\|36 = 3.6 + j10.8\,\Omega$$

Step 3 to Step 4:

$$(20\underline{/-90^\circ})(3.6+j10.8) = 216 - j72 = 227.68\underline{/-18.43^\circ}\,\text{V}$$



P 9.46 Step 1 to Step 2:

$$(4\underline{/0^\circ})(50) = 200\underline{/0^\circ}\,\text{V}$$

Step 2 to Step 3:

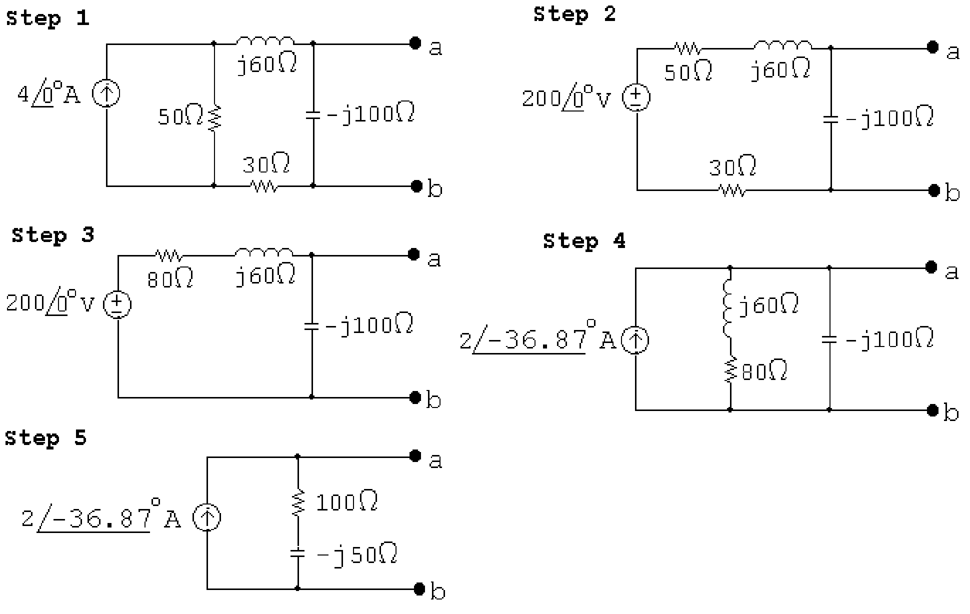
$$50 + 30 + j60 = (80 + j60)\,\Omega$$

Step 3 to Step 4:

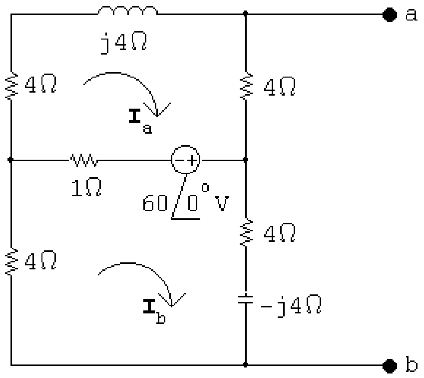
$$\frac{200\underline{/0^\circ}}{(80 + j60)} = 2\underline{/-36.87^\circ}\,\text{A}$$

Step 4 to Step 5:

$$(80 + j60 \| - j100 = 100 - j50\,\Omega$$



P 9.47 Open circuit voltage:



$$(9 + j4)\mathbf{I}_a - \mathbf{I}_b = -60\underline{/0^\circ}$$

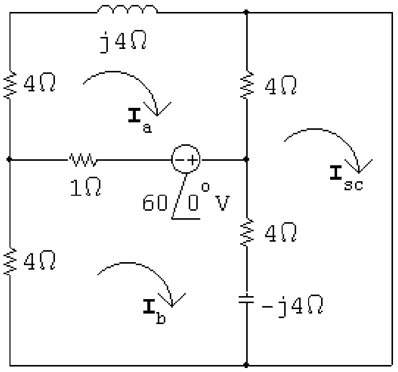
$$-\mathbf{I}_a + (9 - j4)\mathbf{I}_b = 60\underline{/0^\circ}$$

Solving,

$\mathbf{I}_a = -5 + j2.5\,\text{A}; \qquad \mathbf{I}_b = 5 + j2.5\,\text{A}$

$$\mathbf{V}_{\text{Th}} = 4\mathbf{I}_a + (4 - j4)\mathbf{I}_b = 10\underline{/0^\circ}\,\text{V}$$

Short circuit current:



$$(9 + j4)\mathbf{I}_a - 1\mathbf{I}_b - 4\mathbf{I}_{sc} = -60$$

$$-1\mathbf{I}_a + (9 - j4)\mathbf{I}_b - (4 - j4)\mathbf{I}_{sc} = 60$$

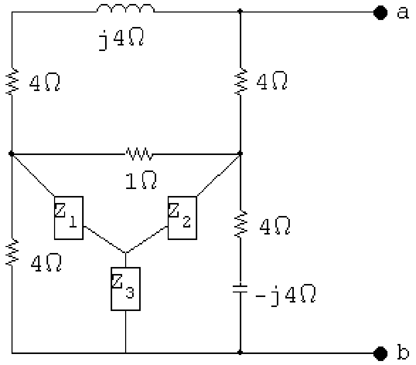
$$-4\mathbf{I}_a - (4 - j4)\mathbf{I}_b + (8 - j4)\mathbf{I}_{sc} = 0$$

Solving,

$$\mathbf{I}_{sc} = 2.07\underline{/0^\circ}$$

$$Z_{\text{Th}} = \frac{\mathbf{V}_{\text{Th}}}{\mathbf{I}_{\text{sc}}} = \frac{10\underline{/0^\circ}}{2.07\underline{/0^\circ}} = 4.83\,\Omega$$

Alternate calculation for Z_{Th}:

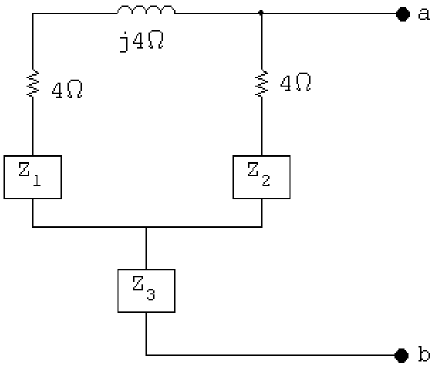


$$\sum Z = 4 + 1 + 4 - j4 = 9 - j4$$

$$Z_1 = \frac{4}{9 - j4}$$

$$Z_2 = \frac{4 - j4}{9 - j4}$$

$$Z_3 = \frac{16 - j16}{9 - j4}$$



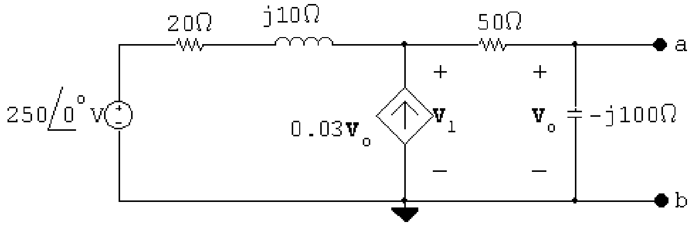
$$Z_{\text{a}} = 4 + j4 + \frac{4}{9 - j4} = \frac{56 + j20}{9 - j4}$$

$$Z_{\text{b}} = 4 + \frac{4 - j4}{9 - j4} = \frac{40 - j20}{9 - j4}$$

$$Z_{\text{a}} \| Z_{\text{b}} = \frac{2640 - j320}{884 - j384}$$

$$Z_3 + Z_{\text{a}} \| Z_{\text{b}} = \frac{16 - j16}{9 - j4} + \frac{2640 - j320}{884 - j384} = 4.83\,\Omega$$

P 9.48 Open circuit voltage:



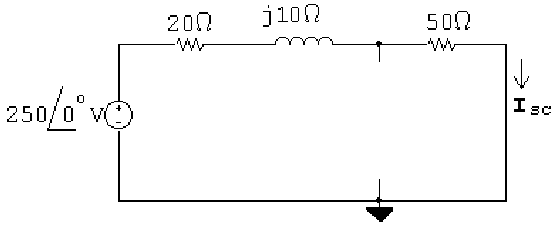
$$\frac{\mathbf{V}_1 - 250}{20 + j10} - 0.03\mathbf{V}_o + \frac{\mathbf{V}_1}{50 - j100} = 0$$

$$\therefore \quad \mathbf{V}_o = \frac{-j100}{50 - j100}\mathbf{V}_1$$

$$\frac{\mathbf{V}_1}{20 + j10} + \frac{j3\mathbf{V}_1}{50 - j100} + \frac{\mathbf{V}_1}{50 - j100} = \frac{250}{20 + j10}$$

$$\mathbf{V}_1 = 500 - j250\,\text{V}; \qquad \mathbf{V}_o = 300 - j400\,\text{V} = \mathbf{V}_{\text{Th}}$$

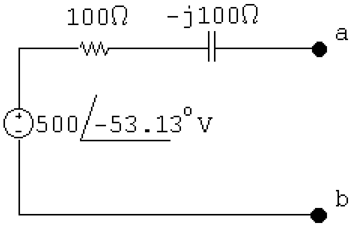
Short circuit current:



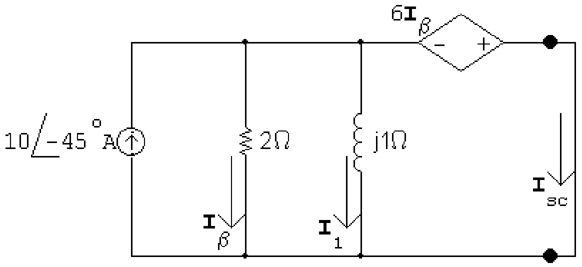
$$\mathbf{I}_{\text{sc}} = \frac{250\underline{/0^\circ}}{70 + j10} = 3.5 - j0.5\,\text{A}$$

$$Z_{\text{Th}} = \frac{\mathbf{V}_{\text{Th}}}{\mathbf{I}_{\text{sc}}} = \frac{300 - j400}{3.5 - j0.5} = 100 - j100\,\Omega$$

The Thévenin equivalent circuit:



P 9.49 Short circuit current

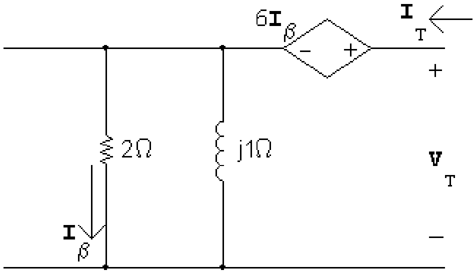


$$\mathbf{I}_\beta = \frac{-6\mathbf{I}_\beta}{2}$$

$$2\mathbf{I}_\beta = -6\mathbf{I}_\beta; \qquad \therefore\ \mathbf{I}_\beta = 0$$

$$\mathbf{I}_1 = 0; \qquad \therefore\ \mathbf{I}_{sc} = 10\underline{/-45^\circ}\,\text{A} = \mathbf{I}_N$$

The Norton impedance is the same as the Thévenin impedance. Find it using a test source

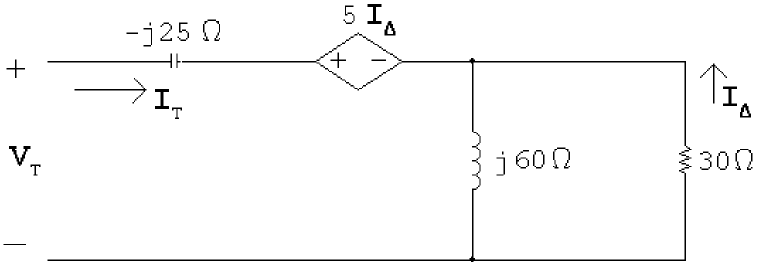


$$\mathbf{V}_T = 6\mathbf{I}_\beta + 2\mathbf{I}_\beta = 8\mathbf{I}_\beta, \qquad \mathbf{I}_\beta = \frac{j1}{2+j1}\mathbf{I}_T$$

$$Z_{\text{Th}} = \frac{\mathbf{V}_T}{\mathbf{I}_T} = \frac{8\mathbf{I}_\beta}{[(2+j1)/j1]\mathbf{I}_\beta} = \frac{j8}{2+j1} = 1.6 + j3.2\,\Omega$$

P 9.50 $j\omega L = j100 \times 10^3(0.6 \times 10^{-3}) = j60\,\Omega$

$$\frac{1}{j\omega C} = \frac{-j}{(100 \times 10^3)(0.4 \times 10^{-6})} = -j25\,\Omega$$



$$\mathbf{V}_T = -j25\mathbf{I}_T + 5\mathbf{I}_\Delta - 30\mathbf{I}_\Delta$$

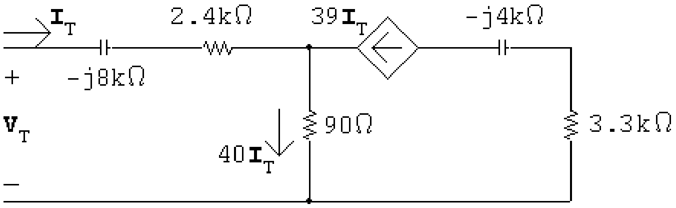
$$\mathbf{I}_\Delta = \frac{-j60}{30+j60}\mathbf{I}_T$$

$$\mathbf{V}_T = -j25\mathbf{I}_T + 25\frac{j60}{30+j60}\mathbf{I}_T$$

$$\frac{\mathbf{V}_T}{\mathbf{I}_T} = Z_{\text{ab}} = 20 - j15 = 25\underline{/-36.87^\circ}\,\Omega$$

P 9.51 $$\frac{1}{\omega C_1} = \frac{10^9}{50{,}000(2.5)} = 8\,\text{k}\Omega$$

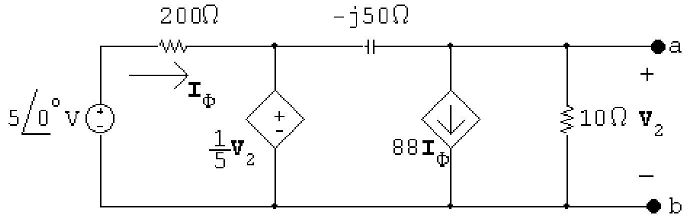
$$\frac{1}{\omega C_2} = \frac{10^9}{50{,}000(5)} = 4\,\text{k}\Omega$$



$$\mathbf{V}_T = (2400 - j8000)\mathbf{I}_T + 40\mathbf{I}_T(90)$$

$$Z_{\text{Th}} = \frac{\mathbf{V}_T}{\mathbf{I}_T} = 6000 - j8000\,\Omega$$

P 9.52 Open circuit voltage:



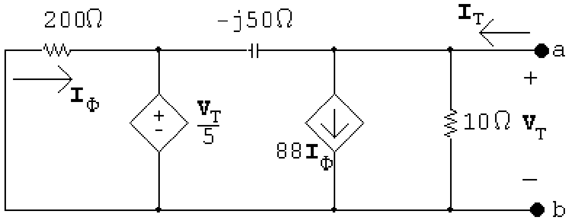
$$\frac{\mathbf{V}_2}{10} + 88\mathbf{I}_\phi + \frac{\mathbf{V}_2 - \frac{1}{5}\mathbf{V}_2}{-j50} = 0$$

$$\mathbf{I}_\phi = \frac{5 - (\mathbf{V}_2/5)}{200}$$

Solving,

$$\mathbf{V}_2 = -66 + j88 = 110\underline{/126.87^\circ}\,\text{V} = \mathbf{V}_{\text{Th}}$$

Find the Thévenin equivalent impedance using a test source:



$$\mathbf{I}_T = \frac{\mathbf{V}_T}{10} + 88\mathbf{I}_\phi + \frac{0.8\mathbf{V}_t}{-j50}$$

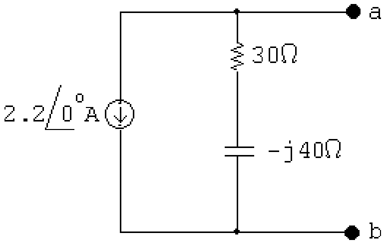
$$\mathbf{I}_\phi = \frac{-\mathbf{V}_T/5}{200}$$

$$\mathbf{I}_T = \mathbf{V}_T\left(\frac{1}{10} - 88\frac{\mathbf{V}_T/5}{200} + \frac{0.8}{-j50}\right)$$

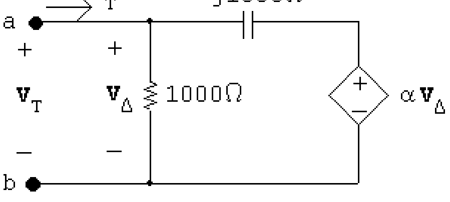
$$\therefore \qquad \frac{\mathbf{V}_T}{\mathbf{I}_T} = 30 - j40 = Z_{\text{Th}}$$

$$\mathbf{I}_{\text{N}} = \frac{\mathbf{V}_{\text{Th}}}{Z_{\text{Th}}} = \frac{-66 + j88}{30 - j40} = -2.2 + j0\,\text{A}$$

The Norton equivalent circuit:



P 9.53 [a]



$$\mathbf{I}_T = \frac{\mathbf{V}_T}{1000} + \frac{\mathbf{V}_T - \alpha\mathbf{V}_T}{-j1000}$$

$$\frac{\mathbf{I}_T}{\mathbf{V}_T} = \frac{1}{1000} - \frac{(1-\alpha)}{j1000} = \frac{j - 1 + \alpha}{j1000}$$

$$\therefore \quad Z_{\text{Th}} = \frac{\mathbf{V}_T}{\mathbf{I}_T} = \frac{j1000}{\alpha - 1 + j}$$

Z_{Th} is real when $\alpha = 1$.

[b] $Z_{\text{Th}} = 1000\,\Omega$

[c] $Z_{\text{Th}} = 500 - j500 = \dfrac{j1000}{\alpha - 1 + j}$

$$= \frac{1000}{(\alpha-1)^2+1} + j\frac{1000(\alpha-1)}{(\alpha-1)^2+1}$$

Equate the real parts:

$$\frac{1000}{(\alpha-1)^2+1} = 500 \quad \therefore \quad (\alpha-1)^2 + 1 = 2$$

$$\therefore \quad (\alpha-1)^2 = 1 \quad \text{so} \quad \alpha = 0$$

Check the imaginary parts:

$$\left.\frac{(\alpha-1)1000}{(\alpha-1)^2+1}\right|_{\alpha=1} = -500$$

Thus, $\alpha = 0$.

[d] $Z_{\text{Th}} = \dfrac{1000}{(\alpha-1)^2+1} + j\dfrac{1000(\alpha-1)}{(\alpha-1)^2+1}$

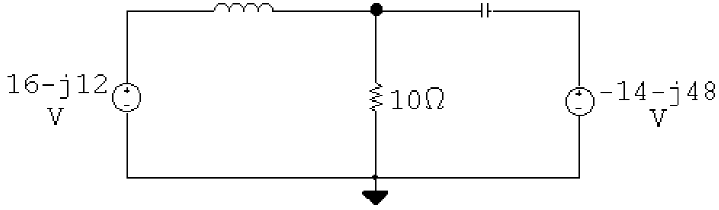
For $\mathbf{Im}(Z_{\text{Th}}) > 0$, α must be greater than 1. So Z_{Th} is inductive for $1 < \alpha \leq 10$.

P 9.54 $j\omega L = j(2000)(1 \times 10^{-3}) = j2\,\Omega$

$$\frac{1}{j\omega C} = -j\frac{10^6}{(2000)(100)} = -j5\,\Omega$$

$$\mathbf{V}_{g1} = 20\underline{/-36.87^\circ} = 16 - j12\,\text{V}$$

$$\mathbf{V}_{g2} = 50\underline{/-106.26^\circ} = -14 - j48\,\text{V}$$



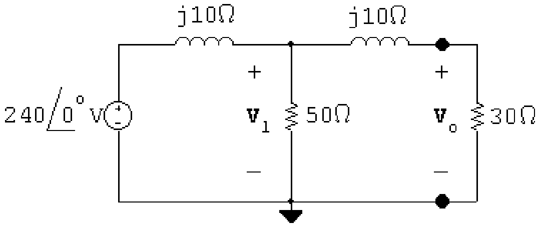
$$\frac{\mathbf{V}_o - (16 - j12)}{j2} + \frac{\mathbf{V}_o}{10} + \frac{\mathbf{V}_o - (-14 - j48)}{-j5} = 0$$

Solving,

$$\mathbf{V}_o = 36\underline{/0^\circ}$$

$$v_o(t) = 36\cos 2000t\,\text{V}$$

P 9.55



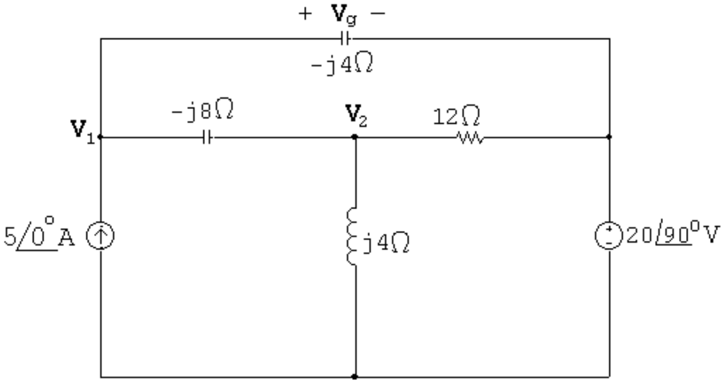
$$\frac{\mathbf{V}_1 - 240}{j10} + \frac{\mathbf{V}_1}{50} + \frac{\mathbf{V}_1}{30 + j10} = 0$$

Solving for $\mathbf{V}_1$ yields

$$\mathbf{V}_1 = 198.63\underline{/-24.44^\circ}\,\text{V}$$

$$\mathbf{V}_o = \frac{30}{30 + j10}(\mathbf{V}_1) = 188.43\underline{/-42.88^\circ}\,\text{V}$$

P 9.56 Set up the frequency domain circuit to use the node voltage method:



At $\mathbf{V}_1$: $$-5\underline{/0^\circ} + \frac{\mathbf{V}_1 - \mathbf{V}_2}{-j8} + \frac{\mathbf{V}_1 - 20\underline{/90^\circ}}{-j4} = 0$$

At $\mathbf{V}_2$: $$\frac{\mathbf{V}_2 - \mathbf{V}_1}{-j8} + \frac{\mathbf{V}_2}{j4} + \frac{\mathbf{V}_2 - 20\underline{/90^\circ}}{12} = 0$$

In standard form:

$$\mathbf{V}_1\left(\frac{1}{-j8}+\frac{1}{-j4}\right)+\mathbf{V}_2\left(-\frac{1}{-j8}\right)=5\underline{/0^\circ}+\frac{20\underline{/90^\circ}}{-j4}$$

$$\mathbf{V}_1\left(-\frac{1}{-j8}\right)+\mathbf{V}_2\left(\frac{1}{-j8}+\frac{1}{j4}+\frac{1}{12}\right)=\frac{20\underline{/90^\circ}}{12}$$

Solving on a calculator:

$$\mathbf{V}_1=-\frac{8}{3}+j\frac{4}{3} \qquad \mathbf{V}_2=-8+j4$$

Thus

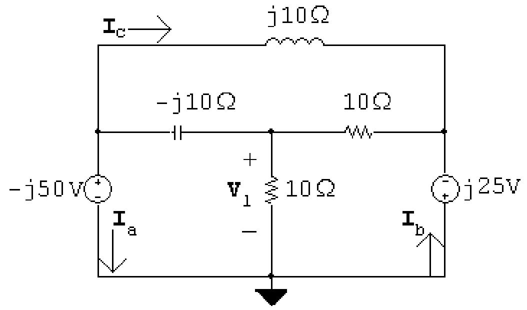
$$\mathbf{V}_g=\mathbf{V}_1-20\underline{/90^\circ}=-\frac{8}{3}-j\frac{56}{3}\,\text{V}$$

P 9.57 $j\omega L=j10^6(10\times10^{-6})=j10\,\Omega$

$$\frac{1}{j\omega C}=\frac{-j}{10^6(100\times10^{-9})}=-j10\,\Omega$$

$$\mathbf{V}_\text{a}=50\underline{/-90^\circ}=-j50\,\text{V}$$

$$\mathbf{V}_\text{b}=25\underline{/90^\circ}=j25\,\text{V}$$



$$\frac{\mathbf{V}_1}{10}+\frac{\mathbf{V}_1+j25}{10}+\frac{\mathbf{V}_1+j50}{-j10}=0$$

Solving,

$$\mathbf{V}_1=25\underline{/-53.13^\circ}\,\text{V}=15-j20\,\text{V}$$

$$\mathbf{I}_{\text{a}} = \frac{\mathbf{V}_1 + j50}{-j10} + \frac{-j25 + j50}{j10}$$

$$= -0.5 + j1.5 = 1.58\underline{/108.43^\circ}\,\text{A}$$

$$i_{\text{a}} = 1.58\cos(10^6 t + 108.43^\circ)\,\text{A}$$

$$\mathbf{I}_{\text{b}} = \frac{-j25 - \mathbf{V}_1}{10} + \frac{-j25 + j50}{j10}$$

$$= 1 - j0.5 = 1.12\underline{/-26.57^\circ}\,\text{A}$$

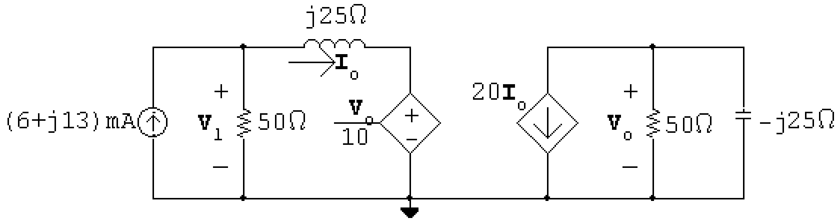
$$i_{\text{b}} = 1.12\cos(10^6 t - 26.57^\circ)\,\text{A}$$

$$\mathbf{I}_{\text{c}} = \frac{-j50 + j25}{j10}$$

$$= -2.5\,\text{A}$$

$$i_{\text{c}} = 2.5\cos(10^6 t + 180^\circ)\,\text{A}$$

P 9.58



$$\frac{\mathbf{V}_o}{50} + \frac{\mathbf{V}_o}{-j25} + 20\mathbf{I}_o = 0$$

$$(2 + j4)\mathbf{V}_o = -2000\mathbf{I}_o$$

$$\mathbf{V}_o = (-200 + j400)\mathbf{I}_o$$

$$\mathbf{I}_o = \frac{\mathbf{V}_1 - (\mathbf{V}_o/10)}{j25}$$

$$\therefore\quad \mathbf{V}_1 = (-20 + j65)\mathbf{I}_o$$

$$0.006 + j0.013 = \frac{\mathbf{V}_1}{50} + \mathbf{I}_o = (-0.4 + j1.3)\mathbf{I}_o + \mathbf{I}_o = (0.6 + j1.3)\mathbf{I}_o$$

$$\therefore\quad \mathbf{I}_o = \frac{0.6 + j1.3(10 \times 10^{-3})}{(0.6 + j1.3)} = 10\underline{/0^\circ}\,\text{mA}$$

$$\mathbf{V}_o = (-200 + j400)\mathbf{I}_o = -2 + j4 = 4.47\underline{/116.57^\circ}\,\text{V}$$

P 9.59 Write a KCL equation at the top node:

$$\frac{\mathbf{V}_o}{-j8} + \frac{\mathbf{V}_o - 2.4\mathbf{I}_\Delta}{j4} + \frac{\mathbf{V}_o}{5} - (10 + j10) = 0$$

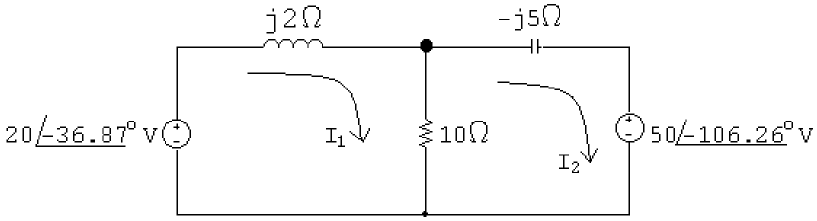
The constraint equation is:

$$\mathbf{I}_\Delta = \frac{\mathbf{V}_o}{-j8}$$

Solving,

$$\mathbf{V}_o = j80 = 80\underline{/90^\circ}\,\text{V}$$

P 9.60 The circuit with the mesh currents identified is shown below:



The mesh current equations are:

$$-20\underline{/-36.87^\circ} + j2\mathbf{I}_1 + 10(\mathbf{I}_1 - \mathbf{I}_2) = 0$$

$$50\underline{/-106.26^\circ} + 10(\mathbf{I}_2 - \mathbf{I}_1) - j5\mathbf{I}_2 = 0$$

In standard form:

$$\mathbf{I}_1(10 + j2) + \mathbf{I}_2(-10) = 20\underline{/-36.87^\circ}$$

$$\mathbf{I}_1(-10) + \mathbf{I}_2(10 - j5) = 50\underline{/-106.26^\circ}$$

Solving on a calculator yields:

$$\mathbf{I}_1 = -6 + j10\,\text{A}; \qquad \mathbf{I}_2 = -9.6 + j10\,\text{A}$$

Thus,

$$\mathbf{V}_o = 10(\mathbf{I}_1 - \mathbf{I}_2) = 36\,\text{V}$$

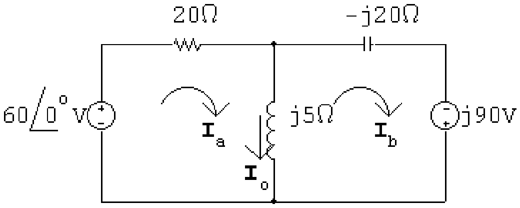
and

$$v_o(t) = 36\cos 2000t\,\text{V}$$

P 9.61 $\mathbf{V}_\mathrm{a} = 60\underline{/0^\circ}\,\mathrm{V}; \qquad \mathbf{V}_\mathrm{b} = 90\underline{/90^\circ}\,\mathrm{V}$

$$j\omega L = j(4\times 10^4)(125\times 10^{-6}) = j5\Omega$$

$$\frac{-j}{\omega C} = \frac{-j10^6}{40{,}000(1.25)} = -j20\,\Omega$$



$$60 = (20+j5)\mathbf{I}_\mathrm{a} - j5\mathbf{I}_\mathrm{b}$$

$$j90 = -j5\mathbf{I}_\mathrm{a} - j15\mathbf{I}_\mathrm{b}$$

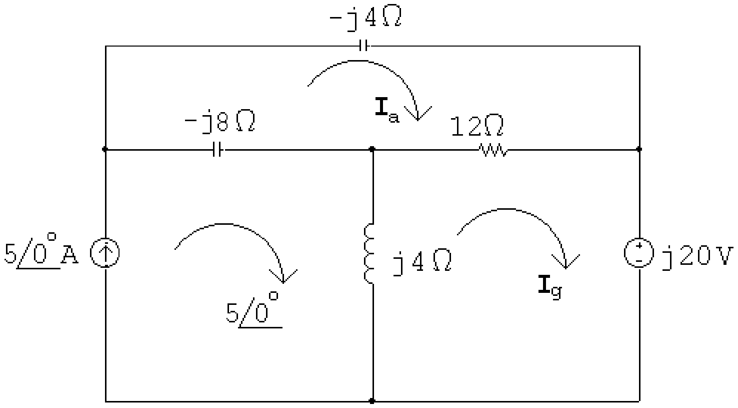
Solving,

$\mathbf{I}_\mathrm{a} = 2.25 - j2.25\,\mathrm{A}; \qquad \mathbf{I}_\mathrm{b} = -6.75 + j0.75\,\mathrm{A}$

$$\mathbf{I}_o = \mathbf{I}_\mathrm{a} - \mathbf{I}_\mathrm{b} = 9 - j3 = 9.49\underline{/-18.43^\circ}\,\mathrm{A}$$

$$i_o(t) = 9.49\cos(40{,}000t - 18.43^\circ)\,\mathrm{A}$$

P 9.62



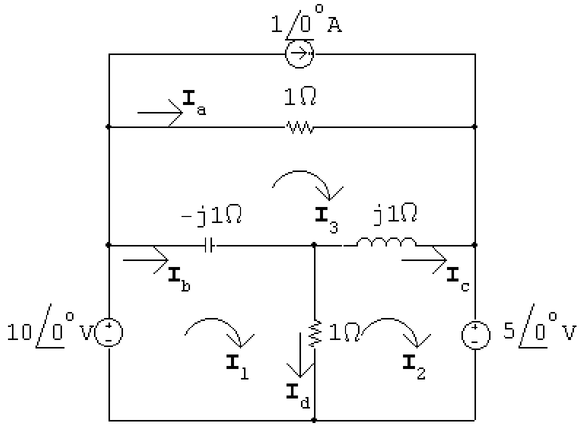
$$(12-j12)\mathbf{I}_\mathrm{a} - 12\mathbf{I}_g - 5(-j8) = 0$$

$$-12\mathbf{I}_\mathrm{a} + (12+j4)\mathbf{I}_g + j20 - 5(j4) = 0$$

Solving,

$$\mathbf{I}_g = 4 - j2 = 4.47\underline{/-26.57^\circ}\,\mathrm{A}$$

P 9.63



$$10\underline{/0^\circ} = (1 - j1)\mathbf{I}_1 - 1\mathbf{I}_2 + j1\mathbf{I}_3$$

$$-5\underline{/0^\circ} = -1\mathbf{I}_1 + (1 + j1)\mathbf{I}_2 - j1\mathbf{I}_3$$

$$1 = j1\mathbf{I}_1 - j1\mathbf{I}_2 + \mathbf{I}_3$$

Solving,

$$\mathbf{I}_1 = 11 + j10\,\text{A}; \qquad \mathbf{I}_2 = 11 + j5\,\text{A}; \qquad \mathbf{I}_3 = 6\,\text{A}$$

$$\mathbf{I}_\text{a} = \mathbf{I}_3 - 1 = 5\,\text{A}$$

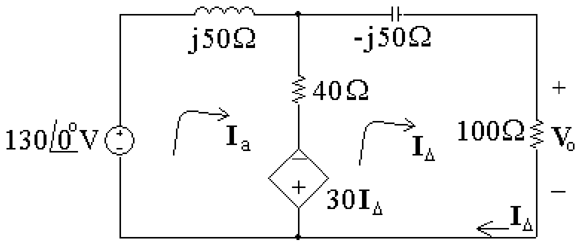
$$\mathbf{I}_\text{b} = \mathbf{I}_1 - \mathbf{I}_3 = 5 + j10\,\text{A}$$

$$\mathbf{I}_\text{c} = \mathbf{I}_2 - \mathbf{I}_3 = 5 + j5\,\text{A}$$

$$\mathbf{I}_\text{d} = \mathbf{I}_1 - \mathbf{I}_2 = j5\,\text{A}$$

P 9.64 $j\omega L = j10{,}000(5 \times 10^{-3}) = j50\,\Omega$

$$\frac{1}{j\omega C} = \frac{-j}{(10{,}000)(2 \times 10^{-6})} = -j50\,\Omega$$



$$130\underline{/0^\circ} = (40 + j50)\mathbf{I}_\text{a} - 40\mathbf{I}_\Delta + 30\mathbf{I}_\Delta$$

$$0 = -40\mathbf{I}_a + 30\mathbf{I}_\Delta + (140 - j50)\mathbf{I}_\Delta$$

Solving,

$$\mathbf{I}_\Delta = (400 - j400)\,\text{mA}$$

$$\mathbf{V}_o = 100\mathbf{I}_\Delta = 40 - j40 = 56.57\underline{/-45^\circ}$$

$$v_o = 56.57\cos(10{,}000t - 45^\circ)\,\text{V}$$

P 9.65 $$\frac{1}{j\omega C} = -j\frac{10^9}{(12{,}500)(800)} = -j100\,\Omega$$

$$j\omega L = j(12{,}500)(0.04) = j500\,\Omega$$

Let $Z_1 = 50 - j100\,\Omega$; $\qquad Z_2 = 250 + j500\,\Omega$

$$\mathbf{I}_g = 125\underline{/0^\circ}\,\text{mA}$$

$$\mathbf{I}_o = \frac{-\mathbf{I}_g Z_2}{Z_1 + Z_2} = \frac{-125\underline{/0^\circ}(250 + j500)}{(300 + j400)}$$

$$= -137.5 - j25\,\text{mA} = 139.75\underline{/-169.7^\circ}\,\text{mA}$$

$$i_o = 139.75\cos(12{,}500t - 169.7^\circ)\,\text{mA}$$

P 9.66 $$Z_o = 12{,}000 - j\frac{10^9}{(20{,}000)(3.125)} = 12{,}000 - j16{,}000\,\Omega$$

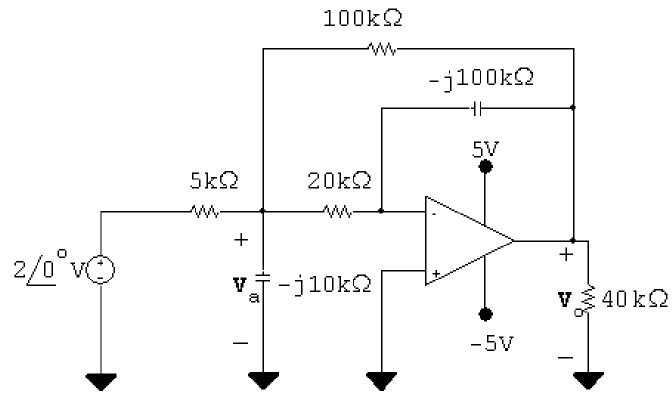
$$Z_T = 6000 + j40{,}000 + 12{,}000 - j16{,}000 = 18{,}000 + j24{,}000\,\Omega = 30{,}000\underline{/53.13^\circ}\,\Omega$$

$$\mathbf{V}_o = \mathbf{V}_g\frac{Z_o}{Z_T} = \frac{(75\underline{/0^\circ})(20{,}000\underline{/-53.13^\circ})}{30{,}000\underline{/53.13^\circ}} = 50\underline{/-106.26^\circ}\,\text{V}$$

$$v_o = 50\cos(20{,}000t - 106.26^\circ)\,\text{V}$$

P 9.67 $$\frac{1}{j\omega C_1} = -j10\,\text{k}\Omega$$

$$\frac{1}{j\omega C_2} = -j100\,\text{k}\Omega$$



$$\frac{\mathbf{V}_\mathrm{a}-2}{5000}+\frac{\mathbf{V}_\mathrm{a}}{-j10{,}000}+\frac{\mathbf{V}_\mathrm{a}}{20{,}000}+\frac{\mathbf{V}_\mathrm{a}-\mathbf{V}_o}{100{,}000}=0$$

$$20\mathbf{V}_\mathrm{a}-40+j10\mathbf{V}_\mathrm{a}+5\mathbf{V}_\mathrm{a}+\mathbf{V}_\mathrm{a}-\mathbf{V}_o=0$$

$$\therefore \qquad (26+j10)\mathbf{V}_\mathrm{a}-\mathbf{V}_o=40$$

$$\frac{0-\mathbf{V}_\mathrm{a}}{20{,}000}+\frac{0-\mathbf{V}_o}{-j100{,}000}=0$$

$$j5\mathbf{V}_\mathrm{a}-\mathbf{V}_o=0$$

Solving,

$$\mathbf{V}_o=1.43+j7.42=7.56\underline{/79.09^\circ}\,\mathrm{V}$$

$$v_o(t)=7.56\cos(10^6t+79.09^\circ)\,\mathrm{V}$$

P 9.68 [a] $\mathbf{V}_g=25\underline{/0^\circ}\,\mathrm{V}$

$$\mathbf{V}_\mathrm{p}=\frac{20}{100}\mathbf{V}_g=5\underline{/0^\circ}; \qquad \mathbf{V}_\mathrm{n}=\mathbf{V}_\mathrm{p}=5\underline{/0^\circ}\,\mathrm{V}$$

$$\frac{5}{80{,}000}+\frac{5-\mathbf{V}_o}{Z_\mathrm{p}}=0$$

$$Z_\mathrm{p}=-j80{,}000\|40{,}000=32{,}000-j16{,}000\,\Omega$$

$$\mathbf{V}_o=\frac{5Z_\mathrm{p}}{80{,}000}+5=7-j=7.07\underline{/-8.13^\circ}$$

$$v_o=7.07\cos(50{,}000t-8.13^\circ)\,\mathrm{V}$$

[b] $\mathbf{V}_\mathrm{p} = 0.2V_m\underline{/0^\circ}; \qquad \mathbf{V}_\mathrm{n} = \mathbf{V}_\mathrm{p} = 0.2V_m\underline{/0^\circ}$

$$\frac{0.2V_m}{80{,}000} + \frac{0.2V_m - \mathbf{V}_o}{32{,}000 - j16{,}000} = 0$$

$$\therefore \quad \mathbf{V}_o = 0.2V_m + \frac{32{,}000 - j16{,}000}{80{,}000}V_m(0.2) = V_m(0.28 - j0.04)$$

$$\therefore \quad |V_m(0.28 - j0.04)| \leq 10$$

$$\therefore \quad V_m \leq 35.36\,\mathrm{V}$$

P 9.69 $\mathbf{V}_g = 4\underline{/0^\circ}\,\mathrm{V}; \qquad \dfrac{1}{j\omega C} = -j20\,\mathrm{k}\Omega$

Let $\mathbf{V}_\mathrm{a}$ = voltage across the capacitor, positive at upper terminal
Then:

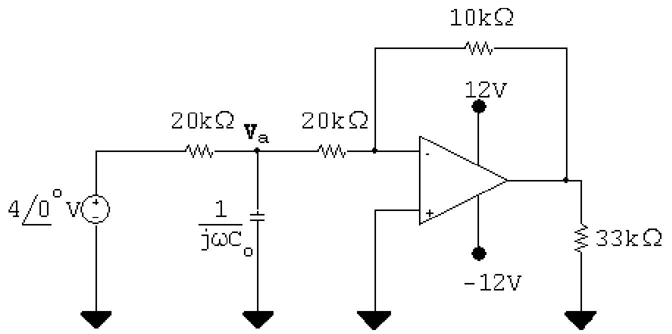
$$\frac{\mathbf{V}_\mathrm{a} - 4\underline{/0^\circ}}{20{,}000} + \frac{\mathbf{V}_\mathrm{a}}{-j20{,}000} + \frac{\mathbf{V}_\mathrm{a}}{20{,}000} = 0; \qquad \therefore \quad \mathbf{V}_\mathrm{a} = (1.6 - j0.8)\,\mathrm{V}$$

$$\frac{0 - \mathbf{V}_\mathrm{a}}{20{,}000} + \frac{0 - \mathbf{V}_o}{10{,}000} = 0; \qquad \mathbf{V}_o = -\frac{\mathbf{V}_\mathrm{a}}{2}$$

$$\therefore \quad \mathbf{V}_o = -0.8 + j0.4 = 0.89\underline{/153.43^\circ}\,\mathrm{V}$$

$$v_o = 0.89\cos(200t + 153.43^\circ)\,\mathrm{V}$$

P 9.70 **[a]**



$$\frac{\mathbf{V}_\mathrm{a} - 4\underline{/0^\circ}}{20{,}000} + j\omega C_o\mathbf{V}_\mathrm{a} + \frac{\mathbf{V}_\mathrm{a}}{20{,}000} = 0$$

$$\mathbf{V}_\mathrm{a} = \frac{4}{2 + j20{,}000\omega C_o}$$

$$\mathbf{V}_o = -\frac{\mathbf{V}_\mathrm{a}}{2}$$

$$\mathbf{V}_o = \frac{-2}{2 + j4 \times 10^6 C_o} = \frac{2\underline{/180^\circ}}{2 + j4 \times 10^6 C_o}$$

$\therefore$ denominator angle $= 45^\circ$

so $4 \times 10^6 C_o = 2 \quad \therefore \quad C = 0.5\,\mu\text{F}$

[b] $\mathbf{V}_o = \frac{2\underline{/180^\circ}}{2 + j2} = 0.707\underline{/135^\circ}\text{ V}$

$v_o = 0.707\cos(200t + 135^\circ)\text{ V}$

P 9.71 **[a]** $\frac{1}{j\omega C} = \frac{-j10^9}{(10^6)(10)} = -j100\,\Omega$

$\mathbf{V}_g = 30\underline{/0^\circ}\text{ V}$

$$\mathbf{V}_\text{p} = \frac{\mathbf{V}_g(1/j\omega C_o)}{25 + (1/j\omega C_o)} = \frac{30\underline{/0^\circ}}{1 + j25\omega C_o} = \mathbf{V}_\text{n}$$

$$\frac{\mathbf{V}_\text{n}}{100} + \frac{\mathbf{V}_\text{n} - \mathbf{V}_o}{-j100} = 0$$

$$\mathbf{V}_o = \frac{1 + j1}{j}\mathbf{V}_\text{n} = (1 - j1)\mathbf{V}_\text{n} = \frac{30(1 - j1)}{1 + j25\omega C_o}$$

$$|\mathbf{V}_o| = \frac{30\sqrt{2}}{\sqrt{1 + 625\omega^2 C_o^2}} = 6$$

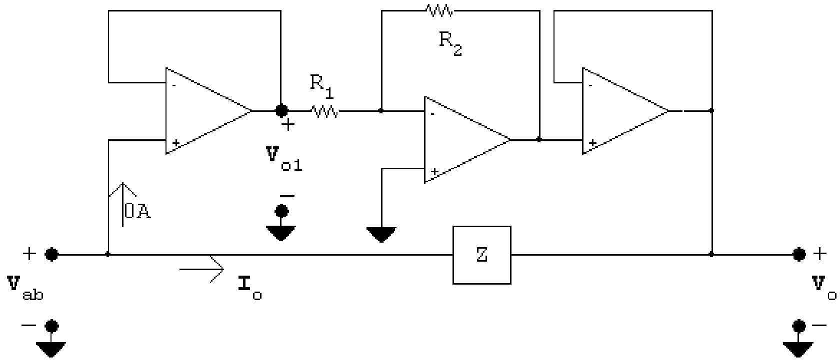
Solving,

$C_o = 280\,\text{nF}$

[b] $\mathbf{V}_o = \frac{30(1 - j1)}{1 + j7} = 6\underline{/-126.87^\circ}$

$v_o = 6\cos(10^6 t - 126.87^\circ)\text{ V}$

P 9.72 **[a]**



Because the op-amps are ideal $\mathbf{I}_{\text{in}} = \mathbf{I}_o$, thus

$$Z_{\text{ab}} = \frac{\mathbf{V}_{\text{ab}}}{\mathbf{I}_{\text{in}}} = \frac{\mathbf{V}_{\text{ab}}}{\mathbf{I}_o}; \qquad \mathbf{I}_o = \frac{\mathbf{V}_{\text{ab}} - \mathbf{V}_o}{Z}$$

$$\mathbf{V}_{o1} = \mathbf{V}_{\text{ab}}; \qquad \mathbf{V}_{o2} = -\left(\frac{R_2}{R_1}\right)\mathbf{V}_{o1} = -K\mathbf{V}_{o1} = -K\mathbf{V}_{\text{ab}}$$

$$\mathbf{V}_o = \mathbf{V}_{o2} = -K\mathbf{V}_{\text{ab}}$$

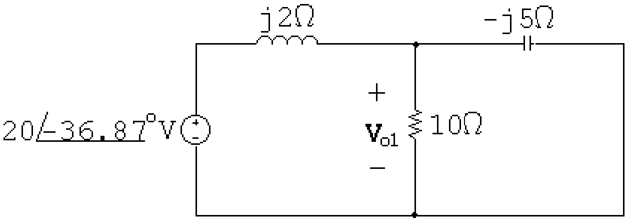
$$\therefore \quad \mathbf{I}_o = \frac{\mathbf{V}_{\text{ab}} - (-K\mathbf{V}_{\text{ab}})}{Z} = \frac{(1+K)\mathbf{V}_{\text{ab}}}{Z}$$

$$\therefore \quad Z_{\text{ab}} = \frac{\mathbf{V}_{\text{ab}}}{(1+K)\mathbf{V}_{\text{ab}}}Z = \frac{Z}{(1+K)}$$

[b] $Z = \dfrac{1}{j\omega C}; \qquad Z_{\text{ab}} = \dfrac{1}{j\omega C(1+K)}; \qquad \therefore \quad C_{\text{ab}} = C(1+K)$

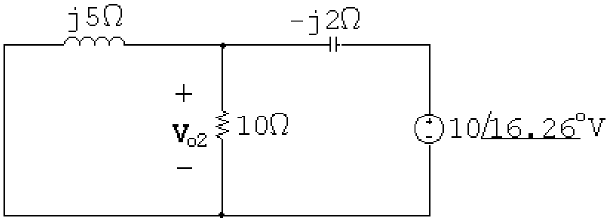
P 9.73 [a] Superposition must be used because the frequencies of the two sources are different.

[b] For $\omega = 2000$ rad/s:



$$10 \| -j5 = 2 - j4\,\Omega \quad \text{so} \quad \mathbf{V}_{o1} = \frac{2-j4}{2-j4+j2}(20\underline{/-36.87^\circ} = 31.62\underline{/-55.3^\circ}\text{ V}$$

For $\omega = 5000$ rad/s:



$$j5\|10 = 2 + j4\,\Omega$$

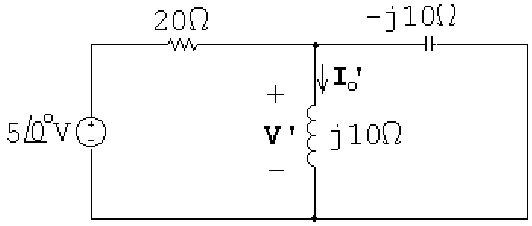
$$\mathbf{V}_{o2} = \frac{2+j4}{2+j4-j2}(10\underline{/16.26^\circ}) = 15.81\underline{/34.69^\circ}\text{ V}$$

Thus,

$$v_o(t) = [31.62\cos(2000t - 55.3^\circ) + 15.81\cos(5000t + 34.69^\circ)]\text{ V}, \quad t \geq 0$$

P 9.74 [**a**] Superposition must be used because the frequencies of the two sources are different.

[**b**] For $\omega = 80{,}000$ rad/s:



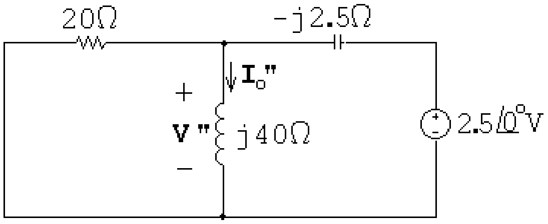
$$\frac{\mathbf{V}_o' - 5}{20} + \frac{\mathbf{V}_o'}{j10} + \frac{\mathbf{V}_o'}{-j10} = 0$$

$$\mathbf{V}_o'\left(\frac{1}{20} + \frac{1}{j10} + \frac{1}{-j10}\right) = \frac{5}{20}$$

$$\therefore \quad \mathbf{V}_o' = 5\underline{/0^\circ}\,\text{V}$$

$$\mathbf{I}_o' = \frac{\mathbf{V}_o'}{j10} = -j0.5 = 500\underline{/-90^\circ}\,\text{mA}$$

For $\omega = 320{,}000$ rad/s:



$$20\|j40 = 16 + j8\,\Omega$$

$$\mathbf{V}'' = \frac{16 + j8}{16 + j8 - j2.5}(2.5\underline{/0^\circ}) = 2.643\underline{/7.59^\circ}\,\text{V}$$

$$\therefore \quad \mathbf{I}_o'' = \frac{\mathbf{V}''}{j40} = 66.08\underline{/-82.4^\circ}\,\text{mA}$$

Thus,

$$i_o(t) = [500\sin 80{,}000t + 66.08\cos(320{,}000t - 82.4^\circ)]\,\text{mA}, \quad t \geq 0$$

P 9.75 [**a**] $j\omega L_L = j100\,\Omega$

$$j\omega L_2 = j500\,\Omega$$

$$Z_{22} = 300 + 500 + j100 + j500 = 800 + j600\,\Omega$$

$$Z_{22}^* = 800 - j600\,\Omega$$

$$\omega M = 270\,\Omega$$

$$Z_r = \left(\frac{270}{1000}\right)^2 [800 - j600] = 58.32 - j43.74\,\Omega$$

[b] $Z_{\text{ab}} = R_1 + j\omega L_1 + Z_r = 41.68 + j180 + 58.32 - j43.74 = 100 + j136.26\,\Omega$

P 9.76 **[a]** $j\omega L_1 = j(200 \times 10^3)(10^{-3}) = j200\,\Omega$

$$j\omega L_2 = j(200 \times 10^3)(4 \times 10^{-3}) = j800\,\Omega$$

$$\frac{1}{j\omega C} = \frac{-j}{(200 \times 10^3)(12.5 \times 10^{-9})} = -j400\,\Omega$$

$$\therefore \quad Z_{22} = 100 + 200 + j800 - j400 = 300 + j400\,\Omega$$

$$\therefore \quad Z_{22}^* = 300 - j400\,\Omega$$

$$M = k\sqrt{L_1 L_2} = 2k \times 10^{-3}$$

$$\omega M = (200 \times 10^3)(2k \times 10^{-3}) = 400k$$

$$Z_r = \left[\frac{400k}{500}\right]^2 (300 - j400) = k^2(192 - j256)\,\Omega$$

$$Z_{\text{in}} = 200 + j200 + 192k^2 - j256k^2$$

$$|Z_{\text{in}}| = [(200 + 192k)^2 + (200 - 256k)^2]^{\frac{1}{2}}$$

$$\frac{d|Z_{\text{in}}|}{dk} = \frac{1}{2}[(200 + 192k)^2 + (200 - 256k)^2]^{-\frac{1}{2}} \times$$

$$[2(200 + 192k^2)384k + 2(200 - 256k^2)(-512k)]$$

$$\frac{d|Z_{\text{in}}|}{dk} = 0 \text{ when}$$

$$768k(200 + 192k^2) - 1024k(200 - 256k^2) = 0$$

$$\therefore \quad k^2 = 0.125; \qquad \therefore \quad k = \sqrt{0.125} = 0.3536$$

[b] $Z_{\text{in}}\ (\text{min}) = 200 + 192(0.125) + j[200 - 0.125(256)]$

$$= 224 + j168 = 280\underline{/36.87^\circ}\,\Omega$$

$$\mathbf{I}_1\ (\text{max}) = \frac{560\underline{/0^\circ}}{224 + j168} = 2\underline{/-36.87^\circ}\,\text{A}$$

$$\therefore \quad i_1\ (\text{peak}) = 2\,\text{A}$$

Note — You can test that the k value obtained from setting $d|Z_{\text{in}}|/dt = 0$ leads to a minimum by noting $0 \leq k \leq 1$. If $k = 1$,

$$Z_{\text{in}} = 392 - j56 = 395.98\underline{/-8.13^\circ}\,\Omega$$

Thus,

$$|Z_{\text{in}}|_{k=1} > |Z_{\text{in}}|_{k=\sqrt{0.125}}$$

If $k = 0$,

$$Z_{\text{in}} = 200 + j200 = 282.84\underline{/45^\circ}\,\Omega$$

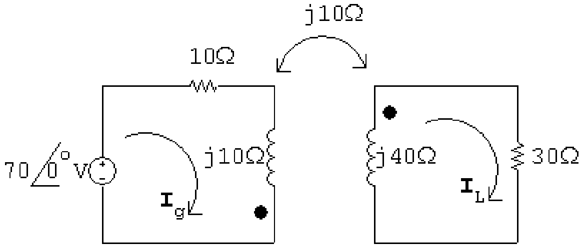
Thus,

$$|Z_{\text{in}}|_{k=0} > |Z_{\text{in}}|_{k=\sqrt{0.125}}$$

P 9.77 **[a]** $j\omega L_1 = j(5000)(2 \times 10^{-3}) = j10\,\Omega$

$$j\omega L_2 = j(5000)(8 \times 10^{-3}) = j40\,\Omega$$

$$j\omega M = j10\,\Omega$$



$$70 = (10 + j10)\mathbf{I}_g + j10\mathbf{I}_L$$

$$0 = j10\mathbf{I}_g + (30 + j40)\mathbf{I}_L$$

Solving,

$$\mathbf{I}_g = 4 - j3\,\text{A}; \qquad \mathbf{I}_L = -1\,\text{A}$$

$$i_g = 5\cos(5000t - 36.87^\circ)\,\text{A}$$

$$i_L = 1\cos(5000t - 180^\circ)\,\text{A}$$

[b] $k = \dfrac{M}{\sqrt{L_1 L_2}} = \dfrac{2}{\sqrt{16}} = 0.5$

[c] When $t = 100\pi\,\mu$s,

$$5000t = (5000)(100\pi) \times 10^{-6} = 0.5\pi = \pi/2\,\text{rad} = 90^\circ$$

$$i_g(100\pi\mu s) = 5\cos(53.13^\circ) = 3\,\text{A}$$

$$i_L(100\pi\mu s) = 1\cos(-90^\circ) = 0\,\text{A}$$

$$w = \frac{1}{2}L_1 i_1^2 + \frac{1}{2}L_2 i_2^2 + M i_1 i_2 = \frac{1}{2}(2 \times 10^{-3})(9) + 0 + 0 = 9\,\mathrm{mJ}$$

When $t = 200\pi\ \mu\mathrm{s}$,

$$5000t = \pi\,\mathrm{rad} = 180°$$

$$i_g(200\pi\mu s) = 5\cos(180 - 53.13) = -4\,\mathrm{A}$$

$$i_L(200\pi\mu s) = 1\cos(180 - 180) = 1\,\mathrm{A}$$

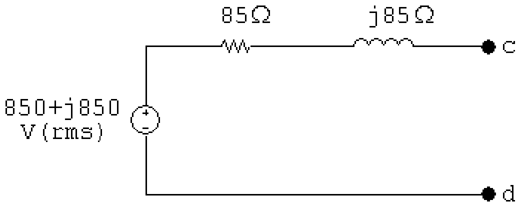
$$w = \frac{1}{2}(2 \times 10^{-3})(16) + \frac{1}{2}(8 \times 10^{-3})(1) + 2 \times 10^{-3}(-4)(1) = 12\,\mathrm{mJ}$$

P 9.78 Remove the voltage source to find the equivalent impedance:

$$Z_{\mathrm{Th}} = 45 + j125 + \left(\frac{20}{|5 + j5|}\right)^2 (5 + j5) = 85 + j85\,\Omega$$

Using voltage division:

$$\mathbf{V}_{\mathrm{Th}} = \mathbf{V}_{\mathrm{cd}} = j20\mathbf{I}_1 = j20\left(\frac{425}{5 + j5}\right) = 850 + j850\,\mathrm{V}$$



P 9.79 $j\omega L_1 = j50\,\Omega$

$$j\omega L_2 = j32\,\Omega$$

$$\frac{1}{j\omega C} = -j20\,\Omega$$

$$j\omega M = j(4 \times 10^3)k\sqrt{(12.5)(8)} \times 10^{-3} = j40k\,\Omega$$

$$Z_{22} = 5 + j32 - j20 = 5 + j12\,\Omega$$

$$Z_{22}^* = 5 - j12\,\Omega$$

$$Z_r = \left[\frac{40k}{|5 + j12|}\right]^2 (5 - j12) = 47.337k^2 - j113.609k^2$$

$Z_{\text{ab}} = 20 + j50 + 47.337k^2 - j113.609k^2 = (20 + 47.337k^2) + j(50 - 113.609k^2)$

Z_{ab} is resistive when

$50 - 113.609k^2 = 0 \quad \text{or} \quad k^2 = 0.44 \quad \text{so} \quad k = 0.66$

$\therefore \quad Z_{\text{ab}} = 20 + (47.337)(0.44) = 40.83\,\Omega$

P 9.80 In Eq. 9.69 replace ω^2M^2 with $k^2\omega^2L_1L_2$ and then write X_{ab} as

$$\begin{aligned} X_{\text{ab}} &= \omega L_1 - \frac{k^2\omega^2L_1L_2(\omega L_2 + \omega L_L)}{R_{22}^2 + (\omega L_2 + \omega L_L)^2} \\ &= \omega L_1\left\{1 - \frac{k^2\omega L_2(\omega L_2 + \omega L_L)}{R_{22}^2 + (\omega L_2 + \omega L_L)^2}\right\} \end{aligned}$$

For X_{ab} to be negative requires

$R_{22}^2 + (\omega L_2 + \omega L_L)^2 < k^2\omega L_2(\omega L_2 + \omega L_L)$

or

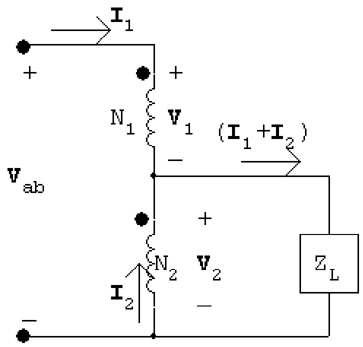
$R_{22}^2 + (\omega L_2 + \omega L_L)^2 - k^2\omega L_2(\omega L_2 + \omega L_L) < 0$

which reduces to

$R_{22}^2 + \omega^2L_2^2(1 - k^2) + \omega L_2\omega L_L(2 - k^2) + \omega^2L_L^2 < 0$

But $k \leq 1$, so it is impossible to satisfy the inequality. Therefore X_{ab} can never be negative if X_L is an inductive reactance.

P 9.81 [a]



$$Z_{\text{ab}} = \frac{\mathbf{V}_{\text{ab}}}{\mathbf{I}_1} = \frac{\mathbf{V}_1 + \mathbf{V}_2}{\mathbf{I}_1}$$

$$\frac{\mathbf{V}_1}{N_1} = \frac{\mathbf{V}_2}{N_2}, \qquad \mathbf{V}_2 = \frac{N_2}{N_1}\mathbf{V}_1$$

$$N_1\mathbf{I}_1 = N_2\mathbf{I}_2, \qquad \mathbf{I}_2 = \frac{N_1}{N_2}\mathbf{I}_1$$

$$\mathbf{V}_2 = (\mathbf{I}_1 + \mathbf{I}_2)Z_L = \mathbf{I}_1\left(1 + \frac{N_1}{N_2}\right)Z_L$$

$$\mathbf{V}_1 + \mathbf{V}_2 = \left(\frac{N_1}{N_2} + 1\right)\mathbf{V}_2 = \left(1 + \frac{N_1}{N_2}\right)^2 Z_L\mathbf{I}_1$$

$$\therefore \quad Z_{\text{ab}} = \frac{(1 + N_1/N_2)^2 Z_L\mathbf{I}_1}{\mathbf{I}_1}$$

$$Z_{\text{ab}} = \left(1 + \frac{N_1}{N_2}\right)^2 Z_L \quad \text{Q.E.D.}$$

[**b**] Assume dot on N_2 is moved to the lower terminal, then

$$\frac{\mathbf{V}_1}{N_1} = \frac{-\mathbf{V}_2}{N_2}, \qquad \mathbf{V}_1 = \frac{-N_1}{N_2}\mathbf{V}_2$$

$$N_1\mathbf{I}_1 = -N_2\mathbf{I}_2, \qquad \mathbf{I}_2 = \frac{-N_1}{N_2}\mathbf{I}_1$$

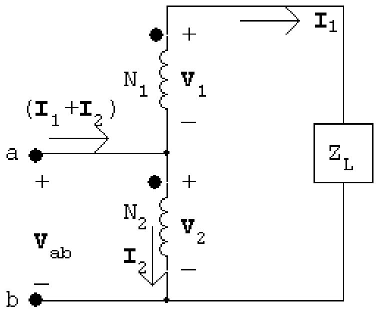
As in part [a]

$$\mathbf{V}_2 = (\mathbf{I}_2 + \mathbf{I}_1)Z_L \quad \text{and} \quad Z_{\text{ab}} = \frac{\mathbf{V}_1 + \mathbf{V}_2}{\mathbf{I}_1}$$

$$Z_{\text{ab}} = \frac{(1 - N_1/N_2)\mathbf{V}_2}{\mathbf{I}_1} = \frac{(1 - N_1/N_2)(1 - N_1/N_2)Z_L\mathbf{I}_1}{\mathbf{I}_1}$$

$$Z_{\text{ab}} = [1 - (N_1/N_2)]^2 Z_L \quad \text{Q.E.D.}$$

P 9.82 [**a**]



$$N_1\mathbf{I}_1 = N_2\mathbf{I}_2, \qquad \mathbf{I}_2 = \frac{N_1}{N_2}\mathbf{I}_1$$

$$Z_{\text{ab}} = \frac{\mathbf{V}_{\text{ab}}}{\mathbf{I}_1 + \mathbf{I}_2} = \frac{\mathbf{V}_2}{\mathbf{I}_1 + \mathbf{I}_2} = \frac{\mathbf{V}_2}{(1 + N_1/N_2)\mathbf{I}_1}$$

$$\frac{\mathbf{V}_1}{\mathbf{V}_2} = \frac{N_1}{N_2}, \qquad \mathbf{V}_1 = \frac{N_1}{N_2}\mathbf{V}_2$$

$$\mathbf{V}_1 + \mathbf{V}_2 = Z_L\mathbf{I}_1 = \left(\frac{N_1}{N_2} + 1\right)\mathbf{V}_2$$

$$Z_{\text{ab}} = \frac{\mathbf{I}_1 Z_L}{(N_1/N_2 + 1)(1 + N_1/N_2)\mathbf{I}_1}$$

$$\therefore \quad Z_{\text{ab}} = \frac{Z_L}{[1 + (N_1/N_2)]^2} \quad \text{Q.E.D.}$$

[b] Assume dot on the N_2 coil is moved to the lower terminal. Then

$$\mathbf{V}_1 = -\frac{N_1}{N_2}\mathbf{V}_2 \quad \text{and} \quad \mathbf{I}_2 = -\frac{N_1}{N_2}\mathbf{I}_1$$

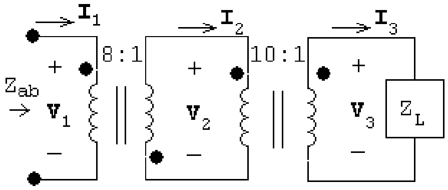
As before

$$Z_{\text{ab}} = \frac{\mathbf{V}_2}{\mathbf{I}_1 + \mathbf{I}_2} \quad \text{and} \quad \mathbf{V}_1 + \mathbf{V}_2 = Z_L\mathbf{I}_1$$

$$\therefore \quad Z_{\text{ab}} = \frac{\mathbf{V}_2}{(1 - N_1/N_2)\mathbf{I}_1} = \frac{Z_L\mathbf{I}_1}{[1 - (N_1/N_2)]^2\mathbf{I}_1}$$

$$Z_{\text{ab}} = \frac{Z_L}{[1 - (N_1/N_2)]^2} \quad \text{Q.E.D.}$$

P 9.83



$$Z_L = \frac{\mathbf{V}_3}{\mathbf{I}_3}$$

$$\frac{\mathbf{V}_2}{10} = \frac{\mathbf{V}_3}{1}; \qquad 10\mathbf{I}_2 = 1\mathbf{I}_3$$

$$\frac{\mathbf{V}_1}{8} = -\frac{\mathbf{V}_2}{1}; \qquad 8\mathbf{I}_1 = -1\mathbf{I}_2$$

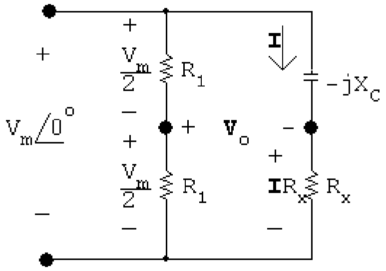
$$Z_{\text{ab}} = \frac{\mathbf{V}_1}{\mathbf{I}_1}$$

Substituting,

$$Z_{\text{ab}} = \frac{\mathbf{V}_1}{\mathbf{I}_1} = \frac{-8\mathbf{V}_2}{-\mathbf{I}_2/8} = \frac{8^2\mathbf{V}_2}{\mathbf{I}_2}$$

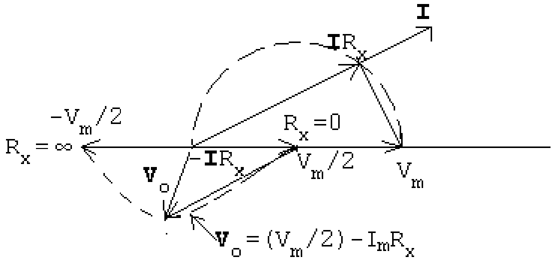
$$= \frac{8^2(10\mathbf{V}_3)}{\mathbf{I}_3/10} = \frac{(8)^2(10)^2\mathbf{V}_3}{\mathbf{I}_3} = (8)^2(10)^2 Z_L = (8)^2(10)^2(80\underline{/60^\circ}) = 512{,}000\underline{/60^\circ}\,\Omega$$

P 9.84 The phasor domain equivalent circuit is



$$V_o = \frac{V_m}{2} - \mathbf{I}R_x; \qquad \mathbf{I} = \frac{V_m}{R_x - jX_C}$$

As R_x varies from 0 to ∞, the amplitude of v_o remains constant and its phase angle increases from 0° to −180°, as shown in the following phasor diagram:



P 9.85 **[a]** $\mathbf{I} = \frac{240}{24} + \frac{240}{j32} = (10 - j7.5)\,\text{A}$

$$\mathbf{V}_s = 240\underline{/0^\circ} + (0.1 + j0.8)(10 - j7.5) = 247 + j7.25 = 247.11\underline{/1.68^\circ}\,\text{V}$$

[b] Use the capacitor to eliminate the j component of $\mathbf{I}$, therefore

$$\mathbf{I}_c = j7.5\,\text{A}, \qquad Z_c = \frac{240}{j7.5} = -j32\,\Omega$$

$$\mathbf{V}_s = 240 + (0.1 + j0.8)10 = 241 + j8 = 241.13\underline{/1.90^\circ}\,\text{V}$$

[c] Let I_c denote the magnitude of the current in the capacitor branch. Then

$$\mathbf{I} = (10 - j7.5 + jI_c) = 10 + j(I_c - 7.5)\,\text{A}$$

$$\mathbf{V}_s = 240\underline{/\alpha} = 240 + (0.1 + j0.8)[10 + j(I_c - 7.5)]$$
$$= (247 - 0.8I_c) + j(7.25 + 0.1I_c)$$

It follows that

$$240\cos\alpha = (247 - 0.8I_c) \quad \text{and} \quad 240\sin\alpha = (7.25 + 0.1I_c)$$

Now square each term and then add to generate the quadratic equation

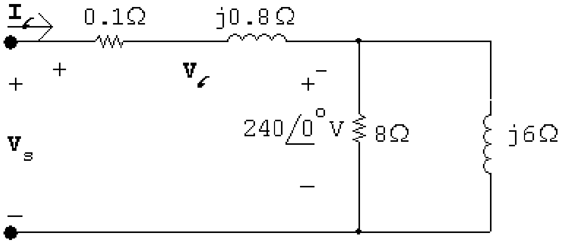
$I_c^2 - 605.77I_c + 5325.48 = 0; \qquad I_c = 302.88 \pm 293.96$

Therefore

$I_c = 8.92\,\text{A}$ (smallest value) and $Z_c = 240/j8.92 = -j26.90\,\Omega$.

Therefore, the capacitive reactance is $-26.90\,\Omega$.

P 9.86 [a]

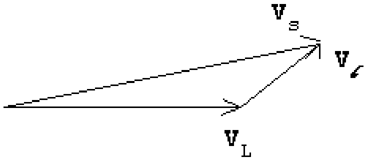


$$\mathbf{I}_\ell = \frac{240}{8} + \frac{240}{j6} = 30 - j40\,\text{A}$$

$$\mathbf{V}_\ell = (0.1 + j0.8)(30 - j40) = 35 + j20 = 40.31\underline{/29.74^\circ}\,\text{V}$$

$$\mathbf{V}_s = 240\underline{/0^\circ} + \mathbf{V}_\ell = 275 + j20 = 275.73\underline{/4.16^\circ}\,\text{V}$$

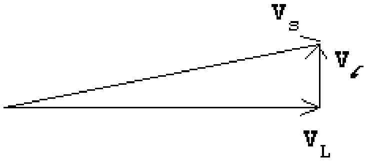
[b]



[c] $$\mathbf{I}_\ell = 30 - j40 + \frac{240}{-j5} = 30 + j8\,\text{A}$$

$$\mathbf{V}_\ell = (0.1 + j0.8)(30 + j8) = -3.4 + j24.8 = 25.03\underline{/97.81^\circ}$$

$$\mathbf{V}_s = 240\underline{/0^\circ} + \mathbf{V}_\ell = 236.6 + j24.8 = 237.9\underline{/5.98^\circ}$$



P 9.87 [a] $$\mathbf{I}_1 = \frac{120}{24} + \frac{240}{8.4 + j6.3} = 23.29 - j13.71 = 27.02\underline{/-30.5^\circ}\,\text{A}$$

$$\mathbf{I}_2 = \frac{120}{12} - \frac{120}{24} = 5\underline{/0^\circ}\,\text{A}$$

$$\mathbf{I}_3 = \frac{120}{12} + \frac{240}{8.4 + j6.3} = 28.29 - j13.71 = 31.44\underline{/-25.87^\circ}\,\text{A}$$

$$\mathbf{I}_4 = \frac{120}{24} = 5\underline{/0^\circ}\,\text{A}; \qquad \mathbf{I}_5 = \frac{120}{12} = 10\underline{/0^\circ}\,\text{A}$$

$$\mathbf{I}_6 = \frac{240}{8.4 + j6.3} = 18.29 - j13.71 = 22.86\underline{/-36.87^\circ}\,\text{A}$$

[**b**] When fuse A is interrupted,

$\mathbf{I}_1 = 0$ $\quad$ $\mathbf{I}_3 = 15\,\text{A}$ $\quad$ $\mathbf{I}_5 = 10\,\text{A}$

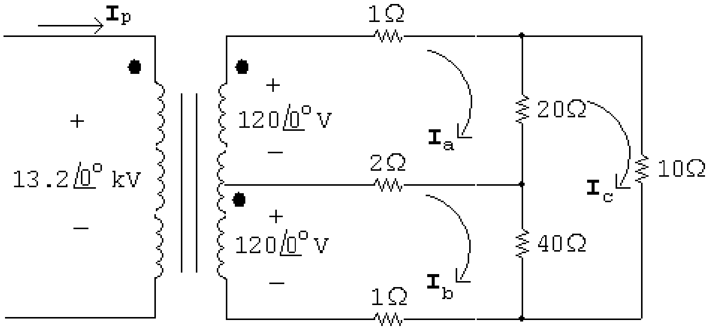
$\mathbf{I}_2 = 10 + 5 = 15\,\text{A}$ $\quad$ $\mathbf{I}_4 = -5\,\text{A}$ $\quad$ $\mathbf{I}_6 = 5\,\text{A}$

[**c**] The clock and television set were fed from the uninterrupted side of the circuit, that is, the 12 Ω load includes the clock and the TV set.

[**d**] No, the motor current drops to 5 A, well below its normal running value of 22.86 A.

[**e**] After fuse A opens, the current in fuse B is only 15 A.

P 9.88 [a] The circuit is redrawn, with mesh currents identified:



The mesh current equations are:

$$120\underline{/0^\circ} = 23\mathbf{I}_a - 2\mathbf{I}_b - 20\mathbf{I}_c$$

$$120\underline{/0^\circ} = -2\mathbf{I}_a + 43\mathbf{I}_b - 40\mathbf{I}_c$$

$$0 = -20\mathbf{I}_a - 40\mathbf{I}_b + 70\mathbf{I}_c$$

Solving,

$\mathbf{I}_a = 24\underline{/0^\circ}\,\text{A}$ $\quad$ $\mathbf{I}_b = 21.96\underline{/0^\circ}\,\text{A}$ $\quad$ $\mathbf{I}_c = 19.40\underline{/0^\circ}\,\text{A}$

The branch currents are:

$$\mathbf{I}_1 = \mathbf{I}_a = 24\underline{/0^\circ}\,\text{A}$$

$$\mathbf{I}_2 = \mathbf{I}_a - \mathbf{I}_b = 2.04\underline{/0^\circ}\,\text{A}$$

$$\mathbf{I}_3 = \mathbf{I}_b = 21.96\underline{/0^\circ}\,\text{A}$$

$\mathbf{I}_4 = \mathbf{I}_c = 19.40\underline{/0^\circ}\,\text{A}$

$\mathbf{I}_5 = \mathbf{I}_a - \mathbf{I}_c = 4.6\underline{/0^\circ}\,\text{A}$

$\mathbf{I}_6 = \mathbf{I}_b - \mathbf{I}_c = 2.55\underline{/0^\circ}\,\text{A}$

[b] Let N_1 be the number of turns on the primary winding; because the secondary winding is center-tapped, let $2N_2$ be the total turns on the secondary. From Fig. 9.58,

$$\frac{13{,}200}{N_1} = \frac{240}{2N_2} \qquad \text{or} \qquad \frac{N_2}{N_1} = \frac{1}{110}$$

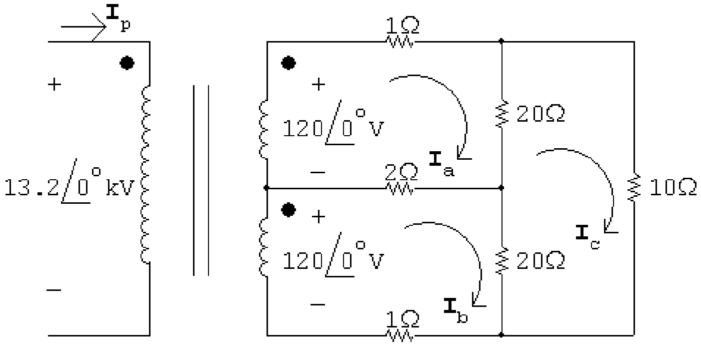
The ampere turn balance requires

$N_1\mathbf{I}_p = N_2\mathbf{I}_1 + N_2\mathbf{I}_3$

Therefore,

$$\mathbf{I}_p = \frac{N_2}{N_1}(\mathbf{I}_1 + \mathbf{I}_3) = \frac{1}{110}(24 + 21.96) = 0.42\underline{/0^\circ}\,\text{A}$$

P 9.89 **[a]**



The three mesh current equations are

$120\underline{/0^\circ} = 23\mathbf{I}_\text{a} - 2\mathbf{I}_\text{b} - 20\mathbf{I}_\text{c}$

$120\underline{/0^\circ} = -2\mathbf{I}_\text{a} + 23\mathbf{I}_\text{b} - 20\mathbf{I}_\text{c}$

$0 = -20\mathbf{I}_\text{a} - 20\mathbf{I}_\text{b} + 50\mathbf{I}_\text{c}$

Solving,

$\mathbf{I}_\text{a} = 24\underline{/0^\circ}\,\text{A}; \qquad \mathbf{I}_\text{b} = 24\underline{/0^\circ}\,\text{A}; \qquad \mathbf{I}_\text{c} = 19.2\underline{/0^\circ}\,\text{A}$

$\therefore\ \mathbf{I}_2 = \mathbf{I}_\text{a} - \mathbf{I}_\text{b} = 0\,\text{A}$

[b] $\mathbf{I}_\text{p} = \dfrac{N_2}{N_1}(\mathbf{I}_1 + \mathbf{I}_3) = \dfrac{N_2}{N_1}(\mathbf{I}_\text{a} + \mathbf{I}_\text{b})$

$$= \frac{1}{110}(24 + 24) = 0.436\underline{/0^\circ}\,\text{A}$$

[c] Yes; when the two 120 V loads are equal, there is no current in the "neutral" line, so no power is lost to this line. Since you pay for power, the cost is lower when the loads are equal.

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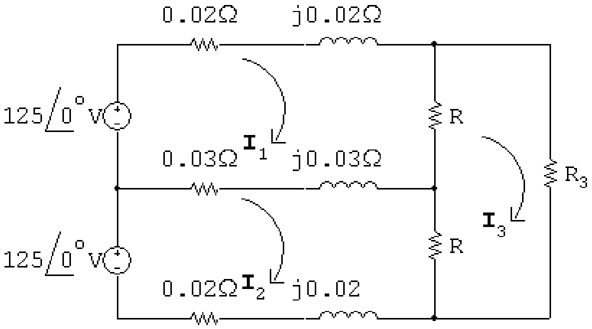
P 9.90 [a]



$$125 = (R + 0.05 + j0.05)\mathbf{I}_1 - (0.03 + j0.03)\mathbf{I}_2 - R\mathbf{I}_3$$

$$125 = -(0.03 + j0.03)\mathbf{I}_1 + (R + 0.05 + j0.05)\mathbf{I}_2 - R\mathbf{I}_3$$

Subtracting the above two equations gives

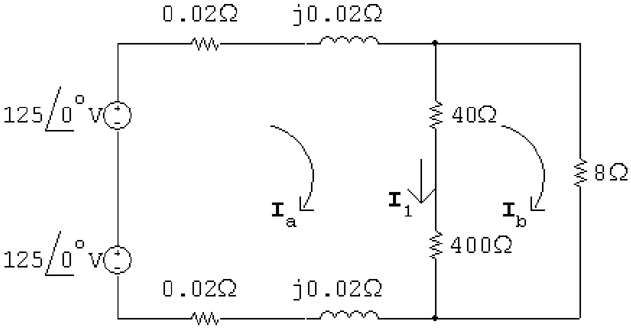
$$0 = (R + 0.08 + j0.08)\mathbf{I}_1 - (R + 0.08 + j0.08)\mathbf{I}_2$$

$$\therefore \quad \mathbf{I}_1 = \mathbf{I}_2 \quad \text{so} \quad \mathbf{I}_\text{n} = \mathbf{I}_1 - \mathbf{I}_2 = 0\,\text{A}$$

[b] $\mathbf{V}_1 = R(\mathbf{I}_1 - \mathbf{I}_3)$; $\mathbf{V}_2 = R(\mathbf{I}_2 - \mathbf{I}_3)$

Since $\mathbf{I}_1 = \mathbf{I}_2$ (from part [a]) $\mathbf{V}_1 = \mathbf{V}_2$

[c]



$$250 = (440.04 + j0.04)\mathbf{I}_\text{a} - 440\mathbf{I}_\text{b}$$

$$0 = -440\mathbf{I}_\text{a} + 448\mathbf{I}_\text{b}$$

Solving,

$$\mathbf{I}_\text{a} = 31.656207 - j0.160343\,\text{A}$$

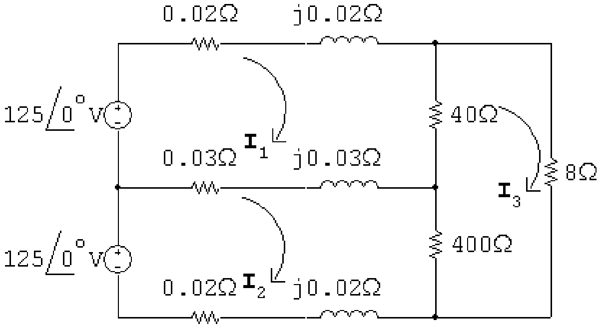
$\mathbf{I}_{\rm b} = 31.090917 - j0.157479\,\text{A}$

$\mathbf{I}_1 = \mathbf{I}_{\rm a} - \mathbf{I}_{\rm b} = 0.56529 - j0.002864\,\text{A}$

$\mathbf{V}_1 = 40\mathbf{I}_1 = 22.612 - j0.11456 = 22.612\underline{/-0.290282^\circ}\,\text{V}$

$\mathbf{V}_2 = 400\mathbf{I}_1 = 226.116 - j1.1456 = 226.1189\underline{/-0.290282^\circ}\,\text{V}$

[**d**]



$125 = (40.05 + j0.05)\mathbf{I}_1 - (0.03 + j0.03)\mathbf{I}_2 - 40\mathbf{I}_3$

$125 = -(0.03 + j0.03)\mathbf{I}_1 + (400.05 + j0.05)\mathbf{I}_2 - 400\mathbf{I}_3$

$0 = -40\mathbf{I}_1 - 400\mathbf{I}_2 + 448\mathbf{I}_3$

Solving,

$\mathbf{I}_1 = 34.19 - j0.182\,\text{A}$

$\mathbf{I}_2 = 31.396 - j0.164\,\text{A}$

$\mathbf{I}_3 = 31.085 - j0.163\,\text{A}$

$\mathbf{V}_1 = 40(\mathbf{I}_1 - \mathbf{I}_3) = 124.2\underline{/-0.35^\circ}\,\text{V}$

$\mathbf{V}_2 = 400(\mathbf{I}_2 - \mathbf{I}_3) = 124.4\underline{/-0.18^\circ}\,\text{V}$

[**e**] Because an open neutral can result in severely unbalanced voltages across the 125 V loads.

P 9.91 [**a**] Let N_1 = primary winding turns and $2N_2$ = secondary winding turns. Then

$$\frac{14{,}000}{N_1} = \frac{250}{2N_2}; \qquad \therefore \quad \frac{N_2}{N_1} = \frac{1}{112} = a$$

In part c),

$\mathbf{I}_{\rm p} = 2a\mathbf{I}_{\rm a}$

$$\therefore\quad \mathbf{I}_\mathrm{p} = \frac{2N_2\mathbf{I}_\mathrm{a}}{N_1} = \frac{1}{56}\mathbf{I}_\mathrm{a}$$

$$= \frac{1}{56}(31.656 - j0.16)$$

$$\mathbf{I}_\mathrm{p} = 565.3 - j2.9\,\mathrm{mA}$$

In part d),

$$\mathbf{I}_\mathrm{p}N_1 = \mathbf{I}_1N_2 + \mathbf{I}_2N_2$$

$$\therefore\quad \mathbf{I}_\mathrm{p} = \frac{N_2}{N_1}(\mathbf{I}_1 + \mathbf{I}_2)$$

$$= \frac{1}{112}(34.19 - j0.182 + 31.396 - j0.164)$$

$$= \frac{1}{112}(65.586 - j0.346)$$

$$\mathbf{I}_\mathrm{p} = 585.6 - j3.1\,\mathrm{mA}$$

[b] Yes, because the neutral conductor carries non-zero current whenever the load is not balanced.

10

Sinusoidal Steady State Power Calculations

Assessment Problems

AP 10.1 [a] $\mathbf{V} = 100\underline{/-45^\circ}\,\text{V}, \qquad \mathbf{I} = 20\underline{/15^\circ}\,\text{A}$

Therefore

$$P = \frac{1}{2}(100)(20)\cos[-45-(15)] = 500\,\text{W}, \qquad \text{A} \rightarrow \text{B}$$

$$Q = 1000\sin -60^\circ = -866.03\ \text{VAR}, \qquad \text{B} \rightarrow \text{A}$$

[b] $\mathbf{V} = 100\underline{/-45^\circ}, \qquad \mathbf{I} = 20\underline{/165^\circ}$

$$P = 1000\cos(-210^\circ) = -866.03\,\text{W}, \qquad \text{B} \rightarrow \text{A}$$

$$Q = 1000\sin(-210^\circ) = 500\ \text{VAR}, \qquad \text{A} \rightarrow \text{B}$$

[c] $\mathbf{V} = 100\underline{/-45^\circ}, \qquad \mathbf{I} = 20\underline{/-105^\circ}$

$$P = 1000\cos(60^\circ) = 500\,\text{W}, \qquad \text{A} \rightarrow \text{B}$$

$$Q = 1000\sin(60^\circ) = 866.03\ \text{VAR}, \qquad \text{A} \rightarrow \text{B}$$

[d] $\mathbf{V} = 100\underline{/0^\circ}, \qquad \mathbf{I} = 20\underline{/120^\circ}$

$$P = 1000\cos(-120^\circ) = -500\,\text{W}, \qquad \text{B} \rightarrow \text{A}$$

$$Q = 1000\sin(-120^\circ) = -866.03\ \text{VAR}, \qquad \text{B} \rightarrow \text{A}$$

AP 10.2

$$\text{pf} = \cos(\theta_v - \theta_i) = \cos[15-(75)] = \cos(-60^\circ) = 0.5 \text{ leading}$$

$$\text{rf} = \sin(\theta_v - \theta_i) = \sin(-60^\circ) = -0.866$$

AP 10.3

From Ex. 9.4 $\quad I_{\text{eff}} = \dfrac{I_\rho}{\sqrt{3}} = \dfrac{0.18}{\sqrt{3}}\,\text{A}$

$$P = I_{\text{eff}}^2 R = \left(\frac{0.0324}{3}\right)(5000) = 54\,\text{W}$$

AP 10.4 **[a]** $Z = (39 + j26)\|(-j52) = 48 - j20 = 52\underline{/-22.62^\circ}\,\Omega$

Therefore $\quad \mathbf{I}_\ell = \dfrac{250\underline{/0^\circ}}{48 - j20 + 1 + j4} = 4.85\underline{/18.08^\circ}\,\text{A (rms)}$

$$\mathbf{V}_\text{L} = Z\mathbf{I}_\ell = (52\underline{/-22.62^\circ})(4.85\underline{/18.08^\circ}) = 252.20\underline{/-4.54^\circ}\,\text{V (rms)}$$

$$\mathbf{I}_\text{L} = \frac{\mathbf{V}_\text{L}}{39 + j26} = 5.38\underline{/-38.23^\circ}\,\text{A (rms)}$$

[b] $S_\text{L} = \mathbf{V}_L\mathbf{I}_L^* = (252.20\underline{/-4.54^\circ})(5.38\underline{/+38.23^\circ}) = 1357\underline{/33.69^\circ}$

$= (1129.09 + j752.73)\,\text{VA}$

$P_\text{L} = 1129.09\,\text{W}; \qquad Q_\text{L} = 752.73\ \text{VAR}$

[c] $P_\ell = |\mathbf{I}_\ell|^2 1 = (4.85)^2 \cdot 1 = 23.52\,\text{W}; \qquad Q_\ell = |\mathbf{I}_\ell|^2 4 = 94.09\ \text{VAR}$

[d] $S_g(\text{delivering}) = 250\mathbf{I}_\ell^* = (1152.62 - j376.36)\,\text{VA}$

Therefore the source is delivering 1152.62 W and absorbing 376.36 magnetizing VAR.

[e] $Q_\text{cap} = \dfrac{|\mathbf{V}_\text{L}|^2}{-52} = \dfrac{(252.20)^2}{-52} = -1223.18\ \text{VAR}$

Therefore the capacitor is delivering 1223.18 magnetizing VAR.

Check: $\quad 94.09 + 752.73 + 376.36 = 1223.18\ \text{VAR}$ $\quad$ and

$1129.09 + 23.52 = 1152.62\,\text{W}$

AP 10.5 Series circuit derivation:

$$S = 250\mathbf{I}^* = (40{,}000 - j30{,}000)$$

Therefore $\quad \mathbf{I}^* = 160 - j120 = 200\underline{/-36.87^\circ}\,\text{A (rms)}$

$$\mathbf{I} = 200\underline{/36.87^\circ}\,\text{A (rms)}$$

$$Z = \frac{\mathbf{V}}{\mathbf{I}} = \frac{250}{200\underline{/36.87^\circ}} = 1.25\underline{/-36.87^\circ} = (1 - j0.75)\,\Omega$$

Therefore $\quad R = 1\,\Omega, \qquad X_\text{C} = -0.75\,\Omega$

Parallel circuit derivation

$$P = \frac{(250)^2}{R}; \qquad \text{therefore} \quad R = \frac{(250)^2}{40{,}000} = 1.5625\,\Omega$$

$$Q = \frac{(250)^2}{X_C}; \qquad \text{therefore} \quad X_C = \frac{(250)^2}{-30{,}000} = -2.083\,\Omega$$

AP 10.6

$$S_1 = 15{,}000(0.6) + j15{,}000(0.8) = 9000 + j12{,}000\,\text{VA}$$

$$S_2 = 6000(0.8) - j6000(0.6) = 4800 - j3600\,\text{VA}$$

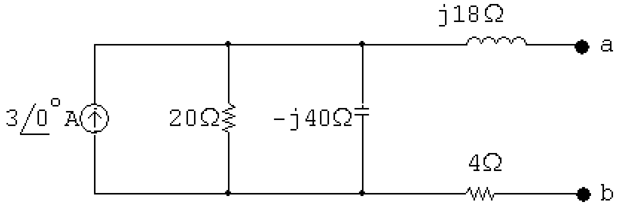
$$S_T = S_1 + S_2 = 13{,}800 + j8400\,\text{VA}$$

$$S_T = 200\mathbf{I}^*; \qquad \text{therefore} \quad \mathbf{I}^* = 69 + j42 \qquad \mathbf{I} = 69 - j42\,\text{A}$$

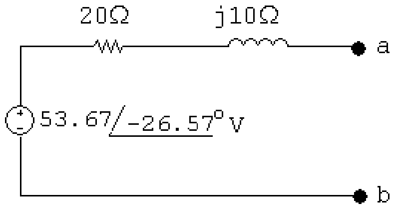
$$\mathbf{V}_s = 200 + j\mathbf{I} = 200 + j69 + 42 = 242 + j69 = 251.64\underline{/15.91^\circ}\,\text{V (rms)}$$

AP 10.7 **[a]** The phasor domain equivalent circuit and the Thévenin equivalent are shown below:

Phasor domain equivalent circuit:



Thévenin equivalent:



$$\mathbf{V}_{\text{Th}} = 3\frac{-j800}{20 - j40} = 48 - j24 = 53.67\underline{/-26.57^\circ}\,\text{V}$$

$$Z_{\text{Th}} = 4 + j18 + \frac{-j800}{20 - j40} = 20 + j10 = 22.36\underline{/26.57^\circ}\,\Omega$$

For maximum power transfer, $Z_L = (20 - j10)\,\Omega$

[**b**] $$\mathbf{I} = \frac{53.67\underline{/-26.57^\circ}}{40} = 1.34\underline{/-26.57^\circ}\,\text{A}$$

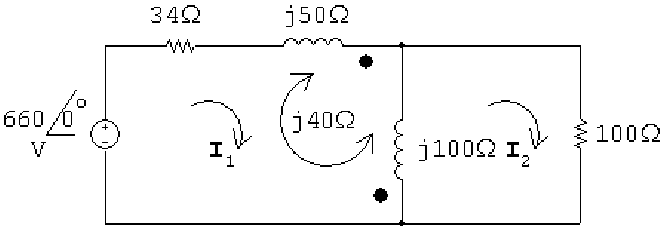
$$\text{Therefore} \quad P = \left(\frac{1.34}{\sqrt{2}}\right)^2 20 = 17.96\,\text{W}$$

[**c**] $R_\text{L} = |Z_\text{Th}| = 22.36\,\Omega$

[**d**] $$\mathbf{I} = \frac{53.67\underline{/-26.57^\circ}}{42.36 + j10} = 1.23\underline{/-39.85^\circ}\,\text{A}$$

$$\text{Therefore} \quad P = \left(\frac{1.23}{\sqrt{2}}\right)^2 (22.36) = 17\,\text{W}$$

AP 10.8



Mesh current equations:

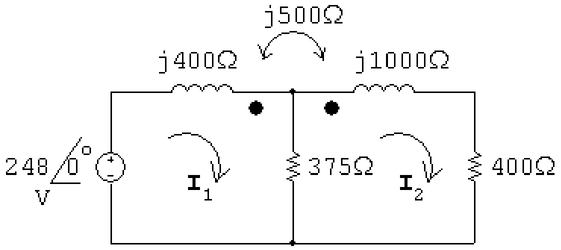
$$660 = (34 + j50)\mathbf{I}_1 + j100(\mathbf{I}_1 - \mathbf{I}_2) + j40\mathbf{I}_1 + j40(\mathbf{I}_1 - \mathbf{I}_2)$$

$$0 = j100(\mathbf{I}_2 - \mathbf{I}_1) - j40\mathbf{I}_1 + 100\mathbf{I}_2$$

Solving,

$$\mathbf{I}_2 = 3.5\underline{/0^\circ}\,\text{A}; \qquad \therefore \quad P = \frac{1}{2}(3.5)^2(100) = 612.50\,\text{W}$$

AP 10.9 [**a**]



$$248 = j400\mathbf{I}_1 - j500\mathbf{I}_2 + 375(\mathbf{I}_1 - \mathbf{I}_2)$$

$$0 = 375(\mathbf{I}_2 - \mathbf{I}_1) + j1000\mathbf{I}_2 - j500\mathbf{I}_1 + 400\mathbf{I}_2$$

Solving,

$$\mathbf{I}_1 = 0.80 - j0.62\,\text{A}; \qquad \mathbf{I}_2 = 0.4 - j0.3 = 0.5\underline{/-36.87^\circ}$$

$$\therefore \quad P = \frac{1}{2}(0.25)(400) = 50\,\text{W}$$

[b] $\mathbf{I}_1 - \mathbf{I}_2 = 0.4 - j0.32\,\text{A}$

$$P_{375} = \frac{1}{2}|\mathbf{I}_1 - \mathbf{I}_2|^2(375) = 49.20\,\text{W}$$

[c] $P_g = \frac{1}{2}(248)(0.8) = 99.20\,\text{W}$

$$\sum P_{\text{abs}} = 50 + 49.2 = 99.20\,\text{W} \quad \text{(checks)}$$

AP 10.10 **[a]** $V_{\text{Th}} = 210\,\text{V}; \qquad \mathbf{V}_2 = \frac{1}{4}\mathbf{V}_1; \qquad \mathbf{I}_1 = \frac{1}{4}\mathbf{I}_2$

Short circuit equations:

$$840 = 80\mathbf{I}_1 - 20\mathbf{I}_2 + \mathbf{V}_1$$

$$0 = 20(\mathbf{I}_2 - \mathbf{I}_1) - \mathbf{V}_2$$

$$\therefore \quad \mathbf{I}_2 = 14\,\text{A}; \qquad R_{\text{Th}} = \frac{210}{14} = 15\,\Omega$$

[b] $P_{\max} = \left(\frac{210}{30}\right)^2 15 = 735\,\text{W}$

AP 10.11 **[a]** $\mathbf{V}_{\text{Th}} = -4(146\underline{/0^\circ}) = -584\underline{/0^\circ}\,\text{V (rms)}$

$$\mathbf{V}_2 = 4\mathbf{V}_1; \qquad \mathbf{I}_1 = -4\mathbf{I}_2$$

Short circuit equations:

$$146\underline{/0^\circ} = 80\mathbf{I}_1 - 20\mathbf{I}_2 + \mathbf{V}_1$$

$$0 = 20(\mathbf{I}_2 - \mathbf{I}_1) - \mathbf{V}_2$$

$$\therefore \quad \mathbf{I}_2 = -146/365 = -0.40\,\text{A}; \qquad R_{\text{Th}} = \frac{-584}{-0.4} = 1460\,\Omega$$

[b] $P = \left(\frac{-584}{2920}\right)^2 1460 = 58.40\,\text{W}$

Problems

P 10.1 **[a]** $P = \frac{1}{2}(100)(10)\cos(50 - 15) = 500\cos 35^\circ = 409.58\,\text{W} \quad \text{(abs)}$

$Q = 500\sin 35^\circ = 286.79\ \text{VAR} \quad \text{(abs)}$

[b] $P = \frac{1}{2}(40)(20)\cos(-15 - 60) = 400\cos(-75^\circ) = 103.53\,\text{W} \quad \text{(abs)}$

$Q = 400\sin(-75^\circ) = -386.37\ \text{VAR} \quad \text{(del)}$

[c] $P = \frac{1}{2}(400)(10)\cos(30 - 150) = 2000\cos(-120^\circ) = -1000\,\text{W} \quad \text{(del)}$

$Q = 2000\sin(-120^\circ) = -1732.05\ \text{VAR} \quad \text{(del)}$

[d] $P = \frac{1}{2}(200)(5)\cos(160 - 40) = 500\cos(120^\circ) = -250\,\text{W} \quad \text{(del)}$

$Q = 500\sin(120^\circ) = 433.01\ \text{VAR} \quad \text{(abs)}$

P 10.2 **[a]** hair dryer = 600 W vacuum = 630 W

sun lamp = 279 W air conditioner = 860 W

television = 240 W $\sum P = 2609\,\text{W}$

Therefore $I_{\text{eff}} = \frac{2609}{120} = 21.74\,\text{A}$

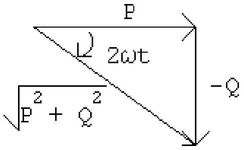
Yes, the breaker will trip.

[b] $\sum P = 2609 - 909 = 1700\,\text{W}; \qquad I_{\text{eff}} = \frac{1700}{120} = 14.17\,\text{A}$

Yes, the breaker will not trip if the current is reduced to 14.17 A.

P 10.3 $p = P + P\cos 2\omega t - Q\sin 2\omega t; \qquad \frac{dp}{dt} = -2\omega P\sin 2\omega t - 2\omega Q\cos 2\omega t$

$$\frac{dp}{dt} = 0 \quad \text{when} \quad -2\omega P\sin 2\omega t = 2\omega Q\cos 2\omega t \quad \text{or} \quad \tan 2\omega t = -\frac{Q}{P}$$



$$\cos 2\omega t = \frac{P}{\sqrt{P^2+Q^2}}; \qquad \sin 2\omega t = -\frac{Q}{\sqrt{P^2+Q^2}}$$

Let $\theta = \tan^{-1}(-Q/P)$, then p is maximum when $2\omega t = \theta$ and p is minimum when $2\omega t = (\theta + \pi)$.

Therefore $p_{\max} = P + P \cdot \dfrac{P}{\sqrt{P^2+Q^2}} - \dfrac{Q(-Q)}{\sqrt{P^2+Q^2}} = P + \sqrt{P^2+Q^2}$

and $p_{\min} = P - P \cdot \dfrac{P}{\sqrt{P^2+Q^2}} - Q \cdot \dfrac{Q}{\sqrt{P^2+Q^2}} = P - \sqrt{P^2+Q^2}$

P 10.4 [**a**] $P = \dfrac{1}{2}\dfrac{(240)^2}{480} = 60\,\text{W}$

$$-\frac{1}{\omega C} = \frac{-9\times 10^6}{(5000)(5)} = -360\,\Omega$$

$$Q = \frac{1}{2}\frac{(240)^2}{(-360)} = -80\,\text{VAR}$$

$$p_{\max} = P + \sqrt{P^2+Q^2} = 60 + \sqrt{(60)^2+(80)^2} = 160\,\text{W (del)}$$

[**b**] $p_{\min} = 60 - \sqrt{60^2+80^2} = -40\,\text{W (abs)}$

[**c**] $P = 60\,\text{W}$ from (a)

[**d**] $Q = -80\,\text{VAR}$ from (a)

[**e**] generates, because $Q < 0$

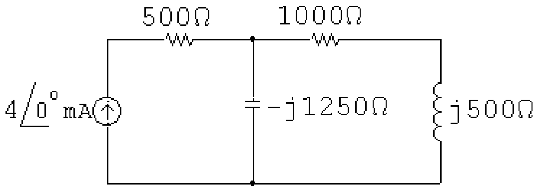
[**f**] pf $= \cos(\theta_v - \theta_i)$

$$\mathbf{I} = \frac{240}{480} + \frac{240}{-j360} = 0.5 + j0.67 = 0.83\underline{/53.13^\circ}\,\text{A}$$

$\therefore$ pf $= \cos(0 - 53.13^\circ) = 0.6$ leading

[**g**] rf $= \sin(-53.13^\circ) = -0.8$

P 10.5 $\mathbf{I}_g = 4\underline{/0^\circ}\,\text{mA}$; $\dfrac{1}{j\omega C} = -j1250\,\Omega$; $j\omega L = j500\,\Omega$

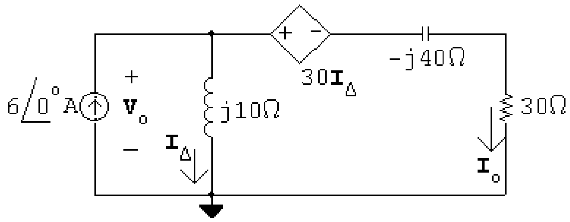


$$Z_{\text{eq}} = 500 + [-j1250\|(1000 + j500)] = 1500 - j500\,\Omega$$

$$P_g = -\frac{1}{2}|I|^2\text{Re}\{Z_{\text{eq}}\} = -\frac{1}{2}(0.004)^2(1500) = -12\,\text{mW}$$

The source delivers 12 mW of power to the circuit.

P 10.6 $j\omega L = j20{,}000(0.5\times 10^{-3}) = j10\,\Omega;\qquad \dfrac{1}{j\omega C} = \dfrac{10^6}{j20{,}000(1.25)} = -j40\,\Omega$



$$-6 + \frac{\mathbf{V}_o}{j10} + \frac{\mathbf{V}_o - 30(\mathbf{V}_o/j10)}{30 - j40} = 0$$

$$\therefore\quad \mathbf{V}_o\left[\frac{1}{j10} + \frac{1+j3}{30-j40}\right] = 6$$

$$\therefore\quad \mathbf{V}_o = 100\underline{/126.87^\circ}\,\text{V}$$

$$\therefore\quad \mathbf{I}_\Delta = \frac{\mathbf{V}_o}{j10} = 10\underline{/36.87^\circ}\,\text{A}$$

$$\mathbf{I}_o = 6\underline{/0^\circ} - \mathbf{I}_\Delta = 6 - 8 - j6 = -2 - j6 = 6.32\underline{/-108.43^\circ}\,\text{A}$$

$$P_{30\Omega} = \frac{1}{2}|\mathbf{I}_o|^2 30 = 600\,\text{W}$$

P 10.7 $Z_\text{f} = -j10{,}000\|20{,}000 = 4000 - j8000\,\Omega$

$$Z_\text{i} = 2000 - j2000\,\Omega$$

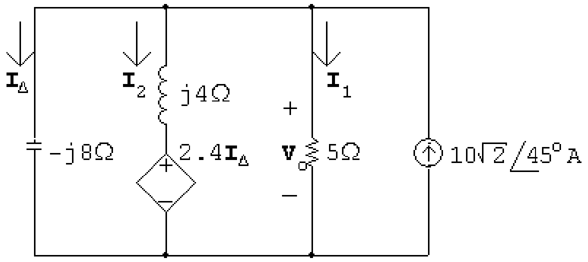
$$\therefore\quad \frac{Z_\text{f}}{Z_\text{i}} = \frac{4000-j8000}{2000-j2000} = 3 - j1$$

$$\mathbf{V}_o = -\frac{Z_\text{f}}{Z_\text{i}}\mathbf{V}_g;\qquad \mathbf{V}_g = 1\underline{/0^\circ}\,\text{V}$$

$$\mathbf{V}_o = -(3-j1)(1) = -3 + j1 = 3.16\underline{/161.57^\circ}\,\text{V}$$

$$P = \frac{1}{2}\frac{V_m^2}{R} = \frac{1}{2}\frac{(10)}{1000} = 5\times 10^{-3} = 5\,\text{mW}$$

P 10.8 [a] From the solution to Problem 9.59 we have:



$$\mathbf{V}_o = j80 = 80\underline{/90^\circ}\,\text{V}$$

$$S_g = -\frac{1}{2}\mathbf{V}_o\mathbf{I}_g^* = -\frac{1}{2}(j80)(10 - j10) = -400 - j400\,\text{VA}$$

Therefore, the independent current source is delivering 400 W and 400 magnetizing vars.

$$\mathbf{I}_1 = \frac{\mathbf{V}_o}{5} = j16\,\text{A}$$

$$P_{5\Omega} = \frac{1}{2}(16)^2(5) = 640\,\text{W}$$

Therefore, the 8 Ω resistor is absorbing 640 W.

$$\mathbf{I}_\Delta = \frac{\mathbf{V}_o}{-j8} = -10\,\text{A}$$

$$Q_{\text{cap}} = \frac{1}{2}(10)^2(-8) = -400\,\text{VAR}$$

Therefore, the $-j8\,\Omega$ capacitor is developing 400 magnetizing vars.

$$2.4\mathbf{I}_\Delta = -24\,\text{V}$$

$$\mathbf{I}_2 = \frac{\mathbf{V}_o - 2.4\mathbf{I}_\Delta}{j4} = \frac{-j80 + 24}{j4}$$

$$= 20 - j6\,\text{A} = 20.88\underline{/-16.7^\circ}\,\text{A}$$

$$Q_{j4} = \frac{1}{2}|\mathbf{I}_2|^2(4) = 872\,\text{VAR}$$

Therefore, the $j4\,\Omega$ inductor is absorbing 872 magnetizing vars.

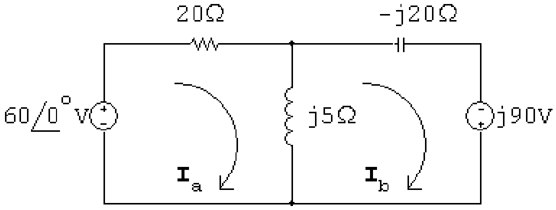
$$S_{\text{d.s.}} = \tfrac{1}{2}(2.4\mathbf{I}_\Delta)\mathbf{I}_2^* = \tfrac{1}{2}(-24)(20 + j6)$$
$$= -240 - j72\,\text{VA}$$

Thus the dependent source is delivering 240 W and 72 magnetizing vars.

[b] $\sum P_{\text{gen}} = 400 + 240 = 640\,\text{W} = \sum P_{\text{abs}}$

[c] $\sum Q_{\text{gen}} = 400 + 400 + 72 = 872\,\text{VAR} = \sum Q_{\text{abs}}$

P 10.9 **[a]** From the solution to Problem 9.61 we have



$\mathbf{I}_{\text{a}} = 2.25 - j2.25\,\text{A}; \quad \mathbf{I}_{\text{b}} = -6.75 + j0.75\,\text{A}; \quad \mathbf{I}_o = 9 - j3\,\text{A}$

$$S_{60V} = -\frac{1}{2}(60)\mathbf{I}_{\text{a}}^* = -30(2.25 + j2.25) = -67.5 - j67.5\,\text{VA}$$

Thus, the 60 V source is developing 67.5 W and 67.5 magnetizing vars.

$$S_{90V} = -\tfrac{1}{2}(j90)\mathbf{I}_{\text{b}}^* = -j45(-6.75 - j0.75)$$
$$= -33.75 + j303.75\,\text{VA}$$

Thus, the 90 V source is delivering 33.75 W and absorbing 303.75 magnetizing vars.

$$P_{20\Omega} = \frac{1}{2}|\mathbf{I}_{\text{a}}|^2(20) = 101.25\,\text{W}$$

Thus the 20 Ω resistor is absorbing 101.25 W.

$$Q_{-j20\Omega} = \frac{1}{2}|\mathbf{I}_{\text{b}}|^2(-20) = -461.25\,\text{VAR}$$

Thus the $-j20\,\Omega$ capacitor is developing 461.25 magnetizing vars.

$$Q_{j5\Omega} = \frac{1}{2}|\mathbf{I}_o|^2(5) = 225\,\text{VAR}$$

Thus the $j5\,\Omega$ inductor is absorbing 225 magnetizing vars.

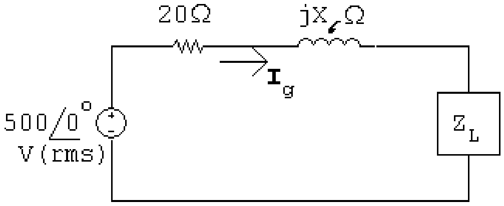
[b] $\sum P_{\text{dev}} = 67.5 + 33.75 = 101.25\,\text{W} = \sum P_{\text{abs}}$

[c] $\sum Q_{\text{dev}} = 67.5 + 461.25 = 528.75\,\text{VAR}$

$$\sum Q_{\text{abs}} = 225 + 303.75 = 528.75\,\text{VAR} = \sum Q_{\text{dev}}$$

P 10.10 **[a]** line loss $= 7500 - 2500 = 5\,\text{kW}$

line loss $= |\mathbf{I}_g|^2 20 \qquad \therefore \quad |\mathbf{I}_g|^2 = 250$

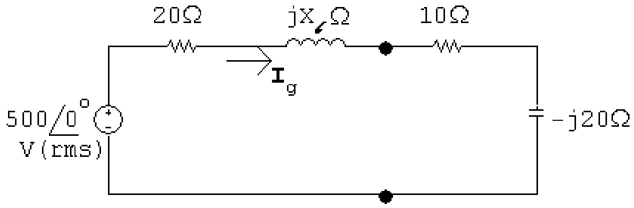


$|\mathbf{I}_g| = \sqrt{250}\,\text{A}$

$|\mathbf{I}_g|^2R_{\rm L} = 2500 \qquad \therefore \quad R_{\rm L} = 10\,\Omega$

$|\mathbf{I}_g|^2X_{\rm L} = -5000 \qquad \therefore \quad X_{\rm L} = -20\,\Omega$

Thus,



$$|Z| = \sqrt{(30)^2 + (X_\ell - 20)^2} \qquad |\mathbf{I}_g| = \frac{500}{\sqrt{900 + (X_\ell - 20)^2}}$$

$$\therefore \quad 900 + (X_\ell - 20)^2 = \frac{25 \times 10^4}{250} = 1000$$

Solving, $\quad (X_\ell - 20) = \pm 10.$

Thus, $\quad X_\ell = 10\,\Omega \qquad$ or $\qquad X_\ell = 30\,\Omega$

[b] If $X_\ell = 30\,\Omega$:

$$\mathbf{I}_g = \frac{500}{30 + j10} = 15 - j5\,\mathrm{A}$$

$S_g = -500\mathbf{I}_g^* = -7500 - j2500\,\mathrm{VA}$

Thus, the voltage source is delivering 7500 W and 2500 magnetizing vars.

$Q_{j30} = |\mathbf{I}_g|^2X_\ell = 250(30) = 7500\,\mathrm{VAR}$

Therefore the line reactance is absorbing 7500 magnetizing vars.

$Q_{-j20} = |\mathbf{I}_g|^2X_{\rm L} = 250(-20) = -5000\,\mathrm{VAR}$

Therefore the load reactance is generating 5000 magnetizing vars.

$\sum Q_{\rm gen} = 7500\,\mathrm{VAR} = \sum Q_{\rm abs}$

If $X_\ell = 10\,\Omega$:

$$\mathbf{I}_g = \frac{500}{30 - j10} = 15 + j5\,\mathrm{A}$$

$S_g = -500\mathbf{I}_g^* = -7500 + j2500\,\mathrm{VA}$

Thus, the voltage source is delivering 7500 W and absorbing 2500 magnetizing vars.

$Q_{j10} = |\mathbf{I}_g|^2(10) = 250(10) = 2500\,\mathrm{VAR}$

Therefore the line reactance is absorbing 2500 magnetizing vars. The load continues to generate 5000 magnetizing vars.

$\sum Q_{\rm gen} = 5000\,\mathrm{VAR} = \sum Q_{\rm abs}$

P 10.11 **[a]** $I_{\text{eff}} = 40/115 \cong 0.35\,\text{A}$

[b] $I_{\text{eff}} = 130/115 \cong 1.13\,\text{A}$

P 10.12 $W_{\text{dc}} = \dfrac{V_{\text{dc}}^2}{R}T; \qquad W_s = \displaystyle\int_{t_o}^{t_o+T} \frac{v_s^2}{R}\,dt$

$$\therefore \quad \frac{V_{\text{dc}}^2}{R}T = \int_{t_o}^{t_o+T} \frac{v_s^2}{R}\,dt$$

$$V_{\text{dc}}^2 = \frac{1}{T}\int_{t_o}^{t_o+T} v_s^2\,dt$$

$$V_{\text{dc}} = \sqrt{\frac{1}{T}\int_{t_o}^{t_o+T} v_s^2\,dt} = V_{\text{rms}} = V_{\text{eff}}$$

P 10.13 $i(t) = 250t \qquad 0 \le t \le 80\,\text{ms}$

$$i(t) = 100 - 1000t \qquad 80\,\text{ms} \le t \le 100\,\text{ms}$$

$$\therefore \quad I_{\text{rms}} = \sqrt{\frac{1}{0.1}\left\{\int_0^{0.08} (250)^2t^2\,dt + \int_{0.08}^{0.1}(100 - 1000t)^2\,dt\right\}}$$

$$\int_0^{0.08} (250)^2t^2\,dt = (250)^2\frac{t^3}{3}\Big|_0^{0.08} = \frac{32}{3}$$

$$(100 - 1000t)^2 = 10^4 - 2\times10^5t + 10^6t^2$$

$$\int_{0.08}^{0.1} 10^4\,dt = 200$$

$$\int_{0.08}^{0.1} 2\times10^5t\,dt = 10^5t^2\Big|_{0.08}^{0.1} = 360$$

$$10^6\int_{0.08}^{0.1} t^2\,dt = \frac{10^6}{3}t^3\Big|_{0.08}^{0.1} = \frac{488}{3}$$

$$\therefore \quad I_{\text{rms}} = \sqrt{10\{(32/3) + 225 - 360 + (488/3)\}} = 11.55\,\text{A}$$

P 10.14 $P = I_{\text{rms}}^2R \qquad \therefore \quad R = \dfrac{1280}{(11.55)^2} = 9.6\,\Omega$

P 10.15 **[a]** Area under one cycle of v_g^2:

$$A = (100)(25 \times 10^{-6}) + 400(25 \times 10^{-6}) + 400(25 \times 10^{-6}) + 100(25 \times 10^{-6})$$
$$= 1000(25 \times 10^{-6})$$

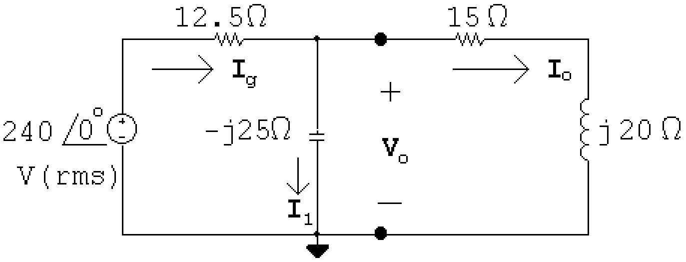
Mean value of v_g^2:

$$\text{M.V.} = \frac{A}{100 \times 10^{-6}} = \frac{1000(25 \times 10^{-6})}{100 \times 10^{-6}} = 250$$

$$\therefore \quad V_{\text{rms}} = \sqrt{250} = 15.81\,\text{V (rms)}$$

[b] $P = \dfrac{V_{\text{rms}}^2}{R} = \dfrac{250}{4} = 62.5\,\text{W}$

P 10.16 **[a]**



$$\frac{\mathbf{V}_o}{-j25} + \frac{\mathbf{V}_o - 240}{12.5} + \frac{\mathbf{V}_o}{15 + j20} = 0$$

$$\therefore \quad \mathbf{V}_o = 183.53 - j14.12 = 184.07\underline{/-4.4^\circ}\,\text{V}$$

$$\mathbf{I}_g = \frac{240 - 183.53 + j14.12}{12.50} = 4.52 + j1.13\,\text{A}$$

$$S_g = -\mathbf{V}_g\mathbf{I}_g^* = -(240)(4.52 - j1.13)$$
$$= -1084.24 + j271.06\,\text{VA}$$

[b] Source is delivering 1084.24 W.

[c] Source is absorbing 271.06 magnetizing VAR.

[d] $Q_{\text{cap}} = \dfrac{(184.07)^2}{-25} = -1355.29\ \text{VAR}$

$$P_{12.5\Omega} = |\mathbf{I}_g|^2(12.5) = 271.06\,\text{W}$$

$$|\mathbf{I}_o| = \frac{184.07}{25} = 7.36\,\text{A}$$

$$P_{15\Omega} = |\mathbf{I}_o|^2(15) = 813.18\,\text{W}$$

$$Q_{\text{ind}} = |\mathbf{I}_o|^2(20) = 1084.24\ \text{VAR}$$

[e] $\sum P_{\text{del}} = 1084.24\,\text{W}$

$\sum P_{\text{diss}} = 271.06 + 813.18 = 1084.24\,\text{W}$

$\therefore \quad \sum P_{\text{del}} = \sum P_{\text{diss}} = 1084.24\,\text{W}$

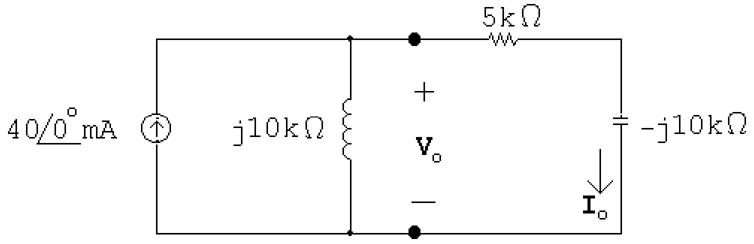
[f] $\sum Q_{\text{abs}} = 271.06 + 1084.24 = 1355.29\,\text{VAR}$

$\sum Q_{\text{dev}} = 1355.29\,\text{VAR}$

$\therefore \quad \sum \text{ mag VAR dev } = \sum \text{ mag VAR abs } = 1355.29\,\text{VAR}$

P 10.17 $\mathbf{I}_g = 40\underline{/0^\circ}\,\text{mA}$

$j\omega L = j10{,}000\,\Omega; \qquad \frac{1}{j\omega C} = -j10{,}000\,\Omega$



$$\mathbf{I}_o = \frac{j10{,}000}{5000}(40\underline{/0^\circ}) = 80\underline{/90^\circ}\,\text{mA}$$

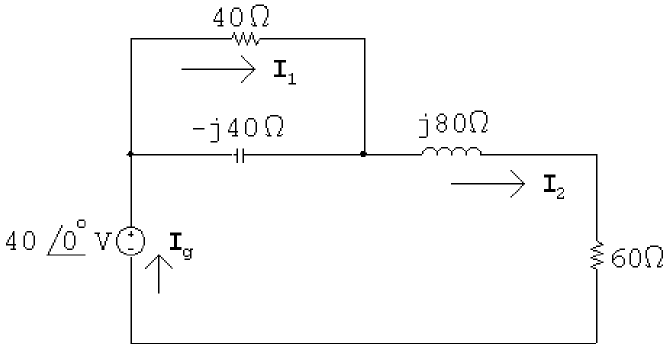
$$P = \frac{1}{2}|\mathbf{I}_o|^2(5000) = \frac{1}{2}(0.08)^2(5000) = 16\,\text{W}$$

$$Q = \frac{1}{2}|\mathbf{I}_o|^2(-10{,}000) = -32\,\text{VAR}$$

$$S = P + jQ = 16 - j32\,\text{VA}$$

$$|S| = 35.78\,\text{VA}$$

P 10.18 [a] $\frac{1}{j\omega C} = -j40\,\Omega; \qquad j\omega L = j80\,\Omega$



$$Z_{\text{eq}} = 40\| - j40 + j80 + 60 = 80 + j60\,\Omega$$

$$\mathbf{I}_g = \frac{40\underline{/0^\circ}}{80 + j60} = 0.32 - j0.24\,\text{A}$$

$$S_g = -\frac{1}{2}\mathbf{V}_g\mathbf{I}_g^* = -\frac{1}{2}40(0.32 + j0.24) = -6.4 - j4.8\,\text{VA}$$

$$P = 6.4\,\text{W}\,(\text{del}); \qquad Q = 4.8\,\text{VAR}\,(\text{del})$$

$$|S| = |S_g| = 8\,\text{VA}$$

[**b**] $\mathbf{I}_1 = \dfrac{-j40}{40 - j40}\mathbf{I}_g = 0.04 - j0.28\,\text{A}$

$$P_{40\Omega} = \frac{1}{2}|\mathbf{I}_1|^2(40) = 1.6\,\text{W}$$

$$P_{60\Omega} = \frac{1}{2}|\mathbf{I}_g|^2(60) = 4.8\,\text{W}$$

$$\sum P_{\text{diss}} = 1.6 + 4.8 = 6.4\,\text{W} = \sum P_{\text{dev}}$$

[**c**] $\mathbf{I}_{-j40\Omega} = \mathbf{I}_g - \mathbf{I}_1 = 0.28 + j0.04\,\text{A}$

$$Q_{-j40\Omega} = \frac{1}{2}|\mathbf{I}_{-j40\Omega}|^2(-40) = -1.6\,\text{VAR}\,(\text{del})$$

$$Q_{j80\Omega} = \frac{1}{2}|\mathbf{I}_g|^2(80) = 6.4\,\text{VAR}\,(\text{abs})$$

$$\sum Q_{\text{abs}} = 6.4 - 1.6 = 4.8\,\text{VAR} = \sum Q_{\text{dev}}$$

P 10.19 $S_{\text{T}} = 40{,}800 + j30{,}600\,\text{VA}$

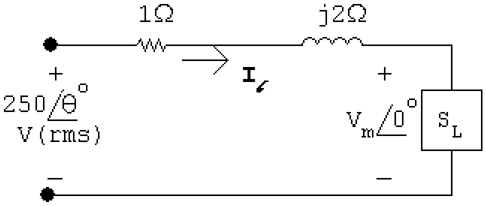
$$S_1 = 20{,}000(0.96 - j0.28) = 19{,}200 - j5600\,\text{VA}$$

$$S_2 = S_{\text{T}} - S_1 = 21{,}600 + j36{,}200 = 42{,}154.48\underline{/59.176^\circ}\,\text{VA}$$

$$\text{rf} = \sin(59.176^\circ) = 0.8587$$

$$\text{pf} = \cos(59.176^\circ) = 0.5124 \text{ lagging}$$

P 10.20 **[a]** Let $\mathbf{V}_{\rm L} = V_m\underline{/0^\circ}$:



$$S_{\rm L} = 2500(0.8 + j0.6) = 2000 + j1500\,{\rm VA}$$

$$\mathbf{I}_\ell^* = \frac{2000}{V_m} + j\frac{1500}{V_m}; \qquad \mathbf{I}_\ell = \frac{2000}{V_m} - j\frac{1500}{V_m}$$

$$250\underline{/\theta} = V_m + \left(\frac{2000}{V_m} - j\frac{1500}{V_m}\right)(1 + j2)$$

$$250V_m\underline{/\theta} = V_m^2 + (2000 - j1500)(1 + j2) = V_m^2 + 5000 + j2500$$

$$250V_m\cos\theta = V_m^2 + 5000; \qquad 250V_m\sin\theta = 2500$$

$$(250)^2V_m^2 = (V_m^2 + 5000)^2 + 2500^2$$

$$62{,}500V_m^2 = V_m^4 + 10{,}000V_m^2 + 31.25 \times 10^6$$

or

$$V_m^4 - 52{,}500V_m^2 + 31.25 \times 10^6 = 0$$

Solving,

$$V_m^2 = 26{,}250 \pm 25{,}647.86; \qquad V_m = 227.81\,{\rm V} \text{ and } V_m = 24.54\,{\rm V}$$

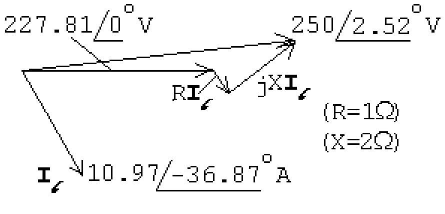
If $V_m = 227.81$ V:

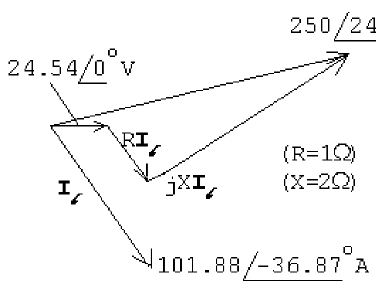
$$\sin\theta = \frac{2500}{(227.81)(250)} = 0.044; \qquad \therefore\quad \theta = 2.52^\circ$$

If $V_m = 24.54$ V:

$$\sin\theta = \frac{2500}{(24.54)(250)} = 0.4075; \qquad \therefore\quad \theta = 24.05^\circ$$

[b]





P 10.21 [**a**] $S_1 = 60{,}000 - j70{,}000\,\text{VA}$

$$S_2 = \frac{|\mathbf{V}_\text{L}|^2}{Z_2^*} = \frac{(2500)^2}{24 - j7} = 240{,}000 - j70{,}000\,\text{VA}$$

$$S_1 + S_2 = 300{,}000\,\text{VA}$$

$$2500\mathbf{I}_\text{L}^* = 300{,}000; \qquad \therefore\ \mathbf{I}_\text{L} = 120\,\text{A(rms)}$$

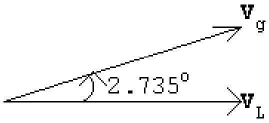
$$\mathbf{V}_g = \mathbf{V}_\text{L} + \mathbf{I}_\text{L}(0.1 + j1) = 2500 + (120)(0.1 + j1)$$

$$= 2512 + j120 = 2514.86\underline{/2.735^\circ}\,\text{Vrms}$$

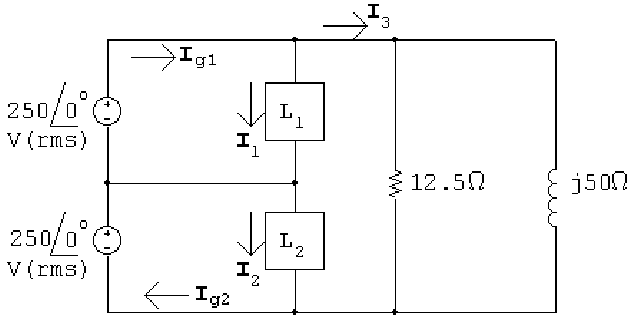
[**b**] $T = \dfrac{1}{f} = \dfrac{1}{60} = 16.67\,\text{ms}$

$$\frac{2.735^\circ}{360^\circ} = \frac{t}{16.67\ \text{ms}}; \qquad \therefore\ t = 126.62\,\mu\text{s}$$

[**c**] $\mathbf{V}_\text{L}$ lags $\mathbf{V}_g$ by 2.735° or $126.62\,\mu$s



P 10.22 [**a**]



$$250\mathbf{I}_1^* = 7500 + j2500; \qquad \therefore\ \mathbf{I}_1 = 30 - j10\,\text{A(rms)}$$

$$250\mathbf{I}_2^* = 2800 - j9600; \qquad \therefore\ \mathbf{I}_2 = 11.2 + j38.4\,\text{A(rms)}$$

$$\mathbf{I}_3 = \frac{500}{12.5} + \frac{500}{j50} = 40 - j10\,\text{A(rms)}$$

$$\mathbf{I}_{g1} = \mathbf{I}_1 + \mathbf{I}_3 = 70 - j20\,\text{A}$$

$$S_{g1} = 250(70 + j20) = 17{,}500 + j5000\,\text{VA}$$

Thus the $\mathbf{V}_{g1}$ source is delivering 17.5 kW and 5000 magnetizing vars.

$$\mathbf{I}_{g2} = \mathbf{I}_2 + \mathbf{I}_3 = 51.2 + j28.4\,\text{A(rms)}$$

$$S_{g2} = 250(51.2 - j28.4) = 12{,}800 - j7100\,\text{VA}$$

Thus the $\mathbf{V}_{g2}$ source is delivering 12.8 kW and absorbing 7100 magnetizing vars.

[b] $\sum P_{\text{gen}} = 17.5 + 12.8 = 30.3\,\text{kW}$

$$\sum P_{\text{abs}} = 7500 + 2800 + \frac{(500)^2}{12.5} = 30.3\,\text{kW} = \sum P_{\text{gen}}$$

$$\sum Q_{\text{del}} = 9600 + 5000 = 14.6\,\text{kVAR}$$

$$\sum Q_{\text{abs}} = 2500 + 7100 + \frac{(500)^2}{50} = 14.6\,\text{kVAR} = \sum Q_{\text{del}}$$

P 10.23 $S_1 = 1200 + 1196 = 2396 + j0\,\text{VA}$

$$\therefore \quad \mathbf{I}_1 = \frac{2396}{120} = 19.967\,\text{A}$$

$$S_2 = 860 + 600 + 240 = 1700 + j0\,\text{VA}$$

$$\therefore \quad \mathbf{I}_2 = \frac{1700}{120} = 14.167\,\text{A}$$

$$S_3 = 4474 + 12{,}200 = 16{,}674 + j0\,\text{VA}$$

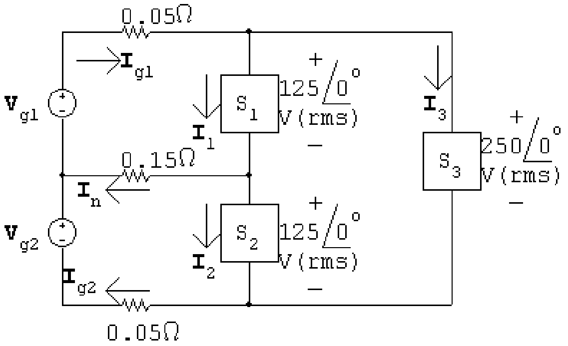
$$\therefore \quad \mathbf{I}_3 = \frac{16{,}674}{240} = 69.475\,\text{A}$$

$$\mathbf{I}_{g1} = \mathbf{I}_1 + \mathbf{I}_3 = 89.44\,\text{A}$$

$$\mathbf{I}_{g2} = \mathbf{I}_2 + \mathbf{I}_3 = 83.64\,\text{A}$$

Breakers will not trip since both feeder currents are less than 100 A.

P 10.24 [**a**]



$$\mathbf{I}_1 = \frac{5000 - j1250}{125} = 40 - j10\,\text{A (rms)}$$

$$\mathbf{I}_2 = \frac{6250 - j2500}{125} = 50 - j20\,\text{A (rms)}$$

$$\mathbf{I}_3 = \frac{8000 + j0}{250} = 32 + j0\,\text{A (rms)}$$

$$\therefore \quad \mathbf{I}_{g1} = 72 - j10\,\text{A (rms)}$$

$$\mathbf{I}_n = \mathbf{I}_1 - \mathbf{I}_2 = -10 + j10\,\text{A (rms)}$$

$$\mathbf{I}_{g2} = 82 - j20\,\text{A}$$

$$\mathbf{V}_{g1} = 0.05\mathbf{I}_{g1} + 125 + j0 + 0.15\mathbf{I}_n = 127.1 - j1\,\text{V(rms)}$$

$$\mathbf{V}_{g2} = -0.15\mathbf{I}_n + 125 + j0 + 0.05\mathbf{I}_{g2} = 130.6 - j2.5\,\text{V(rms)}$$

$$S_{g1} = -[(127.1 - j1)(72 + j10)] = -[9141.2 + j1343]\,\text{VA}$$

$$S_{g2} = -[(130.6 - j2.5)(82 + j20)] = -[10{,}759.2 + j2407]\,\text{VA}$$

Note: Both sources are delivering average power and magnetizing VAR to the circuit.

[**b**] $P_{0.05} = |\mathbf{I}_{g1}|^2(0.05) = 264.2\,\text{W}$

$$P_{0.15} = |\mathbf{I}_n|^2(0.15) = 30\,\text{W}$$

$$P_{0.05} = |\mathbf{I}_{g2}|^2(0.05) = 356.2\,\text{W}$$

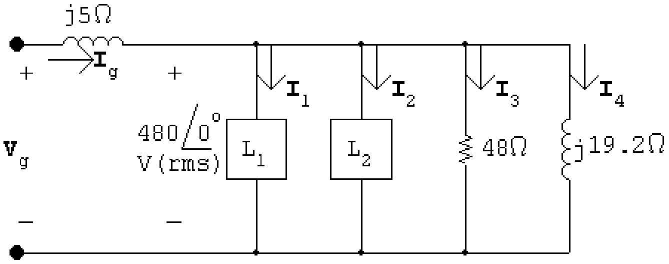
$$\sum P_{\text{dis}} = 264.2 + 30 + 356.2 + 5000 + 8000 + 6250 = 19{,}900.4\,\text{W}$$

$$\sum P_{\text{dev}} = 9141.2 + 10{,}759.2 = 19{,}900.4\,\text{W} = \sum P_{\text{dis}}$$

$$\sum Q_{\text{abs}} = 1250 + 2500 = 3750\,\text{VAR}$$

$$\sum Q_{\text{del}} = 1343 + 2407 = 3750\,\text{VAR} = \sum Q_{\text{abs}}$$

P 10.25



$$480\mathbf{I}_1^* = 7500 + j9000$$

$$\mathbf{I}_1^* = 15.625 + j18.75; \qquad \therefore \quad \mathbf{I}_1 = 15.625 - j18.75\,\text{A(rms)}$$

$$480\mathbf{I}_2^* = 2100 - j1800$$

$$\mathbf{I}_2^* = 4.375 - j3.75; \qquad \therefore \quad \mathbf{I}_2 = 4.375 + j3.75\,\text{A(rms)}$$

$$\mathbf{I}_3 = \frac{480\underline{/0^\circ}}{48} = 10 + j0\,\text{A}; \qquad \mathbf{I}_4 = \frac{480\underline{/0^\circ}}{j19.2} = 0 - j25\,\text{A}$$

$$\mathbf{I}_g = \mathbf{I}_1 + \mathbf{I}_2 + \mathbf{I}_3 + \mathbf{I}_4 = 30 - j40\,\text{A}$$

$$\mathbf{V}_g = 480 + (30 - j40)(j0.5) = 500 + j15 = 500.22\underline{/1.72^\circ}\,\text{V (rms)}$$

P 10.26 **[a]** $Z_1 = 240 + j70 = 250\underline{/16.26^\circ}\,\Omega$

pf $= \cos(16.26^\circ) = 0.96$ lagging

rf $= \sin(16.26^\circ) = 0.28$

$Z_2 = 160 - j120 = 200\underline{/-36.87^\circ}\,\Omega$

pf $= \cos(-36.87^\circ) = 0.8$ leading

rf $= \sin(-36.87^\circ) = -0.6$

$Z_3 = 30 - j40 = 50\underline{/-53.13^\circ}\,\Omega$

pf $= \cos(-53.13^\circ) = 0.6$ leading

rf $= \sin(-53.13^\circ) = -0.8$

[**b**] $Y = Y_1 + Y_2 + Y_3$

$$Y_1 = \frac{1}{250\underline{/16.26^\circ}}; \qquad Y_2 = \frac{1}{200\underline{/-36.87^\circ}}; \qquad Y_3 = \frac{1}{50\underline{/-53.13^\circ}}$$

$$Y = 19.84 + j17.88\,\text{mS}$$

$$Z = \frac{1}{Y} = 37.44\underline{/-42.03^\circ}\,\Omega$$

$$\text{pf } = \cos(-42.03^\circ) = 0.74 \text{ leading}$$

$$\text{rf } = \sin(-42.03^\circ) = -0.67$$

P 10.27 [**a**] $S_1 = 16 + j18\,\text{kVA}; \qquad S_2 = 6 - j8\,\text{kVA}; \qquad S_3 = 8 + j0\,\text{kVA}$

$$S_\text{T} = S_1 + S_2 + S_3 = 30 + j10\,\text{kVA}$$

$$250\mathbf{I}^* = (30 + j10) \times 10^3; \qquad \therefore\ \mathbf{I} = 120 - j40\,\text{A}$$

$$Z = \frac{250}{120 - j40} = 1.875 + j0.625\,\Omega = 1.98\underline{/18.43^\circ}\,\Omega$$

[**b**] $\text{pf } = \cos(18.43^\circ) = 0.9487 \text{ lagging}$

P 10.28 [**a**] From the solution to Problem 10.26 we have

$$\mathbf{I}_\text{L} = 120 - j40\,\text{A (rms)}$$

$$\therefore\ \mathbf{V}_s = 250\underline{/0^\circ} + (120 - j40)(0.01 + j0.08) = 254.4 + j9.2$$
$$= 254.57\underline{/2.07^\circ}\,\text{V (rms)}$$

[**b**] $|\mathbf{I}_\text{L}| = \sqrt{16{,}000}$

$$P_\ell = (16{,}000)(0.01) = 160\,\text{W} \qquad Q_\ell = (16{,}000)(0.08) = 1280\,\text{VAR}$$

[**c**] $P_s = 30{,}000 + 160 = 30.16\,\text{kW} \qquad Q_s = 10{,}000 + 1280 = 11.28\,\text{kVAR}$

[**d**] $\eta = \frac{30}{30.16}(100) = 99.47\%$

P 10.29 [**a**] $\mathbf{I} = \frac{465\underline{/0^\circ}}{124 + j93} = 2.4 - j1.8 = 3\underline{/-36.87^\circ}\,\text{A(rms)}$

$$P = (3)^2(4) = 36\,\text{W}$$

[**b**] $Y_\text{L} = \frac{1}{120 + j90} = 5.33 - j4 \text{ mS}$

$$\therefore\ X_\text{C} = \frac{1}{-4 \times 10^{-3}} = -250\,\Omega$$

[c] $Z_{\rm L} = \dfrac{1}{5.33 \times 10^{-3}} = 187.5\,\Omega$

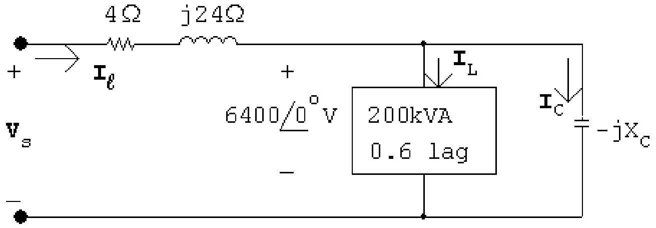
[d] $\mathbf{I} = \dfrac{465\underline{/0^\circ}}{191.5 + j3} = 2.4279\underline{/-0.9^\circ}\,\text{A}$

$$P = (2.4279)^2(4) = 23.58\,\text{W}$$

[e] $\% = \dfrac{23.58}{36}(100) = 65.5\%$

Thus the power loss after the capacitor is added is 65.5% of the power loss before the capacitor is added.

P 10.30



$$\mathbf{I}_{\rm L} = \frac{120{,}000 - j160{,}000}{6400} = 18.75 - j25\,\text{A (rms)}$$

$$\mathbf{I}_{\rm C} = \frac{6400}{-jX_{\rm C}} = j\frac{6400}{X_{\rm C}} = jI_{\rm C}$$

$$\mathbf{I}_\ell = 18.75 - j25 + jI_{\rm C} = 18.75 + j(I_{\rm C} - 25)$$

$$\mathbf{V}_s = 6400 + (4 + j24)[18.75 + j(I_{\rm C} - 25)]$$
$$= (7075 - 24I_{\rm C}) + j(350 + 4I_{\rm C})$$

$$|\mathbf{V}_s|^2 = (7075 - 24I_{\rm C})^2 + (350 + 4I_{\rm C})^2 = (6400)^2$$

$$\therefore\quad 592I_{\rm C}^2 - 336{,}800I_{\rm C} + 9{,}218{,}125 = 0$$

$$I_{\rm C} = 284.46 \pm 255.63 = 28.33\,\text{A(rms)}^*$$

*Select the smaller value of $I_{\rm C}$ to minimize the magnitude of I_ℓ.

$$\therefore\quad X_{\rm C} = -\frac{6400}{28.33} = -221.99$$

$$\therefore\quad C = \frac{1}{(221.99)(120\pi)} = 11.95\,\mu\text{F}$$

P 10.31 **[a]** From Problem 9.75,

$$Z_{\rm ab} = 100 + j136.26 \quad \text{so}$$

$$\mathbf{I}_1 = \frac{50}{100 + j13.74 + 100 + 136.26} = \frac{50}{200 + j150} = 160 - j120\,\text{mA}$$

$$\mathbf{I}_2 = \frac{j\omega M}{Z_{22}}\mathbf{I}_1 = \frac{j270}{800 + j600}(0.16 - j0.12) = 51.84 + j15.12\,\text{mA}$$

$$\mathbf{V}_L = (300 + j100)(0.05184 + j0.01512) = 14.04 + j9.72$$

$$|\mathbf{V}_L| = 17.08\,\text{V}$$

[b] $P_g(\text{ideal}) = 50(0.16) = 8\,\text{W}$

$$P_g(\text{practical}) = 8 - |\mathbf{I}_1|^2(100) = 4\,\text{W}$$

$$P_{\rm L} = |\mathbf{I}_2|^2(300) = 0.8748\,\text{W}$$

$$\%\text{ delivered } = \frac{0.8748}{4}(100) = 21.87\%$$

P 10.32 **[a]** $S_o = \text{ original load } = 1600 + j\frac{1600}{0.8}(0.6) = 1600 + j1200\,\text{kVA}$

$$S_f = \text{ final load } = 1920 + j\frac{1920}{0.96}(0.28) = 1920 + j560\,\text{kVA}$$

$$\therefore\quad Q_{\rm added} = 560 - 1200 = -640\,\text{kVAR}$$

[b] deliver

[c] $S_{\rm a} = \text{ added load } = 320 - j640 = 715.54\underline{/-63.43^\circ}\,\text{kVA}$

$$\text{pf } = \cos(-63.43) = 0.447 \text{ leading}$$

[d] $\mathbf{I}_{\rm L}^* = \frac{(1600 + j1200)\times 10^3}{2400} = 666.67 + j500\,\text{A}$

$$\mathbf{I}_{\rm L} = 666.67 - j500 = 833.33\underline{/-36.87^\circ}\,\text{A(rms)}$$

$$|\mathbf{I}_{\rm L}| = 833.33\,\text{A(rms)}$$

[e] $\mathbf{I}_{\rm L}^* = \frac{(1920 + j560)\times 10^3}{2400} = 800 + j233.33$

$$\mathbf{I}_{\rm L} = 800 - j233.33 = 833.33\underline{/-16.26^\circ}\,\text{A(rms)}$$

$$|\mathbf{I}_{\rm L}| = 833.33\,\text{A(rms)}$$

P 10.33 **[a]** $P_{\rm before} = P_{\rm after} = (833.33)^2(0.05) = 34{,}722.22\,\text{W}$

[b] $\mathbf{V}_s(\text{before}) = 2400 + (666.67 - j500)(0.05 + j0.4)$

$$= 2633.33 + j241.67 = 2644.4\underline{/5.24^\circ}\,\text{V(rms)}$$

$$|\mathbf{V}_s(\text{before})| = 2644.4\,\text{V(rms)}$$

$$\mathbf{V}_s(\text{after}) = 2400 + (800 - j233.33)(0.05 + j0.4)$$

$$= 2533.33 + j308.33 = 2552.028\underline{/6.94^\circ}\,\text{V(rms)}$$

$$|\mathbf{V}_s(\text{after})| = 2552.028\,\text{V(rms)}$$

P 10.34 **[a]** $S_\text{L} = 20{,}000(0.85 + j0.53) = 17{,}000 + j10{,}535.65\,\text{VA}$

$$125\mathbf{I}_\text{L}^* = (17{,}000 + j10{,}535.65); \qquad \mathbf{I}_\text{L}^* = 136 + j84.29\,\text{A(rms)}$$

$$\therefore \quad \mathbf{I}_\text{L} = 136 - j84.29\,\text{A(rms)}$$

$$\mathbf{V}_s = 125 + (136 - j84.29)(0.01 + j0.08) = 133.10 + j10.04$$

$$= 133.48\underline{/4.31^\circ}\,\text{V(rms)}$$

$$|\mathbf{V}_s| = 133.48\,\text{V(rms)}$$

[b] $P_\ell = |\mathbf{I}_\ell|^2(0.01) = (160)^2(0.01) = 256\,\text{W}$

[c] $\dfrac{(125)^2}{X_\text{C}} = -10{,}535.65; \qquad X_\text{C} = -1.48306\,\Omega$

$$-\frac{1}{\omega C} = -1.48306; \qquad C = \frac{1}{(1.48306)(120\pi)} = 1788.59\,\mu\text{F}$$

[d] $\mathbf{I}_\ell = 136 + j0\,\text{A(rms)}$

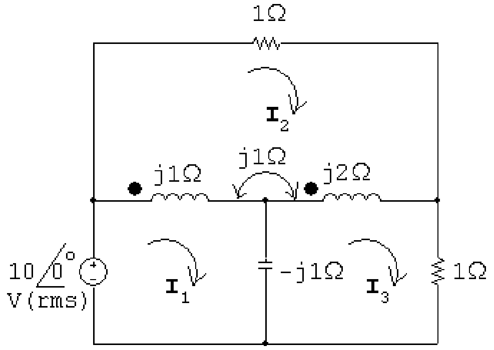
$$\mathbf{V}_s = 125 + 136(0.01 + j0.08) = 126.36 + j10.88$$

$$= 126.83\underline{/4.92^\circ}\,\text{V(rms)}$$

$$|\mathbf{V}_s| = 126.83\,\text{V(rms)}$$

[e] $P_\ell = (136)^2(0.01) = 184.96\,\text{W}$

P 10.35 **[a]**



$$10 = j1(\mathbf{I}_1 - \mathbf{I}_2) + j1(\mathbf{I}_3 - \mathbf{I}_2) - j1(\mathbf{I}_1 - \mathbf{I}_3)$$

$$0 = 1\mathbf{I}_2 + j2(\mathbf{I}_2 - \mathbf{I}_3) + j1(\mathbf{I}_2 - \mathbf{I}_1) + j1(\mathbf{I}_2 - \mathbf{I}_1) + j1(\mathbf{I}_2 - \mathbf{I}_3)$$

$$0 = \mathbf{I}_3 - j1(\mathbf{I}_3 - \mathbf{I}_1) + j2(\mathbf{I}_3 - \mathbf{I}_2) + j1(\mathbf{I}_1 - \mathbf{I}_2)$$

Solving,

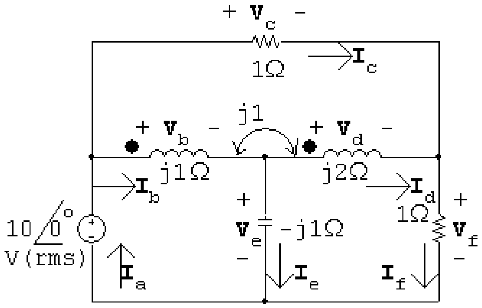
$$\mathbf{I}_1 = 6.25 + j7.5\,\text{A(rms)}; \quad \mathbf{I}_2 = 5 + j2.5\,\text{A(rms)}; \quad \mathbf{I}_3 = 5 - j2.5\,\text{A(rms)}$$

$$\mathbf{I}_\text{a} = \mathbf{I}_1 = 6.25 + j7.5\,\text{A} \qquad \mathbf{I}_\text{b} = \mathbf{I}_1 - \mathbf{I}_2 = 1.25 + j5\,\text{A}$$

$$\mathbf{I}_\text{c} = \mathbf{I}_2 = 5 + j2.5\,\text{A} \qquad \mathbf{I}_\text{d} = \mathbf{I}_3 - \mathbf{I}_2 = -j5\,\text{A}$$

$$\mathbf{I}_\text{e} = \mathbf{I}_1 - \mathbf{I}_3 = 1.25 + j10\,\text{A} \qquad \mathbf{I}_\text{f} = \mathbf{I}_3 = 5 - j2.5\,\text{A}$$

[**b**]



$$\mathbf{V}_\text{a} = 10\,\text{V} \qquad \mathbf{V}_\text{b} = j1\mathbf{I}_\text{b} + j1\mathbf{I}_\text{d} = j1.25\,\text{V}$$

$$\mathbf{V}_\text{c} = 1\mathbf{I}_\text{c} = 5 + j2.5\,\text{V} \qquad \mathbf{V}_\text{d} = j2\mathbf{I}_\text{d} - j1\mathbf{I}_\text{b} = 5 + j1.25\,\text{V}$$

$$\mathbf{V}_\text{e} = -j1\mathbf{I}_\text{e} = 10 - j1.25\,\text{V} \qquad \mathbf{V}_\text{f} = 1\mathbf{I}_\text{f} = 5 - j2.5\,\text{V}$$

$$S_\text{a} = -10\mathbf{I}_\text{a}^* = -62.5 + j75\,\text{VA}$$

$$S_\text{b} = \mathbf{V}_\text{b}\mathbf{I}_\text{b}^* = 6.25 + j1.5625\,\text{VA}$$

$$S_\text{c} = \mathbf{V}_\text{c}\mathbf{I}_\text{c}^* = 31.25 + j0\,\text{VA}$$

$$S_\text{d} = \mathbf{V}_\text{d}\mathbf{I}_\text{d}^* = -6.25 + j25\,\text{VA}$$

$$S_\text{e} = \mathbf{V}_\text{e}\mathbf{I}_\text{e}^* = 0 - j101.5625\,\text{VA}$$

$$S_\text{f} = \mathbf{V}_\text{f}\mathbf{I}_\text{f}^* = 31.25\,\text{VA}$$

[**c**] $\sum P_\text{dev} = 62.5\,\text{W}$

$$\sum P_\text{abs} = 31.25 + 31.25 = 62.5\,\text{W}$$

Note that the total power absorbed by the coupled coils is zero:
$6.25 - 6.25 = 0 = P_\text{b} + P_\text{d}$

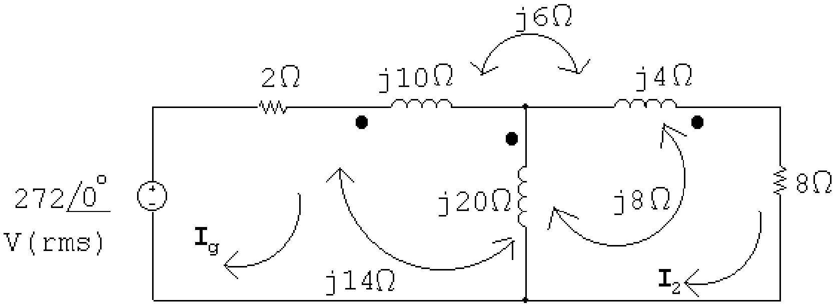
[d] $\sum Q_{\text{dev}} = 101.5625\,\text{VAR}$

Both the source and the capacitor are developing magnetizing vars.

$\sum Q_{\text{abs}} = 75 + 1.5625 + 25 = 101.5625\,\text{VAR}$

$\sum Q$ absorbed by the coupled coils is $Q_{\text{b}} + Q_{\text{d}} = 26.5625$

P 10.36 **[a]**



$$272\underline{/0^\circ} = 2\mathbf{I}_g + j10\mathbf{I}_g + j14(\mathbf{I}_g - \mathbf{I}_2) - j6\mathbf{I}_2$$
$$+j14\mathbf{I}_g - j8\mathbf{I}_2 + j20(\mathbf{I}_g - \mathbf{I}_2)$$
$$0 = j20(\mathbf{I}_2 - \mathbf{I}_g) - j14\mathbf{I}_g + j8\mathbf{I}_2 + j4\mathbf{I}_2$$
$$+j8(\mathbf{I}_2 - \mathbf{I}_g) - j6\mathbf{I}_g + 8\mathbf{I}_2$$

Solving,

$I_g = 20 - j4\,\text{A(rms)}; \qquad \mathbf{I}_2 = 24\underline{/0^\circ}\,\text{A(rms)}$

$P_{8\Omega} = (24)^2(8) = 4608\,\text{W}$

[b] $P_g(\text{developed}) = (272)(20) = 5440\,\text{W}$

[c] $Z_{\text{ab}} = \dfrac{\mathbf{V}_g}{\mathbf{I}_g} - 2 = \dfrac{272}{20 - j4} - 2 = 11.08 + j2.62 = 11.38\underline{/13.28^\circ}\,\Omega$

[d] $P_{2\Omega} = |I_g|^2(2) = 832\,\text{W}$

$\sum P_{\text{diss}} = 832 + 4608 = 5440\,\text{W} = \sum P_{\text{dev}}$

P 10.37 **[a]** $Z_{\text{ab}} = \left(1 + \dfrac{N_1}{N_2}\right)^2 (1 - j2) = 25 - j50\,\Omega$

$\therefore\ \mathbf{I}_1 = \dfrac{100\underline{/0^\circ}}{15 + j50 + 25 - j50} = 2.5\underline{/0^\circ}\,\text{A}$

$\mathbf{I}_2 = \dfrac{N_1}{N_2}\mathbf{I}_1 = 10\underline{/0^\circ}\,\text{A}$

$\therefore\ \mathbf{I}_{\text{L}} = \mathbf{I}_1 + \mathbf{I}_2 = 12.5\underline{/0^\circ}\,\text{A(rms)}$

$P_{1\Omega} = (12.5)^2(1) = 156.25\,\text{W}$

$P_{15\Omega} = (2.5)^2(15) = 93.75\,\text{W}$

[**b**] $P_g = -100(2.5\underline{/0^\circ}) = -250\,\text{W}$

$$\sum P_{\text{abs}} = 156.25 + 93.75 = 250\,\text{W} = \sum P_{\text{dev}}$$

P 10.38 [**a**] $25a_1^2 + 4a_2^2 = 500$

$$\mathbf{I}_{25} = a_1\mathbf{I}; \qquad P_{25} = a_1^2\mathbf{I}^2(25)$$

$$\mathbf{I}_4 = a_2\mathbf{I}; \qquad P_4 = a_2^2\mathbf{I}^2(4)$$

$$P_4 = 4P_{25}; \qquad a_2^2\mathbf{I}^2 4 = 100a_1^2\mathbf{I}^2$$

$$\therefore \qquad 100a_1^2 = 4a_2^2$$

$$25a_1^2 + 100a_1^2 = 500; \qquad a_1 = 2$$

$$25(4) + 4a_2^2 = 500; \qquad a_2 = 10$$

[**b**] $\mathbf{I} = \dfrac{2000\underline{/0^\circ}}{500 + 500} = 2\underline{/0^\circ}\,\text{A(rms)}$

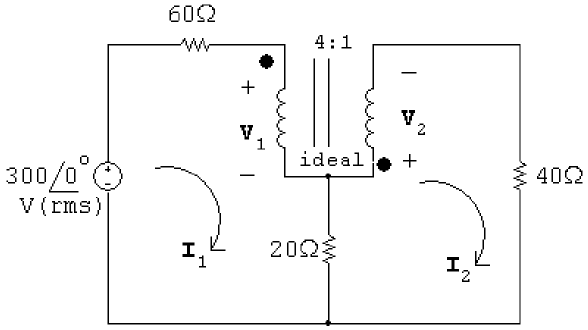
$$\mathbf{I}_{25} = a_1\mathbf{I} = 4\,\text{A}$$

$$P_{25\Omega} = (16)(25) = 400\,\text{W}$$

[**c**] $\mathbf{I}_4 = a_2\mathbf{I} = 10(2) = 20\,\text{A(rms)}$

$$\mathbf{V}_4 = (20)(4) = 80\underline{/0^\circ}\,\text{V(rms)}$$

P 10.39 [**a**]



$$300 = 60\mathbf{I}_1 + \mathbf{V}_1 + 20(\mathbf{I}_1 - \mathbf{I}_2)$$

$$0 = 20(\mathbf{I}_2 - \mathbf{I}_1) + \mathbf{V}_2 + 40\mathbf{I}_2$$

$$\mathbf{V}_2 = \frac{1}{4}\mathbf{V}_1; \qquad \mathbf{I}_2 = -4\mathbf{I}_1$$

Solving,

$$\mathbf{V}_1 = 260\,\text{V (rms)}; \qquad \mathbf{V}_2 = 65\,\text{V (rms)}$$

$$\mathbf{I}_1 = 0.25\,\text{A (rms)}; \qquad \mathbf{I}_2 = -1.0\,\text{A (rms)}$$

$$\mathbf{V}_{5\text{A}} = \mathbf{V}_1 + 20(\mathbf{I}_1 - \mathbf{I}_2) = 285\,\text{V (rms)}$$

$$\therefore \quad P = -(285)(5) = -1425\,\text{W}$$

Thus 1425 W is delivered by the current source to the circuit.

[b] $\mathbf{I}_{20\Omega} = \mathbf{I}_1 - \mathbf{I}_2 = 1.25\,\text{A(rms)}$

$$\therefore \quad P_{20\Omega} = (1.25)^2(20) = 31.25\,\text{W}$$

P 10.40 $Z_\text{L} = |Z_\text{L}|\underline{/\theta^\circ} = |Z_\text{L}|\cos\theta^\circ + j|Z_\text{L}|\sin\theta^\circ$

$$\text{Thus} \quad |\mathbf{I}| = \frac{|\mathbf{V}_\text{Th}|}{\sqrt{(R_\text{Th} + |Z_\text{L}|\cos\theta)^2 + (X_\text{Th} + |Z_\text{L}|\sin\theta)^2}}$$

$$\text{Therefore} \quad P = \frac{0.5|\mathbf{V}_\text{Th}|^2|Z_\text{L}|\cos\theta}{(R_\text{Th} + |Z_\text{L}|\cos\theta)^2 + (X_\text{Th} + |Z_\text{L}|\sin\theta)^2}$$

Let D = demoninator in the expression for P, then

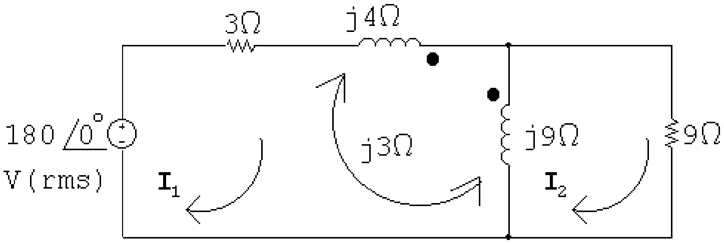
$$\frac{dP}{d|Z_\text{L}|} = \frac{(0.5|\mathbf{V}_\text{Th}|^2\cos\theta)(D\cdot 1 - |Z_\text{L}|dD/d|Z_\text{L}|)}{D^2}$$

$$\frac{dD}{d|Z_\text{L}|} = 2(R_\text{Th} + |Z_\text{L}|\cos\theta)\cos\theta + 2(X_\text{Th} + |Z_\text{L}|\sin\theta)\sin\theta$$

$$\frac{dP}{d|Z_\text{L}|} = 0 \quad \text{when} \quad D = |Z_\text{L}|\left(\frac{dD}{d|Z_\text{L}|}\right)$$

Substituting the expressions for D and $(dD/d|Z_\text{L}|)$ into this equation gives us the relationship $R_\text{Th}^2 + X_\text{Th}^2 = |Z_\text{L}|^2$ or $|Z_\text{Th}| = |Z_\text{L}|$.

P 10.41 **[a]**



$$180 = 3\mathbf{I}_1 + j4\mathbf{I}_1 + j3(\mathbf{I}_2 - \mathbf{I}_1) + j9(\mathbf{I}_1 - \mathbf{I}_2) - j3\mathbf{I}_1$$

$$0 = 9\mathbf{I}_2 + j9(\mathbf{I}_2 - \mathbf{I}_1) + j3\mathbf{I}_1$$

Solving,

$$\mathbf{I}_1 = 18 - j18\,\text{A(rms)}; \qquad \mathbf{I}_2 = 12\underline{/0^\circ}\,\text{A(rms)}$$

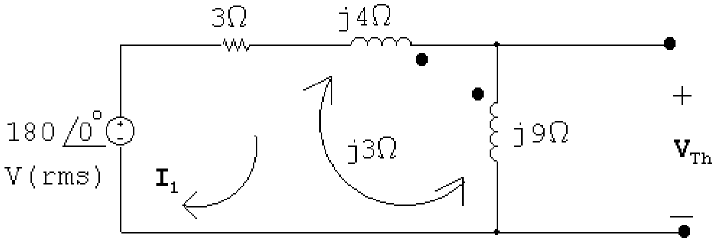
$$\therefore \quad \mathbf{V}_o = (12)(9) = 108\,\text{V(rms)}$$

[b] $P = (12)^2(9) = 1296\,\text{W}$

[c] $S_g = -(180)(18 + j18) = -3240 - j3240\,\text{VA} \quad \therefore \quad P_g = -3240\,\text{W}$

$$\%\ \text{delivered} = \frac{1296}{3240}(100) = 40\%$$

P 10.42 [a] Open circuit voltage:

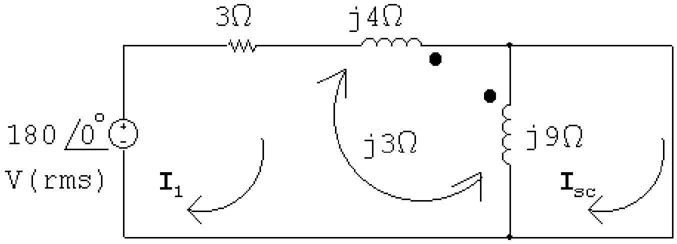


$$180 = 3\mathbf{I}_1 + j4\mathbf{I}_1 - j3\mathbf{I}_1 + j9\mathbf{I}_1 - j3\mathbf{I}_1$$

$$\therefore \quad \mathbf{I}_1 = \frac{180}{3 + j7} = 9.31 - j21.72\,\text{A(rms)}$$

$$\mathbf{V}_{\text{Th}} = j9\mathbf{I}_1 - j3\mathbf{I}_1 = j6\mathbf{I}_1 = 130.34 + j55.86\,\text{V}$$

Short circuit current:



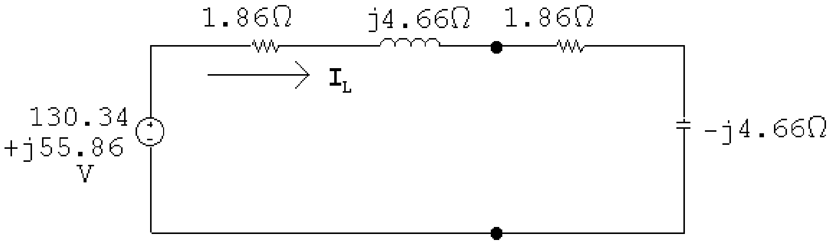
$$180 = 3\mathbf{I}_1 + j4\mathbf{I}_1 + j3(\mathbf{I}_{\text{sc}} - \mathbf{I}_1) + j9(\mathbf{I}_1 - \mathbf{I}_{\text{sc}}) - j3\mathbf{I}_1$$

$$0 = -j9(\mathbf{I}_{\text{sc}} - \mathbf{I}_1) + j3\mathbf{I}_1$$

Solving,

$$\mathbf{I}_{\text{sc}} = 20 - j20\,\text{A}$$

$$Z_{\text{Th}} = \frac{\mathbf{V}_{\text{Th}}}{\mathbf{I}_{\text{sc}}} = \frac{130.34 + j55.86}{20 - j20} = 1.86 + j4.66\,\Omega$$



$$\mathbf{I}_{\text{L}} = \frac{130.34 + j55.86}{3.72} = 35 + j15 = 38.08\underline{/23.20^\circ}\,\text{A}$$

$$P_{\rm L} = (38.08)^2(1.86) = 2700\,\mathrm{W}$$

[b] $$\mathbf{I}_1 = \frac{Z_o + j9}{j6}\mathbf{I}_2 = \frac{1.86 - j4.66 + j9}{j6}(35 + j15) = 30\underline{/0^\circ}\,\mathrm{A(rms)}$$

$$P_{\rm dev} = (180)(30) = 5400\,\mathrm{W}$$

[c] Begin by choosing the capacitor value from Appendix H that is closest to the required reactive impedance, assuming the frequency of the source is 60 Hz:

$$4.66 = \frac{1}{2\pi(60)C} \qquad \text{so} \qquad C = \frac{1}{2\pi(60)(4.66)} = 569.22\,\mu\mathrm{F}$$

Choose the capacitor value closest to this capacitance from Appendix H, which is 470 μF. Then,

$$X_{\rm L} = -\frac{1}{2\pi(60)(470 \times 10^{-6})} = -5.6438\,\Omega$$

Now set $R_{\rm L}$ as close as possible to $\sqrt{R_{\rm Th}^2 + (X_{\rm L} + X_{\rm Th})^2}$:

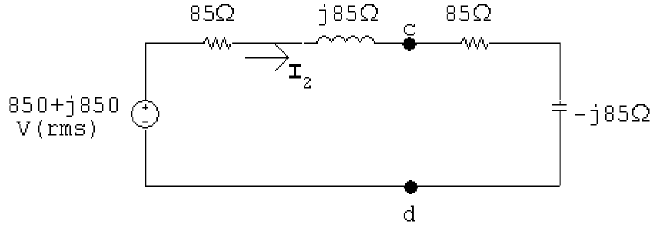
$$R_{\rm L} = \sqrt{1.856^2 + (4.66 - 5.6438)^2} = 2.11\,\Omega$$

The closest single resistor value from Appendix H is 10 Ω. The resulting real power developed by the source is calculated below, using the Thévenin equivalent circuit:

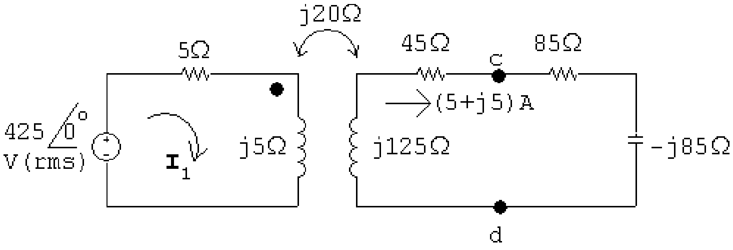
$$\mathbf{I} = \frac{130.34 + j55.86}{1.86 + j4.66 + 10 - j5.6438} = 11.9157\underline{/27.94^\circ}$$

$$P = |130.34 + j55.86|(11.9157) = 1689.7\,\mathrm{W} \qquad \text{(instead of 5400 W)}$$

P 10.43 [a] From Problem 9.78, $Z_{\rm Th} = 85 + j85\,\Omega$ and $\mathbf{V}_{\rm Th} = 850 + j850\,\mathrm{V}$. Thus, for maximum power transfer, $Z_{\rm L} = Z_{\rm Th}^* = 85 - j85\,\Omega$:



$$\mathbf{I}_2 = \frac{850 + j850}{170} = 5 + j5\,\mathrm{A}$$



$$425\underline{/0^\circ} = (5 + j5)\mathbf{I}_1 - j20(5 + j5)$$

$$\therefore \quad \mathbf{I}_1 = \frac{325 + j100}{5 + j5} = 42.5 - j22.5\,\text{A}$$

$$S_g(\text{del}) = 425(42.5 + j22.5) = 18{,}062.5 + j9562.5\,\text{VA}$$

$$P_g = 18{,}062.5\,\text{W}$$

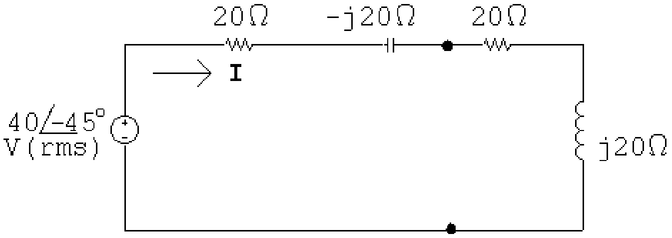
[b] $P_{\text{loss}} = |\mathbf{I}_1|^2(5) + |\mathbf{I}_2|^2(45) = 11{,}562.5 + 2250 = 13{,}812.5\,\text{W}$

$$\%\text{ loss in transformer} = \frac{13{,}812.5}{18{,}062.5}(100) = 76.47\%$$

P 10.44 [a] $Z_{\text{Th}} = -j40 + \dfrac{(40)(j40)}{40 + j40} = 20 - j20\,\Omega$

$$\therefore \quad Z_{\text{L}} = Z_{\text{Th}}^* = 20 + j20\,\Omega$$

[b] $\mathbf{V}_{\text{Th}} = \dfrac{80\underline{/0^\circ}(40)}{40 + j40} = 40(1 - j1) = 40\sqrt{2}\underline{/-45^\circ}\,\text{V}$



$$\mathbf{I} = \frac{40\sqrt{2}\underline{/-45^\circ}}{40} = \sqrt{2}\underline{/-45^\circ}\,\text{A}$$

$$|\mathbf{I}_{\text{rms}}| = 1\,\text{A}$$

$$P_{\text{load}} = (1)^2(20 \times 10^3) = 20\,\text{W}$$

[c] The closest resistor value from Appendix H is 22 Ω. Find the inductor value:

$(5000)L = 20$ so $L = 4\,\text{mH}$

The closest inductor value is 1 mH.

$$\mathbf{I} = \frac{40\underline{/-45^\circ}}{20 - j20 + 22 + j5} = \frac{40\underline{/-45^\circ}}{42 - j15} = 0.8969\underline{/-25.35^\circ}\ \text{A(rms)}$$

$$P_{\text{load}} = (0.8969)^2(22) = 17.70\,\text{W} \qquad \text{(instead of 20 W)}$$

P 10.45 [a] $\dfrac{115.2 - j86.4 - 240}{Z_{\text{Th}}} + \dfrac{115.2 - j80}{90 - j30} = 0$

$$\therefore \quad Z_{\text{Th}} = \frac{240 - 115.2 + j86.4}{1.44 - j0.48} = 60 + j80\,\Omega$$

$$\therefore \quad Z_{\text{L}} = 60 - j80\,\Omega$$

[**b**] $\mathbf{I} = \dfrac{240\underline{/0^\circ}}{120\underline{/0^\circ}} = 2\underline{/0^\circ}\,\mathrm{A(rms)}$

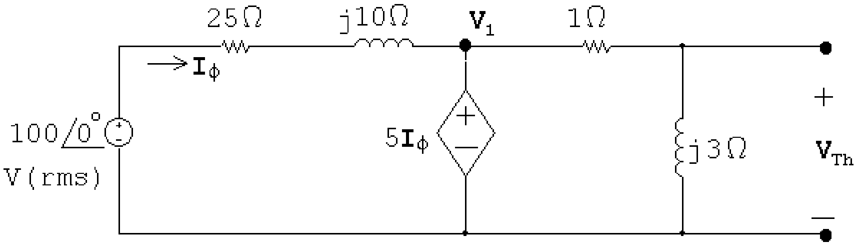
$$P = (2)^2(60) = 240\,\mathrm{W}$$

[**c**] Let $R = 15\,\Omega + 15\,\Omega + 15\,\Omega + 15\,\Omega = 60\,\Omega$

$$\frac{1}{2\pi(60)C} = 80 \qquad \text{so} \qquad C = \frac{1}{2\pi(60)(80)} = 33.16\,\mu\mathrm{F}$$

Let $C = 22\,\mu\mathrm{F}\|10\,\mu\mathrm{F}\|1\,\mu\mathrm{F} = 33\,\mu\mathrm{F}$

P 10.46 [**a**] Open circuit voltage:



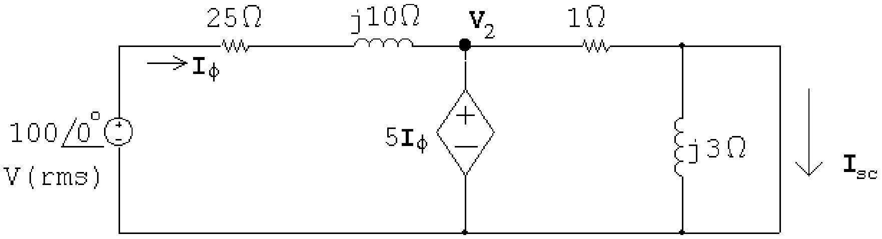
$$\mathbf{V}_1 = 5\mathbf{I}\phi = 5\frac{100 - 5\mathbf{I}_\phi}{25 + j10}$$

$$(25 + j10)\mathbf{I}_\phi = 100 - 5\mathbf{I}\phi$$

$$\mathbf{I}_\phi = \frac{100}{30 + j10} = 3 - j\,\mathrm{A}$$

$$\mathbf{V}_{\mathrm{Th}} = \frac{j3}{1 + j3}(5\mathbf{I}_\phi) = 15\,\mathrm{V}$$

Short circuit current:



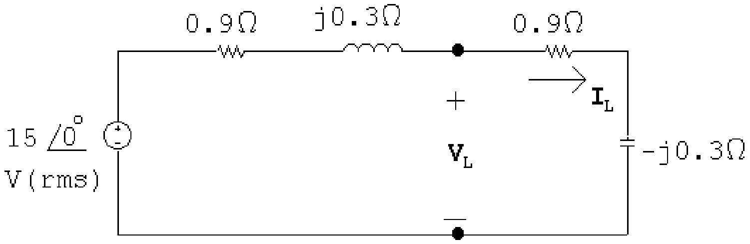
$$\mathbf{V}_2 = 5\mathbf{I}_\phi = \frac{100 - 5\mathbf{I}_\phi}{25 + j10}$$

$$\mathbf{I}_\phi = 3 - j1\,\mathrm{A}$$

$$\mathbf{I}_{\mathrm{sc}} = \frac{5\mathbf{I}_\phi}{1} = 15 - j5\,\mathrm{A}$$

$$Z_{\mathrm{Th}} = \frac{15}{15 - j5} = 0.9 + j0.3\,\Omega$$

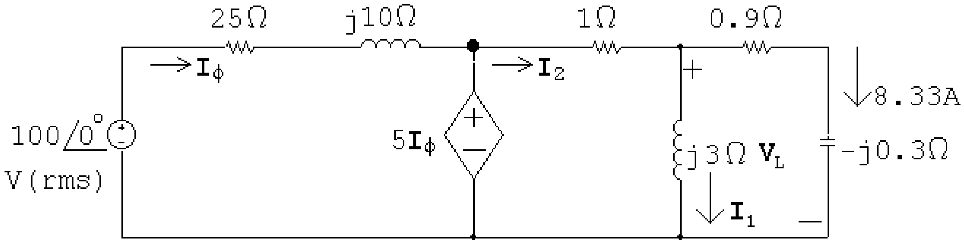
$$Z_L = Z_{\text{Th}}^* = 0.9 - j0.3\,\Omega$$



$$\mathbf{I}_{\text{L}} = \frac{0.3}{1.8} = 8.33\,\text{A(rms)}$$

$$P = |\mathbf{I}_L|^2(0.9) = 62.5\,\text{W}$$

[b] $\mathbf{V}_{\text{L}} = (0.9 - j0.3)(8.33) = 7.5 - j2.5\,\text{V(rms)}$



$$\mathbf{I}_1 = \frac{\mathbf{V}_{\text{L}}}{j3} = -0.833 - j2.5\,\text{A(rms)}$$

$$\mathbf{I}_2 = \mathbf{I}_1 + \mathbf{I}_{\text{L}} = 7.5 - j2.5\,\text{A(rms)}$$

$$5\mathbf{I}_\phi = \mathbf{I}_2 + \mathbf{V}_L \quad \therefore \quad \mathbf{I}_\phi = 3 - j1\,\text{A}$$

$$\mathbf{I}_{\text{d.s.}} = \mathbf{I}_\phi - \mathbf{I}_2 = -4.5 + j1.5\,\text{A}$$

$$S_g = -100(3 + j1) = -300 - j100\,\text{VA}$$

$$S_{\text{d.s.}} = 5(3 - j1)(-4.5 - j1.5) = -75 + j0\,\text{VA}$$

$$P_{\text{dev}} = 300 + 75 = 375\,\text{W}$$

$$\%\ \text{developed} = \frac{62.5}{375}(100) = 16.67\%$$

Checks:

$$P_{25\Omega} = (10)(25) = 250\,\text{W}$$

$$P_{1\Omega} = (67.5)(1) = 67.5\,\text{W}$$

$$P_{0.9\Omega} = 62.5\,\text{W}$$

$$\sum P_{\text{abs}} = 230 + 62.5 + 67.5 = 375 = \sum P_{\text{dev}}$$

$$Q_{j10} = (10)(10) = 100\,\text{VAR}$$

$$Q_{j3} = (6.94)(3) = 20.82\,\text{VAR}$$

$$Q_{-j0.3} = (69.4)(-0.3) = -20.82\,\text{VAR}$$

$$Q_{\text{source}} = -100\,\text{VAR}$$

$$\sum Q = 100 + 20.82 - 20.82 - 100 = 0$$

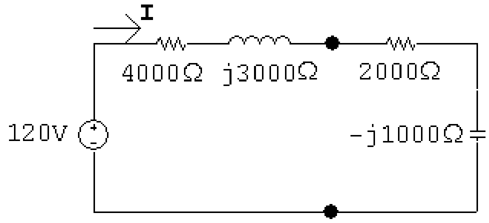
P 10.47 [**a**] First find the Thévenin equivalent:

$$j\omega L = j3000\,\Omega$$

$$Z_{\text{Th}} = 6000\|12{,}000 + j3000 = 4000 + j3000\,\Omega$$

$$\mathbf{V}_{\text{Th}} = \frac{12{,}000}{6000 + 12{,}000}(180) = 120\,\text{V}$$

$$\frac{-j}{\omega C} = -j1000\,\Omega$$



$$\mathbf{I} = \frac{120}{6000 + j2000} = 18 - j6\,\text{mA}$$

$$P = \frac{1}{2}|\mathbf{I}|^2(2000) = 360\,\text{mW}$$

[**b**] Set $C_o = 0.1\,\mu\text{F}$ so $-j/\omega C = -j2000\,\Omega$
Set R_o as close as possible to

$$R_o = \sqrt{4000^2 + (3000 - 2000)^2} = 4123.1\,\Omega$$

$$\therefore\ R_o = 4000\,\Omega$$

[**c**] $$\mathbf{I} = \frac{120}{8000 + j1000} = 14.77 - j1.85\,\text{mA}$$

$$P = \frac{1}{2}|\mathbf{I}|^2(4000) = 443.1\,\text{mW}$$

Yes; $443.1\,\text{mW} > 360\,\text{mW}$

[**d**] $\mathbf{I} = \dfrac{120}{8000} = 15\,\text{mA}$

$$P = \frac{1}{2}(0.015)^2(4000) = 450\,\text{mW}$$

[**e**] $R_o = 4000\,\Omega;\qquad C_o = 66.67\,\text{nF}$

[**f**] Yes; $\qquad 450\,\text{mW} > 443.1\,\text{mW}$

P 10.48 [**a**] Set $C_o = 0.1\,\mu\text{F}$, so $-j/\omega C = -j2000\,\Omega$; also set $R_o = 4123.1\,\Omega$

$$\mathbf{I} = \frac{120}{8123.1 + j1000} = 14.55 - j1.79\,\text{mA}$$

$$P = \frac{1}{2}|\mathbf{I}|^2(4123.1) = 443.18\,\text{mW}$$

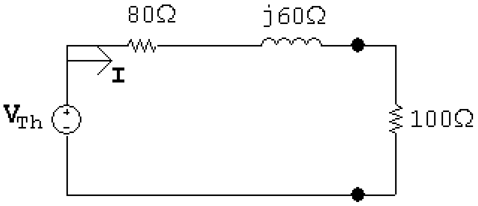
[**b**] Yes; $\qquad 443.18\,\text{mW} > 360\,\text{mW}$

[**c**] Yes; $\qquad 443.18\,\text{mW} < 450\,\text{mW}$

P 10.49 [**a**] $Z_{\text{Th}} = 20 + j60 + \dfrac{(j20)(6 - j18)}{6 + j2} = 80 + j60 = 100\underline{/36.87^\circ}\,\Omega$

$$\therefore \quad R = |Z_{\text{Th}}| = 100\,\Omega$$

[**b**] $\mathbf{V}_{\text{Th}} = \dfrac{j20}{6 - j18 + j20}(480\underline{/0^\circ}) = 480 + j1440\,\text{V(rms)}$

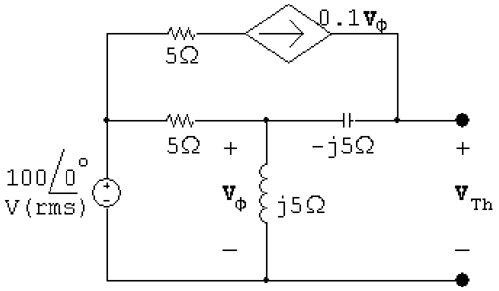


$$\mathbf{I} = \frac{480 + j1440}{180 + j60} = 4.8 + j6.4 = 8\underline{/53.13^\circ}\ \text{A(rms)}$$

$$P = 8^2(100) = 6400\,\text{W}$$

[**c**] Pick the $100\,\Omega$ resistor from Appendix H to match exactly.

P 10.50 [**a**] Open circuit voltage:

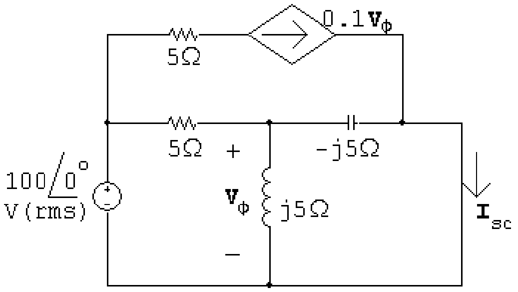


$$\frac{\mathbf{V}_\phi - 100}{5} + \frac{\mathbf{V}_\phi}{j5} - 0.1\mathbf{V}_\phi = 0$$

$$\therefore \quad \mathbf{V}_\phi = 40 + j80\,\mathrm{V(rms)}$$

$$\mathbf{V}_{\mathrm{Th}} = \mathbf{V}_\phi + 0.1\mathbf{V}_\phi(-j5) = \mathbf{V}_\phi(1 - j0.5) = 80 + j60\,\mathrm{V(rms)}$$

Short circuit current:



$$\mathbf{I}_{\mathrm{sc}} = 0.1\mathbf{V}_\phi + \frac{\mathbf{V}_\phi}{-j5} = (0.1 + j0.2)\mathbf{V}_\phi$$

$$\frac{\mathbf{V}_\phi - 100}{5} + \frac{\mathbf{V}_\phi}{j5} + \frac{\mathbf{V}_\phi}{-j5} = 0$$

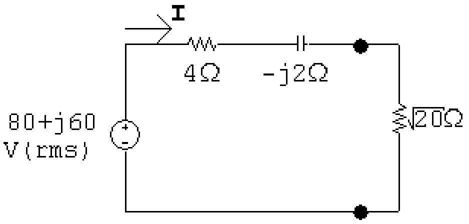
$$\therefore \quad \mathbf{V}_\phi = 100\,\mathrm{V(rms)}$$

$$\mathbf{I}_{\mathrm{sc}} = (0.1 + j0.2)(100) = 10 + j20\,\mathrm{A(rms)}$$

$$Z_{\mathrm{Th}} = \frac{\mathbf{V}_{\mathrm{Th}}}{\mathbf{I}_{\mathrm{sc}}} = \frac{80 + j60}{10 + j20} = 4 - j2\,\Omega$$

$$\therefore \quad R_o = |Z_{\mathrm{Th}}| = 4.47\,\Omega$$

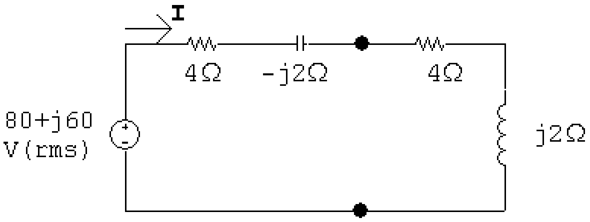
[b]



$$\mathbf{I} = \frac{80 + j60}{4 + \sqrt{20} - j2} = 7.36 + j8.82\,\mathrm{A\ (rms)}$$

$$P = (11.49)^2(\sqrt{20}) = 590.17\,\mathrm{W}$$

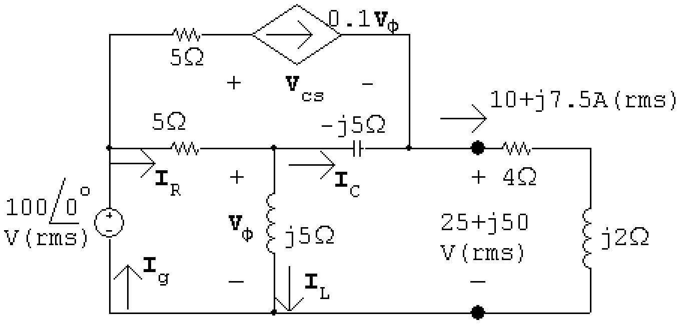
[**c**]



$$\mathbf{I} = \frac{80 + j60}{8} = 10 + j7.5\,\mathrm{A}\,(\mathrm{rms})$$

$$P = (10^2 + 7.5^2)(4) = 625\,\mathrm{W}$$

[**d**]



$$\frac{\mathbf{V}_\phi - 100}{5} + \frac{\mathbf{V}_\phi}{j5} + \frac{\mathbf{V}_o - (25 + j50)}{-j5} = 0$$

$\mathbf{V}_\phi = 50 + j25\,\mathrm{V}\ (\mathrm{rms})$

$0.1\mathbf{V}_\phi = 5 + j2.5\,\mathrm{V}\ (\mathrm{rms})$

$5 + j2.5 + \mathbf{I}_C = 10 + j7.5$

$\mathbf{I}_C = 5 + j5\,\mathrm{A}\ (\mathrm{rms})$

$$\mathbf{I}_L = \frac{\mathbf{V}_\phi}{j5} = 5 - j10\,\mathrm{A}\ (\mathrm{rms})$$

$\mathbf{I}_R = \mathbf{I}_C + \mathbf{I}_L = 10 - j5\,\mathrm{A}\ (\mathrm{rms})$

$\mathbf{I}_g = \mathbf{I}_R + 0.1\mathbf{V}_\phi = 15 - j2.5\,\mathrm{A}\ (\mathrm{rms})$

$S_g = -100\mathbf{I}_g^* = -1500 - j250\,\mathrm{VA}$

$100 = 5(5 + j2.5) + \mathbf{V}_{\mathrm{cs}} + 25 + j50 \quad \therefore \quad \mathbf{V}_{\mathrm{cs}} = 50 - j62.5\,\mathrm{V}\ (\mathrm{rms})$

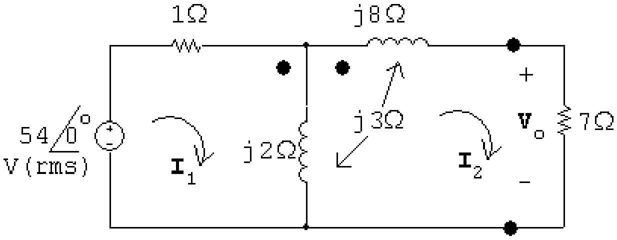
$S_{\mathrm{cs}} = (50 - j62.5)(5 - j2.5) = 93.75 - j437.5\,\mathrm{VA}$

Thus,

$\sum P_{\mathrm{dev}} = 1500$

$$\%\text{ delivered to } R_o = \frac{625}{1500}(100) = 41.67\%$$

P 10.51 [**a**]



$$54 = \mathbf{I}_1 + j2(\mathbf{I}_1 - \mathbf{I}_2) + j3\mathbf{I}_2$$

$$0 = 7\mathbf{I}_2 + j2(\mathbf{I}_2 - \mathbf{I}_1) - j3\mathbf{I}_2 + j8\mathbf{I}_2 + j3(\mathbf{I}_1 - \mathbf{I}_2)$$

Solving,

$\mathbf{I}_1 = 12 - j21\,\text{A (rms)}; \qquad \mathbf{I}_2 = -3\,\text{A (rms)}$

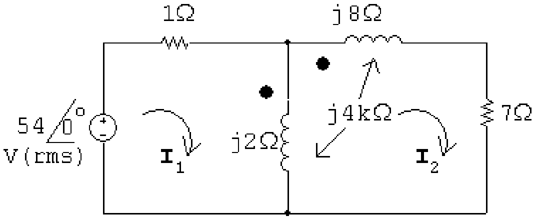
$\mathbf{V}_o = 7\mathbf{I}_2 = -21\underline{/0^\circ}\,\text{V(rms)}$

[**b**] $P = |\mathbf{I}_2|^2(7) = 63\,\text{W}$

[**c**] $P_g = (54)(12) = 648\,\text{W}$

$$\%\text{ delivered } = \frac{63}{648}(100) = 9.72\%$$

P 10.52 [**a**]



$$54 = \mathbf{I}_1 + j2(\mathbf{I}_1 - \mathbf{I}_2) + j4k\mathbf{I}_2$$

$$0 = 7\mathbf{I}_2 + j2(\mathbf{I}_2 - \mathbf{I}_1) - j4k\mathbf{I}_2 + j8\mathbf{I}_2 + j4k(\mathbf{I}_1 - \mathbf{I}_2)$$

Place the equations in standard form:

$$54 = (1 + j2)\mathbf{I}_1 + j(4k - 2)\mathbf{I}_2$$

$$0 = j(4k - 2)\mathbf{I}_1 + [7 + j(10 - 8k)]\mathbf{I}_2$$

$$\mathbf{I}_1 = \frac{54 - \mathbf{I}_2 j(4k - 2)}{(1 + j2)}$$

Substituting,

$$\mathbf{I}_2 = \frac{j54(4k - 2)}{[7 + j(10 - 8k)](1 + j2) - (4k - 2)}$$

For $\mathbf{V}_o = 0, \mathbf{I}_2 = 0$, so if $4k - 2 = 0$, then $k = 0.5$.

[**b**] When $\mathbf{I}_2 = 0$

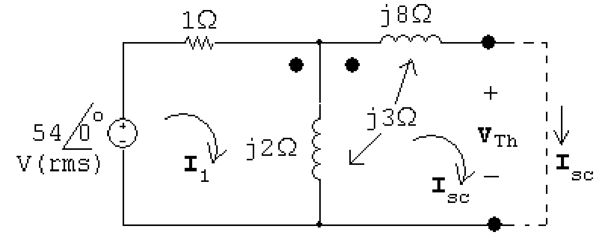
$$\mathbf{I}_1 = \frac{54}{1+j2} = 10.8 - j21.6\,\text{A(rms)}$$

$$P_g = (54)(10.8) = 583.2\,\text{W}$$

Check:

$$P_{\text{loss}} = |\mathbf{I}_1|^2(1) = 583.2\,\text{W}$$

P 10.53 [**a**]



Open circuit:

$$\mathbf{V}_{\text{Th}} = -j3\mathbf{I}_1 + j2\mathbf{I}_1 = -j\mathbf{I}_1$$

$$\mathbf{I}_1 = \frac{54}{1+j2} = 10.8 - j21.6$$

$$\mathbf{V}_{\text{Th}} = -21.6 - j10.8\,\text{V}$$

Short circuit:

$$54 = \mathbf{I}_1 + j2(\mathbf{I}_1 - \mathbf{I}_{\text{sc}}) + j3\mathbf{I}_{\text{sc}}$$

$$0 = j2(\mathbf{I}_{\text{sc}} - \mathbf{I}_1) - j3\mathbf{I}_{\text{sc}} + j8\mathbf{I}_{\text{sc}} + j3(\mathbf{I}_1 - \mathbf{I}_{\text{sc}})$$

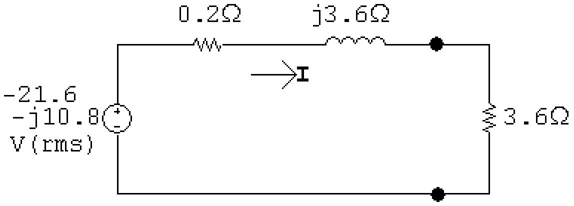
Solving,

$$\mathbf{I}_{\text{sc}} = -3.32 + j5.82$$

$$Z_{\text{Th}} = \frac{\mathbf{V}_{\text{Th}}}{\mathbf{I}_{\text{sc}}} = \frac{-21.6 - j10.8}{-3.32 + j5.82} = 0.2 + j3.6 = 3.6\underline{/86.86^\circ}\,\Omega$$

$$\therefore\quad R_{\text{L}} = |Z_{\text{Th}}| = 3.6\,\Omega$$

[**b**]



$$\mathbf{I} = \frac{-21.6 - j10.8}{3.8 + j3.6} = 4.614\underline{/163.1^\circ}$$

$$P = |\mathbf{I}|^2(3.6) = 76.6\,\text{W}, \quad \text{which is greater than when } R_L = 7\,\Omega$$

P 10.54 **[a]** $\frac{1}{\omega C} = 100\,\Omega; \qquad C = \frac{1}{(60)(200\pi)} = 26.53\,\mu\text{F}$

[b] $\mathbf{V}_{\text{swo}} = 4000 + (40)(1.25 + j10) = 4050 + j400$

$= 4069.71\underline{/5.64^\circ}\,\text{V(rms)}$

$\mathbf{V}_{\text{sw}} = 4000 + (40 - j40)(1.25 + j10) = 4450 + j350 = 4463.73\underline{/4.50^\circ}\,\text{V(rms)}$

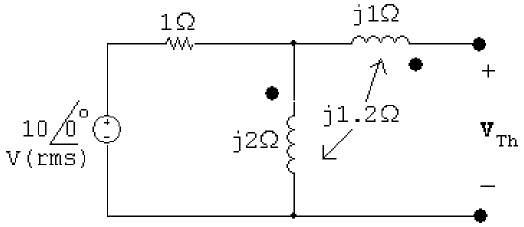
$\%\text{ increase } = \left(\frac{4463.73}{4069.71} - 1\right)(100) = 9.68\%$

[c] $P_{\ell\text{wo}} = (40\sqrt{2})^2(1.25) = 4000\,\text{W}$

$P_{\ell\text{w}} = 40^2(1.25) = 2000\,\text{W}$

$\%\text{ increase } = \left(\frac{4000}{2000} - 1\right)(100) = 100\%$

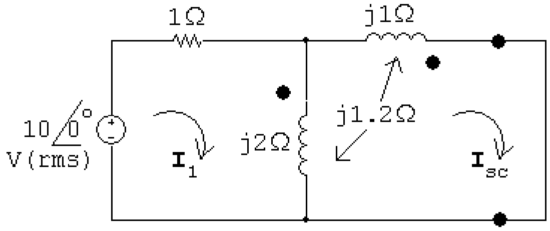
P 10.55 Open circuit voltage:



$\mathbf{I}_1 = \frac{10\underline{/0^\circ}}{1 + j2} = 2 - j4\,\text{A}$

$\mathbf{V}_{\text{Th}} = j2\mathbf{I}_1 + j1.2\mathbf{I}_1 = j3.2\mathbf{I}_1 = 12.8 + j6.4 = 14.31\underline{/26.57^\circ}$

Short circuit current:



$10\underline{/0^\circ} = (1 + j2)\mathbf{I}_1 - j3.2\mathbf{I}_{\text{sc}}$

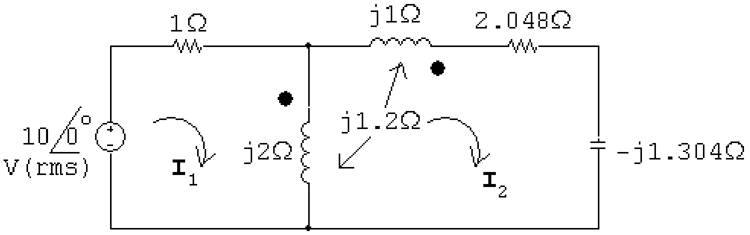
$0 = -j3.2\mathbf{I}_1 + j5.4\mathbf{I}_{\text{sc}}$

Solving,

$\mathbf{I}_{\text{sc}} = 5.89\underline{/-5.92^\circ}\,\text{A}$

$$Z_{\text{Th}} = \frac{14.31\underline{/26.57^\circ}}{5.89\underline{/-5.92^\circ}} = 2.43\underline{/32.49^\circ} = 2.048 + j1.304\,\Omega$$

$$\therefore\quad \mathbf{I}_2 = \frac{14.31\underline{/26.57^\circ}}{4.096} = 3.49\underline{/26.57^\circ}\,\text{A}$$

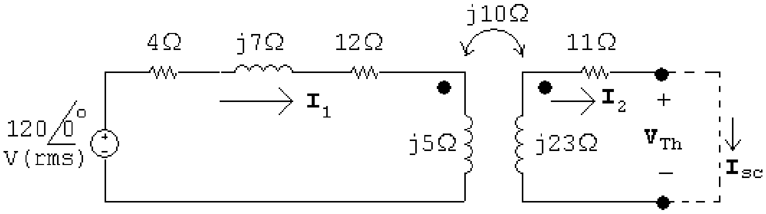


$$10\underline{/0^\circ} = (1 + j2)\mathbf{I}_1 - j3.2\mathbf{I}_2$$

$$\therefore\qquad \mathbf{I}_1 = \frac{10 + j3.2\mathbf{I}_2}{1 + j2} = \frac{10 + j3.2(3.49\underline{/26.57^\circ})}{1 + j2} = 5\,\text{A}$$

$$Z_g = \frac{10\underline{/0^\circ}}{5} = 2 + j0 = 2\underline{/0^\circ}\,\Omega$$

P 10.56 [**a**]



Open circuit:

$$\mathbf{V}_{\text{Th}} = \frac{120}{16 + j12}(j10) = 36 + j48\,\text{V}$$

Short circuit:

$$(16 + j12)\mathbf{I}_1 - j10\mathbf{I}_{\text{sc}} = 120$$

$$-j10\mathbf{I}_1 + (11 + j23)\mathbf{I}_{\text{sc}} = 0$$

Solving,

$$\mathbf{I}_{\text{sc}} = 2.4\,\text{A}$$

$$Z_{\text{Th}} = \frac{36 + j48}{2.4} = 15 + j20\,\Omega$$

$$\therefore\quad Z_{\text{L}} = Z^*_{\text{Th}} = 15 - j20\,\Omega$$

$$\mathbf{I}_{\rm L} = \frac{\mathbf{V}_{\rm Th}}{Z_{\rm Th} + Z_L} = \frac{36 + j48}{30} = 1.2 + j1.6\,\text{A(rms)}$$

$$P_{\rm L} = |\mathbf{I}_{\rm L}|^2(15) = 60\,\text{W}$$

[b] $$\mathbf{I}_1 = \frac{Z_{22}\mathbf{I}_2}{j\omega M} = \frac{26 + j3}{j10}(1.2 + j1.6) = 5.23\underline{/-30.29^\circ}\,\text{A (rms)}$$

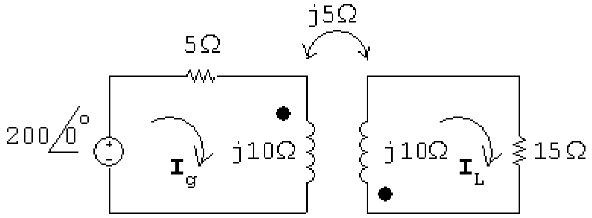
$$P_{\rm transformer} = (120)(5.23)\cos(-30.29^\circ) - (5.23)^2(4) = 432.8\,\text{W}$$

$$\%\text{ delivered } = \frac{60}{432.8}(100) = 13.86\%$$

P 10.57 **[a]** $j\omega L_1 = j(10{,}000)(1 \times 10^{-3}) = j10\,\Omega$

$$j\omega L_2 = j(10{,}000)(1 \times 10^{-3}) = j10\,\Omega$$

$$j\omega M = j10\,\Omega$$



$$200 = (5 + j10)\mathbf{I}_g + j5\mathbf{I}_{\rm L}$$

$$0 = j5\mathbf{I}_g + (15 + j10)\mathbf{I}_{\rm L}$$

Solving,

$$\mathbf{I}_g = 10 - j15\,\text{A}; \qquad \mathbf{I}_{\rm L} = -5\,\text{A}$$

Thus,

$$i_g = 18.03\cos(10{,}000t - 56.31^\circ)\,\text{A}$$

$$i_{\rm L} = 5\cos(10{,}000t - 180^\circ)\,\text{A}$$

[b] $$k = \frac{M}{\sqrt{L_1 L_2}} = \frac{0.5}{\sqrt{1}} = 0.5$$

[c] When $t = 50\pi\,\mu$s:

$$10{,}000t = (10{,}000)(50\pi) \times 10^{-6} = 0.5\pi \text{ rad } = 90^\circ$$

$$i_g(50\pi\,\mu\text{s}) = 18.03\cos(90^\circ - 56.31^\circ) = 15\,\text{A}$$

$$i_{\rm L}(50\pi\,\mu\text{s}) = 5\cos(90^\circ + 180^\circ) = 0\,\text{A}$$

$$w = \frac{1}{2}L_1 i_1^2 + \frac{1}{2}L_2 i_2^2 + M i_1 i_2 = \frac{1}{2}(10^{-3})(15)^2 + 0 + 0 = 112.5\,\text{mJ}$$

When $t = 100\pi\ \mu$s:

$$10{,}000t = (10^4)(100\pi) \times 10^{-6} = \pi = 180^\circ$$

$$i_g(100\pi\ \mu\text{s}) = 18.03\cos(180 - 56.31^\circ) = -10\,\text{A}$$

$$i_{\text{L}}(100\pi\ \mu\text{s}) = 5\cos(180 - 180^\circ) = 5\,\text{A}$$

$$w = \frac{1}{2}(10^{-3})(10)^2 + \frac{1}{2}(10^{-3})(5)^2 + 0.5 \times 10^{-3}(-10)(5) = 37.5\,\text{mJ}$$

[d] From (a), $I_m = 5$ A,

$$\therefore\quad P = \frac{1}{2}(5)^2(15) = 187.5\,\text{W}$$

[e] Open circuit:

$$\mathbf{V}_{\text{Th}} = \frac{200}{5 + j10}(-j5) = -80 - j40\,\text{V}$$

Short circuit:

$$200 = (5 + j10)\mathbf{I}_1 + j5\mathbf{I}_{\text{sc}}$$

$$0 = j5\mathbf{I}_1 + j10\mathbf{I}_{\text{sc}}$$

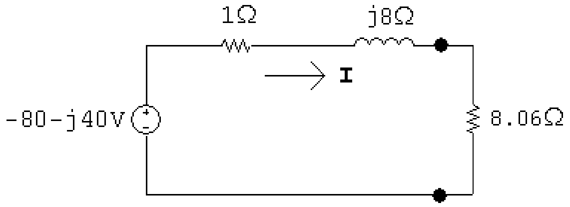
Solving,

$$\mathbf{I}_{\text{sc}} = -\frac{80}{13} + j\frac{120}{13}$$

$$Z_{\text{Th}} = \frac{\mathbf{V}_{\text{Th}}}{\mathbf{I}_{\text{sc}}} = \frac{-80 - j40}{-(80/13) + j(120/13)} = 1 + j8\,\Omega$$

$$\therefore\quad R_{\text{L}} = 8.06\,\Omega$$

[f]



$$\mathbf{I} = \frac{-80 - j40}{1 + j8 + 8.06} = 7.40\underline{/165.12^\circ}\,\text{A}$$

$$P = \frac{1}{2}(7.40)^2(8.06) = 223.42\,\text{W}$$

[g] $Z_{\text{L}} = Z_{\text{Th}}^* = 1 - j8\,\Omega$

[h] $\mathbf{I} = \dfrac{-80 - j40}{2} = 44.72\underline{/-153.43^\circ}$

$$P = \frac{1}{2}(44.72)^2(1) = 1000\,\text{W}$$

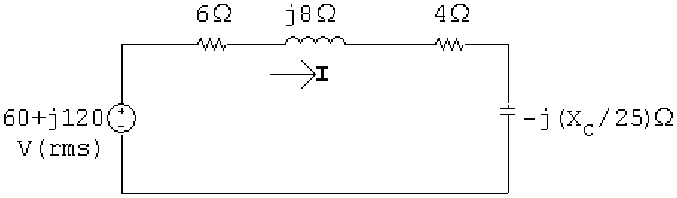
P 10.58 [a] Replace the circuit to the left of the primary winding with a Thévenin equivalent:

$$\mathbf{V}_{\text{Th}} = (15)(20\|j10) = 60 + j120\,\text{V}$$

$$Z_{\text{Th}} = 2 + 20\|j10 = 6 + j8\,\Omega$$

Transfer the secondary impedance to the primary side:

$$Z_p = \frac{1}{25}(100 - jX_{\text{C}}) = 4 - j\frac{X_{\text{C}}}{25}\,\Omega$$



Now maximize **I** by setting $(X_{\text{C}}/25) = 8\,\Omega$:

$$\therefore \quad C = \frac{1}{200(20 \times 10^3)} = 0.25\,\mu\text{F}$$

[b] $\mathbf{I} = \dfrac{60 + j120}{10} = 6 + j12\,\text{A}$

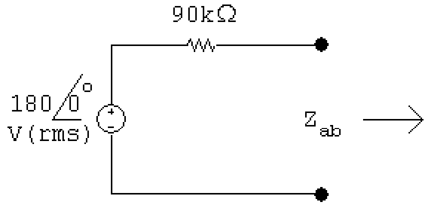
$$P = |\mathbf{I}|^2(4) = 720\,\text{W}$$

[c] $\dfrac{R_o}{25} = 6\,\Omega; \qquad \therefore \quad R_o = 150\,\Omega$

[d] $\mathbf{I} = \dfrac{60 + j120}{12} = 5 + j10\,\text{A}$

$$P = |\mathbf{I}|^2(6) = 750\,\text{W}$$

P 10.59 [a]



For maximum power transfer, $Z_{\text{ab}} = 90\,\text{k}\Omega$

$$Z_{\text{ab}} = \left(1 - \frac{N_1}{N_2}\right)^2 Z_{\text{L}}$$

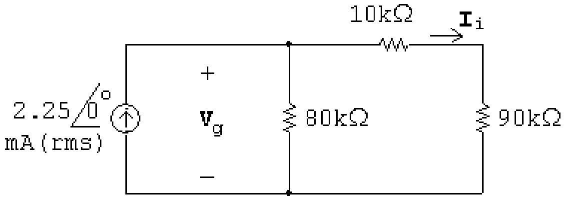
$$\therefore \quad \left(1-\frac{N_1}{N_2}\right)^2 = \frac{90{,}000}{400} = 225$$

$$1-\frac{N_1}{N_2} = \pm 15; \qquad \frac{N_1}{N_2} = 15+1 = 16$$

[**b**] $P = |\mathbf{I}_i|^2(90{,}000) = \left(\frac{180}{180{,}000}\right)^2(90{,}000) = 90\,\text{mW}$

[**c**] $\mathbf{V}_1 = R_i\mathbf{I}_i = (90{,}000)\left(\frac{180}{180{,}000}\right) = 90\,\text{V}$

[**d**]



$$\mathbf{V}_g = (2.25\times10^{-3})(100{,}000\|80{,}000) = 100\,\text{V}$$

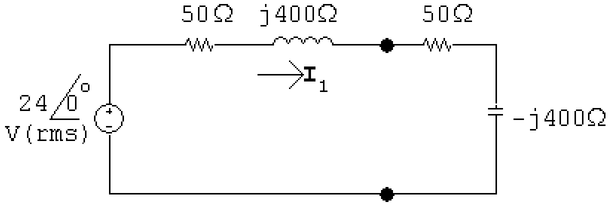
$$P_g(\text{del}) = (2.25\times10^{-3})(100) = 225\,\text{mW}$$

$$\%\ \text{delivered} = \frac{90}{225}(100) = 40\%$$

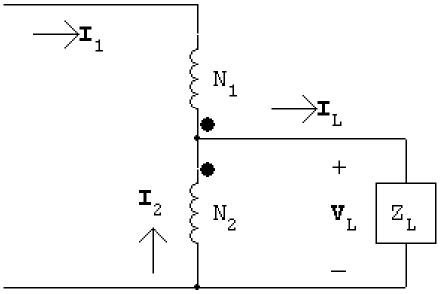
P 10.60 [**a**] $Z_{\text{ab}} = 50 - j400 = \left(1-\frac{N_1}{N_2}\right)^2 Z_{\text{L}}$

$$\therefore \quad Z_{\text{L}} = \frac{1}{(1-6)^2}(50 - j400) = 2 - j16\,\Omega$$

[**b**]



$$\mathbf{I}_1 = \frac{24}{100} = 240\underline{/0^\circ}\,\text{mA}$$



$$N_1\mathbf{I}_1 = -N_2\mathbf{I}_2$$

$$\mathbf{I}_2 = -6\mathbf{I}_1 = -1.44\underline{/0^\circ}\,\text{A}$$

$$\mathbf{I}_\text{L} = \mathbf{I}_1 + \mathbf{I}_2 = -1.68\underline{/0^\circ}\,\text{A}$$

$$\mathbf{V}_\text{L} = (2 - j16)\mathbf{I}_\text{L} = -3.36 + j26.88 = 27.1\underline{/97.13^\circ}\,\text{V(rms)}$$

P 10.61 [a] $Z_\text{Th} = 720 + j1500 + \left(\frac{200}{50}\right)^2 (40 - j30) = 1360 + j1020 = 1700\underline{/36.87^\circ}\,\Omega$

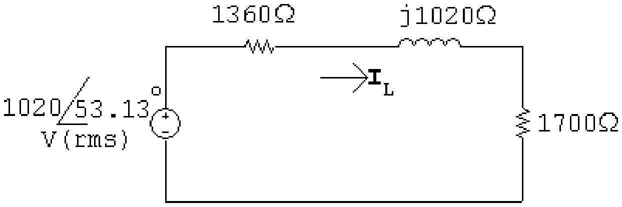
$$\therefore\quad Z_\text{ab} = 1700\,\Omega$$

$$Z_\text{ab} = \frac{Z_\text{L}}{(1 + N_1/N_2)^2}$$

$$(1 + N_1/N_2)^2 = 6800/1700 = 4$$

$$\therefore\quad N_1/N_2 = 1 \quad \text{or} \quad N_2 = N_1 = 1000 \text{ turns}$$

[b] $\mathbf{V}_\text{Th} = \frac{255\underline{/0^\circ}}{40 + j30}(j200) = 1020\underline{/53.13^\circ}\,\text{V}$

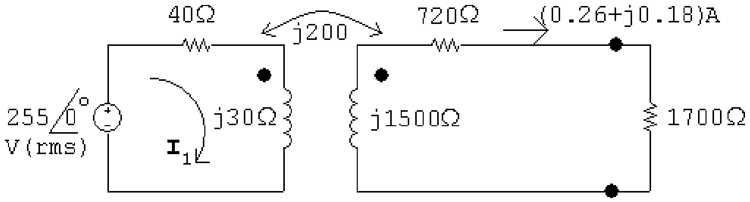


$$\mathbf{I}_L = \frac{1020\underline{/53.13^\circ}}{3060 + j1020} = 0.316\underline{/34.7^\circ}\,\text{A(rms)}$$

Since the transformer is ideal, $P_{6800} = P_{1700}$.

$$P = |\mathbf{I}|^2(1700) = 170\,\text{W}$$

[c]



$$255\underline{/0^\circ} = (40 + j30)\mathbf{I}_1 - j200(0.26 + j0.18)$$

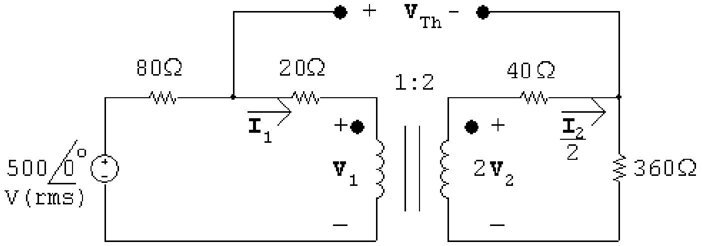
$$\therefore \quad \mathbf{I}_1 = 4.13 - j1.80\,\text{A(rms)}$$

$$P_{\text{gen}} = (255)(4.13) = 1053\,\text{W}$$

$$P_{\text{diss}} = 1053 - 170 = 883\,\text{W}$$

$$\%\text{ dissipated} = \frac{883}{1053}(100) = 83.85\%$$

P 10.62 **[a]** Open circuit voltage:



$$500 = 100\mathbf{I}_1 + \mathbf{V}_1$$

$$\mathbf{V}_2 = 400\mathbf{I}_2$$

$$\frac{\mathbf{V}_1}{1} = \frac{\mathbf{V}_2}{2} \qquad \therefore \qquad \mathbf{V}_2 = 2\mathbf{V}_1$$

$$\mathbf{I}_1 = 2\mathbf{I}_2$$

Substitute and solve:

$$2\mathbf{V}_1 = 400\mathbf{I}_1/2 = 200\mathbf{I}_1 \qquad \therefore \qquad \mathbf{V}_1 = 100\mathbf{I}_1$$

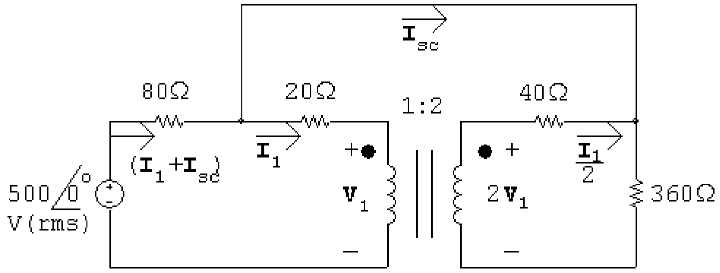
$$500 = 100\mathbf{I}_1 + 100\mathbf{I}_1 \qquad \therefore \qquad \mathbf{I}_1 = 500/200 = 2.5\,\text{A}$$

$$\therefore \qquad \mathbf{I}_2 = \frac{1}{2}\mathbf{I}_1 = 1.25\,\text{A}$$

$$\mathbf{V}_1 = 100(2.5) = 250\,\text{V}; \qquad \mathbf{V}_2 = 2\mathbf{V}_1 = 500\,\text{V}$$

$$V_{\text{Th}} = 20\mathbf{I}_1 + \mathbf{V}_1 - \mathbf{V}_2 + 40\mathbf{I}_2 = -150\,\text{V(rms)}$$

Short circuit current:



$$500 = 80(\mathbf{I}_{\rm sc} + \mathbf{I}_1) + 360(\mathbf{I}_{\rm sc} + 0.5\mathbf{I}_1)$$

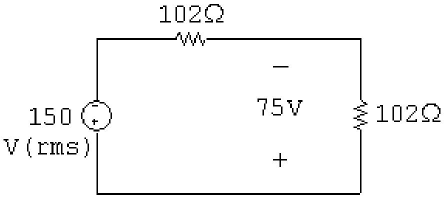
$$2\mathbf{V}_1 = 40\frac{\mathbf{I}_1}{2} + 360(\mathbf{I}_{\rm sc} + 0.5\mathbf{I}_1)$$

$$500 = 80(\mathbf{I}_1 + \mathbf{I}_{\rm sc}) + 20\mathbf{I}_1 + \mathbf{V}_1$$

Solving,

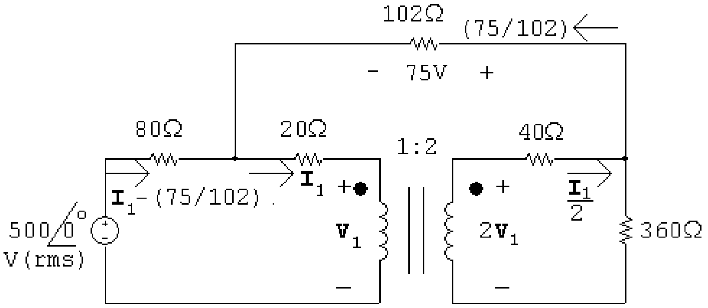
$$\mathbf{I}_{\rm sc} = -1.47\,\mathrm{A}$$

$$R_{\rm Th} = \frac{\mathbf{V}_{\rm Th}}{\mathbf{I}_{\rm sc}} = \frac{-150}{-1.47} = 102\,\Omega$$



$$P = \frac{75^2}{102} = 55.15\,\mathrm{W}$$

[b]



$$500 = 80[\mathbf{I}_1 - (75/102)] - 75 + 360[\mathbf{I}_2 - (75/102)]$$

$$575 + \frac{6000}{102} + \frac{27,000}{102} = 80\mathbf{I}_1 + 180\mathbf{I}_2$$

$$\therefore \quad \mathbf{I}_1 = 3.456\,\mathrm{A}$$

$$P_{\text{source}} = (500)[3.456 - (75/102)] = 1360.35\,\text{W}$$

$$\%\text{ delivered} = \frac{55.15}{1360.35}(100) = 4.05\%$$

[c] $P_{80\Omega} = 80(\mathbf{I}_1 + \mathbf{I}_L)^2 = 592.13\,\text{W}$

$$P_{20\Omega} = 20\mathbf{I}_1^2 = 238.86\,\text{W}$$

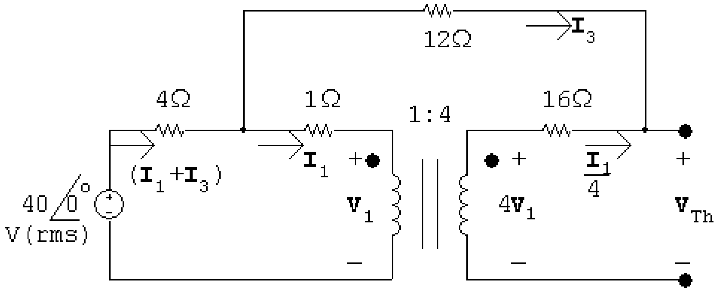
$$P_{40\Omega} = 40\mathbf{I}_2^2 = 119.43\,\text{W}$$

$$P_{102\Omega} = 102\mathbf{I}_L^2 = 55.15\,\text{W}$$

$$P_{360\Omega} = 360(\mathbf{I}_2 + \mathbf{I}_L)^2 = 354.73\,\text{W}$$

$$\sum P_{\text{abs}} = 592.13 + 238.86 + 119.43 + 55.15 + 354.73 = 1360.3\,\text{W} = \sum P_{\text{dev}}$$

P 10.63 [a] Open circuit voltage:



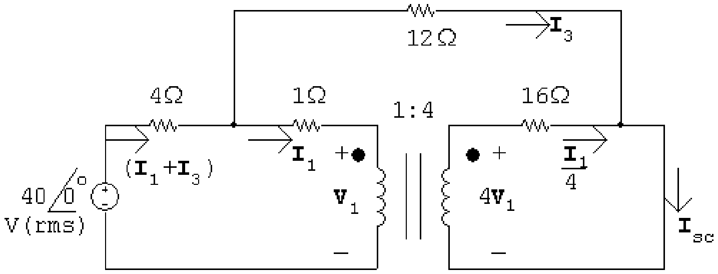
$$40\underline{/0^\circ} = 4(\mathbf{I}_1 + \mathbf{I}_3) + 12\mathbf{I}_3 + \mathbf{V}_{\text{Th}}$$

$$\frac{\mathbf{I}_1}{4} = -\mathbf{I}_3; \qquad \mathbf{I}_1 = -4\mathbf{I}_3$$

Solving,

$$\mathbf{V}_{\text{Th}} = 40\underline{/0^\circ}\,\text{V}$$

Short circuit current:



$$40\underline{/0^\circ} = 4\mathbf{I}_1 + 4\mathbf{I}_3 + \mathbf{I}_1 + \mathbf{V}_1$$

$$4\mathbf{V}_1 = 16(\mathbf{I}_1/4) = 4\mathbf{I}_1; \qquad \therefore\ \mathbf{V}_1 = \mathbf{I}_1$$

$$\therefore\ 40\underline{/0^\circ} = 6\mathbf{I}_1 + 4\mathbf{I}_3$$

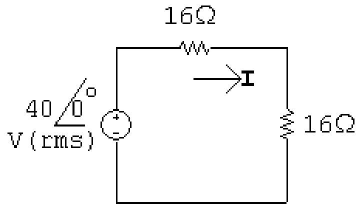
Also,

$$40\underline{/0^\circ} = 4(\mathbf{I}_1 + \mathbf{I}_3) + 12\mathbf{I}_3$$

Solving,

$$\mathbf{I}_1 = 6\,\text{A}; \qquad \mathbf{I}_3 = 1\,\text{A}; \qquad \mathbf{I}_{\text{sc}} = \mathbf{I}_1/4 + \mathbf{I}_3 = 2.5\,\text{A}$$

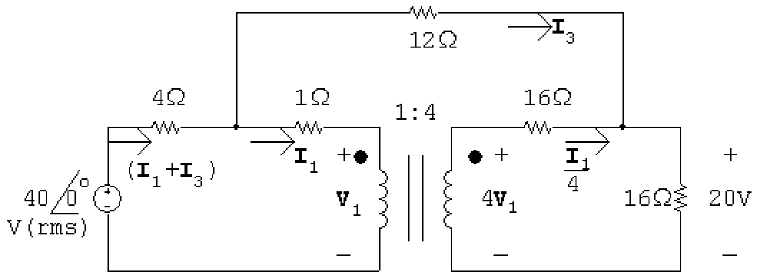
$$R_{\text{Th}} = \frac{\mathbf{V}_{\text{Th}}}{\mathbf{I}_{\text{sc}}} = \frac{40}{2.5} = 16\,\Omega$$



$$\mathbf{I} = \frac{40\underline{/0^\circ}}{32} = 1.25\underline{/0^\circ}\,\text{A(rms)}$$

$$P = (1.25)^2(16) = 25\,\text{W}$$

[b]



$$40 = 4(\mathbf{I}_1 + \mathbf{I}_3) + 12\mathbf{I}_3 + 20$$

$$4\mathbf{V}_1 = 4\mathbf{I}_1 + 16(\mathbf{I}_1/4 + \mathbf{I}_3); \qquad \therefore\ \mathbf{V}_1 = 2\mathbf{I}_1 + 4\mathbf{I}_3$$

$$40 = 4\mathbf{I}_1 + 4\mathbf{I}_3 + \mathbf{I}_1 + \mathbf{V}_1$$

$$\therefore\ \mathbf{I}_1 = 6\,\text{A}; \qquad \mathbf{I}_3 = -0.25\,\text{A}; \qquad \mathbf{I}_1 + \mathbf{I}_3 = 5.75\underline{/0^\circ}\,\text{A}$$

$$P_{40V}(\text{developed}) = 40(5.75) = 230\,\text{W}$$

$$\therefore\ \%\ \text{delivered} = \frac{25}{230}(100) = 10.87\%$$

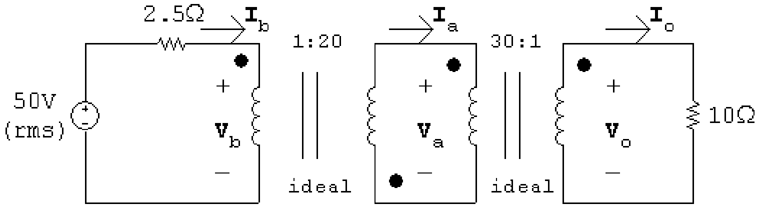
[c] $P_{R_L} = 25\,\text{W}; \qquad P_{16\Omega} = (1.5)^2(16) = 36\,\text{W}$

$$P_{4\Omega} = (5.75)^2(4) = 132.25\,\text{W}; \qquad P_{1\Omega} = (6)^2(1) = 36\,\text{W}$$

$$P_{12\Omega} = (-0.25)^2(12) = 0.75\,\text{W}$$

$$\sum P_{\text{abs}} = 25 + 36 + 132.25 + 36 + 0.75 = 230\,\text{W} = \sum P_{\text{dev}}$$

P 10.64



$$30\mathbf{V}_o = \mathbf{V}_a; \qquad \frac{\mathbf{I}_o}{30} = \mathbf{I}_a; \qquad \text{therefore} \quad \frac{\mathbf{V}_a}{\mathbf{I}_a} = 9\,\text{k}\Omega$$

$$\frac{\mathbf{V}_b}{1} = \frac{-\mathbf{V}_a}{20}; \qquad \mathbf{I}_b = -20\mathbf{I}_a; \qquad \text{therefore} \quad \frac{\mathbf{V}_b}{\mathbf{I}_b} = \frac{9000}{400} = 22.5\,\Omega$$

Therefore $\mathbf{I}_b = [50/(2.5 + 22.5)] = 2\,\text{A}$ (rms); since the ideal transformers are lossless, $P_{10\Omega} = P_{22.5\Omega}$, and the power delivered to the 22.5 Ω resistor is $2^2(22.5)$ or 90 W.

P 10.65 [a] $\dfrac{\mathbf{V}_b}{\mathbf{I}_b} = \dfrac{a^2 10}{400} = 2.5\,\Omega;$ therefore $a^2 = 100,$ $a = 10$

[b] $\mathbf{I}_b = \dfrac{50}{5} = 10\,\text{A};$ $P = (100)(2.5) = 250\,\text{W}$

P 10.66 [a] Begin with the MEDIUM setting, as shown in Fig. 10.31, as it involves only the resistor R_2. Then,

$$P_{\text{med}} = 500\,\text{W} = \frac{V^2}{R_2} = \frac{120^2}{R_2}$$

Thus,

$$R_2 = \frac{120^2}{500} = 28.8\,\Omega$$

[b] Now move to the LOW setting, as shown in Fig. 10.30, which involves the resistors R_1 and R_2 connected in series:

$$P_{\text{low}} = \frac{V^2}{R_1 + R_2} = \frac{V^2}{R_1 + 28.8} = 250\,\text{W}$$

Thus,

$$R_1 = \frac{120^2}{250} - 28.8 = 28.8\,\Omega$$

[c] Note that the HIGH setting has R_1 and R_2 in parallel:

$$P_{\text{high}} = \frac{V^2}{R_1 \| R_2} = \frac{120^2}{28.8 \| 28.8} = 1000\,\text{W}$$

If the HIGH setting has required power other than 1000 W, this problem could not have been solved. In other words, the HIGH power setting was chosen in such a way that it would be satisfied once the two resistor values were calculated to satisfy the LOW and MEDIUM power settings.

P 10.67 **[a]** $P_\text{L} = \dfrac{V^2}{R_1 + R_2}; \qquad R_1 + R_2 = \dfrac{V^2}{P_\text{L}}$

$$P_\text{M} = \frac{V^2}{R_2}; \qquad R_2 = \frac{V^2}{P_\text{M}}$$

$$P_\text{H} = \frac{V^2(R_1 + R_2)}{R_1 R_2}$$

$$R_1 + R_2 = \frac{V^2}{P_\text{L}}; \qquad R_1 = \frac{V^2}{P_\text{L}} - \frac{V^2}{P_\text{M}}$$

$$P_\text{H} = \frac{V^2 V^2/P_\text{L}}{\left(\frac{V^2}{P_\text{L}} - \frac{V^2}{P_\text{M}}\right)\left(\frac{V^2}{P_\text{M}}\right)} = \frac{P_\text{M} P_\text{L} P_\text{M}}{P_\text{L}(P_\text{M} - P_\text{L})}$$

$$P_\text{H} = \frac{P_\text{M}^2}{P_\text{M} - P_\text{L}}$$

[b] $P_\text{H} = \dfrac{(750)^2}{(750 - 250)} = 1125\,\text{W}$

P 10.68 First solve the expression derived in P10.67 for P_M as a function of P_L and P_H. Thus

$$P_\text{M} - P_\text{L} = \frac{P_\text{M}^2}{P_\text{H}} \qquad \text{or} \qquad \frac{P_\text{M}^2}{P_\text{H}} - P_\text{M} + P_\text{L} = 0$$

$$P_\text{M}^2 - P_\text{M} P_\text{H} + P_\text{L} P_\text{H} = 0$$

$$\therefore \quad P_\text{M} = \frac{P_\text{H}}{2} \pm \sqrt{\left(\frac{P_\text{H}}{2}\right)^2 - P_\text{L} P_\text{H}}$$

$$= \frac{P_\text{H}}{2} \pm P_\text{H}\sqrt{\frac{1}{4} - \left(\frac{P_\text{L}}{P_\text{H}}\right)}$$

For the specified values of P_L and P_H

$$P_\text{M} = 500 \pm 1000\sqrt{0.25 - 0.24} = 500 \pm 100$$

$$\therefore \quad P_{M1} = 600\,\text{W}; \qquad P_{M2} = 400\,\text{W}$$

Note in this case we design for two medium power ratings
If $P_{M1} = 600\,\text{W}$

$$R_2 = \frac{(120)^2}{600} = 24\,\Omega$$

$$R_1 + R_2 = \frac{(120)^2}{240} = 60\,\Omega$$

$$R_1 = 60 - 24 = 36\,\Omega$$

CHECK: $P_{\text{H}} = \dfrac{(120)^2(60)}{(36)(24)} = 1000\,\text{W}$

If $P_{M2} = 400\,\text{W}$

$$R_2 = \frac{(120)^2}{400} = 36\,\Omega$$

$R_1 + R_2 = 60\,\Omega$ (as before)

$$R_1 = 24\,\Omega$$

CHECK: $P_{\text{H}} = 1000\,\text{W}$

P 10.69 $R_1 + R_2 + R_3 = \dfrac{(120)^2}{600} = 24\,\Omega$

$$R_2 + R_3 = \frac{(120)^2}{900} = 16\,\Omega$$

$$\therefore\quad R_1 = 24 - 16 = 8\,\Omega$$

$$R_3 + R_1\|R_2 = \frac{(120)^2}{1200} = 12\,\Omega$$

$$\therefore\quad 16 - R_2 + \frac{8R_2}{8 + R_2} = 12$$

$$R_2 - \frac{8R_2}{8 + R_2} = 4$$

$$8R_2 + R_2^2 - 8R_2 = 32 + 4R_2$$

$$R_2^2 - 4R_2 - 32 = 0$$

$$R_2 = 2 \pm \sqrt{4 + 32} = 2 \pm 6$$

$$\therefore\quad R_2 = 8\,\Omega; \qquad \therefore\quad R_3 = 8\,\Omega$$

P 10.70 $R_2 = \dfrac{(220)^2}{500} = 96.8\,\Omega$

$$R_1 + R_2 = \frac{(220)^2}{250} = 193.6\,\Omega$$

$$\therefore \quad R_1 = 96.8\,\Omega$$

CHECK: $R_1 \| R_2 = 48.4\,\Omega$

$$P_{\rm H} = \frac{(220)^2}{48.4} = 1000\,\mathrm{W}$$

P 10.71 Choose $R_1 = 22\,\Omega$ and $R_2 = 33\,\Omega$:

$$P_L = \frac{120^2}{22+33} = 262\,\mathrm{W} \qquad \text{(instead of 240 W)}$$

$$P_M = \frac{120^2}{33} = 436\,\mathrm{W} \qquad \text{(instead of 400 W)}$$

$$P_H = \frac{120^2(55)}{(22)(33)} = 1091\,\mathrm{W} \qquad \text{(instead of 1000 W)}$$

P 10.72 Choose $R_1 = R_2 = 100\,\Omega$:

$$P_L = \frac{220^2}{100+100} = 242\,\mathrm{W} \qquad \text{(instead of 250 W)}$$

$$P_M = \frac{220^2}{100} = 484\,\mathrm{W} \qquad \text{(instead of 500 W)}$$

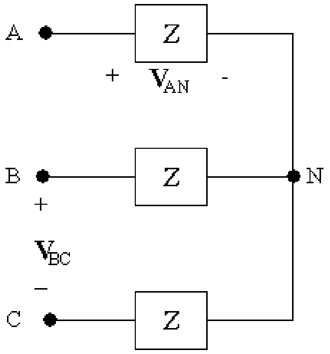
$$P_H = \frac{220^2(200)}{(100)(100)} = 968\,\mathrm{W} \qquad \text{(instead of 1000 W)}$$

11

Balanced Three-Phase Circuits

Assessment Problems

AP 11.1 Make a sketch:



We know $\mathbf{V}_{AN}$ and wish to find $\mathbf{V}_{BC}$. To do this, write a KVL equation to find $\mathbf{V}_{AB}$, and use the known phase angle relationship between $\mathbf{V}_{AB}$ and $\mathbf{V}_{BC}$ to find $\mathbf{V}_{BC}$.

$$\mathbf{V}_{AB} = \mathbf{V}_{AN} + \mathbf{V}_{NB} = \mathbf{V}_{AN} - \mathbf{V}_{BN}$$

Since $\mathbf{V}_{AN}$, $\mathbf{V}_{BN}$, and $\mathbf{V}_{CN}$ form a balanced set, and $\mathbf{V}_{AN} = 240\underline{/-30^\circ}$V, and the phase sequence is positive,

$$\mathbf{V}_{BN} = |\mathbf{V}_{AN}|\underline{/\underline{/\mathbf{V}_{AN}} - 120^\circ} = 240\underline{/-30^\circ - 120^\circ} = 240\underline{/-150^\circ}\text{ V}$$

Then,

$$\mathbf{V}_{AB} = \mathbf{V}_{AN} - \mathbf{V}_{BN} = (240\underline{/-30^\circ}) - (240\underline{/-150^\circ}) = 415.46\underline{/0^\circ}\text{ V}$$

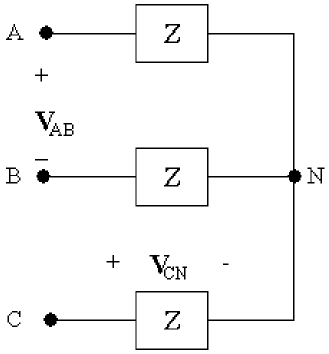
Since $\mathbf{V}_{AB}$, $\mathbf{V}_{BC}$, and $\mathbf{V}_{CA}$ form a balanced set with a positive phase sequence, we can find $\mathbf{V}_{BC}$ from $\mathbf{V}_{AB}$:

$$\mathbf{V}_{BC} = |\mathbf{V}_{AB}|\underline{/(\underline{/\mathbf{V}_{AB}} - 120^\circ)} = 415.69\underline{/0^\circ - 120^\circ} = 415.69\underline{/-120^\circ}\text{ V}$$

Thus,

$\mathbf{V}_{\text{BC}} = 415.69\underline{/-120^\circ}\,\text{V}$

AP 11.2 Make a sketch:



We know $\mathbf{V}_{\text{CN}}$ and wish to find $\mathbf{V}_{\text{AB}}$. To do this, write a KVL equation to find $\mathbf{V}_{\text{BC}}$, and use the known phase angle relationship between $\mathbf{V}_{\text{AB}}$ and $\mathbf{V}_{\text{BC}}$ to find $\mathbf{V}_{\text{AB}}$.

$\mathbf{V}_{\text{BC}} = \mathbf{V}_{\text{BN}} + \mathbf{V}_{\text{NC}} = \mathbf{V}_{\text{BN}} - \mathbf{V}_{\text{CN}}$

Since $\mathbf{V}_{\text{AN}}$, $\mathbf{V}_{\text{BN}}$, and $\mathbf{V}_{\text{CN}}$ form a balanced set, and $\mathbf{V}_{\text{CN}} = 450\underline{/-25^\circ}$ V, and the phase sequence is negative,

$\mathbf{V}_{\text{BN}} = |\mathbf{V}_{\text{CN}}|\underline{/\underline{/\mathbf{V}_{\text{CN}}} - 120^\circ} = 450\underline{/-23^\circ - 120^\circ} = 450\underline{/-145^\circ}\,\text{V}$

Then,

$\mathbf{V}_{\text{BC}} = \mathbf{V}_{\text{BN}} - \mathbf{V}_{\text{CN}} = (450\underline{/-145^\circ}) - (450\underline{/-25^\circ}) = 779.42\underline{/-175^\circ}\,\text{V}$

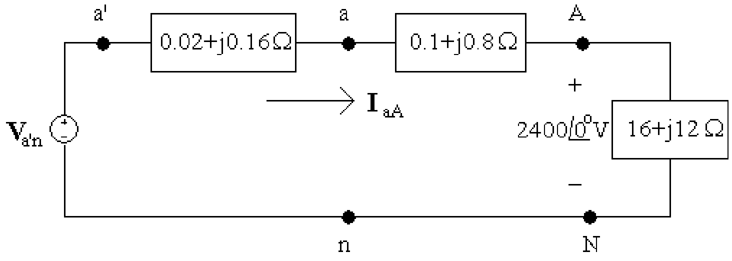
Since $\mathbf{V}_{\text{AB}}$, $\mathbf{V}_{\text{BC}}$, and $\mathbf{V}_{\text{CA}}$ form a balanced set with a negative phase sequence, we can find $\mathbf{V}_{\text{AB}}$ from $\mathbf{V}_{\text{BC}}$:

$\mathbf{V}_{\text{AB}} = |\mathbf{V}_{\text{BC}}|\underline{/\underline{/\mathbf{V}_{\text{BC}}} - 120^\circ} = 779.42\underline{/-295^\circ}\,\text{V}$

But we normally want phase angle values between +180° and −180°. We add 360° to the phase angle computed above. Thus,

$\mathbf{V}_{\text{AB}} = 779.42\underline{/65^\circ}\,\text{V}$

AP 11.3 Sketch the a-phase circuit:



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[a] We can find the line current using Ohm's law, since the a-phase line current is the current in the a-phase load. Then we can use the fact that $\mathbf{I}_{\rm aA}$, $\mathbf{I}_{\rm bB}$, and $\mathbf{I}_{\rm cC}$ form a balanced set to find the remaining line currents. Note that since we were not given any phase angles in the problem statement, we can assume that the phase voltage given, $\mathbf{V}_{\rm AN}$, has a phase angle of 0°.

$$2400\underline{/0^\circ} = \mathbf{I}_{\rm aA}(16 + j12)$$

so

$$\mathbf{I}_{\rm aA} = \frac{2400\underline{/0^\circ}}{16 + j12} = 96 - j72 = 120\underline{/-36.87^\circ}\,\mathrm{A}$$

With an acb phase sequence,

$$\underline{/\mathbf{I}_{\rm bB}} = \underline{/\mathbf{I}_{\rm aA}} + 120^\circ \quad \text{and} \quad \underline{/\mathbf{I}_{\rm cC}} = \underline{/\mathbf{I}_{\rm aA}} - 120^\circ$$

so

$$\mathbf{I}_{\rm aA} = 120\underline{/-36.87^\circ}\,\mathrm{A}$$

$$\mathbf{I}_{\rm bB} = 120\underline{/83.13^\circ}\,\mathrm{A}$$

$$\mathbf{I}_{\rm cC} = 120\underline{/-156.87^\circ}\,\mathrm{A}$$

[b] The line voltages at the source are $\mathbf{V}_{\rm ab}$ $\mathbf{V}_{\rm bc}$, and $\mathbf{V}_{\rm ca}$. They form a balanced set. To find $\mathbf{V}_{\rm ab}$, use the a-phase circuit to find $\mathbf{V}_{\rm AN}$, and use the relationship between phase voltages and line voltages for a y-connection (see Fig. 11.9[b]). From the a-phase circuit, use KVL:

$$\mathbf{V}_{\rm an} = \mathbf{V}_{\rm aA} + \mathbf{V}_{\rm AN} = (0.1 + j0.8)\mathbf{I}_{\rm aA} + 2400\underline{/0^\circ}$$

$$= (0.1 + j0.8)(96 - j72) + 2400\underline{/0^\circ} = 2467.2 + j69.6$$

$$2468.18\underline{/1.62^\circ}\,\mathrm{V}$$

From Fig. 11.9(b),

$$\mathbf{V}_{\rm ab} = \mathbf{V}_{\rm an}(\sqrt{3}\underline{/-30^\circ}) = 4275.02\underline{/-28.38^\circ}\,\mathrm{V}$$

With an acb phase sequence,

$$\underline{/\mathbf{V}_{\rm bc}} = \underline{/\mathbf{V}_{\rm ab}} + 120^\circ \quad \text{and} \quad \underline{/\mathbf{V}_{\rm ca}} = \underline{/\mathbf{V}_{\rm ab}} - 120^\circ$$

so

$$\mathbf{V}_{\rm ab} = 4275.02\underline{/-28.38^\circ}\,\mathrm{V}$$

$$\mathbf{V}_{\rm bc} = 4275.02\underline{/91.62^\circ}\,\mathrm{V}$$

$$\mathbf{V}_{\rm ca} = 4275.02\underline{/-148.38^\circ}\,\mathrm{V}$$

[c] Using KVL on the a-phase circuit

$$\mathbf{V}_{\mathrm{a'n}} = \mathbf{V}_{\mathrm{a'a}} + \mathbf{V}_{\mathrm{an}} = (0.2 + j0.16)\mathbf{I}_{\mathrm{aA}} + \mathbf{V}_{\mathrm{an}}$$

$$= (0.02 + j0.16)(96 - j72) + (2467.2 + j69.9)$$

$$= 2480.64 + j83.52 = 2482.05\underline{/1.93^\circ}\,\mathrm{V}$$

With an acb phase sequence,

$$\underline{/\mathbf{V}_{\mathrm{b'n}}} = \underline{/\mathbf{V}_{\mathrm{a'n}}} + 120^\circ \quad \text{and} \quad \underline{/\mathbf{V}_{\mathrm{c'n}}} = \underline{/\mathbf{V}_{\mathrm{a'n}}} - 120^\circ$$

so

$$\mathbf{V}_{\mathrm{a'n}} = 2482.05\underline{/1.93^\circ}\,\mathrm{V}$$

$$\mathbf{V}_{\mathrm{b'n}} = 2482.05\underline{/121.93^\circ}\,\mathrm{V}$$

$$\mathbf{V}_{\mathrm{c'n}} = 2482.05\underline{/-118.07^\circ}\,\mathrm{V}$$

AP 11.4

$$\mathbf{I}_{\mathrm{cC}} = (\sqrt{3}\underline{/-30^\circ})\mathbf{I}_{\mathrm{CA}} = (\sqrt{3}\underline{/-30^\circ}) \cdot 8\underline{/-15^\circ} = 13.86\underline{/-45^\circ}\,\mathrm{A}$$

AP 11.5

$$\mathbf{I}_{\mathrm{aA}} = 12\underline{/(65^\circ - 120^\circ)} = 12\underline{/-55^\circ}$$

$$\mathbf{I}_{\mathrm{AB}} = \left[\left(\frac{1}{\sqrt{3}}\right)\underline{/-30^\circ}\right]\mathbf{I}_{\mathrm{aA}} = \left(\frac{\underline{/-30^\circ}}{\sqrt{3}}\right) \cdot 12\underline{/-55^\circ}$$

$$= 6.93\underline{/-85^\circ}\,\mathrm{A}$$

AP 11.6 **[a]** $\mathbf{I}_{\mathrm{AB}} = \left[\left(\frac{1}{\sqrt{3}}\right)\underline{/30^\circ}\right][69.28\underline{/-10^\circ}] = 40\underline{/20^\circ}\,\mathrm{A}$

Therefore $\quad Z_\phi = \dfrac{4160\underline{/0^\circ}}{40\underline{/20^\circ}} = 104\underline{/-20^\circ}\,\Omega$

[b] $\mathbf{I}_{\mathrm{AB}} = \left[\left(\frac{1}{\sqrt{3}}\right)\underline{/-30^\circ}\right][69.28\underline{/-10^\circ}] = 40\underline{/-40^\circ}\,\mathrm{A}$

Therefore $\quad Z_\phi = 104\underline{/40^\circ}\,\Omega$

AP 11.7

$$\mathbf{I}_\phi = \frac{110}{3.667} + \frac{110}{j2.75} = 30 - j40 = 50\underline{/-53.13^\circ}\,\mathrm{A}$$

Therefore $\quad |\mathbf{I}_{\mathrm{aA}}| = \sqrt{3}\mathbf{I}_\phi = \sqrt{3}(50) = 86.60\,\mathrm{A}$

AP 11.8 **[a]** $|S| = \sqrt{3}(208)(73.8) = 26{,}587.67\,\mathrm{VA}$

$$Q = \sqrt{(26{,}587.67)^2 - (22{,}659)^2} = 13{,}909.50\ \mathrm{VAR}$$

[b] $$\text{pf} = \frac{22{,}659}{26{,}587.67} = 0.8522 \quad \text{lagging}$$

AP 11.9 **[a]** $$\mathbf{V}_{\rm AN} = \left(\frac{2450}{\sqrt{3}}\right)\underline{/0^\circ}\,\text{V}; \qquad \mathbf{V}_{\rm AN}\mathbf{I}^*_{\rm aA} = S_\phi = 144 + j192\,\text{kVA}$$

Therefore

$$\mathbf{I}^*_{\rm aA} = \frac{(144 + j192)1000}{2450/\sqrt{3}} = (101.8 + j135.7)\,\text{A}$$

$$\mathbf{I}_{\rm aA} = 101.8 - j135.7 = 169.67\underline{/-53.13^\circ}\,\text{A}$$

$$|\mathbf{I}_{\rm aA}| = 169.67\,\text{A}$$

[b] $$P = \frac{(2450)^2}{R}; \qquad \text{therefore} \quad R = \frac{(2450)^2}{144{,}000} = 41.68\,\Omega$$

$$Q = \frac{(2450)^2}{X}; \qquad \text{therefore} \quad X = \frac{(2450)^2}{192{,}000} = 31.26\,\Omega$$

[c] $$Z_\phi = \frac{\mathbf{V}_{\rm AN}}{\mathbf{I}_{\rm aA}} = \frac{2450/\sqrt{3}}{169.67\underline{/-53.13^\circ}} = 8.34\underline{/53.13^\circ} = (5 + j6.67)\,\Omega$$

$$\therefore \quad R = 5\,\Omega, \qquad X = 6.67\,\Omega$$

Problems

P 11.1 $\mathbf{V}_\mathrm{a} = V_m\underline{/0^\circ} = V_m + j0$

$\mathbf{V}_\mathrm{b} = V_m\underline{/-120^\circ} = -V_m(0.5 + j0.866)$

$\mathbf{V}_\mathrm{c} = V_m\underline{/120^\circ} = V_m(-0.5 + j0.866)$

$$\begin{aligned}\mathbf{V}_\mathrm{a} + \mathbf{V}_\mathrm{b} + \mathbf{V}_\mathrm{c} &= (V_m)(1 + j0 - 0.5 - j0.866 - 0.5 + j0.866)\\ &= V_m(0) = 0\end{aligned}$$

P 11.2 [**a**] First, convert the cosine waveforms to phasors:

$\mathbf{V}_\mathrm{a} = 208\underline{/27^\circ}$; $\mathbf{V}_\mathrm{b} = 208\underline{/-147^\circ}$; $\mathbf{V}_\mathrm{c} = 208\underline{/-93^\circ}$

Subtract the phase angle of the a-phase from all phase angles:

$\underline{/\mathbf{V}'_\mathrm{a}} = 27^\circ - 27^\circ = 0^\circ$

$\underline{/\mathbf{V}'_\mathrm{b}} = 147^\circ - 27^\circ = 120^\circ$

$\underline{/\mathbf{V}'_\mathrm{c}} = -93^\circ - 27^\circ = -120^\circ$

Compare the result to Eqs. 11.1 and 11.2:

Therefore acb

[**b**] First, convert the cosine waveforms to phasors:

$\mathbf{V}_\mathrm{a} = 4160\underline{/-18^\circ}$; $\mathbf{V}_\mathrm{b} = 4160\underline{/-138^\circ}$; $\mathbf{V}_\mathrm{c} = 4160\underline{/+102^\circ}$

Subtract the phase angle of the a-phase from all phase angles:

$\underline{/\mathbf{V}'_\mathrm{a}} = -18^\circ + 18^\circ = 0^\circ$

$\underline{/\mathbf{V}'_\mathrm{b}} = -138^\circ + 18^\circ = -120^\circ$

$\underline{/\mathbf{V}'_\mathrm{c}} = 102^\circ + 18^\circ = 120^\circ$

Compare the result to Eqs. 11.1 and 11.2:

Therefore abc

P 11.3 [**a**] $\mathbf{V}_\mathrm{a} = 139\underline{/0^\circ}\,\mathrm{V}$

$\mathbf{V}_\mathrm{b} = 139\underline{/120^\circ}\,\mathrm{V}$

$\mathbf{V}_\mathrm{c} = 139\underline{/-120^\circ}\,\mathrm{V}$

Balanced, negative phase sequence

[**b**] $\mathbf{V}_\mathrm{a} = 381\underline{/0^\circ}\,\mathrm{V}$

$\mathbf{V}_\mathrm{b} = 381\underline{/240^\circ}\,\mathrm{V} = 622\underline{/-120^\circ}\,\mathrm{V}$

$\mathbf{V}_\mathrm{c} = 381\underline{/120^\circ}\,\mathrm{V}$

Balanced, positive phase sequence

[**c**] $\mathbf{V}_\mathrm{a} = 2771\underline{/-120^\circ}\,\mathrm{V}$

$\mathbf{V}_\mathrm{b} = 2771\underline{/0^\circ}\,\mathrm{V}$

$\mathbf{V}_\mathrm{c} = 2771\underline{/120^\circ}\,\mathrm{V}$

Balanced, negative phase sequence

[**d**] $\mathbf{V}_\mathrm{a} = 170\underline{/-60^\circ}\,\mathrm{V}$

$\mathbf{V}_\mathrm{b} = 170\underline{/180^\circ}\,\mathrm{V}$

$\mathbf{V}_\mathrm{c} = 170\underline{/60^\circ}\,\mathrm{V}$

Balanced, positive phase sequence

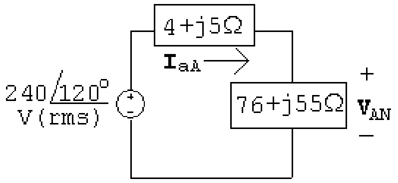
[**e**] Unbalanced, due to unequal amplitudes

[**f**] Unbalanced, due to unequal phase angle separation

P 11.4 $\mathbf{I} = \dfrac{\mathbf{V}_\mathrm{a} + \mathbf{V}_\mathrm{b} + \mathbf{V}_\mathrm{c}}{3(R_\mathrm{W} + jX_\mathrm{W})} = 0$

P 11.5 [**a**] $\mathbf{V}_\mathrm{an} = 1/\sqrt{3}\underline{/30^\circ}\mathbf{V}_\mathrm{ab} = 240\underline{/120^\circ}\,\mathrm{V}$ (rms)

The a-phase circuit is



[**b**] $\mathbf{I}_\mathrm{aA} = \dfrac{240\underline{/120^\circ}}{80 + j60} = 2.4\underline{/83.13^\circ}\,\mathrm{A}$ (rms)

[**c**] $\mathbf{V}_\mathrm{AN} = (76 + j55)\mathbf{I}_\mathrm{aA} = 225.15\underline{/119.02^\circ}\,\mathrm{V}$ (rms)

$\mathbf{V}_\mathrm{AB} = \sqrt{3}\underline{/-30^\circ}\mathbf{V}_\mathrm{AN} = 389.98\underline{/89.02^\circ}\,\mathrm{A}$ (rms)

P 11.6 $Z_{ga} + Z_{la} + Z_{La} = 60 + j80\,\Omega$

$Z_{gb} + Z_{lb} + Z_{Lb} = 40 + j30\Omega$

$Z_{gc} + Z_{lc} + Z_{Lc} = 20 + j15\Omega$

$$\frac{\mathbf{V}_N - 240}{60 + j80} + \frac{\mathbf{V}_N - 240\underline{/120^\circ}}{40 + j30} + \frac{\mathbf{V}_N - 240\underline{/-120^\circ}}{20 + j15} + \frac{\mathbf{V}_N}{10} = 0$$

Solving for $\mathbf{V}_N$ yields

$$\mathbf{V}_N = 42.94\underline{/-156.32^\circ}\text{ V (rms)}$$

$$\mathbf{I}_o = \frac{\mathbf{V}_N}{10} = 4.29\underline{/-156.32^\circ}\text{ A (rms)}$$

P 11.7 $\mathbf{V}_{\text{AN}} = 7620\underline{/30^\circ}\text{ V}$

$$\mathbf{V}_{\text{BN}} = 7620\underline{/150^\circ}\text{ V}$$

$$\mathbf{V}_{\text{CN}} = 7620\underline{/-90^\circ}\text{ V}$$

$$\mathbf{V}_{\text{AB}} = \mathbf{V}_{\text{AN}} - \mathbf{V}_{\text{BN}} = 13{,}198.23\underline{/0^\circ}\text{ V}$$

$$\mathbf{V}_{\text{BC}} = \mathbf{V}_{\text{BN}} - \mathbf{V}_{\text{CN}} = 13{,}198.23\underline{/120^\circ}\text{ V}$$

$$\mathbf{V}_{\text{CA}} = \mathbf{V}_{\text{CN}} - \mathbf{V}_{\text{AN}} = 13{,}198.23\underline{/-120^\circ}\text{ V}$$

$$v_{\text{AB}} = 13{,}198.23\cos\omega t\text{ V}$$

$$v_{\text{BC}} = 13{,}198.23\cos(\omega t + 120^\circ)\text{ V}$$

$$v_{\text{CA}} = 13{,}198.23\cos(\omega t - 120^\circ)\text{ V}$$

P 11.8 [a] $\mathbf{I}_{\text{aA}} = \dfrac{200}{25} = 8\text{ A (rms)}$

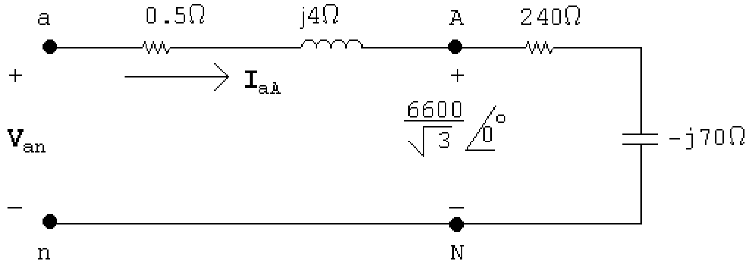
$$\mathbf{I}_{\text{bB}} = \frac{200\underline{/-120^\circ}}{30 - j40} = 4\underline{/-66.87^\circ}\text{ A (rms)}$$

$$\mathbf{I}_{\text{cC}} = \frac{200\underline{/120^\circ}}{80 + j60} = 2\underline{/83.13^\circ}\text{ A (rms)}$$

The magnitudes are unequal and the phase angles are not 120° apart.

b] $\mathbf{I}_o = \mathbf{I}_{\text{aA}} + \mathbf{I}_{\text{bB}} + \mathbf{I}_{\text{cC}} = 9.96\underline{/-9.79^\circ}\text{ A (rms)}$

P 11.9 [a]



$$\mathbf{I}_{\text{aA}} = \frac{6600}{\sqrt{3}(240 - j70)} = 15.24\underline{/16.26^\circ}\text{ A (rms)}$$

$$|\mathbf{I}_{\text{aA}}| = |\mathbf{I}_{\text{L}}| = 15.24\text{ A (rms)}$$

[b] $\mathbf{V}_{\text{an}} = (15.24\underline{/16.26^\circ})(240 - j66) = 3801.24\underline{/0.91^\circ}$

$|\mathbf{V}_{\text{ab}}| = \sqrt{3}(3801.24) = 6583.94\,\text{V (rms)}$

P 11.10 [a] $\mathbf{I}_{\text{aA}} = \dfrac{277\underline{/0^\circ}}{80 + j60} = 2.77\underline{/-36.87^\circ}\,\text{A (rms)}$

$$\mathbf{I}_{\text{bB}} = \frac{277\underline{/-120^\circ}}{80 + j60} = 2.77\underline{/-156.87^\circ}\,\text{A (rms)}$$

$$\mathbf{I}_{\text{cC}} = \frac{277\underline{/120^\circ}}{80 + j60} = 2.77\underline{/83.13^\circ}\,\text{A (rms)}$$

$$\mathbf{I}_o = \mathbf{I}_{\text{aA}} + \mathbf{I}_{\text{bB}} + \mathbf{I}_{\text{cC}} = 0$$

[b] $\mathbf{V}_{\text{AN}} = (78 + j54)\mathbf{I}_{\text{aA}} = 262.79\underline{/-2.17^\circ}\,\text{V (rms)}$

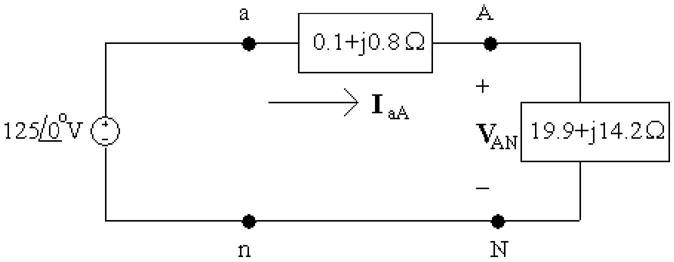
[c] $\mathbf{V}_{\text{AB}} = \mathbf{V}_{\text{AN}} - \mathbf{V}_{\text{BN}}$

$$\mathbf{V}_{\text{BN}} = (77 + j56)\mathbf{I}_{\text{bB}} = 263.73\underline{/-120.84^\circ}\,\text{V (rms)}$$

$$\mathbf{V}_{\text{AB}} = 262.79\underline{/-2.17^\circ} - 263.73\underline{/-120.84^\circ} = 452.89\underline{/28.55^\circ}\,\text{V (rms)}$$

[d] Unbalanced — see conditions for a balanced circuit on p. 504 of the text!

P 11.11 Make a sketch of the a-phase:



[a] Find the a-phase line current from the a-phase circuit:

$$\mathbf{I}_{\text{aA}} = \frac{125\underline{/0^\circ}}{0.1 + j0.8 + 19.9 + j14.2} = \frac{125\underline{/0^\circ}}{20 + j15}$$

$$= 4 - j3 = 5\underline{/-36.87^\circ}\,\text{A (rms)}$$

Find the other line currents using the acb phase sequence:

$$\mathbf{I}_{\text{bB}} = 5\underline{/-36.87^\circ + 120^\circ} = 5\underline{/83.13^\circ}\,\text{A (rms)}$$

$$\mathbf{I}_{\text{cC}} = 5\underline{/-36.87^\circ - 120^\circ} = 5\underline{/-156.87^\circ}\,\text{A (rms)}$$

[b] The phase voltage at the source is $\mathbf{V}_{\rm an} = 125\underline{/0^\circ}$ V. Use Fig. 11.9(b) to find the line voltage, $\mathbf{V}_{\rm an}$, from the phase voltage:

$$\mathbf{V}_{\rm ab} = \mathbf{V}_{\rm an}(\sqrt{3}\underline{/-30^\circ}) = 216.51\underline{/-30^\circ}\text{ V (rms)}$$

Find the other line voltages using the acb phase sequence:

$$\mathbf{V}_{\rm bc} = 216.51\underline{/-30^\circ + 120^\circ} = 216.51\underline{/90^\circ}\text{ V (rms)}$$

$$\mathbf{V}_{\rm ca} = 216.51\underline{/-30^\circ - 120^\circ} = 216.51\underline{/-150^\circ}\text{ V (rms)}$$

[c] The phase voltage at the load in the a-phase is $\mathbf{V}_{\rm AN}$. Calculate its value using $\mathbf{I}_{\rm aA}$ and the load impedance:

$$\mathbf{V}_{\rm AN} = \mathbf{I}_{\rm aA}Z_{\rm L} = (4 - j3)(19.9 + j14.2) = 122.2 - j2.9 = 122.23\underline{/-1.36^\circ}\text{ V (rms)}$$

Find the phase voltage at the load for the b- and c-phases using the acb sequence:

$$\mathbf{V}_{\rm BN} = 122.23\underline{/-1.36^\circ + 120^\circ} = 122.23\underline{/118.64^\circ}\text{ V (rms)}$$

$$\mathbf{V}_{\rm CN} = 122.23\underline{/-1.36^\circ - 120^\circ} = 122.23\underline{/-121.36^\circ}\text{ V (rms)}$$

[d] The line voltage at the load in the a-phase is $\mathbf{V}_{\rm AB}$. Find this line voltage from the phase voltage at the load in the a-phase, $\mathbf{V}_{\rm AN}$, using Fig, 11.9(b):

$$\mathbf{V}_{\rm AB} = \mathbf{V}_{\rm AN}(\sqrt{3}\underline{/-30^\circ}) = 211.72\underline{/-31.36^\circ}\text{ V (rms)}$$

Find the line voltage at the load for the b- and c-phases using the acb sequence:

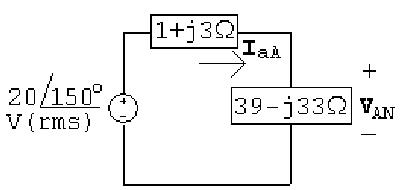
$$\mathbf{V}_{\rm BC} = 211.72\underline{/-31.36^\circ + 120^\circ} = 211.72\underline{/88.64^\circ}\text{ V (rms)}$$

$$\mathbf{V}_{\rm CA} = 211.72\underline{/-31.36^\circ - 120^\circ} = 211.72\underline{/-151.36^\circ}\text{ V (rms)}$$

P 11.12 [a] $\mathbf{V}_{\rm an} = \mathbf{V}_{\rm cn} - \underline{/120^\circ} = 20\underline{/-210^\circ} = 20\underline{/150^\circ}$ V (rms)

$$Z_y = Z_\Delta/3 = 39 - j33\,\Omega$$

The a-phase circuit is



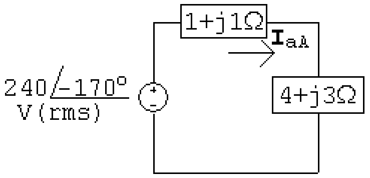
[b] $\mathbf{I}_{\rm aA} = \dfrac{20\underline{/150^\circ}}{40 - j30} = 0.4\underline{/-173.13^\circ}$ A (rms)

[c] $\mathbf{V}_{\rm AN} = (39 + j33)\mathbf{I}_{\rm aA} = 20.44\underline{/146.63^\circ}$ V (rms)

$$\mathbf{V}_{\rm AB} = \sqrt{3}\underline{/30^\circ}\mathbf{V}_{\rm AN} = 35.39\underline{/176.63^\circ}\text{ A (rms)}$$

P 11.13 $Z_y = Z_\Delta/3 = 4 + j3\,\Omega$

The a-phase circuit is



$$\mathbf{I}_{\mathrm{aA}} = \frac{240\underline{/-170^\circ}}{(1+j1)+(4+j3)} = 37.48\underline{/151.34^\circ}\text{ A (rms)}$$

$$\mathbf{I}_{\mathrm{AB}} = \frac{1}{\sqrt{3}}\underline{/-30^\circ}\mathbf{I}_{\mathrm{aA}} = 21.64\underline{/121.34^\circ}\text{ A (rms)}$$

P 11.14 [a] $\mathbf{I}_{\mathrm{AB}} = \dfrac{69{,}000}{864 - j252} = 76.67\underline{/16.26^\circ}\text{ A (rms)}$

$\mathbf{I}_{\mathrm{BC}} = 76.67\underline{/-103.74^\circ}\text{ A (rms)}$

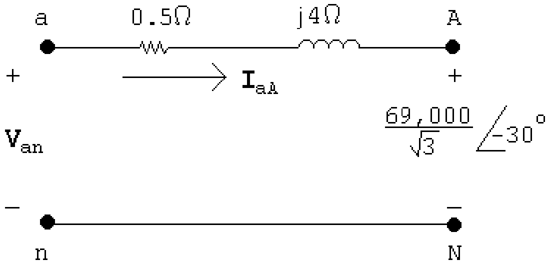
$\mathbf{I}_{\mathrm{CA}} = 76.67\underline{/136.26^\circ}\text{ A (rms)}$

[b] $\mathbf{I}_{\mathrm{aA}} = \sqrt{3}\underline{/-30^\circ}\mathbf{I}_{\mathrm{AB}} = 132.79\underline{/-13.74^\circ}\text{ A (rms)}$

$\mathbf{I}_{\mathrm{bB}} = 132.79\underline{/-133.74^\circ}\text{ A (rms)}$

$\mathbf{I}_{\mathrm{cC}} = 132.79\underline{/106.26^\circ}\text{ A (rms)}$

[c]



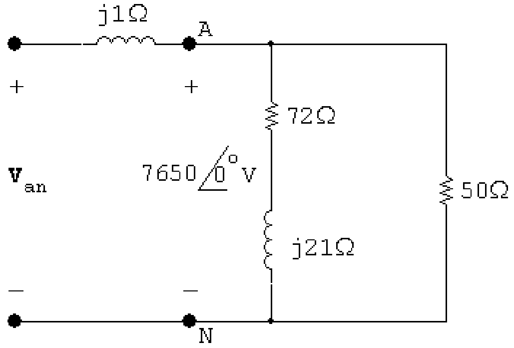
$$\mathbf{V}_{\mathrm{an}} = \frac{13{,}000}{\sqrt{3}}\underline{/-30^\circ} + (0.5+j4)(132.79\underline{/-13.74^\circ})$$
$$= 39{,}755.70\underline{/-29.24^\circ}\text{ V (rms)}$$

$\mathbf{V}_{\mathrm{ab}} = \sqrt{3}\underline{/30^\circ}\mathbf{V}_{\mathrm{an}} = 68{,}858.88\underline{/0.76^\circ}\text{ V (rms)}$

$\mathbf{V}_{\mathrm{bc}} = 68{,}858.88\underline{/-119.24^\circ}\text{ V (rms)}$

$\mathbf{V}_{\mathrm{ca}} = 68{,}858.88\underline{/120.76^\circ}\text{ V (rms)}$

P 11.15 [**a**]



$$\mathbf{I}_{\text{aA}} = \frac{7650}{72 + j21} + \frac{7650}{50} = 252.54\underline{/-6.49^\circ}\,\text{A (rms)}$$

$$|\mathbf{I}_{\text{aA}}| = 252.54\,\text{A (rms)}$$

[**b**] $$\mathbf{I}_{\text{AB}} = \frac{7650\sqrt{3}\underline{/30^\circ}}{150} = 88.33\underline{/30^\circ}\,\text{A (rms)}$$

$$|\mathbf{I}_{\text{AB}}| = 88.33\,\text{A (rms)}$$

[**c**] $$\mathbf{I}_{\text{AN}} = \frac{7650\underline{/0^\circ}}{72 + j21} = 102\underline{/-16.26^\circ}\,\text{A (rms)}$$

$$|\mathbf{I}_{\text{AN}}| = 102\,\text{A (rms)}$$

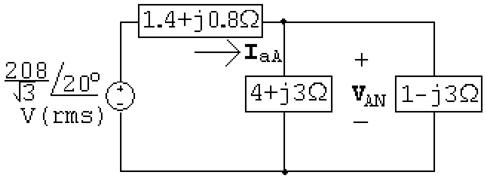
[**d**] $$\mathbf{V}_{\text{an}} = (252.54\underline{/-6.49^\circ})(j1) + 7650\underline{/0^\circ} = 7682.66\underline{/1.87^\circ}\,\text{V (rms)}$$

$$|\mathbf{V}_{\text{ab}}| = \sqrt{3}(7682.66) = 13{,}306.76\,\text{V (rms)}$$

P 11.16 $$\mathbf{V}_{\text{an}} = 1/\sqrt{3}\underline{/-30^\circ}\mathbf{V}_{\text{ab}} = \frac{208}{\sqrt{3}}\underline{/20^\circ}\,\text{V (rms)}$$

$$Z_y = Z_\Delta/3 = 1 - j3\,\Omega$$

The a-phase circuit is



$$Z_{\text{eq}} = (4 + j3)\|(1 - j3) = 2.6 - j1.8\,\Omega$$

$$\mathbf{V}_{\text{AN}} = \frac{2.6 - j1.8}{(1.4 + j0.8) + (2.6 - j1.8)}\left(\frac{208}{\sqrt{3}}\right)\underline{/20^\circ} = 92.1\underline{/-0.66^\circ}\,\text{V (rms)}$$

$$\mathbf{V}_{\text{AB}} = \sqrt{3}\underline{/30^\circ}\mathbf{V}_{\text{AN}} = 159.5\underline{/29.34^\circ}\,\text{V (rms)}$$

P 11.17 [**a**] $$\mathbf{I}_{AB} = \frac{13{,}200/\underline{0^\circ}}{100 - j75} = 105.6/\underline{36.87^\circ}\text{ A (rms)}$$

$$\mathbf{I}_{BC} = 105.6/\underline{156.87^\circ}\text{ A (rms)}$$

$$\mathbf{I}_{CA} = 105.6/\underline{-83.13^\circ}\text{ A (rms)}$$

[**b**] $$\mathbf{I}_{aA} = \sqrt{3}/\underline{-30^\circ}\mathbf{I}_{AB} = 182.9/\underline{66.87^\circ}\text{ A (rms)}$$

$$\mathbf{I}_{bB} = 182.9/\underline{-173.13^\circ}\text{ A (rms)}$$

$$\mathbf{I}_{cC} = 182.9/\underline{-53.13^\circ}\text{ A (rms)}$$

[**c**] $$\mathbf{I}_{ba} = \mathbf{I}_{AB} = 105.6/\underline{36.87^\circ}\text{ A (rms)}$$

$$\mathbf{I}_{cb} = \mathbf{I}_{BC} = 105.6/\underline{156.87^\circ}\text{ A (rms)}$$

$$\mathbf{I}_{ac} = \mathbf{I}_{CA} = 105.6/\underline{-83.13^\circ}\text{ A (rms)}$$

P 11.18 [**a**] $$\mathbf{I}_{AB} = \frac{480/\underline{0^\circ}}{2.4 - j0.7} = 192/\underline{16.26^\circ}\text{ A (rms)}$$

$$\mathbf{I}_{BC} = \frac{480/\underline{120^\circ}}{8 + j6} = 48/\underline{83.13^\circ}\text{ A (rms)}$$

$$\mathbf{I}_{CA} = \frac{480/\underline{-120^\circ}}{20} = 24/\underline{-120^\circ}\text{ A (rms)}$$

[**b**] $$\mathbf{I}_{aA} = \mathbf{I}_{AB} - \mathbf{I}_{CA}$$

$$= 210/\underline{20.79^\circ}$$

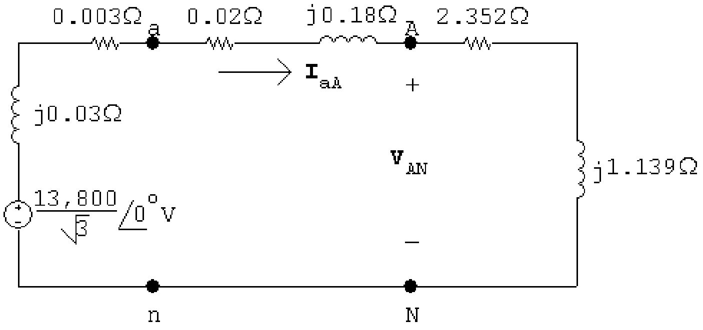
$$\mathbf{I}_{bB} = \mathbf{I}_{BC} - \mathbf{I}_{AB}$$

$$= 178.68/\underline{-178.04^\circ}$$

$$\mathbf{I}_{cC} = \mathbf{I}_{CA} - \mathbf{I}_{BC}$$

$$= 70.7/\underline{-104.53^\circ}$$

P 11.19 [**a**]



[b] $\mathbf{I}_{\rm aA} = \dfrac{13{,}800}{\sqrt{3}(2.375 + j1.349)} = 2917\underline{/-29.6^\circ}\,\text{A (rms)}$

$|\mathbf{I}_{\rm aA}| = 2917\,\text{A (rms)}$

[c] $\mathbf{V}_{\rm AN} = (2.352 + j1.139)(2917\underline{/-29.6^\circ}) = 7622.93\underline{/-3.76^\circ}\,\text{V (rms)}$

$|\mathbf{V}_{\rm AB}| = \sqrt{3}|\mathbf{V}_{\rm AN}| = 13{,}203.31\,\text{V (rms)}$

[d] $\mathbf{V}_{\rm an} = (2.372 + j1.319)(2917\underline{/-29.6^\circ}) = 7616.93\underline{/-0.52^\circ}\,\text{V (rms)}$

$|\mathbf{V}_{\rm ab}| = \sqrt{3}|\mathbf{V}_{\rm an}| = 13{,}712.52\,\text{V (rms)}$

[e] $|\mathbf{I}_{\rm AB}| = \dfrac{|\mathbf{I}_{\rm aA}|}{\sqrt{3}} = 1684.13\,\text{A (rms)}$

[f] $|\mathbf{I}_{\rm ab}| = |\mathbf{I}_{\rm AB}| = 1684.13\,\text{A (rms)}$

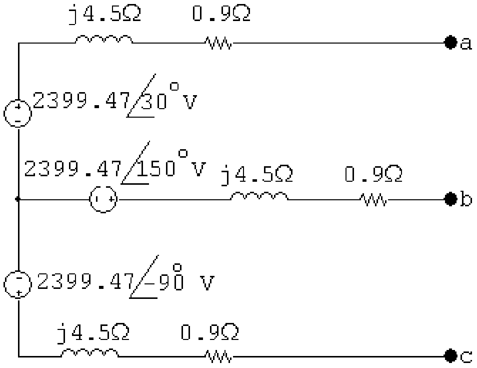
P 11.20 [a] Since the phase sequence is acb (negative) we have:

$\mathbf{V}_{\rm an} = 2399.47\underline{/30^\circ}\,\text{V (rms)}$

$\mathbf{V}_{\rm bn} = 2399.47\underline{/150^\circ}\,\text{V (rms)}$

$\mathbf{V}_{\rm cn} = 2399.47\underline{/-90^\circ}\,\text{V (rms)}$

$Z_Y = \frac{1}{3}Z_\Delta = 0.9 + j4.5\,\Omega/\phi$



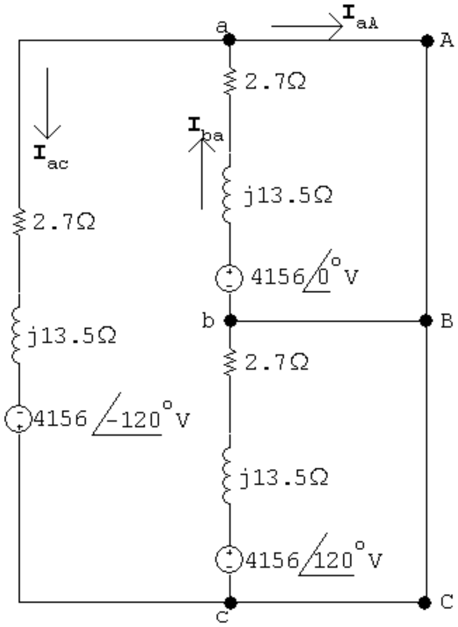
[b] $\mathbf{V}_{\rm ab} = 2399.47\underline{/30^\circ} - 2399.47\underline{/150^\circ} = 2399.47\sqrt{3}\underline{/0^\circ} = 4156\underline{/0^\circ}\,\text{V (rms)}$

Since the phase sequence is negative, it follows that

$\mathbf{V}_{\rm bc} = 4156\underline{/120^\circ}\,\text{V (rms)}$

$\mathbf{V}_{\rm ca} = 4156\underline{/-120^\circ}\,\text{V (rms)}$

[**c**]



$$\mathbf{I}_{\text{ba}} = \frac{4156}{2.7 + j13.5} = 301.87\underline{/-78.69^\circ}\text{ A (rms)}$$

$$\mathbf{I}_{\text{ac}} = 301.87\underline{/-198.69^\circ}\text{ A (rms)}$$

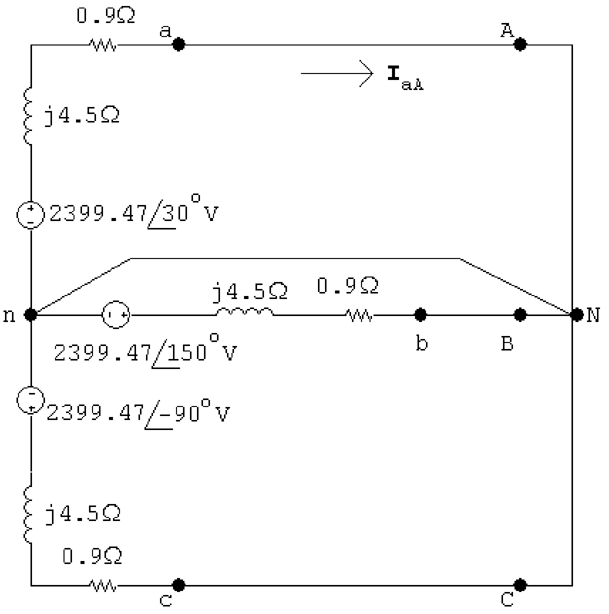
$$\mathbf{I}_{\text{aA}} = \mathbf{I}_{\text{ba}} - \mathbf{I}_{\text{ac}} = 522.86\underline{/-48.69^\circ}\text{ A (rms)}$$

Since we have a balanced three-phase circuit and a negative phase sequence we have:

$$\mathbf{I}_{\text{bB}} = 522.86\underline{/71.31^\circ}\text{ A (rms)}$$

$$\mathbf{I}_{\text{cC}} = 522.86\underline{/-168.69^\circ}\text{ A (rms)}$$

[d]



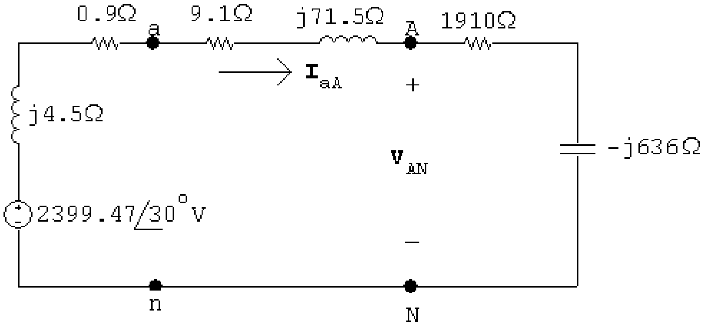
$$\mathbf{I}_{\text{aA}} = \frac{2399.47\underline{/30^\circ}}{0.9 + j4.5} = 522.86\underline{/-48.69^\circ}\,\text{A (rms)}$$

Since we have a balanced three-phase circuit and a negative phase sequence we have:

$$\mathbf{I}_{\text{bB}} = 522.86\underline{/71.31^\circ}\,\text{A (rms)}$$

$$\mathbf{I}_{\text{cC}} = 522.86\underline{/-168.69^\circ}\,\text{A (rms)}$$

P 11.21 **[a]**



[b] $\mathbf{I}_{\text{aA}} = \dfrac{2399.47\underline{/30^\circ}}{1920 - j560} = 1.2\underline{/46.26^\circ}\,\text{A (rms)}$

$$\mathbf{V}_{\text{AN}} = (1910 - j636)(1.2\underline{/46.26^\circ}) = 2415.19\underline{/27.84^\circ}\,\text{V (rms)}$$

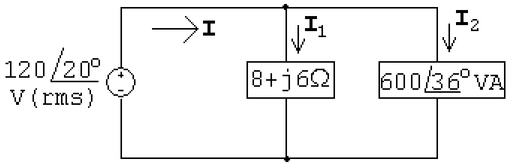
$$|\mathbf{V}_{\text{AB}}| = \sqrt{3}(2415.19) = 4183.24\,\text{V (rms)}$$

[c] $|\mathbf{I}_{\text{ab}}| = \dfrac{1.2}{\sqrt{3}} = 0.69\,\text{A (rms)}$

[d] $\mathbf{V}_{\rm an} = (1919.1 - j564.5)(1.2\underline{/46.26^\circ}) = 2400.48\underline{/29.87^\circ}\,\text{V (rms)}$

$|\mathbf{V}_{\rm ab}| = \sqrt{3}(2400.48) = 4157.76\,\text{V (rms)}$

P 11.22 The a-phase of the circuit is shown below:



$$\mathbf{I}_1 = \frac{120\underline{/20^\circ}}{8+j6} = 12\underline{/-16.87^\circ}\text{A (rms)}$$

$$\mathbf{I}_2^* = \frac{600\underline{/36^\circ}}{120\underline{/20^\circ}} = 5\underline{/16^\circ}\text{A (rms)}$$

$$\mathbf{I} = \mathbf{I}_1 + \mathbf{I}_2 = 12\underline{/-16.87^\circ} + 5\underline{/-16^\circ} = 17\underline{/-16.61^\circ}\ \text{A (rms)}$$

$$S_{\rm a} = \mathbf{VI}^* = (120\underline{/20^\circ})(17\underline{/16.61^\circ}) = 2040\underline{/36.61^\circ}\ \text{VA}$$

$$S_{\rm T} = 3S_{\rm a} = 6120\underline{/36.61^\circ}\ \text{VA}$$

P 11.23 The complex power of the source per phase is $S_s = 20{,}000\underline{/(}\cos^{-1}0.6) = 20{,}000\underline{/53.13^\circ} = 12{,}000 + j16{,}000$ kVA. This complex power per phase must equal the sum of the per-phase impedances of the two loads:

$$S_s = S_1 + S_2 \quad \text{so} \quad 12{,}000 + j16{,}000 = 10{,}000 + S_2$$

$$\therefore\quad S_2 = 2000 + j16{,}000\ \text{VA}$$

$$\text{Also,}\quad S_2 = \frac{|V_{\rm rms}|^2}{Z_2^*}$$

$$|V_{\rm rms}| = \frac{|\text{V}_{\rm load}|}{\sqrt{3}} = 120\ \text{V (rms)}$$

$$\text{Thus,}\quad Z_2^* = \frac{|V_{\rm rms}|^2}{S_2} = \frac{(120)^2}{2000 + j16{,}000} = 0.11 - j0.89\,\Omega$$

$$\therefore\quad Z_2 = 0.11 + j0.89\,\Omega$$

P 11.24 **[a]** $S_{T\Delta} = 14{,}000\underline{/41.41^\circ} - 9000\underline{/53.13^\circ} = 5.5\underline{/22^\circ}$ kVA

$$S_\Delta = S_{T\Delta}/3 = 1833.46\underline{/22^\circ}\text{ VA}$$

[b] $$|\mathbf{V}_{\text{an}}| = \left|\frac{3000\underline{/53.13^\circ}}{10\underline{/-30^\circ}}\right| = 300\text{ V (rms)}$$

$$|\mathbf{V}_{\text{line}}| = |\mathbf{V}_{\text{ab}}| = \sqrt{3}|\mathbf{V}_{\text{an}}| = 300\sqrt{3} = 519.62\text{ V (rms)}$$

P 11.25 $$|I_{\text{line}}| = \frac{1600}{240/\sqrt{3}} = 11.547\text{ A (rms)}$$

$$|Z_y| = \frac{|V|}{|I|} = \frac{240/\sqrt{3}}{11.547} = 12$$

$$Z_y = 12\underline{/-50^\circ}\,\Omega$$

$$Z_\Delta = 3Z_y = 36\underline{/-50^\circ} = 23.14 - j27.58\,\Omega/\phi$$

P 11.26 Let p_{a}, p_{b}, and p_{c} represent the instantaneous power of phases a, b, and c, respectively. Then assuming a positive phase sequence, we have

$$p_{\text{a}} = v_{\text{an}}i_{\text{aA}} = [V_m\cos\omega t][I_m\cos(\omega t - \theta_\phi)]$$

$$p_{\text{b}} = v_{\text{bn}}i_{\text{bB}} = [V_m\cos(\omega t - 120^\circ)][I_m\cos(\omega t - \theta_\phi - 120^\circ)]$$

$$p_{\text{c}} = v_{\text{cn}}i_{\text{cC}} = [V_m\cos(\omega t + 120^\circ)][I_m\cos(\omega t - \theta_\phi + 120^\circ)]$$

The total instantaneous power is $p_T = p_{\text{a}} + p_{\text{b}} + p_{\text{c}}$, so

$$p_T = V_mI_m[\cos\omega t\cos(\omega t - \theta_\phi) + \cos(\omega t - 120^\circ)\cos(\omega t - \theta_\phi - 120^\circ)$$
$$+ \cos(\omega t + 120^\circ)\cos(\omega t - \theta_\phi + 120^\circ)]$$

Now simplify using trigonometric identities. In simplifying, collect the coefficients of $\cos(\omega t - \theta_\phi)$ and $\sin(\omega t - \theta_\phi)$. We get

$$\begin{aligned}p_T &= V_mI_m[\cos\omega t(1 + 2\cos^2 120^\circ)\cos(\omega t - \theta_\phi)\\ &\quad +2\sin\omega t\sin^2 120^\circ\sin(\omega t - \theta_\phi)]\\ &= 1.5V_mI_m[\cos\omega t\cos(\omega t - \theta_\phi) + \sin\omega t\sin(\omega t - \theta_\phi)]\\ &= 1.5V_mI_m\cos\theta_\phi\end{aligned}$$

P 11.27 **[a]** $S_1 = (4.864 + j3.775)\,\text{kVA}$

$$S_2 = 17.636(0.96) + j17.636(0.28) = (16.931 + j4.938)\,\text{kVA}$$

$$\sqrt{3}V_\text{L}I_\text{L}\sin\theta_3 = 13{,}853; \qquad \sin\theta_3 = \frac{13{,}853}{\sqrt{3}(208)(73.8)} = 0.521$$

Therefore $\cos\theta_3 = 0.854$

Therefore

$$P_3 = \frac{13{,}853}{0.521} \times 0.854 = 22{,}693.584\,\text{W}$$

$$S_3 = 22.694 + j13.853\,\text{kVA}$$

$$S_T = S_1 + S_2 + S_3 = 44.49 + j22.57\,\text{kVA}$$

$$S_{T/\phi} = \frac{1}{3}S_T = 14.83 + j7.52\,\text{kVA}$$

$$\frac{208}{\sqrt{3}}\mathbf{I}_\text{aA}^* = (14.83 + j7.52)10^3; \qquad \mathbf{I}_\text{aA}^* = 123.49 + j62.64\,\text{A}$$

$$\mathbf{I}_\text{aA} = 123.49 - j62.64 = 138.46\underline{/-26.90^\circ}\,\text{A} \quad \text{(rms)}$$

[b] pf $= \cos(0^\circ - 26.90^\circ) = 0.892$ lagging

P 11.28 From the solution to Problem 11.18 we have:

$$S_\text{AB} = (480\underline{/0^\circ})(192\underline{/-16.26^\circ}) = 88{,}473.7 - j25{,}804.5\,\text{VA}$$

$$S_\text{BC} = (480\underline{/120^\circ})(48\underline{/-83.13^\circ}) = 18{,}431.98 + j13{,}824.03\,\text{VA}$$

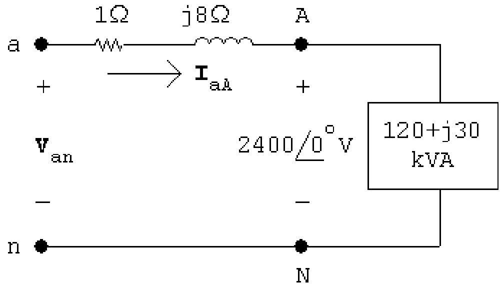
$$S_\text{CA} = (480\underline{/-120^\circ})(24\underline{/120^\circ}) = 11{,}520 + j0\,\text{VA}$$

P 11.29 **[a]** $S_{1/\phi} = 40{,}000(0.96) - j40{,}000(0.28) = 38{,}400 - j11{,}200$ VA

$$S_{2/\phi} = 60{,}000(0.8) + j60{,}000(0.6) = 48{,}000 + j36{,}000 \text{ VA}$$

$$S_{3/\phi} = 33{,}600 + j5200 \text{ VA}$$

$$S_{T/\phi} = S_1 + S_2 + S_3 = 120{,}000 + j30{,}000 \text{ VA}$$



$$\therefore \quad \mathbf{I}_\text{aA}^* = \frac{120{,}000 + j30{,}000}{2400} = 50 + j12.5$$

$$\therefore \quad \mathbf{I}_{\text{aA}} = 50 - j12.5\,\text{A}$$

$$\mathbf{V}_{\text{an}} = 2400 + (50 - j12.5)(1 + j8) = 2550 + j387.5 = 2579.27\underline{/8.64^\circ}\,\text{V (rms)}$$

$$|\mathbf{V}_{\text{ab}}| = \sqrt{3}(2579.27) = 4467.43\,\text{V (rms)}$$

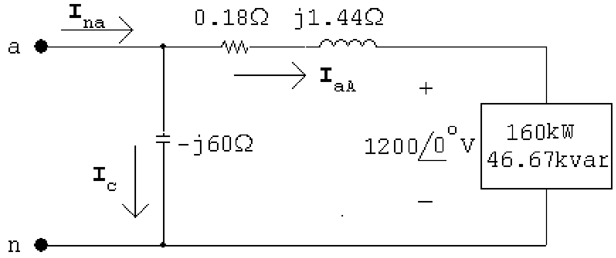
[b] $S_{g/\phi} = (2550 + j387.5)(50 + j12.5) = 122{,}656.25 + j51{,}250\,\text{VA}$

$$\%\ \text{efficiency} = \frac{120{,}000}{122{,}656.25}(100) = 97.83\%$$

P 11.30 **[a]** $\mathbf{I}^*_{\text{aA}} = \dfrac{(160 + j46.67)10^3}{1200} = 133.3 + j38.9$

$$\mathbf{I}_{\text{aA}} = 133.3 - j38.9\,\text{A (rms)}$$

$$\mathbf{V}_{\text{an}} = 1200 + (133.3 - j38.9)(0.18 + j1.44) = 1280 + j185\,\text{V (rms)}$$



$$\mathbf{I}_{\text{C}} = \frac{1280 + j185}{-j60} = -3.1 + j21.3\,\text{A (rms)}$$

$$\mathbf{I}_{\text{na}} = \mathbf{I}_{\text{aA}} + \mathbf{I}_{\text{C}} = 130.25 - j17.556 = 131.4\underline{/-7.68^\circ}\,\text{A (rms)}$$

[b] $S_{g/\phi} = (1280 + j185)(130.25 + j17.556) = 163{,}472 + j46{,}567\,\text{VA}$

$$S_{gT} = 3S_{g/\phi} = -490.4 - j139.7\,\text{kVA}$$

Therefore, the source is delivering 490.4 kW and 139.7 kvars.

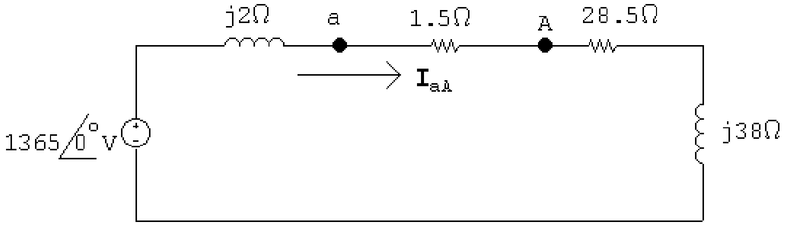
[c] $P_{\text{del}} = 490.4\,\text{kW}$

$$P_{\text{abs}} = 3(160{,}000) + 3|\mathbf{I}_{\text{aA}}|^2(0.18)$$
$$= 490.4\,\text{kW} = P_{\text{del}}$$

[d] $Q_{\text{del}} = 3|\mathbf{I}_{\text{C}}|^2(60) + 139.7 \times 10^3 = 223.3\,\text{kVAR}$

$$Q_{\text{abs}} = 3(46{,}667) + 3|\mathbf{I}_{\text{aA}}|^2(1.44)$$
$$= 223.4\,\text{kVAR} = Q_{\text{del}} \quad \text{(roundoff)}$$

P 11.31 [**a**]



$$\mathbf{I}_{\rm aA} = \frac{1365\underline{/0^\circ}}{30 + j40} = 27.3\underline{/-53.13^\circ}\,\text{A (rms)}$$

$$\mathbf{I}_{\rm CA} = \frac{\mathbf{I}_{\rm aA}}{\sqrt{3}}\underline{/150^\circ} = 15.76\underline{/96.87^\circ}\,\text{A (rms)}$$

[**b**] $S_{g/\phi} = -1365\mathbf{I}^*_{\rm aA} = -22{,}358.75 - j29{,}811.56\,\text{VA}$

$\therefore\quad P_{\rm developed/phase} = 22.359\,\text{kW}$

$P_{\rm absorbed/phase} = |\mathbf{I}_{\rm aA}|^2 28.5 = 21.241\,\text{kW}$

$$\%\text{ delivered } = \frac{21.241}{22.359}(100) = 95\%$$

P 11.32 [**a**] $P_{\rm OUT} = 746 \times 100 = 74{,}600\,\text{W}$

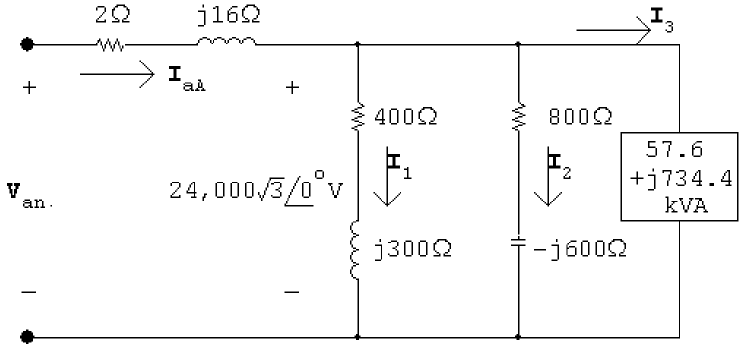
$P_{\rm IN} = 74{,}600/(0.97) = 76{,}907.22\,\text{W}$

$\sqrt{3}V_L I_L \cos\theta = 76{,}907.22$

$$I_L = \frac{76{,}907.22}{\sqrt{3}(208)(0.88)} = 242.58\,\text{A (rms)}$$

[**b**] $Q = \sqrt{3}V_L I_L \sin\phi = \sqrt{3}(208)(242.58)(0.475) = 41{,}511.90\,\text{VAR}$

P 11.33 [**a**]



$$\mathbf{I}_1 = \frac{24{,}000\sqrt{3}\underline{/0^\circ}}{400 + j300} = 66.5 - j49.9\,\text{A (rms)}$$

$$\mathbf{I}_2 = \frac{24{,}000\sqrt{3}\underline{/0^\circ}}{800 - j600} = 33.3 + j24.9\,\text{A (rms)}$$

$$\mathbf{I}_3^* = \frac{57{,}600 + j734{,}400}{24{,}000\sqrt{3}} = 1.4 + j17.7$$

$$\mathbf{I}_3 = 1.4 - j17.7\,\text{A (rms)}$$

$$\mathbf{I}_{\text{aA}} = \mathbf{I}_1 + \mathbf{I}_2 + \mathbf{I}_3 = 101.2 - j42.7\,\text{A} = 109.8\underline{/-22.9^\circ}\,\text{A (rms)}$$

$$\mathbf{V}_{\text{an}} = (2 + j16)(101.2 - j42.7) + 24{,}000\sqrt{3} = 42{,}454.8 + j1533.8\,\text{V (rms)}$$

$$S_\phi = \mathbf{V}_{\text{an}}\mathbf{I}_{\text{aA}}^* = (42{,}454.8 + j1533.8)(101.2 + j42.7)$$

$$= 4{,}230{,}932.5 + j1{,}968{,}040.5\,\text{VA}$$

$$S_T = 3S_\phi = 12{,}692.8 + j5904.1\,\text{kVA}$$

[b] $S_{1/\phi} = 24{,}000\sqrt{3}(66.5 + j49.9) = 2764.4 + j2074.3\,\text{kVA}$

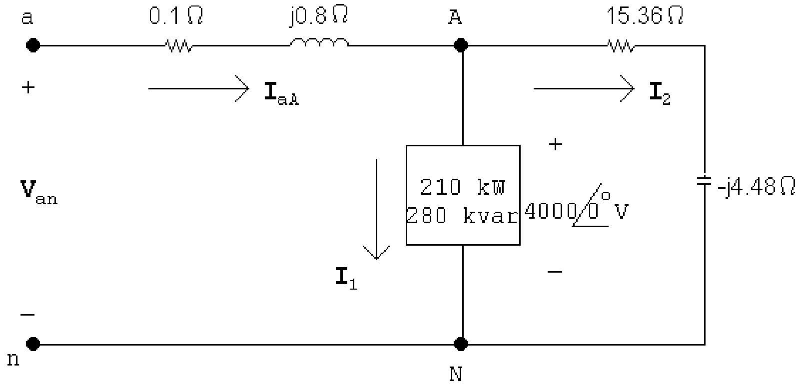
$$S_{2/\phi} = 24{,}000\sqrt{3}(33.3 - j24.9) = 1384.3 - j1035.1\,\text{kVA}$$

$$S_{3/\phi} = 57.6 + j734.4\,\text{kVA}$$

$$S_\phi(\text{load}) = 4206.3 + j1773.6\,\text{kVA}$$

$$\%\text{ delivered } = \left(\frac{4206.3}{4230.9}\right)(100) = 99.4\%$$

P 11.34



$$4000\mathbf{I}_1^* = (210 + j280)10^3$$

$$\mathbf{I}_1^* = \frac{210}{4} + j\frac{280}{4} = 52.5 + j70\,\text{A (rms)}$$

$$\mathbf{I}_1 = 52.5 - j70\,\text{A (rms)}$$

$$\mathbf{I}_2 = \frac{4000\underline{/0^\circ}}{15.36 - j4.48} = 240 + j70\,\text{A (rms)}$$

$$\therefore \quad \mathbf{I}_{\text{aA}} = \mathbf{I}_1 + \mathbf{I}_2 = 292.5 + j0\,\text{A (rms)}$$

$$\mathbf{V}_{\text{an}} = 4000 + j0 + 292.5(0.1 + j0.8) = 4036.04\underline{/3.32^\circ}\,\text{V (rms)}$$

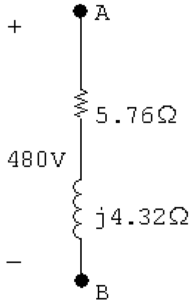
$$|\mathbf{V}_{\text{ab}}| = \sqrt{3}|\mathbf{V}_{\text{an}}| = 6990.62\,\text{V (rms)}$$

P 11.35 Assume a Δ-connect load (series):

$$S_\phi = \frac{1}{3}(96 \times 10^3)(0.8 + j0.6) = 25{,}600 + j19{,}200\,\text{VA}$$

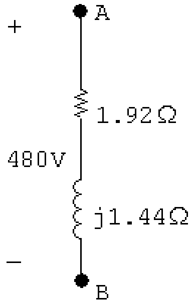
$$Z^*_{\Delta\phi} = \frac{|480|^2}{25{,}600 + j19{,}200} = 5.76 - j4.32\,\Omega/\phi$$

$$Z_{\Delta\phi} = 5.76 + 4.32\,\Omega$$



Now assume a Y-connected load (series):

$$Z_{Y\phi} = \frac{1}{3} Z_{\Delta\phi} = 1.92 + j1.44\,\Omega/\phi$$



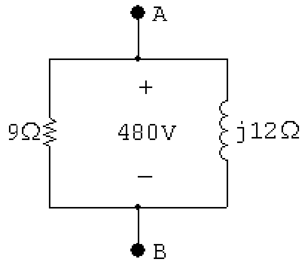
Now assume a Δ-connected load (parallel):

$$P_\phi = \frac{|480|^2}{R_\Delta}$$

$$R_{\Delta\phi} = \frac{|480|^2}{25{,}600} = 9\,\Omega$$

$$Q_\phi = \frac{|480|^2}{X_\Delta}$$

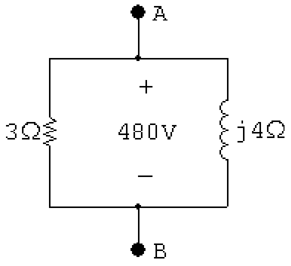
$$X_\Delta\phi = \frac{|480|^2}{19{,}200} = 12\,\Omega$$



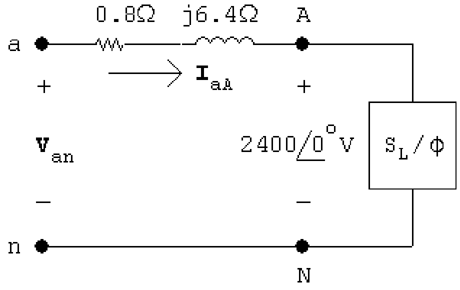
Now assume a Y-connected load (parallel):

$$R_{Y\phi} = \frac{1}{3}R_{\Delta\phi} = 3\,\Omega$$

$$X_{Y\phi} = \frac{1}{3}X_{\Delta\phi} = 4\,\Omega$$



P 11.36 **[a]**



$$S_{L/\phi} = \frac{1}{3}\left[720 + j\frac{720}{0.8}(0.6)\right]10^3 = 240{,}000 + j180{,}000\,\mathrm{VA}$$

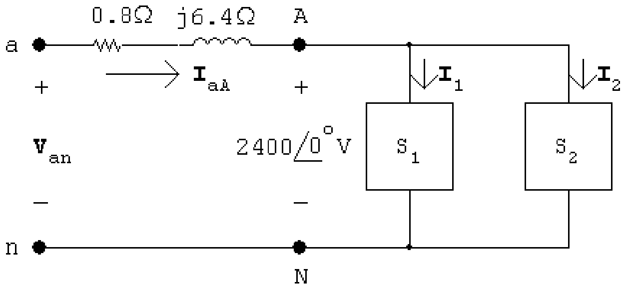
$$\mathbf{I}_{\text{aA}}^{*} = \frac{240{,}000 + j180{,}000}{2400} = 100 + j75\,\text{A (rms)}$$

$$\mathbf{I}_{\text{aA}} = 100 - j75\,\text{A (rms)}$$

$$\begin{aligned}\mathbf{V}_{\text{an}} &= 2400 + (0.8 + j0.6)(100 - j75)\\ &= 2960 + j580 = 3016.29\underline{/11.09^\circ}\,\text{V (rms)}\end{aligned}$$

$$|\mathbf{V}_{\text{ab}}| = \sqrt{3}(3016.29) = 5224.37\,\text{V (rms)}$$

[**b**]



$$\mathbf{I}_1 = 100 - j75\,\text{A} \qquad \text{(from part [a])}$$

$$S_2 = 0 - j\frac{1}{3}(576) \times 10^3 = -j192{,}000\,\text{VAR}$$

$$\mathbf{I}_2^{*} = \frac{-j192{,}000}{2400} = -j80\,\text{A (rms)}$$

$$\therefore \quad \mathbf{I}_2 = j80\,\text{A (rms)}$$

$$\mathbf{I}_{\text{aA}} = 100 - j75 + j80 = 100 + j5\,\text{A (rms)}$$

$$\begin{aligned}\mathbf{V}_{\text{an}} &= 2400 + (100 + j5)(0.8 + j6.4)\\ &= 2448 + j644 = 2531.29\underline{/14.74^\circ}\,\text{V (rms)}\end{aligned}$$

$$|\mathbf{V}_{\text{ab}}| = \sqrt{3}(2531.29) = 4384.33\,\text{V (rms)}$$

[**c**] $|\mathbf{I}_{\text{aA}}| = 125\,\text{A (rms)}$

$$P_{\text{loss}/\phi} = (125)^2(0.8) = 12{,}500\,\text{W}$$

$$P_{g/\phi} = 240{,}000 + 12{,}500 = 252.5\,\text{kW}$$

$$\%\,\eta = \frac{240}{252.5}(100) = 95.05\%$$

[d] $|\mathbf{I}_{\rm aA}| = 100.125\,\text{A (rms)}$

$$P_{\ell/\phi} = (100.125)^2(0.8) = 8020\,\text{W}$$

$$\%\,\eta = \frac{240{,}000}{248{,}200}(100) = 96.77\%$$

[e] $Z_{\rm cap/Y} = -j\dfrac{2400^2}{-192{,}000} = -j30\,\Omega$

$$Z_{\rm cap/\Delta} = 3Z_{\rm cap/Y} = -j90\,\Omega$$

$$\therefore\quad \frac{1}{\omega C} = 90; \qquad C = \frac{1}{(90)(120\pi)} = 29.47\,\mu\text{F}$$

P 11.37 **[a]** From Assessment Problem 11.9, $\mathbf{I}_{\rm aA} = (101.8 - j135.7)\,\text{A (rms)}$

Therefore $\mathbf{I}_{\rm cap} = j135.7\,\text{A (rms)}$

Therefore $Z_{CY} = \dfrac{2450/\sqrt{3}}{j135.7} = -j10.42\,\Omega$

Therefore $C_Y = \dfrac{1}{(10.42)(2\pi)(60)} = 254.5\,\mu\text{F}$

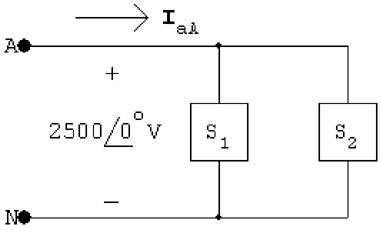
$$Z_{C\Delta} = (-j10.42)(3) = -j31.26\,\Omega$$

Therefore $C_\Delta = \dfrac{254.5}{3} = 84.84\,\mu\text{F}$

[b] $C_Y = 254.5\,\mu\text{F}$

[c] $|\mathbf{I}_{\rm aA}| = 101.8\,\text{A (rms)}$

P 11.38 **[a]**



$$S_g = \frac{1}{3}(150)(0.8 - j0.6) = 40 - j30\,\text{kVA}$$

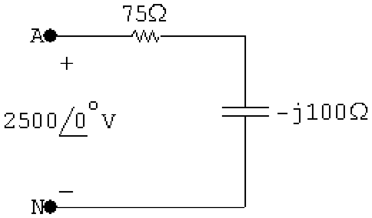
$$S_1 = \frac{1}{3}(30 + j30) = 10 + j10\,\text{kVA}$$

$$S_2 = S_g - S_1 = 30 - j40\,\text{kVA}$$

$$\therefore\quad \mathbf{I}^*_{\rm aA} = \frac{(30 - j40)10^3}{2500} = 12 - j16$$

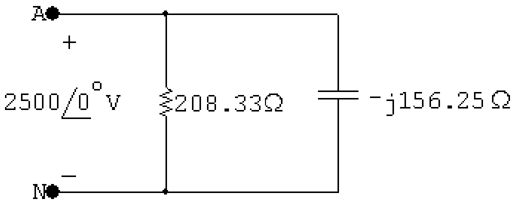
$$\mathbf{I}_{\rm aA} = 12 + j16\,\text{A (rms)}$$

$$Z = \frac{2500}{12 + j16} = 75 - j100\,\Omega$$



[**b**] $$R = \frac{(2500)^2}{30 \times 10^3} = 208.33\,\Omega$$

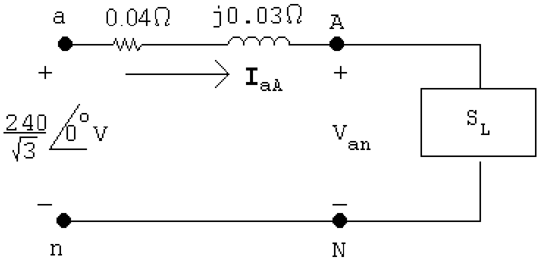
$$X_{\rm L} = \frac{(2500)^2}{-40 \times 10^3} = -156.25\,\Omega$$



P 11.39 [**a**] $$S_{g/\phi} = \frac{1}{3}(41.6)(0.707 + j0.707) \times 10^3 = 9803.73 + j9803.73\,\text{VA}$$

$$\mathbf{I}^*_{\rm aA} = \frac{9803.73 + j9803.73}{240/\sqrt{3}} = 70.76 + j70.76\,\text{A (rms)}$$

$$\mathbf{I}_{\rm aA} = 70.76 - j70.76\,\text{A (rms)}$$



$$\mathbf{V}_{\rm AN} = \frac{240}{\sqrt{3}} - (0.04 + j0.03)(70.76 - j70.76)$$
$$= 133.61 + j0.71 = 133.61\underline{/0.30^\circ}\,\text{V (rms)}$$

$$|\mathbf{V}_{\rm AB}| = \sqrt{3}(133.61) = 231.42\,\text{V (rms)}$$

[b] $S_{L/\phi} = (133.61 + j0.71)(70.76 + j70.76) = 9404 + j9504.5\,\text{VA}$

$S_L = 3S_{L/\phi} = 28{,}212 + j28{,}513\,\text{VA}$

Check:

$S_g = 41{,}600(0.7071 + j0.7071) = 29{,}415 + j29{,}415\,\text{VA}$

$P_\ell = 3|\mathbf{I}_{\text{aA}}|^2(0.04) = 1202\,\text{W}$

$P_g = P_L + P_\ell = 28{,}212 + 1202 = 29{,}414\,\text{W}$ (checks)

$Q_\ell = 3|\mathbf{I}_{\text{aA}}|^2(0.03) = 901\,\text{VAR}$

$Q_g = Q_L + Q_\ell = 28{,}513 + 901 = 29{,}414\,\text{VAR}$ (checks)

P 11.40 $Z_\phi = |Z|\underline{/\theta} = \dfrac{\mathbf{V}_{\text{AN}}}{\mathbf{I}_{\text{aA}}}$

$\theta = \underline{/\mathbf{V}_{\text{AN}}} - \underline{/\mathbf{I}_{\text{aA}}}$

$\theta_1 = \underline{/\mathbf{V}_{\text{AB}}} - \underline{/\mathbf{I}_{\text{aA}}}$

For a positive phase sequence,

$\underline{/\mathbf{V}_{\text{AB}}} = \underline{/\mathbf{V}_{\text{AN}}} + 30^\circ$

Thus,

$\theta_1 = \underline{/\mathbf{V}_{\text{AN}}} + 30^\circ - \underline{/\mathbf{I}_{\text{aA}}} = \theta + 30^\circ$

Similarly,

$Z_\phi = |Z|\underline{/\theta} = \dfrac{\mathbf{V}_{\text{CN}}}{\mathbf{I}_{\text{cC}}}$

$\theta = \underline{/\mathbf{V}_{\text{CN}}} - \underline{/\mathbf{I}_{\text{cC}}}$

$\theta_2 = \underline{/\mathbf{V}_{\text{CB}}} - \underline{/\mathbf{I}_{\text{cC}}}$

For a positive phase sequence,

$\underline{/\mathbf{V}_{\text{CB}}} = \underline{/\mathbf{V}_{\text{BA}}} - 120^\circ = \underline{/\mathbf{V}_{\text{AB}}} + 60^\circ$

$\underline{/\mathbf{I}_{\text{cC}}} = \underline{/\mathbf{I}_{\text{aA}}} + 120^\circ$

Thus,

$$\begin{aligned}\theta_2 &= \underline{/\mathbf{V}_{\text{AB}}} + 60^\circ - (\underline{/\mathbf{I}_{\text{aA}}} + 120^\circ) = \theta_1 - 60^\circ\\ &= \theta + 30^\circ - 60^\circ = \theta - 30^\circ\end{aligned}$$

P 11.41 $\mathbf{I}_{\text{aA}} = \dfrac{\mathbf{V}_{\text{AN}}}{Z_\phi} = |\mathbf{I}_L|\underline{/-\theta_\phi}\,\text{A},$

$$Z_\phi = |Z|\underline{/\theta_\phi}, \qquad \mathbf{V}_{\text{BC}} = |\mathbf{V}_L|\underline{/-90^\circ}\,\text{V},$$

$$\begin{aligned} W_m &= |\mathbf{V}_L|\,|\mathbf{I}_L|\cos[-90^\circ - (-\theta_\phi)] \\ &= |\mathbf{V}_L|\,|\mathbf{I}_L|\cos(\theta_\phi - 90^\circ) \\ &= |\mathbf{V}_L|\,|\mathbf{I}_L|\sin\theta_\phi, \end{aligned}$$

therefore $\sqrt{3}W_m = \sqrt{3}|\mathbf{V}_L|\,|\mathbf{I}_L|\sin\theta_\phi = Q_{\text{total}}$

P 11.42 **[a]** $Z = 16 - j12 = 20\underline{/-36.87^\circ}\,\Omega$

$\mathbf{V}_{\text{AN}} = 680\underline{/0^\circ}\,\text{V}; \qquad \therefore\ \ \mathbf{I}_{\text{aA}} = 34\underline{/36.87^\circ}\,\text{A}$

$\mathbf{V}_{\text{BC}} = \mathbf{V}_{\text{BN}} - \mathbf{V}_{\text{CN}} = 680\sqrt{3}\underline{/-90^\circ}\,\text{V}$

$W_m = (680\sqrt{3})(34)\cos(-90 - 36.87^\circ) = -24{,}027.07\,\text{W}$

$\sqrt{3}W_m = -41{,}616.1\,\text{W}$

[b] $Q_\phi = (34^2)(-12) = -13{,}872\,\text{VAR}$

$Q_T = 3Q_\phi = -41{,}616\,\text{VAR} = \sqrt{3}W_m$

P 11.43 **[a]**
$$\begin{aligned} W_2 - W_1 &= V_L I_L[\cos(\theta - 30^\circ) - \cos(\theta + 30^\circ)] \\ &= V_L I_L[\cos\theta\cos 30^\circ + \sin\theta\sin 30^\circ \\ &\qquad - \cos\theta\cos 30^\circ + \sin\theta\sin 30^\circ] \\ &= 2V_L I_L\sin\theta\sin 30^\circ = V_L I_L\sin\theta, \end{aligned}$$

therefore $\sqrt{3}(W_2 - W_1) = \sqrt{3}V_L I_L\sin\theta = Q_T$

[b] $Z_\phi = (8 + j6)\,\Omega$

$Q_T = \sqrt{3}[2476.25 - 979.75] = 2592\,\text{VAR},$

$Q_T = 3(12)^2(6) = 2592\,\text{VAR};$

$Z_\phi = (8 - j6)\,\Omega$

$Q_T = \sqrt{3}[979.75 - 2476.25] = -2592\,\text{VAR},$

$Q_T = 3(12)^2(-6) = -2592\,\text{VAR};$

$Z_\phi = 5(1 + j\sqrt{3})\,\Omega$

$$Q_T = \sqrt{3}[2160 - 0] = 3741.23\,\text{VAR},$$

$$Q_T = 3(12)^2(5\sqrt{3}) = 3741.23\,\text{VAR};$$

$$Z_\phi = 10\underline{/75^\circ}\,\Omega$$

$$Q_T = \sqrt{3}[-645.53 - 1763.63] = -4172.80\,\text{VAR},$$

$$Q_T = 3(12)^2[-10\sin 75^\circ] = -4172.80\,\text{VAR}$$

P 11.44 $W_{m1} = |\mathbf{V}_{\text{AB}}||\mathbf{I}_{\text{aA}}|\cos(\underline{/\mathbf{V}_{\text{AB}}} - \underline{/\mathbf{I}_{\text{aA}}}) = (199.58)(2.4)\cos(65.68^\circ) = 197.26\,\text{W}$

$W_{m2} = |\mathbf{V}_{\text{CB}}||\mathbf{I}_{\text{cC}}|\cos(\underline{/\mathbf{V}_{\text{CB}}} - \underline{/\mathbf{I}_{\text{cC}}}) = (199.58)(2.4)\cos(5.68^\circ) = 476.64\,\text{W}$

CHECK: $W_1 + W_2 = 673.9 = (2.4)^2(39)(3) = 673.9\,\text{W}$

P 11.45 $\tan\phi = \dfrac{\sqrt{3}(W_2 - W_1)}{W_1 + W_2} = 0.75$

$\therefore\quad \phi = 36.87^\circ$

$\therefore\quad 2400\sqrt{3}|\mathbf{I}_{\text{L}}|\cos 66.87^\circ = 40{,}823.09$

$|\mathbf{I}_{\text{L}}| = 25\,\text{A}$

$|Z| = \dfrac{2400}{25} = 96\,\Omega \qquad \therefore\quad Z = 96\underline{/36.87^\circ}\,\Omega$

P 11.46 **[a]** $W_1 = |\mathbf{V}_{\text{BA}}||\mathbf{I}_{\text{bB}}|\cos\theta$

Negative phase sequence:

$\mathbf{V}_{\text{BA}} = 240\sqrt{3}\underline{/150^\circ}\,\text{V}$

$\mathbf{I}_{\text{aA}} = \dfrac{240\underline{/0^\circ}}{13.33\underline{/-30^\circ}} = 18\underline{/30^\circ}\,\text{A}$

$\mathbf{I}_{\text{bB}} = 18\underline{/150^\circ}\,\text{A}$

$W_1 = (18)(240)\sqrt{3}\cos 0^\circ = 7482.46\,\text{W}$

$W_2 = |\mathbf{V}_{\text{CA}}||\mathbf{I}_{\text{cC}}|\cos\theta$

$\mathbf{V}_{\text{CA}} = 240\sqrt{3}\underline{/-150^\circ}\,\text{V}$

$\mathbf{I}_{\text{cC}} = 18\underline{/-90^\circ}\,\text{A}$

$W_2 = (18)(240)\sqrt{3}\cos(-60^\circ) = 3741.23\,\text{W}$

[b] $P_\phi = (18)^2(40/3)\cos(-30^\circ) = 3741.23\,\text{W}$

$$P_T = 3P_\phi = 11{,}223.69\,\text{W}$$

$$W_1 + W_2 = 7482.46 + 3741.23 = 11{,}223.69\,\text{W}$$

$$\therefore \quad W_1 + W_2 = P_T \qquad \text{(checks)}$$

P 11.47 From the solution to Prob. 11.18 we have

$$\mathbf{I}_{\text{aA}} = 210\underline{/20.79^\circ}\,\text{A} \qquad \text{and} \qquad \mathbf{I}_{\text{bB}} = 178.68\underline{/-178.04^\circ}\,\text{A}$$

[a] $W_1 = |\mathbf{V}_{\text{ac}}|\,|\mathbf{I}_{\text{aA}}|\cos(\theta_{\text{ac}} - \theta_{\text{aA}})$

$$= 480(210)\cos(60^\circ - 20.79^\circ) = 78{,}103.2\,\text{W}$$

[b] $W_2 = |\mathbf{V}_{\text{bc}}|\,|\mathbf{I}_{\text{bB}}|\cos(\theta_{\text{bc}} - \theta_{\text{bB}})$

$$= 480(178.68)\cos(120^\circ + 178.04^\circ) = 40{,}317.7\,\text{W}$$

[c] $W_1 + W_2 = 118{,}421\,\text{W}$

$$P_{\text{AB}} = (192)^2(2.4) = 88{,}473.6\,\text{W}$$

$$P_{\text{BC}} = (48)^2(8) = 18{,}432\,\text{W}$$

$$P_{\text{CA}} = (24)^2(20) = 11{,}520\,\text{W}$$

$$P_{\text{AB}} + P_{\text{BC}} + P_{\text{CA}} = 118{,}425.7$$

therefore $W_1 + W_2 \approx P_{\text{total}}$ (round-off differences)

P 11.48 [a] $Z = \frac{1}{3}Z_\Delta = 4.48 + j15.36 = 16\underline{/73.74^\circ}\,\Omega$

$$\mathbf{I}_{\text{aA}} = \frac{600\underline{/0^\circ}}{16\underline{/73.74^\circ}} = 37.5\underline{/-73.74^\circ}\,\text{A}$$

$$\mathbf{I}_{\text{bB}} = 37.5\underline{/-193.74^\circ}\,\text{A}$$

$$\mathbf{V}_{\text{AC}} = 600\sqrt{3}\underline{/-30^\circ}\,\text{V}$$

$$\mathbf{V}_{\text{BC}} = 600\sqrt{3}\underline{/-90^\circ}\,\text{V}$$

$$W_1 = (600\sqrt{3})(37.5)\cos(-30 + 73.74^\circ) = 28{,}156.15\,\text{W}$$

$$W_2 = (600\sqrt{3})(37.5)\cos(-90 + 193.74^\circ) = -9256.15\,\text{W}$$

[b] $W_1 + W_2 = 18{,}900\,\text{W}$

$$P_T = 3(37.5)^2(13.44/3) = 18{,}900\,\text{W}$$

[c] $\sqrt{3}(W_1 - W_2) = 64{,}800\,\text{VAR}$

$Q_T = 3(37.5)^2(46.08/3) = 64{,}800\,\text{VAR}$

P 11.49 **[a]** $Z_\phi = 100 - j75 = 125\underline{/-36.87^\circ}\,\Omega$

$$S_\phi = \frac{(13{,}200)^2}{125\underline{/36.87^\circ}} = 1{,}115{,}136 + j836{,}352\,\text{VA}$$

[b] $\dfrac{13{,}200}{\sqrt{3}}\underline{/30^\circ}\mathbf{I}^*_{\text{aA}} = S_\phi \quad \text{so} \quad \mathbf{I}_{\text{aA}} = 182.9\underline{/66.87^\circ}$

$W_{m1} = (13{,}200)(182.9)\cos(0 - 66.87^\circ) = 948{,}401.92\,\text{W}$

$W_{m2} = (13{,}200)(182.9)\cos(-60^\circ + 53.13^\circ) = 2{,}397{,}006.08\,\text{W}$

Check: $P_T = 3(1{,}115{,}136)\,\text{W} = W_{m1} + W_{m2}.$

P 11.50 **[a]** Negative phase sequence:

$\mathbf{V}_{\text{AB}} = 240\sqrt{3}\underline{/-30^\circ}\,\text{V}$

$\mathbf{V}_{\text{BC}} = 240\sqrt{3}\underline{/90^\circ}\,\text{V}$

$\mathbf{V}_{\text{CA}} = 240\sqrt{3}\underline{/-150^\circ}\,\text{V}$

$$\mathbf{I}_{\text{AB}} = \frac{240\sqrt{3}\underline{/-30^\circ}}{20\underline{/30^\circ}} = 20.78\underline{/-60^\circ}\,\text{A}$$

$$\mathbf{I}_{\text{BC}} = \frac{240\sqrt{3}\underline{/90^\circ}}{60\underline{/0^\circ}} = 6.93\underline{/90^\circ}\,\text{A}$$

$$\mathbf{I}_{\text{CA}} = \frac{240\sqrt{3}\underline{/-150^\circ}}{40\underline{/-30^\circ}} = 10.39\underline{/-120^\circ}\,\text{A}$$

$\mathbf{I}_{\text{aA}} = \mathbf{I}_{\text{AB}} + \mathbf{I}_{\text{AC}} = 18\underline{/-30^\circ}\,\text{A}$

$\mathbf{I}_{\text{cC}} = \mathbf{I}_{\text{CB}} + \mathbf{I}_{\text{CA}} = \mathbf{I}_{\text{CA}} + \mathbf{I}_{\text{BC}} = 16.75\underline{/-108.06^\circ}$

$W_{m1} = 240\sqrt{3}(18)\cos(-30 + 30^\circ) = 7482.46\,\text{W}$

$W_{m2} = 240\sqrt{3}(16.75)\cos(-90 + 108.07^\circ) = 6621.23\,\text{W}$

[b] $W_{m1} + W_{m2} = 14{,}103.69\,\text{W}$

$P_{\text{A}} = (12\sqrt{3})^2(20\cos 30^\circ) = 7482.46\,\text{W}$

$P_{\text{B}} = (4\sqrt{3})^2(60) = 2880\,\text{W}$

$P_{\text{C}} = (6\sqrt{3})^2[40\cos(-30^\circ)] = 3741.23\,\text{W}$

$P_{\text{A}} + P_{\text{B}} + P_{\text{C}} = 14{,}103.69 = W_{m1} + W_{m2}$

P 11.51 [**a**] $\mathbf{I}_{\mathrm{aA}}^{*} = \dfrac{144(0.96 - j0.28)10^3}{7200} = 20\underline{/-16.26^\circ}\,\mathrm{A}$

$\mathbf{V}_{\mathrm{BN}} = 7200\underline{/-120^\circ}\,\mathrm{V}; \qquad \mathbf{V}_{\mathrm{CN}} = 7200\underline{/120^\circ}\,\mathrm{V}$

$\mathbf{V}_{\mathrm{BC}} = \mathbf{V}_{\mathrm{BN}} - \mathbf{V}_{\mathrm{CN}} = 7200\sqrt{3}\underline{/-90^\circ}\,\mathrm{V}$

$\mathbf{I}_{\mathrm{bB}} = 20\underline{/-103.74^\circ}\,\mathrm{A}$

$W_{\mathrm{m1}} = (7200\sqrt{3})(20)\cos(-90^\circ + 103.74^\circ) = 242{,}278.14\,\mathrm{W}$

[**b**] Current coil in line aA, measure $\mathbf{I}_{\mathrm{aA}}$.
Voltage coil across AC, measure $\mathbf{V}_{\mathrm{AC}}$.

[**c**] $I_{\mathrm{aA}} = 20\underline{/16.76^\circ}\,\mathrm{A}$

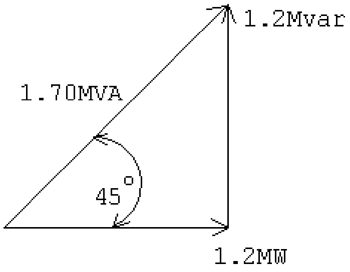
$\mathbf{V}_{\mathrm{CA}} = \mathbf{V}_{\mathrm{AN}} - \mathbf{V}_{\mathrm{CN}} = 7200\sqrt{3}\underline{/-30^\circ}\,\mathrm{V}$

$W_{\mathrm{m2}} = (7200\sqrt{3})(20)\cos(-30^\circ - 16.26^\circ) = 172{,}441.86\,\mathrm{W}$

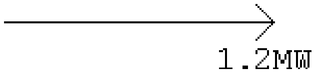
[**d**] $W_{\mathrm{m1}} + W_{\mathrm{m2}} = 414.72\mathrm{kW}$

$P_T = 432{,}000(0.96) = 414.72\,\mathrm{kW} = W_{\mathrm{m1}} + W_{\mathrm{m2}}$

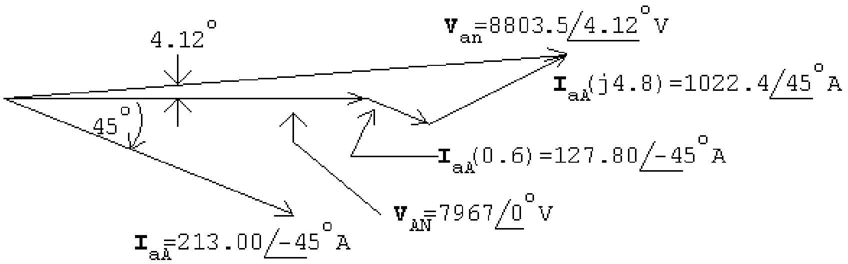
P 11.52 [**a**]



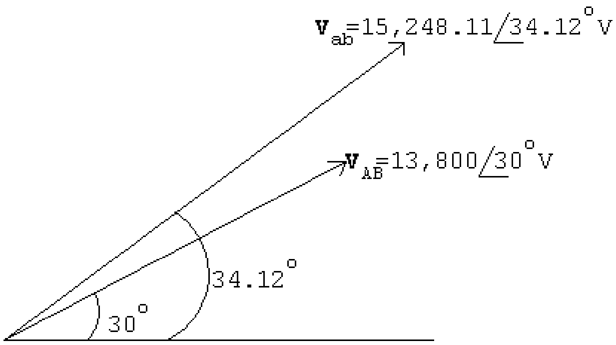
[**b**]



[**c**]



[d]



P 11.53 **[a]** $Q = \dfrac{|\mathbf{V}|^2}{X_{\rm C}}$

$$\therefore \quad |X_{\rm C}| = \frac{(13{,}800)^2}{1.2 \times 10^6} = 158.70\,\Omega$$

$$\therefore \quad \frac{1}{\omega C} = 158.70; \qquad C = \frac{1}{2\pi(60)(158.70)} = 16.71\,\mu\text{F}$$

[b] $|X_{\rm C}| = \dfrac{(13{,}800/\sqrt{3})^2}{1.2 \times 10^6} = \dfrac{1}{3}(158.70)$

$$\therefore \quad C = 3(16.71) = 50.14\,\mu\text{F}$$

P 11.54 **[a]** The capacitor from Appendix H whose value is closest to 16.71 μF is 22 μF.

$$|X_C| = \frac{1}{\omega C} = \frac{1}{2\pi(60)(22 \times 10^{-6})} = 120.57\,\Omega$$

$$Q = \frac{|V|^2}{X_C} = \frac{(13{,}800)^2}{120.57} = 1{,}579{,}497\,\text{VAR}/\phi$$

[b] $\mathbf{I}^*_{\rm aA} = \dfrac{1{,}200{,}000 - j379{,}497}{13{,}800/\sqrt{3}} = 50.2 - j15.9\,\text{A}$

$$\mathbf{V}_{\rm an} = \frac{13{,}800}{\sqrt{3}}\underline{/0^\circ} + (0.6 + j4.8)(50.2 + j15.9) = 7897.8\underline{/1.76^\circ}$$

$$|\mathbf{V}_{\rm ab}| = \sqrt{3}(7897.8) = 13{,}679.4\,\text{V}$$

This voltage falls within the allowable range of 13 kV to 14.6 kV.

P 11.55 **[a]** The capacitor from Appendix H whose value is closest to 50.14 μF is 47 μF.

$$|X_C| = \frac{1}{\omega C} = \frac{1}{2\pi(60)(47 \times 10^{-6})} = 56.4\,\Omega$$

$$Q = \frac{|V|^2}{3X_C} = \frac{(13{,}800)^2}{3(56.4)} = 1{,}124{,}775.6\,\text{VAR}$$

[b] $\mathbf{I}_{\text{aA}}^* = \dfrac{1{,}200{,}000 + j75{,}224}{13{,}800/\sqrt{3}} = 150.6 + j9.4\,\text{A}$

$$\mathbf{V}_{\text{an}} = \frac{13{,}800}{\sqrt{3}}\underline{/0^\circ} + (0.6 + j4.8)(150.6 - j9.4) = 8134.8\underline{/5.06^\circ}$$

$$|\mathbf{V}_{\text{ab}}| = \sqrt{3}(8134.8) = 14{,}089.9\,\text{V}$$

This voltage falls within the allowable range of 13 kV to 14.6 kV.

P 11.56 If the capacitors remain connected when the substation drops its load, the expression for the line current becomes

$$\frac{13{,}800}{\sqrt{3}}\mathbf{I}_{\text{aA}}^* = -j1.2 \times 10^6$$

or $\quad \mathbf{I}_{\text{aA}}^* = -j150.61\,\text{A}$

Hence $\quad \mathbf{I}_{\text{aA}} = j150.61\,\text{A}$

Now,

$$\mathbf{V}_{\text{an}} = \frac{13{,}800}{\sqrt{3}}\underline{/0^\circ} + (0.6 + j4.8)(j150.61) = 7244.49 + j90.37 = 7245.05\underline{/0.71^\circ}\,\text{V}$$

The magnitude of the line-to-line voltage at the generating plant is

$$|\mathbf{V}_{\text{ab}}| = \sqrt{3}(7245.05) = 12{,}548.80\,\text{V}.$$

This is a problem because the voltage is below the acceptable minimum of 13 kV. Thus when the load at the substation drops off, the capacitors must be switched off.

P 11.57 Before the capacitors are added the total line loss is

$$P_{\text{L}} = 3|150.61 + j150.61|^2(0.6) = 81.66\,\text{kW}$$

After the capacitors are added the total line loss is

$$P_{\text{L}} = 3|150.61|^2(0.6) = 40.83\,\text{kW}$$

Note that adding the capacitors to control the voltage level also reduces the amount of power loss in the lines, which in this example is cut in half.

P 11.58 **[a]** $\frac{13{,}800}{\sqrt{3}}\mathbf{I}_{\text{aA}}^{*} = 80 \times 10^3 + j200 \times 10^3 - j1200 \times 10^3$

$$\mathbf{I}_{\text{aA}}^{*} = \frac{80\sqrt{3} - j1000\sqrt{3}}{13.8} = 10.04 - j125.51\,\text{A}$$

$$\therefore \quad \mathbf{I}_{\text{aA}} = 10.04 + j125.51\,\text{A}$$

$$\mathbf{V}_{\text{an}} = \frac{13{,}800}{\sqrt{3}}\underline{/0^\circ} + (0.6 + j4.8)(10.04 + j125.51)$$

$$= 7371.01 + j123.50 = 7372.04\underline{/0.96^\circ}\,\text{V}$$

$$\therefore \quad |\mathbf{V}_{\text{ab}}| = \sqrt{3}(7372.04) = 12{,}768.75\,\text{V}$$

[b] Yes, the magnitude of the line-to-line voltage at the power plant is less than the allowable minimum of 13 kV.

P 11.59 **[a]** $\frac{13{,}800}{\sqrt{3}}\mathbf{I}_{\text{aA}}^{*} = (80 + j200) \times 10^3$

$$\mathbf{I}_{\text{aA}}^{*} = \frac{80\sqrt{3} + j200\sqrt{3}}{13.8} = 10.04 + j25.1\,\text{A}$$

$$\therefore \quad \mathbf{I}_{\text{aA}} = 10.04 - j25.1\,\text{A}$$

$$\mathbf{V}_{\text{an}} = \frac{13{,}800}{\sqrt{3}}\underline{/0^\circ} + (0.6 + j4.8)(10.04 - j25.1)$$

$$= 8093.95 + j33.13 = 8094.02\underline{/0.23^\circ}\,\text{V}$$

$$\therefore \quad |\mathbf{V}_{\text{ab}}| = \sqrt{3}(8094.02) = 14{,}019.25\,\text{V}$$

[b] Yes: $13\,\text{kV} < 14{,}019.25 < 14.6\,\text{kV}$

[c] $P_{\text{loss}} = 3|10.04 + j125.51|^2(0.6) = 28.54\,\text{kW}$

[d] $P_{\text{loss}} = 3|10.04 + j25.1|^2(0.6) = 1.32\,\text{kW}$

[e] Yes, the voltage at the generating plant is at an acceptable level and the line loss is greatly reduced.

12

Introduction to the Laplace Transform

Assessment Problems

AP 12.1 **[a]** $\cosh\beta t = \dfrac{e^{\beta t} + e^{-\beta t}}{2}$

Therefore,

$$\mathcal{L}\{\cosh\beta t\} = \frac{1}{2}\int_{0^-}^{\infty} [e^{(s-\beta)t} + e^{-(s-\beta)t}]dt$$

$$= \frac{1}{2}\left[\frac{e^{-(s-\beta)t}}{-(s-\beta)}\bigg|_{0^-}^{\infty} + \frac{e^{-(s+\beta)t}}{-(s+\beta)}\bigg|_{0^-}^{\infty}\right]$$

$$= \frac{1}{2}\left(\frac{1}{s-\beta} + \frac{1}{s+\beta}\right) = \frac{s}{s^2-\beta^2}$$

[b] $\sinh\beta t = \dfrac{e^{\beta t} - e^{-\beta t}}{2}$

Therefore,

$$\mathcal{L}\{\sinh\beta t\} = \frac{1}{2}\int_{0^-}^{\infty} \left[e^{-(s-\beta)t} - e^{-(s+\beta)t}\right] dt$$

$$= \frac{1}{2}\left[\frac{e^{-(s-\beta)t}}{-(s-\beta)}\right]_{0^-}^{\infty} - \frac{1}{2}\left[\frac{e^{-(s+\beta)t}}{-(s+\beta)}\right]_{0^-}^{\infty}$$

$$= \frac{1}{2}\left(\frac{1}{s-\beta} - \frac{1}{s+\beta}\right) = \frac{\beta}{(s^2-\beta^2)}$$

AP 12.2 **[a]** Let $f(t) = te^{-at}$:

$$F(s) = \mathcal{L}\{te^{-at}\} = \frac{1}{(s+a)^2}$$

Now, $\quad \mathcal{L}\{tf(t)\} = -\dfrac{dF(s)}{ds}$

$$\text{So,}\quad \mathcal{L}\{t\cdot te^{-at}\} = -\frac{d}{ds}\left[\frac{1}{(s+a)^2}\right] = \frac{2}{(s+a)^3}$$

[b] Let $f(t) = e^{-at}\sinh\beta t$, then

$$\mathcal{L}\{f(t)\} = F(s) = \frac{\beta}{(s+a)^2-\beta^2}$$

$$\mathcal{L}\left\{\frac{df(t)}{dt}\right\} = sF(s) - f(0^-) = \frac{s(\beta)}{(s+a)^2-\beta^2} - 0 = \frac{\beta s}{(s+a)^2-\beta^2}$$

[c] Let $f(t) = \cos\omega t$. Then

$$F(s) = \frac{s}{(s^2+\omega^2)} \quad\text{and}\quad \frac{dF(s)}{ds} = \frac{-(s^2-\omega^2)}{(s^2+\omega^2)^2}$$

$$\text{Therefore}\quad \mathcal{L}\{t\cos\omega t\} = -\frac{dF(s)}{ds} = \frac{s^2-\omega^2}{(s^2+\omega^2)^2}$$

AP 12.3

$$F(s) = \frac{6s^2+26s+26}{(s+1)(s+2)(s+3)} = \frac{K_1}{s+1} + \frac{K_2}{s+2} + \frac{K_3}{s+3}$$

$$K_1 = \frac{6-26+26}{(1)(2)} = 3; \qquad K_2 = \frac{24-52+26}{(-1)(1)} = 2$$

$$K_3 = \frac{54-78+26}{(-2)(-1)} = 1$$

Therefore $f(t) = [3e^{-t} + 2e^{-2t} + e^{-3t}]\,u(t)$

AP 12.4

$$F(s) = \frac{7s^2+63s+134}{(s+3)(s+4)(s+5)} = \frac{K_1}{s+3} + \frac{K_2}{s+4} + \frac{K_3}{s+5}$$

$$K_1 = \frac{63-189-134}{1(2)} = 4; \qquad K_2 = \frac{112-252+134}{(-1)(1)} = 6$$

$$K_3 = \frac{175-315+134}{(-2)(-1)} = -3$$

$$f(t) = [4e^{-3t} + 6e^{-4t} - 3e^{-5t}]u(t)$$

AP 12.5

$$F(s) = \frac{10(s^2 + 119)}{(s+5)(s^2 + 10s + 169)}$$

$$s_{1,2} = -5 \pm \sqrt{25 - 169} = -5 \pm j12$$

$$F(s) = \frac{K_1}{s+5} + \frac{K_2}{s+5-j12} + \frac{K_2^*}{s+5+j12}$$

$$K_1 = \frac{10(25+119)}{25-50+169} = 10$$

$$K_2 = \frac{10[(-5+j12)^2 + 119]}{(j12)(j24)} = j4.17 = 4.17\underline{/90^\circ}$$

Therefore

$$\begin{aligned} f(t) &= [10e^{-5t} + 8.33e^{-5t}\cos(12t + 90^\circ)]\,u(t) \\ &= [10e^{-5t} - 8.33e^{-5t}\sin 12t]\,u(t) \end{aligned}$$

AP 12.6

$$F(s) = \frac{4s^2 + 7s + 1}{s(s+1)^2} = \frac{K_0}{s} + \frac{K_1}{(s+1)^2} + \frac{K_2}{s+1}$$

$$K_0 = \frac{1}{(1)^2} = 1; \qquad K_1 = \frac{4-7+1}{-1} = 2$$

$$\begin{aligned} K_2 &= \frac{d}{ds}\left[\frac{4s^2+7s+1}{s}\right]_{s=-1} = \left.\frac{s(8s+7) - (4s^2+7s+1)}{s^2}\right|_{s=-1} \\ &= \frac{1+2}{1} = 3 \end{aligned}$$

Therefore $f(t) = [1 + 2te^{-t} + 3e^{-t}]\,u(t)$

AP 12.7

$$\begin{aligned} F(s) &= \frac{40}{(s^2+4s+5)^2} = \frac{40}{(s+2-j1)^2(s+2+j1)^2} \\ &= \frac{K_1}{(s+2-j1)^2} + \frac{K_2}{(s+2-j1)} + \frac{K_1^*}{(s+2+j1)^2} \\ &\quad + \frac{K_2^*}{(s+2+j1)} \end{aligned}$$

$$K_1 = \frac{40}{(j2)^2} = -10 = 10\underline{/180^\circ} \quad \text{and} \quad K_1^* = -10$$

$$K_2 = \frac{d}{ds}\left[\frac{40}{(s+2+j1)^2}\right]_{s=-2+j1} = \frac{-80(j2)}{(j2)^4} = -j10 = 10\underline{/-90^\circ}$$

$$K_2^* = j10$$

Therefore

$$\begin{aligned} f(t) &= [20te^{-2t}\cos(t+180^\circ) + 20e^{-2t}\cos(t-90^\circ)]\,u(t) \\ &= 20e^{-2t}[\sin t - t\cos t]\,u(t) \end{aligned}$$

AP 12.8

$$F(s) = \frac{5s^2+29s+32}{(s+2)(s+4)} = \frac{5s^2+29s+32}{s^2+6s+8} = 5 - \frac{s+8}{(s+2)(s+4)}$$

$$\frac{s+8}{(s+2)(s+4)} = \frac{K_1}{s+2} + \frac{K_2}{s+4}$$

$$K_1 = \frac{-2+8}{2} = 3; \qquad K_2 = \frac{-4+8}{-2} = -2$$

Therefore,

$$F(s) = 5 - \frac{3}{s+2} + \frac{2}{s+4}$$

$$f(t) = 5\delta(t) + [-3e^{-2t} + 2e^{-4t}]u(t)$$

AP 12.9

$$F(s) = \frac{2s^3+8s^2+2s-4}{s^2+5s+4} = 2s - 2 + \frac{4(s+1)}{(s+1)(s+4)} = 2s - 2 + \frac{4}{s+4}$$

$$f(t) = 2\frac{d\delta(t)}{dt} - 2\delta(t) + 4e^{-4t}\,u(t)$$

AP 12.10

$$\lim_{s\to\infty} sF(s) = \lim_{s\to\infty}\left[\frac{7s^3[1+(9/s)+(134/(7s^2))]}{s^3[1+(3/s)][1+(4/s)][1+(5/s)]}\right] = 7$$

$$\therefore \quad f(0^+) = 7$$

$$\lim_{s\to 0} sF(s) = \lim_{s\to 0}\left[\frac{7s^3+63s^2+134s}{(s+3)(s+4)(s+5)}\right] = 0$$

$$\therefore \quad f(\infty) = 0$$

$$\lim_{s\to\infty} sF(s) = \lim_{s\to\infty}\left[\frac{s^3[4 + (7/s) + (1/s)^2]}{s^3[1 + (1/s)]^2}\right] = 4$$

$$\therefore \quad f(0^+) = 4$$

$$\lim_{s\to 0} sF(s) = \lim_{s\to 0}\left[\frac{4s^2 + 7s + 1}{(s+1)^2}\right] = 1$$

$$\therefore \quad f(\infty) = 1$$

$$\lim_{s\to\infty} sF(s) = \lim_{s\to\infty}\left[\frac{40s}{s^4[1 + (4/s) + (5/s^2)]^2}\right] = 0$$

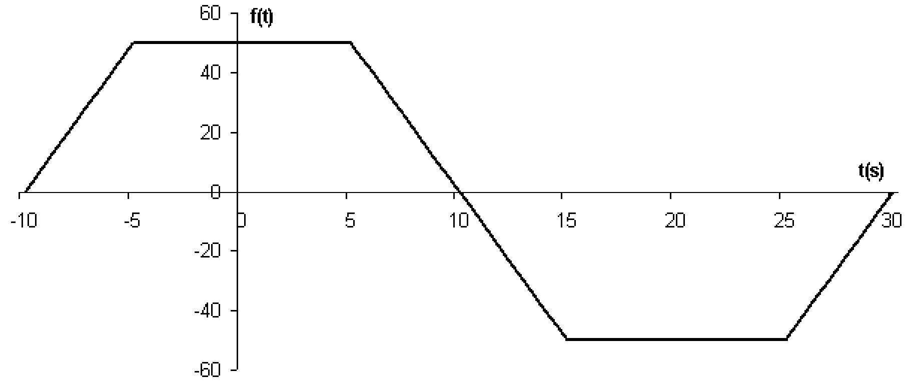
$$\therefore \quad f(0^+) = 0$$

$$\lim_{s\to 0} sF(s) = \lim_{s\to 0}\left[\frac{40s}{(s^2 + 4s + 5)^2}\right] = 0$$

$$\therefore \quad f(\infty) = 0$$

Problems

P 12.1



P 12.2 **[a]** $(10+t)[u(t+10)-u(t)]+(10-t)[u(t)-u(t-10)]$

$$= (t+10)u(t+10) - 2tu(t) + (t-10)u(t-10)$$

[b] $(-24-8t)[u(t+3)-u(t+2)] - 8[u(t+2)-u(t+1)] + 8t[u(t+1)-u(t-1)]$

$$+8[u(t-1)-u(t-2)] + (24-8t)[u(t-2)-u(t-3)]$$

$$= -8(t+3)u(t+3) + 8(t+2)u(t+2) + 8(t+1)u(t+1) - 8(t-1)u(t-1)$$

$$-8(t-2)u(t-2) + 8(t-3)u(t-3)$$

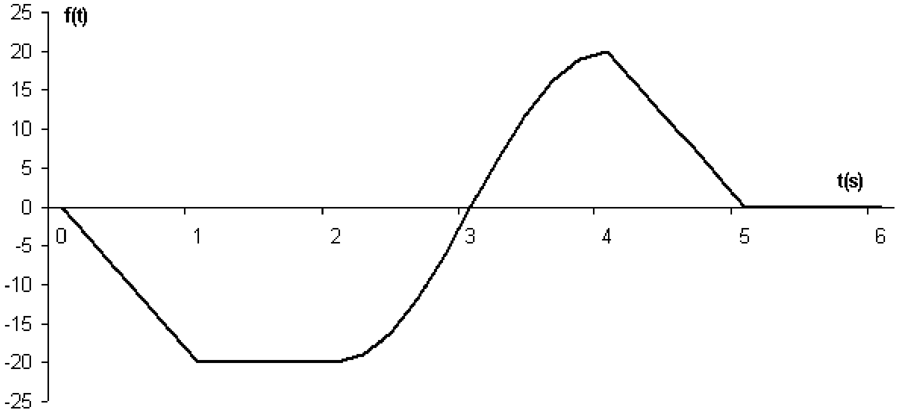
P 12.3 **[a]** $f(t) = 5t[u(t)-u(t-2)] + 10[u(t-2)-u(t-6)]+$

$$(-5t+40)[u(t-6)-u(t-8)]$$

[b] $f(t) = 10\sin\pi t[u(t)-u(t-2)]$

[c] $f(t) = 4t[u(t)-u(t-5)]$

P 12.4 [**a**]

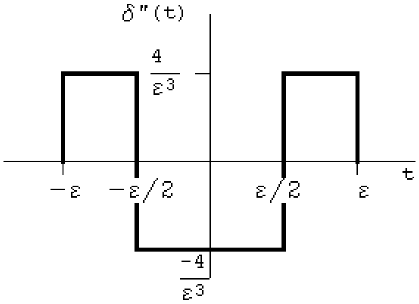


[**b**] $$f(t) = -20t[u(t) - u(t-1)] - 20[u(t-1) - u(t-2)]$$
$$+20\cos(\tfrac{\pi}{2}t)[u(t-2) - u(t-4)]$$
$$+(100 - 20t)[u(t-4) - u(t-5)]$$

P 12.5 As $\varepsilon \rightarrow 0$ the amplitude $\rightarrow \infty$; the duration $\rightarrow 0$; and the area is independent of ε, i.e.,

$$A = \int_{-\infty}^{\infty} \frac{\varepsilon}{\pi} \frac{1}{\varepsilon^2 + t^2}\, dt = 1$$

P 12.6



$$F(s) = \int_{-\varepsilon}^{-\varepsilon/2} \frac{4}{\varepsilon^3} e^{-st}\, dt + \int_{-\varepsilon/2}^{\varepsilon/2} \left(\frac{-4}{\varepsilon^3}\right) e^{-st}\, dt + \int_{\varepsilon/2}^{\varepsilon} \frac{4}{\varepsilon^3} e^{-st}\, dt$$

Therefore $F(s) = \dfrac{4}{s\varepsilon^3}[e^{s\varepsilon} - 2e^{s\varepsilon/2} + 2e^{-s\varepsilon/2} - e^{-s\varepsilon}]$

$$\mathcal{L}\{\delta''(t)\} = \lim_{\varepsilon \rightarrow 0} F(s)$$

After applying L'Hopital's rule three times, we have

$$\lim_{\varepsilon\to 0}\frac{2s}{3}\left[se^{s\varepsilon}-\frac{s}{4}e^{s\varepsilon/2}-\frac{s}{4}e^{-s\varepsilon/2}+se^{-s\varepsilon}\right]=\frac{2s}{3}\left(\frac{3s}{2}\right)$$

Therefore $\mathcal{L}\{\delta''(t)\}=s^2$

P 12.7 **[a]** $A=\left(\frac{1}{2}\right)bh=\left(\frac{1}{2}\right)(2\varepsilon)\left(\frac{1}{\varepsilon}\right)=1$

[b] 0; **[c]** ∞

P 12.8 $F(s)=\int_{-\varepsilon}^{\varepsilon}\frac{1}{2\varepsilon}e^{-st}\,dt=\frac{e^{s\varepsilon}-e^{-s\varepsilon}}{2\varepsilon s}$

$$F(s)=\frac{1}{2s}\lim_{\varepsilon\to 0}\left[\frac{se^{s\varepsilon}+se^{-s\varepsilon}}{1}\right]=\frac{1}{2s}\cdot\frac{2s}{1}=1$$

P 12.9 **[a]** $I=\int_{-1}^{3}(t^3+2)\delta(t)\,dt+\int_{-1}^{3}8(t^3+2)\delta(t-1)\,dt$

$$=(0^3+2)+8(1^3+2)=2+8(3)=26$$

[b] $I=\int_{-2}^{2}t^2\delta(t)\,dt+\int_{-2}^{2}t^2\delta(t+1.5)\,dt+\int_{-2}^{2}\delta(t-3)\,dt$

$$=0^2+(-1.5)^2+0=2.25$$

P 12.10 $f(t)=\frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{(4+j\omega)}{(9+j\omega)}\cdot\pi\delta(\omega)\cdot e^{jt\omega}\,d\omega=\left(\frac{1}{2\pi}\right)\left(\frac{4+j0}{9+j0}\pi e^{-jt0}\right)=\frac{2}{9}$

P 12.11 $\mathcal{L}\left\{\frac{d^nf(t)}{dt^n}\right\}=s^nF(s)-s^{n-1}f(0^-)-s^{n-2}f'(0^-)-\cdots,$

Therefore

$$\mathcal{L}\{\delta^n(t)\}=s^n(1)-s^{n-1}\delta(0^-)-s^{n-2}\delta'(0^-)-s^{n-3}\delta''(0^-)-\cdots=s^n$$

P 12.12 **[a]** Let $dv=\delta'(t-a)\,dt,\qquad v=\delta(t-a)$

$u=f(t),\qquad du=f'(t)\,dt$

Therefore

$$\int_{-\infty}^{\infty}f(t)\delta'(t-a)\,dt=f(t)\delta(t-a)\Big|_{-\infty}^{\infty}-\int_{-\infty}^{\infty}\delta(t-a)f'(t)\,dt$$

$$=0-f'(a)$$

[b] $\mathcal{L}\{\delta'(t)\}=\int_{0^-}^{\infty}\delta'(t)e^{-st}\,dt=-\left[\frac{d(e^{-st})}{dt}\right]_{t=0}=-\left[-se^{-st}\right]_{t=0}=s$

P 12.13 $\mathcal{L}\{e^{-at}f(t)\} = \int_{0^-}^{\infty} [e^{-at}f(t)]e^{-st}\,dt = \int_{0^-}^{\infty} f(t)e^{-(s+a)t}\,dt = F(s+a)$

P 12.14 **[a]** $\mathcal{L}\left\{\frac{d\sin\omega t}{dt}u(t)\right\} = \frac{s\omega}{s^2+\omega^2} - \sin(0) = \frac{s\omega}{s^2+\omega^2}$

[b] $\mathcal{L}\left\{\frac{d\cos\omega t}{dt}u(t)\right\} = \frac{s^2}{s^2+\omega^2} - \cos(0) = \frac{s^2}{s^2+\omega^2} - 1 = \frac{-\omega^2}{s^2+\omega^2}$

[c] $\mathcal{L}\left\{\frac{d^3(t^2)}{dt^3}u(t)\right\} = s^3\left(\frac{2}{s^3}\right) - s^2(0) - s(0) - 2(0) = 2$

[d] $\frac{d\sin\omega t}{dt} = (\cos\omega t)\cdot\omega, \qquad \mathcal{L}\{\omega\cos\omega t\} = \frac{\omega s}{s^2+\omega^2}$

$$\frac{d\cos\omega t}{dt} = -\omega\sin\omega t$$

$$\mathcal{L}\{-\omega\sin\omega t\} = -\frac{\omega^2}{s^2+\omega^2}$$

$$\frac{d^3(t^2u(t))}{dt^3} = 2\delta(t); \qquad \mathcal{L}\{2\delta(t)\} = 2$$

P 12.15 **[a]** $\int_{0^-}^{t} x\,dx = \frac{t^2}{2}$

$$\mathcal{L}\left\{\frac{t^2}{2}\right\} = \frac{1}{2}\int_{0^-}^{\infty} t^2e^{-st}\,dt$$

$$= \frac{1}{2}\left[\frac{e^{-st}}{-s^3}(s^2t^2+2st+2)\Big|_{0^-}^{\infty}\right]$$

$$= \frac{1}{2s^3}(2) = \frac{1}{s^3}$$

$$\therefore \quad \mathcal{L}\left\{\int_{0^-}^{t} x\,dx\right\} = \frac{1}{s^3}$$

[b] $\mathcal{L}\left\{\int_{0^-}^{t} x\,dx\right\} = \frac{\mathcal{L}\{t\}}{s} = \frac{1/s^2}{s} = \frac{1}{s^3}$

$\therefore \quad \mathcal{L}\left\{\int_{0^-}^{t} x\,dx\right\} = \frac{1}{s^3}$ CHECKS

P 12.16 $\mathcal{L}\{f(at)\} = \int_{0^-}^{\infty} f(at)e^{-st}\,dt$

Let $u = at, \qquad du = a\,dt, \qquad u = 0^-$ when $t = 0^-$

and $u = \infty$ when $t = \infty$

Therefore $\mathcal{L}\{f(at)\} = \int_{0^-}^{\infty} f(u)e^{-(u/a)s}\frac{du}{a} = \frac{1}{a}F(s/a)$

P 12.17 **[a]** $\mathcal{L}\{t\} = \dfrac{1}{s^2};\qquad$ therefore $\quad \mathcal{L}\{te^{-at}\} = \dfrac{1}{(s+a)^2}$

[b] $\sin\omega t = \dfrac{e^{j\omega t} - e^{-j\omega t}}{j2}$

Therefore

$$\mathcal{L}\{\sin\omega t\} = \left(\frac{1}{j2}\right)\left(\frac{1}{s-j\omega} - \frac{1}{s+j\omega}\right) = \left(\frac{1}{j2}\right)\left(\frac{2j\omega}{s^2+\omega^2}\right)$$
$$= \frac{\omega}{s^2+\omega^2}$$

[c] $\sin(\omega t + \theta) = (\sin\omega t\cos\theta + \cos\omega t\sin\theta)$

Therefore

$$\mathcal{L}\{\sin(\omega t+\theta)\} = \cos\theta\mathcal{L}\{\sin\omega t\} + \sin\theta\mathcal{L}\{\cos\omega t\}$$
$$= \frac{\omega\cos\theta + s\sin\theta}{s^2+\omega^2}$$

[d] $\mathcal{L}\{t\} = \displaystyle\int_0^\infty te^{-st}\,dt = \frac{e^{-st}}{s^2}(-st-1)\Big|_0^\infty = 0 - \frac{1}{s^2}(0-1) = \frac{1}{s^2}$

[e] $f(t) = \cosh t\cosh\theta + \sinh t\sinh\theta$

From Assessment Problem 12.1(a)

$$\mathcal{L}\{\cosh t\} = \frac{s}{s^2-1}$$

From Assessment Problem 12.1(b)

$$\mathcal{L}\{\sinh t\} = \frac{1}{s^2-1}$$

$$\therefore\quad \mathcal{L}\{\cosh(t+\theta)\} = \cosh\theta\left[\frac{s}{(s^2-1)}\right] + \sinh\theta\left[\frac{1}{s^2-1}\right]$$
$$= \frac{\sinh\theta + s[\cosh\theta]}{(s^2-1)}$$

P 12.18 **[a]** $\mathcal{L}\{f'(t)\} = \displaystyle\int_{-\varepsilon}^{\varepsilon} \frac{e^{-st}}{2\varepsilon}\,dt + \int_{\varepsilon}^{\infty} - ae^{-a(t-\varepsilon)}e^{-st}\,dt$

$$= \frac{1}{2s\varepsilon}(e^{s\varepsilon} - e^{-s\varepsilon}) - \left(\frac{a}{s+a}\right)e^{-s\varepsilon} = F(s)$$

$$\lim_{\varepsilon\to 0} F(s) = 1 - \frac{a}{s+a} = \frac{s}{s+a}$$

[b] $\mathcal{L}\{e^{-at}\} = \dfrac{1}{s+a}$

Therefore $\quad \mathcal{L}\{f'(t)\} = sF(s) - f(0^-) = \dfrac{s}{s+a} - 0 = \dfrac{s}{s+a}$

P 12.19 **[a]** $\mathcal{L}\{40e^{-8(t-3)}u(t-3)\} = \dfrac{40e^{-3s}}{(s+8)}$

[b] First rewrite $f(t)$ as

$$\begin{aligned} f(t) &= (5t-10)u(t-2) + (40-10t)u(t-4) \\ &\quad +(10t-80)u(t-8) + (50-5t)u(t-10) \\ &= 5(t-2)u(t-2) - 10(t-4)u(t-4) \\ &\quad +10(t-8)u(t-8) - 5(t-10)u(t-10) \end{aligned}$$

$$\therefore \quad F(s) = \frac{5[e^{-2s} - 2e^{-4s} + 2e^{-8s} - e^{-10s}]}{s^2}$$

P 12.20 **[a]** $\mathcal{L}\{te^{-at}\} = \displaystyle\int_{0^-}^{\infty} te^{-(s+a)t}\,dt$

$$\begin{aligned} &= \frac{e^{-(s+a)t}}{(s+a)^2}\Big[\ -(s+a)t - 1\Big]_{0^-}^{\infty} \\ &= 0 + \frac{1}{(s+a)^2} \end{aligned}$$

$$\therefore \quad \mathcal{L}\{te^{-at}\} = \frac{1}{(s+a)^2}$$

[b] $\mathcal{L}\left\{\dfrac{d}{dt}(te^{-at})u(t)\right\} = \dfrac{s}{(s+a)^2} - 0$

$$\mathcal{L}\left\{\frac{d}{dt}(te^{-at})u(t)\right\} = \frac{s}{(s+a)^2}$$

[c] $\dfrac{d}{dt}(te^{-at}) = -ate^{-at} + e^{-at}$

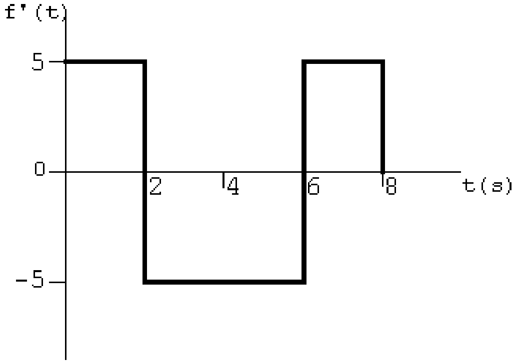
$$\mathcal{L}\{-ate^{-at} + e^{-at}\} = \frac{-a}{(s+a)^2} + \frac{1}{(s+a)} = \frac{-a}{(s+a)^2} + \frac{s+a}{(s+a)^2}$$

$$\therefore \quad \mathcal{L}\left\{\frac{d}{dt}(te^{-at})\right\} = \frac{s}{(s+a)^2} \qquad \text{CHECKS}$$

P 12.21 **[a]**
$$\begin{aligned} f(t) &= 5t[u(t) - u(t-2)] \\ &\quad +(20-5t)[u(t-2) - u(t-6)] \\ &\quad +(5t-40)[u(t-6) - u(t-8)] \\ &= 5tu(t) - 10(t-2)u(t-2) \\ &\quad +10(t-6)u(t-6) - 5(t-8)u(t-8) \end{aligned}$$

$$\therefore \quad F(s) = \frac{5[1 - 2e^{-2s} + 2e^{-6s} - e^{-8s}]}{s^2}$$

[b]



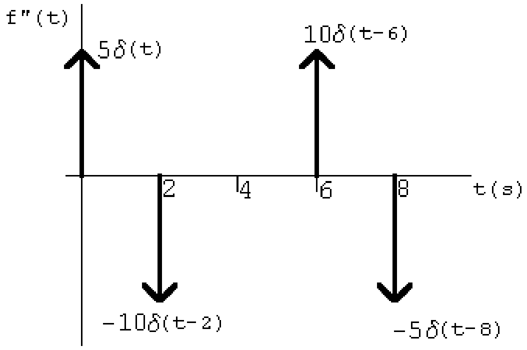
$$f'(t) = 5[u(t) - u(t-2)] - 5[u(t-2) - u(t-6)]$$

$$+5[u(t-6) - u(t-8)]$$

$$= 5u(t) - 10u(t-2) + 10u(t-6) - 5u(t-8)$$

$$\mathcal{L}\{f'(t)\} = \frac{5[1 - 2e^{-2s} + 2e^{-6s} - e^{-8s}]}{s}$$

[c]



$$f''(t) = 5\delta(t) - 10\delta(t-2) + 10\delta(t-6) - 5\delta(t-8)$$

$$\mathcal{L}\{f''(t)\} = 5[1 - 2e^{-2s} + 2e^{-6s} - e^{-8s}]$$

P 12.22 **[a]** $\mathcal{L}\left\{\int_{0^-}^{t} e^{-ax}\,dx\right\} = \frac{F(s)}{s} = \frac{1}{s(s+a)}$

[b] $\int_{0^-}^{t} e^{-ax}\,dx = \frac{1}{a} - \frac{e^{-at}}{a}$

$$\mathcal{L}\left\{\frac{1}{a} - \frac{e^{-at}}{a}\right\} = \frac{1}{a}\left[\frac{1}{s} - \frac{1}{s+a}\right] = \frac{1}{s(s+a)}$$

P 12.23 [a] $\dfrac{dF(s)}{ds} = \dfrac{d}{ds}\left[\int_{0^-}^{\infty} f(t)e^{-st}\,dt\right] = -\int_{0^-}^{\infty} tf(t)e^{-st}\,dt$

Therefore $\mathcal{L}\{tf(t)\} = -\dfrac{dF(s)}{ds}$

[b] $\dfrac{d^2F(s)}{ds^2} = \int_{0^-}^{\infty} t^2f(t)e^{-st}\,dt; \qquad \dfrac{d^3F(s)}{ds^3} = \int_{0^-}^{\infty} -t^3f(t)e^{-st}\,dt$

Therefore $\dfrac{d^nF(s)}{ds^n} = (-1)^n\int_{0^-}^{\infty} t^nf(t)e^{-st}\,dt = (-1)^n\mathcal{L}\{t^nf(t)\}$

[c] $\mathcal{L}\{t^5\} = \mathcal{L}\{t^4t\} = (-1)^4\dfrac{d^4}{ds^4}\left(\dfrac{1}{s^2}\right) = \dfrac{120}{s^6}$

$$\mathcal{L}\{t\sin\beta t\} = (-1)^1\frac{d}{ds}\left(\frac{\beta}{s^2+\beta^2}\right) = \frac{2\beta s}{(s^2+\beta^2)^2}$$

$\mathcal{L}\{te^{-t}\cosh t\}$:

From Assessment Problem 12.1(a),

$$F(s) = \mathcal{L}\{\cosh t\} = \frac{s}{s^2-1}$$

$$\frac{dF}{ds} = \frac{(s^2-1)1 - s(2s)}{(s^2-1)^2} = -\frac{s^2+1}{(s^2-1)^2}$$

Therefore $-\dfrac{dF}{ds} = \dfrac{s^2+1}{(s^2-1)^2}$

Thus

$$\mathcal{L}\{t\cosh t\} = \frac{s^2+1}{(s^2-1)^2}$$

$$\mathcal{L}\{e^{-t}t\cosh t\} = \frac{(s+1)^2+1}{[(s+1)^2-1]^2} = \frac{s^2+2s+2}{s^2(s+2)^2}$$

P 12.24 [a] $\int_s^{\infty} F(u)du = \int_s^{\infty}\left[\int_{0^-}^{\infty} f(t)e^{-ut}\,dt\right]du = \int_{0^-}^{\infty}\left[\int_s^{\infty} f(t)e^{-ut}\,du\right]dt$

$$= \int_{0^-}^{\infty} f(t)\int_s^{\infty} e^{-ut}\,du\,dt = \int_{0^-}^{\infty} f(t)\left[\frac{e^{-tu}}{-t}\Big|_s^{\infty}\right]dt$$

$$= \int_{0^-}^{\infty} f(t)\left[\frac{-e^{-st}}{-t}\right]dt = \mathcal{L}\left\{\frac{f(t)}{t}\right\}$$

[b] $\mathcal{L}\{t\sin\beta t\} = \dfrac{2\beta s}{(s^2+\beta^2)^2}$

therefore $\mathcal{L}\left\{\dfrac{t\sin\beta t}{t}\right\} = \int_s^{\infty}\left[\dfrac{2\beta u}{(u^2+\beta^2)^2}\right]du$

Let $\omega = u^2 + \beta^2$, then $\omega = s^2 + \beta^2$ when $u = s$, and $\omega = \infty$ when $u = \infty$; also $d\omega = 2u\,du$, thus

$$\mathcal{L}\left\{\frac{t\sin\beta t}{t}\right\} = \beta\int_{s^2+\beta^2}^{\infty}\left[\frac{d\omega}{\omega^2}\right] = \beta\left(\frac{-1}{\omega}\right)\Big|_{s^2+\beta^2}^{\infty} = \frac{\beta}{s^2+\beta^2}$$

P 12.25 **[a]** $f_1(t) = e^{-at}\sin\omega t; \qquad F_1(s) = \dfrac{\omega}{(s+a)^2+\omega^2}$

$$F(s) = sF_1(s) - f_1(0^-) = \frac{s\omega}{(s+a)^2+\omega^2} - 0$$

[b] $f_1(t) = e^{-at}\cos\omega t; \qquad F_1(s) = \dfrac{s+a}{(s+a)^2+\omega^2}$

$$F(s) = \frac{F_1(s)}{s} = \frac{s+a}{s[(s+a)^2+\omega^2]}$$

[c] $\dfrac{d}{dt}[e^{-at}\sin\omega t] = \omega e^{-at}\cos\omega t - ae^{-at}\sin\omega t$

$$\text{Therefore}\quad F(s) = \frac{\omega(s+a)-\omega a}{(s+a)^2+\omega^2} = \frac{\omega s}{(s+a)^2+\omega^2}$$

$$\int_{0^-}^{t} e^{-ax}\cos\omega x\,dx = \frac{-ae^{-at}\cos\omega t + we^{-at}\sin\omega t + a}{a^2+\omega^2}$$

Therefore

$$F(s) = \frac{1}{a^2+\omega^2}\left[\frac{-a(s+a)}{(s+a)^2+\omega^2} + \frac{\omega^2}{(s+a)^2+\omega^2} + \frac{a}{s}\right]$$

$$= \frac{s+a}{s[(s+a)^2+\omega^2]}$$

P 12.26 $I_g(s) = \dfrac{1.2s}{s^2+1}; \qquad \dfrac{1}{RC} = 1.6; \qquad \dfrac{1}{LC} = 1; \qquad \dfrac{1}{C} = 1.6$

$$\frac{V(s)}{R} + \frac{1}{L}\frac{V(s)}{s} + C[sV(s) - v(0^-)] = I_g(s)$$

$$V(s)\left[\frac{1}{R} + \frac{1}{Ls} + sC\right] = I_g(s)$$

$$V(s) = \frac{I_g(s)}{\frac{1}{R} + \frac{1}{Ls} + sC} = \frac{LsI_g(s)}{RLs + 1 + s^2LC} = \frac{\frac{1}{C}sI_g(s)}{s^2 + \frac{R}{C}s + \frac{1}{LC}}$$

$$= \frac{(1.6)(1.2)s^2}{(s^2+1.6s+1)(s^2+1)} = \frac{1.92s^2}{(s^2+1.6s+1)(s^2+1)}$$

P 12.27 [**a**] $\frac{1}{L}\int_0^t v_1\,d\tau + \frac{v_1 - v_2}{R} = i_g u(t)$

and

$$C\frac{dv_2}{dt} + \frac{v_2}{R} - \frac{v_1}{R} = 0$$

[**b**] $\frac{V_1}{sL} + \frac{V_1 - V_2}{R} = I_g$

$$\frac{V_2 - V_1}{R} + sCV_2 = 0$$

or

$$(R + sL)V_1(s) - sLV_2(s) = RLsI_g(s)$$

$$-V_1(s) + (RCs + 1)V_2(s) = 0$$

Solving,

$$V_2(s) = \frac{sI_g(s)}{C[s^2 + (R/L)s + (1/LC)]}$$

P 12.28 [**a**] $\frac{v_o - V_{\rm dc}}{R} + \frac{1}{L}\int_0^t v_o\,dx + C\frac{dv_o}{dt} = 0$

$$\therefore \quad v_o + \frac{R}{L}\int_0^t v_o\,dx + RC\frac{dv_o}{dt} = V_{\rm dc}$$

[**b**] $V_o + \frac{R}{L}\frac{V_o}{s} + RCsV_o = \frac{V_{\rm dc}}{s}$

$$\therefore \quad sLV_o + RV_o + RCLs^2V_o = LV_{\rm dc}$$

$$\therefore \quad V_o(s) = \frac{(1/RC)V_{\rm dc}}{s^2 + (1/RC)s + (1/LC)}$$

[**c**] $i_o = \frac{1}{L}\int_0^t v_o\,dx$

$$I_o(s) = \frac{V_o}{sL} = \frac{V_{\rm dc}/RLC}{s[s^2 + (1/RC)s + (1/LC)]}$$

P 12.29 [**a**] For $t \geq 0^+$:

$$Ri_o + L\frac{di_o}{dt} + v_o = 0$$

$$i_o = C\frac{dv_o}{dt} \qquad \frac{di_o}{dt} = C\frac{d^2v_o}{dt^2}$$

$$\therefore \quad RC\frac{dv_o}{dt} + LC\frac{d^2v_o}{dt^2} + v_o = 0$$

or

$$\frac{d^2v_o}{dt^2} + \frac{R}{L}\frac{dv_o}{dt} + \frac{1}{LC}v_o = 0$$

[b] $s^2V_o(s) - sV_{\rm dc} - 0 + \frac{R}{L}[sV_o(s) - V_{\rm dc}] + \frac{1}{LC}V_o(s) = 0$

$$V_o(s)\left[s^2 + \frac{R}{L}s + \frac{1}{LC}\right] = V_{\rm dc}(s + R/L)$$

$$V_o(s) = \frac{V_{\rm dc}[s + (R/L)]}{[s^2 + (R/L)s + (1/LC)]}$$

P 12.30 **[a]** $I_{\rm dc} = \frac{1}{L}\int_0^t v_o\,dx + \frac{v_o}{R} + C\frac{dv_o}{dt}$

[b] $\frac{I_{\rm dc}}{s} = \frac{V_o(s)}{sL} + \frac{V_o(s)}{R} + sCV_o(s)$

$$\therefore \quad V_o(s) = \frac{I_{\rm dc}/C}{s^2 + (1/RC)s + (1/LC)}$$

[c] $i_o = C\frac{dv_o}{dt}$

$$\therefore \quad I_o(s) = sCV_o(s) = \frac{sI_{\rm dc}}{s^2 + (1/RC)s + (1/LC)}$$

P 12.31 **[a]** For $t \geq 0^+$:

$$\frac{v_o}{R} + C\frac{dv_o}{dt} + i_o = 0$$

$$v_o = L\frac{di_o}{dt}; \qquad \frac{dv_o}{dt} = L\frac{d^2i_o}{dt^2}$$

$$\therefore \qquad \frac{L}{R}\frac{di_o}{dt} + LC\frac{d^2i_o}{dt^2} + i_o = 0$$

or $\quad \frac{d^2i_o}{dt^2} + \frac{1}{RC}\frac{di_o}{dt} + \frac{1}{LC}i_o = 0$

[b] $s^2I_o(s) - sI_{\rm dc} - 0 + \frac{1}{RC}[sI_o(s) - I_{\rm dc}] + \frac{1}{LC}I_o(s) = 0$

$$I_o(s)\left[s^2 + \frac{1}{RC}s + \frac{1}{LC}\right] = I_{\rm dc}(s + 1/RC)$$

$$I_o(s) = \frac{I_{\rm dc}[s + (1/RC)]}{[s^2 + (1/RC)s + (1/LC)]}$$

P 12.32 **[a]** $300 = 60i_1 + 25\frac{di_1}{dt} + 10\frac{d}{dt}(i_2 - i_1) + 5\frac{d}{dt}(i_1 - i_2) - 10\frac{di_1}{dt}$

$$0 = 5\frac{d}{dt}(i_2 - i_1) + 10\frac{di_1}{dt} + 40i_2$$

Simplifying the above equations gives:

$$300 = 60i_1 + 10\frac{di_1}{dt} + 5\frac{di_2}{dt}$$

$$0 = 40i_2 + 5\frac{di_1}{dt} + 5\frac{di_2}{dt}$$

[b] $\frac{300}{s} = (10s + 60)I_1(s) + 5sI_2(s)$

$$0 = 5sI_1(s) + (5s + 40)I_2(s)$$

[c] Solving the equations in (b),

$$I_1(s) = \frac{60(s+8)}{s(s+4)(s+24)}$$

$$I_2(s) = \frac{-60}{(s+4)(s+24)}$$

P 12.33 From Problem 12.26:

$$V(s) = \frac{1.92s^2}{(s^2 + 1.6s + 1)(s^2 + 1)}$$

$$s^2 + 1.6s + 1 = (s + 0.8 + j0.6)(s + 0.8 - j0.6); \qquad s^2 + 100 = (s - j1)(s + j1)$$

Therefore

$$\begin{aligned} V(s) &= \frac{1.92s^2}{(s + 0.8 + j0.6)(s + 0.8 - j0.6)(s - j1)(s + j1)} \\ &= \frac{K_1}{s + 0.8 - j0.6} + \frac{K_1^*}{s + 0.8 + j0.6} + \frac{K_2}{s - j1} + \frac{K_2^*}{s + j1} \end{aligned}$$

$$K_1 = \frac{1.92s^2}{(s + 0.8 + j0.6)(s^2 + 1)}\bigg|_{s=-0.8+j0.6} = 1\underline{/-126.87^\circ}$$

$$K_2 = \frac{1.92s^2}{(s + j1)(s^2 + 1.6s + 1)}\bigg|_{s=-j1} = 0.6\underline{/0^\circ}$$

Therefore

$$v(t) = [2e^{-0.8t}\cos(0.6t - 126.87^\circ) + 1.2\cos(t)]u(t)\,\text{V}$$

P 12.34 $\frac{1}{C} = 2 \times 10^6; \qquad \frac{1}{LC} = 4 \times 10^6; \qquad \frac{R}{L} = 5000; \qquad I_g = \frac{0.015}{s}$

$$V_2(s) = \frac{30{,}000}{s^2 + 5000s + 4 \times 10^6}$$

$$s_1 = -1000; \qquad s_2 = -4000$$

$$V_2(s) = \frac{30{,}000}{(s+1000)(s+4000)}$$

$$= \frac{10}{s+1000} - \frac{10}{s+4000}$$

$$v_2(t) = [10e^{-1000t} - 10e^{-4000t}]u(t)\,\text{V}$$

P 12.35 **[a]** $\frac{1}{LC} = \frac{1}{(200 \times 10^{-3})(100 \times 10^{-9})} = 50 \times 10^6$

$$\frac{1}{RC} = \frac{1}{(5000)(100 \times 10^{-9})} = 2000$$

$$V_o(s) = \frac{70{,}000}{s^2 + 2000s + 50 \times 10^6}$$

$$s_{1,2} = -1000 \pm j7000 \text{ rad/s}$$

$$V_o(s) = \frac{70{,}000}{(s+1000-j7000)(s+1000+j7000)}$$

$$= \frac{K_1}{s+1000-j7000} + \frac{K_1^*}{s+1000+j7000}$$

$$K_1 = \frac{70{,}000}{j14{,}000} = 5\underline{/-90^\circ}$$

$$v_o(t) = 10e^{-1000t}\cos(7000t - 90^\circ)]u(t)\,\text{V}$$

$$= [10e^{-1000t}\sin 7000t]u(t)\,\text{V}$$

[b] $I_o(s) = \frac{35(10{,}000)}{s(s+1000-j7000)(s+1000+j7000)}$

$$= \frac{K_1}{s} + \frac{K_2}{s+1000-j7000} + \frac{K_2^*}{s+1000+j7000}$$

$$K_1 = \frac{35(10{,}000)}{50 \times 10^6} = 7\,\text{mA}$$

$$K_2 = \frac{35(10{,}000)}{(-1000+j7000)(j14{,}000)} = 3.54\underline{/171.87^\circ}\,\text{mA}$$

$$i_o(t) = [7 + 7.07e^{-1000t}\cos(7000t + 171.87^\circ)]u(t)\,\text{mA}$$

P 12.36 $\frac{R}{L} = 5000; \qquad \frac{1}{LC} = 4 \times 10^6$

$$V_o(s) = \frac{48(s + 5000)}{s^2 + 5000s + 4 \times 10^6}$$

$$s_{1,2} = -2500 \pm \sqrt{6.25 \times 10^6 - 4 \times 10^6}$$

$$s_1 = -1000 \text{ rad/s}; \qquad s_2 = -4000 \text{ rad/s}$$

$$V_o(s) = \frac{48(s + 5000)}{(s + 1000)(s + 4000)} = \frac{K_1}{s + 1000} + \frac{K_2}{s + 4000}$$

$$K_1 = \frac{48(4000)}{3000} = 64\,\text{V}; \qquad K_2 = \frac{48(1000)}{-3000} = -16\,\text{V}$$

$$V_o(s) = \frac{64}{s + 1000} - \frac{16}{s + 4000}$$

$$v_o(t) = [64e^{-1000t} - 16e^{-4000t}]u(t)\,\text{V}$$

P 12.37 **[a]** $\frac{1}{RC} = \frac{1}{(1 \times 10^3)(2 \times 10^{-6})} = 500$

$$\frac{1}{LC} = \frac{1}{(12.5)(2 \times 10^{-6})} = 40{,}000$$

$$V_o(s) = \frac{500{,}000 I_{\text{dc}}}{s + 500s + 40{,}000}$$

$$= \frac{500{,}000 I_{\text{dc}}}{(s + 100)(s + 400)}$$

$$= \frac{15{,}000}{(s + 100)(s + 400)}$$

$$= \frac{K_1}{s + 100} + \frac{K_2}{s + 400}$$

$$K_1 = \frac{15{,}000}{300} = 50; \qquad K_2 = \frac{15{,}000}{-300} = -50$$

$$V_o(s) = \frac{50}{s + 100} - \frac{50}{s + 400}$$

$$v_o(t) = [50e^{-100t} - 50e^{-400t}]u(t)\,\text{V}$$

[b] $$I_o(s) = \frac{0.03s}{(s+100)(s+400)}$$

$$= \frac{K_1}{s+100} + \frac{K_2}{s+400}$$

$$K_1 = \frac{0.03(-100)}{300} = -0.01$$

$$K_2 = \frac{0.03(-400)}{-300} = 0.04$$

$$I_o(s) = \frac{-0.01}{s+100} + \frac{0.04}{s+400}$$

$$i_o(t) = (40e^{-400t} - 10e^{-100t})u(t)\,\text{mA}$$

[c] $i_o(0) = 40 - 10 = 30\,\text{mA}$

Yes. The initial inductor current is zero by hypothesis, the initial resistor current is zero because the initial capacitor voltage is zero by hypothesis. Thus at $t = 0$ the source current appears in the capacitor.

P 12.38 $\frac{1}{RC} = 8000; \qquad \frac{1}{LC} = 16 \times 10^6$

$$I_o(s) = \frac{0.005(s+8000)}{s^2 + 8000s + 16 \times 10^6}$$

$$s_{1,2} = -4000$$

$$I_o(s) = \frac{0.005(s+8000)}{(s+4000)^2} = \frac{K_1}{(s+4000)^2} + \frac{K_2}{s+4000}$$

$$K_1 = 0.005(s+8000)\Big|_{s=-4000} = 20$$

$$K_2 = \frac{d}{ds}[0.005(s+8000)]_{s=-4000} = 0.005$$

$$I_o(s) = \frac{20}{(s+4000)^2} + \frac{0.005}{s+4000}$$

$$i_o(t) = [20te^{-4000t} + 0.005e^{-4000t}]u(t)\,\text{V}$$

P 12.39 **[a]** $I_1(s) = \frac{K_1}{s} + \frac{K_2}{s+4} + \frac{K_3}{s+24}$

$$K_1 = \frac{(60)(8)}{(4)(24)} = 5; \qquad K_2 = \frac{(60)(4)}{(-4)(20)} = -3$$

$$K_3 = \frac{(60)(-16)}{(-24)(-20)} = -2$$

$$I_1(s) = \left(\frac{5}{s} - \frac{3}{s+4} - \frac{2}{s+24}\right)$$

$$i_1(t) = (5 - 3e^{-4t} - 2e^{-24t})u(t)\,\text{A}$$

$$I_2(s) = \frac{K_1}{s+4} + \frac{K_2}{s+24}$$

$$K_1 = \frac{-60}{20} = -3; \qquad K_2 = \frac{-60}{-20} = 3$$

$$I_2(s) = \left(\frac{-3}{s+4} + \frac{3}{s+24}\right)$$

$$i_2(t) = (3e^{-24t} - 3e^{-4t})u(t)\,\text{A}$$

[b] $i_1(\infty) = 5\,\text{A}; \qquad i_2(\infty) = 0\,\text{A}$

[c] Yes, at $t = \infty$

$$i_1 = \frac{300}{60} = 5\,\text{A}$$

Since i_1 is a dc current at $t = \infty$ there is no voltage induced in the 10 H inductor; hence, $i_2 = 0$. Also note that $i_1(0) = 0$ and $i_2(0) = 0$. Thus our solutions satisfy the condition of no initial energy stored in the circuit.

P 12.40 **[a]** $F(s) = \frac{K_1}{s+1} + \frac{K_2}{s+2} + \frac{K_3}{s+4}$

$$K_1 = \frac{8s^2 + 37s + 32}{(s+2)(s+4)}\bigg|_{s=-1} = 1$$

$$K_2 = \frac{8s^2 + 37s + 32}{(s+1)(s+4)}\bigg|_{s=-2} = 5$$

$$K_3 = \frac{8s^2 + 37s + 32}{(s+1)(s+2)}\bigg|_{s=-4} = 2$$

$$f(t) = [e^{-t} + 5e^{-2t} + 2e^{-4t}]u(t)$$

[b] $F(s) = \dfrac{K_1}{s} + \dfrac{K_2}{s+2} + \dfrac{K_3}{s+4} + \dfrac{K_4}{s+6}$

$$K_1 = \frac{13s^3 + 134s^2 + 392s + 288}{(s+2)(s+4)(s+6)}\bigg|_{s=0} = 6$$

$$K_2 = \frac{13s^3 + 134s^2 + 392s + 288}{s(s+4)(s+6)}\bigg|_{s=-2} = 4$$

$$K_3 = \frac{13s^3 + 134s^2 + 392s + 288}{s(s+2)(s+6)}\bigg|_{s=-4} = 2$$

$$K_4 = \frac{13s^3 + 134s^2 + 392s + 288}{s(s+2)(s+4)}\bigg|_{s=-6} = 1$$

$$f(t) = [6 + 4e^{-2t} + 2e^{-4t} + e^{-6t}]u(t)$$

[c] $F(s) = \dfrac{K_1}{s+1} + \dfrac{K_2}{s+1-2j} + \dfrac{K_2^*}{s+1+2j}$

$$K_1 = \frac{20s^2 + 16s + 12}{s^2 + 2s + 5}\bigg|_{s=-1} = 4$$

$$K_2 = \frac{20s^2 + 16s + 12}{(s+1)(s+1+2j)}\bigg|_{s=-1+2j} = 8 + j6 = 10\underline{/36.87^\circ}$$

$$f(t) = [4e^{-t} + 20e^{-t}\cos(2t + 36.87^\circ)]u(t)$$

[d] $F(s) = \dfrac{K_1}{s} + \dfrac{K_2}{s+7-j} + \dfrac{K_2^*}{s+7+j}$

$$K_1 = \frac{250(s+7)(s+14)}{s^2 + 14s + 50}\bigg|_{s=0} = 490$$

$$K_2 = \frac{250(s+7)(s+14)}{s(s+7+j)}\bigg|_{s=-7+j} = 125\underline{/-163.74^\circ}$$

$$f(t) = [490 + 250e^{-7t}\cos(t - 163.74^\circ)]u(t)$$

P 12.41 **[a]** $F(s) = \dfrac{K_1}{s^2} + \dfrac{K_2}{s} + \dfrac{K_3}{s+5}$

$$K_1 = \frac{100}{s+5}\bigg|_{s=0} = 20$$

$$K_2 = \frac{d}{ds}\left[\frac{100}{s+5}\right] = \frac{-100}{(s+5)^2}\bigg|_{s=0} = -4$$

$$K_3 = \frac{100}{s^2}\bigg|_{s=-5} = 4$$

$$f(t) = [20t - 4 + 4e^{-5t}]u(t)$$

[b] $$F(s) = \frac{K_1}{s} + \frac{K_2}{(s+1)^2} + \frac{K_3}{s+1}$$

$$K_1 = \left.\frac{50(s+5)}{(s+1)^2}\right|_{s=0} = 250$$

$$K_2 = \left.\frac{50(s+5)}{s}\right|_{s=-1} = -200$$

$$K_3 = \frac{d}{ds}\left[\frac{50(s+5)}{s}\right] = \left[\frac{50}{s} - \frac{50(s+5)}{s^2}\right]_{s=-1} = -250$$

$$f(t) = [250 - 200te^{-t} - 250e^{-t}]u(t)$$

[c] $$F(s) = \frac{K_1}{s^2} + \frac{K_2}{s} + \frac{K_3}{s+3-j} + \frac{K_3^*}{s+3+j}$$

$$K_1 = \left.\frac{100(s+3)}{s^2+6s+10}\right|_{s=0} = 30$$

$$K_2 = \frac{d}{ds}\left[\frac{100(s+3)}{s^2+6s+10}\right]$$

$$= \left[\frac{100}{s^2+6s+10} - \frac{100(s+3)(2s+6)}{(s^2+6s+10)^2}\right]_{s=0} = 10 - 18 = -8$$

$$K_3 = \left.\frac{100(s+3)}{s^2(s+3+j)}\right|_{s=-3+j} = 4 + j3 = 5\underline{/36.87^\circ}$$

$$f(t) = [30t - 8 + 10e^{-3t}\cos(t + 36.87^\circ)]u(t)$$

[d] $$F(s) = \frac{K_1}{s} + \frac{K_2}{(s+1)^3} + \frac{K_3}{(s+1)^2} + \frac{K_4}{s+1}$$

$$K_1 = \left.\frac{5(s+2)^2}{(s+1)^3}\right|_{s=0} = 20$$

$$K_2 = \left.\frac{5(s+2)^2}{s}\right|_{s=-1} = -5$$

$$K_3 = \frac{d}{ds}\left[\frac{5(s+2)^2}{s}\right] = \left[\frac{10(s+2)}{s} - \frac{5(s+2)^2}{s^2}\right]_{s=-1}$$

$$= -10 - 5 = -15$$

$$K_4 = \frac{1}{2}\frac{d}{ds}\left[\frac{10(s+2)}{s} - \frac{5(s+2)^2}{s^2}\right]$$

$$= \frac{1}{2}\left[\frac{10}{s} - \frac{10(s+2)}{s^2} - \frac{10(s+2)}{s^2} + \frac{10(s+2)^2}{s^3}\right]_{s=-1}$$

$$= \frac{1}{2}(-10 - 10 - 10 - 10) = -20$$

$$f(t) = [20 - 2.5t^2e^{-t} - 15te^{-t} - 20e^{-t}]u(t)$$

[e] $F(s) = \dfrac{K_1}{s} + \dfrac{K_2}{(s+2-j)^2} + \dfrac{K_2^*}{(s+2+j)^2} + \dfrac{K_3}{s+2-j} + \dfrac{K_3^*}{s+2-j}$

$$K_1 = \left.\frac{400}{(s^2+4s+5)^2}\right|_{s=0} = 16$$

$$K_2 = \left.\frac{400}{s(s+2+j)^2}\right|_{s=-2+j} = 44.72\underline{/26.57^\circ}$$

$$K_3 = \frac{d}{ds}\left[\frac{400}{s(s+2+j)^2}\right] = \left[\frac{400}{s^2(s+2+j)^2} + \frac{-800}{s(s+2+j)^3}\right]_{s=-2+j}$$

$$= 12 + j16 - 20 + j40 = -8 + j56 = 56.57\underline{/98.13^\circ}$$

$$f(t) = [16 + 89.44te^{-2t}\cos(t + 26.57^\circ) + 113.14e^{-2t}\cos(t + 98.13^\circ)]u(t)$$

P 12.42 **[a]**

$$F(s) = \begin{array}{rl} & \qquad\qquad\quad 5 \\ s^2+6s+8 & \overline{)\;5s^2+38s+80} \\ & \;\;\underline{5s^2+30s+40} \\ & \qquad\quad 8s+40 \end{array}$$

$$F(s) = 5 + \frac{8s+40}{s^2+6s+8} = 10 + \frac{K_1}{s+2} + \frac{K_2}{s+4}$$

$$K_1 = \left.\frac{8s+40}{s+4}\right|_{s=-2} = 12$$

$$K_2 = \left.\frac{8s+40}{s+2}\right|_{s=-4} = -4$$

$$f(t) = 5\delta(t) + [12e^{-2t} - 4e^{-4t}]u(t)$$

[b]

$$F(s) = \begin{array}{rl} & \qquad\qquad\qquad 10 \\ s^2+48s+625 & \overline{)\;10s^2+512s+7186} \\ & \;\;\underline{10s^2+480s+6250} \\ & \qquad\qquad 32s+936 \end{array}$$

$$F(s) = 10 + \frac{32s+936}{s^2+48s+625} = 10 + \frac{K_1}{s+24-j7} + \frac{K_2^*}{s+24+j7}$$

$$K_1 = \left.\frac{32s+936}{s+24+j7}\right|_{s=-24+j7} = 16 - j12 = 20\underline{/-36.87^\circ}$$

$$f(t) = 10\delta(t) + [40e^{-24t}\cos(7t - 36.87^\circ)]u(t)$$

[c]

$$F(s) = \begin{array}{r|l} & \quad s-10 \\ \cline{2-2} s^2+15s+50 & s^3+5s^2-50s-100 \\ & s^3+15s^2+50s \\ \cline{2-2} & -10s^2-100s-100 \\ & -10s^2-150s-500 \\ \cline{2-2} & \qquad 50s+400 \end{array}$$

$$F(s) = s - 10 + \frac{K_1}{s+5} + \frac{K_2}{s+10}$$

$$K_1 = \frac{50s+400}{s+10}\bigg|_{s=-5} = 30$$

$$K_2 = \frac{50s+400}{s+5}\bigg|_{s=-10} = 20$$

$$f(t) = \delta'(t) - 10\delta(t) + [30e^{-5t} + 20e^{-10t}]u(t)$$

P 12.43 **[a]** $F(s) = \dfrac{K_1}{s^2} + \dfrac{K_2}{s} + \dfrac{K_3}{s+1-j2} + \dfrac{K_3^*}{s+1+j2}$

$$K_1 = \frac{100(s+1)}{s^2+2s+5}\bigg|_{s=0} = 20$$

$$K_2 = \frac{d}{ds}\left[\frac{100(s+1)}{s^2+2s+5}\right] = \left[\frac{100}{s^2+2s+5} - \frac{100(s+1)(2s+2)}{(s^2+2s+5)^2}\right]_{s=0}$$

$$= 20 - 8 = 12$$

$$K_3 = \frac{100(s+1)}{s^2(s+1+j2)}\bigg|_{s=-1+j2} = -6 + j8 = 10\underline{/126.87^\circ}$$

$$f(t) = [20t + 12 + 20e^{-t}\cos(2t + 126.87^\circ)]u(t)$$

[b] $F(s) = \dfrac{K_1}{s} + \dfrac{K_2}{(s+5)^3} + \dfrac{K_3}{(s+5)^2} + \dfrac{K_4}{s+5}$

$$K_1 = \frac{500}{(s+5)^3}\bigg|_{s=0} = 4$$

$$K_2 = \frac{500}{s}\bigg|_{s=-5} = -100$$

$$K_3 = \frac{d}{ds}\left[\frac{500}{s}\right] = \frac{-500}{s^2}\bigg|_{s=-5} = -20$$

$$K_4 = \frac{1}{2}\frac{d}{ds}\left[\frac{-500}{s^2}\right] = \frac{1}{2}\frac{1000}{s^3}\bigg|_{s=-5} = -4$$

$$f(t) = [4 - 50t^2e^{-5t} - 20te^{-5t} - 4e^{-5t}]u(t)$$

[c] $F(s) = \dfrac{K_1}{s} + \dfrac{K_2}{(s+1)^3} + \dfrac{K_3}{(s+1)^2} + \dfrac{K_4}{s+1}$

$$K_1 = \frac{40(s+2)}{(s+1)^3}\bigg|_{s=0} = 80$$

$$K_2 = \frac{40(s+2)}{s}\bigg|_{s=-1} = -40$$

$$K_3 = \frac{d}{ds}\left[\frac{40(s+2)}{s}\right] = \left[\frac{40}{s} - \frac{40(s+2)}{s^2}\right]_{s=-1} = -40 - 40 = -80$$

$$K_4 = \frac{1}{2}\frac{d}{ds}\left[\frac{40}{s} - \frac{40(s+2)}{s^2}\right]$$

$$= \frac{1}{2}\left[\frac{-40}{s^2} - \frac{40}{s^2} + \frac{80(s+2)}{s^3}\right]_{s=-1} = \frac{1}{2}(-40 - 40 - 80) = -80$$

$$f(t) = [80 - 20t^2e^{-t} - 80te^{-t} - 80e^{-t}]u(t)$$

[d] $F(s) = \dfrac{K_1}{s} + \dfrac{K_2}{(s+1)^4} + \dfrac{K_3}{(s+1)^3} + \dfrac{K_4}{(s+1)^2} + \dfrac{K_5}{s+1}$

$$K_1 = \frac{(s+5)^2}{(s+1)^4}\bigg|_{s=0} = 25$$

$$K_2 = \frac{(s+5)^2}{s}\bigg|_{s=-1} = -16$$

$$K_3 = \frac{d}{ds}\left[\frac{(s+5)^2}{s}\right] = \left[\frac{2(s+5)}{s} - \frac{(s+5)^2}{s^2}\right]_{s=-1}$$

$$= -8 - 16 = -24$$

$$K_4 = \frac{1}{2}\frac{d}{ds}\left[\frac{2(s+5)}{s} - \frac{(s+5)^2}{s^2}\right]$$

$$= \frac{1}{2}\left[\frac{2}{s} - \frac{2(s+5)}{s^2} - \frac{2(s+5)}{s^2} + \frac{3(s+5)^2}{s^3}\right]_{s=-1}$$

$$= \frac{1}{2}(-2 - 8 - 8 - 32) = -25$$

$$K_5 = \frac{1}{6}\frac{d}{ds}\left[\frac{2}{s} - \frac{2(s+5)}{s^2} - \frac{2(s+5)}{s^2} + \frac{3(s+5)^2}{s^3}\right]$$

$$= \frac{1}{6}\left[\frac{-2}{s^2} - \frac{2}{s^2} + \frac{4(s+5)}{s^3} - \frac{2}{s^2} + \frac{4(s+5)}{s^3} + \frac{4(s+5)}{s^3} - \frac{6(s+5)^2}{s^4}\right]_{s=-1}$$

$$= \frac{1}{6}(-2 - 2 - 16 - 2 - 16 - 16 - 96) = -25$$

$$f(t) = [25 - (8/3)t^3e^{-t} - 12t^2e^{-t} - 25te^{-t} - 25e^{-t}]u(t)$$

P 12.44 $$f(t) = \mathcal{L}^{-1}\left\{\frac{K}{s+\alpha-j\beta} + \frac{K^*}{s+\alpha+j\beta}\right\}$$

$$= Ke^{-\alpha t}e^{j\beta t} + K^*e^{-\alpha t}e^{-j\beta t}$$

$$= |K|e^{-\alpha t}[e^{j\theta}e^{j\beta t} + e^{-j\theta}e^{-j\beta t}]$$

$$= |K|e^{-\alpha t}[e^{j(\beta t+\theta)} + e^{-j(\beta t+\theta)}]$$

$$= 2|K|e^{-\alpha t}\cos(\beta t+\theta)$$

P 12.45 **[a]** $$\mathcal{L}\{t^n f(t)\} = (-1)^n\left[\frac{d^nF(s)}{ds^n}\right]$$

Let $f(t) = 1$, then $F(s) = \frac{1}{s}$, thus $\frac{d^nF(s)}{ds^n} = \frac{(-1)^n n!}{s^{(n+1)}}$

Therefore $\mathcal{L}\{t^n\} = (-1)^n\left[\frac{(-1)^n n!}{s^{(n+1)}}\right] = \frac{n!}{s^{(n+1)}}$

It follows that $\mathcal{L}\{t^{(r-1)}\} = \frac{(r-1)!}{s^r}$

and $\mathcal{L}\{t^{(r-1)}e^{-at}\} = \frac{(r-1)!}{(s+a)^r}$

Therefore $\frac{K}{(r-1)!}\mathcal{L}\{t^{r-1}e^{-at}\} = \frac{K}{(s+a)^r} = \mathcal{L}\left\{\frac{Kt^{r-1}e^{-at}}{(r-1)!}\right\}$

[b] $$f(t) = \mathcal{L}^{-1}\left\{\frac{K}{(s+\alpha-j\beta)^r} + \frac{K^*}{(s+\alpha+j\beta)^r}\right\}$$

Therefore

$$f(t) = \frac{Kt^{r-1}}{(r-1)!}e^{-(\alpha-j\beta)t} + \frac{K^*t^{r-1}}{(r-1)!}e^{-(\alpha+j\beta)t}$$

$$= \frac{|K|t^{r-1}e^{-\alpha t}}{(r-1)!}\left[e^{j\theta}e^{j\beta t} + e^{-j\theta}e^{-j\beta t}\right]$$

$$= \left[\frac{2|K|t^{r-1}e^{-\alpha t}}{(r-1)!}\right]\cos(\beta t+\theta)$$

P 12.46 **[a]** $$\lim_{s\to\infty} sV(s) = \lim_{s\to\infty}\left[\frac{1.92s^3}{s^4[1+(1.6/s)+(1/s^2)][1+(1/s^2)]}\right] = 0$$

Therefore $v(0^+) = 0$

[b] No, V has a pair of poles on the imaginary axis.

P 12.47 $sV_o(s) = \dfrac{sV_{\rm dc}/RC}{s^2 + (1/RC)s + (1/LC)}$

$$\lim_{s\to 0} sV_o(s) = 0, \qquad \therefore \quad v_o(\infty) = 0$$

$$\lim_{s\to \infty} sV_o(s) = 0, \qquad \therefore \quad v_o(0^+) = 0$$

$$sI_o(s) = \frac{V_{\rm dc}/RC)}{s^2 + (1/RC)s + (1/LC)}$$

$$\lim_{s\to 0} sI_o(s) = \frac{V_{\rm dc}/RLC}{1/LC} = \frac{V_{\rm dc}}{R}, \qquad \therefore \quad i_o(\infty) = \frac{V_{\rm dc}}{R}$$

$$\lim_{s\to \infty} sI_o(s) = 0, \qquad \therefore \quad i_o(0^+) = 0$$

P 12.48 $sV_o(s) = \dfrac{(I_{\rm dc}/C)s}{s^2 + (1/RC)s + (1/LC)}$

$$\lim_{s\to 0} sV_o(s) = 0, \qquad \therefore \quad v_o(\infty) = 0$$

$$\lim_{s\to \infty} sV_o(s) = 0, \qquad \therefore \quad v_o(0^+) = 0$$

$$sI_o(s) = \frac{s^2 I_{\rm dc}}{s^2 + (1/RC)s + (1/LC)}$$

$$\lim_{s\to 0} sI_o(s) = 0, \qquad \therefore \quad i_o(\infty) = 0$$

$$\lim_{s\to \infty} sI_o(s) = I_{\rm dc}, \qquad \therefore \quad i_o(0^+) = I_{\rm dc}$$

P 12.49 $sI_o(s) = \dfrac{I_{\rm dc}s[s + (1/RC)]}{s^2 + (1/RC)s + (1/LC)}$

$$\lim_{s\to 0} sI_o(s) = 0, \qquad \therefore \quad i_o(\infty) = 0$$

$$\lim_{s\to \infty} sI_o(s) = I_{\rm dc}, \qquad \therefore \quad i_o(0^+) = I_{\rm dc}$$

P 12.50 **[a]** $sF(s) = \dfrac{8s^3 + 37s^2 + 32s}{(s + 1)(s + 2)(s + 4)}$

$$\lim_{s\to 0} sF(s) = 0, \qquad \therefore \quad f(\infty) = 0$$

$$\lim_{s\to \infty} sF(s) = 8, \qquad \therefore \quad f(0^+) = 8$$

[b] $sF(s) = \dfrac{13s^3 + 134s^2 + 392s + 288}{(s+2)(s^2+10s+24)}$

$\lim_{s\to 0} sF(s) = 6; \qquad \therefore \quad f(\infty) = 6$

$\lim_{s\to \infty} sF(s) = 13, \qquad \therefore \quad f(0^+) = 13$

[c] $sF(s) = \dfrac{20s^3 + 16s^2 + 12s}{(s+1)(s^2+2s+5)}$

$\lim_{s\to 0} sF(s) = 0, \qquad \therefore \quad f(\infty) = 0$

$\lim_{s\to \infty} sF(s) = 20, \qquad \therefore \quad f(0^+) = 20$

[d] $sF(s) = \dfrac{250(s+7)(s+14)}{(s^2+14s+50)}$

$\lim_{s\to 0} sF(s) = \dfrac{250(7)(14)}{50} = 490, \qquad \therefore \quad f(\infty) = 490$

$\lim_{s\to \infty} sF(s) = 250, \qquad \therefore \quad f(0^+) = 250$

P 12.51 **[a]** $sF(s) = \dfrac{100}{s(s+5)}$

$F(s)$ has a second-order pole at the origin so we cannot use the final value theorem.

$\lim_{s\to \infty} sF(s) = 0, \qquad \therefore \quad f(0^+) = 0$

[b] $sF(s) = \dfrac{50(s+5)}{(s+1)^2}$

$\lim_{s\to 0} sF(s) = 250, \qquad \therefore \quad f(\infty) = 250$

$\lim_{s\to \infty} sF(s) = 0, \qquad \therefore \quad f(0^+) = 0$

[c] $sF(s) = \dfrac{100(s+3)}{s(s^2+6s+10)}$

$F(s)$ has a second-order pole at the origin so we cannot use the final value theorem.

$\lim_{s\to \infty} sF(s) = 0, \qquad \therefore \quad f(0^+) = 0$

[d] $sF(s) = \dfrac{5(s+2)^2}{(s+1)^3}$

$\lim_{s\to 0} sF(s) = 20, \qquad \therefore \quad f(\infty) = 20$

$\lim_{s\to \infty} sF(s) = 0, \qquad \therefore \quad f(0^+) = 0$

[e] $sF(s) = \dfrac{400}{(s^2 + 4s + 5)^2}$

$\lim_{s\to 0} sF(s) = 16, \qquad \therefore \quad f(\infty) = 16$

$\lim_{s\to\infty} sF(s) = 0, \qquad \therefore \quad f(0^+) = 0$

P 12.52 All of the $F(s)$ functions referenced in this problem are improper rational functions, and thus the corresponding $f(t)$ functions contain impulses ($\delta(t)$). Thus, neither the initial value theorem nor the final value theorem may be applied to these $F(s)$ functions!

P 12.53 **[a]** $sF(s) = \dfrac{100(s+1)}{s(s^2 + 2s + 5)}$

$F(s)$ has a second-order pole at the origin, so we cannot use the final value theorem here.

$\lim_{s\to\infty} sF(s) = 0, \qquad \therefore \quad f(0^+) = 0$

[b] $sF(s) = \dfrac{500}{(s+5)^3}$

$\lim_{s\to 0} sF(s) = 4, \qquad \therefore \quad f(\infty) = 4$

$\lim_{s\to\infty} sF(s) = 0, \qquad \therefore \quad f(0^+) = 0$

[c] $sF(s) = \dfrac{40(s+2)}{(s+1)^3}$

$\lim_{s\to 0} sF(s) = 80, \qquad \therefore \quad f(\infty) = 80$

$\lim_{s\to\infty} sF(s) = 0, \qquad \therefore \quad f(0^+) = 0$

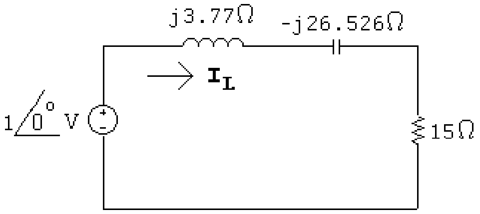
[d] $sF(s) = \dfrac{(s+5)^2}{(s+1)^4}$

$\lim_{s\to 0} sF(s) = 25, \qquad \therefore \quad f(\infty) = 25$

$\lim_{s\to\infty} sF(s) = 0, \qquad \therefore \quad f(0^+) = 0$

P 12.54 [a] $Z_L = j120\pi(0.01) = j3.77\,\Omega;\qquad Z_C = \dfrac{-j}{120\pi(100\times10^{-6})} = -j26.526\,\Omega$

The phasor-transformed circuit is



$$\mathbf{I}_L = \frac{1}{15 + j3.77 - j26.526} = 36.69\underline{/56.61^\circ}\,\text{mA}$$

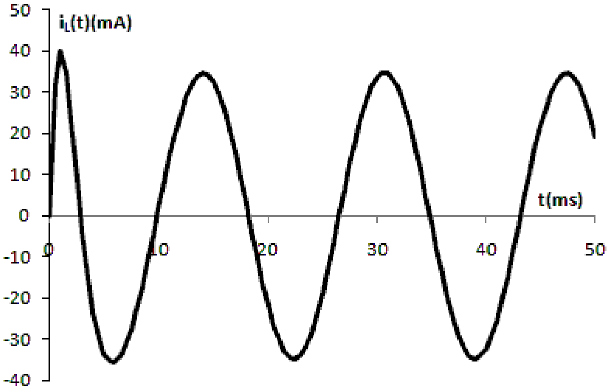
$$\therefore\quad i_{L-ss}(t) = 36.69\cos(120\pi t + 56.61^\circ)\,\text{mA}$$

[b] The steady-state response is the second term in Eq. 12.109, which matches the steady-state response just derived in part (a).

P 12.55 The transient and steady-state components are both proportional to the magnitude of the input voltage. Therefore,

$$K = \frac{40}{42.26} = 0.947$$

So if we make the amplitude of the sinusoidal source 0.947 instead of 1, the current will not exceed the 40 mA limit. A plot of the current through the inductor is shown below with the amplitude of the sinusoidal source set at 0.947.



P 12.56 We begin by using Eq. 12.105, and changing the right-hand side so it is the Laplace transform of Kte^{-100t}:

$$15I_L(s) + 0.01sI_L(s) + 10^4\frac{I_L(s)}{s} = \frac{A}{(s+100)^2}$$

Solving for $I_L(s)$,

$$I_L(s) = \frac{100Ks}{(s^2 + 1500s + 10^6)(s + 100)^2} = \frac{K_1}{s + 750 - j661.44} + \frac{K_1^*}{s + 750 + j661.44}$$

$$+\frac{K_2}{(s + 100)^2} + \frac{K_3}{s + 100}$$

$$K_1 = \frac{100Ks}{(s + 750 + j661.44)(s + 100)^2}\Bigg|_{s=-750+j661.44} = 87.9K\underline{/139.59^\circ}\,\mu\text{A}$$

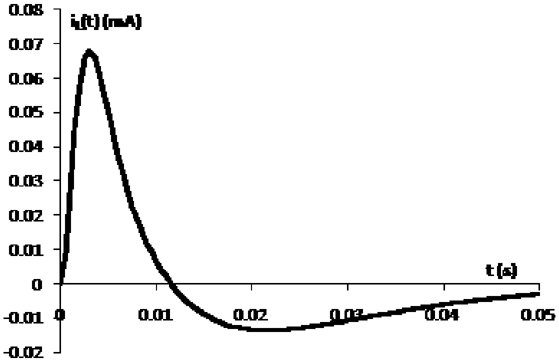
$$K_2 = \frac{100Ks}{(s^2 + 1500s + 10^6)}\Bigg|_{s=-100} = -11.63K\,\text{mA}$$

$$K_3 = \frac{d}{ds}\left[\frac{100Ks}{(s^2 + 1500s + 10^6)}\right]_{s=-100} = 133.86K\,\mu\text{A}$$

Therefore,

$$i_L(t) = K[0.176e^{-750t}\cos(661.44t + 139.59^\circ) - 11.63te^{-100t} + 0.134e^{-100t}]u(t)\,\text{mA}$$

Plot the expression above with $K = 1$:



The maximum value of the inductor current is $0.068K$ mA. Therefore,

$$K = \frac{40}{0.068} = 588$$

So the inductor current rating will not be exceeded if the input to the RLC circuit is $588te^{-100t}$ V.

13

The Laplace Transform in Circuit Analysis

Assessment Problems

AP 13.1 **[a]** $Y = \frac{1}{R} + \frac{1}{sL} + sC = \frac{C[s^2 + (1/RC)s + (1/LC)]}{s}$

$$\frac{1}{RC} = \frac{10^6}{(500)(0.025)} = 80{,}000; \qquad \frac{1}{LC} = 25 \times 10^8$$

$$\text{Therefore} \quad Y = \frac{25 \times 10^{-9}(s^2 + 80{,}000s + 25 \times 10^8)}{s}$$

[b] $z_{1,2} = -40{,}000 \pm \sqrt{16 \times 10^8 - 25 \times 10^8} = -40{,}000 \pm j30{,}000$ rad/s

$-z_1 = -40{,}000 - j30{,}000$ rad/s

$-z_2 = -40{,}000 + j30{,}000$ rad/s

$p_1 = 0$ rad/s

AP 13.2 **[a]** $Z = 2000 + \frac{1}{Y} = 2000 + \frac{4 \times 10^7 s}{s^2 + 80{,}000s + 25 \times 10^8}$

$$= \frac{2000(s^2 + 10^5 s + 25 \times 10^8)}{s^2 + 80{,}000s + 25 \times 10^8} = \frac{2000(s + 50{,}000)^2}{s^2 + 80{,}000s + 25 \times 10^8}$$

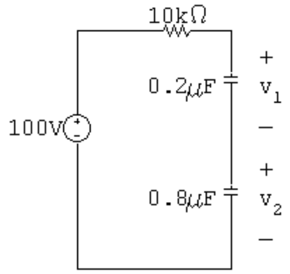
[b] $-z_1 = -z_2 = -50{,}000$ rad/s

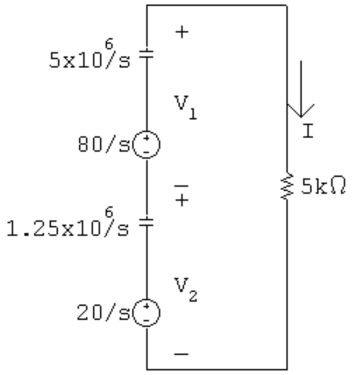
$-p_1 = -40{,}000 - j30{,}000$ rad/s

$-p_2 = -40{,}000 + j30{,}000$ rad/s

AP 13.3 **[a]** At $t = 0^-$, $\quad 0.2v_1 = (0.8)v_2; \quad v_1 = 4v_2; \quad v_1 + v_2 = 100\,\text{V}$

Therefore $\quad v_1(0^-) = 80V = v_1(0^+); \quad v_2(0^-) = 20V = v_2(0^+)$





$$I = \frac{(80/s) + (20/s)}{5000 + [(5 \times 10^6)/s] + (1.25 \times 10^6/s)} = \frac{20 \times 10^{-3}}{s + 1250}$$

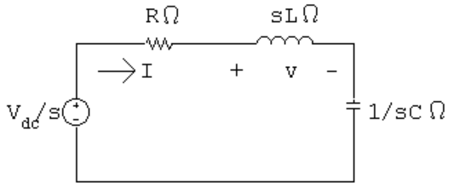
$$V_1 = \frac{80}{s} - \frac{5 \times 10^6}{s}\left(\frac{20 \times 10^{-3}}{s + 1250}\right) = \frac{80}{s + 1250}$$

$$V_2 = \frac{20}{s} - \frac{1.25 \times 10^6}{s}\left(\frac{20 \times 10^{-3}}{s + 1250}\right) = \frac{20}{s + 1250}$$

[b] $i = 20e^{-1250t}u(t)\,\text{mA}; \qquad v_1 = 80e^{-1250t}u(t)\,\text{V}$

$v_2 = 20e^{-1250t}u(t)\,\text{V}$

AP 13.4 **[a]**



$$I = \frac{V_{\text{dc}}/s}{R + sL + (1/sC)} = \frac{V_{\text{dc}}/L}{s^2 + (R/L)s + (1/LC)}$$

$$\frac{V_{\text{dc}}}{L} = 40; \qquad \frac{R}{L} = 1.2; \qquad \frac{1}{LC} = 1.0$$

$$I = \frac{40}{(s + 0.6 - j0.8)(s + 0.6 + j0.8)} = \frac{K_1}{s + 0.6 - j0.8} + \frac{K_1^*}{s + 0.6 + j0.8}$$

$$K_1 = \frac{40}{j1.6} = -j25 = 25\underline{/-90^\circ}; \qquad K_1^* = 25\underline{/90^\circ}$$

[b] $i = 50e^{-0.6t}\cos(0.8t - 90^\circ) = [50e^{-0.6t}\sin 0.8t]u(t)\,\text{A}$

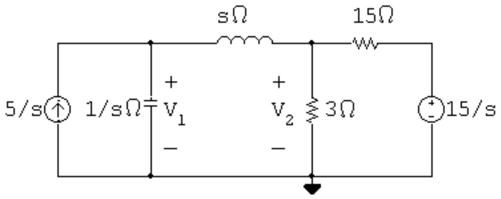
[**c**] $$V = sLI = \frac{160s}{(s+0.6-j0.8)(s+0.6+j0.8)}$$

$$= \frac{K_1}{s+0.6-j0.8} + \frac{K_1^*}{s+0.6+j0.8}$$

$$K_1 = \frac{160(-0.6+j0.8)}{j1.6} = 100\underline{/36.87^\circ}$$

[**d**] $v(t) = [200e^{-0.6t}\cos(0.8t + 36.87^\circ)]u(t)\,\text{V}$

AP 13.5 [**a**]



The two node voltage equations are

$$\frac{V_1 - V_2}{s} + V_1 s = \frac{5}{s} \quad \text{and} \quad \frac{V_2}{3} + \frac{V_2 - V_1}{s} + \frac{V_2 - (15/s)}{15} = 0$$

Solving for V_1 and V_2 yields

$$V_1 = \frac{5(s+3)}{s(s^2+2.5s+1)}, \qquad V_2 = \frac{2.5(s^2+6)}{s(s^2+2.5s+1)}$$

[**b**] The partial fraction expansions of V_1 and V_2 are

$$V_1 = \frac{15}{s} - \frac{50/3}{s+0.5} + \frac{5/3}{s+2} \quad \text{and} \quad V_2 = \frac{15}{s} - \frac{125/6}{s+0.5} + \frac{25/3}{s+2}$$

It follows that

$$v_1(t) = \left[15 - \frac{50}{3}e^{-0.5t} + \frac{5}{3}e^{-2t}\right]u(t)\,\text{V} \quad \text{and}$$

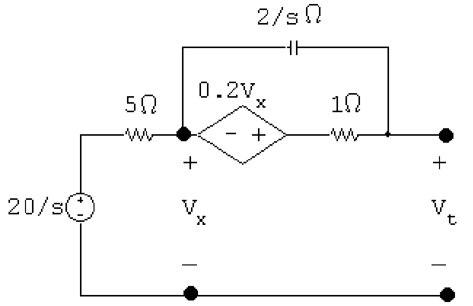
$$v_2(t) = \left[15 - \frac{125}{6}e^{-0.5t} + \frac{25}{3}e^{-2t}\right]u(t)\,\text{V}$$

[**c**] $$v_1(0^+) = 15 - \frac{50}{3} + \frac{5}{3} = 0$$

$$v_2(0^+) = 15 - \frac{125}{6} + \frac{25}{3} = 2.5\,\text{V}$$

[**d**] $v_1(\infty) = 15\,\text{V}; \qquad v_2(\infty) = 15\,\text{V}$

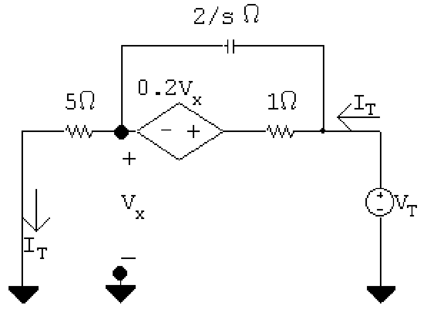
AP 13.6 [**a**]



With no load across terminals $a - b$ $\quad V_x = 20/s$:

$$\frac{1}{2}\left[\frac{20}{s} - V_{\text{Th}}\right]s + \left[1.2\left(\frac{20}{s}\right) - V_{\text{Th}}\right] = 0$$

therefore $\quad V_{\text{Th}} = \dfrac{20(s+2.4)}{s(s+2)}$



$V_x = 5I_T \quad$ and $\quad Z_{\text{Th}} = \dfrac{V_T}{I_T}$

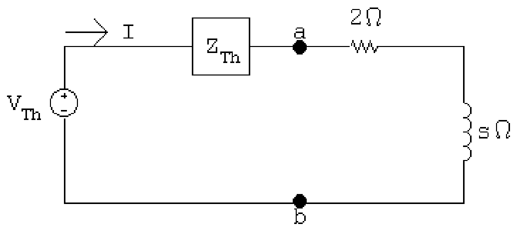
Solving for I_T gives

$$I_T = \frac{(V_T - 5I_T)s}{2} + V_T - 6I_T$$

Therefore

$14I_T = V_T s + 5sI_T + 2V_T; \qquad$ therefore $\quad Z_{\text{Th}} = \dfrac{5(s+2.8)}{s+2}$

[**b**]



$$I = \frac{V_{\text{Th}}}{Z_{\text{Th}} + 2 + s} = \frac{20(s+2.4)}{s(s+3)(s+6)}$$

AP 13.7 [a] $i_2 = 1.25e^{-t} - 1.25e^{-3t}$; therefore $\frac{di_2}{dt} = -1.25e^{-t} + 3.75e^{-3t}$

Therefore $\frac{di_2}{dt} = 0$ when

$1.25e^{-t} = 3.75e^{-3t}$ or $e^{2t} = 3$, $t = 0.5(\ln 3) = 549.31\,\text{ms}$

$i_2(\text{max}) = 1.25[e^{-0.549} - e^{-3(0.549)}] = 481.13\,\text{mA}$

[b] From Eqs. 13.68 and 13.69, we have

$\Delta = 12(s^2 + 4s + 3) = 12(s+1)(s+3)$ and $N_1 = 60(s+2)$

Therefore $I_1 = \frac{N_1}{\Delta} = \frac{5(s+2)}{(s+1)(s+3)}$

A partial fraction expansion leads to the expression

$$I_1 = \frac{2.5}{s+1} + \frac{2.5}{s+3}$$

Therefore we get

$i_1 = 2.5[e^{-t} + e^{-3t}]u(t)\,\text{A}$

[c] $\frac{di_1}{dt} = -2.5[e^{-t} + 3e^{-3t}]$; $\frac{di_1(0.54931)}{dt} = -2.89\,\text{A/s}$

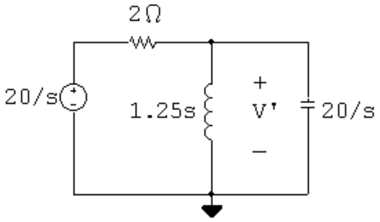
[d] When i_2 is at its peak value,

$$\frac{di_2}{dt} = 0$$

Therefore $L_2\left(\frac{di_2}{dt}\right) = 0$ and $i_2 = -\left(\frac{M}{12}\right)\left(\frac{di_1}{dt}\right)$

[e] $i_2(\text{max}) = \frac{-2(-2.89)}{12} = 481.13\,\text{mA}$ (checks)

AP 13.8 [a] The s-domain circuit with the voltage source acting alone is

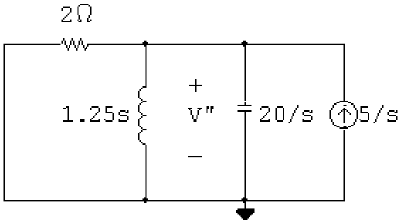


$$\frac{V' - (20/s)}{2} + \frac{V'}{1.25s} + \frac{V's}{20} = 0$$

$$V' = \frac{200}{(s+2)(s+8)} = \frac{100/3}{s+2} - \frac{100/3}{s+8}$$

$$v' = \frac{100}{3}[e^{-2t} - e^{-8t}]u(t)\,\text{V}$$

[b] With the current source acting alone,



$$\frac{V''}{2} + \frac{V''}{1.25s} + \frac{V''s}{20} = \frac{5}{s}$$

$$V'' = \frac{100}{(s+2)(s+8)} = \frac{50/3}{s+2} - \frac{50/3}{s+8}$$

$$v'' = \frac{50}{3}[e^{-2t} - e^{-8t}]u(t)\,\text{V}$$

[c] $v = v' + v'' = [50e^{-2t} - 50e^{-8t}]u(t)\,\text{V}$

AP 13.9 **[a]** $\dfrac{V_o}{s+2} + \dfrac{V_o s}{10} = I_g;$ therefore $\dfrac{V_o}{I_g} = H(s) = \dfrac{10(s+2)}{s^2+2s+10}$

[b] $-z_1 = -2$ rad/s; $-p_1 = -1 + j3$ rad/s; $-p_2 = -1 - j3$ rad/s

AP 13.10 **[a]**

$$V_o = \frac{10(s+2)}{s^2+2s+10} \cdot \frac{1}{s} = \frac{K_o}{s} + \frac{K_1}{s+1-j3} + \frac{K_1^*}{s+1+j3}$$

$$K_o = 2; \qquad K_1 = 5/3\underline{/-126.87^\circ}; \qquad K_1^* = 5/3\underline{/126.87^\circ}$$

$$v_o = [2 + (10/3)e^{-t}\cos(3t - 126.87^\circ)]u(t)\,\text{V}$$

[b] $V_o = \dfrac{10(s+2)}{s^2+2s+10} \cdot 1 = \dfrac{K_2}{s+1-j3} + \dfrac{K_2^*}{s+1+j3}$

$$K_2 = 5.27\underline{/-18.43^\circ}; \qquad K_2^* = 5.27\underline{/18.43^\circ}$$

$$v_o = [10.54e^{-t}\cos(3t - 18.43^\circ)]u(t)\,\text{V}$$

AP 13.11 **[a]**

$$H(s) = \mathcal{L}\{h(t)\} = \mathcal{L}\{v_o(t)\}$$

$$v_o(t) = 10{,}000\cos\theta e^{-70t}\cos 240t - 10{,}000\sin\theta e^{-70t}\sin 240t$$

$$= 9600e^{-70t}\cos 240t - 2800e^{-70t}\sin 240t$$

Therefore $H(s) = \dfrac{9600(s+70)}{(s+70)^2+(240)^2} - \dfrac{2800(240)}{(s+70)^2+(240)^2}$

$$= \frac{9600s}{s^2+140s+62{,}500}$$

[b] $V_o(s) = H(s)\cdot\dfrac{1}{s} = \dfrac{9600}{s^2+140s+62{,}500}$

$$= \frac{K_1}{s+70-j240} + \frac{K_1^*}{s+70+j240}$$

$$K_1 = \frac{9600}{j480} = -j20 = 20\underline{/-90^\circ}$$

Therefore

$$v_o(t) = [40e^{-70t}\cos(240t-90^\circ)]u(t)\,\text{V} = [40e^{-70t}\sin 240t]u(t)\,\text{V}$$

AP 13.12 From Assessment Problem 13.9:

$$H(s) = \frac{10(s+2)}{s^2+2s+10}$$

Therefore $H(j4) = \dfrac{10(2+j4)}{10-16+j8} = 4.47\underline{/-63.43^\circ}$

Thus,

$$v_o = (10)(4.47)\cos(4t-63.43^\circ) = 44.7\cos(4t-63.43^\circ)\,\text{V}$$

AP 13.13 **[a]**

Let $R_1 = 10\,\text{k}\Omega,\quad R_2 = 50\,\text{k}\Omega,\quad C = 400\text{ pF},\quad R_2C = 2\times10^{-5}$

then $V_1 = V_2 = \dfrac{V_gR_2}{R_2+(1/sC)}$

Also $\dfrac{V_1-V_g}{R_1} + \dfrac{V_1-V_o}{R_1} = 0$

therefore $V_o = 2V_1 - V_g$

Now solving for V_o/V_g, we get $H(s) = \dfrac{R_2Cs-1}{R_2Cs+1}$

It follows that $H(j50{,}000) = \dfrac{j-1}{j+1} = j1 = 1\underline{/90^\circ}$

Therefore $v_o = 10\cos(50{,}000t+90^\circ)\,\text{V}$

[b] Replacing R_2 by R_x gives us $H(s) = \dfrac{R_xCs-1}{R_xCs+1}$

Therefore

$$H(j50{,}000) = \frac{j20\times10^{-6}R_x-1}{j20\times10^{-6}R_x+1} = \frac{R_x+j50{,}000}{R_x-j50{,}000}$$

Thus,

$$\frac{50{,}000}{R_x} = \tan 60^\circ = 1.7321, \qquad R_x = 28{,}867.51\,\Omega$$

Problems

P 13.1 $i = \frac{1}{L}\int_{0^-}^{t} v\,d\tau + I_0;$ therefore $I = \left(\frac{1}{L}\right)\left(\frac{V}{s}\right) + \frac{I_0}{s} = \frac{V}{sL} + \frac{I_0}{s}$

P 13.2 $V_{\text{Th}} = V_{\text{ab}} = CV_0\left(\frac{1}{sC}\right) = \frac{V_0}{s};$ $Z_{\text{Th}} = \frac{1}{sC}$

P 13.3 $I_{sc_{\text{ab}}} = I_{\text{N}} = \frac{-LI_0}{sL} = \frac{-I_0}{s};$ $Z_{\text{N}} = sL$

Therefore, the Norton equivalent is the same as the circuit in Fig. 13.4.

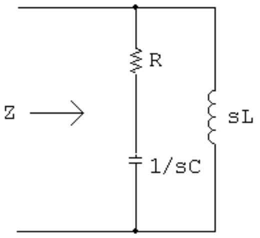
P 13.4 **[a]** $Y = \frac{1}{R} + \frac{1}{sL} + sC = \frac{C[s^2 + (1/RC)s + (1/LC)]}{s}$

$$Z = \frac{1}{Y} = \frac{s/C}{s^2 + (1/RC)s + (1/LC)} = \frac{8\times 10^7 s}{s^2 + 40{,}000s + 256\times 10^6}$$

[b] zero at $z_1 = 0$

poles at $-p_1 = -8000$ rad/s and $-p_2 = -32{,}000$ rad/s

P 13.5 **[a]**



$$Z = \frac{(R + 1/sC)(sL)}{R + sL + (1/sC)} = \frac{(Rs)(s + 1/RC)}{s^2 + (R/L)s + (1/LC)}$$

$\frac{R}{L} = 10{,}000;$ $\frac{1}{RC} = 1600;$ $\frac{1}{LC} = 16\times 10^6$

$$Z = \frac{1000s(s + 1600)}{s^2 + 10{,}000s + 16\times 10^6}$$

[b] $Z = \frac{1000s(s + 1600)}{(s + 2000)(s + 8000)}$

$z_1 = 0;$ $-z_2 = -1600$ rad/s

$-p_1 = -2000$ rad/s; $-p_2 = -8000$ rad/s

P 13.6 **[a]** $Z = R + sL + \frac{1}{sC} = \frac{L[s^2 + (R/L)s + (1/LC)]}{s}$

$$= \frac{[s^2 + 8000s + 25\times 10^6]}{s}$$

[b] $s_{1,2} = -4000 \pm j3000$ rad/s

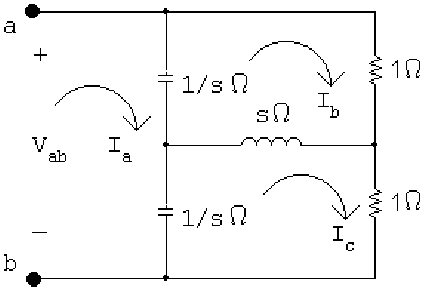
Zeros at $-4000 + j3000$ rad/s and $-4000 - j3000$ rad/s
Pole at 0.

P 13.7 $Z_{\text{ab}} = 1\|[s + (1/s\|1)] = 1\|[s + (1/(s+1))] = \dfrac{s + (1/(s+1))}{1 + s + (1/(s+1))}$

$$= \frac{s^2 + s + 1}{s^2 + 2s + 2} = \frac{(s + 0.5 + j0.866)(s + 0.5 - j0.866)}{(s + 1 + j1)(s + 1 - j1)}$$

Zeros at $-0.5 + j0.866$ rad/s and $-0.5 - j0.866$ rad/s; poles at $-1 + j1$ rad/s and $-1 - j1$ rad/s.

P 13.8 Transform the Y-connection of the two resistors and the inductor into the equivalent delta-connection:



where

$$Z_{\text{a}} = \frac{(s)(1) + (1)(s) + (1)(1)}{s} = \frac{2s+1}{s}$$

$$Z_{\text{b}} = Z_{\text{c}} = \frac{(s)(1) + (1)(s) + (1)(1)}{1} = 2s + 1$$

Then

$$Z_{\text{ab}} = Z_{\text{a}}\|[(1/s\|Z_{\text{c}}) + (1/s\|Z_{\text{b}})] = Z_{\text{a}}\|2(1/s\|Z_{\text{b}})$$

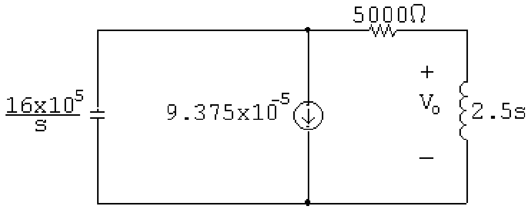
$$1/s\|Z_{\text{b}} = \frac{\frac{1}{s}(2s+1)}{\frac{1}{s} + 2s + 1} = \frac{2s+1}{2s^2 + s + 1}$$

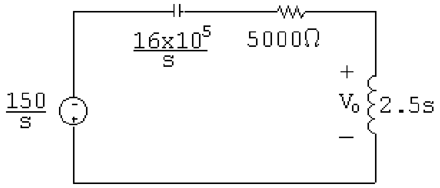
$$Z_{\text{ab}} = \left(\frac{2s+1}{s}\right)\|\frac{2(2s+1)}{2s^2+s+1}$$

$$= \frac{2(2s+1)^2}{(2s+1)(2s^2+s+1) + 2s(2s+1)} = \frac{2}{s+1}$$

No zeros; one pole at -1 rad/s.

P 13.9 **[a]** For $t > 0$:





[b] $$V_o = \frac{2.5s}{(16 \times 10^5)/s + 5000 + 2.5s}\left(\frac{-150}{s}\right)$$

$$= \frac{-150s}{s^2 + 2000s + 64 \times 10^4}$$

$$= \frac{-150s}{(s + 400)(s + 1600)}$$

[c] $$V_o = \frac{K_1}{s + 400} + \frac{K_2}{s + 1600}$$

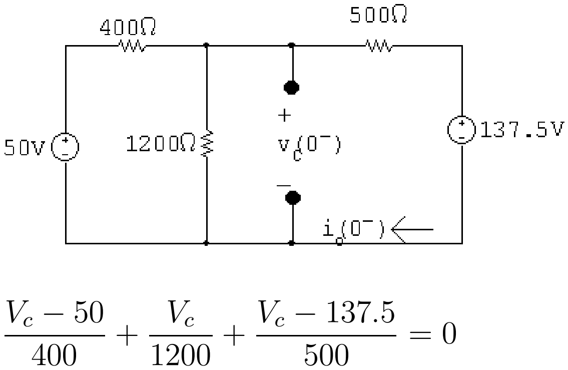
$$K_1 = \frac{-150s}{s + 1600}\bigg|_{s=-400} = 50$$

$$K_2 = \frac{-150s}{s + 400}\bigg|_{s=-1600} = -200$$

$$V_o = \frac{50}{s + 400} - \frac{200}{s + 1600}$$

$$v_o(t) = (50e^{-400t} - 200e^{-1600t})u(t)\,\text{V}$$

P 13.10 **[a]** For $t < 0$:



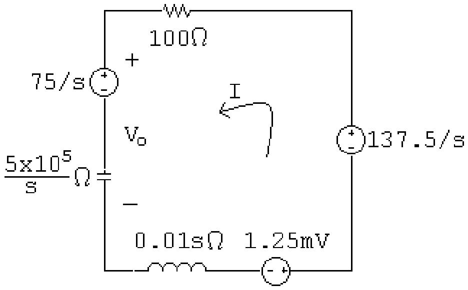
$$\frac{V_c - 50}{400} + \frac{V_c}{1200} + \frac{V_c - 137.5}{500} = 0$$

$$V_c\left(\frac{1}{400} + \frac{1}{1200} + \frac{1}{500}\right) = \frac{50}{400} + \frac{137.5}{500}$$

$$V_c = 75\,\text{V}$$

$$i_L(0^-) = \frac{75 - 137.5}{500} = -0.125\,\text{A}$$

For $t > 0$:



[b] $V_o = \dfrac{5 \times 10^5}{s} I + \dfrac{75}{5}$

$$0 = -\frac{137.5}{s} + 100I + \frac{5 \times 10^5}{s} I + \frac{75}{s} - 1.25 \times 10^{-3} + 0.01sI$$

$$I\left(100 + \frac{5 \times 10^5}{s} + 0.01s\right) = \frac{62.5}{s} + 1.25 \times 10^{-3}$$

$$\therefore \quad I = \frac{6250 + 0.125s}{s^2 + 10^4 s + 5 \times 10^7}$$

$$V_o = \frac{5 \times 10^5}{s}\left(\frac{6250 + 0.125s}{s^2 + 10^4 s + 5 \times 10^7}\right) + \frac{75}{s}$$

$$= \frac{75s^2 + 812{,}500s + 6875 \times 10^6}{s(s^2 + 10^4 s + 5 \times 10^7)}$$

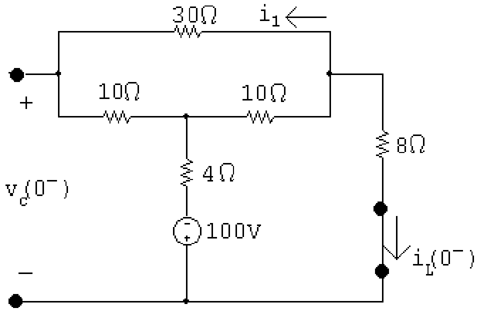
[c] $V_o = \dfrac{K_1}{s} + \dfrac{K_2}{s + 5000 - j5000} + \dfrac{K_2^*}{s + 5000 + j5000}$

$$K_1 = \frac{75s^2 + 812{,}500s + 6875 \times 10^6}{s^2 + 10^4 s + 5 \times 10^7}\bigg|_{s=0} = 137.5$$

$$K_2 = \frac{75s^2 + 812{,}500s + 6875 \times 10^6}{s(s + 5000 + j5000}\bigg|_{s=-5000+j5000} = 40.02\underline{/141.34^\circ}$$

$$v_o(t) = [137.5 + 80.04e^{-5000t}\cos(5000t + 141.34^\circ)]u(t)\,\text{V}$$

P 13.11 **[a]** For $t < 0$:

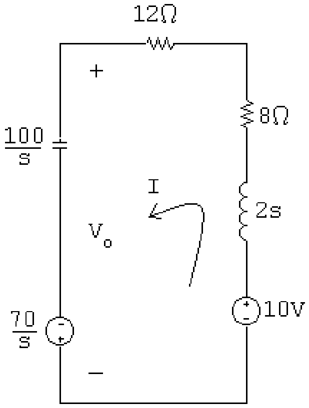


$$i_L(0^-) = \frac{-100}{4 + 10\|40 + 8} = \frac{-100}{20} = -5\,\text{A}$$

$$i_1 = \frac{10}{50}(5) = 1\,\text{A}$$

$$v_C(0^-) = 10(1) + 4(5) - 100 = -70\,\text{V}$$

For $t > 0$:



[b] $(20 + 2s + 100/s)I = 10 + \dfrac{70}{s}$

$$\therefore \qquad I = \frac{5(s+7)}{s^2 + 10s + 50}$$

$$V_o = \frac{100}{s}I - \frac{70}{s}$$

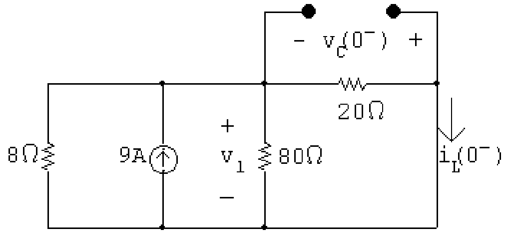
$$= \frac{-70s^2 - 200s}{s(s^2 + 10s + 50)} = \frac{-70(s + 20/7)}{s^2 + 10s + 50}$$

$$= \frac{K_1}{s + 5 - j5} + \frac{K_1^*}{s + 5 + j5}$$

$$K_1 = \frac{-70(s + 20/7)}{s + 5 + j5}\bigg|_{s=-5+j5} = 38.1\underline{/-156.8^\circ}$$

[c] $v_o(t) = 76.2e^{-5t}\cos(5t - 156.8°)u(t)\,\text{V}$

P 13.12 **[a]** For $t < 0$:



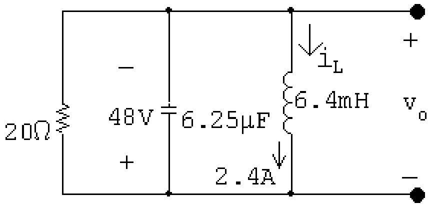
$$\frac{1}{R_e} = \frac{1}{8} + \frac{1}{80} + \frac{1}{20} = 0.1875; \qquad R_e = 5.33\,\Omega$$

$$v_1 = (9)(5.33) = 48\,\text{V}$$

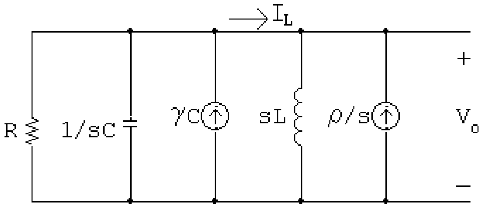
$$i_\text{L}(0^-) = \frac{48}{20} = 2.4\,\text{A}$$

$$v_\text{C}(0^-) = -v_1 = -48\,\text{V}$$

For $t = 0^+$:



s-domain circuit:



where

$$R = 20\,\Omega; \qquad C = 6.25\,\mu\text{F}; \qquad \gamma = -48\,\text{V};$$

$$L = 6.4\,\text{mH}; \qquad \text{and} \qquad \rho = -2.4\,\text{A}$$

[b] $$\frac{V_o}{R} + V_osC - \gamma C + \frac{V_o}{sL} - \frac{\rho}{s} = 0$$

$$\therefore \quad V_o = \frac{\gamma[s + (\rho/\gamma C)]}{s^2 + (1/RC)s + (1/LC)}$$

$$\frac{\rho}{\gamma C} = \frac{-2.4}{(-48)(6.25 \times 10^{-6})} = 8000$$

$$\frac{1}{RC} = \frac{1}{(20)(6.25\times10^{-6})} = 8000$$

$$\frac{1}{LC} = \frac{1}{(6.4\times10^{-3})(6.25\times10^{-6})} = 25\times10^{6}$$

$$V_o = \frac{-48(s+8000)}{s^2+8000s+25\times10^6}$$

[c] $$I_{\rm L} = \frac{V_o}{sL} - \frac{\rho}{s} = \frac{V_o}{0.0064s} + \frac{2.4}{s}$$

$$= \frac{-7500(s+8000)}{s(s^2+8000s+25\times10^6)} - \frac{2.4}{s} = \frac{2.4(s+4875)}{(s^2+8000s+25\times10^6)}$$

[d] $$V_o = \frac{-48(s+8000)}{s^2+8000s+25\times10^6}$$

$$= \frac{K_1}{s++4000-j3000} + \frac{K_1^*}{s+4000+j3000}$$

$$K_1 = \frac{-48(s+8000)}{s+4000+j3000}\bigg|_{s=-4000+j3000} = 40\underline{/126.87^\circ}$$

$$v_o(t) = [80e^{-4000t}\cos(3000t+126.87^\circ)]u(t)\,\text{V}$$

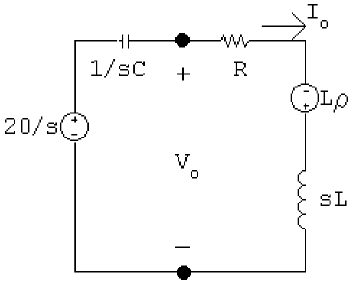
[e] $$I_{\rm L} = \frac{2.4(s+4875)}{s^2+8000s+25\times10^6}$$

$$= \frac{K_1}{s+4000-j3000} + \frac{K_1^*}{s+4000+j3000}$$

$$K_1 = \frac{2.4(s+4875)}{s+4000+j3000}\bigg|_{s=-4000+j3000} = 1.25\underline{/-16.26^\circ}$$

$$i_{\rm L}(t) = [2.5e^{-4000t}\cos(3000t-16.26^\circ)]u(t)\,\text{A}$$

P 13.13 **[a]** $$i_o(0^-) = \frac{20}{4000} = 5\,\text{mA}$$



$$I_o = \frac{20/s+L\rho}{R+sL+1/sC} = \frac{sC(20/s+L\rho)}{s^2LC+RsC+1}$$

$$= \frac{20/L + s\rho}{s^2 + sR/L + 1/LC} = \frac{40 + s(0.005)}{s^2 + 8000s + 16 \times 10^6}$$

$$V_o = RI_o - L\rho + sLI_o = \frac{4000(40 + 0.005s)}{s^2 + 8000s + 16 \times 10^6} - 0.0025 + \frac{0.0025s(s + 8000)}{s^2 + 8000s + 16 \times 10^6}$$

$$= \frac{20s + 120{,}000}{(s + 4000)^2} = \frac{20}{(s + 4000)^2} + \frac{40{,}000}{s + 4000}$$

$$v_o(t) = [20te^{-4000t} + 40{,}000e^{-4000t}]u(t)\,\text{V}$$

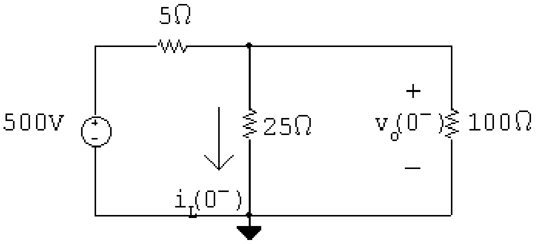
[b] $$I_o = \frac{0.005(s + 8000)}{s^2 + 8000s + 16 \times 10^6}$$

$$= \frac{K_1}{(s + 4000)^2} + \frac{K_2}{s + 4000}$$

$$K_1 = 20 \qquad K_2 = 0.005$$

$$i_o(t) = [20te^{-4000t} + 0.005e^{-4000t}]u(t)\,\text{A}$$

P 13.14 For $t < 0$:

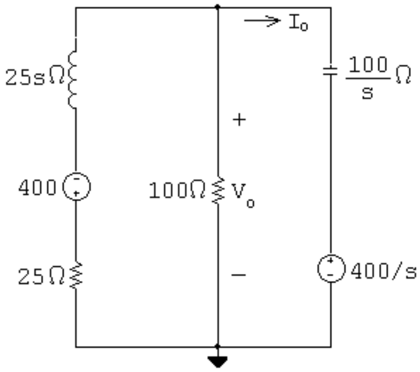


$$\frac{v_o(0^-) - 500}{5} + \frac{v_o(0^-)}{25} + \frac{v_o(0^-)}{100} = 0$$

$$25v_o(0^-) = 10{,}000 \qquad \therefore \qquad v_o(0^-) = 400\,\text{V}$$

$$i_L(0^-) = \frac{v_o(0^-)}{25} = \frac{400}{25} = 16\,\text{A}$$

For $t > 0$:



$$\frac{V_o + 400}{25 + 25s} + \frac{V_o}{100} + \frac{V_o - (400/s)}{100/s} = 0$$

$$V_o\left(\frac{1}{25+25s}+\frac{1}{100}+\frac{s}{100}\right)=4-\frac{400}{25+25s}$$

$$\therefore \qquad V_o=\frac{400(s-3)}{s^2+2s+5}$$

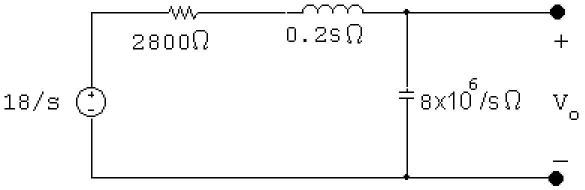
$$I_o=\frac{V_o-(400/s)}{100/s}=\frac{-20s-20}{s^2+2s+5}$$

$$=\frac{K_1}{s+1-j2}+\frac{K_1^*}{s+1+j2}$$

$$K_1=\frac{-20(s+1)}{s+1+j2}\bigg|_{s=-1+j2}=-10$$

$$i_o(t)=[-20e^{-t}\cos 2t]u(t)\,\text{A}$$

P 13.15



$$V_o=\frac{(18/s)(8\times10^6/s)}{2800+0.2s+(8\times10^6/s)}$$

$$=\frac{720\times10^6}{s(s^2+14{,}000s+40\times10^6)}$$

$$=\frac{720\times10^6}{s(s+4000)(s+10{,}000)}$$

$$=\frac{K_1}{s}+\frac{K_2}{s+4000}+\frac{K_3}{s+10{,}000}$$

$$K_1=\frac{720\times10^6}{4\times10^7}=18$$

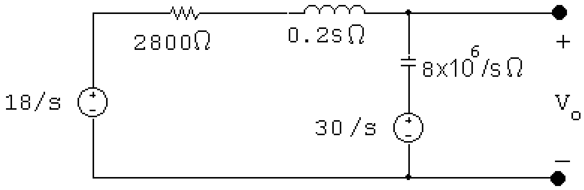
$$K_2=\frac{720\times10^6}{(-4000)(6000)}=-30$$

$$K_3=\frac{720\times10^6}{(-6000)(-10{,}000)}=12$$

$$V_o=\frac{18}{s}-\frac{30}{s+4000}+\frac{12}{s+10{,}000}$$

$$v_o(t)=[18-30e^{-4000t}+12e^{-10{,}000t}]u(t)\,\text{V}$$

P 13.16 With a non-zero initial voltage on the capacitor, the s-domain circuit becomes:



$$\frac{V_o - 18/s}{0.2s + 2800} + \frac{(V_o - 30/s)s}{8\times 10^6} = 0$$

$$V_o\left[\frac{5}{s+14{,}000} + \frac{s}{8\times 10^6}\right] = \frac{30}{80\times 10^6} + \frac{90}{s(s+14{,}000)}$$

$$\therefore \quad V_o = \frac{30s^2 + 420{,}000s + 720\times 10^6}{s(s+4000)(s+10{,}000)}$$

$$= \frac{K_1}{s} + \frac{K_2}{s+4000} + \frac{K_3}{s+10{,}000}$$

$$K_1 = \frac{720\times 10^6}{40\times 10^6} = 18$$

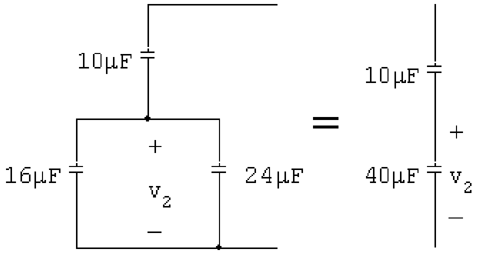
$$K_2 = \frac{30s^2 + 420{,}000s + 720\times 10^6}{s(s+10{,}000)}\bigg|_{s=-4000} = 20$$

$$K_3 = \frac{30s^2 + 420{,}000s + 720\times 10^6}{s(s+4000)}\bigg|_{s=-10{,}000} = -8$$

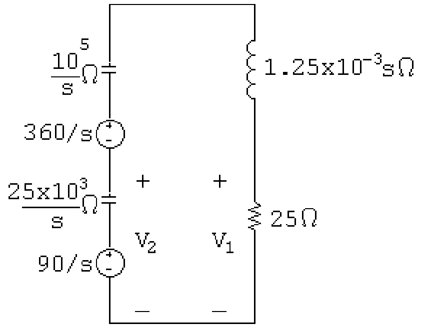
$$V_o = \frac{18}{s} + \frac{20}{s+4000} - \frac{8}{s+10{,}000}$$

$$v_o(t) = [18 + 20e^{-4000t} - 8e^{-10{,}000t}]u(t)\,\mathrm{V}$$

P 13.17 **[a]** For $t < 0$:



$$V_2 = \frac{10}{10+40}(450) = 90\,\mathrm{V}$$



[b] $$V_1 = \frac{25(450/s)}{(125{,}000/s) + 25 + 1.25 \times 10^{-3}s}$$

$$= \frac{9 \times 10^6}{s^2 + 20,000s + 10^8} = \frac{9 \times 10^6}{(s + 10{,}000)^2}$$

$$v_1(t) = (9 \times 10^6 te^{-10{,}000t})u(t)\,\text{V}$$

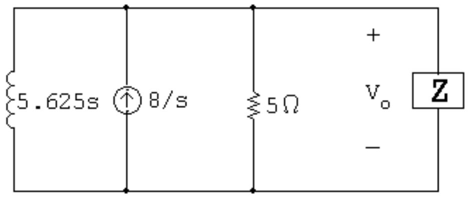
[c] $$V_2 = \frac{90}{s} - \frac{(25{,}000/s)(450/s)}{(125{,}000/s) + 1.25 \times 10^{-3}s + 25}$$

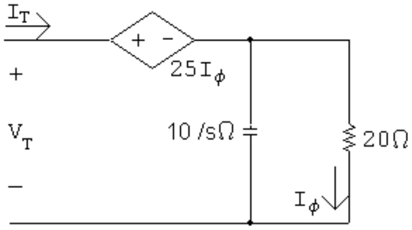
$$= \frac{90(s + 20{,}000)}{s^2 + 20{,}000s + 10^8}$$

$$= \frac{900{,}000}{(s + 10{,}000)^2} + \frac{90}{s + 10{,}000}$$

$$v_2(t) = [9 \times 10^5 te^{-10{,}000t} + 90e^{-10{,}000t}]u(t)\,\text{V}$$

P 13.18 **[a]** $i_L(0^-) = i_L(0^+) = \dfrac{24}{3} = 8\,\text{A}$ directed upward





$$V_T = 25I_\phi + \left[\frac{20(10/s)}{20 + (10/s)}\right] I_T = \frac{25I_T(10/s)}{20 + (10/s)} + \left(\frac{200}{10 + 20s}\right) I_T$$

$$\frac{V_T}{I_T} = Z = \frac{250 + 200}{20s + 10} = \frac{45}{2s + 1}$$

$$\frac{V_o}{5}+\frac{V_o(2s+1)}{45}+\frac{V_o}{5.625s}=\frac{8}{s}$$

$$\frac{[9s+(2s+1)s+8]V_o}{45s}=\frac{8}{s}$$

$$V_o[2s^2+10s+8]=360$$

$$V_o=\frac{360}{2s^2+10s+8}=\frac{180}{s^2+5s+4}$$

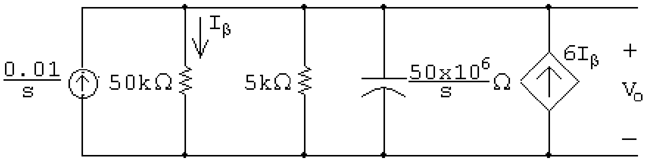
[b] $V_o=\dfrac{180}{(s+1)(s+4)}=\dfrac{K_1}{s+1}+\dfrac{K_2}{s+4}$

$$K_1=\frac{180}{3}=60; \qquad K_2=\frac{180}{-3}=-60$$

$$V_o=\frac{60}{s+1}-\frac{60}{s+4}$$

$$v_o(t)=[60e^{-t}-60e^{-4t}]u(t)\,\mathrm{V}$$

P 13.19 $v_C(0^-)=v_C(0^+)=0$



$$\frac{0.01}{s}=\frac{V_o}{50{,}000}+\frac{V_o}{5000}+\frac{V_os}{50\times10^6}-\frac{6V_o}{50{,}000}$$

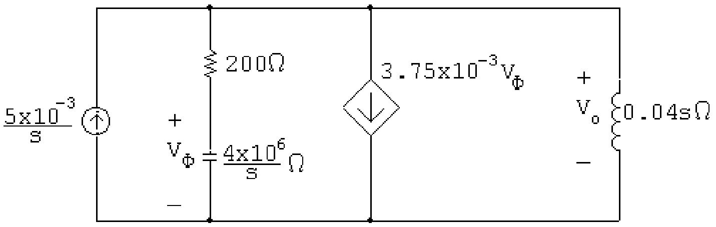
$$\frac{500\times10^3}{s}=(1000+10{,}000+s-6000)V_o$$

$$V_o=\frac{500\times10^3}{s(s+5000)}=\frac{K_1}{s}+\frac{K_2}{s+5000}$$

$$=\frac{100}{s}-\frac{100}{s+5000}$$

$$v_o(t)=[100-100e^{-5000t}]u(t)\,\mathrm{V}$$

P 13.20



$$\frac{5\times 10^{-3}}{s}=\frac{V_o}{200+4\times 10^6/s}+3.75\times 10^{-3}V_\phi+\frac{V_o}{0.04s}$$

$$V_\phi=\frac{4\times 10^6/s}{200+4\times 10^6/s}V_o=\frac{4\times 10^6 V_o}{200s+4\times 10^6}$$

$$\therefore \quad \frac{5\times 10^{-3}}{s}=\frac{V_o s}{200s+4\times 10^6}+\frac{15{,}000V_o}{200s+4\times 10^6}+\frac{25V_o}{s}$$

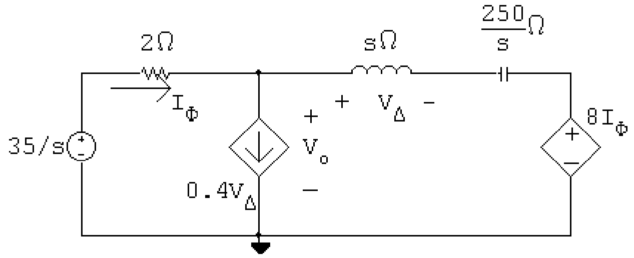
$$\therefore \ V_o=\frac{s+20{,}000}{s^2+20{,}000s+10^8}=\frac{K_1}{(s+10{,}000)^2}+\frac{K_2}{s+10{,}000}$$

$K_1 = 10{,}000; \qquad K_2 = 1$

$$V_o=\frac{10{,}000}{(s+10{,}000)^2}+\frac{1}{s+10{,}000}$$

$$v_o(t)=[10{,}000te^{-10{,}000t}+e^{-10{,}000t}]u(t)\,\mathrm{V}$$

P 13.21 [a]



$$\frac{V_o-35/s}{2}+0.4V_\Delta+\frac{V_o-8I_\phi}{s+(250/s)}=0$$

$$V_\Delta=\left[\frac{V_o-8I_\phi}{s+(250/s)}\right]s; \qquad I_\phi=\frac{(35/s)-V_o}{2}$$

Solving for V_o yields:

$$V_o=\frac{29.4s^2+56s+1750}{s(s^2+2s+50)}=\frac{29.4s^2+56s+1750}{s(s+1-j7)(s+1+j7)}$$

$$V_o = \frac{K_1}{s} + \frac{K_2}{s+1-j7} + \frac{K_2^*}{s+1+j7}$$

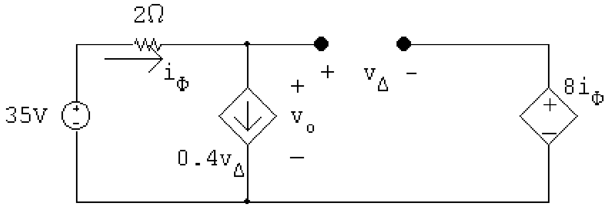
$$K_1 = \frac{29.4s^2 + 56s + 1750}{s^2 + 2s + 50}\bigg|_{s=0} = 35$$

$$K_2 = \frac{29.4s^2 + 56s + 1750}{s(s+1+j7)}\bigg|_{s=-1+j7}$$

$$= -2.8 + j0.6 = 2.86\underline{/167.91^\circ}$$

$$\therefore \quad v_o(t) = [35 + 5.73e^{-t}\cos(7t + 167.91^\circ)]u(t)\,\text{V}$$

[b] At $t = 0^+$ $\qquad v_o = 35 + 5.73\cos(167.91^\circ) = 29.4\,\text{V}$

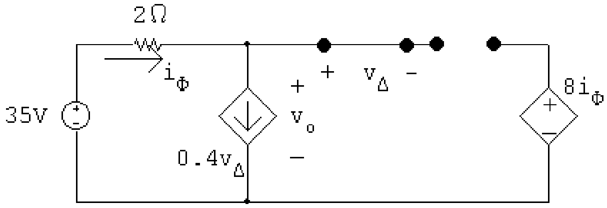


$$\frac{v_o - 35}{2} + 0.4v_\Delta = 0; \qquad v_o - 35 + 0.8v_\Delta = 0$$

$$v_o = v_\Delta + 8i_\phi = v_\Delta + 8(0.4v_\Delta) = 4.2\,\text{V}$$

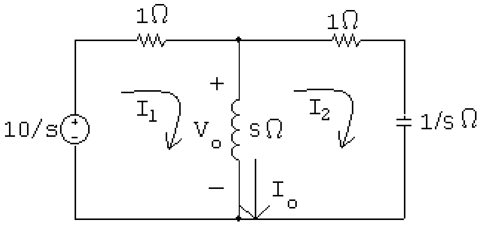
$$v_o + (0.8)\frac{v_o}{4.2} = 35; \qquad \therefore \quad v_o(0^+) = 29.4\,\text{V(checks)}$$

At $t = \infty$, the circuit is



$$v_\Delta = 0, \quad i_\phi = 0 \qquad \therefore \quad v_o = 35\,\text{V(checks)}$$

P 13.22 **[a]**



$$I_1 + s(I_1 - I_2) = \frac{10}{s} \qquad \text{and} \qquad I_2 + \frac{1}{s}I_2 + s(I_2 - I_1) = 0$$

Solving the second equation for I_1:

$$I_1 = \frac{s^2+s+1}{s^2}I_2$$

Substituting into the first equation and solving for I_2:

$$\left[(s+1)\frac{s^2+s+1}{s^2} - s\right] I_2 = \frac{10}{s}$$

$$\therefore \qquad I_2 = \frac{10s}{2s^2+2s+1}$$

$$\therefore \qquad I_1 = \frac{s^2+s+1}{s^2}\cdot\frac{10s}{2s^2+2s+1} = \frac{10(s^2+s+1)}{s(2s^2+2s+1)}$$

$$I_o = I_1 - I_2 = \frac{10(s^2+s+1)}{s(2s^2+2s+1)} - \frac{10s}{2s^2+2s+1} = \frac{5(s+1)}{s(s^2+s+0.5)}$$

$$= \frac{K_1}{s} + \frac{K_2}{s+0.5-j0.5} + \frac{K_2^*}{s+0.5+j0.5}$$

$$K_1 = 10; \qquad K_2 = 5\underline{/-180^\circ}$$

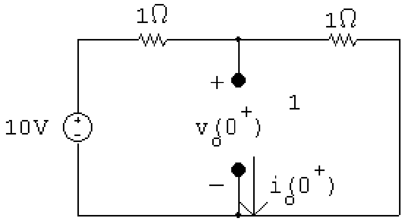
$$\therefore \qquad i_o(t) = [10 - 10e^{-0.5t}\cos 0.5t]u(t)\,\text{A}$$

[b] $$V_o = sI_o = \frac{5(s+1)}{s^2+s+0.5} = \frac{K_1}{s+0.5-j0.5} + \frac{K_1^*}{s+0.5+j0.5}$$

$$K_1 = 3.54\underline{/-45^\circ}$$

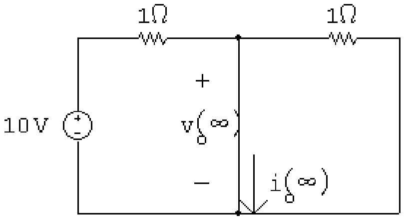
$$\therefore \qquad v_o(t) = 7.07e^{-0.5t}\cos(0.5t-45^\circ)u(t)\,\text{V}$$

[c] At $t = 0^+$ the circuit is



$$\therefore \;\; v_o(0^+) = 5\,\text{V} = 7.07\cos(-45^\circ); \qquad I_o(0^+) = 0$$

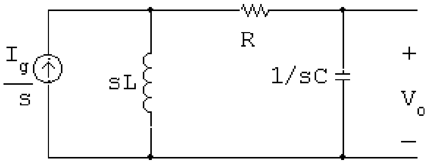
Both values agree with our solutions for v_o and i_o.
At $t = \infty$ the circuit is

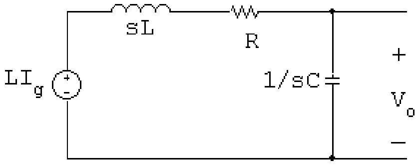


$$\therefore \;\; v_o(\infty) = 0; \qquad i_o(\infty) = 10\,\text{A}$$

Both values agree with our solutions for v_o and i_o.

P 13.23 **[a]**





$$V_o = \frac{(1/sC)(sL)(I_g/s)}{R + sL + (1/sC)} = \frac{I_g/C}{s^2 + (R/L)s + (1/LC)}$$

$$\frac{I_g}{C} = \frac{0.015}{0.1} = 0.15$$

$$\frac{R}{L} = 7; \qquad \frac{1}{LC} = 10$$

$$V_o = \frac{0.15}{s^2 + 7s + 10}$$

[b] $sV_o = \dfrac{0.15s}{s^2 + 7s + 10}$

$$\lim_{s\to 0} sV_o = 0; \qquad \therefore \quad v_o(\infty) = 0$$

$$\lim_{s\to\infty} sV_o = 0; \qquad \therefore \quad v_o(0^+) = 0$$

[c] $V_o = \dfrac{0.15}{(s+2)(s+5)} = \dfrac{0.05}{s+2} + \dfrac{-0.05}{s+5}$

$$v_o = [50e^{-2t} - 50e^{-5t}]u(t)\,\text{mV}$$

P 13.24 $I_L = \dfrac{I_g}{s} + \dfrac{V_o}{1/sC} = \dfrac{I_g}{s} - sCV_o$

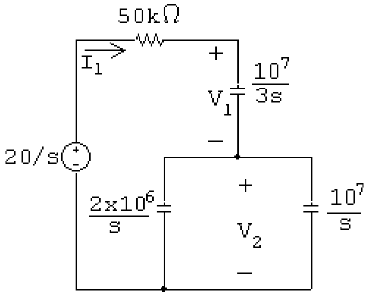
$$I_L = \frac{15}{s} - \frac{15s}{(s+2)(s+5)} = \frac{15}{s} - \left[\frac{-10}{s+2} + \frac{25}{s+5}\right]$$

$$i_L(t) = [15 + 10e^{-2t} - 25e^{-5t}]u(t)\,\text{mA}$$

Check:

$$i_L(0^+) = 0 \quad \text{(ok)}; \qquad i_L(\infty) = 15\,\text{mA} \quad \text{(ok)}$$

P 13.25 **[a]**



[b] $Z_{\text{eq}} = 50{,}000 + \dfrac{10^7}{3s} + \dfrac{20 \times 10^{12}/s^2}{12 \times 10^6/s}$

$$= 50{,}000 + \frac{10^7}{3s} + \frac{20 \times 10^{12}}{12 \times 10^6 s}$$

$$= \frac{100{,}000s + 10^7}{2s}$$

$$I_1 = \frac{20/s}{Z_{\text{eq}}} = \frac{0.4 \times 10^{-3}}{s + 100}$$

$$V_1 = \frac{10^7}{3s} I_1 = \frac{4000/3}{s(s + 100)}$$

$$V_2 = \frac{10^7}{6s} \cdot \frac{0.4 \times 10^{-4}}{s + 100} = \frac{2000/3}{s(s + 100)}$$

[c] $i_1(t) = 0.4e^{-100t}u(t)\,\text{mA}$

$$V_1 = \frac{40/3}{s} - \frac{40/3}{s + 100}; \qquad v_1(t) = (40/3)(1 - 1e^{-100t})u(t)\,\text{V}$$

$$V_2 = \frac{20/3}{s} - \frac{20/3}{s + 100}; \qquad v_2(t) = (20/3)(1 - 1e^{-100t})u(t)\,\text{V}$$

[d] $i_1(0^+) = 0.4\,\text{mA}$

$$i_1(0^+) = \frac{20}{50} \times 10^{-3} = 0.44\,\text{mA(checks)}$$

$$v_1(0^+) = 0; \qquad v_2(0^+) = 0\text{(checks)}$$

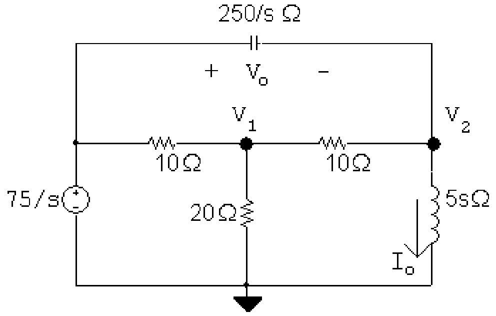
$$v_1(\infty) = 40/3\,\text{V}; \qquad v_2(\infty) = 20/3\,\text{V(checks)}$$

$$v_1(\infty) + v_2(\infty) = 20\,\text{V(checks)}$$

$$(0.3 \times 10^{-6})v_1(\infty) = 4\,\mu\text{C}$$

$$(0.6 \times 10^{-6})v_2(\infty) = 4\,\mu\text{C(checks)}$$

P 13.26 **[a]**



$$\frac{V_1 - 75/s}{10} + \frac{V_1}{20} + \frac{V_1 - V_2}{10} = 0$$

$$\frac{V_2}{5s} + \frac{V_2 - V_1}{10} + \frac{(V_2 - 75/s)s}{250} = 0$$

Thus,

$$5V_1 - 2V_2 = \frac{150}{s}$$

$$-25sV_1 + (s^2 + 25s + 50)V_2 = 75s$$

$$\Delta = \begin{vmatrix} 5 & -2 \\ -25s & s^2 + 25s + 50 \end{vmatrix} = 5(s + 5)(s + 10)$$

$$N_2 = \begin{vmatrix} 5 & 150/s \\ -25s & 75s \end{vmatrix} = 375(s + 10)$$

$$V_2 = \frac{N_2}{\Delta} = \frac{375(s + 10)}{5(s + 5)(s + 10)} = \frac{75}{s + 5}$$

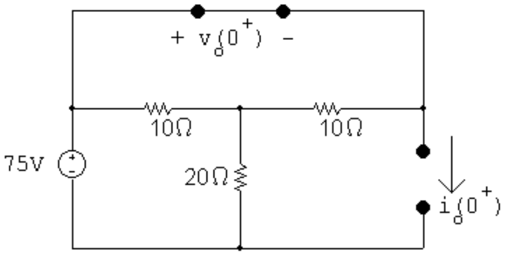
$$V_o = \frac{75}{s} - \frac{75}{s + 5} = \frac{375}{s(s + 5)}$$

$$I_o = \frac{V_2}{5s} = \frac{15}{s(s + 5)} = \frac{3}{s} - \frac{3}{s + 5}$$

[b] $v_o(t) = (75 - 75e^{-5t})u(t)\,\text{V}$

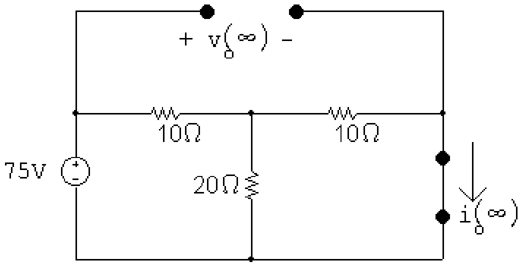
$i_o(t) = (3 - 3e^{-5t})u(t)\,\text{A}$

[c] At $t = 0^+$ the circuit is



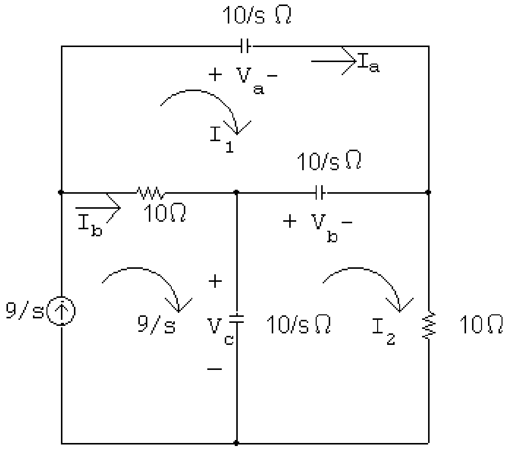
$v_o(0^+) = 0; \qquad i_o(0^+) = 0 \qquad$ Checks

At $t = \infty$ the circuit is



$$v_o(\infty) = 75\,\text{V}; \qquad i_o(\infty) = \frac{75}{10 + (200/30)} \cdot \frac{20}{30} = 3\,\text{A} \quad \text{Checks}$$

P 13.27 **[a]**



$$\frac{10}{s}I_1 + \frac{10}{s}(I_1 - I_2) + 10(I_1 - 9/s) = 0$$

$$\frac{10}{s}(I_2 - 9/s) + \frac{10}{s}(I_2 - I_1) + 10I_2 = 0$$

Simplifying,

$$(s + 2)I_1 - I_2 = 9$$

$$-I_1 + (s + 2)I_2 = \frac{9}{s}$$

$$\Delta = \begin{vmatrix} (s+2) & -1 \\ -1 & (s+2) \end{vmatrix} = s^2 + 4s + 3 = (s+1)(s+3)$$

$$N_1 = \begin{vmatrix} 9 & -1 \\ 9/s & (s+2) \end{vmatrix} = \frac{9s^2 + 18s + 9}{s} = \frac{9}{s}(s+1)^2$$

$$I_1 = \frac{N_1}{\Delta} = \frac{9}{s}\left[\frac{(s+1)^2}{(s+1)(s+3)}\right] = \frac{9(s+1)}{s(s+3)}$$

$$N_2 = \begin{vmatrix} (s+2) & 9 \\ -1 & 9/s \end{vmatrix} = \frac{18}{s}(s+1)$$

$$I_2 = \frac{N_2}{\Delta} = \frac{18(s+1)}{s(s+1)(s+3)} = \frac{18}{s(s+3)}$$

$$I_{\rm a} = I_1 = \frac{9(s+1)}{s(s+3)} = \frac{3}{s} + \frac{6}{s+3}$$

$$I_{\rm b} = \frac{9}{s} - I_1 = \frac{9}{s} - \frac{9(s+1)}{s(s+3)} = \frac{6}{s} - \frac{6}{s+3}$$

[b] $i_{\rm a}(t) = 3(1 + 2e^{-3t})u(t)\,\mathrm{A}$

$i_{\rm b}(t) = 6(1 - e^{-3t})u(t)\,\mathrm{A}$

[c] $$V_{\rm a} = \frac{10}{s}I_{\rm b} = \frac{10}{s}\left(\frac{3}{s} + \frac{6}{s+3}\right)$$

$$= \frac{30}{s^2} + \frac{60}{s(s+3)} = \frac{30}{s^2} + \frac{20}{s} - \frac{20}{s+3}$$

$$V_{\rm b} = \frac{10}{s}(I_2 - I_1) = \frac{10}{s}\left[\left(\frac{6}{s} - \frac{6}{s+3}\right) - \left(\frac{3}{s} + \frac{6}{s+3}\right)\right]$$

$$= \frac{10}{s}\left[\frac{3}{s} - \frac{12}{s+3}\right] = \frac{30}{s^2} - \frac{40}{s} + \frac{40}{s+3}$$

$$V_{\rm c} = \frac{10}{s}(9/s - I_2) = \frac{10}{s}\left(\frac{9}{s} - \frac{6}{s} + \frac{6}{s+3}\right)$$

$$= \frac{30}{s^2} + \frac{20}{s} - \frac{20}{s+3}$$

[d] $v_{\rm a}(t) = [30t + 20 - 20e^{-3t}]u(t)\,\mathrm{V}$

$v_{\rm b}(t) = [30t - 40 + 40e^{-3t}]u(t)\,\mathrm{V}$

$v_{\rm c}(t) = [30t + 20 - 20e^{-3t}]u(t)\,\mathrm{V}$

[e] Calculating the time when the capacitor voltage drop first reaches 1000 V:

$$30t + 20 - 20e^{-3t} = 1000 \quad \text{or} \quad 30t - 40 + 40e^{-3t} = 1000$$

Note that in either of these expressions the exponential tem is negligible when compared to the other terms. Thus,

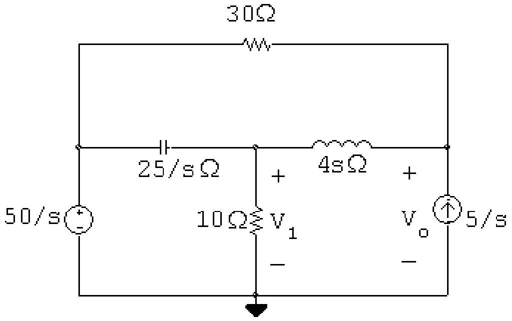
$$30t + 20 = 1000 \quad \text{or} \quad 30t - 40 = 1000$$

Thus,

$$t = \frac{980}{30} = 32.67\,\text{s} \quad \text{or} \quad t = \frac{1040}{30} = 34.67\,\text{s}$$

Therefore, the breakdown will occur at $t = 32.67$ s.

P 13.28 **[a]**



$$\frac{V_1}{10} + \frac{V_1 - 50/s}{25/s} + \frac{V_1 - V_o}{4s} = 0$$

$$\frac{-5}{s} + \frac{V_o - V_1}{4s} + \frac{V_o - 50/s}{30} = 0$$

Simplfying,

$$(4s^2 + 10s + 25)V_1 - 25V_o = 200s$$

$$-15V_1 + (2s + 15)V_o = 400$$

$$\Delta = \begin{vmatrix} (4s^2 + 10s + 25) & -25 \\ -15 & (2s + 15) \end{vmatrix} = 8s(s+5)^2$$

$$N_o = \begin{vmatrix} (4s^2 + 10s + 25) & 200s \\ -15 & 400 \end{vmatrix} = 200(8s^2 + 35s + 50)$$

$$V_o = \frac{N_o}{\Delta} = \frac{200(8s^2 + 35s + 50)}{8s(s+5)^2} = \frac{25(8s^2 + 35s + 50)}{s(s+5)^2} = \frac{K_1}{s} + \frac{K_2}{(s+5)^2} + \frac{K_3}{s+5}$$

$$K_1 = \frac{(25)(50)}{25} = 50; \quad K_2 = \frac{25(200 - 175 + 50)}{-5} = -375$$

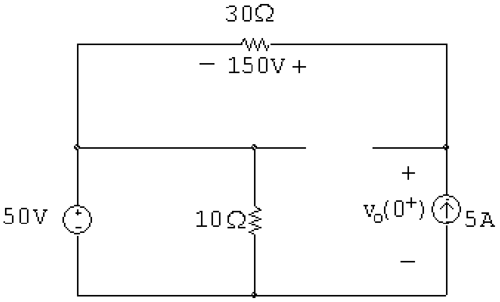
$$K_3 = 25\frac{d}{ds}\left[\frac{8s^2+35s+50}{s}\right]_{s=-5} = 25\left[\frac{s(16s+35)-(8s^2+35s+50)}{s^2}\right]_{s=-5}$$

$$= -5(-45) - 75 = 150$$

$$\therefore \quad V_o = \frac{50}{s} - \frac{375}{(s+5)^2} + \frac{150}{s+5}$$

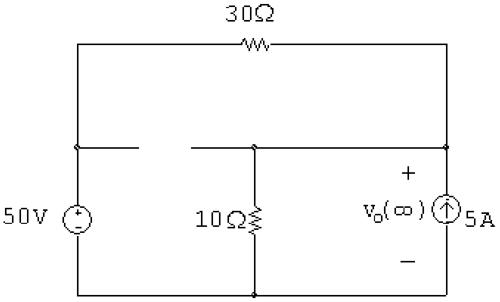
[b] $v_o(t) = [50 - 375te^{-5t} + 150e^{-5t}]u(t)\,\mathrm{V}$

[c] At $t = 0^+$:



$$v_o(0^+) = 50 + 150 = 200\,\mathrm{V}(\text{checks})$$

At $t = \infty$:

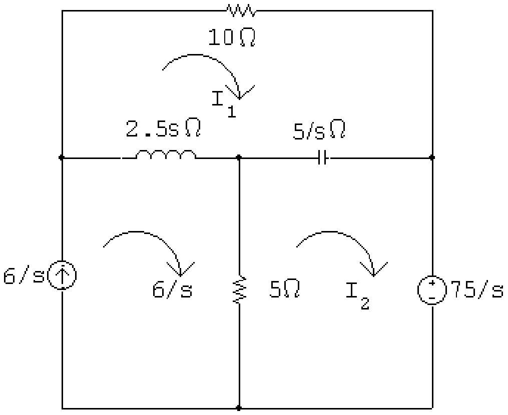


$$\frac{v_o(\infty)}{10} - 5 + \frac{v_o(\infty)-50}{30} = 0$$

$$\therefore \quad 3v_o(\infty) - 150 + v_o(\infty) - 50 = 0; \qquad \therefore \quad 4v_o(\infty) = 200$$

$$\therefore \qquad v_o(\infty) = 50\,\mathrm{V}(\text{checks})$$

P 13.29 **[a]**



$$0 = 2.5s(I_1 - 6/s) + \frac{5}{s}(I_1 - I_2) + 10I_1$$

$$\frac{-75}{s} = \frac{5}{s}(I_2 - I_1) + 5(I_2 - 6/s)$$

or

$$(s^2 + 4s + 2)I_1 - 2I_2 = 6s$$

$$-I_1 + (s + 1)I_2 = -9$$

$$\Delta = \begin{vmatrix} (s^2 + 4s + 2) & -2 \\ -1 & (s + 1) \end{vmatrix} = 5(s + 2)(s + 3)$$

$$N_1 = \begin{vmatrix} 6s & -2 \\ 9 & (s + 1) \end{vmatrix} = 6(s^2 + s - 3)$$

$$I_1 = \frac{N_1}{\Delta} = \frac{6(s^2 + s - 3)}{s(s + 2)(s + 3)}$$

$$N_2 = \begin{vmatrix} (s^2 + 4s + 2) & 6s \\ -1 & 9 \end{vmatrix} = -9s^2 - 30s - 18$$

$$I_2 = \frac{N_2}{\Delta} = \frac{-9s^2 - 30s - 18}{s(s + 2)(s + 3)}$$

[b] $sI_1 = \dfrac{6(s^2 + s - 3)}{(s + 2)(s + 3)}$

$$\lim_{s\to\infty} sI_1 = i_1(0^+) = 6\,\text{A}; \qquad \lim_{s\to 0} sI_1 = i_1(\infty) = -3\,\text{A}$$

$$sI_2 = \frac{-9s^2 - 30s - 18}{(s + 2)(s + 3)}$$

$$\lim_{s\to\infty} sI_2 = i_2(0^+) = -9\,\text{A}; \qquad \lim_{s\to 0} sI_2 = i_2(\infty) = -3\,\text{A}$$

[c] $$I_1 = \frac{6(s^2+s-3)}{s(s+2)(s+3)} = \frac{K_1}{s} + \frac{K_2}{s+2} + \frac{K_3}{s+3}$$

$$K_1 = \frac{6(-3)}{6} = -3; \qquad K_2 = \frac{6(4-2-3)}{(-2)(1)} = 3$$

$$K_3 = \frac{6(9-3-3)}{(-3)(-1)} = 6$$

$$i_1(t) = [-3 + 3e^{-2t} + 6e^{-3t}]u(t)\,\text{A}$$

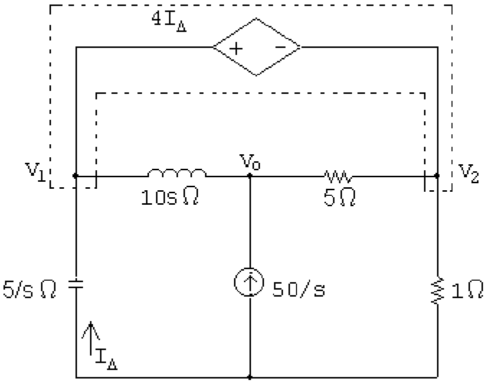
$$I_2 = \frac{-9s^2-30s-18}{s(s+2)(s+3)} = \frac{K_1}{s} + \frac{K_2}{s+2} + \frac{K_3}{s+3}$$

$$K_1 = \frac{-18}{6} = -3; \qquad K_2 = \frac{-36+60-18}{(-2)(1)} = -3$$

$$K_3 = \frac{-81+90-18}{(-3)(-1)} = -3$$

$$i_2(t) = [-3 - 3e^{-2t} - 3e^{-3t}]u(t)\,\text{A}$$

P 13.30 [a]



At V_o:

$$\frac{V_o - V_1}{10s} - \frac{50}{s} + \frac{V_o - V_2}{5} = 0$$

$$\therefore \quad V_o(2s+1) - 2sV_2 - V_1 = 500$$

Supernode:

$$\frac{V_1 s}{5} + \frac{V_1 - V_o}{10s} + \frac{V_2}{1} + \frac{V_2 - V_1}{5} = 0$$

$$\therefore \quad -V_o(2s+1) + 12sV_2 + (2s^2+1)V_1 = 0$$

Constraint:

$$V_1 - V_2 = 4I_\Delta = 4\left(-\frac{V_1 s}{5}\right)$$

$$\therefore \qquad V_2 = (0.8s + 1)V_1$$

Simplifying:

$$V_o(2s + 1) - V_1(1.6s^2 + 2s + 1) = 500$$

$$-V_o(2s + 1) - V_1(11.6s^2 + 12s + 1) = 0$$

$$\Delta = \begin{vmatrix} 2s+1 & -(1.6s^2 + 2s + 1) \\ -(2s+1) & (11.6s^2 + 12s + 1) \end{vmatrix} = 20(s^2 + 1.5s + 0.5)$$

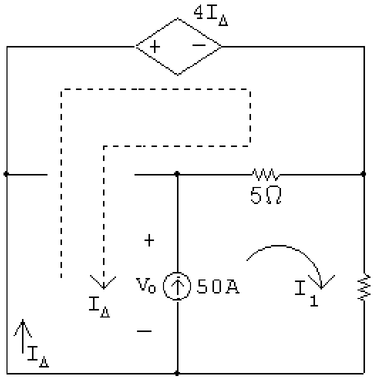
$$N_o = \begin{vmatrix} 500 & -(1.6s^2 + 2s + 1) \\ 0 & (11.6s^2 + 12s + 1) \end{vmatrix} = 500(11.6s^2 + 12s + 1)$$

$$V_o = \frac{N_o}{\Delta} = \frac{25(11.6s^2 + 12s + 1)}{s(s + 0.5)(s + 1)}$$

[b] $v_o(0^+) = \lim_{s\to\infty} sV_o = 25(11.6) = 290\,\text{V}$

$$v_o(\infty) = \lim_{s\to 0} sV_o = \frac{25}{0.5} = 50\,\text{V}$$

[c] At $t = 0^+$ the circuit is



$$4I_\Delta + 1I_1 = 0; \quad I_1 - I_\Delta = 50$$

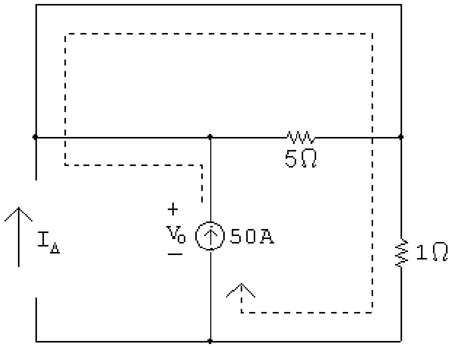
$$\therefore \quad 4I_\phi + 50 + I_\Delta = 0; \qquad 5I_\Delta = -50$$

$$\therefore \quad I_\Delta = I_o(0^+) = -10\,\text{A}$$

Also $I_1 = 50 - 10 = 40\,\text{A}$

$$V_o(0^+) = 5(I_1 - I_\Delta) + 1I_1 = 6I_1 - 5I_\Delta = 240 - 5(-10) = 290\,\text{V (checks)}$$

At $t = \infty$ the circuit is



$$V_o(\infty) = 50(1) = 50\,\text{V(checks)}$$

[d] $V_o = \dfrac{25(11.6s^2 + 12s + 1)}{s(s + 0.5)(s + 1)} = \dfrac{K_1}{s} + \dfrac{K_2}{s + 0.5} + \dfrac{K_3}{s + 1}$

$$K_1 = \frac{25}{(0.5)(1)} = 50; \qquad K_2 = \frac{-52.5}{(-0.5)(0.5)} = 210$$

$$K_3 = \frac{15}{(-1)(-0.5)} = 30$$

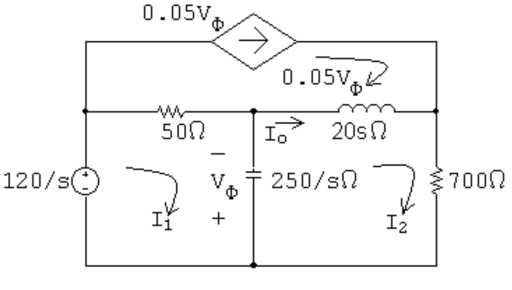
$$V_o = \frac{50}{s} + \frac{210}{s + 0.5} + \frac{30}{s + 1}$$

$$v_o(t) = (50 + 210e^{-0.5t} + 30e^{-t})u(t)\,\text{V}$$

$$v_o(\infty) = 50\,\text{V(checks)}$$

$$v_o(0^+) = 50 + 210 + 30 = 290\,\text{V(checks)}$$

P 13.31 **[a]**



$$\frac{120}{s} = 50(I_1 - 0.05V_\phi) + \frac{250}{s}(I_1 - I_2)$$

$$\frac{250}{s} = 50I_1 - 2.5\left(\frac{250}{s}\right)(I_2 - I_1) + \frac{250}{s}I_1 - \frac{250}{s}I_2$$

Simplfying,

$$(50s + 875)I_1 - 875I_2 = 120$$

$$250(s-1)I_1 + (20s^2 + 450s + 250)I_2 = 0$$

$$\Delta = \begin{vmatrix} (50s+875) & -875 \\ 250(s-1) & (20s^2+450s+250) \end{vmatrix} = 1000s(s^2+40s+625)$$

$$N_1 = \begin{vmatrix} 120 & -875 \\ 0 & (20s^2+450s+250) \end{vmatrix} = 1200(2s^2+45s+25)$$

$$N_2 = \begin{vmatrix} (50s+875) & 120 \\ 250(s-1) & 0 \end{vmatrix} = -30{,}000(s-1)$$

$$I_1 = \frac{N_1}{\Delta} = \frac{1200(2s^2+45s+25)}{s(s^2+40s+625)}$$

$$I_2 = \frac{N_2}{\Delta} = \frac{-30{,}000(s-1)}{s(s^2+40s+625)}$$

$$I_o = I_2 - 0.05V_\phi = I_2 - 0.05\left[\frac{250}{s}(I_2 - I_1)\right]$$

$$I_2 - I_1 = \frac{-2400(s+35)}{s(s^2+40s+625)}$$

$$\frac{250}{s}(I_2 - I_1) = \frac{-600{,}000(s+35)}{s(s^2+40s+625)}$$

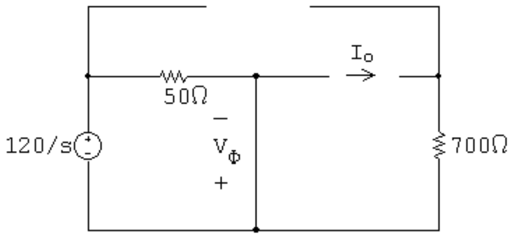
$$\therefore \quad I_o = \frac{-30{,}000(s-1)}{s(s^2+40s+625)} + \frac{30{,}000(s+35)}{s(s^2+40s+625)} = \frac{1080}{s(s^2+40s+625)}$$

[b] $sI_o = \dfrac{1080}{(s^2+40s+625)}$

$$i_o(0^+) = \lim_{s\to\infty} sI_o = 0$$

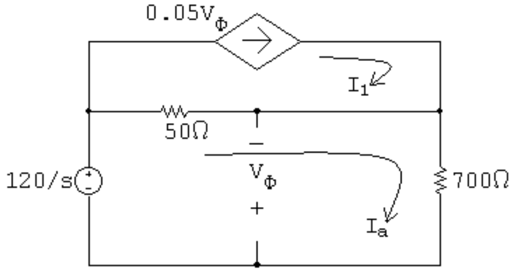
$$i_o(\infty) = \lim_{s\to 0} sV_o = \frac{1080}{625} = 1728\,\text{mA}$$

[c] At $t = 0^+$ the circuit is



$$i(0^+) = 0 \text{ (checks)}$$

At $t = \infty$ the circuit is



$$120 = 50(i_\mathrm{a} - i_1) + 700i_\mathrm{a}$$

$$= 50(i_\mathrm{a} - 0.05v_\phi) + 700i_\mathrm{a} = 750i_\mathrm{a} - 2.5v\phi$$

$$v_\phi = -700i_\mathrm{a} \quad \therefore \quad 120 = 750i_\mathrm{a} + 1750i_\mathrm{a} = 2500i_\mathrm{a}$$

$$i_\mathrm{a} = \frac{120}{2500} = 48\,\mathrm{mA}$$

$$v_\phi = -700i_\mathrm{a} = -33.60\,\mathrm{V}$$

$$i_o(\infty) = 48 \times 10^{-3} - 0.05(-33.60) = 48 \times 10^{-3} + 1.68 = 1728\,\mathrm{mA} \text{ (checks)}$$

[d] $I_o = \dfrac{1080}{s(s^2 + 40s + 625)} = \dfrac{K_1}{s} + \dfrac{K_2}{s + 20 - j15} + \dfrac{K_2^*}{s + 20 + j15}$

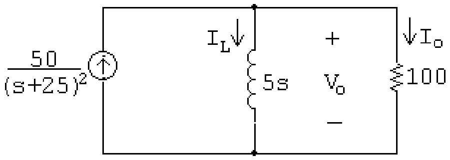
$$K_1 = \frac{1080}{625} = 1.728$$

$$K_2 = \frac{1080}{(-20 + j15)(j30)} = 1.44\underline{/126.87^\circ}$$

$$i_o(t) = [1728 + 2880e^{-20t}\cos(15t + 126.87^\circ)]u(t)\,\mathrm{mA}$$

Check: $i_o(0^+) = 0\,\mathrm{mA}$; $i_o(\infty) = 1728\,\mathrm{mA}$

P 13.32 **[a]**



$$100\|5s = \frac{500s}{5s + 100} = \frac{100s}{s + 20}$$

$$V_o = \frac{100s}{s + 20}\left[\frac{50}{(s + 25)^2}\right] = \frac{5000s}{(s + 20)(s + 25)^2}$$

$$I_o = \frac{V_o}{100} = \frac{50s}{(s + 20)(s + 25)^2}$$

$$I_L = \frac{V_o}{5s} = \frac{1000}{(s + 20)(s + 25)^2}$$

[b] $V_o = \frac{K_1}{s+20} + \frac{K_2}{(s+25)^2} + \frac{K_3}{s+25}$

$$K_1 = \left.\frac{5000s}{(s+25)^2}\right|_{s=-20} = -4000$$

$$K_2 = \left.\frac{5000s}{(s+20)}\right|_{s=-25} = 25{,}000$$

$$K_3 = \frac{d}{ds}\left[\frac{5000s}{s+20}\right]_{s=-25} = \left[\frac{5000}{s+20} - \frac{5000s}{(s+20)^2}\right]_{s=-25} = 4000$$

$$v_o(t) = [-4000e^{-20t} + 25{,}000te^{-25t} + 4000e^{-25t}]u(t)\,\mathrm{V}$$

$$I_o = \frac{K_1}{s+20} + \frac{K_2}{(s+25)^2} + \frac{K_3}{s+25}$$

$$K_1 = \left.\frac{50s}{(s+25)^2}\right|_{s=-20} = -40$$

$$K_2 = \left.\frac{50s}{(s+20)}\right|_{s=-25} = 250$$

$$K_3 = \frac{d}{ds}\left[\frac{50s}{s+20}\right]_{s=-25} = \left[\frac{50}{s+20} - \frac{50s}{(s+20)^2}\right]_{s=-25} = 40$$

$$i_o(t) = [-40e^{-20t} + 250te^{-25t} + 40e^{-25t}]u(t)\,\mathrm{V}$$

$$I_L = \frac{K_1}{s+20} + \frac{K_2}{(s+25)^2} + \frac{K_3}{s+25}$$

$$K_1 = \left.\frac{1000}{(s+25)^2}\right|_{s=-20} = 40$$

$$K_2 = \left.\frac{1000}{(s+20)}\right|_{s=-25} = -200$$

$$K_3 = \frac{d}{ds}\left[\frac{1000}{s+20}\right]_{s=-25} = \left[-\frac{1000}{(s+20)^2}\right]_{s=-25} = -40$$

$$i_L(t) = [40e^{-20t} - 200te^{-25t} - 40e^{-25t}]u(t)\,\mathrm{V}$$

P 13.33 $v_C = 12 \times 10^5 te^{-5000t}\,\mathrm{V}, \quad C = 5\,\mu\mathrm{F};$ therefore

$$i_C = C\left(\frac{dv_C}{dt}\right) = 6e^{-5000t}(1 - 5000t)\,\mathrm{A}$$

$i_C > 0$ when $1 > 5000t$ or $i_C > 0$ when $0 < t < 200\,\mu\mathrm{s}$

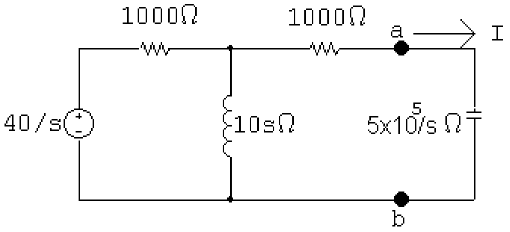
$$\text{and} \quad i_C < 0 \quad \text{when} \quad t > 200\,\mu\text{s}$$

$$i_C = 0 \quad \text{when} \quad 1 - 5000t = 0, \quad \text{or} \quad t = 200\,\mu\text{s}$$

$$\frac{dv_C}{dt} = 12 \times 10^5 e^{-5000t}[1 - 5000t]$$

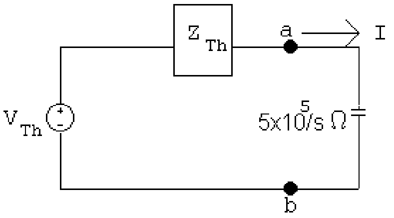
$$\therefore \quad i_C = 0 \quad \text{when} \quad \frac{dv_C}{dt} = 0$$

P 13.34



$$V_{\text{Th}} = \frac{10s}{10s + 1000} \cdot \frac{40}{s} = \frac{400}{10s + 1000} = \frac{40}{s + 100}$$

$$Z_{\text{Th}} = 1000 + 1000\|10s = 1000 + \frac{10{,}000s}{10s + 1000} = \frac{2000(s + 50)}{s + 100}$$



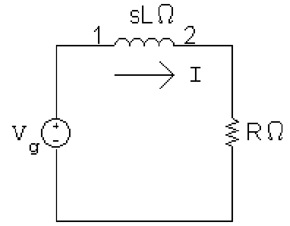
$$I = \frac{40/(s + 100)}{(5 \times 10^5)/s + 2000(s + 50)/(s + 100)} = \frac{40s}{2000s^2 + 600{,}000s + 5 \times 10^7}$$

$$= \frac{0.02s}{s^2 + 300s + 25{,}000} = \frac{K_1}{s + 150 - j50} + \frac{K_1^*}{s + 150 + j50}$$

$$K_1 = \frac{0.02s}{s + 150 + j50}\bigg|_{s=-150+j50} = 31.62 \times 10^{-3}\underline{/71.57^\circ}$$

$$i(t) = 63.25e^{-150t}\cos(50t + 71.57^\circ)u(t)\,\text{mA}$$

P 13.35 **[a]** The s-domain equivalent circuit is



$$I = \frac{V_g}{R+sL} = \frac{V_g/L}{s+(R/L)}, \qquad V_g = \frac{V_m(\omega\cos\phi + s\sin\phi)}{s^2+\omega^2}$$

$$I = \frac{K_0}{s+R/L} + \frac{K_1}{s-j\omega} + \frac{K_1^*}{s+j\omega}$$

$$K_0 = \frac{V_m(\omega L\cos\phi - R\sin\phi)}{R^2+\omega^2L^2}, \qquad K_1 = \frac{V_m\underline{/\phi - 90^\circ - \theta(\omega)}}{2\sqrt{R^2+\omega^2L^2}}$$

where $\tan\theta(\omega) = \omega L/R$. Therefore, we have

$$i(t) = \frac{V_m(\omega L\cos\phi - R\sin\phi)}{R^2+\omega^2L^2}e^{-(R/L)t} + \frac{V_m\sin[\omega t + \phi - \theta(\omega)]}{\sqrt{R^2+\omega^2L^2}}$$

[b] $$i_{ss}(t) = \frac{V_m}{\sqrt{R^2+\omega^2L^2}}\sin[\omega t + \phi - \theta(\omega)]$$

[c] $$i_{\text{tr}} = \frac{V_m(\omega L\cos\phi - R\sin\phi)}{R^2+\omega^2L^2}e^{-(R/L)t}$$

[d] $$\mathbf{I} = \frac{\mathbf{V}_g}{R+j\omega L}, \qquad \mathbf{V}_g = V_m\underline{/\phi - 90^\circ}$$

Therefore $$\mathbf{I} = \frac{V_m\underline{/\phi - 90^\circ}}{\sqrt{R^2+\omega^2L^2}\underline{/\theta(\omega)}} = \frac{V_m}{\sqrt{R^2+\omega^2L^2}}\underline{/\phi - \theta(\omega) - 90^\circ}$$

Therefore $$i_{ss} = \frac{V_m}{\sqrt{R^2+\omega^2L^2}}\sin[\omega t + \phi - \theta(\omega)]$$

[e] The transient component vanishes when

$$\omega L\cos\phi = R\sin\phi \quad \text{or} \quad \tan\phi = \frac{\omega L}{R} \quad \text{or} \quad \phi = \theta(\omega)$$

P 13.36 **[a]** $$W = \frac{1}{2}L_1i_1^2 + \frac{1}{2}L_2i_2^2 + Mi_1i_2$$

$$W = 4(15)^2 + 9(100) + 150(6) = 2700\,\text{J}$$

[b] $$120i_1 + 8\frac{di_1}{dt} - 6\frac{di_2}{dt} = 0$$

$$270i_2 + 18\frac{di_2}{dt} - 6\frac{di_1}{dt} = 0$$

Laplace transform the equations to get

$120I_1 + 8(sI_1 - 15) - 6(sI_2 + 10) = 0$

$270I_2 + 18(sI_2 + 10) - 6(sI_1 - 15) = 0$

In standard form,

$(8s + 120)I_1 - 6sI_2 = 180$

$-6sI_1 + (18s + 270)I_2 = -270$

$$\Delta = \begin{vmatrix} 8s + 120 & -6s \\ -6s & 18s + 270 \end{vmatrix} = 108(s + 10)(s + 30)$$

$$N_1 = \begin{vmatrix} 180 & -6s \\ -270 & 18s + 270 \end{vmatrix} = 1620(s + 30)$$

$$N_2 = \begin{vmatrix} 8s + 120 & 180 \\ -6s & -270 \end{vmatrix} = -1080(s + 30)$$

$$I_1 = \frac{N_1}{\Delta} = \frac{1620(s + 30)}{108(s + 10)(s + 30)} = \frac{15}{s + 10}$$

$$I_2 = \frac{N_2}{\Delta} = \frac{-1080(s + 30)}{108(s + 10)(s + 30)} = \frac{-10}{s + 10}$$

[c] $i_1(t) = 15e^{-10t}u(t)\,\text{A}; \qquad i_2(t) = -10e^{-10t}u(t)\,\text{A}$

[d] $W_{120\Omega} = \int_0^\infty (225e^{-20t})(120)\,dt = 27{,}000\frac{e^{-20t}}{-20}\Big|_0^\infty = 1350\,\text{J}$

$W_{270\Omega} = \int_0^\infty (100e^{-20t})(270)\,dt = 27{,}000\frac{e^{-20t}}{-20}\Big|_0^\infty = 1350\,\text{J}$

$W_{120\Omega} + W_{270\Omega} = 2700\,\text{J}$

[e] $W = \frac{1}{2}L_1i_1^2 + \frac{1}{2}L_2i_2^2 + Mi_1i_2 = 900 + 900 - 900 = 900\,\text{J}$

With the dot reversed the s-domain equations are

$(8s + 120)I_1 + 6sI_2 = 60$

$6sI_1 + (18s + 270)I_2 = -90$

As before, $\Delta = 108(s + 10)(s + 30)$. Now,

$$N_1 = \begin{vmatrix} 60 & -6s \\ -90 & 18s + 270 \end{vmatrix} = 1620(s + 10)$$

$$N_2 = \begin{vmatrix} 8s + 120 & 60 \\ -6s & -90 \end{vmatrix} = -1080(s + 10)$$

$$I_1 = \frac{N_1}{\Delta} = \frac{15}{s+30}; \qquad I_2 = \frac{N_2}{\Delta} = \frac{-10}{s+30}$$

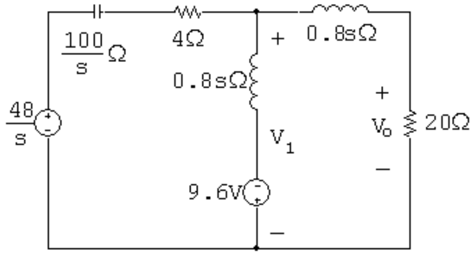
$$i_1(t) = 15e^{-30t}u(t)\,\text{A}; \qquad i_2(t) = -10e^{-30t}u(t)\,\text{A}$$

$$W_{270\Omega} = \int_0^\infty (100e^{-60t})(270)\,dt = 450\,\text{J}$$

$$W_{120\Omega} = \int_0^\infty (225e^{-60t})(120)\,dt = 450\,\text{J}$$

$$W_{120\Omega} + W_{270\Omega} = 900\,\text{J}$$

P 13.37 The s-domain equivalent circuit is



$$\frac{V_1 - 48/s}{4 + (100/s)} + \frac{V_1 + 9.6}{0.8s} + \frac{V_1}{0.8s + 20} = 0$$

$$V_1 = \frac{-1200}{s^2 + 10s + 125}$$

$$V_o = \frac{20}{0.8s + 20}V_1 = \frac{-30{,}000}{(s+25)(s+5-j10)(s+5+j10)}$$

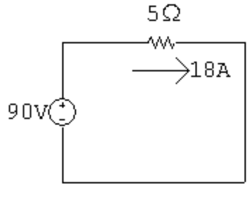
$$= \frac{K_1}{s+25} + \frac{K_2}{s+5-j10} + \frac{K_2^*}{s+5+j10}$$

$$K_1 = \left.\frac{-30{,}000}{s^2+10s+125}\right|_{s=-25} = -60$$

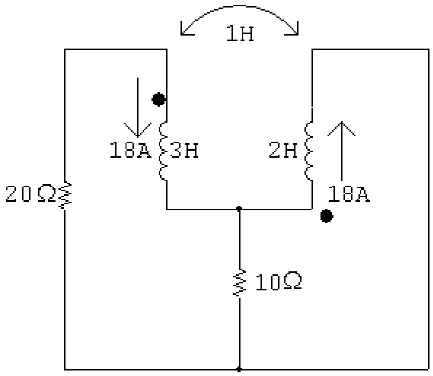
$$K_2 = \left.\frac{-30{,}000}{(s+25)(s+5+j10)}\right|_{s=-5+j10} = 67.08\underline{/63.43^\circ}$$

$$v_o(t) = [-60e^{-25t} + 134.16e^{-5t}\cos(10t + 63.43^\circ)]u(t)\,\text{V}$$

P 13.38 For $t < 0$:



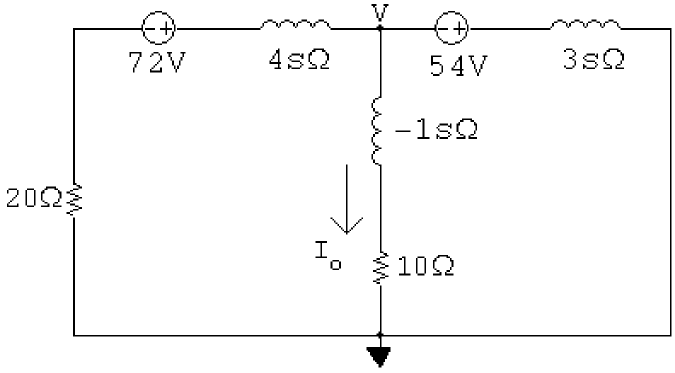
For $t > 0^+$:



Note that because of the dot locations on the coils, the sign of the mutual inductance is negative! (See Example C.1 in Appendix C.)

$L_1 - M = 3 + 1 = 4\,\text{H}; \qquad L_2 - M = 2 + 1 = 3\,\text{H}$

$18 \times 4 = 72; \qquad 18 \times 3 = 54$



$$\frac{V - 72}{4s + 20} + \frac{V}{-s + 10} + \frac{V + 54}{3s} = 0$$

$$V\left(\frac{1}{4s + 20} + \frac{1}{-s + 10} + \frac{1}{3s}\right) = \frac{72}{4s + 20} - \frac{54}{3s}$$

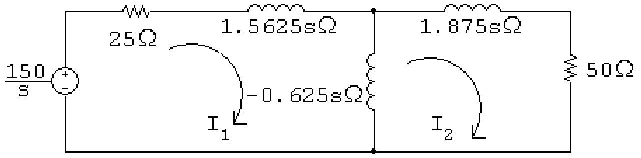
$$V\left[\frac{3s(-s + 10) + 3s(4s + 20) + (4s + 20)(-s + 10)}{3s(-s + 10)(4s + 20)}\right] = \frac{72(3s) - 54(4s + 20)}{3s(4s + 20)}$$

$$V = \frac{[72(3s) - 54(4s + 20)](-s + 10)}{5s^2 + 110s + 200}$$

$$I_o = \frac{V}{-s + 10} = \frac{-108}{(s + 2)(s + 20)} = \frac{-1.2}{s + 2} + \frac{1.2}{s + 20}$$

$$i_o(t) = 1.2[e^{-20t} - e^{-2t}]u(t)\,\text{A}$$

P 13.39 **[a]**



$$\frac{150}{s} = (25 + 0.9375s)I_1 + 0.625sI_2$$

$$0 = 0.625sI_2 + (50 - 1.25s)I_1$$

$$\Delta = \begin{vmatrix} 0.9375s + 25 & 0.625s \\ 0.625s & 1.25s + 50 \end{vmatrix} = 0.78125(s^2 + 100s + 1600)$$

$$N_1 = \begin{vmatrix} 150 & 0.625s \\ 0 & 1.25s + 50 \end{vmatrix} = \frac{187.5(s + 40)}{s}$$

$$I_1 = \frac{N_1}{\Delta} = \frac{240(s + 40)}{s(s + 20)(s + 80)}$$

[b] $$sI_1 = \frac{240(s + 40)}{(s + 20)(s + 80)}$$

$$\lim_{s\to 0} sI_1 = i_1(\infty) = 6\,\text{A}$$

$$\lim_{s\to\infty} sI_1 = i_1(0) = 0$$

[c] $$I_1 = \frac{K_1}{s} + \frac{K_2}{s + 20} + \frac{K_3}{s + 80}$$

$$K_1 = 6; \qquad K_2 = -4; \qquad K_3 = -2$$

$$i_1(t) = (6 - 4e^{-20t} - 2e^{-80t})u(t)\,\text{A}$$

P 13.40 **[a]** From the solution to Problem 13.39 we have

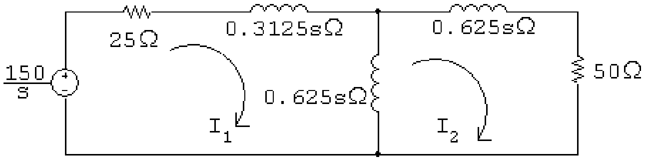
$$N_2 = \begin{vmatrix} 0.9375s + 25 & 150 \\ 0.625s & 0 \end{vmatrix} = -93.75$$

$$I_2 = \frac{-120}{(s + 20)(s + 80)} = \frac{K_1}{s + 20} + \frac{K_2}{s + 80}$$

$$K_1 = \frac{-120}{60} = -2; \qquad K_2 = \frac{-120}{-60} = 2$$

$$i_2(t) = (-2e^{-20t} + 2e^{-80t})u(t)\,\text{A}$$

[b] Reversing the dot on the 1.25 H coil will reverse the sign of M, thus the circuit becomes



The two simulanteous equations are

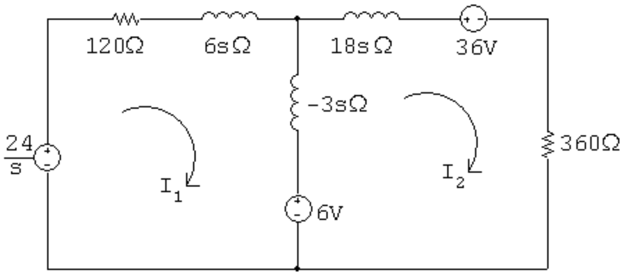
$$\frac{150}{s} = (25 + 0.9375s)I_1 - 0.625sI_2$$

$$0 = -0.625sI_1 + (1.25s + 50)I_2$$

When these equations are compared to those derived in Problem 13.39 we see the only difference is the algebraic sign of the 0.625s term. Thus reversing the dot will have no effect on I_1 and will reverse the sign of I_2. Hence,

$$i_2(t) = (2e^{-20t} - 2e^{-80t})u(t)\,\text{A}$$

P 13.41 **[a]** s-domain equivalent circuit is



Note: $\quad i_2(0^+) = -\frac{20}{10} = -2\text{ A}$

[b] $\frac{24}{s} = (120 + 3s)I_1 + 3sI_2 + 6$

$$0 = -6 + 3sI_1 + (360 + 15s)I_2 + 36$$

In standard form,

$$(s + 40)I_1 + sI_2 = (8/s) - 2$$

$$sI_1 + (5s + 120)I_2 = -10$$

$$\Delta = \begin{vmatrix} s + 40 & s \\ s & 5s + 120 \end{vmatrix} = 4(s + 20)(s + 60)$$

$$N_1 = \begin{vmatrix} (8/s) - 2 & s \\ -10 & 5s + 120 \end{vmatrix} = \frac{-200(s - 4.8)}{s}$$

$$I_1 = \frac{N_1}{\Delta} = \frac{-50(s-4.8)}{s(s+20)(s+60)}$$

[c] $sI_1 = \dfrac{-50(s-4.8)}{(s+20)(s+60)}$

$$\lim_{s\to\infty} sI_1 = i_1(0^+) = 0\,\text{A}$$

$$\lim_{s\to 0} sI_1 = i_1(\infty) = \frac{(-50)(-4.8)}{(20)(60)} = 0.2\,\text{A}$$

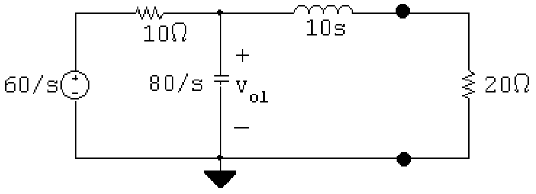
[d] $I_1 = \dfrac{K_1}{s} + \dfrac{K_2}{s+20} + \dfrac{K_3}{s+60}$

$$K_1 = \frac{240}{1200} = 0.2; \qquad K_2 = \frac{-50(-20)+240}{(-20)(40)} = -1.55$$

$$K_3 = \frac{-50(-60)+240}{(-60)(-40)} = 1.35$$

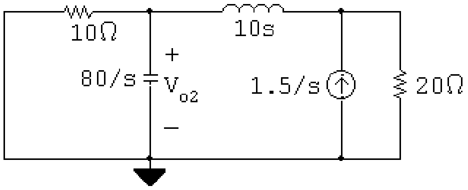
$$i_1(t) = [0.2 - 1.55e^{-20t} + 1.35e^{-60t}]u(t)\,\text{A}$$

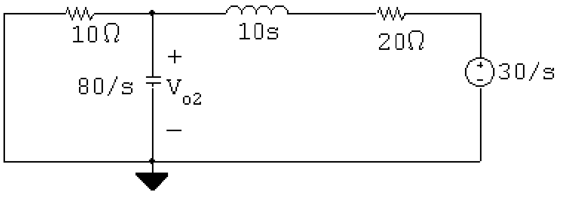
P 13.42 **[a]** Voltage source acting alone:



$$\frac{V_{o1} - 60/s}{10} + \frac{V_{01}s}{80} + \frac{V_{01}}{20+10s} = 0$$

$$\therefore\quad V_{01} = \frac{480(s+2)}{s(s+4)(s+6)}$$





$$\frac{V_{o2}}{10} + \frac{V_{02}s}{80} + \frac{V_{02} - 30/s}{10(s+2)} = 0$$

$$\therefore \quad V_{02} = \frac{240}{s(s+4)(s+6)}$$

$$V_o = V_{o1} + V_{o2} = \frac{480(s+2)+240}{s(s+4)(s+6)} = \frac{480(s+2.5)}{s(s+4)(s+6)}$$

[b] $V_o = \frac{K_1}{s} + \frac{K_2}{s+4} + \frac{K_3}{s+6}$

$$K_1 = \frac{(480)(2.5)}{(4)(6)} = 50; \qquad K_2 = \frac{480(-1.5)}{(-4)(2)} = 90; \qquad K_3 = \frac{480(-3.5)}{(-6)(-2)} = -140$$

$$v_o(t) = [50 + 90e^{-4t} - 140e^{-6t}]u(t)\,\mathrm{V}$$

P 13.43 $\Delta = \begin{vmatrix} Y_{11} & Y_{12} \\ Y_{12} & Y_{22} \end{vmatrix} = Y_{11}Y_{22} - Y_{12}^2$

$$N_2 = \begin{vmatrix} Y_{11} & [(V_g/R_1) + \gamma C - (\rho/s)] \\ Y_{12} & (I_g - \gamma C) \end{vmatrix}$$

$$V_2 = \frac{N_2}{\Delta}$$

Substitution and simplification lead directly to Eq. 13.90.

P 13.44 **[a]** $V_o = -\frac{Z_f}{Z_i}V_g$

$$Z_f = \frac{10^4(80 \times 10^6/s)}{10^4 + 80 \times 10^6/s} = \frac{80 \times 10^6}{s + 8000}$$

$$Z_i = 4000 + \frac{10^9}{62.5s} = \frac{4000(s+4000)}{s}$$

$$V_g = \frac{16{,}000}{s^2}$$

$$\therefore \quad V_o = \frac{-320 \times 10^6}{s(s+4000)(s+8000)}$$

[b] $V_o = \frac{K_1}{s} + \frac{K_2}{s+4000} + \frac{K_3}{s+8000}$

$$K_1 = \frac{-20{,}000(16{,}000)}{(4000)(8000)} = -10$$

$$K_2 = \frac{-320 \times 10^6}{(-4000)(4000)} = 20$$

$$K_3 = \frac{-320 \times 10^6}{(-8000)(-4000)} = -10$$

$$\therefore \quad v_o(t) = (-10 + 20e^{-4000t} - 10e^{-8000t})u(t)\,\text{V}$$

[c] $-10 + 20e^{-4000t_s} - 10e^{-8000t_s} = -5$

$$\therefore \quad 20e^{-4000t_s} - 10e^{-8000t_s} = 5$$

Let $x = e^{-4000t_s}$. Then

$20x - 10x^2 = 5; \qquad$ or $x^2 - 2x + 0.5 = 0$

Solving,

$x = 1 \pm \sqrt{0.5} \quad$ so $\quad x = 0.2929$

$\therefore \quad e^{-4000t_s} = 0.2929; \qquad \therefore \quad t_s = 306.99\,\mu\text{s}$

[d] $v_g = \text{m}\,tu(t); \qquad V_g = \dfrac{\text{m}}{s^2}$

$$V_o = \frac{-20{,}000\text{m}}{s(s+4000)(s+8000)}$$

$$K_1 = \frac{-20{,}000\text{m}}{(4000)(8000)} = \frac{-20{,}000\text{m}}{32 \times 10^6}$$

$$\therefore \quad -5 = \frac{-20{,}000\text{m}}{32 \times 10^6} \qquad \therefore \quad \text{m} = 8000\,\text{V/s}$$

Thus, m must be less than or equal to 8000 V/s to avoid saturation.

P 13.45 **[a]** Let v_a be the voltage across the $0.5\,\mu$F capacitor, positive at the upper terminal.
Let v_b be the voltage across the $100\,\text{k}\Omega$ resistor, positive at the upper terminal.
Also note

$$\frac{10^6}{0.5s} = \frac{2 \times 10^6}{s} \quad \text{and} \quad \frac{10^6}{0.25s} = \frac{4 \times 10^6}{s}; \qquad V_g = \frac{0.5}{s}$$

$$\frac{sV_\text{a}}{s \times 10^6} + \frac{V_\text{a} - (0.5/s)}{200{,}000} + \frac{V_\text{a}}{200{,}000} = 0$$

$$sV_\text{a} + 10V_\text{a} - \frac{5}{s} + 10V_\text{a} = 0$$

$$V_\text{a} = \frac{5}{s(s+20)}$$

$$\frac{0 - V_\text{a}}{200{,}000} + \frac{(0 - V_\text{b})s}{4 \times 10^6} = 0$$

$$\therefore \qquad V_\mathrm{b} = -\frac{20}{s}V_\mathrm{a} = \frac{-100}{s^2(s+20)}$$

$$\frac{V_\mathrm{b}}{100{,}000} + \frac{(V_\mathrm{b}-0)s}{4\times 10^6} + \frac{(V_\mathrm{b}-V_o)s}{4\times 10^6} = 0$$

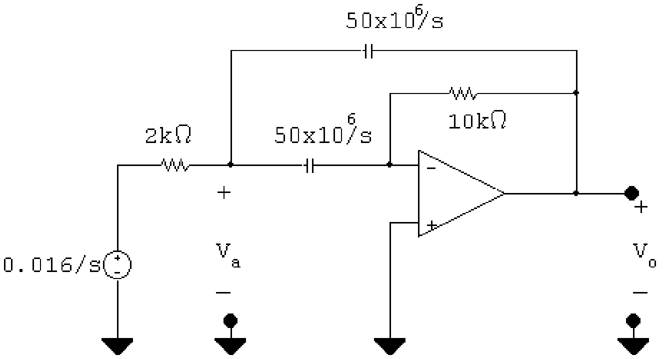
$$40V_\mathrm{b} + sV_\mathrm{b} + sV_\mathrm{b} = sV_o$$

$$\therefore \qquad V_o = \frac{2(s+20)V_\mathrm{b}}{s}; \qquad V_o = 2\left(\frac{-100}{s^3}\right) = \frac{-200}{s^3}$$

[**b**] $v_o(t) = -100t^2u(t)\,\mathrm{V}$

[**c**] $-100t^2 = -4; \qquad t = 0.2\,\mathrm{s} = 200\,\mathrm{ms}$

P 13.46



$$\frac{V_\mathrm{a} - 0.016/s}{2000} + \frac{V_\mathrm{a}s}{50\times 10^6} + \frac{(V_\mathrm{a}-V_o)s}{50\times 10^6} = 0$$

$$\frac{(0-V_\mathrm{a})s}{50\times 10^6} + \frac{(0-V_o)}{10{,}000} = 0$$

$$V_\mathrm{a} = \frac{-5000V_o}{s}$$

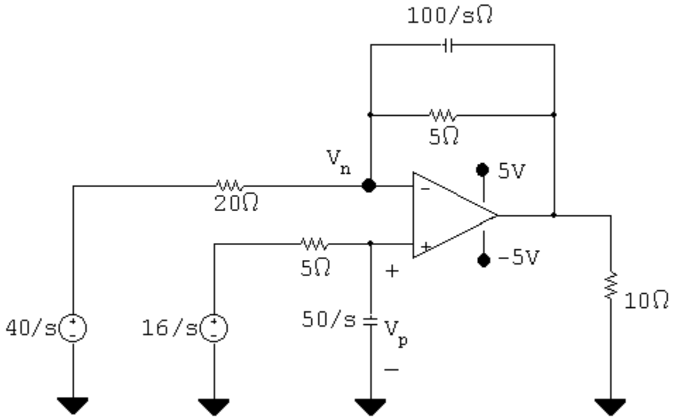
$$\therefore \quad \frac{-5000V_o}{s}(2s+25{,}000) - sV_o = 25{,}000\left(\frac{0.016}{s}\right)$$

$$V_o = \frac{-4000}{(s+5000-j10{,}000)(s+5000+j10{,}000)}$$

$$K_1 = \frac{-400}{j10{,}000} = j0.02 = 0.02\underline{/90^\circ}$$

$$v_o(t) = 40e^{-5000t}\cos(10{,}000t+90^\circ) = -40e^{-5000t}\sin(10{,}000t)u(t)\,\mathrm{mV}$$

P 13.47 [**a**]



$$V_p = \frac{50/s}{5+50/s}V_{g2} = \frac{50}{5s+50}V_{g2}$$

$$\frac{V_p - 40/s}{20} + \frac{V_p - V_o}{5} + \frac{V_p - V_o}{100/s} = 0$$

$$V_p\left(\frac{1}{20} + \frac{1}{5} + \frac{s}{100}\right) - V_o\left(\frac{1}{5} + \frac{s}{100}\right) = \frac{2}{s}$$

$$\frac{s+25}{100}\left(\frac{50}{5s+50}\right)\frac{16}{s} - \frac{2}{s} = V_o\left(\frac{1}{5} + \frac{s}{100}\right) = V_o\left(\frac{s+20}{100}\right)$$

$$V_o = \frac{100}{s+20}\left[\frac{16(s+25)}{10(s+10)(s)} - \frac{2}{s}\right] = \frac{-40s+2000}{s(s+10)(s+20)}$$

$$= \frac{K_1}{s} + \frac{K_2}{s+10} + \frac{K_3}{s+20}$$

$$K_1 = 10; \qquad K_2 = -24; \qquad K_3 = 14$$

$$\therefore \qquad v_o(t) = [10 - 24e^{-10t} + 14e^{-20t}]u(t)\,\text{V}$$

[**b**] $10 - 24x + 14x^2 = 5$

$$14x^2 - 24x + 5 = 0$$

$$x = 0 \quad \text{or} \quad 0.242691$$

$$e^{-10t} = 0.242691 \qquad \therefore \qquad t = 141.60\,\text{ms}$$

P 13.48 Let v_{o1} equal the output voltage of the first op amp. Then

$$V_{o1} = \frac{-Z_{f1}}{Z_{A1}}V_g \quad \text{where} \quad Z_{f1} = 25 \times 10^3\,\Omega$$

$$Z_{A1} = 25{,}000 + \frac{25{,}000(20 \times 10^4/s)}{25{,}000 + (20 \times 10^4/s)}$$

$$= \frac{25{,}000(s+16)}{(s+8)}\,\Omega$$

$$\therefore \qquad V_{o1} = \frac{-(s+8)}{(s+16)}V_g$$

$$v_g(t) = 16u(t)\,\text{mV}; \quad \therefore \qquad V_g = \frac{16\times 10^{-3}}{s}$$

$$V_{o1} = \frac{-16\times 10^{-3}(s+8)}{s(s+16)} = \frac{-0.008}{s} + \frac{-0.008}{s+16}$$

$$\therefore \qquad v_{o1} = -0.008(1+e^{-16t})\,\text{V}$$

The op amp will saturate when $v_{o1} = \pm 6$ V. Hence, saturation will occur when

$$-0.008(1+e^{-16t}) = -6 \qquad \text{so} \qquad e^{-16t} = 749$$

$$\text{Thus} \qquad t = \frac{\ln 749}{-16} = -0.414\,\text{s}$$

Thus, the first op amp never saturates. We must investigate the output of the second op amp:

$$V_o = \frac{-Z_{f2}}{Z_{A2}}V_{o1} \quad \text{where} \quad Z_{f2} = \frac{2\times 10^8}{s}\,\Omega \text{ and } Z_{A2} = 25{,}000\,\Omega$$

$$\therefore \qquad V_o = \frac{-8000}{s}V_{o1} = \frac{-8000}{s}\left[\frac{-(s+8)}{(s+16)}\right]V_g$$

$$= \frac{8000(s+8)}{s(s+16)}V_g$$

$$v_g(t) = 16u(t)\,\text{mV}; \quad \therefore \qquad V_g = \frac{16\times 10^{-3}}{s}$$

$$V_o = \frac{128(s+8)}{s^2(s+16)} = \frac{K_1}{s^2} + \frac{K_2}{s} + \frac{K_3}{s+16}$$

$$K_1 = \frac{128(8)}{16} = 64$$

$$K_2 = 128\frac{d}{ds}\left[\frac{s+8}{s+16}\right]_{s=0} = 4$$

$$K_3 = \frac{128(-8)}{256} = -4$$

$$v_o(t) = [64t + 4 - 4e^{-16t}]u(t)\text{ V}$$

The op amp will saturate when $v_o = \pm 6$ V. Hence, saturation will occur when

$$64t + 4 - 4e^{-16t} = 6 \quad \text{or} \quad 16t - 0.5 = e^{-16t}$$

This equation can be solved by trial and error. First note that $t > 0.5/16$ or $t > 31.25$ ms.
Try 40 ms:

$$0.64 - 0.5 = 0.14; \qquad e^{-0.64} = 0.53$$

Try 50 ms:

$$0.80 - 0.5 = 0.30; \qquad e^{-0.80} = 0.45$$

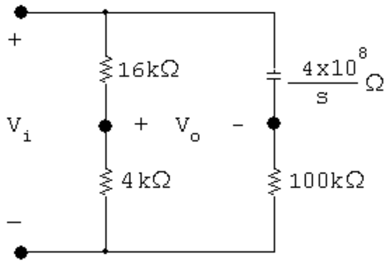
Try 60 ms:

$$0.96 - 0.5 = 0.46; \qquad e^{-0.96} = 0.38$$

Further trial and error gives

$$t_{\text{sat}} \cong 56.5\,\text{ms}$$

P 13.49 **[a]**



$$\frac{4}{20}V_i = V_o + \frac{100{,}000V_i}{100{,}000 + (4\times10^8/s)}$$

$$0.2V_i - \frac{sV_i}{s + 4000} = V_o$$

$$\therefore \quad \frac{V_o}{V_i} = H(s) = \frac{-0.8(s - 1000)}{(s + 4000)}$$

[b] $-z_1 = 1000\,\text{rad/s}$

$-p_1 = -4000\,\text{rad/s}$

P 13.50 **[a]** $\dfrac{V_o}{V_i} = \dfrac{1/sC}{R + 1/sC} = \dfrac{1}{RCs + 1}$

$$H(s) = \frac{(1/RC)}{s + (1/RC)} = \frac{250}{s + 250}; \qquad -p_1 = -250\,\text{rad/s}$$

[b] $\dfrac{V_o}{V_i} = \dfrac{R}{R + 1/sC} = \dfrac{RCs}{RCs + 1} = \dfrac{s}{s + (1/RC)}$

$$= \frac{s}{s + 250}; \qquad z_1 = 0, \quad -p_1 = -250\,\text{rad/s}$$

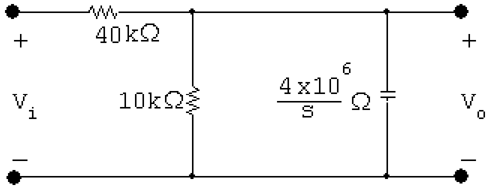
[c] $\dfrac{V_o}{V_i} = \dfrac{sL}{R + sL} = \dfrac{s}{s + R/L} = \dfrac{s}{s + 8000}$

$z_1 = 0; \qquad -p_1 = -8000\,\text{rad/s}$

[d] $\dfrac{V_o}{V_i} = \dfrac{R}{R + sL} = \dfrac{R/L}{s + (R/L)} = \dfrac{8000}{s + 8000}$

$-p_1 = -8000\,\text{rad/s}$

[e]



$$\frac{V_o s}{4 \times 10^6} + \frac{V_o}{10{,}000} + \frac{V_o - V_i}{40{,}000} = 0$$

$$sV_o + 400V_o + 100V_o = 100V_i$$

$$H(s) = \frac{V_o}{V_i} = \frac{100}{s + 500}$$

$-p_1 = -500\,\text{rad/s}$

P 13.51 **[a]** $\dfrac{1/sC}{R + 1/sC} = \dfrac{1}{RsC + 1} = \dfrac{1/RC}{s + 1/RC}$

There are no zeros, and a single pole at $-1/RC$ rad/sec.

[b] $\dfrac{R}{R + sL} = \dfrac{R/L}{s + R/L}$

There are no zeros, and a single pole at $-R/L$ rad/sec.

[c] There are several possible solutions. One is

$$R = 10\,\Omega; \qquad L = 10\,\text{mH}; \qquad C = 100\,\mu\text{F}$$

P 13.52 **[a]** $\dfrac{R}{R + 1/sC} = \dfrac{RsC}{RsC + 1} = \dfrac{s}{s + 1/RC}$

There is a single zero at 0 rad/sec, and a single pole at $-1/RC$ rad/sec.

[b] $\dfrac{sL}{R + sL} = \dfrac{s}{s + R/L}$

There is a single zero at 0 rad/sec, and a single pole at $-R/L$ rad/sec.

[c] There are several possible solutions. One is

$$R = 100\,\Omega; \qquad L = 10\,\text{mH}; \qquad C = 1\,\mu\text{F}$$

P 13.53 **[a]** $\dfrac{R}{1/sC + sL + R} = \dfrac{(R/L)s}{s^2 + (R/L)s + 1/LC}$

There is a single zero at 0 rad/sec, and two poles:

$$p_1 = -(R/2L) + \sqrt{(R/2L)^2 - (1/LC)}; \qquad p_2 = -(R/2L) - \sqrt{(R/2L)^2 - (1/LC)}$$

[b] There are several possible solutions. One is

$$R = 250\,\Omega; \quad L = 10\,\text{mH}; \quad C = 1\,\mu\text{F}$$

These component values yield the following poles:

$$-p_1 = -5000 \text{ rad/sec} \quad \text{and} \quad -p_2 = -20{,}000 \text{ rad/sec}$$

[c] There are several possible solutions. One is

$$R = 200\,\Omega; \quad L = 10\,\text{mH}; \quad C = 1\,\mu\text{F}$$

These component values yield the following poles:

$$-p_1 = -10{,}000 \text{ rad/sec} \quad \text{and} \quad -p_2 = -10{,}000 \text{ rad/sec}$$

[d] There are several possible solutions. One is

$$R = 120\,\Omega; \quad L = 10\,\text{mH}; \quad C = 1\,\mu\text{F}$$

These component values yield the following poles:

$$-p_1 = -6000 + j8000 \text{ rad/sec} \quad \text{and} \quad -p_2 = -6000 - j8000 \text{ rad/sec}$$

P 13.54 **[a]** $Z_i = 1000 + \dfrac{5 \times 10^6}{s} = \dfrac{1000(s + 5000)}{s}$

$$Z_f = \frac{40 \times 10^6}{s} \| 40{,}000 = \frac{40 \times 10^6}{s + 1000}$$

$$H(s) = -\frac{Z_f}{Z_i} = \frac{-40 \times 10^6/(s + 1000)}{1000(s + 5000)/s} = \frac{-40{,}000s}{(s + 1000)(s + 5000)}$$

[b] Zero at $z_1 = 0$; Poles at $-p_1 = -1000$ rad/s and $-p_2 = -5000$ rad/s

P 13.55 **[a]** Let $R_1 = 250\,\text{k}\Omega$; $R_2 = 125\,\text{k}\Omega$; $C_2 = 1.6\,\text{nF}$; and $C_f = 0.4\,\text{nF}$. Then

$$Z_f = \frac{(R_2 + 1/sC_2)1/sC_f}{\left(R_2 + \frac{1}{sC_2} + \frac{1}{sC_f}\right)} = \frac{(s + 1/R_2C_2)}{C_f s\left(s + \frac{C_2+C_f}{C_2C_fR_2}\right)}$$

$$\frac{1}{C_f} = 2.5 \times 10^9$$

$$\frac{1}{R_2C_2} = \frac{62.5 \times 10^7}{125 \times 10^3} = 5000\,\text{rad/s}$$

$$\frac{C_2 + C_f}{C_2C_fR_2} = \frac{2 \times 10^{-9}}{(0.64 \times 10^{-18})(125 \times 10^3)} = 25{,}000\,\text{rad/s}$$

$$\therefore \quad Z_f = \frac{2.5 \times 10^9(s + 5000)}{s(s + 25{,}000)}\Omega$$

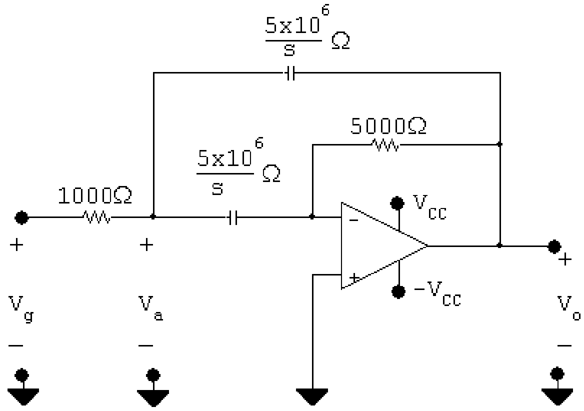
$$Z_i = R_1 = 250 \times 10^3\,\Omega$$

$$H(s) = \frac{V_o}{V_g} = \frac{-Z_f}{Z_i} = \frac{-10^4(s + 5000)}{s(s + 25{,}000)}$$

[b] $-z_1 = -5000\,\text{rad/s}$

$-p_1 = 0$; $-p_2 = -25{,}000\,\text{rad/s}$

P 13.56 **[a]**



$$\frac{V_\text{a} - V_g}{1000} + \frac{sV_\text{a}}{5 \times 10^6} + \frac{(V_\text{a} - V_o)s}{5 \times 10^6} = 0$$

$$5000V_\text{a} - 5000V_g + 2sV_\text{a} - sV_o = 0$$

$$(5000 + 2s)V_\text{a} - sV_o = 5000V_g$$

$$\frac{(0 - V_\text{a})s}{5 \times 10^6} + \frac{0 - V_o}{5000} = 0$$

$$-sV_{\text{a}} - 1000V_o = 0; \quad \therefore \quad V_{\text{a}} - \frac{-1000}{s}V_o$$

$$(2s + 5000)\left(\frac{-1000}{s}\right)V_o - sV_o = 5000V_g$$

$$1000V_o(2s + 5000) + s^2V_o = -5000sV_g$$

$$V_o(s^2 + 2000s + 5 \times 10^6) = -5000sV_g$$

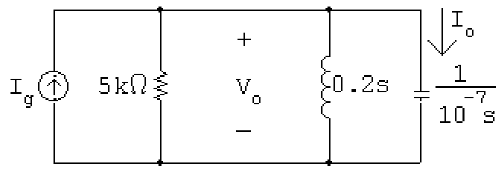
$$\frac{V_o}{V_g} = \frac{-5000s}{s^2 + 2000s + 5 \times 10^6}$$

$$s_{1,2} = -1000 \pm \sqrt{10^6 - 5 \times 10^6} = -1000 \pm j2000$$

$$\frac{V_o}{V_g} = \frac{-5000s}{(s + 1000 - j2000)(s + 1000 + j2000)}$$

[b] $z_1 = 0; \qquad -p_1 = -1000 + j2000; \qquad -p_2 = -1000 - j2000$

P 13.57 **[a]**



$$\frac{V_o}{5000} + \frac{V_o}{0.2s} + V_o(10^{-7})s = I_g$$

$$\therefore \quad V_o = \frac{10 \times 10^6 s}{s^2 + 2000s + 50 \times 10^6} \cdot I_g$$

$$I_g = \frac{0.1s}{s^2 + 10^8}; \qquad I_o = 10^{-7}sV_o$$

$$\therefore \quad H(s) = \frac{s^2}{s^2 + 2000s + 50 \times 10^6}$$

[b] $I_o = \dfrac{(s^2)(0.1s)}{(s + 1000 - j7000)(s + 1000 + j7000)(s^2 + 10^8)}$

$$I_o = \frac{0.1s^3}{(s + 1000 - j7000)(s + 1000 + j7000)(s + j10^4)(s - j10^4)}$$

[c] Damped sinusoid of the form

$$Me^{-1000t}\cos(7000t + \theta_1)$$

[d] Steady-state sinusoid of the form

$$N\cos(10^4t + \theta_2)$$

[e] $I_o = \frac{K_1}{s+1000-j7000} + \frac{K_1^*}{s+1000+j7000} + \frac{K_2}{s-j10^4} + \frac{K_2^*}{s+j10^4}$

$$K_1 = \frac{0.1(-1000+j7000)^3}{(j14{,}000)(-1000-j5000)(-1000+j17{,}000)} = 46.9\times10^{-3}\underline{/-140.54^\circ}$$

$$K_2 = \frac{0.1(j10^4)^3}{(j20{,}000)(1000+j3000)(1000+j17{,}000)} = 92.85\times10^{-3}\underline{/21.8^\circ}$$

$$i_o(t) = [93.8e^{-1000t}\cos(7000t-140.54^\circ) + 185.7\cos(10^4t+21.8^\circ)]\,\mathrm{mA}$$

Test:

$$Z = \frac{1}{Y}; \qquad Y = \frac{1}{5000} + \frac{1}{j2000} + \frac{1}{-j1000} = \frac{2+j5}{10{,}000}$$

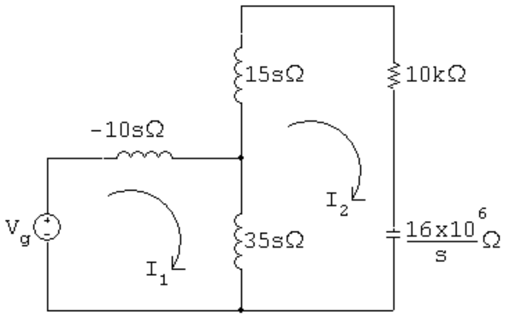
$$\therefore\quad Z = \frac{10{,}000}{2+j5} = 1856.95\underline{/-68.2^\circ}\,\Omega$$

$$\mathbf{V}_o = \mathbf{I}_g Z = (0.1\underline{/0^\circ})(1856.95\underline{/-68.2^\circ}) = 185.695\underline{/-68.2^\circ}\,\mathrm{V}$$

$$\mathbf{I}_o = (10^{-7})(j10^4)\mathbf{V}_o = 185.7\underline{/21.8^\circ}\,\mathrm{mA}$$

$$i_{oss} = 185.7\cos(10^4t+21.8^\circ)\,\mathrm{mA(checks)}$$

P 13.58



$$V_g = 25sI_1 - 35sI_2$$

$$0 = -35sI_1 + \left(50s + 10{,}000 + \frac{16\times10^6}{s}\right)I_2$$

$$\Delta = \begin{vmatrix} 25s & -35s \\ -35s & 50s+10{,}000+16\times10^6/s \end{vmatrix} = 25(s+2000)(s+8000)$$

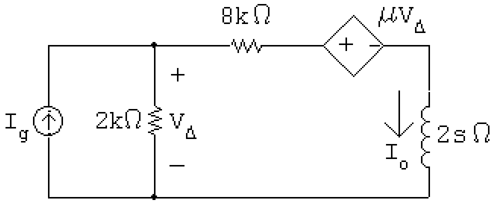
$$N_2 = \begin{vmatrix} 25s & V_g \\ -35s & 0 \end{vmatrix} = 35sV_g$$

$$I_2 = \frac{N_2}{\Delta} = \frac{35sV_g}{25(s+2000)(s+8000)}$$

$$H(s) = \frac{I_2}{V_g} = \frac{1.4s}{(s+2000)(s+8000)}$$

$$\therefore \quad z_1 = 0; \qquad -p_1 = -2000 \text{ rad/s}; \qquad -p_2 = -8000 \text{ rad/s}$$

P 13.59 **[a]**



$$2000(I_o - I_g) + 8000I_o + \mu(I_g - I_o)(2000) + 2sI_o = 0$$

$$\therefore \quad I_o = \frac{1000(1-\mu)}{s+1000(5-\mu)}I_g$$

$$\therefore \quad H(s) = \frac{1000(1-\mu)}{s+1000(5-\mu)}$$

[b] $\mu < 5$

[c]

μ	$H(s)$	I_o
-3	$4000/(s+8000)$	$20{,}000/s(s+8000)$
0	$1000/(s+5000)$	$5000/s(s+5000)$
4	$-3000/(s+1000)$	$-15{,}000/s(s+1000)$
5	$-4000/s$	$-20{,}000/s^2$
6	$-5000/(s-1000)$	$-25{,}000/s(s-1000)$

$\mu = -3$:

$$I_o = \frac{2.5}{s} - \frac{2.5}{(s+8000)}; \qquad i_o = [2.5 - 2.5e^{-8000t}]u(t)\text{ A}$$

$\mu = 0$:

$$I_o = \frac{1}{s} - \frac{1}{s+5000}; \qquad i_o = [1 - e^{-5000t}]u(t)\text{ A}$$

$\mu = 4$:

$$I_o = \frac{-15}{s} - \frac{15}{s+1000}; \qquad i_o = [-15 + 15e^{-1000t}]u(t)\text{ A}$$

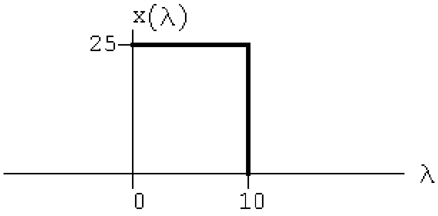
$\mu = 5$:

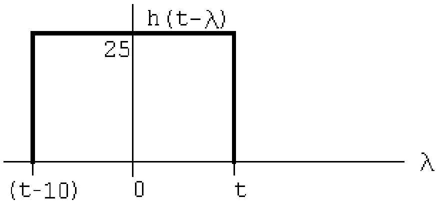
$$I_o = \frac{-20{,}000}{s^2}; \qquad i_o = -20{,}000t\,u(t)\,\mathrm{A}$$

$\mu = 6$:

$$I_o = \frac{25}{s} - \frac{25}{s-1000}; \qquad i_o = 25[1 - e^{1000t}]u(t)\,\mathrm{A}$$

P 13.60 [**a**]



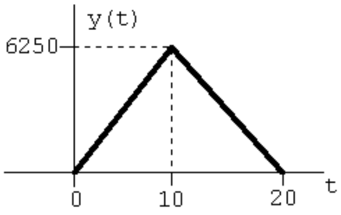


$y(t) = 0 \qquad t < 0$

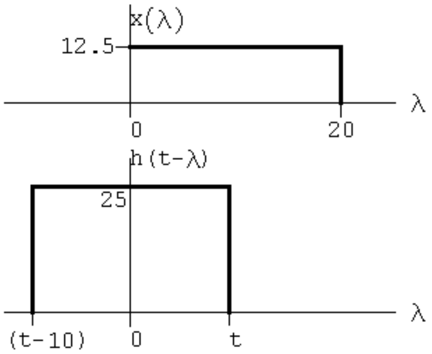
$$0 \le t \le 10: \qquad y(t) = \int_0^t 625\,d\lambda = 625t$$

$$10 \le t \le 20: \qquad y(t) = \int_{t-10}^{10} 625\,d\lambda = 625(10 - t + 10) = 625(20 - t)$$

$$20 \le t < \infty: \qquad y(t) = 0$$



[b]



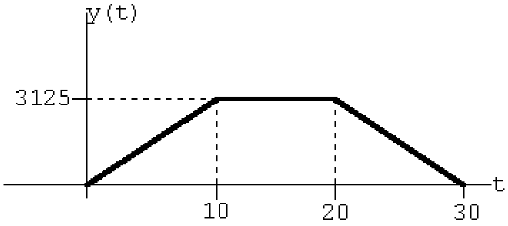
$$y(t) = 0 \qquad t < 0$$

$$0 \le t \le 10: \qquad y(t) = \int_0^t 312.5\,d\lambda = 312.5t$$

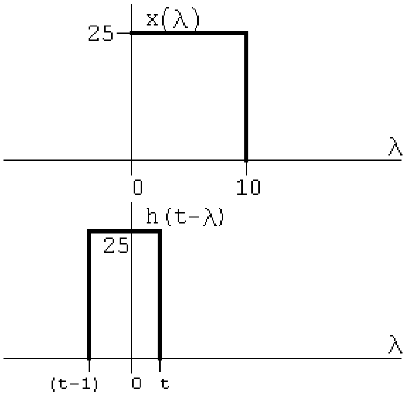
$$10 \le t \le 20: \qquad y(t) = \int_0^{10} 312.5\,d\lambda = 3125$$

$$20 \le t \le 30: \qquad y(t) = \int_{t-20}^{10} 312.5\,d\lambda = 312.5(30-t)$$

$$30 \le t < \infty: \qquad y(t) = 0$$



[c]



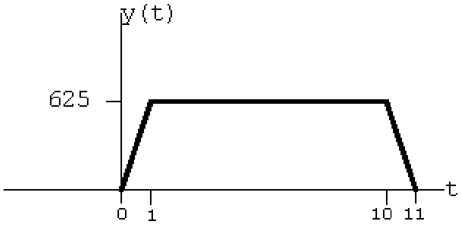
$$y(t) = 0 \qquad t < 0$$

$$0 \le t \le 1: \qquad y(t) = \int_0^t 625\,d\lambda = 625t$$

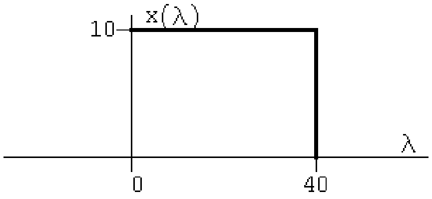
$$1 \le t \le 10: \qquad y(t) = \int_0^1 625\,d\lambda = 625$$

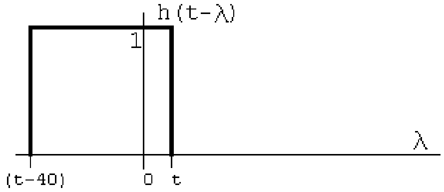
$$10 \le t \le 11: \qquad y(t) = \int_{t-10}^1 625\,d\lambda = 625(11-t)$$

$$11 \le t < \infty: \qquad y(t) = 0$$



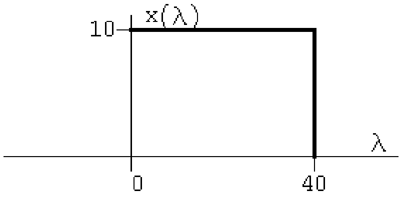
P 13.61 **[a]** $0 \le t \le 40$:

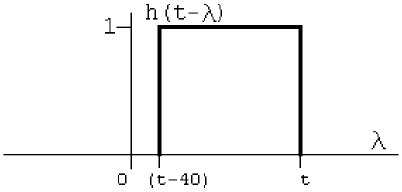




$$y(t) = \int_0^t (10)(1)(d\lambda) = 10\lambda \Big|_0^t = 10t$$

$40 \le t \le 80$:

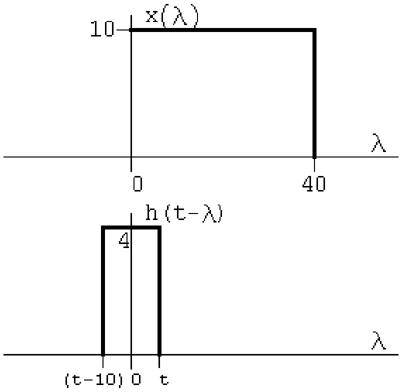




$$y(t) = \int_{t-40}^{40} (10)(1)(d\lambda) = 10\lambda \Big|_{t-40}^{40} = 10(80-t)$$

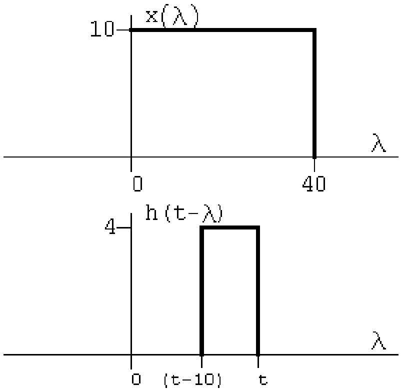
$$t \ge 80: \qquad y(t) = 0$$

[b] $0 \le t \le 10$:



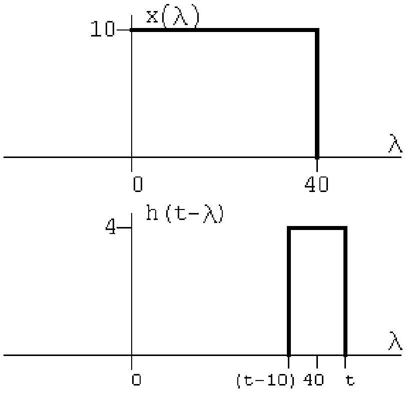
$$y(t) = \int_0^t 40\,d\lambda = 40\lambda \Big|_0^t = 40t$$

$10 \le t \le 40$:



$$y(t) = \int_{t-10}^t 40\,d\lambda = 40\lambda \Big|_{t-10}^t = 400$$

$40 \le t \le 50$:



$$y(t) = \int_{t-10}^{40} 40\,d\lambda = 40\lambda \Big|_{t-10}^{40} = 40(50 - t)$$

$t \ge 50 : \qquad y(t) = 0$

[**c**] The expressions are

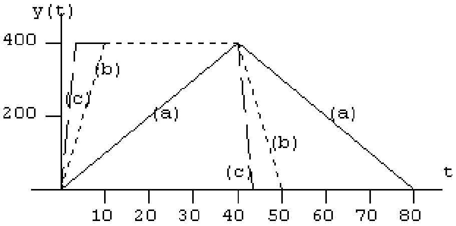
$$0 \le t \le 1: \qquad y(t) = \int_0^t 400\,d\lambda = 400\lambda \Big|_0^t = 400t$$

$$1 \le t \le 40: \qquad y(t) = \int_{t-1}^t 400\,d\lambda = 400\lambda \Big|_{t-1}^t = 400$$

$$40 \le t \le 41: \qquad y(t) = \int_{t-1}^{40} 400\,d\lambda = 400\lambda \Big|_{t-1}^{40} = 400(41-t)$$

$$41 \le t < \infty: \qquad y(t) = 0$$

[**d**]



[**e**] Yes, note that $h(t)$ is approaching $40\delta(t)$, therefore $y(t)$ must approach $40x(t)$, i.e.

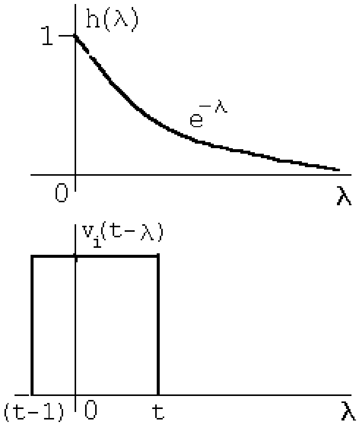
$$y(t) = \int_0^t h(t-\lambda)x(\lambda)\,d\lambda \to \int_0^t 40\delta(t-\lambda)x(\lambda)\,d\lambda$$

$$\to 40x(t)$$

This can be seen in the plot, e.g., in part (c), $y(t) \cong 40x(t)$.

P 13.62 $H(s) = \dfrac{V_o}{V_i} = \dfrac{1}{s+1}; \qquad h(t) = e^{-t}$

For $0 \le t \le 1$:



$$v_o = \int_0^t e^{-\lambda}\,d\lambda = (1 - e^{-t})\,\text{V}$$

For $1 \le t \le \infty$:

$$v_o = \int_{t-1}^{t} e^{-\lambda}\,d\lambda = (e-1)e^{-t}\,\text{V}$$

P 13.63 $H(s) = \dfrac{V_o}{V_i} = \dfrac{s}{s+1} = 1 - \dfrac{1}{s+1}; \qquad h(t) = \delta(t) - e^{-t}$

$$h(\lambda) = \delta(\lambda) - e^{-\lambda}$$

For $0 \le t \le 1$:

$$v_o = \int_0^t [\delta(\lambda) - e^{-\lambda}]\,d\lambda = [1 + e^{-\lambda}]\,\Big|_0^t = e^{-t}\,\text{V}$$

For $1 \le t \le \infty$:

$$v_o = \int_{t-1}^{t} (-e^{-\lambda})\,d\lambda = e^{-\lambda}\,\Bigg|_{t-1}^{t} = (1-e)e^{-t}\,\text{V}$$

P 13.64 **[a]** From Problem 13.50(a)

$$H(s) = \frac{250}{s+250}$$

$$h(\lambda) = 250e^{-250\lambda}$$

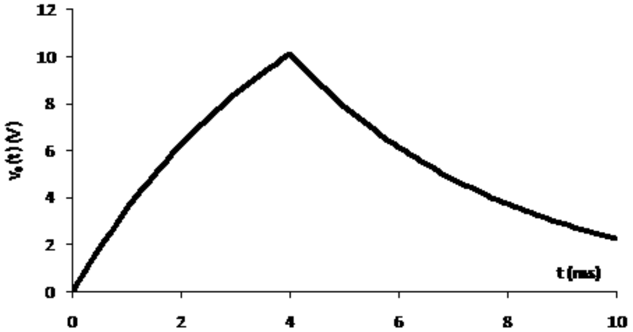
$0 \le t \le 4\,\text{ms}$:

$$v_o = \int_0^t 16(250)e^{-250\lambda}\,d\lambda = 16(1 - e^{-250t})\,\text{V}$$

$4\,\text{ms} \le t \le \infty$:

$$v_o = \int_{t-0.004}^{t} 16(250)e^{-250\lambda}\,d\lambda = 16(e-1)e^{-250t}\,\text{V}$$

[b]



P 13.65 [**a**] $H(s) = \dfrac{2500}{s+2500} \qquad \therefore \quad h(\lambda) = 2500e^{-2500\lambda}$

$0 \le t \le 4\,\text{ms}$:

$$v_o = \int_0^t 16(2500)e^{-2500\lambda}\,d\lambda = 16(1 - e^{-2500t})\,\text{V}$$

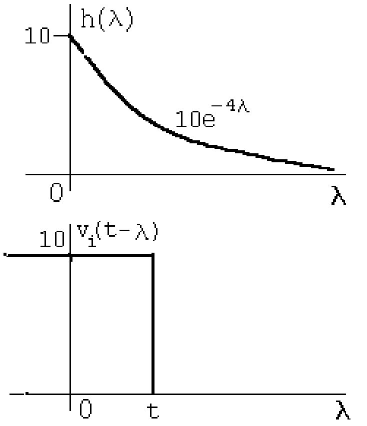
$4\,\text{ms} \le t \le \infty$:

$$v_o = \int_{t-0.004}^t 16(2500)e^{-2500\lambda}\,d\lambda = 16(e^{10} - 1)e^{-2500t}\,\text{V}$$

[**b**] decrease

[**c**] The circuit with $R = 10\,\text{k}\Omega$.

P 13.66 [**a**]

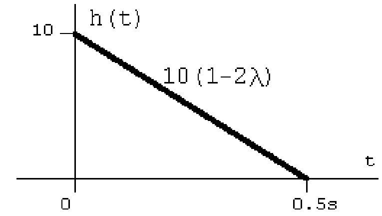


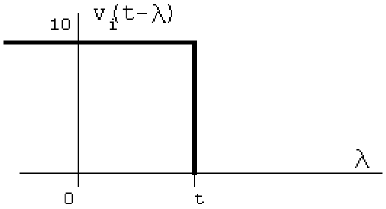
$$v_o = \int_0^t 10(10e^{-4\lambda})\,d\lambda$$

$$= 100\frac{e^{-4\lambda}}{-4}\bigg|_0^t = -25[e^{-4t} - 1]$$

$$= 25(1 - e^{-4t})\,\text{V}, \qquad 0 \le t \le \infty$$

[b]





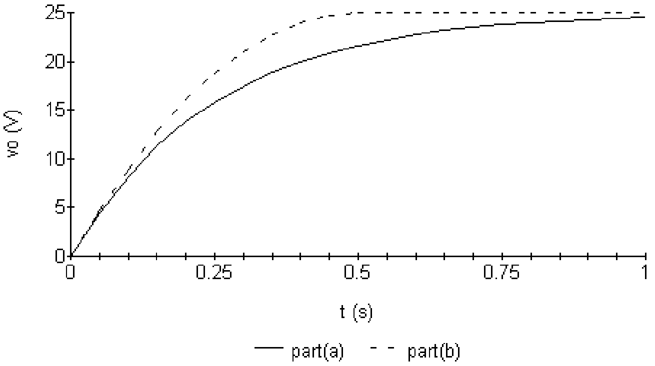
$0 \le t \le 0.5$:

$$v_o = \int_0^t 100(1-2\lambda)\,d\lambda = 100(\lambda - \lambda^2)\Big|_0^t = 100t(1-t)$$

$0.5 \le t \le \infty$:

$$v_o = \int_0^{0.5} 100(1-2\lambda)\,d\lambda = 100(\lambda - \lambda^2)\Big|_0^{0.5} = 25$$

[c]



P 13.67 **[a]** $-1 \le t \le 4$:

$$v_o = \int_0^{t+1} 10\lambda\,d\lambda = 5\lambda^2\Big|_0^{t+1} = 5t^2 + 10t + 5\,\text{V}$$

$4 \le t \le 9$:

$$v_o = \int_{t-4}^{t+1} 10\lambda\,d\lambda = 5\lambda^2\Big|_{t-4}^{t+1} = 50t - 75\,\text{V}$$

$9 \le t \le 14$:

$$v_o = 10\int_{t-4}^{10} \lambda\,d\lambda + 10\int_{10}^{t+1} 10\,d\lambda$$

$$= 5\lambda^2 \Big|_{t-4}^{10} + 100\lambda \Big|_{10}^{t+1} = -5t^2 + 140t - 480\,\text{V}$$

$14 \leq t \leq 19$:

$$v_o = 100 \int_{t-4}^{t+1} d\lambda = 500\,\text{V}$$

$19 \leq t \leq 24$:

$$v_o = \int_{t-4}^{20} 100\lambda\, d\lambda + \int_{20}^{t+2} 10(30 - \lambda)\, d\lambda$$

$$= 100\lambda \Big|_{t-2}^{20} + 300\lambda \Big|_{20}^{t+1} - 5\lambda^2 \Big|_{20}^{t+2}$$

$$= -5t^2 + 190t - 1305\,\text{V}$$

$24 \leq t \leq 29$:

$$v_o = 10 \int_{t-4}^{t+1} (30 - \lambda)\, d\lambda = 300\lambda \Big|_{t-4}^{t+1} - 5\lambda^2 \Big|_{t-4}^{t+1}$$

$$= 1575 - 50t\,\text{V}$$

$29 \leq t \leq 34$:

$$v_o = 10 \int_{t-4}^{30} (30 - \lambda)\, d\lambda = 300\lambda \Big|_{t-4}^{30} - 5\lambda^2 \Big|_{t-2}^{30}$$

$$= 5t^2 - 340t + 5780\,\text{V}$$

Summary:

$v_o = 0 \qquad -\infty \leq t \leq -1$

$v_o = 5t^2 + 10t + 5\,\text{V} \qquad -1 \leq t \leq 4$

$v_o = 50t - 75\,\text{V} \qquad 4 \leq t \leq 9$

$v_o = -5t^2 + 140t - 480\,\text{V} \qquad 9 \leq t \leq 14$

$v_o = 500\,\text{V} \qquad 14 \leq t \leq 19$

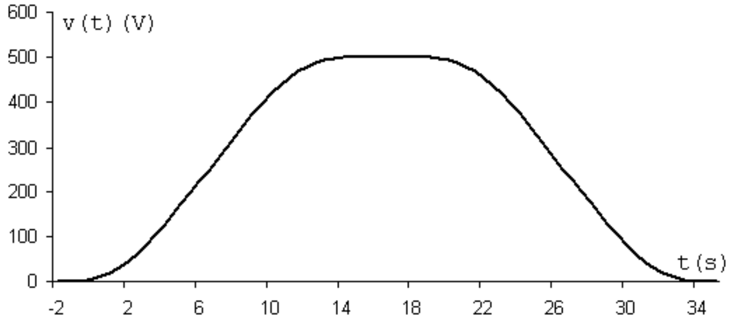
$v_o = -5t^2 + 190t - 1305\,\text{V} \qquad 19 \leq t \leq 24$

$v_o = 1575 - 50t\,\text{V} \qquad 24 \leq t \leq 29$

$v_o = 5t^2 - 340t + 5780\,\text{V} \qquad 29 \leq t \leq 34$

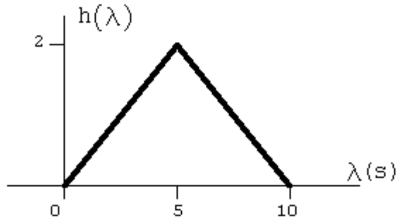
$v_o = 0 \qquad 34 \leq t \leq \infty$

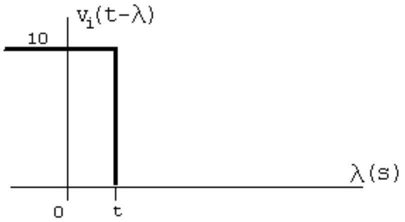
[b]



P 13.68 **[a]** $h(\lambda) = \frac{2}{5}\lambda \qquad 0 \leq \lambda \leq 5$

$$h(\lambda) = \left(4 - \frac{2}{5}\lambda\right) \qquad 5 \leq \lambda \leq 10$$





$0 \leq t \leq 5$:

$$v_o = 10\int_0^t \frac{2}{5}\lambda\, d\lambda = 2t^2$$

$5 \leq t \leq 10$:

$$v_o = 10\int_0^5 \frac{2}{5}\lambda\, d\lambda + 10\int_5^t \left(4 - \frac{2}{5}\lambda\right)\, d\lambda$$

$$= \frac{4\lambda^2}{2}\bigg|_0^5 + 40\lambda\bigg|_5^t - \frac{4\lambda^2}{2}\bigg|_5^t$$

$$= -100 + 40t - 2t^2$$

$10 \leq t \leq \infty$:

$$v_o = 10\int_0^5 \frac{2}{5}\lambda\, d\lambda + 10\int_5^{10} \left(4 - \frac{2}{5}\lambda\right)\, d\lambda$$

$$= \frac{4\lambda^2}{2}\Big|_0^5 + 40\lambda\Big|_5^{10} - \frac{4\lambda^2}{2}\Big|_5^{10}$$

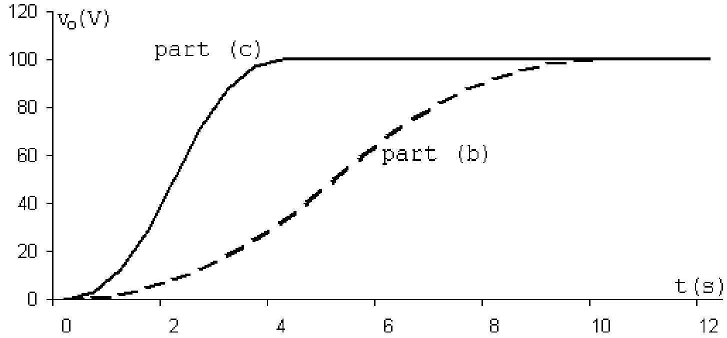
$$= 50 + 200 - 150 = 100$$

$v_o = 2t^2\,\text{V} \qquad 0 \leq t \leq 5$

$v_o = 40t - 100 - 2t^2\,\text{V} \qquad 5 \leq t \leq 10$

$v_o = 100\,\text{V} \qquad 10 \leq t \leq \infty$

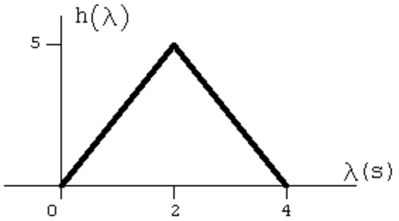
[**b**]

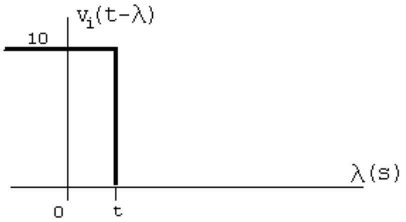


[**c**] Area $= \frac{1}{2}(10)(2) = 10 \quad \therefore \quad \frac{1}{2}(4)h = 10$ so $h = 5$

$h(\lambda) = \frac{5}{2}\lambda \qquad 0 \leq \lambda \leq 2$

$h(\lambda) = \left(10 - \frac{5}{2}\lambda\right) \qquad 2 \leq \lambda \leq 4$





$0 \leq t \leq 2$:

$$v_o = 10\int_0^t \frac{5}{2}\lambda\,d\lambda = 12.5t^2$$

$2 \le t \le 4$:

$$v_o = 10\int_0^2 \frac{5}{2}\lambda\,d\lambda + 10\int_2^t \left(10 - \frac{5}{2}\lambda\right)\,d\lambda$$

$$= \frac{25\lambda^2}{2}\Big|_0^2 + 100\lambda\Big|_2^t - \frac{25\lambda^2}{2}\Big|_2^t$$

$$= -100 + 100t - 12.5t^2$$

$4 \le t \le \infty$:

$$v_o = 10\int_0^2 \frac{5}{2}\lambda\,d\lambda + 10\int_2^4 \left(10 - \frac{5}{2}\lambda\right)\,d\lambda$$

$$= \frac{25\lambda^2}{2}\Big|_0^2 + 100\lambda\Big|_2^4 - \frac{25\lambda^2}{2}\Big|_2^4$$

$$= 50 + 200 - 150 = 100$$

$v_o = 12.5t^2\,\text{V} \qquad 0 \le t \le 2$

$v_o = 100t - 100 - 12.5t^2\,\text{V} \qquad 2 \le t \le 4$

$v_o = 100\,\text{V} \qquad 4 \le t \le \infty$

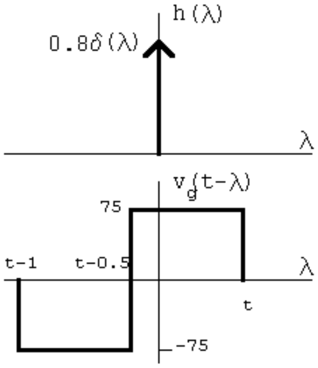
[d] The waveform in part (c) is closer to replicating the input waveform because in part (c) $h(\lambda)$ is closer to being an ideal impulse response. That is, the area was preserved as the base was shortened.

P 13.69 **[a]** $V_o = \dfrac{16}{20}V_g$

$$\therefore \qquad H(s) = \frac{V_o}{V_g} = \frac{4}{5}$$

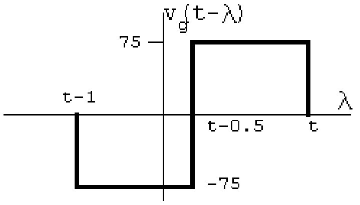
$h(\lambda) = 0.8\delta(\lambda)$

[b]



$$0 < t < 0.5\,\text{s}: \qquad v_o = \int_0^t 75[0.8\delta(\lambda)]\,d\lambda = 60\,\text{V}$$

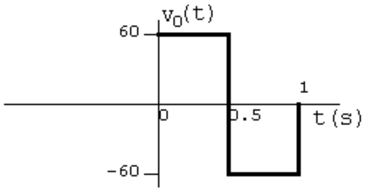
$0.5\,\text{s} \leq t \leq 1.0\,\text{s}$:



$$v_o = \int_0^{t-0.5} -75[0.8\delta(\lambda)]\,d\lambda = -60\,\text{V}$$

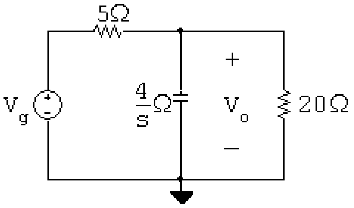
$1\,\text{s} < t < \infty: \qquad v_o = 0$

[c]



Yes, because the circuit has no memory.

P 13.70 **[a]**

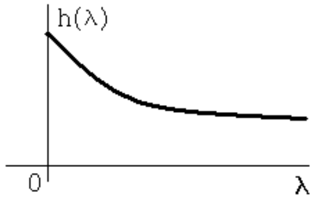


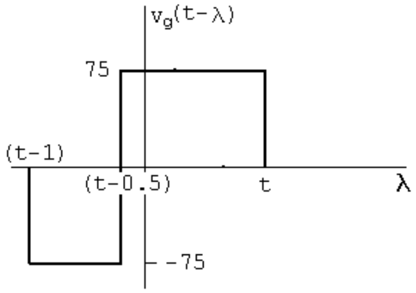
$$\frac{V_o - V_g}{5} + \frac{V_o s}{4} + \frac{V_o}{20} = 0$$

$$(5s+5)V_o = 4V_g$$

$$H(s) = \frac{V_o}{V_g} = \frac{0.8}{s+1}; \qquad h(\lambda) = 0.8e^{-\lambda}u(\lambda)$$

[b]



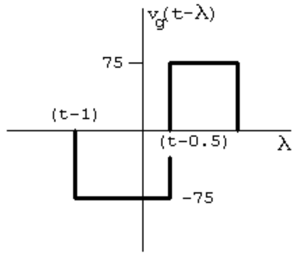


$0 \leq t \leq 0.5\,\text{s};$

$$v_o = \int_0^t 75(0.8e^{-\lambda})\,d\lambda = 60\frac{e^{-\lambda}}{-1}\Big|_0^t$$

$$v_o = 60 - 60e^{-t}\,\text{V}, \qquad 0 \leq t \leq 0.5\,\text{s}$$

$0.5\,\text{s} \leq t \leq 1\,\text{s}:$



$$v_o = \int_0^{t-0.5}(-75)(0.8e^{-\lambda})\,d\lambda + \int_{t-0.5}^{t} 75(0.8e^{-\lambda})\,d\lambda$$

$$= -60\frac{e^{-\lambda}}{-1}\Big|_0^{t-0.5} + 60\frac{e^{-\lambda}}{-1}\Big|_{t-0.5}^{t}$$

$$= 120e^{-(t-0.5)} - 60e^{-t} - 60\,\text{V}, \qquad 0.5\,\text{s} \leq t \leq 1\,\text{s}$$

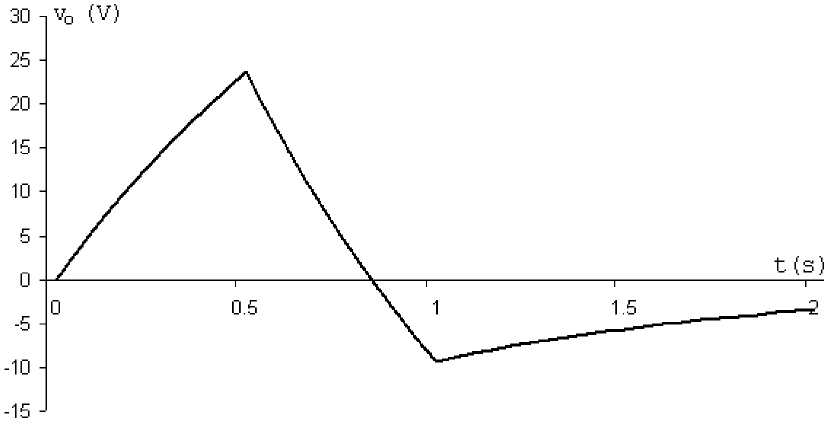
$1\,\text{s} \leq t \leq \infty;$

$$v_o = \int_{t-1}^{t-0.5}(-75)(0.8e^{-\lambda})\,d\lambda + \int_{t-0.5}^{t} 75(0.8e^{-\lambda})\,d\lambda$$

$$= -60\frac{e^{-\lambda}}{-1}\Big|_{t-1}^{t-0.5} + 60\frac{e^{-\lambda}}{-1}\Big|_{t-0.5}^{t}$$

$$= 120e^{-(t-0.5)} - 60e^{-(t-1)} - 60e^{-t}\,\text{V}, \qquad 1\,\text{s} \leq t \leq \infty$$

[c]



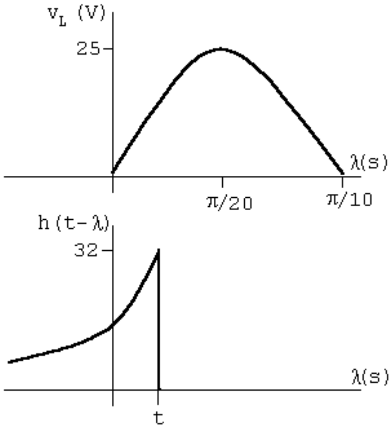
[d] No, the circuit has memory because of the capacitive storage element.

P 13.71 $v_i = 25\,\sin 10\lambda\,[u(\lambda) - u(\lambda - \pi/10)]$

$$H(s) = \frac{32}{s + 32}$$

$$h(\lambda) = 32e^{-32\lambda}$$

$$h(t - \lambda) = 32e^{-32(t-\lambda)} = 32e^{-32t}e^{32\lambda}$$



$$v_o = 800e^{-32t}\int_0^t e^{32\lambda}\sin 10\lambda\,d\lambda$$

$$= 800e^{-32t}\left[\frac{e^{32\lambda}}{32^2 + 10^2}(32\sin 10\lambda - 10\cos 10\lambda\,\Big|_0^t\right]$$

$$= \frac{800e^{-32t}}{1124}[e^{32t}(32\sin 10t - 10\cos 10t) + 10]$$

$$= \frac{800}{1124}[32\sin 10t - 10\cos 10t + 10e^{-32t}]$$

$$v_o(0.075) = 10.96\,\mathrm{V}$$

P 13.72 $$H(s) = \frac{16s}{40+4s+16s} = \frac{0.8s}{s+2} = 0.8\left(1 - \frac{2}{s+2}\right) = 0.8 - \frac{1.6}{s+2}$$

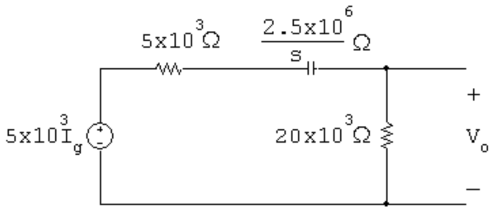
$$h(\lambda) = 0.8\delta(\lambda) - 1.6e^{-2\lambda}u(\lambda)$$

$$v_o = \int_0^t 75[0.8\delta(\lambda) - 1.6e^{-2\lambda}]\,d\lambda = \int_0^t 60\delta(\lambda)\,d\lambda - 120\int_0^t e^{-2\lambda}\,d\lambda$$

$$= 60 - 120\frac{e^{-2\lambda}}{-2}\Big|_0^t = 60 + 60(e^{-2t} - 1)$$

$$= 60e^{-2t}u(t)\,\mathrm{V}$$

P 13.73



$$V_o = \frac{5\times 10^3 I_g}{25\times 10^3 + 2.5\times 10^6/s}(20\times 10^3)$$

$$\frac{V_o}{I_g} = H(s) = \frac{4000s}{s+100}$$

$$H(s) = 4000\left[1 - \frac{100}{s+100}\right] = 4000 - \frac{4\times 10^5}{s+100}$$

$$h(t) = 4000\delta(t) - 4\times 10^5 e^{-100t}$$

$$v_o = \int_0^{10^{-3}} (-20\times 10^{-3})[4000\delta(\lambda) - 4\times 10^5 e^{-100\lambda}]\,d\lambda$$

$$+\int_{10^{-3}}^{5\times 10^{-3}} (10\times 10^{-3})[-4\times 10^5 e^{-100\lambda}]\,d\lambda$$

$$= -80 + 8000\int_0^{10^{-3}} e^{-100\lambda}\,d\lambda - 4000\int_{10^{-3}}^{5\times 10^{-3}} e^{-100\lambda}\,d\lambda$$

$$= -80 - 80(e^{-0.1} - 1) + 40(e^{-0.5} - e^{-0.1})$$

$$= 40e^{-0.5} - 120e^{-0.1} = -84.32\,\mathrm{V}$$

Alternate:

$$I_g = \int_0^{4\times10^{-3}} (10\times10^{-3})e^{-st}\,dt + \int_{4\times10^{-3}}^{6\times10^{-3}} (-20\times10^{-3})e^{-st}\,dt$$

$$= \left[\frac{10}{s} - \frac{30}{s}e^{-4\times10^{-3}s} + \frac{20}{s}e^{-6\times10^{-3}s}\right]\times 10^{-3}$$

$$V_o = I_gH(s) = \frac{40}{s+100}[1 - 3e^{-4\times10^{-3}s} + 2e^{-6\times10^{-3}s}]$$

$$= \frac{40}{s+100} - \frac{120e^{-4\times10^{-3}s}}{s+100} + \frac{80e^{-6\times10^{-3}s}}{s+100}]$$

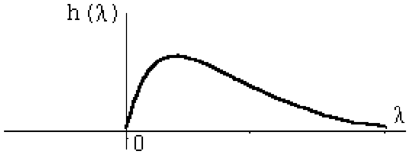
$$v_o(t) = 40e^{-100t} - 120e^{-100(t-4\times10^{-3})}u(t-4\times10^{-3})$$
$$+80e^{-100(t-6\times10^{-3})}u(t-6\times10^{-3})$$

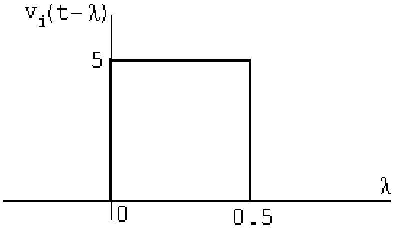
$$v_o(5\times10^{-3}) = 40e^{-0.5} - 120e^{-0.1} + 80(0) = -84.32\,\text{V} \quad \text{(checks)}$$

P 13.74 **[a]** $H(s) = \dfrac{V_o}{V_i} = \dfrac{1/LC}{s^2 + (R/L)s + (1/LC)}$

$$= \frac{100}{s^2+20s+100} = \frac{100}{(s+10)^2}$$

$$h(\lambda) = 100\lambda e^{-10\lambda}u(\lambda)$$





$0 \le t \le 0.5$:

$$v_o = 500\int_0^t \lambda e^{-10\lambda}\,d\lambda$$

$$= 500\left\{\frac{e^{-10\lambda}}{100}(-10\lambda - 1)\Big|_0^t\right\}$$

$$= 5[1 - e^{-10t}(10t+1)]$$

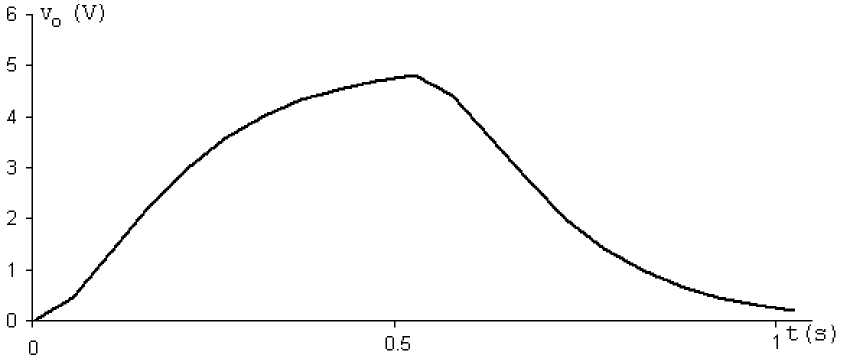
$0.5 \le t \le \infty$:

$$v_o = 500\int_{t-0.5}^{t} \lambda e^{-10\lambda}\, d\lambda$$

$$= 500\left\{\frac{e^{-10\lambda}}{100}(-10\lambda - 1)\Bigg|_{t-0.5}^{t}\right\}$$

$$= 5e^{-10t}[e^{5}(10t-4) - 10t - 1]$$

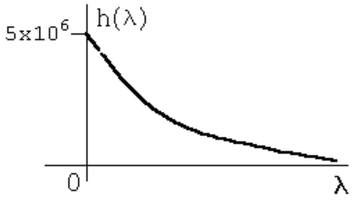
[b]

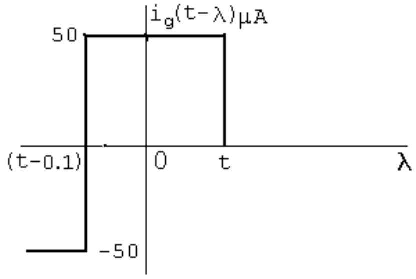


P 13.75 **[a]** $I_o = \dfrac{V_o}{10^5} + \dfrac{V_o s}{5\times10^6} = \dfrac{V_o(s+50)}{5\times10^6}$

$$\frac{V_o}{I_g} = H(s) = \frac{5\times10^6}{s+50}$$

$$h(\lambda) = 5\times10^6 e^{-50\lambda}u(\lambda)$$



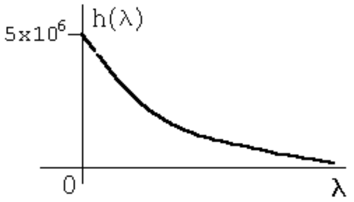


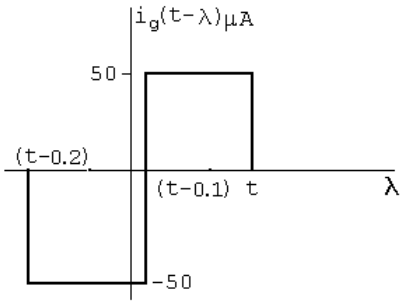
$0 \le t \le 0.1\,\text{s}$:

$$v_o = \int_0^t (50 \times 10^{-6})(5 \times 10^6)e^{-50\lambda}\,d\lambda = 250\frac{e^{-50\lambda}}{-50}\Bigg|_0^t$$

$$= 5(1 - e^{-50t})\,\text{V}$$

$0.1\,\text{s} \le t \le 0.2\,\text{s}$:





$$v_o = \int_0^{t-0.1} (-50 \times 10^{-6})(5 \times 10^6 e^{-50\lambda}\,d\lambda)$$

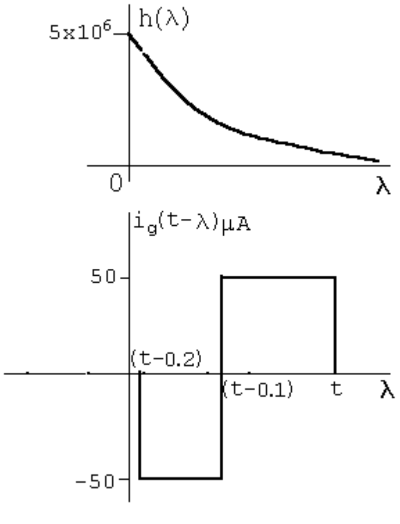
$$+\int_{t-0.1}^{t} (50 \times 10^{-6})(5 \times 10^6 e^{-50\lambda}\,d\lambda)$$

$$= -250\frac{e^{-50\lambda}}{-50}\Bigg|_0^{t-0.1} + 250\frac{e^{-50\lambda}}{-50}\Bigg|_{t-0.1}^{t}$$

$$= 5\left[e^{-50(t-0.1)} - 1\right] - 5\left[e^{-50t} - e^{-50(t-0.1)}\right]$$

$$v_o = [10e^{-50(t-0.1)} - 5e^{-50t} - 5]\,\text{V}$$

$0.2\,\mathrm{s} \le t \le \infty$:



$$v_o = \int_{t-0.2}^{t-0.1} -250e^{-50\lambda}\,d\lambda + \int_{t-0.1}^{t} 250e^{-50\lambda}\,d\lambda$$

$$= \left[5e^{-50\lambda}\Big|_{t-0.2}^{t-0.1} - 5e^{-50\lambda}\Big|_{t-0.1}^{t}\right]$$

$$v_o = [10e^{-50(t-0.1)} - 5e^{-50(t-0.2)} - 5e^{-50t}]\,\mathrm{V}$$

[b] $I_o = \dfrac{V_o s}{5\times10^6} = \dfrac{s}{5\times10^6}\cdot\dfrac{5\times10^6 I_g}{s+50}$

$$\frac{I_o}{I_g} = H(s) = \frac{s}{s+50} = 1 - \frac{50}{s+50}$$

$$h(\lambda) = \delta(\lambda) - 50e^{-50\lambda}$$

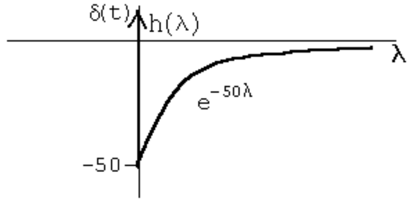
$0 < t < 0.1\,\mathrm{s}$:

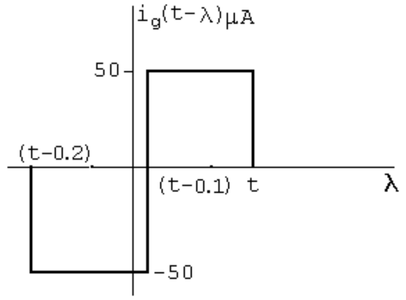
$$i_o = \int_0^t (50\times10^{-6})[\delta(\lambda) - 50e^{-50\lambda}]\,d\lambda$$

$$= 50\times10^{-6} - 25\times10^{-3}\frac{e^{-50\lambda}}{-50}\Big|_0^t$$

$$= 50\times10^{-6} + 50\times10^{-6}[e^{-50t} - 1] = 50e^{-50t}\,\mu\mathrm{A}$$

$0.1\,\text{s} < t < 0.2\,\text{s}$:





$$i_o = \int_0^{t-0.1} (-50 \times 10^{-6})[\delta(\lambda) - 50e^{-50\lambda}]\,d\lambda$$

$$+\int_{t-0.1}^{t} (50 \times 10^{-6})(-50e^{-50\lambda})\,d\lambda$$

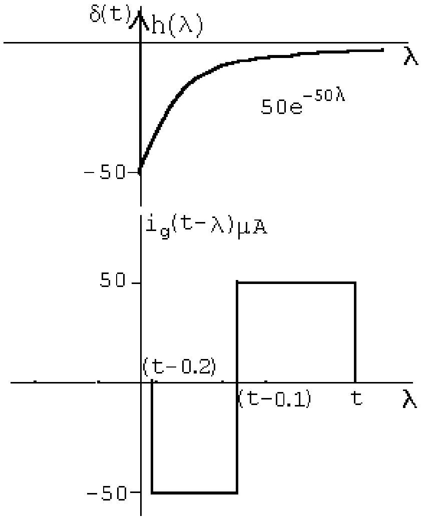
$$= -50 \times 10^{-6} + 2.5 \times 10^{-3} \frac{e^{-50\lambda}}{-50}\Big|_0^{t-0.1} - 2.5 \times 10^{-3} \frac{e^{-50\lambda}}{-50}\Big|_{t-0.1}^{t}$$

$$= -50 \times 10^{-6} - 50 \times 10^{-6} e^{-50(t-0.1)} + 50 \times 10^{-6}$$

$$+50 \times 10^{-6} e^{-50t} - 50 \times 10^{-6} e^{-50(t-0.1)}$$

$$= 50e^{-50t} - 100e^{-50(t-0.1)}\,\mu\text{A}$$

$0.2\,\mathrm{s} < t < \infty$:



$$i_o = \int_{t-0.2}^{t-0.1}(-50 \times 10^{-6})(-50e^{-50\lambda})\,d\lambda$$

$$+\int_{t-0.1}^{t}(50 \times 10^{-6})(-50e^{-50\lambda})\,d\lambda$$

$$= 50e^{-50t} - 100e^{-50(t-0.1)} + 50e^{-50(t-0.2)}\,\mu\mathrm{A}$$

[c] At $t = 0.1^-$:

$$v_o = 5(1 - e^{-5}) = 4.97\,\mathrm{V}; \qquad i_{100\mathrm{k}\Omega} = \frac{4.97}{0.1} = 49.66\,\mu\mathrm{A}$$

$\therefore\ i_o = 50 - 49.66 = 0.34\,\mu\mathrm{A}$

From the solution for i_o we have

$i_o(0.1^-) = 50e^{-5} = 0.34\,\mu\mathrm{A}$ (checks)

At $t = 0.1^+$:

$v_o(0.1^+) = v_o(0.1^-) = 4.97\,\mathrm{V}$

$i_{100\mathrm{k}\Omega} = 49.66\,\mu\mathrm{A}$

$\therefore\ i_o(0.1^+) = -(50 + 49.66) = -99.66\,\mu\mathrm{A}$

From the solution for i_o we have

$i_o(0.1^+) = 50e^{-5} - 100 = 99.66\,\mu\mathrm{A}$ (checks)

At $t = 0.2^-$:

$v_o = 10e^{-5} - 5e^{-10} - 5 = -4.93\,\mathrm{V}$

$i_{100\text{k}\Omega} = 49.33\,\mu\text{A}$

$i_o = -50 + 49.33 = -0.67\,\mu\text{A}$

From the solution for i_o,

$v_o(0.2^-) = 50e^{-10} - 100e^{-5} = -0.67\,\mu\text{A}$ (checks)

At $t = 0.2^+$:

$v_o(0.2^+) = v_o(0.2^-) = -4.93\,\text{V}; \qquad i_{100\text{k}\Omega} = -49.33\,\mu\text{A}$

$i_o = 0 + 49.33 = 49.33\,\mu\text{A}$

From the solution for i_o,

$i_o(0.2^+) = 50e^{-10} - 100e^{-5} + 50 = 49.33\,\mu\text{A}$(checks)

P 13.76 **[a]** $Y(s) = \int_0^\infty y(t)e^{-st}\,dt$

$$Y(s) = \int_0^\infty e^{-st}\left[\int_0^\infty h(\lambda)x(t-\lambda)\,d\lambda\right]dt$$

$$= \int_0^\infty\int_0^\infty e^{-st}h(\lambda)x(t-\lambda)\,d\lambda\,dt$$

$$= \int_0^\infty h(\lambda)\int_0^\infty e^{-st}x(t-\lambda)\,dt\,d\lambda$$

But $x(t-\lambda) = 0$ when $t < \lambda$.

Therefore $Y(s) = \int_0^\infty h(\lambda)\int_\lambda^\infty e^{-st}x(t-\lambda)\,dt\,d\lambda$

Let $u = t - \lambda$; $du = dt$; $u = 0$ when $t = \lambda$; $u = \infty$ when $t = \infty$.

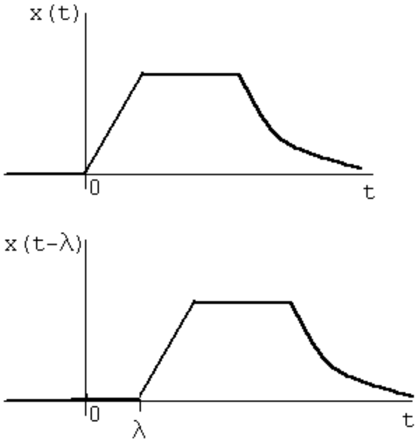
$$Y(s) = \int_0^\infty h(\lambda)\int_0^\infty e^{-s(u+\lambda)}x(u)\,du\,d\lambda$$

$$= \int_0^\infty h(\lambda)e^{-s\lambda}\int_0^\infty e^{-su}x(u)\,du\,d\lambda$$

$$= \int_0^\infty h(\lambda)e^{-s\lambda}X(s)\,d\lambda = H(s)\,X(s)$$

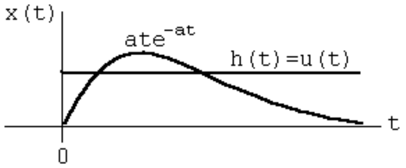
Note on $x(t-\lambda) = 0, \qquad t < \lambda$

We are using one-sided Laplace transforms; therefore $h(t)$ and $x(t)$ are assumed zero for $t < 0$.



[b] $F(s) = \dfrac{a}{s(s+a)^2} = \dfrac{1}{s} \cdot \dfrac{a}{(s+a)^2} = H(s)X(s)$

$\therefore \quad h(t) = u(t), \qquad x(t) = at\,e^{-at}u(t)$



$$\therefore \quad f(t) = \int_0^t (1)a\lambda e^{-a\lambda}\,d\lambda = a\left[\frac{e^{-a\lambda}}{a^2}(-a\lambda - 1)\right|_0^t$$

$$= \frac{1}{a}[e^{-at}(-at-1) - 1(-1)] = \frac{1}{a}[1 - e^{-at} - ate^{-at}]$$

$$= \left[\frac{1}{a} - \frac{1}{a}e^{-at} - te^{-at}\right]u(t)$$

Check:

$$F(s) = \frac{a}{s(s+a)^2} = \frac{K_0}{s} + \frac{K_1}{(s+a)^2} + \frac{K_2}{s+a}$$

$$K_0 = \frac{1}{a}; \qquad K_1 = -1; \qquad K_2 = \frac{d}{ds}\left(\frac{a}{s}\right)_{s=-a} = -\frac{1}{a}$$

$$f(t) = \left[\frac{1}{a} - te^{-at} - \frac{1}{a}e^{-at}\right]u(t)$$

P 13.77 $H(j3) = \dfrac{4(3+j3)}{-9+j24+41} = 0.42\underline{/8.13^\circ}$

$\therefore \quad v_o(t) = 16.97\cos(3t + 8.13^\circ)\text{ V}$

P 13.78 $V_o = \dfrac{50}{s+8000} - \dfrac{20}{s+5000} = \dfrac{30(s+3000)}{(s+5000)(s+8000)}$

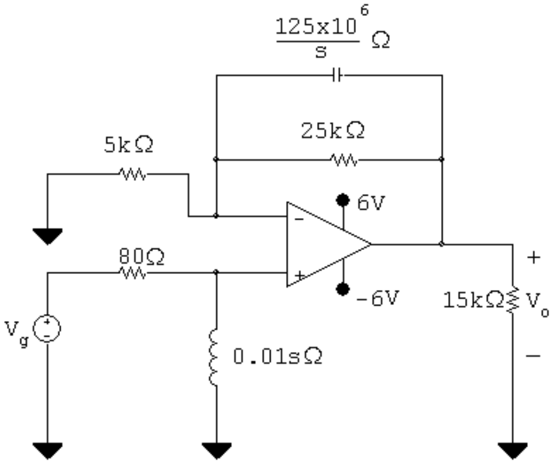
$$V_o = H(s)V_g = H(s)\left(\frac{30}{s}\right)$$

$$\therefore \quad H(s) = \frac{s(s+3000)}{(s+5000)(s+8000)}$$

$$H(j6000) = \frac{(j6000)(3000+j6000)}{(5000+j6000)(8000+j6000)} = 0.52\underline{/66.37^\circ}$$

$$\therefore \quad v_o(t) = 61.84\cos(6000t + 66.37^\circ)\,\text{V}$$

P 13.79 [**a**]



$$V_p = \frac{0.01s}{80+0.01s}V_g = \frac{s}{s+8000}V_g$$

$$V_n = V_p$$

$$\frac{V_n}{5000} + \frac{V_n - V_o}{25{,}000} + (V_n - V_o)8\times10^{-9}s = 0$$

$$5V_n + V_n - V_o + (V_n - V_o)2\times10^{-4}s = 0$$

$$6V_n + 2\times10^{-4}sV_n = V_o + 2\times10^{-4}sV_o$$

$$2\times10^{-4}V_n(s+30{,}000) = 2\times10^{-4}V_o(s+5000)$$

$$V_o = \frac{s+30{,}000}{s+5000}V_i = \left(\frac{s+30{,}000}{s+5000}\right)\left(\frac{sV_g}{s+8000}\right)$$

$$H(s) = \frac{V_o}{V_g} = \frac{s(s+30{,}000)}{(s+5000)(s+8000)}$$

[b] $v_g = 0.6u(t); \qquad V_g = \dfrac{0.6}{s}$

$$V_o = \frac{0.6(s+30{,}000)}{(s+5000)(s+8000)} = \frac{K_1}{s+5000} + \frac{K_2}{s+8000}$$

$$K_1 = \frac{0.6(25{,}000)}{3000} = 5; \qquad K_2 = \frac{0.6(22{,}000)}{-3000} = -4.4$$

$$\therefore \quad v_o(t) = (5e^{-5000t} - 4.4e^{-8000t})u(t)\,\text{V}$$

[c] $V_g = 2\cos 10{,}000t\,\text{V}$

$$H(j\omega) = \frac{j10{,}000(30{,}000 + j10{,}000)}{(5000 + j10{,}000)(8000 + j10{,}000)} = 2.21\underline{/-6.34^\circ}$$

$$\therefore \qquad v_o = 4.42\cos(10{,}000t - 6.34^\circ)\,\text{V}$$

P 13.80 **[a]** $H(s) = \dfrac{-Z_f}{Z_i}$

$$Z_f = \frac{(1/C_f)}{s + (1/R_fC_f)} = \frac{10^8}{s+1000}$$

$$Z_i = \frac{R_i[s + (1/R_iC_i)]}{s} = \frac{10{,}000(s+400)}{s}$$

$$H(s) = \frac{-10^4 s}{(s+400)(s+1000)}$$

[b] $H(j400) = \dfrac{-10^4(j400)}{(400+j400)(1000+j400)} = 6.565\underline{/-156.8^\circ}$

$$v_o(t) = 13.13\cos(400t - 156.8^\circ)\,\text{V}$$

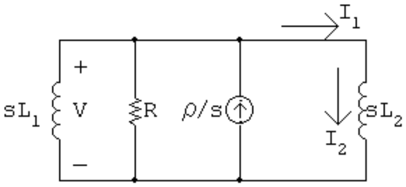
P 13.81 Original charge on C_1; $\quad q_1 = V_0C_1$

The charge transferred to C_2; $\quad q_2 = V_0C_e = \dfrac{V_0C_1C_2}{C_1+C_2}$

The charge remaining on C_1; $\quad q_1' = q_1 - q_2 = \dfrac{V_0C_1^2}{C_1+C_2}$

Therefore $\quad V_2 = \dfrac{q_2}{C_2} = \dfrac{V_0C_1}{C_1+C_2} \quad$ and $\quad V_1 = \dfrac{q_1'}{C_1} = \dfrac{V_0C_1}{C_1+C_2}$

P 13.82 **[a]** The s-domain circuit is



The node-voltage equation is $\dfrac{V}{sL_1} + \dfrac{V}{R} + \dfrac{V}{sL_2} = \dfrac{\rho}{s}$

Therefore $V = \dfrac{\rho R}{s + (R/L_e)}$ where $L_e = \dfrac{L_1 L_2}{L_1 + L_2}$

Therefore $v = \rho R e^{-(R/L_e)t} u(t)\,\text{V}$

[b] $I_1 = \dfrac{V}{R} + \dfrac{V}{sL_2} = \dfrac{\rho[s + (R/L_2)]}{s[s + (R/L_e)]} = \dfrac{K_0}{s} + \dfrac{K_1}{s + (R/L_e)}$

$K_0 = \dfrac{\rho L_1}{L_1 + L_2}$; $K_1 = \dfrac{\rho L_2}{L_1 + L_2}$

Thus we have $i_1 = \dfrac{\rho}{L_1 + L_2}[L_1 + L_2 e^{-(R/L_e)t}]u(t)\,\text{A}$

[c] $I_2 = \dfrac{V}{sL_2} = \dfrac{(\rho R/L_2)}{s[s + (R/L_e)]} = \dfrac{K_2}{s} + \dfrac{K_3}{s + (R/L_e)}$

$K_2 = \dfrac{\rho L_1}{L_1 + L_2}$; $K_3 = \dfrac{-\rho L_1}{L_1 + L_2}$

Therefore $i_2 = \dfrac{\rho L_1}{L_1 + L_2}[1 - e^{-(R/L_e)t}]u(t)$

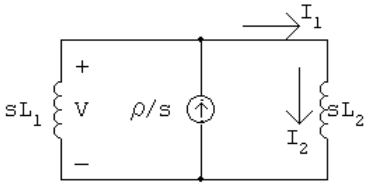
[d] $\lambda(t) = L_1 i_1 + L_2 i_2 = \rho L_1$

P 13.83 **[a]** As $R \to \infty$, $v(t) \to \rho L_e \delta(t)$ since the area under the impulse generating function is ρL_e.

$i_1(t) \to \dfrac{\rho L_1}{L_1 + L_2} u(t)\,\text{A}$ as $R \to \infty$

$i_2(t) \to \dfrac{\rho L_1}{L_1 + L_2} u(t)\,\text{A}$ as $R \to \infty$

[b] The s-domain circuit is



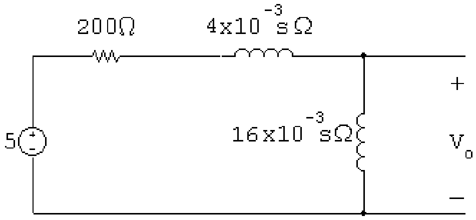
$\dfrac{V}{sL_1} + \dfrac{V}{sL_2} = \dfrac{\rho}{s}$; therefore $V = \dfrac{\rho L_1 L_2}{L_1 + L_2} = \rho L_e$

Therefore $v(t) = \rho L_e \delta(t)$

$$I_1 = I_2 = \frac{V}{sL_2} = \left(\frac{\rho L_1}{L_1 + L_2}\right)\left(\frac{1}{s}\right)$$

Therefore $i_1 = i_2 = \frac{\rho L_1}{L_1 + L_2} u(t)$ A

P 13.84 **[a]**



$$V_o = \frac{5}{200 + 20 \times 10^{-3}s} \cdot 16 \times 10^{-3}s$$

$$= \frac{4s}{s + 10{,}000} = 4 - \frac{40{,}000}{s + 10{,}000}$$

$$v_o(t) = 4\delta(t) - 40{,}000e^{-10{,}000t}u(t)\text{ V}$$

[b] At $t = 0$ the voltage impulse establishes a current in the inductors; thus

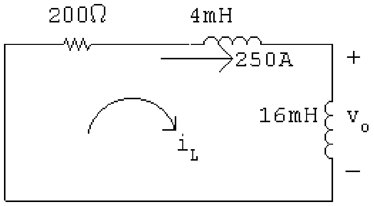
$$i_L(0) = \frac{10^3}{20}\int_{0^-}^{0^+} 5\delta(t)\,dt = 250\text{ A}$$

It follows that since $i_L(0^-) = 0$ that

$$\frac{di_L}{dt}(0) = 250\delta(t)$$

$$\therefore \quad v_o(0) = (16 \times 10^{-3})(250\delta(t)) = 4\delta(t)$$

This agrees with our solution.
At $t = 0^+$ our circuit is



$$\therefore \quad i_L(t) = 250e^{-t/\tau}\text{ A}, \qquad t \geq 0^+$$

$$\tau = L/R = 0.1\text{ ms}$$

$$\therefore \quad i_L(t) = 250e^{-10{,}000t}\text{ A}, \qquad t \geq 0^+$$

$$v_o(t) = 16 \times 10^{-3}\frac{di_L}{dt} = -40{,}000e^{-10{,}000t}\,\text{V}, \qquad t \geq 0^+$$

which agrees with our solution.

P 13.85 **[a]** $Z_1 = \dfrac{1/C_1}{s + 1/R_1C_1} = \dfrac{25 \times 10^{10}}{s + 20 \times 10^4}\Omega$

$$Z_2 = \frac{1/C_2}{s + 1/R_2C_2} = \frac{6.25 \times 10^{10}}{s + 12{,}500}\Omega$$

$$\frac{V_0}{Z_2} + \frac{V_0 - 10/s}{Z_1} = 0$$

$$\frac{V_0(s + 12{,}500)}{6.25 \times 10^{10}} + \frac{V_0(s + 20 \times 10^4)}{25 \times 10^{10}} = \frac{10}{s}\,\frac{(s + 20 \times 10^4)}{25 \times 10^{10}}$$

$$V_0 = \frac{2(s + 200{,}000)}{s(s + 50{,}000)} = \frac{K_1}{s} + \frac{K_2}{s + 50{,}000}$$

$$K_1 = \frac{2(200{,}000)}{50{,}000} = 8$$

$$K_2 = \frac{2(150{,}000)}{-50{,}000} = -6$$

$$\therefore\quad v_o = [8 - 6e^{-50{,}000t}]u(t)\,\text{V}$$

[b] $I_0 = \dfrac{V_0}{Z_2} = \dfrac{2(s + 200{,}000)(s + 12{,}500)}{s(s + 50{,}000)6.25 \times 10^{10}}$

$$= 32 \times 10^{-12}\left[1 + \frac{162{,}500s + 25 \times 10^8}{s(s + 50{,}000)}\right]$$

$$= 32 \times 10^{-12}\left[1 + \frac{K_1}{s} + \frac{K_2}{s + 50{,}000}\right]$$

$$K_1 = 50{,}000; \qquad K_2 = 112{,}500$$

$$i_o = 32\delta(t) + [1.6 \times 10^6 + 3.6 \times 10^6 e^{-50{,}000t}]u(t)\ \text{pA}$$

[c] When $C_1 = 64\,\text{pF}$

$$Z_1 = \frac{156.25 \times 10^8}{s + 12{,}500}\Omega$$

$$\frac{V_0(s + 12{,}500)}{625 \times 10^8} + \frac{V_0(s + 12{,}500)}{156.25 \times 10^8} = \frac{10}{s}\,\frac{(s + 12{,}500)}{156.25 \times 10^8}$$

$$\therefore\quad V_0 + 4V_0 = \frac{40}{s}$$

$$V_0 = \frac{8}{s}$$

$$v_o = 8u(t)\,\text{V}$$

$$I_0 = \frac{V_0}{Z_2} = \frac{8}{s}\,\frac{(s+12{,}500)}{6.25\times10^{10}} = 128\times10^{-12}\left[1+\frac{12{,}500}{s}\right]$$

$$i_o(t) = 128\delta(t) + 1.6\times10^{6}u(t)\,\text{pA}$$

P 13.86 Let $a = \dfrac{1}{R_1C_1} = \dfrac{1}{R_2C_2}$

Then $Z_1 = \dfrac{1}{C_1(s+a)}$ and $Z_2 = \dfrac{1}{C_2(s+a)}$

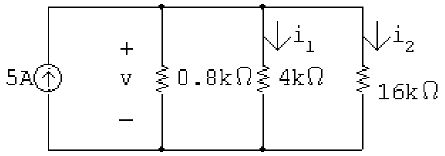
$$\frac{V_o}{Z_2} + \frac{V_o}{Z_1} = \frac{10/s}{Z_1}$$

$$V_oC_2(s+a) + V_0C_1(s+a) = (10/s)C_1(s+a)$$

$$V_o = \frac{10}{s}\left(\frac{C_1}{C_1+C_2}\right)$$

Thus, v_o is the input scaled by the factor $\dfrac{C_1}{C_1+C_2}$.

P 13.87 **[a]** For $t < 0$:



$$R_{\text{eq}} = 0.8\,\text{k}\Omega\|4\,\text{k}\Omega\|16\,\text{k}\Omega = 0.64\,\text{k}\Omega; \qquad v = 5(640) = 3200\,\text{V}$$

$$i_1(0^-) = \frac{3200}{4000} = 0.8\,\text{A}; \qquad i_2(0^-) = \frac{3200}{1600} = 0.2\,\text{A}$$

[b] For $t > 0$:

$$i_1 + i_2 = 0$$

$$8(\Delta i_1) = 2(\Delta i_2)$$

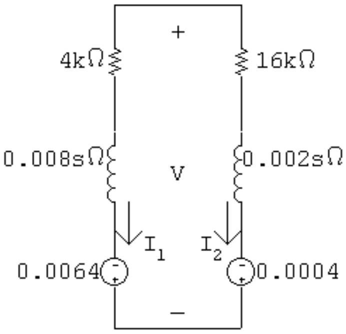
$$i_1(0^-) + \Delta i_1 + i_2(0^-) + \Delta i_2 = 0; \qquad \text{therefore} \quad \Delta i_1 = -0.2\,\text{A}$$

$$\Delta i_2 = -0.8\,\text{A}; \qquad i_1(0^+) = 0.8 - 0.2 = 0.6\,\text{A}$$

[c] $i_2(0^-) = 0.2\,\text{A}$

[d] $i_2(0^+) = 0.2 - 0.8 = -0.6\,\text{A}$

[e] The s-domain equivalent circuit for $t > 0$ is



$$I_1 = \frac{0.006}{0.01s + 20{,}000} = \frac{0.6}{s + 2 \times 10^6}$$

$$i_1(t) = 0.6e^{-2\times10^6 t}u(t)\,\text{A}$$

[f] $i_2(t) = -i_1(t) = -0.6e^{-2\times10^6 t}u(t)\,\text{A}$

[g] $$V = -0.0064 + (0.008s + 4000)I_1 = \frac{-0.0016(s + 6.5 \times 10^6)}{s + 2 \times 10^6}$$

$$= -1.6 \times 10^{-3} - \frac{7200}{s + 2 \times 10^6}$$

$$v(t) = [-1.6 \times 10^{-3}\delta(t)] - [7200e^{-2\times10^6 t}u(t)]\,\text{V}$$

P 13.88 [a] For $t < 0$, $0.5v_1 = 2v_2$; therefore $v_1 = 4v_2$

$v_1 + v_2 = 100$; therefore $v_1(0^-) = 80\,\text{V}$

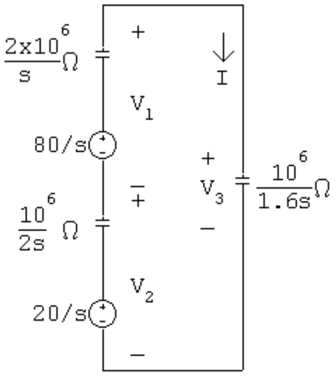
[b] $v_2(0^-) = 20\,\text{V}$

[c] $v_3(0^-) = 0\,\text{V}$

[d] For $t > 0$:

$$I = \frac{100/s}{3.125/s} \times 10^{-6} = 32 \times 10^{-6}$$

$$i(t) = 32\delta(t)\,\mu\text{A}$$



[e] $v_1(0^+) = -\frac{10^6}{0.5}\int_{0^-}^{0^+} 32\times 10^{-6}\delta(t)\,dt + 80 = -64 + 80 = 16\,\text{V}$

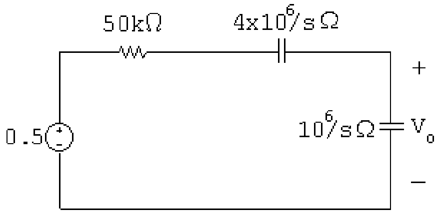
[f] $v_2(0^+) = -\frac{10^6}{2}\int_{0^-}^{0^+} 32\times 10^{-6}\delta(t)\,dt + 20 = -16 + 20 = 4\,\text{V}$

[g] $V_3 = \frac{0.625\times 10^6}{s}\cdot 32\times 10^{-6} = \frac{20}{s}$

$v_3(t) = 20u(t)\,\text{V}; \qquad v_3(0^+) = 20\,\text{V}$

Check: $v_1(0^+) + v_2(0^+) = v_3(0^+)$

P 13.89 **[a]**



$$V_o = \frac{0.5}{50{,}000 + 5\times 10^6/s}\cdot\frac{10^6}{s}$$

$$\frac{500{,}000}{50{,}000s + 5\times 10^6} = \frac{10}{s+100}$$

$v_o = 10e^{-100t}u(t)\,\text{V}$

[b] At $t = 0$ the current in the $1\,\mu$F capacitor is $10\delta(t)\,\mu$A

$\therefore \qquad v_o(0^+) = 10^6\int_{0^-}^{0^+} 10\times 10^{-6}\delta(t)\,dt = 10\,\text{V}$

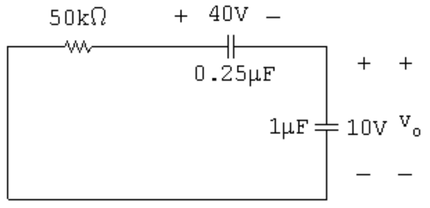
After the impulsive current has charged the $1\,\mu$F capacitor to 10 V it discharges through the 50 kΩ resistor.

$C_e = \frac{C_1C_2}{C_1+C_2} = \frac{0.25}{1.25} = 0.2\,\mu\text{F}$

$\tau = (50{,}000)(0.2\times 10^{-6}) = 10^{-2}$

$\frac{1}{\tau} = 100$ (checks)

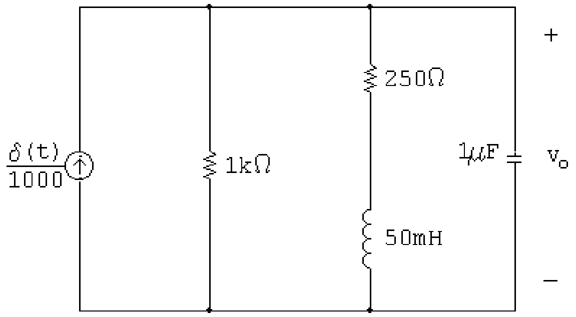
Note – after the impulsive current passes the circuit becomes



The solution for v_o in this circuit is also

$$v_o = 10e^{-100t}u(t)\,\text{V}$$

P 13.90 **[a]** After making a source transformation, the circuit is as shown. The impulse current will pass through the capacitive branch since it appears as a short circuit to the impulsive current,



$$\text{Therefore} \quad v_o(0^+) = 10^6 \int_{0^-}^{0^+} \left[\frac{\delta(t)}{1000}\right] dt = 1000\,\text{V}$$

$$\text{Therefore} \quad w_C = (0.5)Cv^2 = 0.5\,\text{J}$$

[b] $i_L(0^+) = 0; \qquad \text{therefore} \quad w_L = 0\,\text{J}$

[c] $V_o(10^{-6})s + \dfrac{V_o}{250 + 0.05s} + \dfrac{V_o}{1000} = 10^{-3}$

Therefore

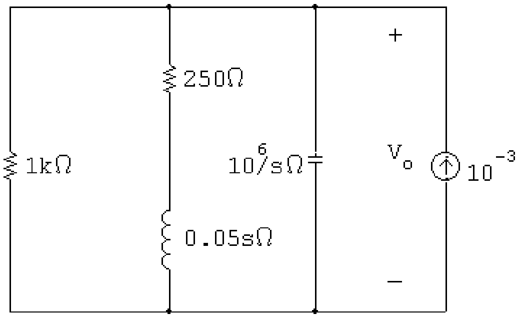
$$V_o = \frac{1000(s + 5000)}{s^2 + 6000s + 25 \times 10^6}$$

$$= \frac{K_1}{s + 3000 - j4000} + \frac{K_1^*}{s + 3000 + j4000}$$

$$K_1 = 559.02\underline{/-26.57^\circ}; \qquad K_1^* = 559.02\underline{/26.57^\circ}$$

$$v_o = [1118.03e^{-3000t}\cos(4000t - 26.57^\circ)]u(t)\,\text{V}$$

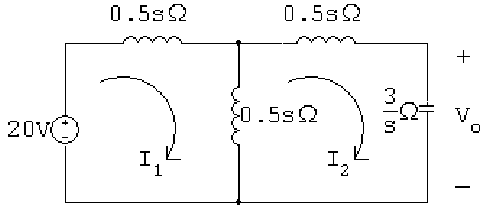
[d] The s-domain circuit is



$$\frac{V_o s}{10^6} + \frac{V_o}{250 + 0.05s} + \frac{V_o}{1000} = 10^{-3}$$

Note that this equation is identical to that derived in part [c], therefore the solution for V_o will be the same.

P 13.91 **[a]**



$$20 = sI_1 - 0.5sI_2$$

$$0 = -0.5sI_1 + \left(s + \frac{3}{s}\right) I_2$$

$$\Delta = \begin{vmatrix} s & -0.5s \\ -0.5s & (s+3/s) \end{vmatrix} = s^2 + 3 - 0.25s^2 = 0.75(s^2+4)$$

$$N_1 = \begin{vmatrix} 20 & -0.5s \\ 0 & (s+3/s) \end{vmatrix} = 20s + \frac{60}{s} = \frac{20s^2+60}{s} = \frac{20(s^2+3)}{s}$$

$$I_1 = \frac{N_1}{\Delta} = \frac{20(s^2+3)}{s(0.75)(s^2+4)} = \frac{80}{3}\cdot\frac{s^2+3}{s(s^2+4)}$$

$$= \frac{K_0}{s} + \frac{K_1}{s-j2} + \frac{K_1^*}{s+j2}$$

$$K_0 = \frac{80}{3}\left(\frac{3}{4}\right) = 20; \qquad K_1 = \frac{80}{3}\left[\frac{-4+3}{(j2)(j4)}\right] = \frac{10}{3}\underline{/0^\circ}$$

$$\therefore\quad i_1 = \left[20 + \frac{20}{3}\cos 2t\right] u(t)\text{ A}$$

[b] $$N_2 = \begin{vmatrix} s & 20 \\ -0.5s & 0 \end{vmatrix} = 10s$$

$$I_2 = \frac{N_2}{\Delta} = \frac{10s}{0.75(s^2+4)} = \frac{40}{3}\left(\frac{s}{s^2+4}\right) = \frac{K_1}{s-j2} + \frac{K_1^*}{s+j2}$$

$$K_1 = \frac{40}{3}\left(\frac{j2}{j4}\right) = \frac{20}{3}\underline{/0^\circ}$$

$$i_2 = \left[\frac{40}{3}\cos 2t\right] u(t)\text{ A}$$

[c] $V_0 = \frac{3}{s}I_2 = \left(\frac{3}{s}\right)\frac{40}{3}\left(\frac{s}{s^2+4}\right) = \frac{40}{s^2+4} = \frac{K_1}{s-j2} + \frac{K_1^*}{s+j2}$

$$K_1 = \frac{40}{j4} = -j10 = 10\underline{/90^\circ}$$

$$v_o = 20\cos(2t - 90^\circ) = 20\sin 2t$$

$$v_o = [20\sin 2t]u(t)\,\text{V}$$

[d] Let us begin by noting i_1 jumps from 0 to (80/3) A between 0^- and 0^+ and in this same interval i_2 jumps from 0 to (40/3) A. Therefore in the derivatives of i_1 and i_2 there will be impulses of $(80/3)\delta(t)$ and $(40/3)\delta(t)$, respectively. Thus

$$\frac{di_1}{dt} = \frac{80}{3}\delta(t) - \frac{40}{3}\sin 2t\,\text{A/s}$$

$$\frac{di_2}{dt} = \frac{40}{3}\delta(t) - \frac{80}{3}\sin 2t\,\text{A/s}$$

From the circuit diagram we have

$$20\delta(t) = 1\frac{di_1}{dt} - 0.5\frac{di_2}{dt}$$

$$= \frac{80}{3}\delta(t) - \frac{40}{3}\sin 2t - \frac{20\delta(t)}{3} + \frac{40}{3}\sin 2t$$

$$= 20\delta(t)$$

Thus our solutions for i_1 and i_2 are in agreement with known circuit behavior.

Let us also note the impulsive voltage will impart energy into the circuit. Since there is no resistance in the circuit, the energy will not dissipate. Thus the fact that i_1, i_2, and v_o exist for all time is consistent with known circuit behavior.

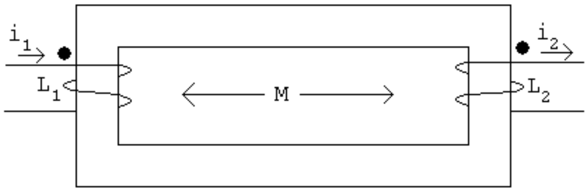
Also note that although i_1 has a dc component, i_2 does not. This follows from known transformer behavior.

Finally we note the flux linkage prior to the appearance of the impulsive voltage is zero. Now since $v = d\lambda/dt$, the impulsive voltage source must be matched to an instantaneous change in flux linkage at $t = 0^+$ of 20. For the given polarity dots and reference directions of i_1 and i_2 we have

$$\lambda(0^+) = L_1 i_1(0^+) + M i_1(0^+) - L_2 i_2(0^+) - M i_2(0^+)$$

$$\lambda(0^+) = 1\left(\frac{80}{3}\right) + 0.5\left(\frac{80}{3}\right) - 1\left(\frac{40}{3}\right) - 0.5\left(\frac{40}{3}\right)$$

$$= \frac{120}{3} - \frac{60}{3} = 20 \quad \text{(checks)}$$



P 13.92 **[a]** The circuit parameters are

$$R_\text{a} = \frac{120^2}{1200} = 12\,\Omega \qquad R_\text{b} = \frac{120^2}{1800} = 8\,\Omega \qquad X_\text{a} = \frac{120^2}{350} = \frac{288}{7}\,\Omega$$

The branch currents are

$$\mathbf{I}_1 = \frac{120\underline{/0^\circ}}{12} = 10\underline{/0^\circ}\text{ A(rms)} \qquad \mathbf{I}_2 = \frac{120\underline{/0^\circ}}{j1440/35} = -j\frac{35}{12} = \frac{35}{12}\underline{/-90^\circ}\text{ A(rms)}$$

$$\mathbf{I}_3 = \frac{120\underline{/0^\circ}}{8} = 15\underline{/0^\circ}\text{ A(rms)}$$

$$\therefore\quad \mathbf{I}_o = \mathbf{I}_1 + \mathbf{I}_2 + \mathbf{I}_3 = 25 - j\frac{35}{12} = 25.17\underline{/-6.65^\circ}\text{ A(rms)}$$

Therefore,

$$i_2 = \left(\frac{35}{12}\right)\sqrt{2}\cos(\omega t - 90^\circ)\text{ A} \qquad \text{and} \qquad i_L = 25.17\sqrt{2}\cos(\omega t - 6.65^\circ)\text{ A}$$

Thus,

$$i_2(0^-) = i_2(0^+) = 0\text{ A} \qquad \text{and} \qquad i_L(0^-) = i_L(0^+) = 25\sqrt{2}\text{ A}$$

[b] Begin by using the s-domain circuit in Fig. 13.60 to solve for V_0 symbolically. Write a single node voltage equation:

$$\frac{V_0 - (V_g + L_\ell I_o)}{sL_\ell} + \frac{V_0}{R_a} + \frac{V_0}{sL_a} = 0$$

$$\therefore\quad V_0 = \frac{(R_a/L_\ell)V_g + I_oR_a}{s + [R_a(L_a + L_\ell)]/L_aL_\ell}$$

where $L_\ell = 1/120\pi$ H, $L_a = 12/35\pi$ H, $R_a = 12\,\Omega$, and $I_0R_a = 300\sqrt{2}$ V. Thus,

$$V_0 = \frac{1440\pi(122.92\sqrt{2}s - 3000\pi\sqrt{2})}{(s + 1475\pi)(s^2 + 14{,}400\pi^2)} + \frac{300\sqrt{2}}{s + 1475\pi}$$

$$= \frac{K_1}{s + 1475\pi} + \frac{K_2}{s - j120\pi} + \frac{K_2^*}{s + j120\pi} + \frac{300\sqrt{2}}{s + 1475\pi}$$

The coefficients are

$$K_1 = -121.18\sqrt{2}\,\text{V} \qquad K_2 = 61.03\sqrt{2}\underline{/6.85^\circ}\text{ V} \qquad K_2^* = 61.03\sqrt{2}\underline{/-6.85^\circ}$$

Note that $K_1 + 300\sqrt{2} = 178.82\sqrt{2}$ V. Thus, the inverse transform of V_0 is

$$v_0 = 178.82\sqrt{2}e^{-1475\pi t} + 122.06\sqrt{2}\cos(120\pi t + 6.85^\circ)\text{ V}$$

Initially,

$$v_0(0^+) = 178.82\sqrt{2} + 122.06\sqrt{2}\cos 6.85^\circ = 300\sqrt{2}\text{ V}$$

Note that at $t = 0^+$ the initial value of i_L, which is $25\sqrt{2}$ A, exists in the $12\,\Omega$ resistor R_a. Thus, the initial value of V_0 is $(25\sqrt{2})(12) = 300\sqrt{2}$ V.

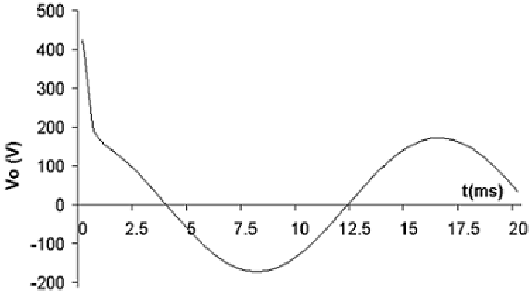
[c] The phasor domain equivalent circuit has a $j1\,\Omega$ inductive impedance in series with the parallel combination of a $12\,\Omega$ resistive impedance and a $j1440/35\,\Omega$ inductive impedance (remember that $\omega = 120\pi$ rad/s). Note that $\mathbf{V}_g = 120\underline{/0^\circ} + (25.17\underline{/-6.65^\circ})(j1) = 125.43\underline{/11.50^\circ}$ V(rms). The node voltage equation in the phasor domain circuit is

$$\frac{\mathbf{V}_0 - 125.43\underline{/11.50^\circ}}{j1} + \frac{\mathbf{V}_0}{12} + \frac{35\mathbf{V}_0}{1440} = 0$$

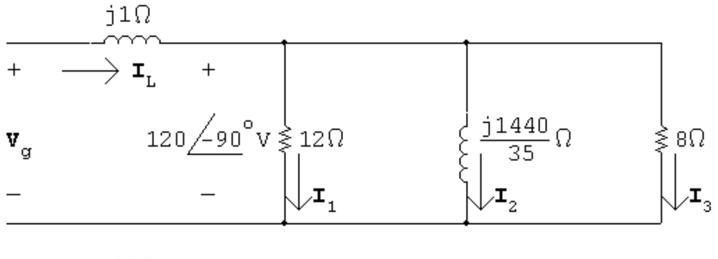
$$\therefore \quad \mathbf{V}_0 = 122.06\underline{/6.85^\circ}\text{ V(rms)}$$

Therefore, $v_0 = 122.06\sqrt{2}\cos(120\pi t + 6.85^\circ)$ V, agreeing with the steady-state component of the result in part (b).

[d] A plot of v_0, generated in Excel, is shown below.



P 13.93 [a] At $t = 0^-$ the phasor domain equivalent circuit is



$$\mathbf{I}_1 = \frac{-j120}{12} = -j10 = 10\underline{/-90^\circ}\text{A (rms)}$$

$$\mathbf{I}_2 = \frac{-j120(35)}{j1440} = -\frac{35}{12} = \frac{35}{12}\underline{/180^\circ}\text{A (rms)}$$

$$\mathbf{I}_3 = \frac{-j120}{8} = -j15 = 15\underline{/-90^\circ}\text{A (rms)}$$

$$\mathbf{I}_L = \mathbf{I}_1 + \mathbf{I}_2 + \mathbf{I}_3 = -\frac{35}{12} - j25 = 25.17\underline{/-96.65^\circ}\text{A (rms)}$$

$$i_L = 25.17\sqrt{2}\cos(120\pi t - 96.65^\circ)\text{A}$$

$$i_L(0^-) = i_L(0^+) = -2.92\sqrt{2}\text{A}$$

$$i_2 = \frac{35}{12}\sqrt{2}\cos(120\pi t + 180^\circ)\text{A}$$

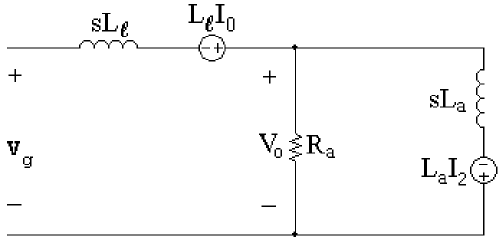
$$i_2(0^-) = i_2(0^+) = -\frac{35}{12}\sqrt{2} = -2.92\sqrt{2}\text{A}$$

$$\mathbf{V}_g = \mathbf{V}_o + j1\mathbf{I}_L$$

$$\begin{aligned}\mathbf{V}_g &= -j120 + 25 - j\frac{35}{12}\\ &= 25 - j122.92 = 125.43\underline{/-78.50^\circ}\text{V (rms)}\\ v_g &= 125.43\sqrt{2}\cos(120\pi t - 78.50^\circ)\text{V}\\ &= 125.43\sqrt{2}[\cos 120\pi t\cos 78.50^\circ + \sin 120\pi t\sin 78.50^\circ]\\ &= 25\sqrt{2}\cos 120\pi t + 122.92\sqrt{2}\sin 120\pi t\end{aligned}$$

$$\therefore\quad V_g = \frac{25\sqrt{2}s + 122.92\sqrt{2}(120\pi)}{s^2 + (120\pi)^2}$$

s-domain circuit:



where

$$L_l = \frac{1}{120\pi}\text{ H};\qquad L_a = \frac{12}{35\pi}\text{ H};\qquad R_a = 12\,\Omega$$

$$i_L(0) = -2.92\sqrt{2}\text{ A};\qquad i_2(0) = -2.92\sqrt{2}\text{ A}$$

The node voltage equation is

$$0 = \frac{V_o - (V_g + i_L(0)L_l)}{sL_l} + \frac{V_o}{R_a} + \frac{V_o + i_2(0)L_a}{sL_a}$$

Solving for V_o yields

$$V_o = \frac{V_gR_a/L_l}{[s + R_a(L_l + L_a)/L_aL_l]} + \frac{R_a[i_L(0) - i_2(0)]}{[s + R_a(L_l + L_a)/L_lL_a]}$$

$$\frac{R_a}{L_l} = 1440\pi$$

$$\frac{R_a(L_l + L_a)}{L_l L_a} = \frac{12(\frac{1}{120\pi} + \frac{12}{35\pi})}{(\frac{12}{35\pi})(\frac{1}{120\pi})} = 1475\pi$$

$$i_L(0) - i_2(0) = -2.92\sqrt{2} + 2.92\sqrt{2} = 0$$

$$\therefore \quad V_o = \frac{1440\pi[25\sqrt{2}s + 122.92\sqrt{2}(120\pi)]}{(s + 1475\pi)[s^2 + (120\pi)^2]}$$

$$= \frac{K_1}{s + 1475\pi} + \frac{K_2}{s - j120\pi} + \frac{K_2^*}{s + j120\pi}$$

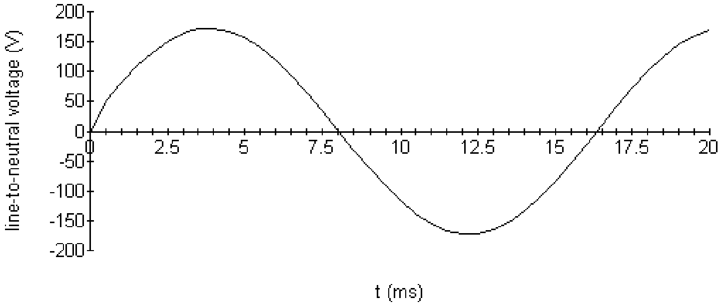
$$K_1 = -14.55\sqrt{2} \qquad\qquad K_2 = 61.03\sqrt{2}\underline{/-83.15^\circ}$$

$$\therefore \quad v_o(t) = -14.55\sqrt{2}e^{-1475\pi t} + 122.06\sqrt{2}\cos(120\pi t - 83.15^\circ)\text{V}$$

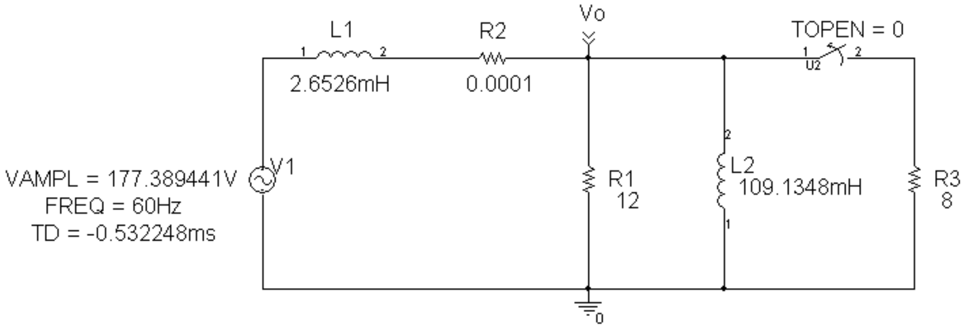
Check:

$$v_o(0) = (-14.55 + 14.55)\sqrt{2} = 0$$

[b]



PSpice schematic



PSpice output file

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**** 07/15/01 07:40:45 ************** PSpice Lite (Mar 2000) *****************

 ** Profile: "SCHEMATIC1-tran"  [ C:\shortcircuits\solutions\p9_76-SCHEMATIC1-tran.sim ]


 ****     CIRCUIT DESCRIPTION


******************************************************************************
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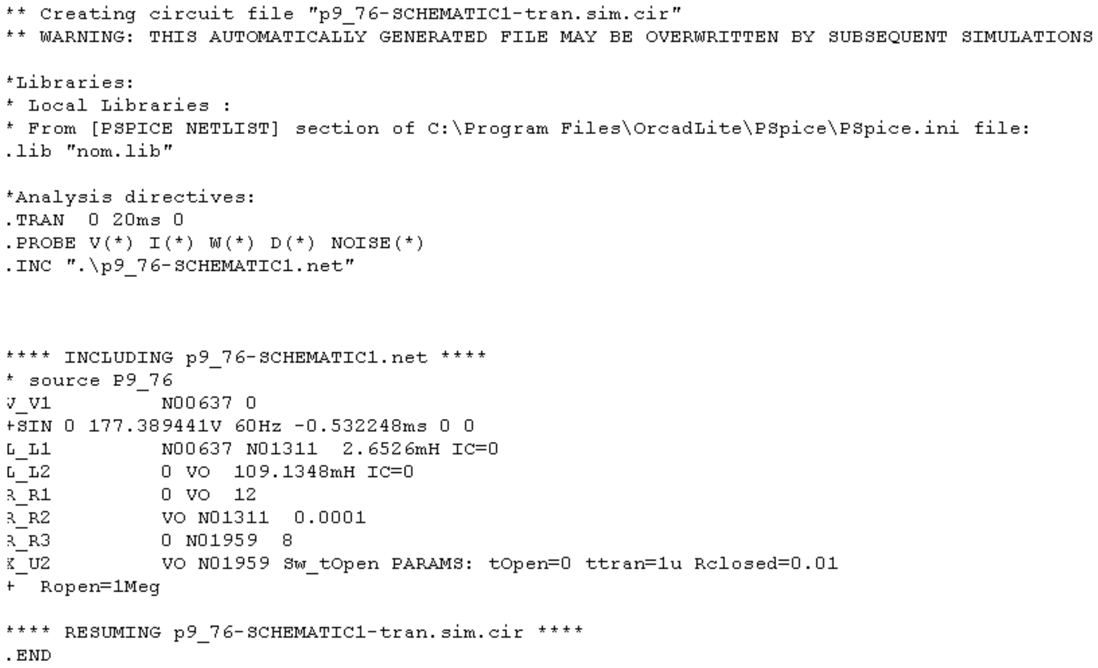

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** Creating circuit file "p9_76-SCHEMATIC1-tran.sim.cir"
** WARNING: THIS AUTOMATICALLY GENERATED FILE MAY BE OVERWRITTEN BY SUBSEQUENT SIMULATIONS

*Libraries:
* Local Libraries :
* From [PSPICE NETLIST] section of C:\Program Files\OrcadLite\PSpice\PSpice.ini file:
.lib "nom.lib"

*Analysis directives:
.TRAN  0 20ms 0
.PROBE V(*) I(*) W(*) D(*) NOISE(*)
.INC ".\p9_76-SCHEMATIC1.net"



**** INCLUDING p9_76-SCHEMATIC1.net ****
* source P9_76
V_V1         N00637 0
+SIN 0 177.389441V 60Hz -0.532248ms 0 0
L_L1         N00637 N01311  2.6526mH IC=0
L_L2         0 VO  109.1348mH IC=0
R_R1         0 VO  12
R_R2         VO N01311  0.0001
R_R3         0 N01959  8
X_U2         VO N01959 Sw_tOpen PARAMS: tOpen=0 ttran=1u Rclosed=0.01
+  Ropen=1Meg

**** RESUMING p9_76-SCHEMATIC1-tran.sim.cir ****
.END
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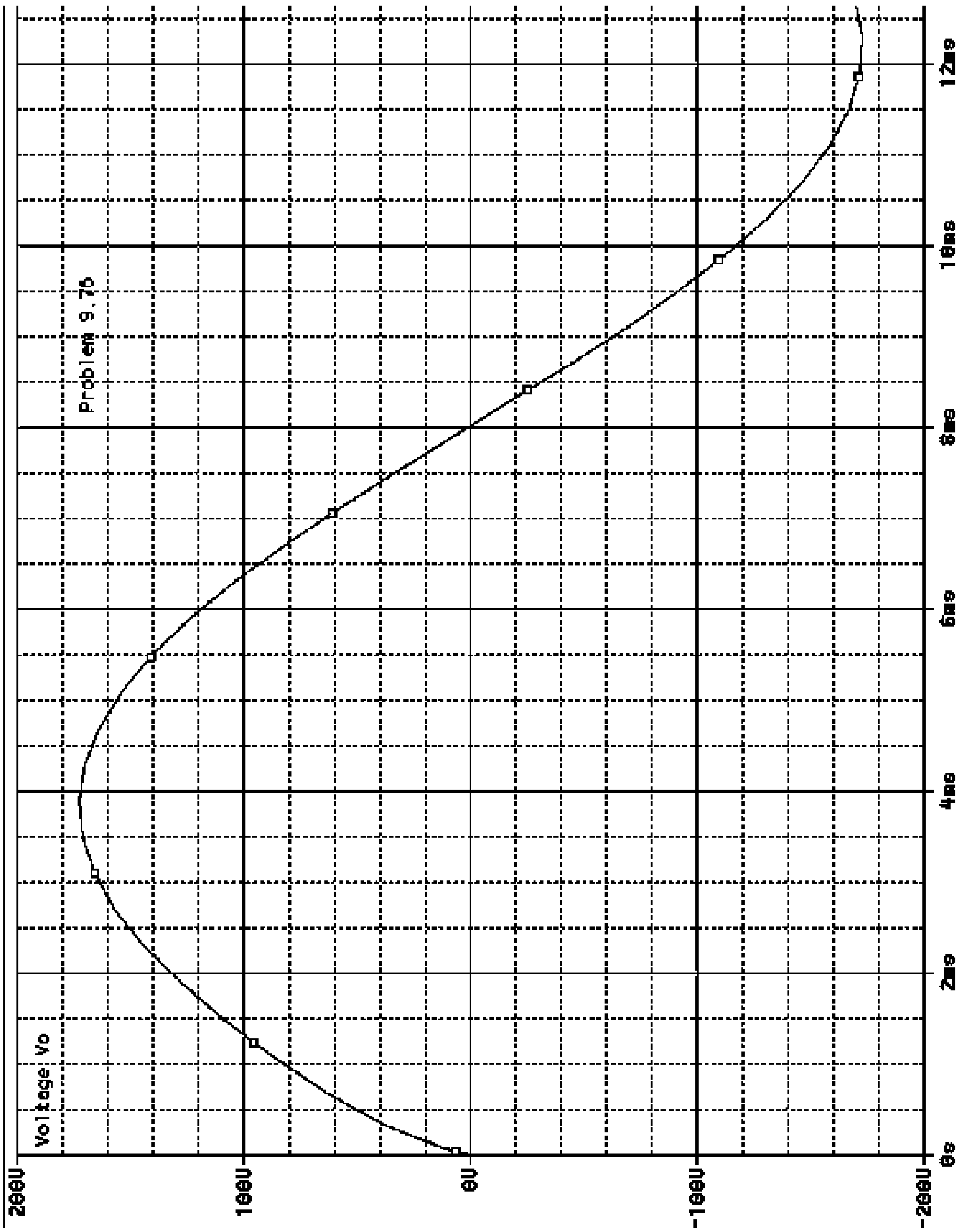
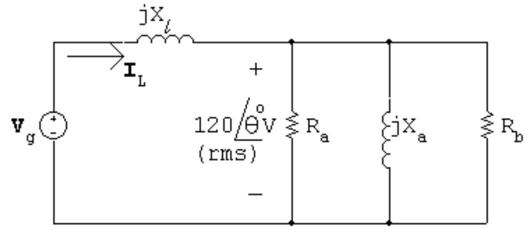


[c] In Problem 13.92, the line-to-neutral voltage spikes at $300\sqrt{2}$ V. Here the line-to-neutral voltage has no spike. Thus the amount of voltage disturbance depends on what part of the cycle the sinusoidal steady-state voltage is switched.

P 13.94 **[a]** First find $\mathbf{V}_g$ before R_b is disconnected. The phasor domain circuit is



$$\mathbf{I}_L = \frac{120\underline{/\theta^\circ}}{R_a} + \frac{120\underline{/\theta^\circ}}{R_b} + \frac{120\underline{/\theta^\circ}}{jX_a}$$
$$= \frac{120\underline{/\theta^\circ}}{R_aR_bX_a}[(R_a + R_b)X_a = jR_aR_b]$$

Since $X_l = 1\,\Omega$ we have

$$\mathbf{V}_g = 120\underline{/\theta^\circ} + \frac{120\underline{/\theta^\circ}}{R_aR_bX_a}[R_aR_b + j(R_a + R_b)X_a]$$

$$R_a = 12\,\Omega; \qquad R_b = 8\,\Omega; \qquad X_a = \frac{1440}{35}\,\Omega$$

$$\mathbf{V}_g = \frac{120\underline{/\theta^\circ}}{1400}(1475 + j300)$$
$$= \frac{25}{12}\underline{/\theta^\circ}(59 + j12) = 125.43\underline{/(\theta + 11.50)^\circ}$$

$$v_g = 125.43\sqrt{2}\cos(120\pi t + \theta + 11.50^\circ)\mathrm{V}$$

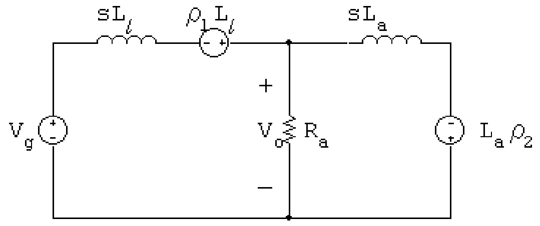
Let $\beta = \theta + 11.50^\circ$. Then

$$v_g = 125.43\sqrt{2}(\cos 120\pi t\cos\beta - \sin 120\pi t\sin\beta)\mathrm{V}$$

Therefore

$$V_g = \frac{125.43\sqrt{2}(s\cos\beta - 120\pi\sin\beta)}{s^2 + (120\pi)^2}$$

The s-domain circuit becomes



where $\rho_1 = i_L(0^+)$ and $\rho_2 = i_2(0^+)$.
The s-domain node voltage equation is

$$\frac{V_o - (V_g + \rho_1 L_l)}{sL_l} + \frac{V_o}{R_a} + \frac{V_o + \rho_2 L_a}{sL_a} = 0$$

Solving for V_o yields

$$V_o = \frac{V_g R_a/L_l + (\rho_1 - \rho_2)R_a}{[s + \frac{(L_a+L_l)R_a}{L_a L_l}]}$$

Substituting the numerical values

$$L_l = \frac{1}{120\pi}\ \text{H}; \qquad L_a = \frac{12}{35\pi}\ \text{H}; \qquad R_a = 12\,\Omega; \qquad R_b = 8\,\Omega;$$

gives

$$V_o = \frac{1440\pi V_g + 12(\rho_1 - \rho_2)}{(s + 1475\pi)}$$

Now determine the values of ρ_1 and ρ_2.

$$\rho_1 = i_L(0^+) \quad \text{and} \quad \rho_2 = i_2(0^+)$$

$$\mathbf{I}_L = \frac{120\underline{/\theta^\circ}}{R_a R_b X_a}[(R_a + R_b)X_a - jR_a R_b]$$

$$= \frac{120\underline{/\theta^\circ}}{96(1440/35)}\left[\frac{(20)(1440)}{35} - j96\right]$$

$$= 25.17\underline{/(\theta - 6.65)^\circ}\text{A(rms)}$$

$$\therefore \quad i_L = 25.17\sqrt{2}\cos(120\pi t + \theta - 6.65^\circ)\text{A}$$

$$i_L(0^+) = \rho_1 = 25.17\sqrt{2}\cos(\theta - 6.65^\circ)\text{A}$$

$$\therefore \quad \rho_1 = 25\sqrt{2}\cos\theta + 2.92\sqrt{2}\sin\theta\text{A}$$

$$\mathbf{I}_2 = \frac{120\underline{/\theta^\circ}}{j(1440/35)} = \frac{35}{12}\underline{/(\theta - 90)^\circ}$$

$$i_2 = \frac{35}{12}\sqrt{2}\cos(120\pi t + \theta - 90^\circ)\text{A}$$

$$\rho_2 = i_2(0^+) = \frac{35}{12}\sqrt{2}\sin\theta = 2.92\sqrt{2}\sin\theta\text{A}$$

$$\therefore \quad \rho_1 = \rho_2 = 25\sqrt{2}\cos\theta$$

$$(\rho_1 - \rho_2)R_a = 300\sqrt{2}\cos\theta$$

$$\therefore \quad \text{V}_o = \frac{1440\pi}{s + 1475\pi}\cdot V_g + \frac{300\sqrt{2}\cos\theta}{s + 1475\pi}$$

$$= \frac{1440\pi}{s + 1475\pi}\left[\frac{125.43\sqrt{2}(s\cos\beta - 120\pi\sin\beta)}{s^2 + 14{,}400\pi^2}\right] + \frac{300\sqrt{2}\cos\theta}{s + 1475\pi}$$

$$= \frac{K_1 + 300\sqrt{2}\cos\theta}{s + 1475\pi} + \frac{K_2}{s - j120\pi} + \frac{K_2^*}{s + j120\pi}$$

Now

$$K_1 = \frac{(1440\pi)(125.43\sqrt{2})[-1475\pi\cos\beta - 120\pi\sin\beta]}{1475^2\pi^2 + 14{,}400\pi^2}$$

$$= \frac{-1440(125.43\sqrt{2})[1475\cos\beta + 120\sin\beta]}{1475^2 + 14{,}000}$$

Since $\beta = \theta + 11.50°$, K_1 reduces to

$$K_1 = -121.18\sqrt{2}\cos\theta + 14.55\sqrt{2}\sin\theta$$

From the partial fraction expansion for V_o we see $v_o(t)$ will go directly into steady state when $K_1 = -300\sqrt{2}\cos\theta$. It follow that

$$14.55\sqrt{2}\sin\theta = -178.82\sqrt{2}\cos\theta$$

or $\quad \tan\theta = -12.29$

Therefore, $\quad \theta = -85.35°$

[b] When $\theta = -85.35°$, $\beta = -73.85°$

$$\therefore\quad K_2 = \frac{1440\pi(125.43\sqrt{2})[-120\pi\sin(-73.85°) + j120\pi\cos(-73.85°)}{(1475\pi + j120\pi)(j240\pi)}$$

$$= \frac{720\sqrt{2}(120.48 + j34.88)}{-120 + j1475}$$

$$= 61.03\sqrt{2}\underline{/-78.50°}$$

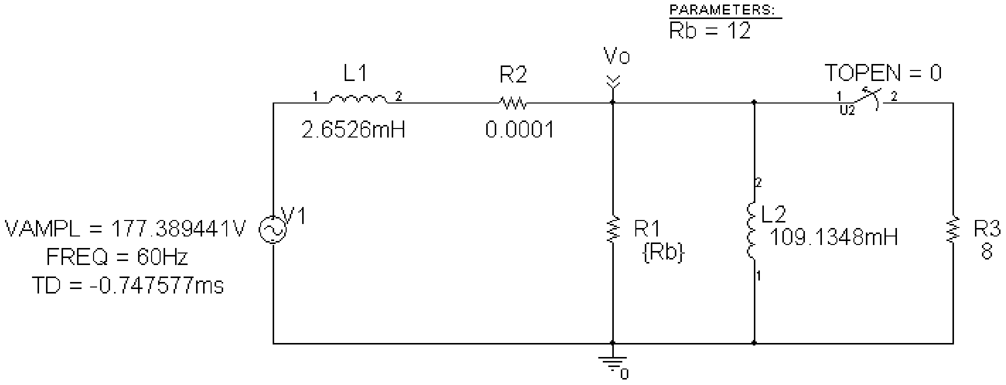
$$\therefore\quad v_o = 122.06\sqrt{2}\cos(120\pi t - 78.50°)\ \text{V} \quad t > 0$$

$$= 172.61\cos(120\pi t - 78.50°)\ \text{V} \quad t > 0$$

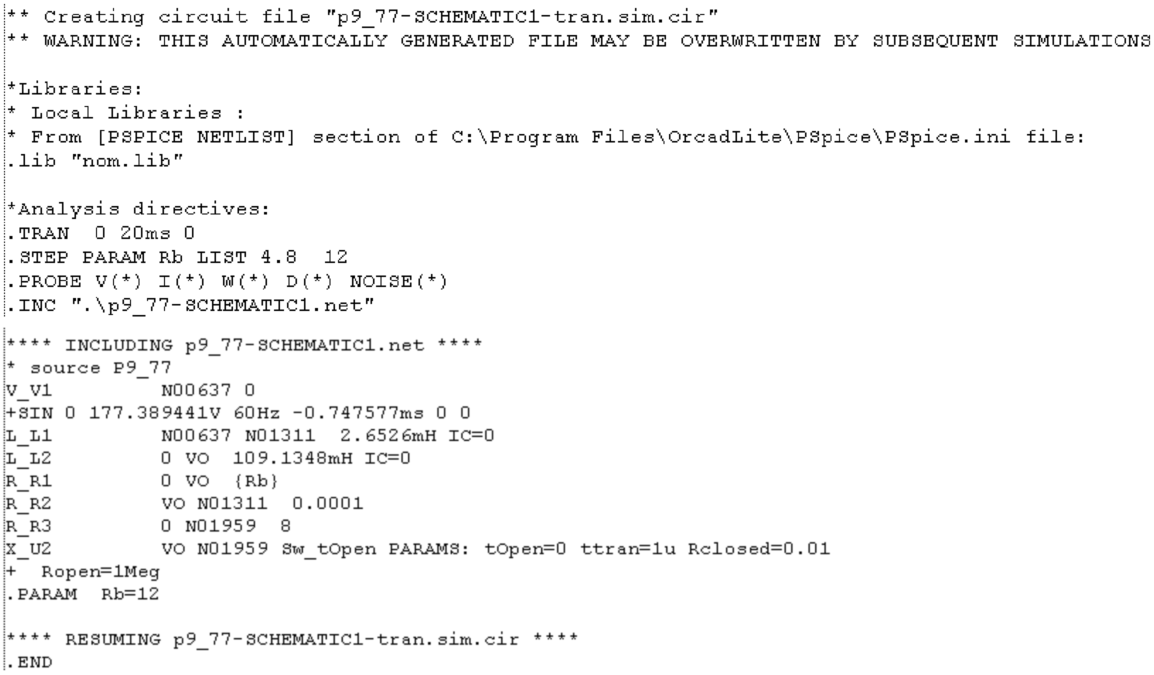
[c] $v_{o1} = 169.71\cos(120\pi t - 85.35°)\text{V} \qquad t < 0$

$v_{o2} = 172.61\cos(120\pi t - 78.50°)\text{V} \qquad t > 0$

PSpice schematic



PSpice output file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** Creating circuit file "p9_77-SCHEMATIC1-tran.sim.cir"
** WARNING: THIS AUTOMATICALLY GENERATED FILE MAY BE OVERWRITTEN BY SUBSEQUENT SIMULATIONS

*Libraries:
* Local Libraries :
* From [PSPICE NETLIST] section of C:\Program Files\OrcadLite\PSpice\PSpice.ini file:
.lib "nom.lib"

*Analysis directives:
.TRAN  0 20ms 0
.STEP PARAM Rb LIST 4.8  12
.PROBE V(*) I(*) W(*) D(*) NOISE(*)
.INC ".\p9_77-SCHEMATIC1.net"

**** INCLUDING p9_77-SCHEMATIC1.net ****
* source P9_77
V_V1         N00637 0
+SIN 0 177.389441V 60Hz -0.747577ms 0 0
L_L1         N00637 N01311  2.6526mH IC=0
L_L2         0 VO  109.1348mH IC=0
R_R1         0 VO  {Rb}
R_R2         VO N01311  0.0001
R_R3         0 N01959  8
X_U2         VO N01959 Sw_tOpen PARAMS: tOpen=0 ttran=1u Rclosed=0.01
+  Ropen=1Meg
.PARAM  Rb=12

**** RESUMING p9_77-SCHEMATIC1-tran.sim.cir ****
.END
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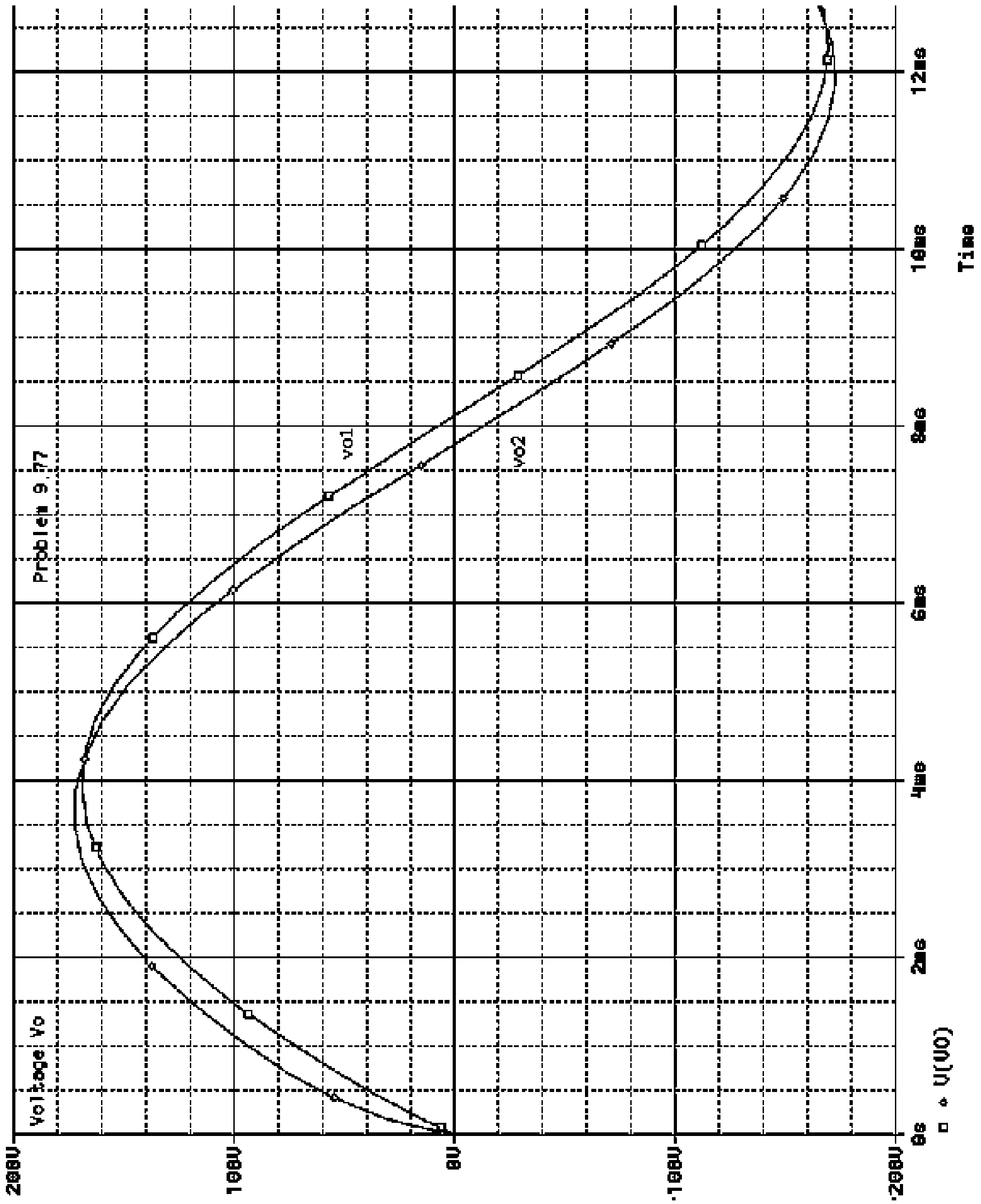

Voltage Vo
Problem 9.77
vo1
vo2
200V
100V
0V
-100V
-200V
0s
2ms
4ms
6ms
8ms
10ms
12ms
Time
□ ◇ V(VO)

14

Introduction to Frequency-Selective Circuits

Assessment Problems

AP 14.1

$$f_c = 8\,\text{kHz}, \quad \omega_c = 2\pi f_c = 16\pi\,\text{krad/s}$$

$$\omega_c = \frac{1}{RC}; \qquad R = 10\,\text{k}\Omega;$$

$$\therefore \quad C = \frac{1}{\omega_c R} = \frac{1}{(16\pi \times 10^3)(10^4)} = 1.99\,\text{nF}$$

AP 14.2 **[a]** $\omega_c = 2\pi f_c = 2\pi(2000) = 4\pi\,\text{krad/s}$

$$L = \frac{R}{\omega_c} = \frac{5000}{4000\pi} = 0.40\,\text{H}$$

[b] $H(j\omega) = \dfrac{\omega_c}{\omega_c + j\omega} = \dfrac{4000\pi}{4000\pi + j\omega}$

When $\omega = 2\pi f = 2\pi(50{,}000) = 100{,}000\pi$ rad/s

$$H(j100{,}000\pi) = \frac{4000\pi}{4000\pi + j100{,}000\pi} = \frac{1}{1 + j25} = 0.04\underline{/87.71^\circ}$$

$$\therefore \quad |H(j100{,}000\pi)| = 0.04$$

[c] $\therefore \quad \theta(100{,}000\pi) = -87.71^\circ$

AP 14.3

$$\omega_c = \frac{R}{L} = \frac{5000}{3.5 \times 10^{-3}} = 1.43\,\text{Mrad/s}$$

AP 14.4 **[a]** $\omega_c = \dfrac{1}{RC} = \dfrac{10^6}{R} = \dfrac{10^6}{100} = 10\,\text{krad/s}$

[b] $\omega_c = \dfrac{10^6}{5000} = 200\,\text{rad/s}$

[c] $\omega_c = \dfrac{10^6}{3 \times 10^4} = 33.33\,\text{rad/s}$

AP 14.5 Let Z represent the parallel combination of $(1/SC)$ and R_L. Then

$$Z = \frac{R_L}{(R_L Cs + 1)}$$

$$\text{Thus} \quad H(s) = \frac{Z}{R+Z} = \frac{R_L}{R(R_L Cs + 1) + R_L}$$

$$= \frac{(1/RC)}{s + \frac{R+R_L}{R_L}\left(\frac{1}{RC}\right)} = \frac{(1/RC)}{s + \frac{1}{K}\left(\frac{1}{RC}\right)}$$

$$\text{where} \quad K = \frac{R_L}{R + R_L}$$

AP 14.6

$$\omega_o^2 = \frac{1}{LC} \quad \text{so} \quad L = \frac{1}{\omega_o^2 C} = \frac{1}{(24\pi \times 10^3)^2(0.1 \times 10^{-6})} = 1.76\,\text{mH}$$

$$Q = \frac{\omega_o}{\beta} = \frac{\omega_o}{R/L} \quad \text{so} \quad R = \frac{\omega_o L}{Q} = \frac{(24\pi \times 10^3)(1.76 \times 10^{-3})}{6} = 22.10\,\Omega$$

AP 14.7

$$\omega_o = 2\pi(2000) = 4000\pi\ \text{rad/s};$$

$$\beta = 2\pi(500) = 1000\pi\ \text{rad/s}; \qquad R = 250\,\Omega$$

$$\beta = \frac{1}{RC} \quad \text{so} \quad C = \frac{1}{\beta R} = \frac{1}{(1000\pi)(250)} = 1.27\,\mu\text{F}$$

$$\omega_o^2 = \frac{1}{LC} \quad \text{so} \quad L = \frac{1}{\omega_o^2 C} = \frac{10^6}{(4000\pi)^2(1.27)} = 4.97\,\text{mH}$$

AP 14.8

$$\omega_o^2 = \frac{1}{LC} \quad \text{so} \quad L = \frac{1}{\omega_o^2 C} = \frac{1}{(10^4\pi)^2(0.2 \times 10^{-6})} = 5.07\,\text{mH}$$

$$\beta = \frac{1}{RC} \quad \text{so} \quad R = \frac{1}{\beta C} = \frac{1}{400\pi(0.2 \times 10^{-6})} = 3.98\,\text{k}\Omega$$

AP 14.9

$$\omega_o^2 = \frac{1}{LC} \quad \text{so} \quad L = \frac{1}{\omega_o^2 C} = \frac{1}{(400\pi)^2(0.2 \times 10^{-6})} = 31.66\,\text{mH}$$

$$Q = \frac{f_o}{\beta} = \frac{5 \times 10^3}{200} = 25 = \omega_o RC$$

$$\therefore \quad R = \frac{Q}{\omega_o C} = \frac{25}{(400\pi)(0.2 \times 10^{-6})} = 9.95\,\text{k}\Omega$$

AP 14.10

$$\omega_o = 8000\pi\,\text{rad/s}$$

$$C = 500\,\text{nF}$$

$$\omega_o^2 = \frac{1}{LC} \quad \text{so} \quad L = \frac{1}{\omega_o^2 C} = 3.17\,\text{mH}$$

$$Q = \frac{\omega_o}{\beta} = \frac{\omega_o L}{R} = \frac{1}{\omega_o CR}$$

$$\therefore \quad R = \frac{1}{\omega_o CQ} = \frac{1}{(8000\pi)(500)(5 \times 10^{-9})} = 15.92\,\Omega$$

AP 14.11

$$\omega_o = 2\pi f_o = 2\pi(20{,}000) = 40\pi\,\text{krad/s}; \qquad R = 100\,\Omega; \qquad Q = 5$$

$$Q = \frac{\omega_o}{\beta} = \frac{\omega_o}{(R/L)} \quad \text{so} \quad L = \frac{QR}{\omega_o} = \frac{5(100)}{(40\pi \times 10^3)} = 3.98\,\text{mH}$$

$$\omega_o^2 = \frac{1}{LC} \quad \text{so} \quad C = \frac{1}{\omega_o^2 L} = \frac{1}{(40\pi \times 10^3)^2(3.98 \times 10^{-3})} = 15.92\,\text{nF}$$

Problems

P 14.1 **[a]** $\omega_c = \frac{R}{L} = \frac{127}{10 \times 10^{-3}} = 12.7\,\text{krad/s}$

$$\therefore \quad f_c = \frac{\omega_c}{2\pi} = \frac{12{,}700}{2\pi} = 2021.27\,\text{Hz}$$

[b] $H(s) = \frac{\omega_c}{s + \omega_c} = \frac{12{,}700}{s + 12{,}700}$

$$H(j\omega) = \frac{12{,}700}{12{,}700 + j\omega}$$

$$H(j\omega_c) = \frac{12{,}700}{12{,}700 + j12{,}700} = 0.7071\underline{/-45^\circ}$$

$$H(j0.2\omega_c) = \frac{12{,}700}{12{,}700 + j2540} = 0.981\underline{/-11.31^\circ}$$

$$H(j5\omega_c) = \frac{12{,}700}{12{,}700 + j63{,}500} = 0.196\underline{/-78.69^\circ}$$

[c] $v_o(t)|_{\omega_c} = 7.07\cos(12{,}700t - 45^\circ)\,\text{V}$

$$v_o(t)|_{0.2\omega_c} = 9.81\cos(2540t - 11.31^\circ)\,\text{V}$$

$$v_o(t)|_{5\omega_c} = 1.96\cos(63{,}500t - 78.69^\circ)\,\text{V}$$

P 14.2 **[a]** $\frac{R}{L} = 10{,}000\pi \text{ rad/s}$

$$R = (0.001)(10{,}000)(\pi) = 31.42\,\Omega$$

[b] $R_e = 31.42\|68 = 21.49\,\Omega$

$$\omega_{\text{loaded}} = \frac{R_e}{L} = 21{,}488.34 \text{ rad/s}$$

$$\therefore \quad f_{\text{loaded}} = 3419.98\,\text{Hz}$$

[c] The $33\,\Omega$ resistor in Appendix H is closest to the desired value of $31.42\,\Omega$. Therefore,

$$\omega_c = 33\,\text{krad/s} \quad \text{so} \quad f_c = 5252.11\,\text{Hz}$$

P 14.3 **[a]** $H(s) = \frac{V_o}{V_i} = \frac{R}{sL + R + R_l} = \frac{(R/L)}{s + (R + R_l)/L}$

[b] $H(j\omega) = \dfrac{(R/L)}{\left(\frac{R+R_l}{L}\right) + j\omega}$

$$|H(j\omega)| = \frac{(R/L)}{\sqrt{\left(\frac{R+R_l}{L}\right)^2 + \omega^2}}$$

$|H(j\omega)|_{\text{max}}$ occurs when $\omega = 0$

[c] $|H(j\omega)|_{\text{max}} = \dfrac{R}{R+R_l}$

[d] $|H(j\omega_c)| = \dfrac{R}{\sqrt{2}(R+R_l)} = \dfrac{R/L}{\sqrt{\left(\frac{R+R_l}{L}\right)^2 + \omega_c^2}}$

$$\therefore \quad \omega_c^2 = \left(\frac{R+R_l}{L}\right)^2; \qquad \therefore \quad \omega_c = (R+R_l)/L$$

[e] $\omega_c = \dfrac{127+75}{0.01} = 20{,}200$ rad/s

$$H(j\omega) = \frac{12{,}700}{20{,}200 + j\omega}$$

$$H(j0) = 0.6287$$

$$H(j20{,}200) = \frac{0.6287}{\sqrt{2}}\underline{/-45^\circ} = 0.4446\underline{/-45^\circ}$$

$$H(j6060) = \frac{12{,}700}{20{,}200 + j6060} = 0.6022\underline{/-16.70^\circ}$$

$$H(j60{,}600) = \frac{12{,}700}{20{,}200 + j60{,}600} = 0.1988\underline{/-71.57^\circ}$$

P 14.4 **[a]** $\omega_c = \dfrac{1}{RC} = \dfrac{1}{(10^3)(100 \times 10^{-}9)} = 10\,\text{krad/s}$

$$f_c = \frac{\omega_c}{2\pi} = 1591.55\,\text{Hz}$$

[b] $H(j\omega) = \dfrac{\omega_c}{s+\omega_c} = \dfrac{10{,}000}{s+10{,}000}$

$$H(j\omega) = \frac{10{,}000}{10{,}000 + j\omega}$$

$$H(j\omega_c) = \frac{10{,}000}{10{,}000 + j10{,}000} = 0.7071\underline{/-45^\circ}$$

$$H(j0.1\omega_c) = \frac{10{,}000}{10{,}000 + j1000} = 0.9950\underline{/-5.71^\circ}$$

$$H(j10\omega_c) = \frac{10{,}000}{10{,}000 + j100{,}000} = 0.0995\underline{/-84.29^\circ}$$

[c] $v_o(t)|_{\omega_c} = 200(0.7071)\cos(10{,}000t - 45^\circ)$

$$= 141.42\cos(10{,}000t - 45^\circ)\,\text{mV}$$

$$v_o(t)|_{0.1\omega_c} = 200(0.9950)\cos(1000t - 5.71^\circ)$$

$$= 199.01\cos(1000t - 5.71^\circ)\,\text{mV}$$

$$v_o(t)|_{10\omega_c} = 200(0.0995)\cos(100{,}000t - 84.29^\circ)$$

$$= 19.90\cos(100{,}000t - 84.29^\circ)\,\text{mV}$$

P 14.5 **[a]** $f_c = \dfrac{\omega_c}{2\pi} = \dfrac{50{,}000}{2\pi} = \dfrac{50}{2\pi} \times 10^3 = 7957.75\,\text{Hz}$

[b] $\dfrac{1}{RC} = 50 \times 10^3$

$$R = \frac{1}{(50 \times 10^3)(0.5 \times 10^{-6})} = 40\,\Omega$$

[c] With a load resistor added in parallel with the capacitor the transfer function becomes

$$H(s) = \frac{R_L\|(1/sC)}{R + R_L\|(1/sC)} = \frac{R_L/sC}{R[R_L + (1/sC)] + R_L/sC}$$

$$= \frac{R_L}{RR_LsC + R + R_L} = \frac{1/RC}{s + [(R + R_L)/RR_LC]}$$

This transfer function is in the form of a low-pass filter, with a cutoff frequency equal to the quantity added to s in the denominator. Therefore,

$$\omega_c = \frac{R + R_L}{RR_LC} = \frac{1}{RC}\left(1 + \frac{R}{R_L}\right)$$

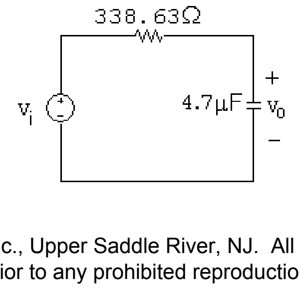
$$\therefore \quad \frac{R}{R_L} = 0.05 \qquad \therefore \quad R_L = 20R = 800\,\Omega$$

[d] $H(j0) = \dfrac{R_L}{R + R_L} = \dfrac{800}{840} = 0.9524$

P 14.6 **[a]** $\omega_c = 2\pi(100) = 628.32$ rad/s

[b] $\omega_c = \dfrac{1}{RC}$ so $R = \dfrac{1}{\omega_c C} = \dfrac{1}{(628.32)(4.7 \times 10^{-6})} = 338.63\,\Omega$

[c]



[d] $$H(s) = \frac{V_o}{V_i} = \frac{1/sC}{R + 1/sC} = \frac{1/RC}{s + 1/RC} = \frac{628.32}{s + 628.32}$$

[e] $$H(s) = \frac{V_o}{V_i} = \frac{(1/sC)\|R_L}{R + (1/sC)\|R_L} = \frac{1/RC}{s + \left(\dfrac{R+R_L}{R_L}\right)1/RC} = \frac{628.32}{s + 2(628.32)}$$

[f] $\omega_c = 2(628.32) = 1256.64$ rad/s

[g] $H(0) = 1/2$

P 14.7 **[a]** Let $$Z = \frac{R_L(1/SC)}{R_L + 1/SC} = \frac{R_L}{R_L Cs + 1}$$

Then $$H(s) = \frac{Z}{Z + R}$$

$$= \frac{R_L}{RR_L Cs + R + R_L}$$

$$= \frac{(1/RC)}{s + \left(\dfrac{R + R_L}{RR_L C}\right)}$$

[b] $$|H(j\omega)| = \frac{(1/RC)}{\sqrt{\omega^2 + [(R + R_L)/RR_L C]^2}}$$

$|H(j\omega)|$ is maximum at $\omega = 0$.

[c] $$|H(j\omega)|_{\max} = \frac{R_L}{R + R_L}$$

[d] $$|H(j\omega_c)| = \frac{R_L}{\sqrt{2}(R + R_L)} = \frac{(1/RC)}{\sqrt{\omega_C^2 + [(R + R_L)/RR_L C]^2}}$$

$$\therefore \quad \omega_c = \frac{R + R_L}{RR_L C} = \frac{1}{RC}(1 + (R/R_L))$$

[e] $$\omega_c = \frac{1}{(10^3)(10^{-7})}[1 + (10^3/10^4)] = 10{,}000(1 + 0.1) = 11{,}000 \text{ rad/s}$$

$$H(j0) = \frac{10{,}000}{11{,}000} = 0.9091\underline{/0^\circ}$$

$$H(j\omega_c) = \frac{10{,}000}{11{,}000 + j11{,}000} = 0.6428\underline{/-45^\circ}$$

$$H(j0.1\omega_c) = \frac{10{,}000}{11{,}000 + j1100} = 0.9046\underline{/-5.71^\circ}$$

$$H(j10\omega_c) = \frac{10{,}000}{11{,}000 + j110{,}000} = 0.0905\underline{/-84.29^\circ}$$

P 14.8 **[a]** $Z_L = j\omega L = j0L = 0$ so it is a short circuit.

At $\omega = 0, \quad V_o = V_i$

[b] $Z_L = j\omega L = j\infty L = \infty$ so it is an open circuit.

At $\omega = \infty, \quad V_o = 0$

[c] This is a low pass filter, with a gain of 1 at low frequencies and a gain of 0 at high frequencies.

[d] $H(s) = \dfrac{V_o}{V_i} = \dfrac{R}{R + sL} = \dfrac{R/L}{s + R/L}$

[e] $\omega_c = \dfrac{R}{L} = \dfrac{330}{0.01} = 33$ krad/s

P 14.9 **[a]** $H(s) = \dfrac{V_o}{V_i} = \dfrac{R\|R_L}{R\|R_L + sL} = \dfrac{\frac{R}{L}\left(\frac{R_L}{R+R_L}\right)}{s + \frac{R}{L}\left(\frac{R_L}{R+R_L}\right)}$

[b] $\omega_{c(UL)} = \dfrac{R}{L}; \qquad \omega_{c(L)} = \dfrac{R}{L}\left(\dfrac{R_L}{R+R_L}\right)$ so the cutoff frequencies are different.

$H(0)_{(UL)} = 1; \qquad H(0)_{(L)} = 1$ so the passband gains are the same.

[c] $\omega_{c(UL)} = 33{,}000$ rad/s

$\omega_{c(L)} = 33{,}000 - 0.05(33{,}000) = 31{,}350$ rad/s

$31{,}350 = \dfrac{330}{0.01}\left(\dfrac{R_L}{330 + R_L}\right)$ so $\dfrac{R_L}{330 + R_L} = 0.95$

$\therefore \quad 0.05R_L = 313.5$ so $R_L \geq 6270\,\Omega$

P 14.10 **[a]** $\dfrac{1}{RC} = \dfrac{1}{(50 \times 10^3)(5 \times 10^{-9})} = 4000\,\text{rad/s}$

$f_c = \dfrac{4000}{2\pi} = 636.62\,\text{Hz}$

[b] $H(s) = \dfrac{s}{s + \omega_c} \quad \therefore \quad H(j\omega) = \dfrac{j\omega}{4000 + j\omega}$

$H(j\omega_c) = H(j4000) = \dfrac{j4000}{4000 + j4000} = 0.7071\underline{/45^\circ}$

$H(j0.2\omega_c) = H(j800) = \dfrac{j800}{4000 + j800} = 0.1961\underline{/78.69^\circ}$

$H(j5\omega_c) = H(j20\omega_c) = \dfrac{j20{,}000}{4000 + j20{,}000} = 0.9806\underline{/11.31^\circ}$

[c] $v_o(t)|_{\omega_c} = (0.7071)(500)\cos(4000t + 45^\circ)$

$$= 353.55\cos(4000t + 45^\circ)\,\mathrm{mV}$$

$$v_o(t)|_{0.2\omega_c} = (0.1961)(500)\cos(800t + 78.60^\circ)$$

$$= 98.06\cos(800t + 78.69^\circ)\,\mathrm{mV}$$

$$v_o(t)|_{5\omega_c} = (0.9806)(500)\cos(20{,}000t + 11.31^\circ)$$

$$= 490.29\cos(20{,}000t + 11.31^\circ)\,\mathrm{mV}$$

P 14.11 **[a]** $H(s) = \dfrac{V_o}{V_i} = \dfrac{R}{R + R_c + (1/sC)}$

$$= \frac{R}{R+R_c} \cdot \frac{s}{[s + (1/(R+R_c)C)]}$$

[b] $H(j\omega) = \dfrac{R}{R+R_c} \cdot \dfrac{j\omega}{j\omega + (1/(R+R_c)C)}$

$$|H(j\omega)| = \frac{R}{R+R_c} \cdot \frac{\omega}{\sqrt{\omega^2 + \frac{1}{(R+R_c)^2C^2}}}$$

The magnitude will be maximum when $\omega = \infty$.

[c] $|H(j\omega)|_{\max} = \dfrac{R}{R+R_c}$

[d] $|H(j\omega_c)| = \dfrac{R\omega_c}{(R+R_c)\sqrt{\omega_c^2 + [1/(R+R_c)C]^2}}$

$$\therefore \quad |H(j\omega)| = \frac{R}{\sqrt{2}(R+R_c)} \qquad \text{when}$$

$$\therefore \quad \omega_c^2 = \frac{1}{(R+R_c)^2C^2}$$

$$\text{or } \omega_c = \frac{1}{(R+R_c)C}$$

[e] $\omega_c = \dfrac{1}{(62.5\times 10^3)(5\times 10^{-9})} = 3200\ \mathrm{rad/s}$

$$\frac{R}{R+R_c} = \frac{50}{62.5} = 0.8$$

$$\therefore \qquad H(j\omega) = \frac{0.8j\omega}{3200 + j\omega}$$

$$H(j\omega_c) = \frac{(0.8)j3200}{3200 + j3200} = 0.5657\underline{/45^\circ}$$

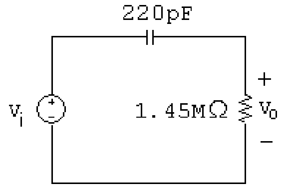
$$H(j0.2\omega_c) = \frac{(0.8)j640}{3200 + j640} = 0.1569\underline{/78.69^\circ}$$

$$H(j5\omega_c) = \frac{(0.8)j16{,}000}{3200 + j16{,}000} = 0.7845\underline{/11.31^\circ}$$

P 14.12 **[a]** $\omega_c = 2\pi(500) = 3141.59$ rad/s

[b] $\omega_c = \dfrac{1}{RC}$ so $R = \dfrac{1}{\omega_c C} = \dfrac{1}{(3141.59)(220 \times 10^{-12})} = 1.45\,\text{M}\Omega$

[c]



[d] $H(s) = \dfrac{V_o}{V_i} = \dfrac{R}{R + 1/sC} = \dfrac{s}{s + 1/RC} = \dfrac{s}{s + 3141.59}$

[e] $H(s) = \dfrac{V_o}{V_i} = \dfrac{R\|R_L}{R\|R_L + (1/sC)} = \dfrac{s}{s + \left(\dfrac{R + R_L}{R_L}\right) 1/RC} = \dfrac{s}{s + 2(3141.59)}$

[f] $\omega_c = 2(3141.59) = 6283.19$ rad/s

[g] $H(\infty) = 1$

P 14.13 **[a]** $\omega_c = \dfrac{1}{RC} = 2\pi(300) = 600\pi$ rad/s

$$\therefore \quad R = \frac{1}{\omega_c C} = \frac{1}{(600\pi)(100 \times 10^{-9})} = 5305.16\,\Omega = 5.305\,\text{k}\Omega$$

[b] $R_e = 5305.16\|47{,}000 = 4767.08\,\Omega$

$$\omega_c = \frac{1}{R_e C} = \frac{1}{(4767.08)(100 \times 10^{-9})} = 2097.7 \text{ rad/s}$$

$$f_c = \frac{\omega_c}{2\pi} = \frac{2097.7}{2\pi} = 333.86\,\text{Hz}$$

P 14.14 **[a]** $R = \omega_c L = (1500 \times 10^3)(100 \times 10^{-6}) = 150\,\Omega$ (a value from Appendix H)

[b] With a load resistor in parallel with the inductor, the transfer function becomes

$$H(s) = \frac{sL\|R_L}{R + sL\|R_L} = \frac{sLR_L}{R(sL + R_L) + sLR_L} = \frac{s[R_L/(R + R_L)]}{s + [RR_L/(R + R_L)]}$$

This transfer function is in the form of a high-pass filter whose cutoff frequency is the quantity added to s in the denominator. Thus,

$$\omega_c = \frac{RR_L}{L(R + R_L)}$$

Substituting in the values of R and L from part (a), we can solve for the value of load resistance that gives a cutoff frequency of 1200 krad/s:

$$\frac{150R_L}{100\times 10^{-6}(150+R_L)} = 1200\times 10^3 \qquad \text{so} \qquad R_L = 600\,\Omega$$

The smallest resistor from Appendix H that is larger than $600\,\Omega$ is $680\,\Omega$.

P 14.15 **[a]** For $\omega = 0$, the inductor behaves as a short circuit, so $V_o = 0$.
For $\omega = \infty$, the inductor behaves as an open circuit, so $V_o = V_i$.
Thus, the circuit is a high-pass filter.

[b] $H(s) = \dfrac{sL}{R+sL} = \dfrac{s}{s+R/L} = \dfrac{s}{s+15{,}000}$

[c] $\omega_c = \dfrac{R}{L} = 15{,}000$ rad/s

[d] $|H(jR/L)| = \left|\dfrac{jR/L}{jR/L+R/L}\right| = \left|\dfrac{j}{j+1}\right| = \dfrac{1}{\sqrt{2}}$

P 14.16 **[a]** $H(s) = \dfrac{V_o}{V_i} = \dfrac{R_L\|sL}{R+R_L\|sL} = \dfrac{s\left(\dfrac{R_L}{R+R_L}\right)}{s+\dfrac{R}{L}\left(\dfrac{R_L}{R+R_L}\right)}$

$$= \frac{\frac{1}{2}s}{s+\frac{1}{2}(15{,}000)}$$

[b] $\omega_c = \dfrac{R}{L}\left(\dfrac{R_L}{R+R_L}\right) = \dfrac{1}{2}(15{,}000) = 7500$ rad/s

[c] $\omega_{c(L)} = \dfrac{1}{2}\omega_{c(UL)}$

[d] The gain in the passband is also reduced by a factor of 1/2 for the loaded filter.

P 14.17 By definition $Q = \omega_o/\beta$ therefore $\beta = \omega_o/Q$. Substituting this expression into Eqs. 14.34 and 14.35 yields

$$\omega_{c1} = -\frac{\omega_o}{2Q} + \sqrt{\left(\frac{\omega_o}{2Q}\right)^2 + \omega_o^2}$$

$$\omega_{c2} = \frac{\omega_o}{2Q} + \sqrt{\left(\frac{\omega_o}{2Q}\right)^2 + \omega_o^2}$$

Now factor ω_o out to get

$$\omega_{c1} = \omega_o \left[-\frac{1}{2Q} + \sqrt{1 + \left(\frac{1}{2Q}\right)^2} \right]$$

$$\omega_{c2} = \omega_o \left[\frac{1}{2Q} + \sqrt{1 + \left(\frac{1}{2Q}\right)^2} \right]$$

P 14.18 $\omega_o = \sqrt{\omega_{c1}\omega_{c2}} = \sqrt{(121)(100)} = 110\,\text{krad/s}$

$$f_o = \frac{\omega_o}{2\pi} = 17.51\,\text{kHz}$$

$$\beta = 121 - 100 = 21\,\text{krad/s} \quad \text{or} \quad 2.79\,\text{kHz}$$

$$Q = \frac{\omega_o}{\beta} = \frac{110}{21} = 5.24$$

P 14.19 $\beta = \dfrac{\omega_o}{Q} = \dfrac{50{,}000}{4} = 12.5\,\text{krad/s}; \qquad \dfrac{12{,}500}{2\pi} = 1.99\,\text{kHz}$

$$\omega_{c2} = 50{,}000 \left[\frac{1}{8} + \sqrt{1 + \left(\frac{1}{8}\right)^2} \right] = 56.64\,\text{krad/s}$$

$$f_{c2} = \frac{56.64\,\text{k}}{2\pi} = 9.01\,\text{kHz}$$

$$\omega_{c1} = 50{,}000 \left[-\frac{1}{8} + \sqrt{1 + \left(\frac{1}{8}\right)^2} \right] = 44.14\,\text{krad/s}$$

$$f_{c1} = \frac{44.14\,\text{k}}{2\pi} = 7.02\,\text{kHz}$$

P 14.20 **[a]** $\omega_o^2 = \dfrac{1}{LC} \quad \text{so} \quad L = \dfrac{1}{[8000(2\pi)]^2(5 \times 10^{-9})} = 79.16\,\text{mH}$

$$R = \frac{\omega_o L}{Q} = \frac{8000(2\pi)(79.16 \times 10^{-3})}{2} = 1.99\,\text{k}\Omega$$

[b] $f_{c1} = 8000 \left[-\dfrac{1}{4} + \sqrt{1 + \dfrac{1}{16}} \right] = 6.25\,\text{kHz}$

[c] $f_{c2} = 8000\left[\frac{1}{4} + \sqrt{1 + \frac{1}{16}}\right] = 10.25\,\text{kHz}$

[d] $\beta = f_{c2} - f_{c1} = 4\,\text{kHz}$

or

$$\beta = \frac{f_o}{Q} = \frac{8000}{2} = 4\,\text{kHz}$$

P 14.21 **[a]** We need ω_c close to $2\pi(8000) = 50{,}265.48$ rad/s. There are several possible approaches – this one starts by choosing $L = 10$ mH. Then,

$$C = \frac{1}{[2\pi(8000)]^2(0.01)} = 39.58\,\text{nF}$$

Use the closest value from Appendix H, which is $0.047\,\mu$F to give

$$\omega_c = \sqrt{\frac{1}{(0.01)(47 \times 10^{-9})}} = 46{,}126.56 \text{ rad/s} \quad \text{or} \quad f_c = 7341.27\,\text{Hz}$$

Then, $R = \frac{\omega_o L}{Q} = \frac{(46{,}126.56)(0.01)}{2} = 230\,\Omega$

Use the closest value from Appendix H, which is $220\,\Omega$ to give

$$Q = \frac{(46{,}126.56)(0.01)}{220} = 2.1$$

[b] $\%$ error in $f_c = \frac{7341.27 - 8000}{8000}(100) = -8.23\%$

$\%$ error in $Q = \frac{2.1 - 2}{2}(100) = 5\%$

P 14.22 **[a]** $\omega_o^2 = \frac{1}{LC} = \frac{1}{(10 \times 10^{-3})(10 \times 10^{-9})} = 10^{10}$

$$\omega_o = 10^5 \text{ rad/s} = 100\,\text{krad/s}$$

[b] $f_o = \frac{\omega_o}{2\pi} = \frac{10^5}{2\pi} = 15.9\,\text{kHz}$

[c] $Q = \omega_o RC = (100 \times 10^3)(8000)(10 \times 10^{-9}) = 8$

[d] $\omega_{c1} = \omega_o\left[-\frac{1}{2Q} + \sqrt{1 + \left(\frac{1}{2Q}\right)^2}\right] = 10^5\left[-\frac{1}{16} + \sqrt{1 + \frac{1}{256}}\right] = 93.95\,\text{krad/s}$

[e] $\therefore\ f_{c1} = \frac{\omega_{c1}}{2\pi} = 14.95\,\text{kHz}$

[f] $\omega_{c2} = \omega_o\left[\frac{1}{2Q} + \sqrt{1 + \left(\frac{1}{2Q}\right)^2}\right] = 10^5\left[\frac{1}{16} + \sqrt{1 + \frac{1}{256}}\right] = 106.45\,\text{krad/s}$

[g] $\therefore \quad f_{c2} = \dfrac{\omega_{c2}}{2\pi} = 16.94\,\text{kHz}$

[h] $\beta = \dfrac{\omega_o}{Q} = \dfrac{10^5}{8} = 12.5\,\text{krad/s or } 1.99\,\text{kHz}$

P 14.23 **[a]** $L = \dfrac{1}{\omega_o^2 C} = \dfrac{1}{(50 \times 10^{-9})(20 \times 10^3)^2} = 50\,\text{mH}$

$$R = \frac{Q}{\omega_o C} = \frac{5}{(20 \times 10^3)(50 \times 10^{-9})} = 5\,\text{k}\Omega$$

[b] $\omega_{c2} = \omega_o \left[\dfrac{1}{2Q} + \sqrt{1 + \left(\dfrac{1}{2Q}\right)^2}\right] = 20{,}000 \left[\dfrac{1}{10} + \sqrt{1 + \dfrac{1}{100}}\right]$

$$= 22.10\,\text{krad/s} \quad \therefore \quad f_{c2} = \frac{\omega_{c2}}{2\pi} = 3.52\,\text{kHz}$$

$$\omega_{c1} = \omega_o \left[-\frac{1}{2Q} + \sqrt{1 + \left(\frac{1}{2Q}\right)^2}\right] = 20{,}000 \left[-\frac{1}{10} + \sqrt{1 + \frac{1}{100}}\right]$$

$$= 18.10\,\text{krad/s} \quad \therefore \quad f_{c1} = \frac{\omega_{c1}}{2\pi} = 2.88\,\text{kHz}$$

[c] $\beta = \dfrac{\omega_o}{Q} = \dfrac{20{,}000}{5} = 4000 \text{ rad/s} \quad \text{or} \quad 636.62\,\text{Hz}$

P 14.24 **[a]** We need $\omega_c = 20{,}000$ rad/s. There are several possible approaches – this one starts by choosing $L = 1$ mH. Then,

$$C = \frac{1}{20{,}000^2(0.001)} = 2.5\,\mu\text{F}$$

Use the closest value from Appendix H, which is $2.2\,\mu$F to give

$$\omega_c = \sqrt{\frac{1}{(0.001)(2.2 \times 10^{-6})}} = 21{,}320 \text{ rad/s}$$

Then, $\quad R = \dfrac{Q}{\omega_o C} = \dfrac{5}{(21320)(2.2 \times 10^{-6})} = 106.6\,\Omega$

Use the closest value from Appendix H, which is $100\,\Omega$ to give

$$Q = 100(21{,}320)(2.2 \times 10^{-6}) = 4.69$$

[b] $\%$ error in $\omega_c = \dfrac{21{,}320 - 20{,}000}{20{,}000}(100) = 6.6\%$

$$\% \text{ error in } Q = \frac{4.69 - 5}{5}(100) = -6.2\%$$

P 14.25 **[a]** $\omega_o^2 = \dfrac{1}{LC} = \dfrac{1}{(40 \times 10^{-3})(40 \times 10^{-9})} = 625 \times 10^6$

$$\omega_o = 25 \times 10^3 \text{ rad/s} = 25\,\text{krad/s}$$

$$f_o = \frac{25{,}000}{2\pi} = 3978.87\,\text{Hz}$$

[b] $Q = \dfrac{\omega_o L}{R + R_i} = \dfrac{(25 \times 10^3)(40 \times 10^{-3})}{200} = 5$

[c] $\omega_{c1} = \omega_o \left[-\dfrac{1}{2Q} + \sqrt{1 + \left(\dfrac{1}{2Q}\right)^2}\right] = 25{,}000\left[-\dfrac{1}{10} + \sqrt{1 + \dfrac{1}{100}}\right]$

$$= 22.62\,\text{krad/s} \quad \text{or} \quad 3.60\,\text{kHz}$$

[d] $w_{c2} = \omega_o \left[\dfrac{1}{2Q} + \sqrt{1 + \left(\dfrac{1}{2Q}\right)^2}\right] = 25{,}000\left[\dfrac{1}{10} + \sqrt{1 + \dfrac{1}{100}}\right]$

$$= 27.62\,\text{krad/s} \quad \text{or} \quad 4.4\,\text{kHz}$$

[e] $\beta = \omega_{c2} - \omega_{c1} = 27.62 - 22.62 = 5\,\text{krad/s}$

or

$$\beta = \frac{\omega_o}{Q} = \frac{25{,}000}{5} = 5\,\text{krad/s} \quad \text{or} \qquad 795.77\,\text{Hz}$$

P 14.26 **[a]** $H(s) = \dfrac{(R/L)s}{s^2 + \frac{(R+R_i)}{L}s + \frac{1}{LC}}$

For the numerical values in Problem 14.25 we have

$$H(s) = \frac{4500s}{s^2 + 5000s + 625 \times 10^6}$$

$$\therefore \quad H(j\omega) = \frac{4500j\omega}{(625 \times 10^6 - \omega^2) + j5000\omega}$$

$$H(j\omega_o) = \frac{j4500(25 \times 10^3)}{j5000(25 \times 10^3)} = 0.9\underline{/0^\circ}$$

$$\therefore \quad v_o(t) = 500(0.9)\cos 25{,}000 = 450\cos 25{,}000t\,\text{mV}$$

[b] From the solution to Problem 14.25,

$$\omega_{c1} = 22.62\,\text{krad/s}$$

$$H(j22.62\,\text{k}) = \frac{j4500(22.62 \times 10^3)}{(113.12 + j113.12) \times 10^6} = 0.6364\underline{/45^\circ}$$

$$\therefore \quad v_o(t) = 500(0.6364)\cos(22{,}620t + 45^\circ) = 318.2\cos(22{,}620t + 45^\circ)\,\text{mV}$$

[c] From the solution to Problem 14.25,

$$\omega_{c2} = 27.62 \text{ krad/s}$$

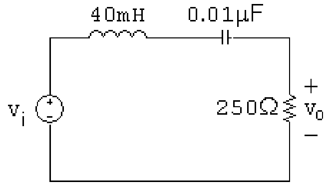
$$H(j27.62\,\text{k}) = \frac{j4500(27.62 \times 10^3)}{(-138.12 + j138.12) \times 10^6} = 0.6364\underline{/-45^\circ}$$

$$\therefore \quad v_o(t) = 500(0.6364)\cos(27{,}620t - 45^\circ) = 318.2\cos(27{,}620t - 45^\circ)\,\text{mV}$$

P 14.27 **[a]** $\omega_o = \sqrt{1/LC}$ so $L = \dfrac{1}{\omega_o^2 C} = \dfrac{1}{(50{,}000)^2(0.01 \times 10^{-6})} = 40\,\text{mH}$

$$Q = \frac{\omega_o}{\beta} \quad \text{so} \quad \beta = \frac{\omega_o}{Q} = \frac{50{,}000}{8} = 6250 \text{ rad/s}$$

$$\beta = \frac{R}{L} \quad \text{so} \quad R = L\beta = (40 \times 10^{-3})(6250) = 250\,\Omega$$



[b] From part (a), $\beta = 6250$ rad/s. Then,

$$\omega_{c1,2} = \pm\frac{\beta}{2} + \sqrt{\frac{\beta}{2} + \omega_o^2} = \pm\frac{6250}{2} + \sqrt{\left(\frac{6250}{2}\right)^2 + 50{,}000^2} = \pm 3125 + 50{,}097.56$$

$$\omega_{c1} = 46{,}972.56 \text{ rad/s} \qquad \omega_{c2} = 53{,}222.56 \text{ rad/s}$$

P 14.28 $H(j\omega) = \dfrac{j\omega(6250)}{50{,}000^2 - \omega^2 + j\omega(6250)}$

[a] $H(j50{,}000) = \dfrac{j50{,}000(6250)}{50{,}000^2 - 50{,}000^2 + j(50{,}000)(6250)} = 1$

$$V_o = (1)V_i \quad \therefore \quad v_o(t) = 50\cos 50{,}000t\,\text{mV}$$

[b] $H(j46{,}972.56) = \dfrac{j46{,}972.56(6250)}{50{,}000^2 - 46{,}972.56^2 + j(46{,}972.56)(6250)} = \dfrac{1}{\sqrt{2}}\underline{/45^\circ}$

$$V_o = \frac{1}{\sqrt{2}}\underline{/45^\circ}V_i \quad \therefore \quad v_o(t) = 35.36\cos(46{,}972.56t + 45^\circ)\,\text{mV}$$

[c] $H(j53{,}222.56) = \dfrac{j53{,}222.56(6250)}{50{,}000^2 - 53{,}222.56^2 + j(53{,}222.56)(6250)} = \dfrac{1}{\sqrt{2}}\underline{/-45^\circ}$

$$V_o = \frac{1}{\sqrt{2}}\underline{/-45^\circ}V_i \quad \therefore \quad v_o(t) = 35.36\cos(53{,}222.56t - 45^\circ)\,\text{mV}$$

[d] $H(j10{,}000) = \dfrac{j10{,}000(6250)}{50{,}000^2 - 10{,}000^2 + j(10{,}000)(6250)} = 0.026\underline{/88.5^\circ}$

$V_o = 0.026\underline{/88.5^\circ}V_i \quad \therefore \quad v_o(t) = 1.3\cos(10{,}000t + 88.5^\circ)\,\text{mV}$

[e] $H(j250{,}000) = \dfrac{j250{,}000(6250)}{50{,}000^2 - 250{,}000^2 + j(250{,}000)(6250)} = 0.026\underline{/-88.5^\circ}$

$V_o = 0.026\underline{/-88.5^\circ}V_i \quad \therefore \quad v_o(t) = 1.3\cos(250{,}000t - 88.5^\circ)\,\text{mV}$

P 14.29 $H(s) = 1 - \dfrac{(R/L)s}{s^2 + (R/L)s + (1/LC)} = \dfrac{s^2 + (1/LC)}{s^2 + (R/L)s + (1/LC)}$

$$H(j\omega) = \frac{50{,}000^2 - \omega^2}{50{,}000^2 - \omega^2 + j\omega(6250)}$$

[a] $H(j50{,}000) = \dfrac{50{,}000^2 - 50{,}000^2}{50{,}000^2 - 50{,}000^2 + j(50{,}000)(6250)} = 0$

$V_o = (0)V_i \quad \therefore \quad v_o(t) = 0\,\text{mV}$

[b] $H(j46{,}972.56) = \dfrac{50{,}000^2 - 46{,}972.56^2}{50{,}000^2 - 46{,}972.56^2 + j(46{,}972.56)(6250)} = \dfrac{1}{\sqrt{2}}\underline{/-45^\circ}$

$V_o = \dfrac{1}{\sqrt{2}}\underline{/-45^\circ}V_i \quad \therefore \quad v_o(t) = 35.36\cos(46{,}972.56t - 45^\circ)\,\text{mV}$

[c] $H(j53{,}222.56) = \dfrac{50{,}000^2 - 53{,}222.56^2}{50{,}000^2 - 53{,}222.56^2 + j(53{,}222.56)(6250)} = \dfrac{1}{\sqrt{2}}\underline{/45^\circ}$

$V_o = \dfrac{1}{\sqrt{2}}\underline{/45^\circ}V_i \quad \therefore \quad v_o(t) = 35.36\cos(53{,}222.56t + 45^\circ)\,\text{mV}$

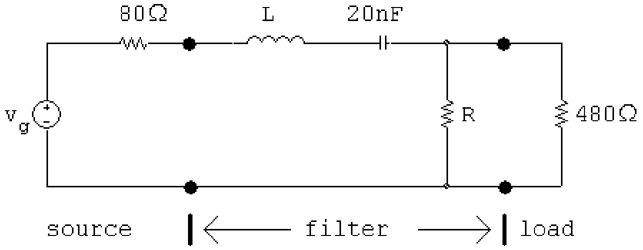
[d] $H(j10{,}000) = \dfrac{50{,}000^2 - 10{,}000^2}{50{,}000^2 - 10{,}000^2 + j(10{,}000)(6250)} = 0.9997\underline{/-1.49^\circ}$

$V_o = 0.9997\underline{/-1.49^\circ}V_i \quad \therefore \quad v_o(t) = 49.98\cos(10{,}000t - 1.49^\circ)\,\text{mV}$

[e] $H(j250{,}000) = \dfrac{50{,}000^2 - 250{,}000^2}{50{,}000^2 - 250{,}000^2 + j(250{,}000)(6250)} = 0.9997\underline{/1.49^\circ}$

$V_o = 0.9997\underline{/1.49^\circ}V_i \quad \therefore \quad v_o(t) = 49.98\cos(250{,}000t + 1.49^\circ)\,\text{mV}$

P 14.30 **[a]**



[b] $$L = \frac{1}{\omega_o^2 C} = \frac{1}{(50 \times 10^3)^2(20 \times 10^{-4})} = 20\,\text{mH}$$

$$R = \frac{\omega_o L}{Q} = \frac{(50 \times 10^3)(20 \times 10^{-3})}{6.25} = 160\,\Omega$$

[c] $R_e = 160\|480 = 120\,\Omega$

$$R_e + R_i = 120 + 80 = 200\,\Omega$$

$$Q_{\text{system}} = \frac{\omega_o L}{R_e + R_i} = \frac{(50 \times 10^3)(20 \times 10^{-3})}{200} = 5$$

[d] $$\beta_{\text{system}} = \frac{\omega_o}{Q_{\text{system}}} = \frac{50 \times 10^3}{5} = 10\,\text{krad/s}$$

$$\beta_{\text{system}}(\text{Hz}) = \frac{10{,}000}{2\pi} = 1591.55\,\text{Hz}$$

P 14.31 **[a]** $\frac{V_o}{V_i} = \frac{Z}{Z + R}$ where $Z = \frac{1}{Y}$

and $Y = sC + \frac{1}{sL} + \frac{1}{R_L} = \frac{LCR_Ls^2 + sL + R_L}{R_LLs}$

$$H(s) = \frac{R_LLs}{R_LRLCs^2 + (R + R_L)Ls + RR_L}$$

$$= \frac{(1/RC)s}{s^2 + \left[\left(\frac{R+R_L}{R_L}\right)\left(\frac{1}{RC}\right)\right]s + \frac{1}{LC}}$$

$$= \frac{\left(\frac{R_L}{R+R_L}\right)\left(\frac{R+R_L}{R_L}\right)\left(\frac{1}{RC}\right)s}{s^2 + \left[\left(\frac{R+R_L}{R_L}\right)\left(\frac{1}{RC}\right)\right]s + \frac{1}{LC}}$$

$$= \frac{K\beta s}{s^2 + \beta s + \omega_o^2}, \qquad K = \frac{R_L}{R + R_L}, \qquad \beta = \frac{1}{(R\|R_L)C}$$

[b] $$\beta = \left(\frac{R + R_L}{R_L}\right)\frac{1}{RC}$$

[c] $\beta_U = \dfrac{1}{RC}$

$$\therefore \quad \beta_L = \left(\frac{R + R_L}{R_L}\right)\beta_U = \left(1 + \frac{R}{R_L}\right)\beta_U$$

[d] $Q = \dfrac{\omega_o}{\beta} = \dfrac{\omega_o RC}{\left(\frac{R+R_L}{R_L}\right)}$

[e] $Q_U = \omega_o RC$

$$\therefore \quad Q_L = \left(\frac{R_L}{R + R_L}\right)Q_U = \frac{1}{[1 + (R/R_L)]}Q_U$$

[f] $H(j\omega) = \dfrac{Kj\omega\beta}{\omega_o^2 - \omega^2 + j\omega\beta}$

$$H(j\omega_o) = K$$

Let ω_c represent a corner frequency. Then

$$|H(j\omega_c)| = \frac{K}{\sqrt{2}} = \frac{K\omega_c\beta}{\sqrt{(\omega_o^2 - \omega_c^2)^2 + \omega_c^2\beta^2}}$$

$$\therefore \quad \frac{1}{\sqrt{2}} = \frac{\omega_c\beta}{\sqrt{(\omega_o^2 - \omega_c^2)^2 + \omega_c^2\beta^2}}$$

Squaring both sides leads to

$$(\omega_o^2 - \omega_c^2)^2 = \omega_c^2\beta^2 \text{ or } (\omega_o^2 - \omega_c^2) = \pm\omega_c\beta$$

$$\therefore \quad \omega_c^2 \pm \omega_c\beta - \omega_o^2 = 0$$

or

$$\omega_c = \mp\frac{\beta}{2} \pm \sqrt{\frac{\beta^2}{4} + \omega_o^2}$$

The two positive roots are

$$\omega_{c1} = -\frac{\beta}{2} + \sqrt{\frac{\beta^2}{4} + \omega_o^2} \quad \text{and} \quad \omega_{c2} = \frac{\beta}{2} + \sqrt{\frac{\beta^2}{4} + \omega_o^2}$$

where

$$\beta = \left(1 + \frac{R}{R_L}\right)\frac{1}{RC} \text{ and } \omega_o^2 = \frac{1}{LC}$$

P 14.32 **[a]** $\omega_o^2 = \dfrac{1}{LC} = \dfrac{1}{(5 \times 10^{-3})(200 \times 10^{-12})} = 10^{12}$

$$\omega_o = 1\,\text{Mrad/s}$$

[b] $\beta = \frac{R+R_L}{R_L}\cdot\frac{1}{RC} = \left(\frac{500\times 10^3}{400\times 10^3}\right)\left(\frac{1}{(100\times 10^3)(200\times 10^{-12})}\right) = 62.5\,\text{krad/s}$

[c] $Q = \frac{\omega_o}{\beta} = \frac{10^6}{62.5\times 10^3} = 16$

[d] $H(j\omega_o) = \frac{R_L}{R+R_L} = 0.8\underline{/0^\circ}$

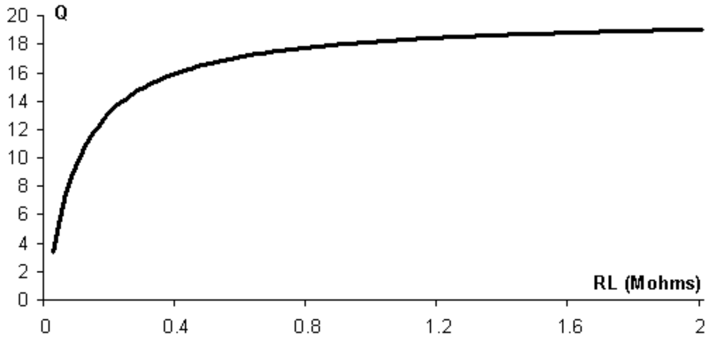
$$\therefore\quad v_o(t) = 250(0.8)\cos(10^6 t) = 200\cos 10^6 t\,\text{mV}$$

[e] $\beta = \left(1+\frac{R}{R_L}\right)\frac{1}{RC} = \left(1+\frac{100}{R_L}\right)(50\times 10^3)\ \text{rad/s}$

$$\omega_o = 10^6\ \text{rad/s}$$

$$Q = \frac{\omega_o}{\beta} = \frac{20}{1+(100/R_L)}\qquad \text{where } R_L \text{ is in kilohms}$$

[f]



P 14.33 $\omega_o^2 = \frac{1}{LC} = \frac{1}{(2\times 10^{-6})(50\times 10^{-12})} = 10^{16}$

$$\omega_o = 100\,\text{Mrad/s}$$

$$Q_U = \omega_o RC = (100\times 10^6)(2.4\times 10^3)(50\times 10^{-12}) = 12$$

$$\therefore\quad \left(\frac{R_L}{R+R_L}\right)12 = 7.5;\qquad \therefore\quad R_L = \frac{7.5}{4.5}R = 4\,\text{k}\Omega$$

P 14.34 [a] In analyzing the circuit qualitatively we visualize v_i as a sinusoidal voltage and we seek the steady-state nature of the output voltage v_o.

At zero frequency the inductor provides a direct connection between the input and the output, hence $v_o = v_i$ when $\omega = 0$.

At infinite frequency the capacitor provides the direct connection, hence $v_o = v_i$ when $\omega = \infty$.

At the resonant frequency of the parallel combination of L and C the impedance of the combination is infinite and hence the output voltage will be zero when $\omega = \omega_o$.
At frequencies on either side of ω_o the amplitude of the output voltage will be nonzero but less than the amplitude of the input voltage.
Thus the circuit behaves like a band-reject filter.

[b] Let Z represent the impedance of the parallel branches L and C, thus

$$Z = \frac{sL(1/sC)}{sL + 1/sC} = \frac{sL}{s^2LC + 1}$$

Then

$$H(s) = \frac{V_o}{V_i} = \frac{R}{Z + R} = \frac{R(s^2LC + 1)}{sL + R(s^2LC + 1)}$$

$$= \frac{[s^2 + (1/LC)]}{s^2 + \left(\frac{1}{RC}\right) s + \left(\frac{1}{LC}\right)}$$

$$H(s) = \frac{s^2 + \omega_o^2}{s^2 + \beta s + \omega_o^2}$$

[c] From part (b) we have

$$H(j\omega) = \frac{\omega_o^2 - \omega^2}{\omega_o^2 - \omega^2 + j\omega\beta}$$

It follows that $H(j\omega) = 0$ when $\omega = \omega_o$.

$$\therefore \quad \omega_o = \frac{1}{\sqrt{LC}}$$

[d] $$|H(j\omega)| = \frac{\omega_o^2 - \omega^2}{\sqrt{(\omega_o^2 - \omega^2)^2 + \omega^2\beta^2}}$$

$|H(j\omega)| = \dfrac{1}{\sqrt{2}}$ when $\omega^2\beta^2 = (\omega_o^2 - \omega^2)^2$

or $\pm\,\omega\beta = \omega_o^2 - \omega^2$, thus

$$\omega^2 \pm \beta\omega - \omega_o^2 = 0$$

The two positive roots of this quadratic are

$$\omega_{c_1} = \frac{-\beta}{2} + \sqrt{\left(\frac{\beta}{2}\right)^2 + \omega_o^2}$$

$$\omega_{c_2} = \frac{\beta}{2} + \sqrt{\left(\frac{\beta}{2}\right)^2 + \omega_o^2}$$

Also note that since $\beta = \omega_o/Q$

$$\omega_{c_1} = \omega_o \left[\frac{-1}{2Q} + \sqrt{1 + \left(\frac{1}{2Q}\right)^2} \right]$$

$$\omega_{c_2} = \omega_o \left[\frac{1}{2Q} + \sqrt{1 + \left(\frac{1}{2Q}\right)^2} \right]$$

[e] It follows from the equations derived in part (d) that

$$\beta = \omega_{c_2} - \omega_{c_1} = 1/RC$$

[f] By definition $Q = \omega_o/\beta = \omega_o RC$.

P 14.35 **[a]** $\omega_o^2 = \dfrac{1}{LC} = \dfrac{1}{(50 \times 10^{-6})(20 \times 10^{-9})} = 10^{12}$

$$\therefore \quad \omega_o = 1\,\text{Mrad/s}$$

[b] $f_o = \dfrac{\omega_o}{2\pi} = 159.15\,\text{kHz}$

[c] $Q = \omega_o RC = (10^6)(750)(20 \times 10^{-9}) = 15$

[d] $\omega_{c1} = \omega_o \left[-\dfrac{1}{2Q} + \sqrt{1 + \left(\dfrac{1}{2Q}\right)^2} \right] = 10^6 \left[-\dfrac{1}{30} + \sqrt{1 + \dfrac{1}{900}} \right]$

$$= 967.22\,\text{krad/s}$$

[e] $f_{c1} = \dfrac{\omega_{c1}}{2\pi} = 153.94\,\text{kHz}$

[f] $\omega_{c2} = \omega_o \left[\dfrac{1}{2Q} + \sqrt{1 + \left(\dfrac{1}{2Q}\right)^2} \right] = 10^6 \left[\dfrac{1}{30} + \sqrt{1 + \dfrac{1}{900}} \right]$

$$= 1.03\,\text{Mrad/s}$$

[g] $f_{c2} = \dfrac{\omega_{c1}}{2\pi} = 164.55\,\text{kHz}$

[h] $\beta = f_{c2} - f_{c1} = 10.61\,\text{kHz}$

P 14.36 **[a]** $\omega_o = 2\pi f_o = 8\pi\,\text{krad/s}$

$$L = \frac{1}{\omega_o^2 C} = \frac{1}{(8000\pi)^2(0.5 \times 10^{-6})} = 3.17\,\text{mH}$$

$$R = \frac{Q}{\omega_o C} = \frac{5}{(8000\pi)(0.5 \times 10^{-6})} = 397.89\,\Omega$$

[b] $$f_{c2} = f_o\left[\frac{1}{2Q} + \sqrt{1 + \left(\frac{1}{2Q}\right)^2}\right] = 4000\left[\frac{1}{10} + \sqrt{1 + \frac{1}{100}}\right]$$

$$= 4.42\,\text{kHz}$$

$$f_{c1} = f_o\left[-\frac{1}{2Q} + \sqrt{1 + \left(\frac{1}{2Q}\right)^2}\right] = 4000\left[-\frac{1}{10} + \sqrt{1 + \frac{1}{100}}\right]$$

$$= 3.62\,\text{kHz}$$

[c] $\beta = f_{c2} - f_{c1} = 800\,\text{Hz}$

or

$$\beta = \frac{f_o}{Q} = \frac{4000}{5} = 800\,\text{Hz}$$

P 14.37 **[a]** $R_e = 397.89 \| 1000 = 284.63\,\Omega$

$$Q = \omega_o R_e C = (8000\pi)(284.63)(0.5 \times 10^{-6}) = 3.58$$

[b] $$\beta = \frac{f_o}{Q} = \frac{4000}{3.58} = 1.12\,\text{kHz}$$

[c] $$f_{c2} = 4000\left[\frac{1}{7.16} + \sqrt{1 + \frac{1}{7.16^2}}\right] = 4.60\,\text{kHz}$$

[d] $$f_{c1} = 4000\left[-\frac{1}{7.16} + \sqrt{1 + \frac{1}{7.16^2}}\right] = 3.48\,\text{kHz}$$

P 14.38 **[a]** We need $\omega_c = 2\pi(4000) = 25{,}132.74$ rad/s. There are several possible approaches – this one starts by choosing $L = 100\,\mu\text{H}$. Then,

$$C = \frac{1}{[2\pi(4000)]^2(100 \times 10^{-6})} = 15.83\,\mu\text{F}$$

Use the closest value from Appendix H, which is $22\,\mu$F, to give

$$\omega_c = \sqrt{\frac{1}{100 \times 10^{-6})(22 \times 10^{-6})}} = 21{,}320.07 \text{ rad/s} \quad \text{so} \quad f_c = 3393.19\,\text{Hz}$$

$$\text{Then,} \quad R = \frac{Q}{\omega_o C} = \frac{5}{(21320.07)(22 \times 10^{-6})} = 10.66\,\Omega$$

Use the closest value from Appendix H, which is $10\,\Omega$, to give

$$Q = 10(21{,}320.07)(22 \times 10^{-6}) = 4.69$$

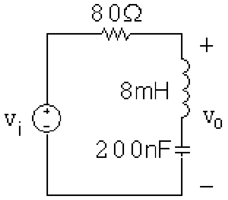
[b] $$\%\text{ error in } f_c = \frac{3393.19 - 4000}{4000}(100) = -15.2\%$$

$$\%\text{ error in } Q = \frac{4.69 - 5}{5}(100) = -6.2\%$$

P 14.39 **[a]** $\omega_o = \sqrt{1/LC}$ so $L = \dfrac{1}{\omega_o^2 C} = \dfrac{1}{(25{,}000)^2(200\times 10^{-9})} = 8\,\text{mH}$

$$Q = \frac{\omega_o}{\beta} \quad \text{so} \quad \beta = \frac{\omega_o}{Q} = \frac{25{,}000}{2.5} = 10{,}000 \text{ rad/s}$$

$$\beta = \frac{R}{L} \quad \text{so} \quad R = L\beta = (8\times 10^{-3})(10{,}000) = 80\,\Omega$$



[b] From part (a), $\beta = 10{,}000$ rad/s.

$$\omega_{c1,2} = \pm\frac{\beta}{2} + \sqrt{\frac{\beta}{2} + \omega_o^2} = \pm\frac{10{,}000}{2} + \sqrt{\left(\frac{10{,}000}{2}\right)^2 + 25{,}000^2} = \pm 5000 + 25{,}495.1$$

$\omega_{c1} = 20{,}495.1$ rad/s $\qquad \omega_{c2} = 30{,}495.1$ rad/s

P 14.40 $H(j\omega) = \dfrac{\omega_o^2 - \omega^2}{\omega_o^2 - \omega^2 + j\omega\beta} = \dfrac{25{,}000^2 - \omega^2}{25{,}000^2 - \omega^2 + j\omega(10{,}000)}$

[a] $H(j25{,}000) = \dfrac{25{,}000^2 - 25{,}000^2}{25{,}000^2 - 25{,}000^2 + j(25{,}000)(10{,}000)} = 0$

$V_o = (0)V_i \quad \therefore \quad v_o(t) = 0\,\text{mV}$

[b] $H(j20{,}495.1) = \dfrac{25{,}000^2 - 20{,}495.1^2}{25{,}000^2 - 20{,}495.1^2 + j(20{,}495.1)(10{,}000)} = \dfrac{1}{\sqrt{2}}\underline{/-45^\circ}$

$V_o = \dfrac{1}{\sqrt{2}}\underline{/-45^\circ} V_i \quad \therefore \quad v_o(t) = 176.78\cos(20{,}495.1t - 45^\circ)\,\text{mV}$

[c] $H(j30{,}495.1) = \dfrac{25{,}000^2 - 30{,}495.1^2}{25{,}000^2 - 30{,}495.1^2 + j(30{,}495.1)(10{,}000)} = \dfrac{1}{\sqrt{2}}\underline{/45^\circ}$

$V_o = \dfrac{1}{\sqrt{2}}\underline{/45^\circ} V_i \quad \therefore \quad v_o(t) = 176.78\cos(30{,}495.1t + 45^\circ)\,\text{mV}$

[d] $H(j5000) = \dfrac{25{,}000^2 - 5000^2}{25{,}000^2 - 5000^2 + j(5000)(10{,}000)} = 0.9965\underline{/-4.76^\circ}$

$V_o = 0.9965\underline{/-4.76^\circ} V_i \quad \therefore \quad v_o(t) = 249.1\cos(5000t - 4.76^\circ)\,\text{mV}$

[e] $$H(j125{,}000) = \frac{25{,}000^2 - 125{,}000^2}{25{,}000^2 - 125{,}000^2 + j(125{,}000)(10{,}000)} = 0.9965\underline{/4.76^\circ}$$

$$V_o = 0.9965\underline{/4.76^\circ}V_i \quad \therefore \quad v_o(t) = 249.1\cos(125{,}000t + 4.76^\circ)\,\text{mV}$$

P 14.41 $$H(j\omega) = \frac{j\omega\beta}{\omega_o^2 - \omega^2 + j\omega\beta} = \frac{j\omega(10{,}000)}{25{,}000^2 - \omega^2 + j\omega(10{,}000)}$$

[a] $$H(j25{,}000) = \frac{j(25{,}000)(10{,}000)}{25{,}000^2 - 25{,}000^2 + j(25{,}000)(10{,}000)} = 1$$

$$V_o = (1)V_i \quad \therefore \quad v_o(t) = 250\cos 25{,}000t\,\text{mV}$$

[b] $$H(j20{,}495.1) = \frac{j(20{,}495.1)(10{,}000)}{25{,}000^2 - 20{,}495.1^2 + j(20{,}495.1)(10{,}000)} = \frac{1}{\sqrt{2}}\underline{/45^\circ}$$

$$V_o = \frac{1}{\sqrt{2}}\underline{/45^\circ}V_i \quad \therefore \quad v_o(t) = 176.78\cos(20{,}495.1t + 45^\circ)\,\text{mV}$$

[c] $$H(j30{,}495.1) = \frac{j(30{,}495.1)(10{,}000)}{25{,}000^2 - 30{,}495.1^2 + j(30{,}495.1)(10{,}000)} = \frac{1}{\sqrt{2}}\underline{/-45^\circ}$$

$$V_o = \frac{1}{\sqrt{2}}\underline{/-45^\circ}V_i \quad \therefore \quad v_o(t) = 176.78\cos(30{,}495.1t - 45^\circ)\,\text{mV}$$

[d] $$H(j5000) = \frac{j(5000)(10{,}000)}{25{,}000^2 - 5000^2 + j(5000)(10{,}000)} = 0.083\underline{/85.24^\circ}$$

$$V_o = 0.083\underline{/85.24^\circ}V_i \quad \therefore \quad v_o(t) = 20.75\cos(5000t + 85.24^\circ)\,\text{mV}$$

[e] $$H(j125{,}000) = \frac{j(125{,}000)(10{,}000)}{25{,}000^2 - 125{,}000^2 + j(125{,}000)(10{,}000)} = 0.083\underline{/-85.24^\circ}$$

$$V_o = 0.083\underline{/-85.24^\circ}V_i \quad \therefore \quad v_o(t) = 20.75\cos(125{,}000t - 85.24^\circ)\,\text{mV}$$

P 14.42 **[a]** Let $$Z = \frac{R_L(sL + (1/sC))}{R_L + sL + (1/sC)}$$

$$Z = \frac{R_L(s^2LC + 1)}{s^2LC + R_LCs + 1}$$

Then $$H(s) = \frac{V_o}{V_i} = \frac{s^2R_LCL + R_L}{(R + R_L)LCs^2 + RR_LCs + R + R_L}$$

Therefore

$$H(s) = \left(\frac{R_L}{R + R_L}\right) \cdot \frac{[s^2 + (1/LC)]}{\left[s^2 + \left(\frac{RR_L}{R+R_L}\right)\frac{s}{L} + \frac{1}{LC}\right]}$$

$$= \frac{K(s^2 + \omega_o^2)}{s^2 + \beta s + \omega_o^2}$$

where $$K = \frac{R_L}{R + R_L}; \quad \omega_o^2 = \frac{1}{LC}; \quad \beta = \left(\frac{RR_L}{R + R_L}\right)\frac{1}{L}$$

[b] $\omega_o = \dfrac{1}{\sqrt{LC}}$

[c] $\beta = \left(\dfrac{RR_L}{R+R_L}\right)\dfrac{1}{L}$

[d] $Q = \dfrac{\omega_o}{\beta} = \dfrac{\omega_o L}{[RR_L/(R+R_L)]}$

[e] $H(j\omega) = \dfrac{K(\omega_o^2-\omega^2)}{(\omega_o^2-\omega^2)+j\beta\omega}$

$$H(j\omega_o) = 0$$

[f] $H(j0) = \dfrac{K\omega_o^2}{\omega_o^2} = K$

[g] $H(j\omega) = \dfrac{K\left[(\omega_o/\omega)^2-1\right]}{\left\{\left[(\omega_o/\omega)^2-1\right]+j\beta/\omega\right\}}$

$$H(j\infty) = \frac{-K}{-1} = K$$

[h] $H(j\omega) = \dfrac{K(\omega_o^2-\omega^2)}{(\omega_o^2-\omega^2)+j\beta\omega}$

$$H(j0) = H(j\infty) = K$$

Let ω_c represent a corner frequency. Then

$$|H(j\omega_c)| = \frac{K}{\sqrt{2}}$$

$$\therefore\quad \frac{K}{\sqrt{2}} = \frac{K(\omega_o^2-\omega_c^2)}{\sqrt{(\omega_o^2-\omega_c^2)^2+\omega_c^2\beta^2}}$$

Squaring both sides leads to

$$(\omega_o^2-\omega_c^2)^2 = \omega_c^2\beta^2 \text{ or } (\omega_o^2-\omega_c^2) = \pm\omega_c\beta$$

$$\therefore\quad \omega_c^2 \pm \omega_c\beta - \omega_o^2 = 0$$

or

$$\omega_c = \mp\frac{\beta}{2} \pm \sqrt{\frac{\beta^2}{4}+\omega_o^2}$$

The two positive roots are

$$\omega_{c1} = -\frac{\beta}{2} + \sqrt{\frac{\beta^2}{4}+\omega_o^2} \quad \text{and} \quad \omega_{c2} = \frac{\beta}{2} + \sqrt{\frac{\beta^2}{4}+\omega_o^2}$$

where

$$\beta = \frac{RR_L}{R+R_L}\cdot\frac{1}{L} \text{ and } \omega_o^2 = \frac{1}{LC}$$

P 14.43 **[a]** $\omega_o^2 = \dfrac{1}{LC} = \dfrac{1}{(10^{-6})(4\times10^{-12})} = 0.25\times10^{18} = 25\times10^{16}$

$$\omega_o = 5\times10^8 = 500\,\text{Mrad/s}$$

$$\beta = \frac{RR_L}{R+R_L}\cdot\frac{1}{L} = \frac{(30)(150)}{180}\cdot\frac{1}{10^{-6}} = 25\,\text{Mrad/s} = 3.98\,\text{MHz}$$

$$Q = \frac{\omega_o}{\beta} = \frac{500\,\text{M}}{25\,\text{M}} = 20$$

[b] $H(j0) = \dfrac{R_L}{R+R_L} = \dfrac{150}{180} = 0.8333$

$$H(j\infty) = \frac{R_L}{R+R_L} = 0.8333$$

[c] $f_{c2} = \dfrac{250}{\pi}\left[\dfrac{1}{40} + \sqrt{1+\dfrac{1}{1600}}\right] = 81.59\,\text{MHz}$

$$f_{c1} = \frac{250}{\pi}\left[-\frac{1}{40} + \sqrt{1+\frac{1}{1600}}\right] = 77.61\,\text{MHz}$$

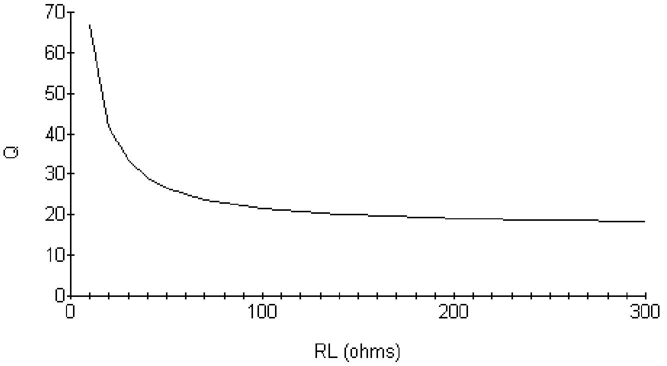
Check: $\beta = f_{c2} - f_{c1} = 3.98\,\text{MHz}.$

[d] $Q = \dfrac{\omega_o}{\beta} = \dfrac{500\times10^6}{\frac{RR_L}{R+R_L}\cdot\frac{1}{L}}$

$$= \frac{500(R+R_L)}{RR_L} = \frac{50}{3}\left(1+\frac{30}{R_L}\right)$$

where R_L is in ohms.

[e]



P 14.44 **[a]** $\omega_o^2 = \dfrac{1}{LC} = 625 \times 10^6$

$$\therefore \quad L = \frac{1}{(625 \times 10^6)(25 \times 10^{-9})} = 64\,\text{mH}$$

$$\frac{R_L}{R+R_L} = 0.9; \qquad \therefore \quad 0.1R_L = 0.9R$$

$$\therefore \qquad R_L = 9R \quad \therefore \quad R = \frac{500}{9} = 55.6\,\Omega$$

[b] $\beta = \left(\dfrac{R_L}{R+R_L}\right) R \cdot \dfrac{1}{L} = 781.25$ rad/s

$$Q = \frac{\omega_o}{\beta} = \frac{25{,}000}{781.25} = 32$$

P 14.45 **[a]** $|H(j\omega)| = \dfrac{10^{10}}{\sqrt{(10^{10}-\omega^2)^2 + (50{,}000\omega)^2}} = 1$

$$\therefore \qquad 10^{20} = (10^{10}-\omega^2)^2 + (50{,}000\omega)^2$$

$$= -2 \times 10^{10}\omega^2 + \omega^4 + 25 \times 10^8\omega^2$$

$$\therefore \qquad \omega^2 = 175 \times 10^8 \quad \text{so} \quad \omega = 132{,}287.57\,\text{rad/s}$$

[b] From the equation for $|H(j\omega)|$ in part (a), the frequency for which the magnitude is maximum is the frequency for which the denominator is minimum. This is the frequency at which

$$(10^{10}-\omega^2)^2 = 0 \quad \text{so} \quad \omega = \sqrt{10^{10}} = 100{,}000\,\text{rad/s}$$

[c] $|H(j100{,}000)| = \dfrac{10^{10}}{\sqrt{(10^{10} - 100{,}000^2)^2 + [50{,}000(100{,}000)]^2}} = 2$

P 14.46 **[a]** Use the cutoff frequencies to calculate the bandwidth:

$$\omega_{c1} = 2\pi(697) = 4379.38 \text{ rad/s} \qquad \omega_{c2} = 2\pi(941) = 5912.48 \text{ rad/s}$$

Thus $\quad \beta = \omega_{c2} - \omega_{c1} = 1533.10$ rad/s

Calculate inductance using Eq. (14.32) and capacitance using Eq. (14.31):

$$L = \frac{R}{\beta} = \frac{600}{1533.10} = 0.39\,\text{H}$$

$$C = \frac{1}{L\omega_{c1}\omega_{c2}} = \frac{1}{(0.39)(4379.38)(5912.48)} = 0.10\,\mu\text{F}$$

[b] At the outermost two frequencies in the low-frequency group (687 Hz and 941 Hz) the amplitudes are

$$|\mathrm{V}_{697\mathrm{Hz}}| = |V_{941\mathrm{Hz}}| = \frac{|V_{\mathrm{peak}}|}{\sqrt{2}} = 0.707|V_{\mathrm{peak}}|$$

because these are cutoff frequencies. We calculate the amplitudes at the other two low frequencies using Eq. (14.32):

$$|V| = (|V_{\mathrm{peak}}|)(|H(j\omega)|) = |V_{\mathrm{peak}}|\frac{\omega\beta}{\sqrt{(\omega_o^2 - \omega^2)^2 + (\omega\beta)^2}}$$

Therefore

$$|V_{770\mathrm{Hz}}| = |V_{\mathrm{peak}}| = \frac{(4838.05)(1533.10)}{\sqrt{(5088.52^2 - 4838.05^2)^2 + [(4838.05)(1533.10)]^2}}$$

$$= 0.948|V_{\mathrm{peak}}|$$

and

$$|V_{852\mathrm{Hz}}| = |V_{\mathrm{peak}}| = \frac{(5353.27)(1533.10)}{\sqrt{(5088.52^2 - 5353.27^2)^2 + [(5353.27)(1533.10)]^2}}$$

$$= 0.948|V_{\mathrm{peak}}|$$

It is not a coincidence that these two magnitudes are the same. The frequencies in both bands of the DTMF system were carefully chosen to produce this type of predictable behavior with linear filters. In other words, the frequencies were chosen to be equally far apart with respect to the response produced by a linear filter. Most musical scales consist of tones designed with this dame property – note intervals are selected to place the notes equally far apart. That is why the DTMF tones remind us of musical notes! Unlike musical scales, DTMF frequencies were selected to be harmonically unrelated, to lower the risk of misidentifying a tone's frequency if the circuit elements are not perfectly linear.

[c] The high-band frequency closest to the low-frequency band is 1209 Hz. The amplitude of a tone with this frequency is

$$|V_{1209\mathrm{Hz}}| = |V_{\mathrm{peak}}| = \frac{(7596.37)(1533.10)}{\sqrt{(5088.52^2 - 7596.37^2)^2 + [(7596.37)(1533.10)]^2}}$$

$$= 0.344|V_{\mathrm{peak}}|$$

This is less than one half the amplitude of the signals with the low-band cutoff frequencies, ensuring adequate separation of the bands.

P 14.47 The cutoff frequencies and bandwidth are

$$\omega_{c_1} = 2\pi(1209) = 7596\,\text{rad/s}$$

$$\omega_{c_2} = 2\pi(1633) = 10.26\,\text{krad/s}$$

$$\beta = \omega_{c_2} - \omega_{c_1} = 2664\,\text{rad/s}$$

Telephone circuits always have $R = 600\,\Omega$. Therefore, the filters inductance and capacitance values are

$$L = \frac{R}{\beta} = \frac{600}{2664} = 0.225\,\text{H}$$

$$C = \frac{1}{\omega_{c_1}\omega_{c_2}L} = 0.057\,\mu\text{F}$$

At the highest of the low-band frequencies, 941 Hz, the amplitude is

$$|V_\omega| = |V_{\text{peak}}|\frac{\omega\beta}{\sqrt{(\omega_o^2 - \omega^2)^2 + \omega^2\beta^2}}$$

where $\omega_o = \sqrt{\omega_{c_1}\omega_{c_2}}$. Thus,

$$|V_\omega| = \frac{|V_{\text{peak}}|(5912)(2664)}{\sqrt{[(8828)^2 - (5912)^2]^2 + [(5912)(2664)]^2}}$$

$$= 0.344\;|V_{\text{peak}}|$$

Again it is not coincidental that this result is the same as the response of the low-band filter to the lowest of the high-band frequencies.

P 14.48 From Problem 14.46 the response to the largest of the DTMF low-band tones is $0.948|V_{\text{peak}}|$. The response to the 20 Hz tone is

$$|V_{20\text{Hz}}| = \frac{|V_{\text{peak}}|(125.6)(1533)}{[(5089^2 - 125.6^2)^2 + [(125.6)(1533)]^2]^{1/2}}$$

$$= 0.00744|V_{\text{peak}}|$$

$$\therefore\quad \frac{|V_{20\text{Hz}}|}{|V_{770\text{Hz}}|} = \frac{|V_{20\text{Hz}}|}{|V_{852\text{Hz}}|} = \frac{0.00744|V_{\text{peak}}|}{0.948|V_{\text{peak}}|} = 0.5$$

$$\therefore\quad |V_{20\text{Hz}}| = 63.7|V_{770\text{Hz}}|$$

Thus, the 20Hz signal can be 63.7 times as large as the DTMF tones.

15

Active Filter Circuits

Assessment Problems

AP 15.1

$$H(s) = \frac{-(R_2/R_1)s}{s + (1/R_1C)}$$

$$\frac{1}{R_1C} = 1\,\text{rad/s}; \qquad R_1 = 1\,\Omega, \quad \therefore \quad C = 1\,\text{F}$$

$$\frac{R_2}{R_1} = 1, \quad \therefore \quad R_2 = R_1 = 1\,\Omega$$

$$\therefore \qquad H_{\text{prototype}}(s) = \frac{-s}{s+1}$$

AP 15.2

$$H(s) = \frac{-(1/R_1C)}{s + (1/R_2C)} = \frac{-20{,}000}{s + 5000}$$

$$\frac{1}{R_1C} = 20{,}000; \quad C = 5\,\mu\text{F}$$

$$\therefore \quad R_1 = \frac{1}{(20{,}000)(5 \times 10^{-6})} = 10\,\Omega$$

$$\frac{1}{R_2C} = 5000$$

$$\therefore \quad R_2 = \frac{1}{(5000)(5 \times 10^{-6})} = 40\,\Omega$$

AP 15.3

$$\omega_c = 2\pi f_c = 2\pi \times 10^4 = 20{,}000\pi \text{ rad/s}$$

$$\therefore \quad k_f = 20{,}000\pi = 62{,}831.85$$

$$C' = \frac{C}{k_f k_m} \qquad \therefore \qquad 0.5 \times 10^{-6} = \frac{1}{k_f k_m}$$

$$\therefore \quad k_m = \frac{1}{(0.5 \times 10^{-6})(62{,}831.85)} = 31.83$$

AP 15.4 For a 2nd order Butterworth high pass filter

$$H(s) = \frac{s^2}{s^2 + \sqrt{2}s + 1}$$

For the circuit in Fig. 15.25

$$H(s) = \frac{s^2}{s^2 + \left(\frac{2}{R_2C}\right) s + \left(\frac{1}{R_1R_2C^2}\right)}$$

Equate the transfer functions. For $C = 1$F,

$$\frac{2}{R_2C} = \sqrt{2}, \quad \therefore \quad R_2 = \sqrt{2} = 0.707\,\Omega$$

$$\frac{1}{R_1R_2C^2} = 1, \quad \therefore \quad R_1 = \frac{1}{\sqrt{2}} = 1.414\,\Omega$$

AP 15.5

$Q = 8, K = 5, \omega_o = 1000\,\text{rad/s}, C = 1\,\mu\text{F}$

For the circuit in Fig 15.26

$$H(s) = \frac{-\left(\dfrac{1}{R_1C}\right) s}{s^2 + \left(\dfrac{2}{R_3C}\right) s + \left(\dfrac{R_1 + R_2}{R_1R_2R_3C^2}\right)}$$

$$= \frac{K\beta s}{s^2 + \beta s + \omega_o^2}$$

$$\beta = \frac{2}{R_3C}, \qquad \therefore \qquad R_3 = \frac{2}{\beta C}$$

$$\beta = \frac{\omega_o}{Q} = \frac{1000}{8} = 125\,\text{rad/s}$$

$$\therefore \quad R_3 = \frac{2 \times 10^6}{(125)(1)} = 16\,\text{k}\Omega$$

$$K\beta = \frac{1}{R_1C}$$

$$\therefore \quad R_1 = \frac{1}{K\beta C} = \frac{1}{5(125)(1 \times 10^{-6})} = 1.6\,\text{k}\Omega$$

$$\omega_o^2 = \frac{R_1 + R_2}{R_1R_2R_3C^2}$$

$$10^6 = \frac{(1600 + R_2)}{(1600)(R_2)(16{,}000)(10^{-6})^2}$$

Solving for R_2,

$$R_2 = \frac{(1600 + R_2)10^6}{256 \times 10^5}, \quad 246R_2 = 16{,}000, \quad R_2 = 65.04\,\Omega$$

AP 15.6

$$\omega_o = 1000\,\text{rad/s}; \qquad Q = 4;$$

$$C = 2\,\mu\text{F}$$

$$H(s) = \frac{s^2 + (1/R^2C^2)}{s^2 + \left[\dfrac{4(1-\sigma)}{RC}\right]s + \left(\dfrac{1}{R^2C^2}\right)}$$
$$= \frac{s^2 + \omega_o^2}{s^2 + \beta s + \omega_o^2}; \qquad \omega_o = \frac{1}{RC}; \qquad \beta = \frac{4(1-\sigma)}{RC}$$

$$R = \frac{1}{\omega_o C} = \frac{1}{(1000)(2 \times 10^{-6})} = 500\,\Omega$$

$$\beta = \frac{\omega_o}{Q} = \frac{1000}{4} = 250$$

$$\therefore \qquad \frac{4(1-\sigma)}{RC} = 250$$

$$4(1-\sigma) = 250RC = 250(500)(2 \times 10^{-6}) = 0.25$$

$$1 - \sigma = \frac{0.25}{4} = 0.0625; \quad \therefore \qquad \sigma = 0.9375$$

Problems

P 15.1 Summing the currents at the inverting input node yields

$$\frac{0 - V_i}{Z_i} + \frac{0 - V_o}{Z_f} = 0$$

$$\therefore \quad \frac{V_o}{Z_f} = -\frac{V_i}{Z_i}$$

$$\therefore \quad H(s) = \frac{V_o}{V_i} = -\frac{Z_f}{Z_i}$$

P 15.2 **[a]** $Z_f = \dfrac{R_2(1/sC_2)}{[R_2 + (1/sC_2)]} = \dfrac{R_2}{R_2C_2s + 1}$

$$= \frac{(1/C_2)}{s + (1/R_2C_2)}$$

Likewise

$$Z_i = \frac{(1/C_1)}{s + (1/R_1C_1)}$$

$$\therefore \quad H(s) = \frac{-(1/C_2)[s + (1/R_1C_1)]}{[s + (1/R_2C_2)](1/C_1)}$$

$$= -\frac{C_1}{C_2}\frac{[s + (1/R_1C_1)]}{[s + (1/R_2C_2)]}$$

[b] $H(j\omega) = \dfrac{-C_1}{C_2}\left[\dfrac{j\omega + (1/R_1C_1)}{j\omega + (1/R_2C_2)}\right]$

$$H(j0) = \frac{-C_1}{C_2}\left(\frac{R_2C_2}{R_1C_1}\right) = \frac{-R_2}{R_1}$$

[c] $H(j\infty) = -\dfrac{C_1}{C_2}\left(\dfrac{j}{j}\right) = \dfrac{-C_1}{C_2}$

[d] As $\omega \to 0$ the two capacitor branches become open and the circuit reduces to a resistive inverting amplifier having a gain of $-R_2/R_1$.

As $\omega \to \infty$ the two capacitor branches approach a short circuit and in this case we encounter an indeterminate situation; namely $v_n \to v_i$ but $v_n = 0$ because of the ideal op amp. At the same time the gain of the ideal op amp is infinite so we have the indeterminate form $0 \cdot \infty$. Although $\omega = \infty$ is indeterminate we can reason that for finite large values of ω $H(j\omega)$ will approach $-C_1/C_2$ in value. In other words, the circuit approaches a purely capacitive inverting amplifier with a gain of $(-1/j\omega C_2)/(1/j\omega C_1)$ or $-C_1/C_2$.

P 15.3 [a] $Z_f = \dfrac{(1/C_2)}{s + (1/R_2C_2)}$

$$Z_i = R_1 + \frac{1}{sC_1} = \frac{R_1}{s}[s + (1/R_1C_1)]$$

$$H(s) = -\frac{(1/C_2)}{[s + (1/R_2C_2)]} \cdot \frac{s}{R_1[s + (1/R_1C_1)]}$$

$$= -\frac{1}{R_1C_2}\frac{s}{[s + (1/R_1C_1)][s + (1/R_2C_2)]}$$

[b] $H(j\omega) = -\dfrac{1}{R_1C_2}\dfrac{j\omega}{\left(j\omega + \frac{1}{R_1C_1}\right)\left(j\omega + \frac{1}{R_2C_2}\right)}$

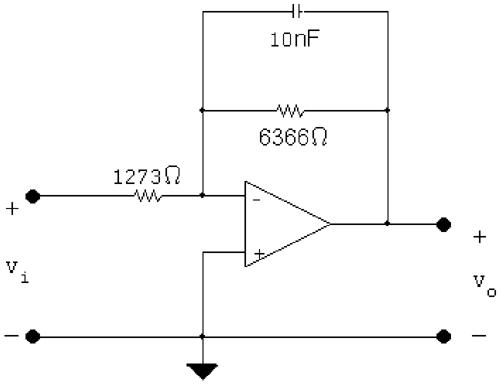
$$H(j0) = 0$$

[c] $H(j\infty) = 0$

[d] As $\omega \rightarrow 0$ the capacitor C_1 disconnects v_i from the circuit. Therefore $v_o = v_n = 0$.
As $\omega \rightarrow \infty$ the capacitor short circuits the feedback network, thus $Z_F = 0$ and therefore $v_o = 0$.

P 15.4 [a] $\omega_c = \dfrac{1}{R_2C}$ so $R_2 = \dfrac{1}{\omega_c C} = \dfrac{1}{2\pi(2500)(10 \times 10^{-9})} = 6366\,\Omega$

$$K = \frac{R_2}{R_1} \quad \text{so} \quad R_1 = \frac{R_2}{K} = \frac{6366}{5} = 1273\,\Omega$$



[b] Both the cutoff frequency and the passband gain are changed.

P 15.5 [a] $5(2) = 10\,\text{V}$ so $V_{cc} \geq 10\,\text{V}$

[b] $H(j\omega) = \dfrac{-5(2\pi)(2500)}{j\omega + 2\pi(2500)}$

$$H(j5000\pi) = \frac{-5(5000\pi)}{5000\pi + j5000\pi} = -2.5 + j2.5 = \frac{5}{\sqrt{2}}\underline{/135^\circ}$$

$$V_o = \frac{10}{\sqrt{2}}\underline{/135^\circ}V_i \quad \text{so} \quad v_o(t) = 7.07\cos(5000\pi t + 135^\circ)\,\text{V}$$

[c] $$H(j1000\pi) = \frac{-5(5000\pi)}{5000\pi + j1000\pi} = 4.9\underline{/168.7^\circ}$$

$$V_o = 4.9\underline{/168.7^\circ}V_i \quad \text{so} \quad v_o(t) = 9.8\cos(1000\pi t + 168.7^\circ)\,\text{V}$$

[d] $$H(j25{,}000\pi) = \frac{-5(5000\pi)}{5000\pi + j25{,}000\pi} = 0.98\underline{/101.3^\circ}$$

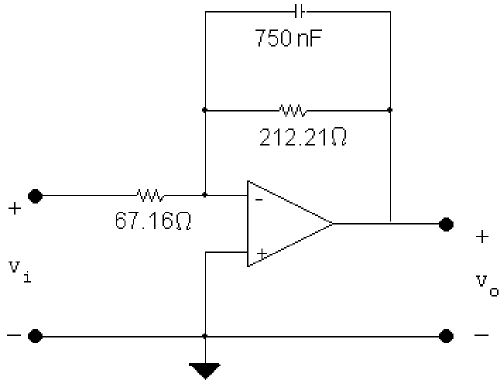
$$V_o = 0.98\underline{/101.3^\circ}V_i \quad \text{so} \quad v_o(t) = 1.96\cos(25{,}000\pi t + 101.3^\circ)\,\text{V}$$

P 15.6 **[a]** $$K = 10^{(10/20)} = 3.16 = \frac{R_2}{R_1}$$

$$R_2 = \frac{1}{\omega_c C} = \frac{1}{(2\pi)(10^3)(750 \times 10^{-9})} = 212.21\,\Omega$$

$$R_1 = \frac{R_2}{K} = \frac{212.21}{3.16} = 67.16\,\Omega$$

[b]



P 15.7 **[a]** $$\frac{1}{RC} = 2\pi(1000) \quad \text{so} \quad RC = 1.5915 \times 10^{-4}$$

There are several possible approaches. Here, choose $R_f = 150\,\Omega$. Then

$$C = \frac{1.5915 \times 10^{-4}}{150} = 1.06 \times 10^{-6}$$

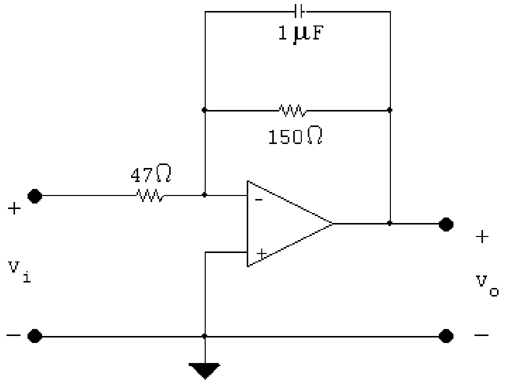
Choose $C = 1\,\mu$F. This gives

$$\omega_c = \frac{1}{(150)(10^{-6})} = 6.67 \times 10^3 \text{ rad/s} \quad \text{so} \quad f_c = 1061\,\text{Hz}$$

To get a passband gain of 10 dB, choose

$$R_i = \frac{R_f}{3.16} = \frac{150}{3.16} = 47.47\,\Omega$$

Choose $R_i = 47\,\Omega$ to give $K = 20\log_{10}(150/47) = 10.08$ dB. The resulting circuit is

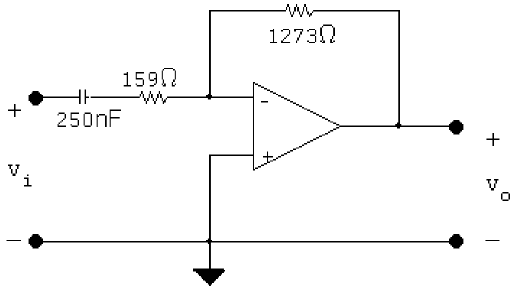


[b] $$\%\text{ error in } f_c = \frac{1061-1000}{1000}(100) = 6.1\%$$

$$\%\text{ error in passband gain} = \frac{10.08-10}{10}(100) = 0.8\%$$

P 15.8 **[a]** $$\omega_c = \frac{1}{R_1C} \quad \text{so} \quad R_1 = \frac{1}{\omega_c C} = \frac{1}{2\pi(4000)(250\times10^{-9})} = 159\,\Omega$$

$$K = \frac{R_2}{R_1} \quad \text{so} \quad R_2 = KR_1 = (8)(159) = 1273\,\Omega$$



[b] The passband gain changes but the cutoff frequency is unchanged.

P 15.9 **[a]** $8(0.25) = 2\,\text{V} \quad \text{so} \quad V_{cc} \geq 2\,\text{V}$

[b] $$H(j\omega) = \frac{-8j\omega}{j\omega + 8000\pi}$$

$$H(j600\pi) = \frac{-8(j8000\pi)}{8000\pi + j8000\pi} = \frac{8}{\sqrt{2}}\underline{/-135^\circ}$$

$$V_o = \frac{8}{\sqrt{2}}\underline{/-135^\circ}V_i \quad \text{so} \quad v_o(t) = 1.41\cos(8000\pi t - 135^\circ)\,\text{V}$$

[c] $$H(j1600\pi) = \frac{-8(j1600\pi)}{8000\pi + j1600\pi} = 1.57\underline{/-101.3^\circ}$$

$$V_o = 1.57\underline{/-101.3^\circ}V_i \quad \text{so} \quad v_o(t) = 392.2\cos(1600\pi t - 101.3^\circ)\,\text{mV}$$

[d] $H(j40{,}000\pi) = \dfrac{-8(j40{,}000\pi)}{8000\pi + j40{,}000\pi} = 7.84\underline{/-168.7^\circ}$

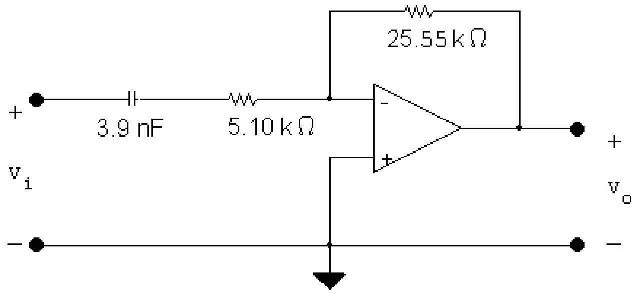
$V_o = 7.84\underline{/-168.7^\circ}V_i$ so $v_o(t) = 1.96\cos(40{,}000\pi t - 168.7^\circ)\,\text{V}$

P 15.10 **[a]** $R_1 = \dfrac{1}{\omega_c C} = \dfrac{1}{(2\pi)(8\times10^3)(3.9\times10^{-9})} = 5.10\,\text{k}\Omega$

$K = 10^{(14/20)} = 5.01 = \dfrac{R_2}{R_1}$

$\therefore\ R_2 = 5.01R_1 = 25.55\,\text{k}\Omega$

[b]



P 15.11 **[a]** $\dfrac{1}{RC} = 2\pi(8000)$ so $RC = 19.89\times10^{-6}$

There are several possible approaches. Here, choose $C = 0.047\,\mu\text{F}$. Then

$R_i = \dfrac{19.89\times10^{-6}}{0.047\times10^{-6}} = 423$

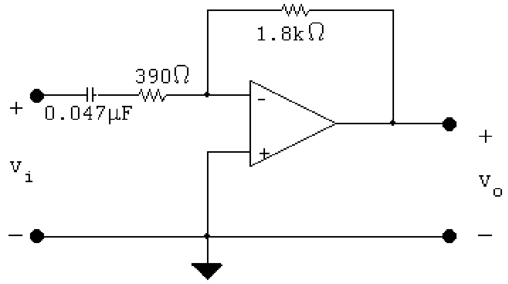
Choose $R_i = 390\,\Omega$. This gives

$\omega_c = \dfrac{1}{(0.047\times10^{-6})(390)} = 54.56$ krad/s so $f_c = 8.68\,\text{kHz}$

To get a passband gain of 14 dB, choose

$R_f = 5R_i = 5(390) = 1950\,\Omega$

Choose $R_f = 1.8\,\text{k}\Omega$ to give a passband gain of $20\log_{10}(1800/390) = 13.3$ dB. The resulting circuit is



[b] $\%\text{ error in } f_c = \dfrac{8683.76-8000}{8000}(100) = 8.5\%$

$$\%\text{ error in passband gain} = \frac{13.3-14}{14}(100) = -5.1\%$$

P 15.12 For the RC circuit

$$H(s) = \frac{V_o}{V_i} = \frac{s}{s+(1/RC)}$$

$$R' = k_mR; \qquad C' = \frac{C}{k_mk_f}$$

$$\therefore \quad R'C' = \frac{RC}{k_f} = \frac{1}{k_f}; \qquad \frac{1}{R'C'} = k_f$$

$$H'(s) = \frac{s}{s+(1/R'C')} = \frac{s}{s+k_f} = \frac{(s/k_f)}{(s/k_f)+1}$$

For the RL circuit

$$H(s) = \frac{s}{s+(R/L)}$$

$$R' = k_mR; \qquad L' = \frac{k_mL}{k_f}$$

$$\frac{R'}{L'} = k_f\left(\frac{R}{L}\right) = k_f$$

$$H'(s) = \frac{s}{s+k_f} = \frac{(s/k_f)}{(s/k_f)+1}$$

P 15.13 For the RC circuit

$$H(s) = \frac{V_o}{V_i} = \frac{(1/RC)}{s+(1/RC)}$$

$$R' = k_mR; \qquad C' = \frac{C}{k_mk_f}$$

$$\therefore \quad R'C' = k_mR\frac{C}{k_mk_f} = \frac{1}{k_f}RC = \frac{1}{k_f}$$

$$\frac{1}{R'C'} = k_f$$

$$H'(s) = \frac{(1/R'C')}{s + (1/R'C')} = \frac{k_f}{s + k_f}$$

$$H'(s) = \frac{1}{(s/k_f) + 1}$$

For the RL circuit $\quad H(s) = \dfrac{R/L}{s + R/L} \quad$ so

$$R' = k_m R; \qquad L' = \frac{k_m}{k_f}L$$

$$\frac{R'}{L'} = \frac{k_m R}{\frac{k_m}{k_f}L} = k_f\left(\frac{R}{L}\right) = k_f$$

$$H'(s) = \frac{(R'/L')}{s + (R'/L')} = \frac{k_f}{s + k_f}$$

$$H'(s) = \frac{1}{(s/k_f) + 1}$$

P 15.14 $H(s) = \dfrac{(R/L)s}{s^2 + (R/L)s + (1/LC)} = \dfrac{\beta s}{s^2 + \beta s + \omega_o^2}$

For the prototype circuit $\omega_o = 1$ and $\beta = \omega_o/Q = 1/Q$.
For the scaled circuit

$$H'(s) = \frac{(R'/L')s}{s^2 + (R'/L')s + (1/L'C')}$$

where $R' = k_m R;\ L' = \dfrac{k_m}{k_f}L;$ and $C' = \dfrac{C}{k_f k_m}$

$$\therefore \quad \frac{R'}{L'} = \frac{k_m R}{\frac{k_m}{k_f}L} = k_f\left(\frac{R}{L}\right) = k_f\beta$$

$$\frac{1}{L'C'} = \frac{k_f k_m}{\frac{k_m}{k_f}LC} = \frac{k_f^2}{LC} = k_f^2$$

$$Q' = \frac{\omega_o'}{\beta'} = \frac{k_f\omega_o}{k_f\beta} = Q$$

therefore the Q of the scaled circuit is the same as the Q of the unscaled circuit. Also note $\beta' = k_f\beta$.

$$\therefore \quad H'(s) = \frac{\left(\frac{k_f}{Q}\right)s}{s^2 + \left(\frac{k_f}{Q}\right)s + k_f^2}$$

$$H'(s) = \frac{\left(\frac{1}{Q}\right)\left(\frac{s}{k_f}\right)}{\left[\left(\frac{s}{k_f}\right)^2 + \frac{1}{Q}\left(\frac{s}{k_f}\right) + 1\right]}$$

P 15.15 [**a**] $L = 1\,\text{H}; \qquad C = 1\,\text{F}$

$$R = \frac{1}{Q} = \frac{1}{20} = 0.05\,\Omega$$

[**b**] $k_f = \frac{\omega_o'}{\omega_o} = 40{,}000; \qquad k_m = \frac{R'}{R} = \frac{5000}{0.05} = 100{,}000$

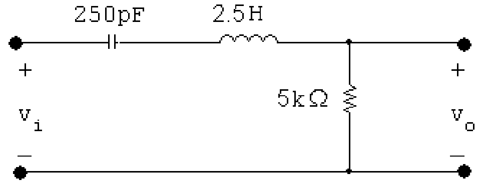
Thus,

$$R' = k_m R = (0.05)(100{,}000) = 5\,\text{k}\Omega$$

$$L' = \frac{k_m}{k_f}L = \frac{100{,}000}{40{,}000}(1) = 2.5\,\text{H}$$

$$C' = \frac{C}{k_m k_f} = \frac{1}{(40{,}000)(100{,}000)} = 250\,\text{pF}$$

[**c**]



P 15.16 [**a**] Since $\omega_o^2 = 1/LC$ and $\omega_o = 1$ rad/s,

$$C = \frac{1}{L} = \frac{1}{Q}\,\text{F}$$

[**b**] $H(s) = \frac{(R/L)s}{s^2 + (R/L)s + (1/LC)}$

$$H(s) = \frac{(1/Q)s}{s^2 + (1/Q)s + 1}$$

[c] In the prototype circuit

$$R = 1\,\Omega; \qquad L = 16\,\text{H}; \qquad C = \frac{1}{L} = 0.0625\,\text{F}$$

$$\therefore \quad k_m = \frac{R'}{R} = 10{,}000; \qquad k_f = \frac{\omega_o'}{\omega_o} = 25{,}000$$

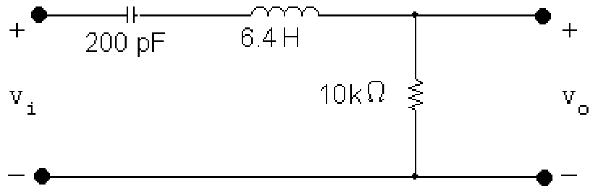
Thus

$$R' = k_m R = 10\,\text{k}\Omega$$

$$L' = \frac{k_m}{k_f}L = \frac{10{,}000}{25{,}000}(16) = 6.4\,\text{H}$$

$$C' = \frac{C}{k_m k_f} = \frac{0.0625}{(10{,}000)(25{,}000)} = 250\,\text{pF}$$

[d]



[e] $$H'(s) = \frac{\frac{1}{16}\left(\frac{s}{25{,}000}\right)}{\left(\frac{s}{25{,}000}\right)^2 + \frac{1}{16}\left(\frac{s}{25{,}000}\right) + 1}$$

$$H'(s) = \frac{1562.5s}{s^2 + 1562.5s + 625 \times 10^6}$$

P 15.17 **[a]** Using the first prototype

$$\omega_o = 1 \text{ rad/s}; \qquad C = 1\,\text{F}; \qquad L = 1\,\text{H}; \qquad R = 25\,\Omega$$

$$k_m = \frac{R'}{R} = \frac{40{,}000}{25} = 1600; \qquad k_f = \frac{\omega_o'}{\omega_o} = 50{,}000$$

Thus,

$$R' = k_m R = 40\,\text{k}\Omega; \qquad L' = \frac{k_m}{k_f}L = \frac{1600}{50{,}000}(1) = 32\,\text{mH};$$

$$C' = \frac{C}{k_m k_f} = \frac{1}{(1600)(50{,}000)} = 12.5\,\text{nF}$$

Using the second prototype

$$\omega_o = 1 \text{ rad/s}; \qquad C = 25\,\text{F}$$

$$L = \frac{1}{25} = 40\,\text{mH}; \qquad R = 1\,\Omega$$

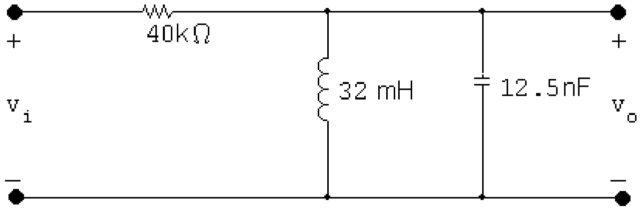
$$k_m = \frac{R'}{R} = 40{,}000; \qquad k_f = \frac{\omega_o'}{\omega_o} = 50{,}000$$

Thus,

$$R' = k_m R = 40\,\text{k}\Omega; \qquad L' = \frac{k_m}{k_f}L = \frac{40{,}000}{50{,}000}(0.04) = 32\,\text{mH};$$

$$C' = \frac{C}{k_m k_f} = \frac{25}{(40{,}000)(50{,}000)} = 12.5\,\text{nF}$$

[**b**]



P 15.18 For the scaled circuit

$$H'(s) = \frac{s^2 + \left(\frac{1}{L'C'}\right)}{s^2 + \left(\frac{R'}{L'}\right)s + \left(\frac{1}{L'C'}\right)}$$

$$L' = \frac{k_m}{k_f}L; \qquad C' = \frac{C}{k_m k_f}$$

$$\therefore \quad \frac{1}{L'C'} = \frac{k_f^2}{LC}; \qquad R' = k_m R$$

$$\therefore \quad \frac{R'}{L'} = k_f\left(\frac{R}{L}\right)$$

It follows then that

$$\begin{aligned} H'(s) &= \frac{s^2 + \left(\frac{k_f^2}{LC}\right)}{s^2 + \left(\frac{R}{L}\right)k_f s + \frac{k_f^2}{LC}} \\ &= \frac{\left(\frac{s}{k_f}\right)^2 + \left(\frac{1}{LC}\right)}{\left[\left(\frac{s}{k_f}\right)^2 + \left(\frac{R}{L}\right)\left(\frac{s}{k_f}\right) + \left(\frac{1}{LC}\right)\right]} \\ &= H(s)\big|_{s=s/k_f} \end{aligned}$$

P 15.19 For the circuit in Fig. 15.31

$$H(s) = \frac{s^2 + \left(\frac{1}{LC}\right)}{s^2 + \frac{s}{RC} + \left(\frac{1}{LC}\right)}$$

It follows that

$$H'(s) = \frac{s^2 + \frac{1}{L'C'}}{s^2 + \frac{s}{R'C'} + \frac{1}{L'C'}}$$

where $R' = k_mR; \qquad L' = \frac{k_m}{k_f}L;$

$$C' = \frac{C}{k_mk_f}$$

$$\therefore \quad \frac{1}{L'C'} = \frac{k_f^2}{LC}$$

$$\frac{1}{R'C'} = \frac{k_f}{RC}$$

$$\begin{aligned} H'(s) &= \frac{s^2 + \left(\frac{k_f^2}{LC}\right)}{s^2 + \left(\frac{k_f}{RC}\right)s + \frac{k_f^2}{LC}} \\ &= \frac{\left(\frac{s}{k_f}\right)^2 + \frac{1}{LC}}{\left(\frac{s}{k_f}\right)^2 + \left(\frac{1}{RC}\right)\left(\frac{s}{k_f}\right) + \frac{1}{LC}} \\ &= H(s)\big|_{s=s/k_f} \end{aligned}$$

P 15.20 **[a]** For the circuit in Fig. P15.20(a)

$$H(s) = \frac{V_o}{V_i} = \frac{s + \frac{1}{s}}{\frac{1}{Q} + s + \frac{1}{s}} = \frac{s^2+1}{s^2 + \left(\frac{1}{Q}\right)s + 1}$$

For the circuit in Fig. P15.18(b)

$$\begin{aligned} H(s) = \frac{V_o}{V_i} &= \frac{Qs + \frac{Q}{s}}{1 + Qs + \frac{Q}{s}} \\ &= \frac{Q(s^2+1)}{Qs^2 + s + Q} \end{aligned}$$

$$H(s) = \frac{s^2+1}{s^2 + \left(\frac{1}{Q}\right)s + 1}$$

[b] $$H'(s) = \frac{\left(\frac{s}{50{,}000}\right)^2 + 1}{\left(\frac{s}{50{,}000}\right)^2 + \frac{1}{5}\left(\frac{s}{50{,}000}\right) + 1}$$

$$= \frac{s^2 + 25 \times 10^8}{s^2 + 10{,}000s + 25 \times 10^8}$$

P 15.21 For prototype circuit (a):

$$H(s) = \frac{V_o}{V_i} = \frac{Q}{Q + \frac{1}{s+\frac{1}{s}}} = \frac{Q}{Q + \frac{s}{s^2+1}}$$

$$H(s) = \frac{Q(s^2+1)}{Q(s^2+1)+s} = \frac{s^2+1}{s^2 + \left(\frac{1}{Q}\right)s + 1}$$

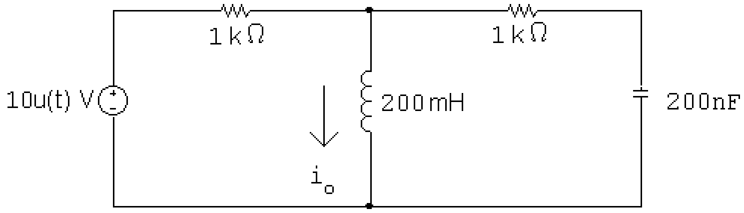
For prototype circuit (b):

$$H(s) = \frac{V_o}{V_i} = \frac{1}{1 + \frac{(s/Q)}{(s^2+1)}}$$

$$= \frac{s^2+1}{s^2 + \left(\frac{1}{Q}\right)s + 1}$$

P 15.22 **[a]** $$k_m = \frac{R'}{R} = \frac{1000}{1} = 1000; \qquad k_f = \frac{C}{k_m C'} = \frac{1}{(1000)(200 \times 10^{-9})} = 5000$$

$$L' = \frac{k_m}{k_f}(L) = \frac{1000}{5000}(1) = 200\,\text{mH}$$



[b] $$\frac{V - 10/s}{1000} + \frac{V}{0.2s} + \frac{V}{1000 + (5 \times 10^6/s)} = 0$$

$$V\left(\frac{1}{1000} + \frac{5}{s} + \frac{s}{1000s + 5 \times 10^6}\right) = \frac{1}{100s}$$

$$V = \frac{10(s+5000)}{2s^2 + 10{,}000s + 25 \times 10^6} = \frac{5(s+5000)}{s^2 + 5000s + 12.5 \times 10^6}$$

$$I_o = \frac{V}{0.2s} = \frac{25(s+5000)}{s(s^2 + 5000s + 12.5 \times 10^6)}$$

$$= \frac{K_1}{s} + \frac{K_2}{s+2500-j2500} + \frac{K_2^*}{s+2500+j2500}$$

$K_1 = 0.01; \qquad K_2 = -0.005$

$i_o(t) = 10 - 10e^{-2500t}\cos 2500t\,\text{mA}$

Since $k_m = 1000$ and the source voltage didn't change, the amplitude of the current is reduced by a factor of 1000. Since $k_f = 5000$ the coefficients of t are multiplied by 5000.

P 15.23 $k_m = \dfrac{R'}{R} = \dfrac{5000}{50} = 100; \qquad k_f = \dfrac{\omega_o'}{\omega_o} = 5000$

$$C' = \frac{C}{k_m k_f} = \frac{4\times 10^{-3}}{(100)(5000)} = 8\,\text{nF}$$

$50\,\Omega \rightarrow 5\,\text{k}\Omega; \qquad 700\,\Omega \rightarrow 70\,\text{k}\Omega$

$$L' = \frac{k_m}{k_f}L = \frac{100}{5000}(20) = 0.4\,\text{H}$$

$$0.05v_\phi \rightarrow \frac{0.05}{100}v_\phi = 5\times 10^{-4}v_\phi$$

The original expression for the current:

$i_o(t) = 1728 + 2880e^{-20t}\cos(15t - 233.13^\circ)\,\text{mA}$

The frequency components will be multiplied by $k_f = 5000$:

$20 \rightarrow 20(5000) = 10^5; \qquad 15 \rightarrow 15(5000) = 75{,}000$

The magnitudes will be reduced by $k_m = 100$:

$1728 \rightarrow 1728/100 = 17.28; \qquad 2880 \rightarrow 2880/100 = 28.80$

The expression for the current in the scaled circuit is thus,

$i_o(t) = 17.28 + 28.80e^{-10^5t}\cos(75{,}000t - 233.13^\circ)\,\text{mA}$

P 15.24 From the solution to Problem 14.22, $\omega_o = 100$ krad/s and $\beta = 12.5$ krad/s. Compute the two scale factors:

$$k_f = \frac{\omega_o'}{\omega_o} = \frac{2\pi(200 \times 10^3)}{100 \times 10^3} = 4\pi$$

$$k_m = \frac{1}{k_f}\frac{C}{C'} = \frac{1}{4\pi}\frac{10 \times 10^{-9}}{2.5 \times 10^{-9}} = \frac{1}{\pi}$$

Thus,

$$R' = k_m R = \frac{8000}{\pi} = 2546.48\,\Omega \qquad L' = \frac{k_m}{k_f}L = \frac{1/\pi}{4\pi}(10 \times 10^{-3}) = 253.3\,\mu\text{H}$$

Calculate the cutoff frequencies:

$$\omega_{c1}' = k_f\omega_{c1} = 4\pi(93.95 \times 10^3) = 1180.6 \text{ krad/s}$$

$$\omega_{c2}' = k_f\omega_{c2} = 4\pi(106.45 \times 10^3) = 1337.7 \text{ krad/s}$$

To check, calculate the bandwidth:

$$\beta' = \omega_{c2}' - \omega_{c1}' = 157.1 \text{ krad/s} = 4\pi\beta \text{ (checks!)}$$

P 15.25 From the solution to Problem 14.35, $\omega_o = 10^6$ rad/s and $\beta = 2\pi(10.61)$ krad/s. Calculate the scale factors:

$$k_f = \frac{\omega_o'}{\omega_o} = \frac{50 \times 10^3}{10^6} = 0.05$$

$$k_m = \frac{k_f L'}{L} = \frac{0.05(200 \times 10^{-6})}{50 \times 10^{-6}} = 0.2$$

Thus,

$$R' = k_m R = (0.2)(750) = 150\,\Omega \qquad C' = \frac{C}{k_m k_f} = \frac{20 \times 10^{-9}}{(0.2)(0.05)} = 2\,\mu\text{F}$$

Calculate the bandwidth:

$$\beta' = k_f\beta = (0.05)[2\pi(10.6 \times 10^3)] = 3330 \text{ rad/s}$$

To check, calculate the quality factor:

$$Q = \frac{\omega_o}{\beta} = \frac{10^6}{2\pi(10.61 \times 10^3)} = 15$$

$$Q' = \frac{\omega_o'}{\beta'} = \frac{50 \times 10^3}{3330} = 15 \text{ (checks)}$$

P 15.26 **[a]** From Eq 15.1 we have

$$H(s) = \frac{-K\omega_c}{s+\omega_c}$$

where $K = \dfrac{R_2}{R_1}, \quad \omega_c = \dfrac{1}{R_2C}$

$$\therefore \quad H'(s) = \frac{-K'\omega_c'}{s+\omega_c'}$$

where $K' = \dfrac{R_2'}{R_1'} \qquad \omega_c' = \dfrac{1}{R_2'C'}$

By hypothesis $R_1' = k_mR_1; \qquad R_2' = k_mR_2,$

and $C' = \dfrac{C}{k_fk_m}$. It follows that

$K' = K$ and $\omega_c' = k_f\omega_c$, therefore

$$H'(s) = \frac{-Kk_f\omega_c}{s+k_f\omega_c} = \frac{-K\omega_c}{\left(\frac{s}{k_f}\right)+\omega_c}$$

[b] $H(s) = \dfrac{-K}{s+1}$

[c] $H'(s) = \dfrac{-K}{\left(\frac{s}{k_f}\right)+1} = \dfrac{-Kk_f}{s+k_f}$

P 15.27 **[a]** From Eq. 15.4

$H(s) = \dfrac{-Ks}{s+\omega_c}$ where $K = \dfrac{R_2}{R_1}$ and

$$\omega_c = \frac{1}{R_1C}$$

$\therefore \quad H'(s) = \dfrac{-K's}{s+\omega_c'}$ where $K' = \dfrac{R_2'}{R_1'}$

and $\omega_c' = \dfrac{1}{R_1'C'}$

By hypothesis

$R_1' = k_mR_1; \qquad R_2' = k_mR_2; \qquad C' = \dfrac{C}{k_mk_f}$

It follows that

$K' = K$ and $\omega_c' = k_f\omega_c$

$$\therefore \quad H'(s) = \frac{-Ks}{s+k_f\omega_c} = \frac{-K(s/k_f)}{\left(\frac{s}{k_f}\right)+\omega_c}$$

[b] $H(s) = \dfrac{-Ks}{s+1}$

[c] $H'(s) = \dfrac{-K(s/k_f)}{\left(\frac{s}{k_f}+1\right)} = \dfrac{-Ks}{s+k_f}$

P 15.28 **[a]** $H_{\text{hp}} = \dfrac{s}{s+1}; \qquad k_f = \dfrac{\omega_o'}{\omega} = \dfrac{1000(2\pi)}{1} = 2000\pi$

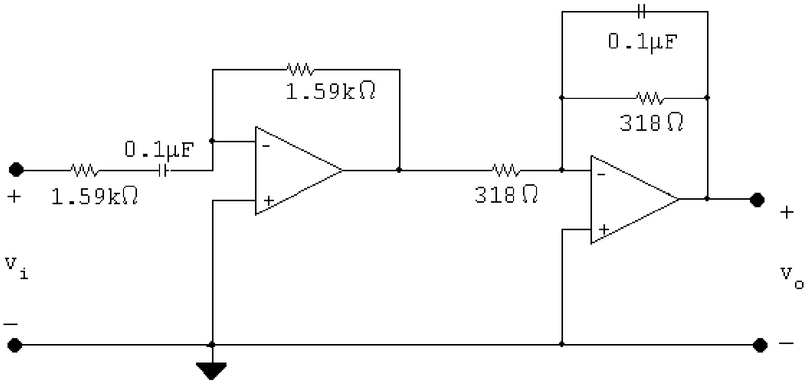
$$\therefore \quad H'_{\text{hp}} = \frac{s}{s+2000\pi}$$

$$\frac{1}{R_H C_H} = 2000\pi; \qquad \therefore \quad R_H = \frac{1}{(2000\pi)(0.1\times10^{-6})} = 1.59\,\text{k}\Omega$$

$$H_{\text{lp}} = \frac{1}{s+1}; \qquad k_f = \frac{\omega_o'}{\omega} = \frac{5000(2\pi)}{1} = 10{,}000\pi$$

$$\therefore \quad H'_{\text{lp}} = \frac{10{,}000\pi}{s+10{,}000\pi}$$

$$\frac{1}{R_L C_L} = 10{,}000\pi; \qquad \therefore \quad R_L = \frac{1}{(10{,}000\pi)(0.1\times10^{-6})} = 318.3\,\Omega$$



[b] $H'(s) = \dfrac{s}{s+2000\pi}\cdot\dfrac{10{,}000\pi}{s+10{,}000\pi}$

$$= \frac{10{,}000\pi s}{(s+2000\pi)(s+10{,}000\pi)}$$

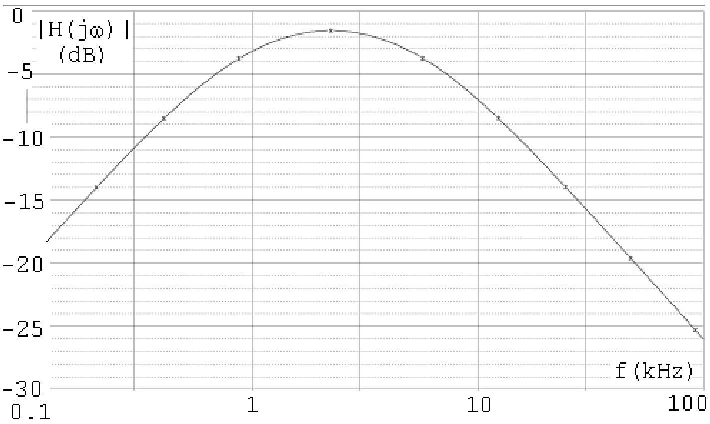
[c] $\omega_o = \sqrt{\omega_{c1}\omega_{c1}} = \sqrt{(2000\pi)(10{,}000\pi)} = 1000\pi\sqrt{20}$ rad/s

$$H'(j\omega_o) = \frac{(10{,}000\pi)(j1000\pi\sqrt{20})}{(2000\pi + j1000\pi\sqrt{20})(10{,}000\pi + j1000\pi\sqrt{20})}$$

$$= \frac{j10\sqrt{20}}{(2+j\sqrt{20})(10+j\sqrt{20})} = 0.8333\underline{/0^\circ}$$

[d] $G = 20\log_{10}(0.8333) = -1.58$ dB

[e]



P 15.29 [a] For the high-pass section:

$$k_f = \frac{\omega_o'}{\omega} = \frac{4000(2\pi)}{1} = 8000\pi$$

$$H'(s) = \frac{s}{s + 8000\pi}$$

$$\therefore \qquad \frac{1}{R_1(10 \times 10^{-9})} = 8000\pi; \qquad R_1 = 3.98\,\text{k}\Omega \quad \therefore \qquad R_2 = 3.98\,\text{k}\Omega$$

For the low-pass section:

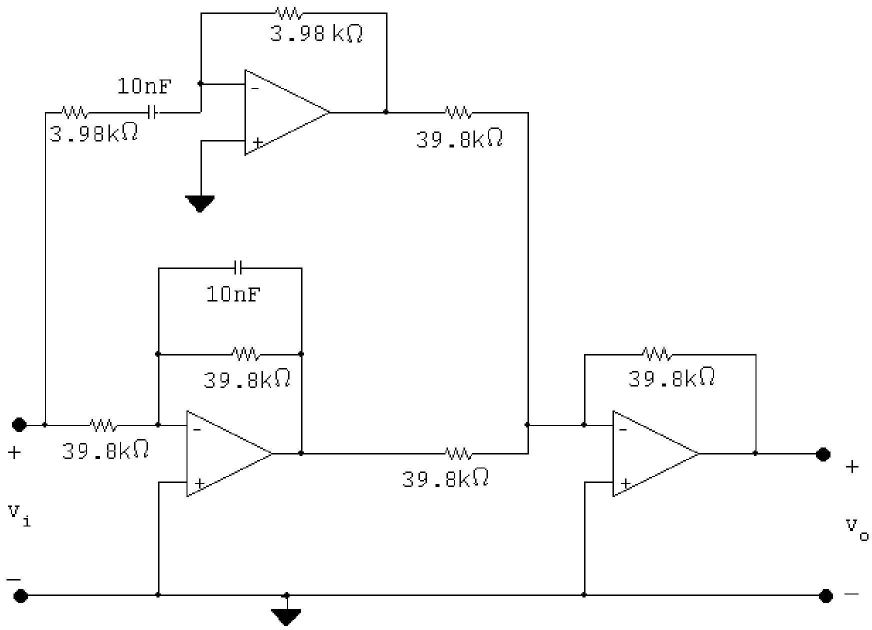
$$k_f = \frac{\omega_o'}{\omega} = \frac{400(2\pi)}{1} == 800\pi$$

$$H'(s) = \frac{800\pi}{s + 800\pi}$$

$$\therefore \qquad \frac{1}{R_2(10 \times 10^{-9})} = 800\pi; \qquad R_2 = 39.8\,\text{k}\Omega \quad \therefore \qquad R_1 = 39.8\,\text{k}\Omega$$

0 dB gain corresponds to $K = 1$. In the summing amplifier we are free to choose R_f and R_i so long as $R_f/R_i = 1$. To keep from having many different resistance values in the circuit we opt for $R_f = R_i = 39.8\,\text{k}\Omega$.

[**b**]



[**c**] $H'(s) = \frac{s}{s + 8000\pi} + \frac{800\pi}{s + 800\pi}$

$$= \frac{s^2 + 1600\pi s + 64 \times 10^5\pi^2}{(s + 800\pi)(s + 8000\pi)}$$

[**d**] $\omega_o = \sqrt{(8000\pi)(800\pi)} = 800\pi\sqrt{10}$

$$H'(j800\pi\sqrt{10}) = \frac{-(800\pi\sqrt{10})^2 + 1600\pi(j800\pi\sqrt{10}) + 64 \times 10^5\pi^2}{(800\pi + j800\pi\sqrt{10})(8000\pi + j800\pi\sqrt{10})}$$

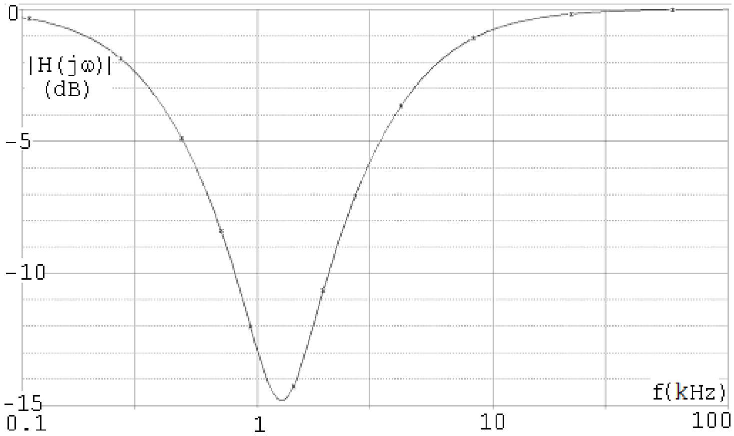
$$= \frac{j128 \times 10^4\sqrt{10}\pi^2}{(800\pi)^2(1 + j\sqrt{10})(10 + j\sqrt{10})}$$

$$= \frac{j2\sqrt{10}}{(1 + j\sqrt{10})(10 + j\sqrt{10})}$$

$$= 0.1818\underline{/0^\circ}$$

[**e**] $G = 20\log_{10} 0.1818 = -14.81$ dB

[f]



P 15.30 $\omega_o = 2\pi f_o = 400\pi\,\text{rad/s}$

$$\beta = 2\pi(1000) = 2000\pi\,\text{rad/s}$$

$$\therefore\quad \omega_{c_2} - \omega_{c_1} = 2000\pi$$

$$\sqrt{\omega_{c_1}\omega_{c_2}} = \omega_o = 400\pi$$

Solve for the cutoff frequencies:

$$\omega_{c_1}\omega_{c_2} = 16\times 10^4\pi^2$$

$$\omega_{c_2} = \frac{16\times 10^4\pi^2}{\omega_{c_1}}$$

$$\therefore\quad \frac{16\times 10^4\pi^2}{\omega_{c_1}} - \omega_{c_1} = 2000\pi$$

or $\omega_{c_1}^2 + 2000\pi\omega_{c_1} - 16\times 10^4\pi^2 = 0$

$$\omega_{c_1} = -1000\pi \pm \sqrt{10^6\pi^2 + 0.16\times 10^6\pi^2}$$

$$\omega_{c_1} = 1000\pi(-1\pm\sqrt{1.16}) = 242.01\,\text{rad/s}$$

$$\therefore\quad \omega_{c_2} = 2000\pi + 242.01 = 6525.19\,\text{rad/s}$$

Thus, $f_{c1} = 38.52$ Hz and $f_{c2} = 1038.52$ Hz

Check: $\beta = f_{c2} - f_{c1} = 1000\text{Hz}$

$$\omega_{c2} = \frac{1}{R_L C_L} = 6525.19$$

$$R_L = \frac{1}{(6525.19)(5 \times 10^{-6})} = 30.65\,\Omega$$

$$\omega_{c1} = \frac{1}{R_H C_H} = 242.01$$

$$R_H = \frac{1}{(242.01)(5 \times 10^{-6})} = 826.43\,\Omega$$

P 15.31 $\omega_o = 1000\,\text{rad/s}; \qquad \text{GAIN} = 6$

$\beta = 4000 \text{ rad/s}; \qquad C = 0.2\,\mu\text{F}$

$\beta = \omega_{c_2} - \omega_{c_1} = 4000$

$\omega_o = \sqrt{\omega_{c_1}\omega_{c_2}} = 1000$

Solve for the cutoff frequencies:

$\therefore \quad \omega_{c_1}^2 + 4000\omega_{c_1} - 10^6 = 0$

$\omega_{c_1} = -2000 \pm 1000\sqrt{5} = 236.07\,\text{rad/s}$

$\omega_{c_2} = 4000 + \omega_{c_1} = 4236.07\,\text{rad/s}$

Check: $\quad \beta = \omega_{c2} - \omega_{c1} = 4000 \text{ rad/s}$

$$\omega_{c_1} = \frac{1}{R_L C_L}$$

$$\therefore \quad R_L = \frac{1}{(0.2 \times 10^{-6})(236.07)} = 21.18\,\text{k}\Omega$$

$$\frac{1}{R_H C_H} = 4236.07$$

$$R_H = \frac{1}{(0.2 \times 10^{-6})(4236.07)} = 1.18\,\text{k}\Omega$$

$$\frac{R_f}{R_i} = 6$$

If $R_i = 1\,\text{k}\Omega \qquad R_f = 6R_i = 6\,\text{k}\Omega$

P 15.32 $H(s) = \dfrac{V_o}{V_i} = \dfrac{-Z_f}{Z_i}$

$$Z_f = \frac{1}{sC_2}\|R_2 = \frac{(1/C_2)}{s + (1/R_2C_2)}; \qquad Z_i = R_1 + \frac{1}{sC_1} = \frac{sR_1C_1 + 1}{sC_1}$$

$$\therefore \quad H(s) = \frac{\dfrac{-1/C_2}{s + (1/R_2C_2)}}{\dfrac{s + (1/R_1C_1)}{s/R_1}} = \frac{-(1/R_1C_2)s}{[s + (1/R_1C_1)][s + (1/R_2C_2)]}$$

$$= \frac{-K\beta s}{s^2 + \beta s + \omega_o^2}$$

[a] $H(s) = \dfrac{-250s}{(s+50)(s+20)} = \dfrac{-250s}{s^2 + 70s + 1000} = \dfrac{-3.57(70s)}{s^2 + 70s + (\sqrt{1000})^2}$

$\omega_o = \sqrt{1000} = 31.6$ rad/s

$\beta = 70$ rad/s

$K = -3.57$

[b] $Q = \dfrac{\omega_o}{\beta} = 0.45$

$$\omega_{c1,2} = \pm\frac{\beta}{2} + \sqrt{\left(\frac{\beta}{2}\right)^2 + \omega_o^2} = \pm 35 + \sqrt{35^2 + 1000} = \pm 35 + 47.17$$

$\omega_{c1} = 12.17$ rad/s $\qquad \omega_{c2} = 82.17$ rad/s

P 15.33 **[a]** $H(s) = \dfrac{(1/sC)}{R + (1/sC)} = \dfrac{(1/RC)}{s + (1/RC)}$

$$H(j\omega) = \frac{(1/RC)}{j\omega + (1/RC)}$$

$$|H(j\omega)| = \frac{(1/RC)}{\sqrt{\omega^2 + (1/RC)^2}}$$

$$|H(j\omega)|^2 = \frac{(1/RC)^2}{\omega^2 + (1/RC)^2}$$

[b] Let V_a be the voltage across the capacitor, positive at the upper terminal. Then

$$\frac{V_a - V_{in}}{R_1} + sCV_a + \frac{V_a}{R_2 + sL} = 0$$

Solving for V_a yields

$$V_a = \frac{(R_2 + sL)V_{in}}{R_1LCs^2 + (R_1R_2C + L)s + (R_1 + R_2)}$$

But

$$v_o = \frac{sLV_a}{R_2 + sL}$$

Therefore

$$V_o = \frac{sLV_{in}}{R_1LCs^2 + (L + R_1R_2C)s + (R_1 + R_2)}$$

$$H(s) = \frac{sL}{R_1LCs^2 + (L + R_1R_2C)s + (R_1 + R_2)}$$

$$H(j\omega) = \frac{j\omega L}{[(R_1 + R_2) - R_1LC\omega^2] + j\omega(L + R_1R_2C)}$$

$$|H(j\omega)| = \frac{\omega L}{\sqrt{[R_1 + R_2 - R_1LC\omega^2]^2 + \omega^2(L + R_1R_2C)^2}}$$

$$|H(j\omega)|^2 = \frac{\omega^2L^2}{(R_1 + R_2 - R_1LC\omega^2)^2 + \omega^2(L + R_1R_2C)^2}$$

$$= \frac{\omega^2L^2}{R_1^2L^2C^2\omega^4 + (L^2 + R_1^2R_2^2C^2 - 2R_1^2LC)\omega^2 + (R_1 + R_2)^2}$$

[c] Let V_a be the voltage across R_2 positive at the upper terminal. Then

$$\frac{V_a - V_{in}}{R_1} + \frac{V_a}{R_2} + V_asC + V_asC = 0$$

$$(0 - V_a)sC + (0 - V_a)sC + \frac{0 - V_o}{R_3} = 0$$

$$\therefore \quad V_a = \frac{R_2V_{in}}{2R_1R_2Cs + R_1 + R_2}$$

and $V_a = -\dfrac{V_o}{2R_3Cs}$

It follows directly that

$$H(s) = \frac{V_o}{V_{in}} = \frac{-2R_2R_3Cs}{2R_1R_2Cs + (R_1 + R_2)}$$

$$H(j\omega) = \frac{-2R_2R_3C(j\omega)}{(R_1 + R_2) + j\omega(2R_1R_2C)}$$

$$|H(j\omega)| = \frac{2R_2R_3C\omega}{\sqrt{(R_1 + R_2)^2 + \omega^2 4R_1^2R_2^2C^2}}$$

$$|H(j\omega)|^2 = \frac{4R_2^2R_3^2C^2\omega^2}{(R_1 + R_2)^2 + 4R_1^2R_2^2C^2\omega^2}$$

P 15.34 For the scaled circuit

$$H'(s) = \frac{1/(R')^2C_1'C_2'}{s^2 + \frac{2}{R'C_1'}s + \frac{1}{(R')^2C_1'C_2'}}$$

where

$$R' = k_mR; \qquad C_1' = C_1/k_fk_m; \qquad C_2' = C_2/k_fk_m$$

It follows that

$$\frac{1}{(R')^2C_1'C_2'} = \frac{k_f^2}{R^2C_1C_2}$$

$$\frac{2}{R'C_1'} = \frac{2k_f}{RC_1}$$

$$\therefore \quad H'(s) = \frac{k_f^2/RC_1C_2}{s^2 + \frac{2k_f}{RC_1}s + \frac{k_f^2}{R^2C_1C_2}}$$

$$= \frac{1/RC_1C_2}{\left(\frac{s}{k_f}\right)^2 + \frac{2}{RC_1}\left(\frac{s}{k_f}\right) + \frac{1}{R^2C_1C_2}}$$

P 15.35 **[a]** $y = 20\log_{10}\frac{1}{\sqrt{1+\omega^{2n}}} = -10\log_{10}(1 + \omega^{2n})$

From the laws of logarithms we have

$$y = \left(\frac{-10}{\ln 10}\right)\ln(1 + \omega^{2n})$$

Thus

$$\frac{dy}{d\omega} = \left(\frac{-10}{\ln 10}\right)\frac{2n\omega^{2n-1}}{(1 + \omega^{2n})}$$

$$x = \log_{10}\omega = \frac{\ln\omega}{\ln 10}$$

$$\therefore \quad \ln\omega = x\ln 10$$

$$\frac{1}{\omega}\frac{d\omega}{dx} = \ln 10, \qquad \frac{d\omega}{dx} = \omega \ln 10$$

$$\frac{dy}{dx} = \left(\frac{dy}{d\omega}\right)\left(\frac{d\omega}{dx}\right) = \frac{-20n\omega^{2n}}{1+\omega^{2n}}\,\text{dB/decade}$$

at $\omega = \omega_c = 1\,\text{rad/s}$

$$\frac{dy}{dx} = -10n\,\text{dB/decade.}$$

[b] $y = 20\log_{10}\frac{1}{[\sqrt{1+\omega^2}]^n} = -10n\log_{10}(1+\omega^2)$

$$= \frac{-10n}{\ln 10}\ln(1+\omega^2)$$

$$\frac{dy}{d\omega} = \frac{-10n}{\ln 10}\left(\frac{1}{1+\omega^2}\right)2\omega = \frac{-20n\omega}{(\ln 10)(1+\omega^2)}$$

As before

$$\frac{d\omega}{dx} = \omega(\ln 10); \qquad \therefore \quad \frac{dy}{dx} = \frac{-20n\omega^2}{(1+\omega^2)}$$

At the corner $\omega_c = \sqrt{2^{1/n}-1} \qquad \therefore \quad \omega_c^2 = 2^{1/n}-1$

$$\frac{dy}{dx} = \frac{-20n[2^{1/n}-1]}{2^{1/n}}\,\text{dB/decade.}$$

[c] For the Butterworth Filter

n	dy/dx (dB/decade)
1	−10
2	−20
3	−30
4	−40
∞	−∞

For the cascade of identical sections

n	dy/dx (dB/decade)
1	−10
2	−11.72
3	−12.38
4	−12.73
∞	−13.86

[d] It is apparent from the calculations in part (c) that as n increases the amplitude characteristic at the cutoff frequency decreases at a much faster rate for the Butterworth filter.
Hence the transition region of the Butterworth filter will be much narrower than that of the cascaded sections.

P 15.36 **[a]** $n \cong \frac{(-0.05)(-30)}{\log_{10}(7000/2000)} \cong 2.76$

$$\therefore \quad n = 3$$

[b] Gain $= 20\log_{10}\dfrac{1}{\sqrt{1+(7000/2000)^6}} = -32.65$ dB

P 15.37 **[a]** $H(s) = \dfrac{1}{(s+1)(s^2+s+1)}$

[b] $f_c = 2000\,\text{Hz}; \qquad \omega_c = 4000\pi\ \text{rad/s}; \qquad k_f = 4000\pi$

$$H'(s) = \frac{1}{(\frac{s}{k_f}+1)[(\frac{s}{k_f})^2+\frac{s}{k_f}+1]}$$

$$= \frac{k_f^3}{(s+k_f)(s^2+k_fs+k_f^2)}$$

$$= \frac{(4000\pi)^3}{(s+4000\pi)[s^2+4000\pi s+(4000\pi)^2]}$$

[c] $H'(j14{,}000\pi) = \dfrac{64}{(4+j14)(-180+j52)}$

$$= 0.02332\underline{/-236.77^\circ}$$

Gain $= 20\log_{10}(0.02332) = -32.65$ dB

P 15.38 **[a]** In the first-order circuit $R = 1\,\Omega$ and $C = 1$ F.

$$k_m = \frac{R'}{R} = \frac{1000}{1} = 1000; \qquad k_f = \frac{\omega_o'}{\omega_o} = \frac{2\pi(2000)}{1} = 4000\pi$$

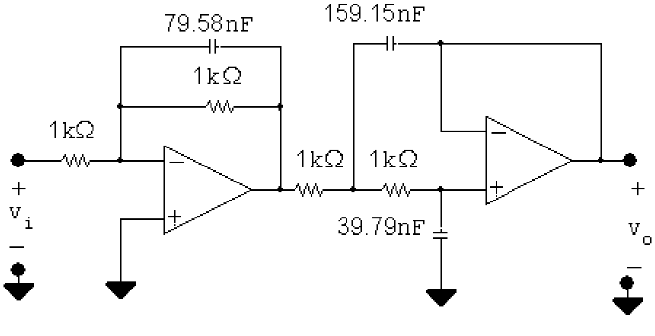
$$R' = k_mR = 1000\,\Omega; \qquad C' = \frac{C}{k_mk_f} = \frac{1}{(1000)(4000\pi)} = 79.58\,\text{nF}$$

In the second-order circuit $R = 1\,\Omega$, $2/C_1 = 1$ so $C_1 = 2$ F, and $C_2 = 1/C_1 = 0.5$ F. Therefore in the scaled second-order circuit

$$R' = k_mR = 1000\,\Omega; \qquad C_1' = \frac{C_1}{k_mk_f} = \frac{2}{(1000)(4000\pi)} = 159.15\,\text{nF}$$

$$C_2' = \frac{C_2}{k_mk_f} = \frac{0.5}{(1000)(4000\pi)} = 39.79\,\text{nF}$$

[b]



P 15.39 **[a]** $n = \dfrac{(-0.05)(-48)}{\log_{10}(2000/500)} = 3.99 \quad \therefore \quad n = 4$

From Table 15.1 the transfer function of the first section is

$$H_1(s) = \frac{s^2}{s^2 + 0.765s + 1}$$

For the prototype circuit

$$\frac{2}{R_2} = 0.765; \qquad R_2 = 2.61\,\Omega; \qquad R_1 = \frac{1}{R_2} = 0.383\,\Omega$$

The transfer function of the second section is

$$H_2(s) = \frac{s^2}{s^2 + 1.848s + 1}$$

For the prototype circuit

$$\frac{2}{R_2} = 1.848; \qquad R_2 = 1.082\,\Omega; \qquad R_1 = \frac{1}{R_2} = 0.9240\,\Omega$$

The scaling factors are:

$$k_f = \frac{\omega_o'}{\omega_o} = \frac{2\pi(2000)}{1} = 4000\pi$$

$$C' = \frac{C}{k_m k_f} \quad \therefore \quad 10 \times 10^{-9} = \frac{1}{4000\pi k_m}$$

$$\therefore \quad k_m = \frac{1}{4000\pi(10 \times 10^{-9})} = 7957.75$$

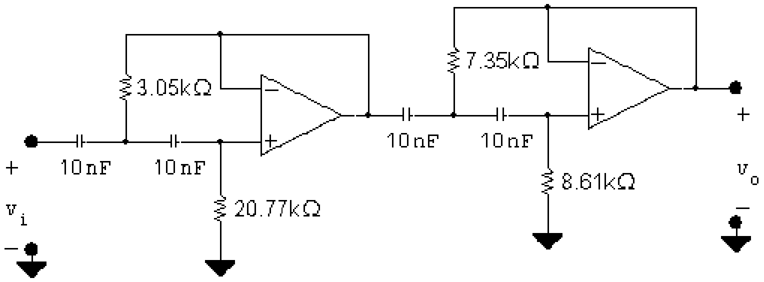
Therefore in the first section

$$R_1' = k_m R_1 = 3.05\,\text{k}\Omega; \qquad R_2' = k_m R_2 = 20.77\,\text{k}\Omega$$

In the second section

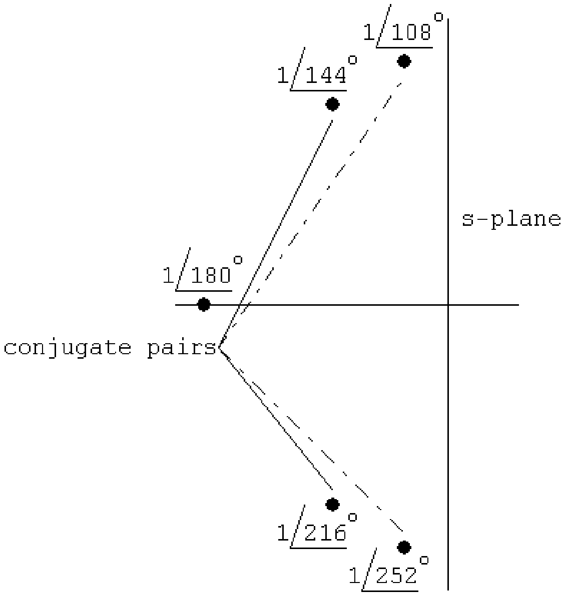
$$R_1' = k_m R_1 = 7.35\,\text{k}\Omega; \qquad R_2' = k_m R_2 = 8.61\,\text{k}\Omega$$

[b]



P 15.40 $n = 5$: $1 + (-1)^5 s^{10} = 0; \qquad s^{10} = 1$

$$s^{10} = 1\underline{/(0 + 360k)^\circ} \qquad \text{so} \qquad s = 1\underline{/36k^\circ}$$



k	s_{k+1}	k	s_{k+1}
0	$1\underline{/0^\circ}$	5	$1\underline{/180^\circ}$
1	$1\underline{/36^\circ}$	6	$1\underline{/216^\circ}$
2	$1\underline{/72^\circ}$	7	$1\underline{/252^\circ}$
3	$1\underline{/108^\circ}$	8	$1\underline{/288^\circ}$
4	$1\underline{/144^\circ}$	9	$1\underline{/324^\circ}$

Group by conjugate pairs to form denominator polynomial.

$$(s+1)[s-(\cos 108^\circ + j\sin 108^\circ)][(s-(\cos 252^\circ + j\sin 252^\circ)]$$

$$\cdot\,[(s-(\cos 144^\circ + j\sin 144^\circ)][(s-(\cos 216^\circ + j\sin 216^\circ)]$$

$$= (s+1)(s+0.309-j0.951)(s+0.309+j0.951)\cdot$$

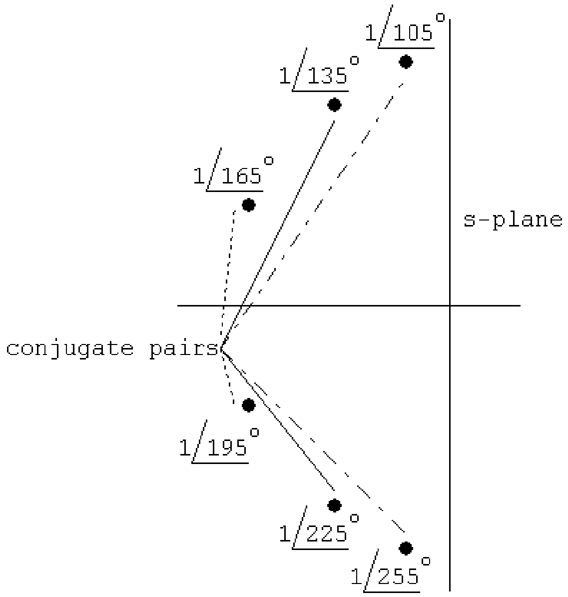
$$(s+0.809-j0.588)(s+0.809+j0.588)$$

which reduces to

$$(s+1)(s^2+0.618s+1)(s^2+1.618s+1)$$

$n = 6$: $1 + (-1)^6 s^{12} = 0 \qquad s^{12} = -1$

$$s^{12} = 1\underline{/180^\circ + 360k}$$



k	s_{k+1}	k	s_{k+1}
0	$1\underline{/15^\circ}$	6	$1\underline{/195^\circ}$
1	$1\underline{/45^\circ}$	7	$1\underline{/225^\circ}$
2	$1\underline{/75^\circ}$	8	$1\underline{/255^\circ}$
3	$1\underline{/105^\circ}$	9	$1\underline{/285^\circ}$
4	$1\underline{/135^\circ}$	10	$1\underline{/315^\circ}$
5	$1\underline{/165^\circ}$	11	$1\underline{/345^\circ}$

Grouping by conjugate pairs yields

$$(s + 0.2588 - j0.9659)(s + 0.2588 + j0.9659)\times$$

$$(s + 0.7071 - j0.7071)(s + 0.7071 + j0.7071)\times$$

$$(s + 0.9659 - j0.2588)(s + 0.9659 + j0.2588)$$

or $(s^2 + 0.5176s + 1)(s^2 + 1.4142s + 1)(s^2 + 1.9318s + 1)$

P 15.41 $$H'(s) = \frac{s^2}{s^2 + \dfrac{2}{k_mR_2(C/k_mk_f)}s + \dfrac{1}{k_mR_1k_mR_2(C^2/k_m^2k_f^2)}}$$

$$H'(s) = \frac{s^2}{s^2 + \dfrac{2k_f}{R_2C}s + \dfrac{k_f^2}{R_1R_2C^2}}$$

$$= \frac{(s/k_f)^2}{(s/k_f)^2 + \dfrac{2}{R_2C}\left(\dfrac{s}{k_f}\right) + \dfrac{1}{R_1R_2C^2}}$$

P 15.42 **[a]** $n = \dfrac{(-0.05)(-48)}{\log_{10}(32/8)} = 3.99 \quad \therefore \quad n = 4$

From Table 15.1 the transfer function is

$$H(s) = \frac{1}{(s^2 + 0.765s + 1)(s^2 + 1.848s + 1)}$$

The capacitor values for the first stage prototype circuit are

$$\frac{2}{C_1} = 0.765 \quad \therefore \quad C_1 = 2.61\,\text{F}$$

$$C_2 = \frac{1}{C_1} = 0.38\,\text{F}$$

The values for the second stage prototype circuit are

$$\frac{2}{C_1} = 1.848 \quad \therefore \quad C_1 = 1.08\,\text{F}$$

$$C_2 = \frac{1}{C_1} = 0.92\,\text{F}$$

The scaling factors are

$$k_m = \frac{R'}{R} = 1000; \qquad k_f = \frac{\omega_o'}{\omega_o} = 16{,}000\pi$$

Therefore the scaled values for the components in the first stage are

$$R_1 = R_2 = R = 1000\,\Omega$$

$$C_1 = \frac{2.61}{(16{,}000\pi)(1000)} = 52.01\,\text{nF}$$

$$C_2 = \frac{0.38}{(16{,}000\pi)(1000)} = 7.61\,\text{nF}$$

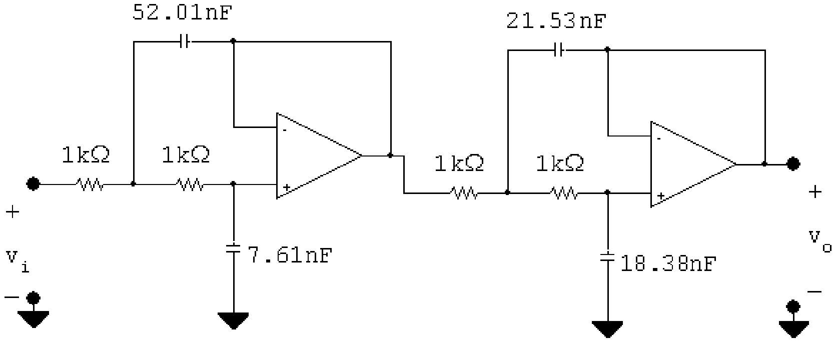
The scaled values for the second stage are

$$R_1 = R_2 = R = 1000\,\Omega$$

$$C_1 = \frac{1.08}{(16{,}000\pi)(1000)} = 21.53\,\text{nF}$$

$$C_2 = \frac{0.92}{(16{,}000\pi)(1000)} = 18.38\,\text{nF}$$

[**b**]



P 15.43 [**a**] The cascade connection is a bandpass filter.

[**b**] The cutoff frequencies are 2 kHz and 8 kHz.

The center frequency is $\sqrt{(2)(8)} = 4$ kHz.

The Q is $4/(8-2) = 2/3 = 0.67$

[**c**] For the high pass section $k_f = 4000\pi$. The prototype transfer function is

$$H_{\text{hp}}(s) = \frac{s^4}{(s^2+0.765s+1)(s^2+1.848s+1)}$$

$$\therefore \qquad H'_{\text{hp}}(s) = \frac{(s/4000\pi)^4}{[(s/4000\pi)^2 + 0.765(s/4000\pi)+1]}$$

$$\cdot \frac{1}{[(s/4000\pi)^2 + 1.848(s/4000\pi)+1]}$$

$$= \frac{s^4}{(s^2+3060\pi s+16\times 10^6\pi^2)(s^2+7392\pi s+16\times 10^6\pi^2)}$$

For the low pass section $k_f = 16{,}000\pi$

$$H_{\text{lp}}(s) = \frac{1}{(s^2+0.765s+1)(s^2+1.848s+1)}$$

$$\therefore \qquad H'_{\text{lp}}(s) = \frac{1}{[(s/16{,}000\pi)^2 + 0.765(s/16{,}000\pi)+1]}$$

$$\cdot \frac{1}{[(s/16{,}000\pi)^2 + 1.848(s/16{,}000\pi)+1]}$$

$$= \frac{(16{,}000\pi)^4}{([s^2+12{,}240\pi s+(16{,}000\pi)^2)][s^2+29{,}568\pi s+(16{,}000\pi)^2]}$$

The cascaded transfer function is

$H'(s) = H'_{\mathrm{hp}}(s)H'_{\mathrm{lp}}(s)$

For convenience let

$D_1 = s^2 + 3060\pi s + 16 \times 10^6\pi^2$

$D_2 = s^2 + 7392\pi s + 16 \times 10^6\pi^2$

$D_3 = s^2 + 12{,}240\pi s + 256 \times 10^6\pi^2$

$D_4 = s^2 + 29{,}568\pi s + 256 \times 10^6\pi^2$

Then

$$H'(s) = \frac{65{,}536 \times 10^{12}\pi^4 s^4}{D_1D_2D_3D_4}$$

[d] $\omega_o = 2\pi(4000) = 8000\pi$ rad/s

$s = j8000\pi$

$s^4 = 4096 \times 10^{12}\pi^4$

$D_1 = (16 \times 10^6\pi^2 - 64 \times 10^6\pi^2) + j(8000\pi)(3060\pi)$

$\quad = 10^6\pi^2(-48 - j24.48) = 10^6\pi^2(53.88\underline{/152.98^\circ})$

$D_2 = (16 \times 10^6\pi^2 - 64 \times 10^6\pi^2) + j(8000\pi)(7392\pi)$

$\quad = 10^6\pi^2(-48 + j59.136) = 10^6\pi^2(76.16\underline{/129.07^\circ})$

$D_1 = (256 \times 10^6\pi^2 - 64 \times 10^6\pi^2) + j(8000\pi)(12{,}240\pi)$

$\quad = 10^6\pi^2(192 + j97.92) = 10^6\pi^2(215.53\underline{/27.02^\circ})$

$D_1 = (256 \times 10^6\pi^2 - 64 \times 10^6\pi^2) + j(8000\pi)(29{,}568\pi)$

$\quad = 10^6\pi^2(192 + j236.544) = 10^6\pi^2(304.66\underline{/50.93^\circ})$

$$H'(j\omega_o) = \frac{(65{,}536)(4096)\pi^8 \times 10^{24}}{(\pi^8 \times 10^{24})[(53.88)(76.16)(215.53)(304.66)\underline{/360^\circ}]}$$

$\quad = 0.996\underline{/-360^\circ} = 0.996\underline{/0^\circ}$

P 15.44 **[a]** From the statement of the problem, $K = 10$ ($= 20$ dB). Therefore for the prototype bandpass circuit

$$R_1 = \frac{Q}{K} = \frac{16}{10} = 1.6\,\Omega$$

$$R_2 = \frac{Q}{2Q^2 - K} = \frac{16}{502}\,\Omega$$

$$R_3 = 2Q = 32\,\Omega$$

The scaling factors are

$$k_f = \frac{\omega_o'}{\omega_o} = 2\pi(6400) = 12{,}800\pi$$

$$k_m = \frac{C}{C'k_f} = \frac{1}{(20\times10^{-9})(12{,}800\pi)} = 1243.40$$

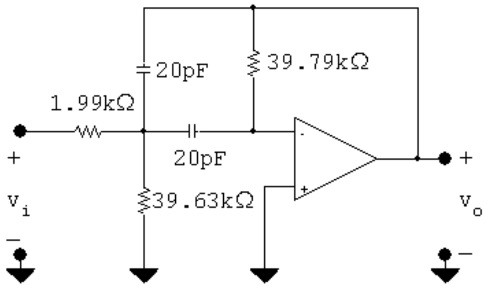
Therefore,

$$R_1' = k_mR_1 = (1.6)(1243.30) = 1.99\,\text{k}\Omega$$

$$R_2' = k_mR_2 = (16/502)(1243.40) = 39.63\,\Omega$$

$$R_3' = k_mR_3 = 32(1243.40) = 39.79\,\text{k}\Omega$$

[b]



P 15.45 From Eq 15.56 we can write

$$H(s) = \frac{-\left(\frac{2}{R_3C}\right)\left(\frac{R_3C}{2}\right)\left(\frac{1}{R_1C}\right)s}{s^2 + \frac{2}{R_3C}s + \frac{R_1+R_2}{R_1R_2R_3C^2}}$$

or

$$H(s) = \frac{-\left(\frac{R_3}{2R_1}\right)\left(\frac{2}{R_3C}s\right)}{s^2 + \frac{2}{R_3C}s + \frac{R_1+R_2}{R_1R_2R_3C^2}}$$

Therefore

$$\frac{2}{R_3C} = \beta = \frac{\omega_o}{Q}; \qquad \frac{R_1+R_2}{R_1R_2R_3C^2} = \omega_o^2;$$

and $K = \dfrac{R_3}{2R_1}$

By hypothesis $C = 1\,\mathrm{F}$ and $\omega_o = 1\,\mathrm{rad/s}$

$$\therefore \quad \frac{2}{R_3} = \frac{1}{Q} \text{ or } R_3 = 2Q$$

$$R_1 = \frac{R_3}{2K} = \frac{Q}{K}$$

$$\frac{R_1 + R_2}{R_1 R_2 R_3} = 1$$

$$\frac{Q}{K} + R_2 = \left(\frac{Q}{K}\right)(2Q)R_2$$

$$\therefore \quad R_2 = \frac{Q}{2Q^2 - K}$$

P 15.46 [**a**] First we will design a unity gain filter and then provide the passband gain with an inverting amplifier. For the high pass section the cut-off frequency is 500 Hz. The order of the Butterworth is

$$n = \frac{(-0.05)(-20)}{\log_{10}(500/200)} = 2.51$$

$$\therefore \quad n = 3$$

$$H_{hp}(s) = \frac{s^3}{(s+1)(s^2+s+1)}$$

For the prototype first-order section

$$R_1 = R_2 = 1\,\Omega, \quad C = 1\,\mathrm{F}$$

For the prototype second-order section

$$R_1 = 0.5\,\Omega, \quad R_2 = 2\,\Omega, \quad C = 1\,\mathrm{F}$$

The scaling factors are

$$k_f = \frac{\omega_o'}{\omega_o} = 2\pi(500) = 1000\pi$$

$$k_m = \frac{C}{C'k_f} = \frac{1}{(15 \times 10^{-9})(1000\pi)} = \frac{10^6}{15\pi}$$

In the scaled first-order section

$$R_1' = R_2' = k_m R_1 = \frac{10^6}{15\pi}(1) = 21.22\,\mathrm{k}\Omega$$

$$C' = 15\,\mathrm{nF}$$

In the scaled second-order section

$$R_1' = 0.5k_m = 10.61\,\mathrm{k}\Omega$$

$$R_2' = 2k_m = 42.44\,\mathrm{k}\Omega$$

$$C' = 15\,\mathrm{nF}$$

For the low-pass section the cut-off frequency is 4500 Hz. The order of the Butterworth filter is

$$n = \frac{(-0.05)(-20)}{\log_{10}(11{,}250/4500)} = 2.51; \qquad \therefore \quad n = 3$$

$$H_{\mathrm{lp}}(s) = \frac{1}{(s+1)(s^2+s+1)}$$

For the prototype first-order section

$$R_1 = R_2 = 1\,\Omega, \quad C = 1\,\mathrm{F}$$

For the prototype second-order section

$$R_1 = R_2 = 1\,\Omega; \qquad C_1 = 2\,\mathrm{F}; \qquad C_2 = 0.5\,\mathrm{F}$$

The low-pass scaling factors are

$$k_m = \frac{R'}{R} = 10^4; \qquad k_f = \frac{\omega_o'}{\omega_o} = (4500)(2\pi) = 9000\pi$$

For the scaled first-order section

$$R_1' = R_2' = 10\,\mathrm{k}\Omega; \qquad C' = \frac{C}{k_f k_m} = \frac{1}{(9000\pi)(10^4)} = 3.54\,\mathrm{nF}$$

For the scaled second-order section

$$R_1' = R_2' = 10\,\mathrm{k}\Omega$$

$$C_1' = \frac{C_1}{k_f k_m} = \frac{2}{(9000\pi)(10^4)} = 7.07\,\mathrm{nF}$$

$$C_2' = \frac{C_2}{k_f k_m} = \frac{0.5}{(9000\pi)(10^4)} = 1.77\,\mathrm{nF}$$

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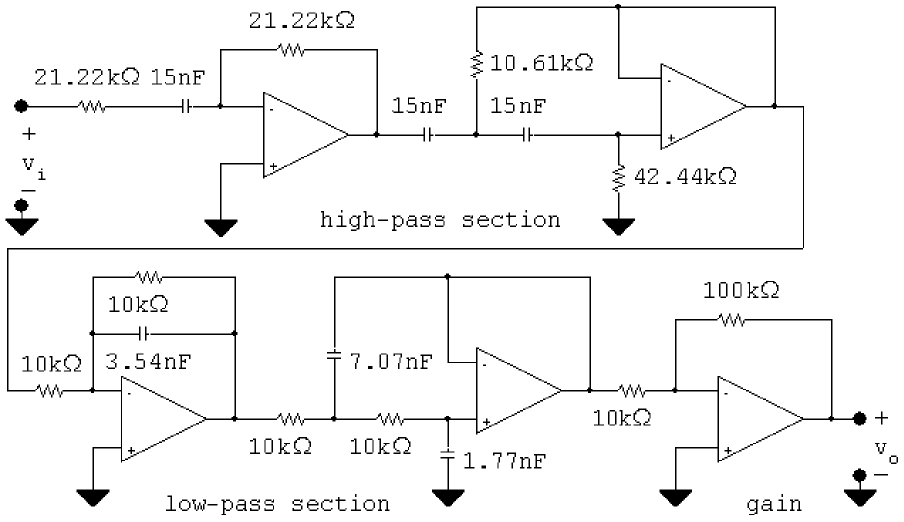
$$20\log_{10} K = 20\ \mathrm{dB}, \qquad \therefore \quad K = 10$$

Since we are using $10\,\mathrm{k}\Omega$ resistors in the low-pass stage, we will use $R_f = 100\,\mathrm{k}\Omega$ and $R_i = 10\,\mathrm{k}\Omega$ in the inverting amplifier stage.

[**b**]



P 15.47 [**a**] Unscaled high-pass stage

$$H_{hp}(s) = \frac{s^3}{(s+1)(s^2+s+1)}$$

The frequency scaling factor is $k_f = (\omega_o'/\omega_o) = 1000\pi$. Therefore the scaled transfer function is

$$H'_{hp}(s) = \frac{(s/1000\pi)^3}{\left(\frac{s}{1000\pi}+1\right)\left[\left(\frac{s}{1000\pi}\right)^3 + \frac{s}{1000\pi}+1\right]}$$

$$= \frac{s^3}{(s+1000\pi)[s^2+1000\pi s+10^6\pi^2]}$$

Unscaled low-pass stage

$$H_{lp}(s) = \frac{1}{(s+1)(s^2+s+1)}$$

The frequency scaling factor is $k_f = (\omega_o'/\omega_o) = 9000\pi$. Therefore the scaled transfer function is

$$H'_{lp}(s) = \frac{1}{\left(\frac{s}{9000\pi}+1\right)\left[\left(\frac{s}{9000\pi}\right)^2 + \left(\frac{s}{9000\pi}\right)+1\right]}$$

$$= \frac{(9000\pi)^3}{(s+9000\pi)(s^2+9000\pi s+81\times10^6\pi^2)}$$

Thus the transfer function for the filter is

$$H'(s) = 10H'_{hp}(s)H'_{lp}(s) = \frac{729\times10^{10}\pi^3 s^3}{D_1D_2D_3D_4}$$

where

$$D_1 = s + 1000\pi$$

$$D_2 = s + 9000\pi$$

$$D_3 = s^2 + 1000\pi s + 10^6\pi^2$$

$$D_4 = s^2 + 9000\pi s + 81 \times 10^6\pi^2$$

[b] At 200 Hz $\quad \omega = 400\pi$ rad/s

$$D_1(j400\pi) = 400\pi(2.5 + j1)$$

$$D_2(j400\pi) = 400\pi(22.5 + j1)$$

$$D_3(j400\pi) = 4 \times 10^5\pi^2(2.1 + j1.0)$$

$$D_4(j400\pi) = 4 \times 10^5\pi^2(202.1 + j9)$$

Therefore

$$D_1D_2D_3D_4(j400\pi) = 256\pi^6 10^{14}(28{,}534.82\underline{/52.36^\circ})$$

$$H'(j400\pi) = \frac{(729\pi^3 \times 10^{10})(64 \times 10^6\pi^3)}{256\pi^6 \times 10^{14}(28{,}534.82\underline{/52.36^\circ})}$$

$$= 0.639\underline{/-52.36^\circ}$$

$$\therefore \quad 20\log_{10}|H'(j400\pi)| = 20\log_{10}(0.639) = -3.89 \text{ dB}$$

At $f = 1500$ Hz, $\quad \omega = 3000\pi$ rad/s

Then

$$D_1(j3000\pi) = 1000\pi(1 + j3)$$

$$D_2(j3000\pi) = 3000\pi(3 + j1)$$

$$D_3(j3000\pi) = 10^6\pi^2(-8 + j3)$$

$$D_4(j3000\pi) = 10^6\pi^2(8 + j3)$$

$$H'(j3000\pi) = \frac{(729 \times \pi^3 \times 10^{10})(27 \times 10^9\pi^3)}{27 \times 10^{18}\pi^6(730\underline{/270^\circ})}$$

$$= 9.99\underline{/90^\circ}$$

$$\therefore \quad 20\log_{10}|H'(j3000\pi)| = 19.99 \text{ dB}$$

[c] From the transfer function the gain is down 19.99 + 3.89 or 23.88 dB at 200 Hz. Because the upper cut-off frequency is nine times the lower cut-off frequency we would expect the high-pass stage of the filter to predict the loss in gain at 200 Hz. For a 3nd order Butterworth

$$\text{GAIN} = 20\log_{10}\frac{1}{\sqrt{1+(500/200)^6}} = -23.89 \text{ dB}.$$

1500 Hz is in the passband for this bandpass filter. Hence we expect the gain at 1500 Hz to nearly equal 20 dB as specified in Problem 15.39. Thus our scaled transfer function confirms that the filter meets the specifications.

P 15.48 **[a]** From Table 15.1

$$H_{lp}(s) = \frac{1}{(s^2+0.518s+1)(s^2+\sqrt{2}s+1)(s^2+1.932s+1)}$$

$$H_{hp}(s) = \frac{1}{\left(\frac{1}{s^2}+0.518\left(\frac{1}{s}\right)+1\right)\left(\frac{1}{s^2}+\sqrt{2}\left(\frac{1}{s}\right)+1\right)\left(\frac{1}{s^2}+1.932\left(\frac{1}{s}\right)+1\right)}$$

$$H_{hp}(s) = \frac{s^6}{(s^2+0.518s+1)(s^2+\sqrt{2}s+1)(s^2+1.932s+1)}$$

P 15.49 **[a]** $k_f = 25{,}000$

$$H'_{\text{hp}}(s) = \frac{(s/25{,}000)^6}{[(s/25{,}000)^2+0.518(s/25{,}000)+1]}$$

$$\cdot\frac{1}{[(s/25{,}000)^2+\sqrt{2}s/25{,}000+1][(s/25{,}000)^2+1.932s/25{,}000+1]}$$

$$= \frac{s^6}{(s^2+12{,}950s+625\times10^6)(s^2+35{,}355s+625\times10^6)}$$

$$\cdot\frac{1}{(s^2+48{,}300s+625\times10^6)}$$

[b] $$H'(j25{,}000) = \frac{-(25{,}000)^6}{[12{,}950(j25{,}000)][35{,}355(j25{,}000)][48{,}300(j25{,}000)]}$$

$$= \frac{-(25{,}000)^3}{(12{,}950)(35{,}355)(48{,}300)j^3}$$

$$= 0.7066\underline{/-90^\circ}$$

$$20\log_{10}|H'(j25{,}000)| = -3.02 \text{ dB}$$

P 15.50 [**a**] At very low frequencies the two capacitor branches are open and because the op amp is ideal the current in R_3 is zero. Therefore at low frequencies the circuit behaves as an inverting amplifier with a gain of R_2/R_1. At very high frequencies the capacitor branches are short circuits and hence the output voltage is zero.

[**b**] Let the node where R_1, R_2, R_3, and C_2 join be denoted as a, then

$$(V_a - V_i)G_1 + V_a sC_2 + (V_a - V_o)G_2 + V_a G_3 = 0$$

$$-V_a G_3 - V_o sC_1 = 0$$

or

$$(G_1 + G_2 + G_3 + sC_2)V_a - G_2 V_o = G_1 V_i$$

$$V_a = \frac{-sC_1}{G_3} V_o$$

Solving for V_o/V_i yields

$$\begin{aligned} H(s) &= \frac{-G_1G_3}{(G_1 + G_2 + G_3 + sC_2)sC_1 + G_2G_3} \\ &= \frac{-G_1G_3}{s^2C_1C_2 + (G_1 + G_2 + G_3)C_1 s + G_2G_3} \\ &= \frac{-G_1G_3/C_1C_2}{s^2 + \left[\frac{(G_1+G_2+G_3)}{C_2}\right] s + \frac{G_2G_3}{C_1C_2}} \\ &= \frac{-\frac{G_1G_2G_3}{G_2C_1C_2}}{s^2 + \left[\frac{(G_1+G_2+G_3)}{C_2}\right] s + \frac{G_2G_3}{C_1C_2}} \\ &= \frac{-Kb_o}{s^2 + b_1 s + b_o} \end{aligned}$$

where $K = \dfrac{G_1}{G_2}; \qquad b_o = \dfrac{G_2G_3}{C_1C_2}$

and $b_1 = \dfrac{G_1 + G_2 + G_3}{C_2}$

[**c**] Rearranging we see that

$$G_1 = KG_2$$

$$G_3 = \frac{b_o C_1 C_2}{G_2} = \frac{b_o C_1}{G_2}$$

since by hypothesis $C_2 = 1\,\text{F}$

$$b_1 = \frac{G_1 + G_2 + G_3}{C_2} = G_1 + G_2 + G_3$$

$$\therefore\ b_1 = KG_2 + G_2 + \frac{b_oC_1}{G_2}$$

$$b_1 = G_2(1+K) + \frac{b_oC_1}{G_2}$$

Solving this quadratic equation for G_2 we get

$$G_2 = \frac{b_1}{2(1+K)} \pm \sqrt{\frac{b_1^2 - b_oC_1 4(1+K)}{4(1+K)^2}}$$

$$= \frac{b_1 \pm \sqrt{b_1^2 - 4b_o(1+K)C_1}}{2(1+K)}$$

For G_2 to be realizable

$$C_1 < \frac{b_1^2}{4b_o(1+K)}$$

[d] 1. Select $C_2 = 1\,\text{F}$

2. Select C_1 such that $C_1 < \dfrac{b_1^2}{4b_o(1+K)}$

3. Calculate G_2 (R_2)

4. Calculate G_1 (R_1); $G_1 = KG_2$

5. Calculate G_3 (R_3); $G_3 = b_oC_1/G_2$

P 15.51 **[a]** In the second order section of a third order Butterworth filter $b_o = b_1 = 1$
Therefore,

$$C_1 \leq \frac{b_1^2}{4b_o(1+K)} = \frac{1}{(4)(1)(5)} = 0.05\,\text{F}$$

$$\therefore \qquad C_1 = 0.05\,\text{F} \quad \text{(limiting value)}$$

[b] $G_2 = \dfrac{1}{2(1+4)} = 0.1\,\text{S}$

$$G_3 = \frac{1}{0.1}(0.05) = 0.5\,\text{S}$$

$$G_1 = 4(0.1) = 0.4\,\text{S}$$

Therefore,

$$R_1 = \frac{1}{G_1} = 2.5\,\Omega; \qquad R_2 = \frac{1}{G_2} = 10\,\Omega; \qquad R_3 = \frac{1}{G_3} = 2\,\Omega$$

[c] $k_f = \dfrac{\omega_o'}{\omega_o} = 2\pi(2500) = 5000\pi$

$$k_m = \frac{C_2}{C_2' k_f} = \frac{1}{(10 \times 10^{-9})k_f} = 6366.2$$

$$C_1' = \frac{0.05}{k_f k_m} = 0.5 \times 10^{-9} = 500\,\text{pF}$$

$$R_1' = (2.5)(6366.2) = 15.92\,\text{k}\Omega$$

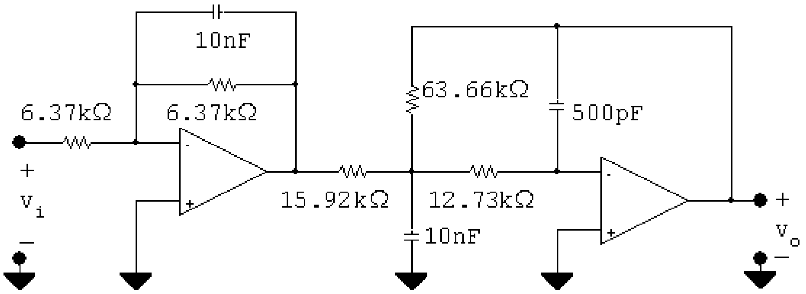
$$R_2' = (10)(6366.2) = 63.66\,\text{k}\Omega$$

$$R_3' = (2)(6366.2) = 12.73\,\text{k}\Omega$$

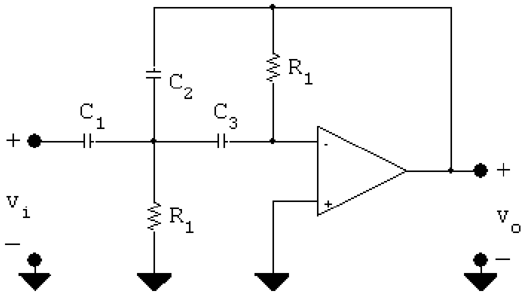
[d] $R_1' = R_2' = (6366.2)(1) = 6.37\,\text{k}\Omega$

$$C' = \frac{C}{k_f k_m} = \frac{1}{10^8} = 10\,\text{nF}$$

[e]



P 15.52 **[a]** By hypothesis the circuit becomes:



For very small frequencies the capacitors behave as open circuits and therefore v_o is zero. As the frequency increases, the capacitive branch impedances become small compared to the resistive branches. When this happens the circuit becomes an inverting amplifier with the capacitor C_2 dominating the feedback path. Hence the gain of the amplifier approaches $(1/j\omega C_2)/(1/j\omega C_1)$ or C_1/C_2. Therefore the circuit behaves like a high-pass filter with a passband gain of C_1/C_2.

[b] Summing the currents away from the upper terminal of R_2 yields

$$V_aG_2 + (V_a - V_i)sC_1 + (V_a - V_o)sC_2 + V_asC_3 = 0$$

or

$$V_a[G_2 + s(C_1 + C_2 + C_3)] - V_osC_2 = sC_1V_i$$

Summing the currents away from the inverting input terminal gives

$$(0 - V_a)sC_3 + (0 - V_o)G_1 = 0$$

or

$$sC_3V_a = -G_1V_o; \qquad V_a = \frac{-G_1V_o}{sC_3}$$

Therefore we can write

$$\frac{-G_1V_o}{sC_3}[G_2 + s(C_1 + C_2 + C_3)] - sC_2V_o = sC_1V_i$$

Solving for V_o/V_i gives

$$H(s) = \frac{V_o}{V_i} = \frac{-C_1C_3s^2}{C_2C_3s^2 + G_1(C_1 + C_2 + C_3)s + G_1G_2]}$$

$$= \frac{\frac{-C_1}{C_2}s^2}{\left[s^2 + \frac{G_1}{C_2C_3}(C_1 + C_2 + C_3)s + \frac{G_1G_2}{C_2C_3}\right]}$$

$$= \frac{-Ks^2}{s^2 + b_1s + b_o}$$

Therefore the circuit implements a second-order high-pass filter with a passband gain of C_1/C_2.

[c] $C_1 = K$:

$$b_1 = \frac{G_1}{(1)(1)}(K + 2) = G_1(K + 2)$$

$$\therefore \quad G_1 = \frac{b_1}{K + 2}; \qquad R_1 = \left(\frac{K + 2}{b_1}\right)$$

$$b_o = \frac{G_1G_2}{(1)(1)} = G_1G_2$$

$$\therefore \quad G_2 = \frac{b_o}{G_1} = \frac{b_o}{b_1}(K + 2)$$

$$\therefore \quad R_2 = \frac{b_1}{b_o(K + 2)}$$

[d] From Table 15.1 the transfer function of the second-order section of a third-order high-pass Butterworth filter is

$$H(s) = \frac{Ks^2}{s^2 + s + 1}$$

Therefore $b_1 = b_o = 1$

Thus

$$C_1 = K = 8\,\text{F}$$

$$R_1 = \frac{8+2}{1} = 10\,\Omega$$

$$R_2 = \frac{1}{1(8+2)} = 0.1\,\Omega$$

P 15.53 **[a]** Low-pass filter:

$$n = \frac{(-0.05)(-30)}{\log_{10}(1000/400)} = 3.77; \qquad \therefore \quad n = 4$$

In the first prototype second-order section: $b_1 = 0.765,\ b_o = 1,\ C_2 = 1\,\text{F}$

$$C_1 \le \frac{b_1^2}{4b_o(1+K)} \le \frac{(0.765)^2}{(4)(2)} \le 0.0732$$

choose $C_1 = 0.03\,\text{F}$

$$G_2 = \frac{0.765 \pm \sqrt{(0.765)^2 - 4(2)(0.03)}}{4} = \frac{0.765 \pm 0.588}{4}$$

Arbitrarily select the larger value for G_2, then

$$G_2 = 0.338\ \text{S}; \quad \therefore \qquad R_2 = \frac{1}{G_2} = 2.96\,\Omega$$

$$G_1 = KG_2 = 0.338\ \text{S}; \quad \therefore \qquad R_1 = \frac{1}{G_1} = 2.96\,\Omega$$

$$G_3 = \frac{b_oC_1}{G_2} = \frac{(1)(0.03)}{0.338} = 0.089 \quad \therefore \qquad R_3 = 1/G_3 = 11.3\,\Omega$$

Therefore in the first second-order prototype circuit

$$R_1 = R_2 = 2.96\,\Omega; \qquad R_3 = 11.3\,\Omega$$

$$C_1 = 0.03\,\text{F}; \qquad C_2 = 1\,\text{F}$$

In the second second-order prototype circuit:
$b_1 = 1.848,\ b_0 = 1,\ C_2 = 1\,\text{F}$

$$\therefore \quad C_1 \le \frac{(1.848)^2}{8} \le 0.427$$

choose $C_1 = 0.30\,\mathrm{F}$

$$G_2 = \frac{1.848 \pm \sqrt{(1.848)^2 - 8(0.3)}}{4} = \frac{1.848 \pm 1.008}{4}$$

Arbitrarily select the larger value, then

$$G_2 = 0.7139\ \ \mathrm{S}; \quad \therefore \qquad R_2 = \frac{1}{G_2} = 1.4008\,\Omega$$

$$G_1 = KG_2 = 0.7139\ \ \mathrm{S}; \quad \therefore \qquad R_1 = \frac{1}{G_1} = 1.4008\,\Omega$$

$$G_3 = \frac{b_oC_1}{G_2} = \frac{(1)(0.30)}{0.7139} = 0.4202\ \ \mathrm{S} \quad \therefore \qquad R_3 = 1/G_3 = 2.3796\,\Omega$$

In the low-pass section of the filter

$$k_f = \frac{\omega_o'}{\omega_o} = 2\pi(400) = 800\pi$$

$$k_m = \frac{C^2}{C'^2k_f} = \frac{1}{(10 \times 10^{-9})k_f} = \frac{125{,}000}{\pi}$$

Therefore in the first scaled second-order section

$$R_1' = R_2' = 2.96k_m = 118\,\mathrm{k}\Omega$$

$$R_3' = 11.3k_m = 448\,\mathrm{k}\Omega$$

$$C_1' = \frac{0.03}{k_fk_m} = 300\,\mathrm{pF}$$

$$C_2' = 10\,\mathrm{nF}$$

In the second scaled second-order section

$$R_1' = R_2' = 1.4008k_m = 55.74\,\mathrm{k}\Omega$$

$$R_3' = 2.38k_m = 94.68\,\mathrm{k}\Omega$$

$$C_1' = \frac{0.3}{k_fk_m} = 3\,\mathrm{nF}$$

$$C_2' = 10\,\mathrm{nF}$$

High-pass filter section

$$n = \frac{(-0.05)(-30)}{\log_{10}(6400/2560)} = 3.77; \qquad n = 4$$

In the first prototype second-order section:
$b_1 = 0.765;\ b_o = 1;\ C_2 = C_3 = 1\,\mathrm{F}$

$$C_1 = K = 1\,\mathrm{F}$$

$$R_1 = \frac{K+2}{b_1} = \frac{3}{0.765} = 3.92\,\Omega$$

$$R_2 = \frac{b_1}{b_o(K+2)} = \frac{0.765}{3} = 0.255\,\Omega$$

In the second prototype second-order section: $b_1 = 1.848$; $b_o = 1$; $C_2 = C_3 = 1\,\text{F}$

$$C_1 = K = 1\,\text{F}$$

$$R_1 = \frac{K+2}{b_1} = \frac{3}{1.848} = 1.623\,\Omega$$

$$R_2 = \frac{b_1}{b_o(K+2)} = \frac{1.848}{3} = 0.616\,\Omega$$

In the high-pass section of the filter

$$k_f = \frac{\omega_o'}{\omega_o} = 2\pi(6400) = 12{,}800\pi$$

$$k_m = \frac{C}{C'k_f} = \frac{1}{(10\times10^{-9})(12{,}800\pi)} = \frac{7812.5}{\pi}$$

In the first scaled second-order section

$$R_1' = 3.92k_m = 9.75\,\text{k}\Omega$$

$$R_2' = 0.255k_m = 634\,\Omega$$

$$C_1' = C_2' = C_3' = 10\,\text{nF}$$

In the second scaled second-order section

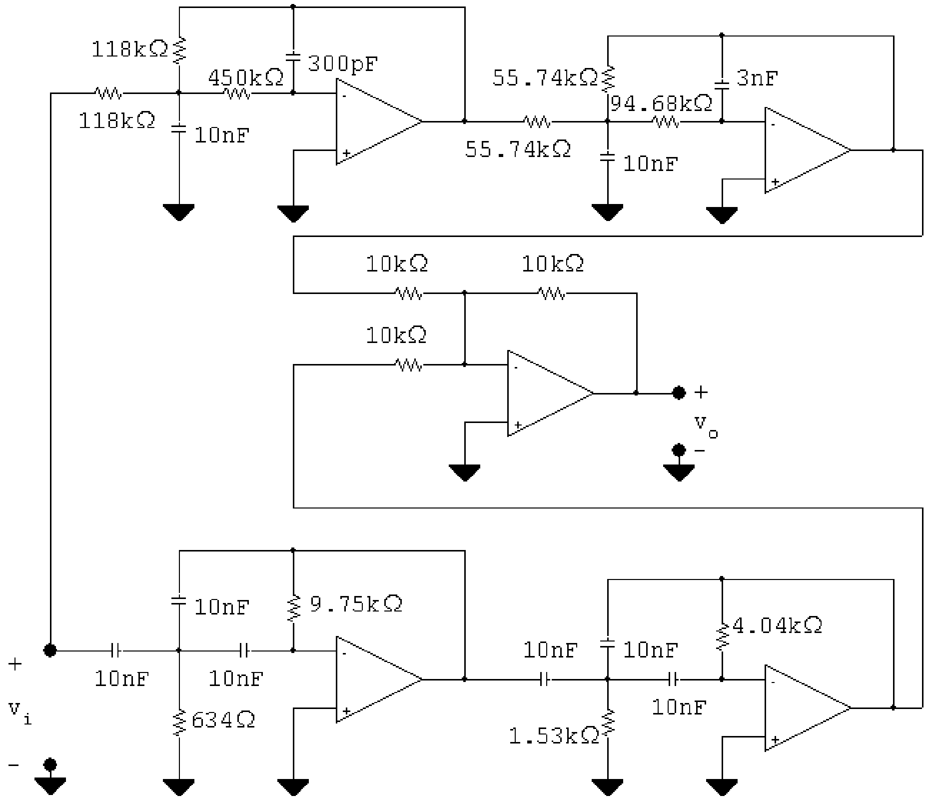
$$R_1' = 1.623k_m = 4.04\,\text{k}\Omega$$

$$R_2' = 0.616k_m = 1.53\,\text{k}\Omega$$

$$C_1' = C_2' = C_3' = 10\,\text{nF}$$

In the gain section, let $R_i = 10\,\text{k}\Omega$ and $R_f = 10\,\text{k}\Omega$.

[b]



P 15.54 **[a]** The prototype low-pass transfer function is

$$H_{lp}(s) = \frac{1}{(s^2 + 0.765s + 1)(s^2 + 1.848s + 1)}$$

The low-pass frequency scaling factor is

$$k_{f_{lp}} = 2\pi(400) = 800\pi$$

The scaled transfer function for the low-pass filter is

$$H'_{lp}(s) = \frac{1}{\left[\left(\frac{s}{800\pi}\right)^2 + \frac{0.765s}{800\pi} + 1\right]\left[\left(\frac{s}{800\pi}\right)^2 + \frac{1.848s}{800\pi} + 1\right]}$$

$$= \frac{4096 \times 10^8\pi^4}{[s^2 + 612\pi s + (800\pi)^2]\,[s^2 + 1478.4\pi s + (800\pi)^2]}$$

The prototype high-pass transfer function is

$$H_{hp}(s) = \frac{s^4}{(s^2 + 0.765s + 1)(s^2 + 1.848s + 1)}$$

The high-pass frequency scaling factor is

$$k_{f_{hp}} = 2\pi(6400) = 12{,}800\pi$$

The scaled transfer function for the high-pass filter is

$$H'_{hp}(s) = \frac{(s/12{,}800\pi)^4}{\left[\left(\frac{s}{12{,}800\pi}\right)^2 + \frac{0.765s}{12{,}800\pi} + 1\right]\left[\left(\frac{s}{12{,}800\pi}\right)^2 + \frac{1.848s}{12{,}800\pi} + 1\right]}$$

$$= \frac{s^4}{[s^2 + 9792\pi s + (12{,}800\pi)^2][s^2 + 23{,}654.4\pi s + (12{,}800\pi)^2]}$$

The transfer function for the filter is

$$H'(s) = \left[H'_{lp}(s) + H'_{hp}(s)\right]$$

[b] $f_o = \sqrt{f_{c1}f_{c2}} = \sqrt{400)(6400)} = 1600\,\text{Hz}$

$$\omega_o = 2\pi f_o = 3200\pi\ \text{rad/s}$$

$$(j\omega_o)^2 = -1024 \times 10^4\pi^2$$

$$(j\omega_o)^4 = 1{,}048{,}576 \times 10^8\pi^4$$

$$H'_{lp}(j\omega_o) = \frac{4096 \times 10^8\pi^4}{[-960 \times 10^4\pi^2 + j612(3200\pi^2)]} \times$$

$$\frac{1}{[-960 \times 10^4\pi^2 + j1478.4(3200\pi^2)]}$$

$$= \frac{40{,}000}{(-3000 + j612)(-3000 + j1478.4)}$$

$$= 3906.2 \times 10^{-6}\underline{/-322.24^\circ}$$

$$H'_{hp}(j\omega_o) = \frac{1{,}048{,}576 \times 10^8\pi^4}{[15{,}360 \times 10^4\pi^2 + j9792(3200\pi^2)]}$$

$$\frac{1}{[15{,}360 \times 10^4\pi^2 + j23{,}654.4(3200\pi^2)]}$$

$$= \frac{10.24 \times 10^6}{(48{,}000 + j9792)(48{,}000 + j23{,}654.4)}$$

$$= 3906.2 \times 10^{-6}\underline{/-37.76^\circ}$$

$$\therefore\quad H'(j\omega_o) = -3906.2 \times 10^{-6}(1\underline{/-322.24^\circ} + 1\underline{/-37.76^\circ})$$

$$= -3906.2 \times 10^{-6}(1.58\underline{/0^\circ}) = -6176.35 \times 10^{-6}\underline{/0^\circ}$$

$$G = 20\log_{10}|H'(j\omega_o)| = 20\log_{10}(6176.35 \times 10^{-6}) = -44.19\ \text{dB}$$

P 15.55 **[a]** At low frequencies the capacitor branches are open; $v_o = v_i$. At high frequencies the capacitor branches are short circuits and the output voltage is zero. Hence the circuit behaves like a unity-gain low-pass filter.

[b] Let v_a represent the voltage-to-ground at the right-hand terminal of R_1. Observe this will also be the voltage at the left-hand terminal of R_2. The s-domain equations are

$$(V_a - V_i)G_1 + (V_a - V_o)sC_1 = 0$$

$$(V_o - V_a)G_2 + sC_2V_o = 0$$

or

$$(G_1 + sC_1)V_a - sC_1V_o = G_1V_i$$

$$-G_2V_a + (G_2 + sC_2)V_o = 0$$

$$\therefore \quad V_a = \frac{G_2 + sC_2V_o}{G_2}$$

$$\therefore \quad \left[(G_1 + sC_1)\frac{(G_2 + sC_2)}{G_2} - sC_1\right]V_o = G_1V_i$$

$$\therefore \quad \frac{V_o}{V_i} = \frac{G_1G_2}{(G_1 + sC_1)(G_2 + sC_2) - C_1G_2s}$$

which reduces to

$$\frac{V_o}{V_i} = \frac{G_1G_2/C_1C_2}{s^2 + \frac{G_1}{C_1}s + \frac{G_1G_2}{C_1C_2}} = \frac{b_o}{s^2 + b_1s + b_o}$$

[c] There are four circuit components and two restraints imposed by $H(s)$; therefore there are two free choices.

[d] $b_1 = \dfrac{G_1}{C_1} \quad \therefore \quad G_1 = b_1C_1$

$$b_o = \frac{G_1G_2}{C_1C_2} \quad \therefore \quad G_2 = \frac{b_o}{b_1}C_2$$

[e] No, all physically realizeable capacitors will yield physically realizeable resistors.

[f] From Table 15.1 we know the transfer function of the prototype 4th order Butterworth filter is

$$H(s) = \frac{1}{(s^2 + 0.765s + 1)(s^2 + 1.848s + 1)}$$

In the first section $b_o = 1, \quad b_1 = 0.765$

$$\therefore \quad G_1 = (0.765)(1) = 0.765 \text{ S}$$

$$R_1 = 1/G_1 = 1.307\,\Omega$$

$$G_2 = \frac{1}{0.765}(1) = 1.307 \text{ S}$$

$R_2 = 1/G_2 = 0.765\,\Omega$

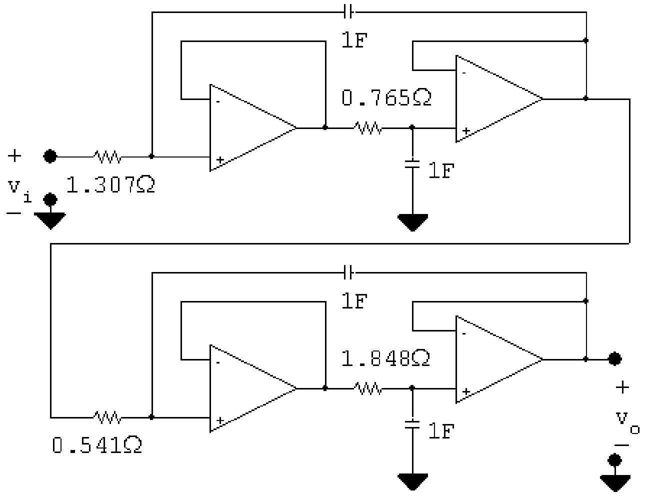
In the second section $b_o = 1, \quad b_1 = 1.848$

$\therefore \quad G_1 = 1.848\,\text{S}$

$R_1 = 1/G_1 = 0.541\,\Omega$

$$G_2 = \left(\frac{1}{1.848}\right)(1) = 0.541\,\text{S}$$

$R_2 = 1/G_2 = 1.848\,\Omega$



P 15.56 **[a]** $k_f = \dfrac{\omega_o'}{\omega_o} = 2\pi(3000) = 6000\pi$

$$k_m = \frac{C}{C'k_f} = \frac{1}{(4.7 \times 10^{-9})(6000\pi)} = \frac{10^6}{28.2\pi}$$

In the first section

$R_1' = 1.307k_m = 14.75\,\text{k}\Omega$

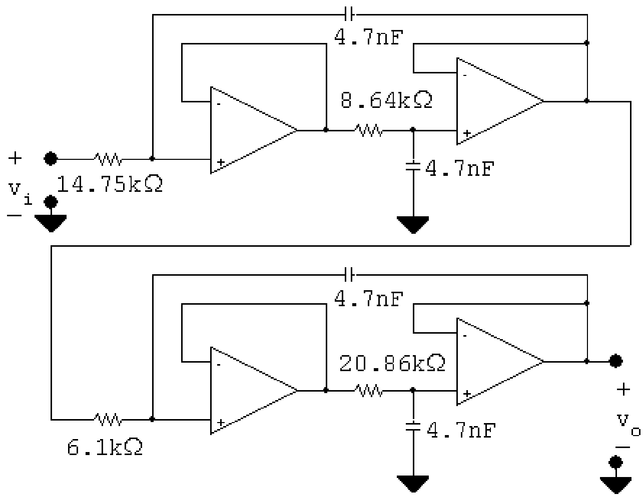
$R_2' = 0.765k_m = 8.64\,\text{k}\Omega$

In the second section

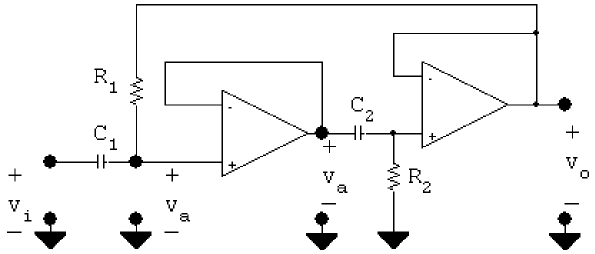
$R_1' = 0.541k_m = 6.1\,\text{k}\Omega$

$R_2' = 1.848k_m = 20.86\,\text{k}\Omega$

[b]



P 15.57 **[a]** Interchanging the *R*s and *C*s yields the following circuit.



At low frequencies the capacitors appear as open circuits and hence the output voltage is zero. As the frequency increases the capacitor branches approach short circuits and $v_a = v_i = v_o$. Thus the circuit is a unity-gain, high-pass filter.

[b] The s-domain equations are

$$(V_a - V_i)sC_1 + (V_a - V_o)G_1 = 0$$

$$(V_o - V_a)sC_2 + V_oG_2 = 0$$

It follows that

$$V_a(G_1 + sC_1) - G_1V_o = sC_1V_i$$

and $V_a = \dfrac{(G_2 + sC_2)V_o}{sC_2}$

Thus

$$\left\{\left[\frac{(G_2 + sC_2)}{sC_2}\right](G_1 + sC_1) - G_1\right\}V_o = sC_1V_i$$

$$V_o\{s^2C_1C_2 + sC_1G_2 + G_1G_2\} = s^2C_1C_2V_i$$

$$H(s) = \frac{V_o}{V_i} = \frac{s^2}{\left(s^2 + \frac{G_2}{C_2}s + \frac{G_1G_2}{C_1C_2}\right)}$$

$$= \frac{V_o}{V_i} = \frac{s^2}{s^2 + b_1 s + b_o}$$

[c] There are 4 circuit components: R_1, R_2, C_1 and C_2.
There are two transfer function constraints: b_1 and b_o.
Therefore there are two free choices.

[d] $b_o = \frac{G_1G_2}{C_1C_2}$; $\qquad b_1 = \frac{G_2}{C_2}$

$$\therefore\quad G_2 = b_1C_2; \qquad R_2 = \frac{1}{b_1C_2}$$

$$G_1 = \frac{b_o}{b_1}C_1 \;\therefore\quad R_1 = \frac{b_1}{b_oC_1}$$

[e] No, all realizeable capacitors will produce realizeable resistors.

[f] The second-order section in a 3rd-order Butterworth high-pass filter is $s^2/(s^2 + s + 1)$. Therefore $b_o = b_1 = 1$ and

$$R_1 = \frac{1}{(1)(1)} = 1\,\Omega.$$

$$R_2 = \frac{1}{(1)(1)} = 1\,\Omega.$$

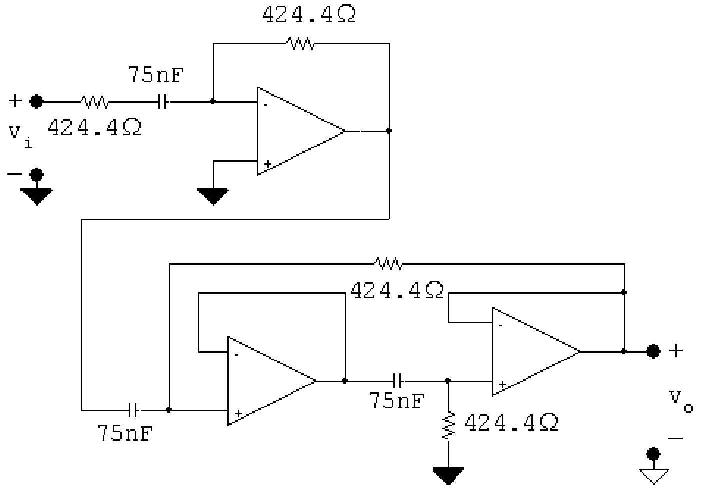
P 15.58 **[a]** $k_f = \frac{\omega_o'}{\omega_o} = 10^4\pi$

$$k_m = \frac{C}{C'k_f} = \frac{1}{(75 \times 10^{-9})(10^4\pi)} = \frac{10^5}{75\pi}$$

$$C_1' = C_2' = 75\,\text{nF}; \qquad R_1' = R_2' = k_mR = 424.4\,\Omega$$

[b] $R = 424.4\,\Omega$; $\qquad C = 75\,\text{nF}$

[c]



[d] $H_{\text{hp}}(s) = \dfrac{s^3}{(s+1)(s^2+s+1)}$

$$H'_{\text{hp}}(s) = \frac{(s/10^4\pi)^3}{[(s/10^4\pi)+1][(s/10^4\pi)^2+(s/10^4\pi)+1]}$$

$$= \frac{s^3}{(s+10^4\pi)(s^2+10^4\pi s+10^8\pi^2)}$$

[e] $H'_{\text{hp}}(j10^4\pi) = \dfrac{(j10^4\pi)^3}{(j10^4\pi+10^4\pi)[(j10^4\pi)^2+10^4\pi(j10^4\pi)+10^8\pi^2]} = 0.7071\underline{/135^\circ}$

$\therefore \quad |H'_{\text{hp}}| = 0.7071 = -3.01$ dB

P 15.59 **[a]** It follows directly from Eqs 15.64 and 15.65 that

$$H(s) = \frac{s^2+1}{s^2+4(1-\sigma)s+1}$$

Now note from Eq 15.69 that $(1-\sigma)$ equals $1/4Q$, hence

$$H(s) = \frac{s^2+1}{s^2+\frac{1}{Q}s+1}$$

[b] For Example 15.13 $\omega_o = 5000$ rad/s and $Q = 5$. Therefore $k_f = 5000$ and

$$H'(s) = \frac{(s/5000)^2+1}{(s/5000)^2+\dfrac{1}{5}\left(\dfrac{s}{5000}\right)+1}$$

$$= \frac{s^2+25\times10^6}{s^2+1000s+25\times10^6}$$

P 15.60 **[a]** $\omega_o = 2000\pi$ rad/s

$\therefore \quad k_f = \dfrac{\omega'_o}{\omega_o} = 2000\pi$

$$k_m = \frac{C}{C'k_f} = \frac{1}{(15\times10^{-9})(2000\pi)} = \frac{10^5}{3\pi}$$

$$R' = k_m R = \frac{10^5}{3\pi}(1) = 10{,}610\,\Omega \qquad \text{so} \qquad R'/2 = 5305\,\Omega$$

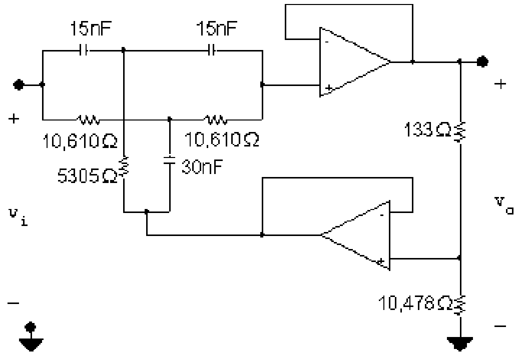
$$\sigma = 1 - \frac{1}{4Q} = 1 - \frac{1}{4(20)} = 0.9875$$

$$\sigma R' = 10{,}478\,\Omega; \qquad (1-\sigma)R' = 133\,\Omega$$

$$C' = 15\,\text{nF}$$

$$2C' = 30\,\text{nF}$$

[**b**]



[**c**] $k_f = 2000\pi$

$$H(s) = \frac{(s/2000\pi)^2 + 1}{(s/2000\pi)^2 + \frac{1}{20}(s/2000\pi) + 1}$$

$$= \frac{s^2 + 4\times10^6\pi^2}{s^2 + 100\pi s + 4\times10^6\pi^2}$$

P 15.61 To satisfy the gain specification of 20 dB at $\omega = 0$ and $\alpha = 1$ requires

$$\frac{R_1 + R_2}{R_1} = 10 \qquad \text{or} \qquad R_2 = 9R_1$$

Use the specified resistor of 11.1 kΩ for R_1 and a 100 kΩ potentiometer for R_2. Since $(R_1 + R_2)/R_1 \gg 1$ the value of C_1 is

$$C_1 = \frac{1}{2\pi(40)(10^5)} = 39.79\ \text{nF}$$

Choose a capacitor value of 40 nF. Using the selected values of R_1 and R_2 the maximum gain for $\alpha = 1$ is

$$20\log_{10}\left(\frac{111.1}{11.1}\right)_{\alpha=1} = 20.01\ \text{dB}$$

When $C_1 = 40$ nF the frequency $1/R_2C_1$ is

$$\frac{1}{R_2C_1} = \frac{10^9}{10^5(40)} = 250 \text{ rad/s} = 39.79 \text{ Hz}$$

The magnitude of the transfer function at 250 rad/s is

$$|H(j250)|_{\alpha=1} = \left|\frac{111.1 \times 10^3 + j250(11.1)(100)(40)10^{-3}}{11.1 \times 10^3 + j250(11.1)(100)(40)10^{-3}}\right| = 7.11$$

Therefore the gain at 39.79 Hz is

$$20\log_{10}(7.11)_{\alpha=1} = 17.04 \text{ dB}$$

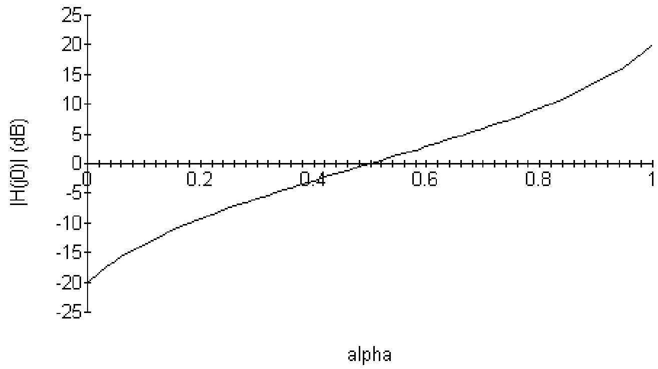
P 15.62 $20\log_{10}\left(\frac{R_1 + R_2}{R_1}\right) = 13.98$

$$\therefore \quad \frac{R_1 + R_2}{R_1} = 5; \qquad \therefore \quad R_2 = 4R_1$$

Choose $\quad R_1 = 100\,\text{k}\Omega$. Then $\quad R_2 = 400\,\text{k}\Omega$

$$\frac{1}{R_2C_1} = 100\pi \text{ rad/s}; \qquad \therefore \quad C_1 = \frac{1}{(100\pi)(400 \times 10^3)} = 7.96\,\text{nF}$$

P 15.63 $|H(j0)| = \frac{R_1 + \alpha R_2}{R_1 + (1-\alpha)R_2} = \frac{11.1 + \alpha(100)}{11.1 + (1-\alpha)100}$

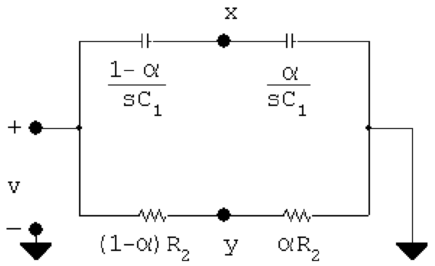


P 15.64 **[a]** Combine the impedances of the capacitors in series in Fig. P15.64(b) to get

$$\frac{1}{sC_{\text{eq}}} = \frac{1-\alpha}{sC_1} + \frac{\alpha}{sC_1} = \frac{1}{sC_1}$$

which is identical to the impedance of the capacitor in Fig. P15.60(a).

[**b**]



$$V_x = \frac{\alpha/sC_1}{(1-\alpha)/sC_1 + \alpha/sC_1}V = \alpha V$$

$$V_y = \frac{\alpha R_2}{(1-\alpha)R_2 + \alpha R_2} = \alpha V = V_x$$

[**c**] Since x and y are both at the same potential, they can be shorted together, and the circuit in Fig. 15.34 can thus be drawn as shown in Fig. 15.53(c).

[**d**] The feedback path between V_o and V_s containing the resistance $R_4 + 2R_3$ has no effect on the ratio V_o/V_s, as this feedback path is not involved in the nodal equation that defines the voltage ratio. Thus, the circuit in Fig. P15.64(c) can be simplified into the form of Fig. 15.2, where the input impedance is the equivalent impedance of R_1 in series with the parallel combination of $(1-\alpha)/sC_1$ and $(1-\alpha)R_2$, and the feedback impedance is the equivalent impedance of R_1 in series with the parallel combination of α/sC_1 and αR_2:

$$Z_i = R_1 + \frac{\frac{(1-\alpha)}{sC_1}\cdot(1-\alpha)R_2}{(1-\alpha)R_2 + \frac{(1-\alpha)}{sC_1}}$$

$$= \frac{R_1 + (1-\alpha)R_2 + R_1R_2C_1s}{1 + R_2C_1s}$$

$$Z_f = R_1 + \frac{\frac{\alpha}{sC_1}\cdot\alpha R_2}{\alpha R_2 + \frac{\alpha}{sC_1}}$$

$$= \frac{R_1 + \alpha R_2 + R_1R_2C_1s}{1 + R_2C_1s}$$

P 15.65 As $\omega \to 0$

$$|H(j\omega)| \to \frac{2R_3 + R_4}{2R_3 + R_4} = 1$$

Therefore the circuit would have no effect on low frequency signals. As $\omega \to \infty$

$$|H(j\omega)| \to \frac{[(1-\beta)R_4 + R_o](\beta R_4 + R_3)}{[(1-\beta)R_4 + R_3](\beta R_4 + R_o)}$$

When $\beta = 1$

$$|H(j\infty)|_{\beta=1} = \frac{R_o(R_4 + R_3)}{R_3(R_4 + R_o)}$$

If $R_4 \gg R_o$

$$|H(j\infty)|_{\beta=1} \cong \frac{R_o}{R_3} > 1$$

Thus, when $\beta = 1$ we have amplification or "boost". When $\beta = 0$

$$|H(j\infty)|_{\beta=0} = \frac{R_3(R_4 + R_o)}{R_o(R_4 + R_3)}$$

If $R_4 \gg R_o$

$$|H(j\infty)|_{\beta=0} \cong \frac{R_3}{R_0} < 1$$

Thus, when $\beta = 0$ we have attenuation or "cut".
Also note that when $\beta = 0.5$

$$|H(j\omega)|_{\beta=0.5} = \frac{(0.5R_4 + R_o)(0.5R_4 + R_3)}{(0.5R_4 + R_3)(0.5R_4 + R_o)} = 1$$

Thus, the transition from amplification to attenuation occurs at $\beta = 0.5$. If $\beta > 0.5$ we have amplification, and if $\beta < 0.5$ we have attenuation.
Also note the amplification an attenuation are symmetric about $\beta = 0.5$. i.e.

$$|H(j\omega)|_{\beta=0.6} = \frac{1}{|H(j\omega)|_{\beta=0.4}}$$

Yes, the circuit can be used as a treble volume control because

- The circuit has no effect on low frequency signals
- Depending on β the circuit can either amplify ($\beta > 0.5$) or attenuate ($\beta < 0.5$) signals in the treble range
- The amplification (boost) and attenuation (cut) are symmetric around $\beta = 0.5$. When $\beta = 0.5$ the circuit has no effect on signals in the treble frequency range.

P 15.66 **[a]** $|H(j\infty)|_{\beta=1} = \dfrac{R_o(R_4 + R_3)}{R_3(R_4 + R_o)} = \dfrac{(65.9)(505.9)}{(5.9)(565.9)} = 9.99$

$$\therefore \quad \text{maximum boost } = 20\log_{10} 9.99 = 19.99 \text{ dB}$$

[b] $|H(j\infty)|_{\beta=0} = \dfrac{R_3(R_4 + R_3)}{R_o(R_4 + R_o)}$

$\therefore$ maximum cut $= -19.99$ dB

[c] $R_4 = 500\,\text{k}\Omega$; $\qquad R_o = R_1 + R_3 + 2R_2 = 65.9\,\text{k}\Omega$

$\therefore \quad R_4 = 7.59 R_o$

Yes, R_4 is significantly greater than R_o.

[d]
$$|H(j/R_3C_2)|_{\beta=1} = \left|\frac{(2R_3 + R_4) + j\frac{R_o}{R_3}(R_4 + R_3)}{(2R_3 + R_4) + j(R_4 + R_o)}\right|$$

$$= \left|\frac{511.8 + j\frac{65.9}{5.9}(505.9)}{511.8 + j565.9}\right|$$

$$= 7.44$$

$$20\log_{10}|H(j/R_3C_2)|_{\beta=1} = 20\log_{10} 7.44 = 17.43 \text{ dB}$$

[e] When $\beta = 0$

$$|H(j/R_3C_2)|_{\beta=0} = \frac{(2R_3 + R_4) + j(R_4 + R_o)}{(2R_3 + R_4) + j\dfrac{R_o}{R_3}(R_4 + R_3)}$$

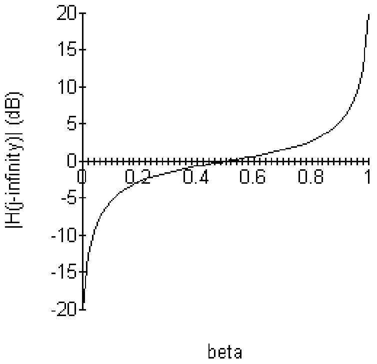
Note this is the reciprocal of $|H(j/R_3C_2)|_{\beta=1}$.

$\therefore \quad 20\log_{10}|H(j/R_3C_2)|_{\beta+0} = -17.43$ dB

[f] The frequency $1/R_3C_2$ is very nearly where the gain is 3 dB off from its maximum boost or cut. Therefore for frequencies higher than $1/R_3C_2$ the circuit designer knows that gain or cut will be within 3 dB of the maximum.

P 15.67
$$|H(j\infty)| = \frac{[(1-\beta)R_4 + R_o][\beta R_4 + R_3]}{[(1-\beta R_4 + R_3][\beta R_4 + R_o]}$$

$$= \frac{[(1-\beta)500 + 65.9][\beta 500 + 5.9]}{[(1-\beta)500 + 5.9][\beta 500 + 65.9]}$$


20
15
10
5
0
-5
-10
-15
-20
0
0.2
0.4
0.6
0.8
1
|H(j-infinity)| (dB)
beta

16

Fourier Series

Assessment Problems

AP 16.1

$$a_v = \frac{1}{T}\int_0^{2T/3} V_m\,dt + \frac{1}{T}\int_{2T/3}^{T}\left(\frac{V_m}{3}\right)\,dt = \frac{7}{9}V_m = 7\pi\,\mathrm{V}$$

$$a_k = \frac{2}{T}\left[\int_0^{2T/3} V_m \cos\,k\omega_0 t\,dt + \int_{2T/3}^{T}\left(\frac{V_m}{3}\right)\cos\,\mathrm{k}\omega_0 t\,dt\right]$$

$$= \left(\frac{4V_m}{3k\omega_0 T}\right)\sin\left(\frac{4k\pi}{3}\right) = \left(\frac{6}{k}\right)\sin\left(\frac{4k\pi}{3}\right)$$

$$b_k = \frac{2}{T}\left[\int_0^{2T/3} V_m \sin k\omega_0 t\,dt + \int_{2T/3}^{T}\left(\frac{V_m}{3}\right)\sin k\omega_0 t\,dt\right]$$

$$= \left(\frac{4V_m}{3k\omega_0 T}\right)\left[1-\cos\left(\frac{4k\pi}{3}\right)\right] = \left(\frac{6}{k}\right)\left[1-\cos\left(\frac{4k\pi}{3}\right)\right]$$

AP 16.2 **[a]** $a_v = 7\pi = 21.99\,\mathrm{V}$

[b] $a_1 = -5.196$ $a_2 = 2.598$ $a_3 = 0$ $a_4 = -1.299$ $a_5 = 1.039$

$b_1 = 9$ $b_2 = 4.5$ $b_3 = 0$ $b_4 = 2.25$ $b_5 = 1.8$

[c] $w_0 = \left(\frac{2\pi}{T}\right) = 50\,\mathrm{rad/s}$

[d] $f_3 = 3f_0 = 23.87\,\mathrm{Hz}$

[e] $$v(t) = 21.99 - 5.2\cos 50t + 9\sin 50t + 2.6\sin 100t + 4.5\cos 100t$$
$$-1.3\sin 200t + 2.25\cos 200t + 1.04\sin 250t + 1.8\cos 250t + \cdots\,\mathrm{V}$$

AP 16.3 Odd function with both half- and quarter-wave symmetry.

$$v_g(t) = \left(\frac{6V_m}{T}\right)t, \qquad 0 \le t \le T/6; \qquad a_v = 0, \qquad a_k = 0 \quad \text{for all } k$$

$$b_k = 0 \quad \text{for } k \text{ even}$$

$$\begin{aligned} b_k &= \frac{8}{T}\int_0^{T/4} f(t)\sin k\omega_0 t\,dt, \qquad k \text{ odd} \\ &= \frac{8}{T}\int_0^{T/6}\left(\frac{6V_m}{T}\right) t\sin k\omega_0 t\,dt + \frac{8}{T}\int_{T/6}^{T/4} V_m \sin k\omega_0 t\,dt \\ &= \left(\frac{12V_m}{k^2\pi^2}\right)\sin\left(\frac{k\pi}{3}\right) \end{aligned}$$

$$v_g(t) = \frac{12V_m}{\pi^2}\sum_{n=1,3,5,\ldots}^{\infty}\frac{1}{n^2}\sin\frac{n\pi}{3}\sin n\omega_0 t\,\text{V}$$

AP 16.4 **[a]** $A_1 = -5.2 - j9 = 10.4\underline{/-120^\circ}$; $\qquad A_2 = 2.6 - j4.5 = 5.2\underline{/-60^\circ}$

$A_3 = 0$; $\qquad A_4 = -1.3 - j2.25 = 2.6\underline{/-120^\circ}$

$A_5 = 1.04 - j1.8 = 2.1\underline{/-60^\circ}$

$\theta_1 = -120^\circ$; $\qquad \theta_2 = -60^\circ$; $\qquad \theta_3$ not defined;

$\theta_4 = -120^\circ$; $\qquad \theta_5 = -60^\circ$

[b] $v(t) = 21.99 + 10.4\cos(50t - 120^\circ) + 5.2\cos(100t - 60^\circ)$

$+2.6\cos(200t - 120^\circ) + 2.1\cos(250t - 60^\circ) + \cdots\,\text{V}$

AP 16.5 The Fourier series for the input voltage is

$$\begin{aligned} v_i &= \frac{8A}{\pi^2}\sum_{n=1,3,5,\ldots}^{\infty}\left(\frac{1}{n^2}\sin\frac{n\pi}{2}\right)\sin n\omega_0(t + T/4) \\ &= \frac{8A}{\pi^2}\sum_{n=1,3,5,\ldots}^{\infty}\left(\frac{1}{n^2}\sin^2\frac{n\pi}{2}\right)\cos n\omega_0 t \\ &= \frac{8A}{\pi^2}\sum_{n=1,3,5,\ldots}^{\infty}\frac{1}{n^2}\cos n\omega_0 t \end{aligned}$$

$$\frac{8A}{\pi^2} = \frac{8(281.25\pi^2)}{\pi^2} = 2250\,\text{mV}$$

$$\omega_0 = \frac{2\pi}{T} = \frac{2\pi}{200\pi}\times 10^3 = 10$$

$$\therefore \qquad v_i = 2250 \sum_{n=1,3,5,\ldots}^{\infty} \frac{1}{n^2} \cos 10nt \, \text{mV}$$

From the circuit we have

$$\mathbf{V}_o = \frac{\mathbf{V}_i}{R + (1/j\omega C)} \cdot \frac{1}{j\omega C} = \frac{\mathbf{V}_i}{1 + j\omega RC}$$

$$\mathbf{V}_o = \frac{1/RC}{1/RC + j\omega}\mathbf{V}_i = \frac{100}{100 + j\omega}\mathbf{V}_i$$

$$\mathbf{V}_{i1} = 2250\underline{/0^\circ}\, \text{mV}; \qquad \omega_0 = 10 \text{ rad/s}$$

$$\mathbf{V}_{i3} = \frac{2250}{9}\underline{/0^\circ} = 250\underline{/0^\circ}\, \text{mV}; \qquad 3\omega_0 = 30 \text{ rad/s}$$

$$\mathbf{V}_{i5} = \frac{2250}{25}\underline{/0^\circ} = 90\underline{/0^\circ}\, \text{mV}; \qquad 5\omega_0 = 50 \text{ rad/s}$$

$$\mathbf{V}_{o1} = \frac{100}{100 + j10}(2250\underline{/0^\circ}) = 2238.83\underline{/-5.71^\circ}\, \text{mV}$$

$$\mathbf{V}_{o3} = \frac{100}{100 + j30}(250\underline{/0^\circ}) = 239.46\underline{/-16.70^\circ}\, \text{mV}$$

$$\mathbf{V}_{o5} = \frac{100}{100 + j50}(90\underline{/0^\circ}) = 80.50\underline{/-26.57^\circ}\, \text{mV}$$

$$\therefore \qquad v_o = 2238.33 \cos(10t - 5.71^\circ) + 239.46 \cos(30t - 16.70^\circ)$$

$$+80.50 \cos(50t - 26.57^\circ) + \ldots \text{ mV}$$

AP 16.6 **[a]** $\omega_o = \dfrac{2\pi}{T} = \dfrac{2\pi}{0.2\pi}(10^3) = 10^4$ rad/s

$$v_g(t) = 840 \sum_{n=1,3,5,\ldots}^{\infty} \frac{1}{n} \sin \frac{n\pi}{2} \cos n10{,}000t \, \text{V}$$

$$= 840 \cos 10{,}000t - 280 \cos 30{,}000t + 168 \cos 50{,}000t$$

$$- 120 \cos 70{,}000t + \cdots \text{ V}$$

$$\mathbf{V}_{g1} = 840\underline{/0^\circ}\, \text{V}; \qquad \mathbf{V}_{g3} = 280\underline{/180^\circ}\, \text{V}$$

$$\mathbf{V}_{g5} = 168\underline{/0^\circ}\, \text{V}; \qquad \mathbf{V}_{g7} = 120\underline{/180^\circ}\, \text{V}$$

$$H(s) = \frac{V_o}{V_g} = \frac{\beta s}{s^2 + \beta s + \omega_c^2}$$

$$\beta = \frac{1}{RC} = \frac{10^9}{10^4(20)} = 5000 \text{ rad/s}$$

$$\omega_c^2 = \frac{1}{LC} = \frac{(10^9)(10^3)}{400} = 25 \times 10^8$$

$$H(s) = \frac{5000s}{s^2 + 5000s + 25 \times 10^8}$$

$$H(j\omega) = \frac{j5000\omega}{25 \times 10^8 - \omega^2 + j5000\omega}$$

$$H_1 = \frac{j5 \times 10^7}{24 \times 10^8 + j5 \times 10^7} = 0.02\underline{/88.81^\circ}$$

$$H_3 = \frac{j15 \times 10^7}{16 \times 10^8 + j15 \times 10^7} = 0.09\underline{/84.64^\circ}$$

$$H_5 = \frac{j25 \times 10^7}{25 \times 10^7} = 1\underline{/0^\circ}$$

$$H_7 = \frac{j35 \times 10^7}{-24 \times 10^8 + j35 \times 10^7} = 0.14\underline{/-81.70^\circ}$$

$$\mathbf{V}_{o1} = \mathbf{V}_{g1}H_1 = 17.50\underline{/88.81^\circ} \text{ V}$$

$$\mathbf{V}_{o3} = \mathbf{V}_{g3}H_3 = 26.14\underline{/-95.36^\circ} \text{ V}$$

$$\mathbf{V}_{o5} = \mathbf{V}_{g5}H_5 = 168\underline{/0^\circ} \text{ V}$$

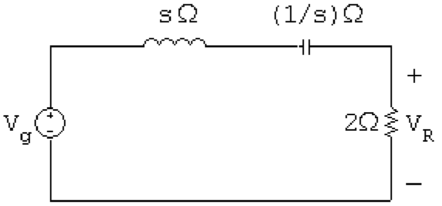
$$\mathbf{V}_{o7} = \mathbf{V}_{g7}H_7 = 17.32\underline{/98.30^\circ} \text{ V}$$

$$v_o = 17.50\cos(10{,}000t + 88.81^\circ) + 26.14\cos(30{,}000t - 95.36^\circ) + 168\cos(50{,}000t) + 17.32\cos(70{,}000t + 98.30^\circ) + \cdots \text{ V}$$

[b] The 5th harmonic because the circuit is a passive bandpass filter with a Q of 10 and a center frequency of 50 krad/s.

AP 16.7

$$w_0 = \frac{2\pi \times 10^3}{2094.4} = 3 \,\text{rad/s}$$



$$j\omega_0 k = j3k$$

$$V_R = \frac{2}{2+s+1/s}(V_g) = \frac{2sV_g}{s^2+2s+1}$$

$$H(s) = \left(\frac{V_R}{V_g}\right) = \frac{2s}{s^2+2s+1}$$

$$H(j\omega_0 k) = H(j3k) = \frac{j6k}{(1-9k^2)+j6k}$$

$$v_{g_1} = 25.98\sin\omega_0 t\,\text{V}; \qquad V_{g_1} = 25.98\underline{/0^\circ}\,\text{V}$$

$$H(j3) = \frac{j6}{-8+j6} = 0.6\underline{/-53.13^\circ}; \qquad V_{R_1} = 15.588\underline{/-53.13^\circ}\,\text{V}$$

$$P_1 = \frac{(15.588/\sqrt{2})^2}{2} = 60.75\,\text{W}$$

$$v_{g_3} = 0, \quad \text{therefore} \quad P_3 = 0\,\text{W}$$

$$v_{g_5} = -1.04\sin 5\omega_0 t\,\text{V}; \qquad V_{g_5} = 1.04\underline{/180^\circ}$$

$$H(j15) = \frac{j30}{-224+j30} = 0.1327\underline{/-82.37^\circ}$$

$$V_{R_5} = (1.04\underline{/180^\circ})(0.1327\underline{/-82.37^\circ}) = 138\underline{/97.63^\circ}\,\text{mV}$$

$$P_5 = \frac{(0.1396/\sqrt{2})^2}{2} = 4.76\,\text{mW}; \qquad \text{therefore} \quad P \cong P_1 \cong 60.75\,\text{W}$$

AP 16.8 Odd function with half- and quarter-wave symmetry, therefore $a_v = 0$, $a_k = 0$ for all k, $b_k = 0$ for k even; for k odd we have

$$b_k = \frac{8}{T}\int_0^{T/8} 2\sin\,\text{k}\omega_0 t\,dt + \frac{8}{T}\int_{T/8}^{T/4} 8\sin k\omega_0 t\,dt$$

$$= \left(\frac{8}{\pi k}\right)\left[1+3\cos\left(\frac{k\pi}{4}\right)\right], \quad k \text{ odd}$$

$$\text{Therefore} \quad C_n = \left(\frac{-j4}{n\pi}\right)\left[1+3\cos\left(\frac{n\pi}{4}\right)\right], \quad n \text{ odd}$$

AP 16.9 **[a]** $I_{\text{rms}} = \sqrt{\frac{2}{T}\left[(2)^2\left(\frac{T}{8}\right)(2) + (8)^2\left(\frac{3T}{8}-\frac{T}{8}\right)\right]} = \sqrt{34} = 5.7683\,\text{A}$

[b] $C_1 = \dfrac{-j12.5}{\pi}; \quad C_3 = \dfrac{j1.5}{\pi}; \quad C_5 = \dfrac{j0.9}{\pi};$

$$C_7 = \frac{-j1.8}{\pi}; \quad C_9 = \frac{-j1.4}{\pi}; \quad C_{11} = \frac{j0.4}{\pi}$$

$$I_{\text{rms}} = \sqrt{I_{dc}^2 + 2\sum_{n=1,3,5,\ldots}^{\infty} |C_n|^2} \cong \sqrt{\frac{2}{\pi^2}\left(12.5^2 + 1.5^2 + 1.8^2 + 1.4^2 + 0.4^2\right)}$$

$$\cong 5.777\,\text{A}$$

[c] $\%\ \text{Error} = \dfrac{5.777 - 5.831}{5.831} \times 100 = -1.08\%$

[d] Using just the terms $C_1 - C_9$,

$$I_{\text{rms}} = \sqrt{I_{dc}^2 + 2\sum_{n=1,3,5,\ldots}^{\infty} |C_n|^2} \cong \sqrt{\frac{2}{\pi^2}\left(12.5^2 + 1.5^2 + 1.8^2 + 1.4^2\right)}$$

$$\cong 5.774\,\text{A}$$

$$\%\ \text{Error} = \frac{5.774 - 5.831}{5.831} \times 100 = -0.98\%$$

Thus, the % error is still less than 1%.

AP 16.10 $T = 32\,\text{ms}$, therefore 8 ms requires shifting the function $T/4$ to the right.

$$i = \sum_{\substack{n=-\infty \\ n(\text{odd})}}^{\infty} -j\frac{4}{n\pi}\left(1 + 3\cos\frac{n\pi}{4}\right) e^{jn\omega_0(t-T/4)}$$

$$= \frac{4}{\pi} \sum_{\substack{n=-\infty \\ n(\text{odd})}}^{\infty} \frac{1}{n}\left(1 + 3\cos\frac{n\pi}{4}\right) e^{-j(n+1)(\pi/2)} e^{jn\omega_0 t}$$

Problems

P 16.1 **[a]** Odd function with half- and quarter-wave symmetry, $a_v = 0$, $a_k = 0$ for all k, $b_k = 0$ for even k; for k odd we have

$$b_k = \frac{8}{T}\int_0^{T/4} V_m \sin k\omega_0 t\,dt = \frac{4V_m}{k\pi}, \qquad k \text{ odd}$$

$$\text{and} \quad v(t) = \frac{4V_m}{\pi}\sum_{n=1,3,5,\ldots}^{\infty}\frac{1}{n}\sin n\omega_0 t\,\text{V}$$

[b] Even function: $b_k = 0$ for k

$$a_v = \frac{2}{T}\int_0^{T/2} V_m \sin\frac{\pi}{T}t\,dt = \frac{2V_m}{\pi}$$

$$a_k = \frac{4}{T}\int_0^{T/2} V_m \sin\frac{\pi}{T}t\cos k\omega_0 t\,dt = \frac{2V_m}{\pi}\left(\frac{1}{1-2k} + \frac{1}{1+2k}\right)$$

$$= \frac{4V_m/\pi}{1-4k^2}$$

$$\text{and} \quad v(t) = \frac{2V_m}{\pi}\left[1 + 2\sum_{n=1}^{\infty}\frac{1}{1-4n^2}\cos n\omega_0 t\right]\text{V}$$

[c] $a_v = \dfrac{1}{T}\displaystyle\int_0^{T/2} V_m \sin\left(\frac{2\pi}{T}\right)t\,dt = \frac{V_m}{\pi}$

$$a_k = \frac{2}{T}\int_0^{T/2} V_m \sin\frac{2\pi}{T}t\cos k\omega_0 t\,dt = \frac{V_m}{\pi}\left(\frac{1+\cos k\pi}{1-k^2}\right)$$

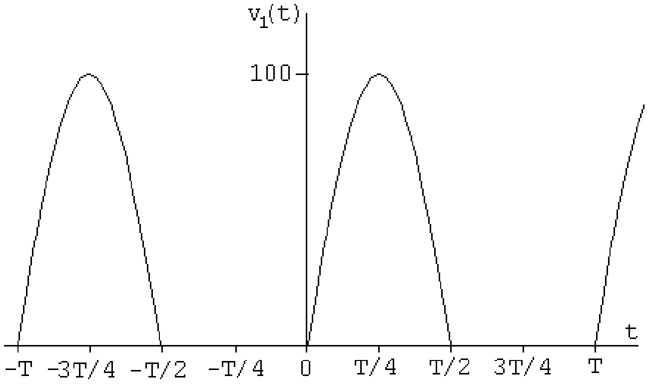
Note: $a_k = 0$ for k-odd, $a_k = \dfrac{2V_m}{\pi(1-k^2)}$ for k even,

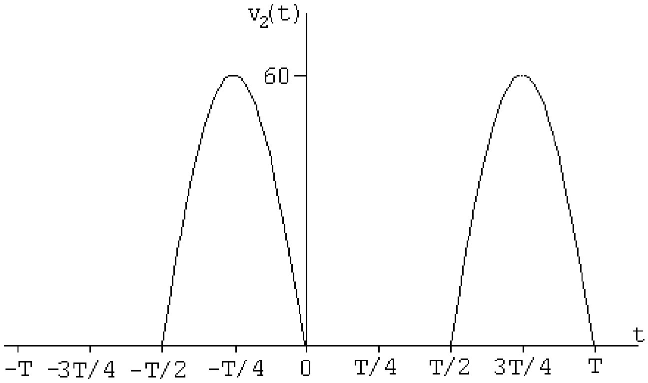
$$b_k = \frac{2}{T}\int_0^{T/2} V_m \sin\frac{2\pi}{T}t\sin k\omega_0 t\,dt = 0 \quad \text{for} \quad k = 2, 3, 4, \ldots$$

For $k = 1$, we have $b_1 = \dfrac{V_m}{2}$; therefore

$$v(t) = \frac{V_m}{\pi} + \frac{V_m}{2}\sin\omega_0 t + \frac{2V_m}{\pi}\sum_{n=2,4,6,\ldots}^{\infty}\frac{1}{1-n^2}\cos n\omega_0 t\,\text{V}$$

P 16.2 In studying the periodic function in Fig. P16.2 note that it can be visualized as the combination of two half-wave rectified sine waves, as shown in the figure below. Hence we can use the Fourier series for a half-wave rectified sine wave which is given as the answer to Problem 16.1(c).





$$v_1(t) = \frac{100}{\pi} + 50\sin\omega_o t + \frac{200}{\pi}\sum_{n=2,4,6,\ldots}^{\infty}\frac{\cos n\omega_o t}{(n^2-1)}\,\text{V}$$

$$v_2(t) = \frac{60}{\pi} + 30\sin\omega_o(t - T/2) + \frac{120}{\pi}\sum_{n=2,4,6,\ldots}^{\infty}\frac{\cos n\omega_o(t-T/2)}{(n^2-1)}\,\text{V}$$

Observe the following:

$$\sin\omega_o(t - T/2) = \sin\left(\omega_o t - \frac{2\pi}{T}\frac{T}{2}\right) = \sin(\omega_o t - \pi) = -\sin\omega_o t$$

$$\cos n\omega_o(t - T/2) = \cos\left(n\omega_o t - \frac{2\pi n}{T}\frac{T}{2}\right) = \cos(n\omega_o t - n\pi) = \cos n\omega_o t$$

Using the observations above and that fact that n is even,

$$v_2(t) = \frac{60}{\pi} - 30\sin\omega_o t - \frac{120}{\pi}\sum_{n=2,4,6,\ldots}^{\infty}\frac{\cos(n\omega_o t)}{(n^2-1)}\,\text{V}$$

Thus,

$$v(t) = v_1(t) + v_2(t) = \frac{160}{\pi} + 20\sin\omega_o t - \frac{320}{\pi}\sum_{n=2,4,6,\ldots}^{\infty}\frac{\cos(n\omega_o t)}{(n^2-1)}\,\text{V}$$

P 16.3 **[a]** $\omega_{\rm oa} = \dfrac{2\pi}{200 \times 10^{-6}} = 31,415.93$ rad/s

$$\omega_{\rm ob} = \frac{2\pi}{40 \times 10^{-6}} = 157.08 \text{ krad/s}$$

[b] $f_{\rm oa} = \dfrac{1}{T} = \dfrac{1}{200 \times 10^{-6}} = 5000\,\text{Hz};\qquad f_{\rm ob} = \dfrac{1}{40 \times 10^{-6}} = 25{,}000\,\text{Hz}$

[c] $a_{\rm va} = 0; \qquad a_{\rm vb} = \dfrac{100(10 \times 10^{-6})}{40 \times 10^{-6}} = 25\,\text{V}$

[d] The periodic function in Fig. P16.1(a) has half-wave symmetry. Therefore,

$$a_{\rm v} = 0; \quad a_{\rm ka} = 0 \quad \text{for } k \text{ even}; \quad b_{\rm ka} = 0 \quad \text{for } k \text{ even}$$

For k odd,

$$\begin{aligned} a_{\rm ka} &= \frac{4}{T}\int_0^{T/4} 40\cos\frac{2\pi kt}{T}\,dt + \frac{4}{T}\int_{T/4}^{T/2} 80\cos\frac{2\pi kt}{T}\,dt \\ &= \frac{160}{T}\frac{T}{2\pi k}\sin\frac{2\pi kt}{T}\Big|_0^{T/4} + \frac{320}{T}\frac{T}{2\pi k}\sin\frac{2\pi kt}{T}\Big|_{T/4}^{T/2} \\ &= \frac{80}{\pi k}\sin\frac{\pi k}{2} + \frac{160}{\pi k}\left(\sin\pi k - \sin\frac{\pi k}{2}\right) \\ &= -\frac{80}{\pi k}\sin\frac{\pi k}{2}, \quad k \text{ odd} \end{aligned}$$

$$\begin{aligned} b_{\rm ka} &= \frac{4}{T}\int_0^{T/4} 40\sin\frac{2\pi kt}{T}\,dt + \frac{4}{T}\int_{T/4}^{T/2} 80\sin\frac{2\pi kt}{T}\,dt \\ &= \frac{-160}{T}\frac{T}{2\pi k}\cos\frac{2\pi kt}{T}\Big|_0^{T/4} - \frac{320}{T}\frac{T}{2\pi k}\cos\frac{2\pi kt}{T}\Big|_{T/4}^{T/2} \\ &= \frac{-80}{\pi k}(0-1) + \frac{160}{\pi k}(-1-0) \\ &= \frac{240}{\pi k} \end{aligned}$$

The periodic function in Fig. P16.1(b) is even; therefore, $b_k = 0$ for all k. Also,

$$a_{\rm vb} = 25\,\text{V}$$

$$\begin{aligned} a_{\rm kb} &= \frac{4}{T}\int_0^{T/8} 100\cos\frac{2\pi kt}{T}\,dt \\ &= \frac{400}{T}\frac{T}{2\pi k}\sin\frac{2\pi k}{T}t\Big|_0^{T/8} \\ &= \frac{200}{\pi k}\sin\frac{\pi k}{4} \end{aligned}$$

[e] For the periodic function in Fig. P16.1(a),

$$v(t) = \frac{80}{\pi}\sum_{n=1,3,5}^{\infty}\left(-\frac{1}{n}\sin\frac{n\pi}{2}\cos n\omega_o t + \frac{3}{n}\sin n\omega_o t\right)\ \text{V}$$

For the periodic function in Fig. P16.1(b),

$$v(t) = 25 + \frac{200}{\pi}\sum_{n=1}^{\infty}\left(\frac{1}{n}\sin\frac{n\pi}{4}\cos n\omega_o t\right)\ \text{V}$$

P 16.4 $a_{\text{v}} = \frac{1}{T}\int_0^{T/4} V_m\,dt + \frac{1}{T}\int_{T/4}^{T}\frac{V_m}{2}\,dt = \frac{5}{8}V_m = 37.5\pi\ \text{V}$

$$a_{\text{k}} = \frac{2}{T}\left[\int_0^{T/4} V_m\cos k\omega_0 t\,dt + \int_{T/4}^{T}\frac{V_m}{2}\cos k\omega_0 t\,dt\right]$$

$$= \frac{V_m}{k\omega_0 T}\sin\frac{k\pi}{2} = \frac{30}{k}\sin\frac{k\pi}{2}$$

$$b_{\text{k}} = \frac{2}{T}\left[\int_0^{T/4} V_m\sin k\omega_0 t\,dt + \int_{T/4}^{T}\frac{V_m}{2}\sin k\omega_0 t\,dt\right]$$

$$= \frac{V_m}{k\omega_0 T}\left[1-\cos\frac{k\pi}{2}\right] = \frac{30}{k}\left[1-\cos\frac{k\pi}{2}\right]$$

P 16.5 **[a]** $I_6 = \int_{t_o}^{t_o+T}\sin m\omega_0 t\,dt = -\frac{1}{m\omega_0}\cos m\omega_0 t\Big|_{t_o}^{t_o+T}$

$$= \frac{-1}{m\omega_0}[\cos m\omega_0(t_o+T) - \cos m\omega_0 t_o]$$

$$= \frac{-1}{m\omega_0}[\cos m\omega_0 t_o\cos m\omega_0 T - \sin m\omega_0 t_o\sin m\omega_0 T - \cos m\omega_0 t_o]$$

$$= \frac{-1}{m\omega_0}[\cos m\omega_0 t_o - 0 - \cos m\omega_0 t_o] = 0 \quad \text{for all } m,$$

$$I_7 = \int_{t_o}^{t_o+T}\cos m\omega_0 t_o\,dt = \frac{1}{m\omega_0}[\sin m\omega_0 t]\Big|_{t_o}^{t_o+T}$$

$$= \frac{1}{m\omega_0}[\sin m\omega_0(t_o+T) - \sin m\omega_0 t_o]$$

$$= \frac{1}{m\omega_0}[\sin m\omega_0 t_o - \sin m\omega_0 t_o] = 0 \quad \text{for all } m$$

[b] $I_8 = \int_{t_o}^{t_o+T}\cos m\omega_0 t\sin n\omega_0 t\,dt = \frac{1}{2}\int_{t_o}^{t_o+T}[\sin(m+n)\omega_0 t - \sin(m-n)\omega_0 t]\,dt$

But $(m+n)$ and $(m-n)$ are integers, therefore from I_6 above, $I_8 = 0$ for all m, n.

[c] $I_9 = \int_{t_o}^{t_o+T} \sin m\omega_0 t \sin n\omega_0 t\, dt = \frac{1}{2}\int_{t_o}^{t_o+T} [\cos(m-n)\omega_0 t - \cos(m+n)\omega_0 t]\, dt$

If $m \neq n$, both integrals are zero (I_7 above). If $m = n$, we get

$$I_9 = \frac{1}{2}\int_{t_o}^{t_o+T} dt - \frac{1}{2}\int_{t_o}^{t_o+T} \cos 2m\omega_0 t\, dt = \frac{T}{2} - 0 = \frac{T}{2}$$

[d] $I_{10} = \int_{t_o}^{t_o+T} \cos m\omega_0 t \cos n\omega_0 t\, dt$

$$= \frac{1}{2}\int_{t_o}^{t_o+T} [\cos(m-n)\omega_0 t + \cos(m+n)\omega_0 t]\, dt$$

If $m \neq n$, both integrals are zero (I_7 above). If $m = n$, we have

$$I_{10} = \frac{1}{2}\int_{t_o}^{t_o+T} dt + \frac{1}{2}\int_{t_o}^{t_o+T} \cos 2m\omega_0 t\, dt = \frac{T}{2} + 0 = \frac{T}{2}$$

P 16.6 $f(t)\sin k\omega_0 t = a_v \sin k\omega_0 t + \sum_{n=1}^{\infty} a_n \cos n\omega_0 t \sin k\omega_0 t + \sum_{n=1}^{\infty} b_n \sin n\omega_0 t \sin k\omega_0 t$

Now integrate both sides from t_o to $t_o + T$. All the integrals on the right-hand side reduce to zero except in the last summation when $n = k$, therefore we have

$$\int_{t_o}^{t_o+T} f(t)\sin k\omega_0 t\, dt = 0 + 0 + b_k\left(\frac{T}{2}\right) \quad \text{or} \quad b_k = \frac{2}{T}\int_{t_o}^{t_o+T} f(t)\sin k\omega_0 t\, dt$$

P 16.7 $a_v = \frac{1}{T}\int_{t_o}^{t_o+T} f(t)\, dt = \frac{1}{T}\left\{\int_{-T/2}^{0} f(t)\, dt + \int_{0}^{T/2} f(t)\, dt\right\}$

Let $t = -x, \quad dt = -dx, \quad x = \frac{T}{2}$ when $t = \frac{-T}{2}$

and $x = 0$ when $t = 0$

Therefore $\frac{1}{T}\int_{-T/2}^{0} f(t)\, dt = \frac{1}{T}\int_{T/2}^{0} f(-x)(-dx) = -\frac{1}{T}\int_{0}^{T/2} f(x)\, dx$

Therefore $a_v = -\frac{1}{T}\int_{0}^{T/2} f(t)\, dt + \frac{1}{T}\int_{0}^{T/2} f(t)\, dt = 0$

$$a_k = \frac{2}{T}\int_{-T/2}^{0} f(t)\cos k\omega_0 t\, dt + \frac{2}{T}\int_{0}^{T/2} f(t)\cos k\omega_0 t\, dt$$

Again, let $t = -x$ in the first integral and we get

$$\frac{2}{T}\int_{-T/2}^{0} f(t)\cos k\omega_0 t\, dt = -\frac{2}{T}\int_{0}^{T/2} f(x)\cos k\omega_0 x\, dx$$

Therefore $a_k = 0$ for all k.

$$b_k = \frac{2}{T}\int_{-T/2}^{0} f(t)\sin k\omega_0 t\,dt + \frac{2}{T}\int_{0}^{T/2} f(t)\sin k\omega_0 t\,dt$$

Using the substitution $t = -x$, the first integral becomes

$$\frac{2}{T}\int_{0}^{T/2} f(x)\sin k\omega_0 x\,dx$$

Therefore we have $b_k = \dfrac{4}{T}\displaystyle\int_{0}^{T/2} f(t)\sin k\omega_0 t\,dt$

P 16.8 $b_k = \dfrac{2}{T}\displaystyle\int_{-T/2}^{0} f(t)\sin k\omega_0 t\,dt + \dfrac{2}{T}\displaystyle\int_{0}^{T/2} f(t)\sin k\omega_0 t\,dt$

Now let $t = x - T/2$ in the first integral, then $dt = dx$, $x = 0$ when $t = -T/2$ and $x = T/2$ when $t = 0$, also
$\sin k\omega_0(x - T/2) = \sin(k\omega_0 x - k\pi) = \sin k\omega_0 x\cos k\pi$. Therefore

$$\frac{2}{T}\int_{-T/2}^{0} f(t)\sin k\omega_0 t\,dt = -\frac{2}{T}\int_{0}^{T/2} f(x)\sin k\omega_0 x\cos k\pi\,dx \quad \text{and}$$

$$b_k = \frac{2}{T}(1 - \cos k\pi)\int_{0}^{T/2} f(x)\sin k\omega_0 t\,dt$$

Now note that $1 - \cos k\pi = 0$ when k is even, and $1 - \cos k\pi = 2$ when k is odd. Therefore $b_k = 0$ when k is even, and

$$b_k = \frac{4}{T}\int_{0}^{T/2} f(t)\sin k\omega_0 t\,dt \quad \text{when } k \text{ is odd}$$

P 16.9 Because the function is even and has half-wave symmetry, we have $a_v = 0$, $a_k = 0$ for k even, $b_k = 0$ for all k and

$$a_k = \frac{4}{T}\int_{0}^{T/2} f(t)\cos k\omega_0 t\,dt, \qquad k \text{ odd}$$

The function also has quarter-wave symmetry;
therefore $f(t) = -f(T/2 - t)$ in the interval $T/4 \leq t \leq T/2$;
thus we write

$$a_k = \frac{4}{T}\int_{0}^{T/4} f(t)\cos k\omega_0 t\,dt + \frac{4}{T}\int_{T/4}^{T/2} f(t)\cos k\omega_0 t\,dt$$

Now let $t = (T/2 - x)$ in the second integral, then $dt = -dx$, $x = T/4$ when $t = T/4$ and $x = 0$ when $t = T/2$. Therefore we get

$$\frac{4}{T}\int_{T/4}^{T/2} f(t)\cos k\omega_0 t\,dt = -\frac{4}{T}\int_{0}^{T/4} f(x)\cos k\pi\cos k\omega_0 x\,dx$$

Therefore we have

$$a_k = \frac{4}{T}(1 - \cos k\pi)\int_0^{T/4} f(t)\cos k\omega_0 t\,dt$$

But k is odd, hence

$$a_k = \frac{8}{T}\int_0^{T/4} f(t)\cos k\omega_0 t\,dt, \qquad k \text{ odd}$$

P 16.10 Because the function is odd and has half-wave symmetry, $a_v = 0$, $a_k = 0$ for all k, and $b_k = 0$ for k even. For k odd we have

$$b_k = \frac{4}{T}\int_0^{T/2} f(t)\sin k\omega_0 t\,dt$$

The function also has quarter-wave symmetry, therefore $f(t) = f(T/2 - t)$ in the interval $T/4 \leq t \leq T/2$. Thus we have

$$b_k = \frac{4}{T}\int_0^{T/4} f(t)\sin k\omega_0 t\,dt + \frac{4}{T}\int_{T/4}^{T/2} f(t)\sin k\omega_0 t\,dt$$

Now let $t = (T/2 - x)$ in the second integral and note that $dt = -dx$, $x = T/4$ when $t = T/4$ and $x = 0$ when $t = T/2$, thus

$$\frac{4}{T}\int_{T/4}^{T/2} f(t)\sin k\omega_0 t\,dt = -\frac{4}{T}\cos k\pi\int_0^{T/4} f(x)(\sin k\omega_0 x)\,dx$$

But k is odd, therefore the expression for b_k becomes

$$b_k = \frac{8}{T}\int_0^{T/4} f(t)\sin k\omega_0 t\,dt$$

P 16.11 [**a**] $\omega_o = \dfrac{2\pi}{T} = 2\pi$ rad/s

[**b**] yes

[**c**] no

[**d**] no

P 16.12 [**a**] $f = \dfrac{1}{T} = \dfrac{1}{16 \times 10^{-3}} = 62.5\,\text{Hz}$

[**b**] no

[**c**] yes

[**d**] yes

[**e**] yes

[f] $a_v = 0,$ function is odd

$a_k = 0,$ for all k; the function is odd

$b_k = 0,$ for k even, the function has half-wave symmetry

$$b_k = \frac{8}{T}\int_0^{T/4} f(t)\sin k\omega_o t, \qquad k \text{ odd}$$

$$= \frac{8}{T}\left\{\int_0^{T/8} 5t\sin k\omega_o t\,dt + \int_{T/8}^{T/4} 0.01\sin k\omega_o t\,dt\right\}$$

$$= \frac{8}{T}\{\text{Int1} + \text{Int2}\}$$

$$\text{Int1} = 5\int_0^{T/8} t\sin k\omega_o t\,dt$$

$$= 5\left[\frac{1}{k^2\omega_o^2}\sin k\omega_o t - \frac{t}{k\omega_o}\cos k\omega_o t\,\Big|_0^{T/8}\right]$$

$$= \frac{5}{k^2\omega_o^2}\sin\frac{k\pi}{4} - \frac{0.625T}{k\omega_o}\cos\frac{k\pi}{4}$$

$$\text{Int2} = 0.01\int_{T/8}^{T/4}\sin k\omega_o t\,dt = \frac{-0.01}{k\omega_o}\cos k\omega_o t\,\Big|_{T/8}^{T/4} = \frac{0.01}{k\omega_o}\cos\frac{k\pi}{4}$$

$$\text{Int1} + \text{Int2} = \frac{5}{k^2\omega_o^2}\sin\frac{k\pi}{4} + \left(\frac{0.01}{k\omega_o} - \frac{0.625T}{k\omega_o}\right)\cos\frac{k\pi}{4}$$

$$0.625T = 0.625(16\times 10^{-3}) = 0.01$$

$$\therefore \quad \text{Int1} + \text{Int2} = \frac{5}{k^2\omega_o^2}\sin\frac{k\pi}{4}$$

$$b_k = \left[\frac{8}{T}\cdot\frac{5}{4\pi^2k^2}\cdot T^2\right]\sin\frac{k\pi}{4} = \frac{0.16}{\pi^2k^2}\sin\frac{k\pi}{4}, \qquad k \text{ odd}$$

$$i(t) = \frac{160}{\pi^2}\sum_{n=1,3,5,\ldots}^{\infty}\frac{\sin(n\pi/4)}{n^2}\sin n\omega_o t\,\text{mA}$$

P 16.13 **[a]** $v(t)$ is even and has both half- and quarter-wave symmetry, therefore $a_v = 0$, $b_k = 0$ for all k, $a_k = 0$ for k-even; for odd k we have

$$a_k = \frac{8}{T}\int_0^{T/4} V_m\cos k\omega_0 t\,dt = \frac{4V_m}{\pi k}\sin\left(\frac{k\pi}{2}\right)$$

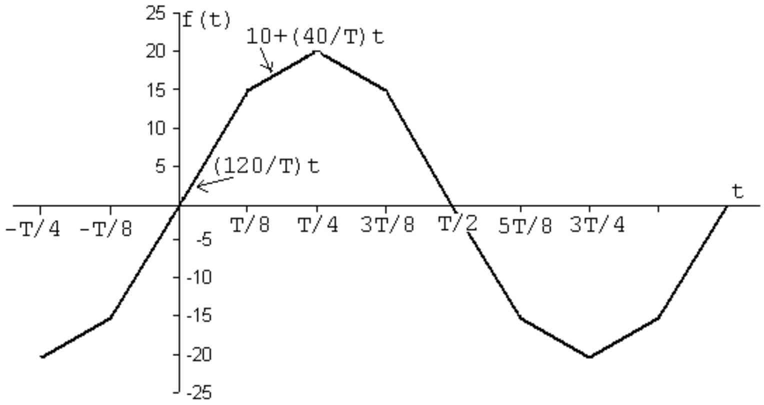
$$v(t) = \frac{4V_m}{\pi}\sum_{n=1,3,5,\ldots}^{\infty}\left[\frac{1}{n}\sin\frac{n\pi}{2}\right]\cos n\omega_0 t\,\text{V}$$

[b] $v(t)$ is even and has both half- and quarter-wave symmetry, therefore $a_v = 0$, $b_k = 0$ for k-even, $a_k = 0$ for all k; for k-odd we have

$$a_k = \frac{8}{T}\int_0^{T/4}\left(\frac{4V_p}{T}t - V_p\right)\cos k\omega_0 t\,dt = \frac{-8V_p}{\pi^2 k^2}$$

$$\text{Therefore}\quad v(t) = \frac{-8V_p}{\pi^2}\sum_{n=1,3,5,\ldots}^{\infty}\frac{1}{n^2}\cos n\omega_0 t\,\mathrm{V}$$

P 16.14 **[a]**



[b] $a_v = 0;\qquad a_k = 0,\quad \text{for all } k;\qquad b_k = 0,\quad \text{for } k \text{ even}$

$$b_k = \frac{8}{T}\int_0^{T/4} f(t)\sin k\omega_0 t\,dt,\quad \text{for } k \text{ odd}$$

$$= \frac{8}{T}\int_0^{T/4}\frac{120t}{T}\sin k\omega_0 t\,dt + \frac{8}{T}\int_{T/4}^{T/2}\left(10 + \frac{40}{T}t\right)\sin k\omega_0 t\,dt$$

$$= \frac{960}{T^2}\int_0^{T/8} t\sin k\omega_0 t\,dt + \frac{80}{T}\int_{T/8}^{T/4}\sin k\omega_0 t\,dt + \frac{320}{T^2}\int_{T/8}^{T/4} t\sin k\omega_0 t\,dt$$

$$= \frac{960}{T^2}\left[\frac{\sin k\omega_0 t}{k^2\omega_0^2} - \frac{t\cos k\omega_0 t}{k\omega_0}\right]_0^{T/8} - \frac{80}{T}\frac{\cos k\omega_0 t}{k\omega_0}\Big|_{T/8}^{T/4}$$

$$+\frac{320}{T^2}\left[\frac{\sin k\omega_0 t}{k^2\omega_0^2} - \frac{t\cos k\omega_0 t}{k\omega_0}\right]_{T/8}^{T/4}$$

$$k\omega_0\frac{T}{4} = \frac{k\pi}{2};\qquad k\omega_0\frac{T}{8} = \frac{k\pi}{4}$$

$$b_k = \frac{960}{T^2}\left[\frac{\sin(k\pi/4)}{k^2\omega_0^2} - \frac{T}{8k\omega_0}\cos(k\pi/4)\right] - \frac{80}{k\omega_0 T}\left[\cos(k\pi/2) - \cos(k\pi/4)\right]$$

$$+\frac{320}{T^2}\left[\frac{\sin(k\pi/2)}{k^2\omega_0^2} - \frac{T}{4}\frac{\cos(k\pi/2)}{k\omega_0} - \frac{\sin(k\pi/4)}{k^2\omega_0^2} + \frac{T\cos(k\pi/4)}{8k\omega_0}\right]$$

$$= \frac{960}{(k\omega_0 T)^2}\sin(k\pi/4) + \frac{320}{(k\omega_0 T)^2}\sin(k\pi/2) - \frac{320}{(k\omega_o T)^2}\sin(k\pi/4)$$

$$= \frac{640}{(k\omega_0 T)^2}\sin(k\pi/4) + \frac{320}{(k\omega_0 T)^2}\sin(k\pi/2)$$

$$k\omega_0 T = 2k\pi; \qquad (k\omega_0 T)^2 = 4k^2\pi^2$$

$$b_k = \frac{160}{\pi^2 k^2}\sin(k\pi/4) + \frac{80}{\pi^2 k^2}\sin(k\pi/2)$$

[c] $b_k = \frac{80}{\pi^2 k^2}[2\sin(k\pi/4) + \sin(k\pi/2)]$

$$b_1 = \frac{80}{\pi^2}[2\sin(\pi/4) + \sin(\pi/2)] \cong 19.57$$

$$b_3 = \frac{80}{9\pi^2}[2\sin(3\pi/4) + \sin(3\pi/2)] \cong 0.37$$

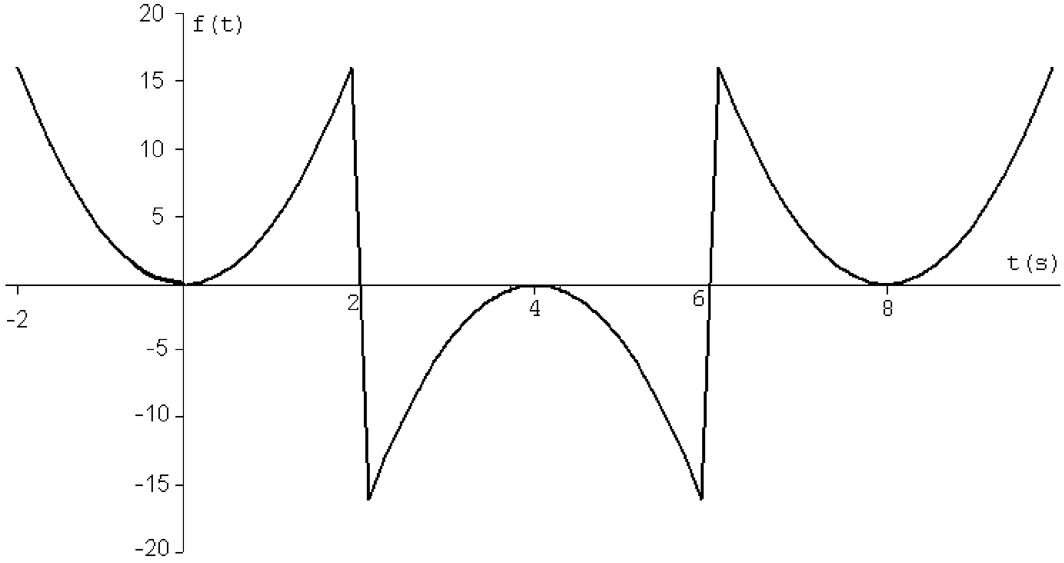
$$b_5 = \frac{80}{25\pi^2}[2\sin(5\pi/4) + \sin(5\pi/2)] \cong -0.13$$

$$f(t) = 19.57\sin(\omega_0 t) + 0.37\sin(3\omega_0 t) - 0.13\sin(5\omega_0 t) + \ldots$$

[d] $t = \frac{T}{4}; \qquad \omega_0 t = \frac{2\pi}{T}\cdot\frac{T}{4} = \frac{\pi}{2}$

$$f(T/4) \cong 19.57\sin(\pi/2) + 0.37\sin(3\pi/2) - 0.13\sin(5\pi/2) \cong 19.81$$

P 16.15 **[a]**



[b] Even, since $f(t) = f(-t)$

[c] Yes, since $f(t) = -f(T/2 - t)$ in the interval $0 < t < 4$.

[d] $a_v = 0, \quad a_k = 0, \quad$ for k even (half-wave symmetry)

$b_k = 0, \quad$ for all k (function is even)

Because of the quarter-wave symmetry, the expression for a_k is

$$a_k = \frac{8}{T}\int_0^{T/4} f(t)\cos k\omega_0 t\,dt, \qquad k \text{ odd}$$

$$= \frac{8}{8}\int_0^2 4t^2\cos k\omega_0 t\,dt = 4\left[\frac{2t}{k^2\omega_0^2}\cos k\omega_0 t + \frac{k\omega_0^2 t^2 - 2}{k^3\omega_0^3}\sin k\omega_0 t\right]_0^2$$

$$k\omega_0(2) = k\left(\frac{2\pi}{8}\right)(2) = \frac{k\pi}{2}$$

$\cos(k\pi/2) = 0, \quad$ since k is odd

$$\therefore \qquad a_k = 4\left[0 + \frac{4k^2\omega_0^2 - 2}{k^3\omega_0^3}\sin(k\pi/2)\right] = \frac{16k^2\omega_0^2 - 8}{k^3\omega_0^3}\sin(k\pi/2)$$

$$\omega_0 = \frac{2\pi}{8} = \frac{\pi}{4}; \qquad \omega_0^2 = \frac{\pi^2}{16}; \qquad \omega_0^3 = \frac{\pi^3}{64}$$

$$a_k = \left(\frac{k^2\pi^2 - 8}{k^3\pi^3}\right)(64)\sin(k\pi/2)$$

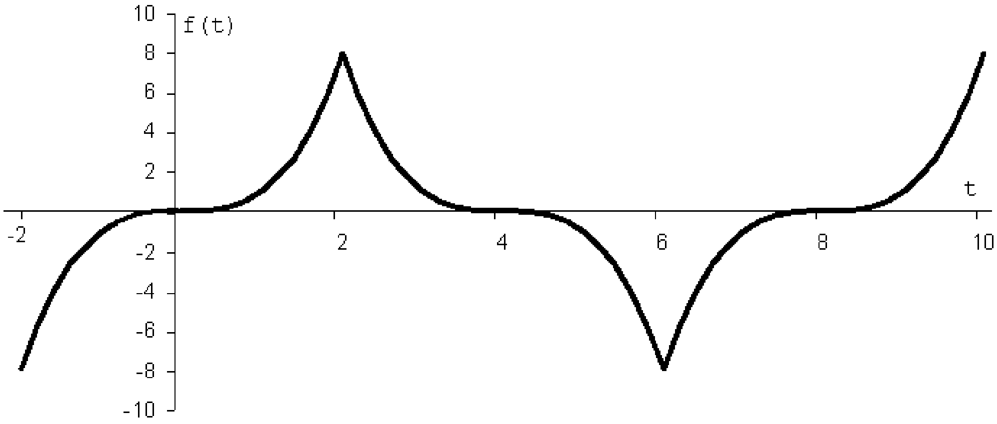
$$f(t) = 64\sum_{n=1,3,5,\ldots}^{\infty}\left[\frac{n^2\pi^2 - 8}{\pi^3 n^3}\right]\sin(n\pi/2)\cos(n\omega_0 t)$$

[e] $\cos n\omega_0(t-2) = \cos(n\omega_0 t - n\pi/2) = \sin(n\pi/2)\sin n\omega_0 t$

$$f(t) = 64\sum_{n=1,3,5,\ldots}^{\infty}\left[\frac{n^2\pi^2 - 8}{\pi^3 n^3}\right]\sin^2(n\pi/2)\sin(n\omega_0 t)$$

$$= 64\sum_{n=1,3,5,\ldots}^{\infty}\left[\frac{n^2\pi^2 - 8}{\pi^3 n^3}\right]\sin(n\omega_0 t)$$

P 16.16 **[a]**



[b] Odd, since $f(-t) = -f(t)$

[c] $f(t)$ has quarter-wave symmetry, since $f(T/2 - t) = f(t)$ in the interval $0 < t < 4$.

[d] $a_v = 0$, (half-wave symmetry); $a_k = 0$, for all k (function is odd)

$$b_k = 0, \quad \text{for } k \text{ even} \quad \text{(half-wave symmetry)}$$

$$b_k = \frac{8}{T}\int_0^{T/4} f(t)\sin k\omega_0 t\,dt, \quad k \text{ odd}$$

$$= \frac{8}{8}\int_0^2 t^3 \sin k\omega_0 t\,dt$$

$$= \left[\frac{3t^2}{k^2\omega_0^2}\sin k\omega_0 t - \frac{6}{k^4\omega_0^4}\sin k\omega_0 t - \frac{t^3}{k\omega_0}\cos k\omega_0 t + \frac{6t}{k^3\omega_0^3}\cos k\omega_0 t\right]_0^2$$

$$k\omega_0(2) = k\left(\frac{2\pi}{8}\right)(2) = \frac{k\pi}{2}$$

$$\cos(k\pi/2) = 0, \quad \text{since } k \text{ is odd}$$

$$\therefore \quad b_k = \left[\frac{12}{k^2\omega_0^2}\sin(k\pi/2) - \frac{6}{k^4\omega_0^4}\sin(k\pi/2)\right]$$

$$k\omega_0 = k\left(\frac{2\pi}{8}\right) = \frac{k\pi}{4}; \qquad k^2\omega_0^2 = \frac{k^2\pi^2}{16}; \qquad k^4\omega_0^4 = \frac{k^4\pi^4}{256}$$

$$\therefore \quad b_k = \frac{192}{\pi^2 k^2}\left[1 - \frac{8}{\pi^2 k^2}\right]\sin(k\pi/2), \quad k \text{ odd}$$

$$f(t) = \frac{192}{\pi^2}\sum_{n=1,3,5,\ldots}^{\infty}\left[\frac{1}{n^2}\left(1 - \frac{8}{\pi^2 n^2}\right)\sin(n\pi/2)\right]\sin n\omega_0 t$$

[e] $\sin n\omega_0(t-2) = \sin(n\omega_0 t - n\pi/2) = -\cos n\omega_0 t \sin(n\pi/2)$

$$f(t) = \frac{-192}{\pi^2}\sum_{n=1,3,5,\ldots}^{\infty}\left[\frac{1}{n^2}\left(1 - \frac{8}{\pi^2 n^2}\right)\right]\cos n\omega_0 t$$

P 16.17 **[a]** $i(t)$ is odd, therefore $a_v = 0$ and $b_k = 0$ for all k.

$$f(t) = i(t) = I_m - \frac{2I_m}{T}t, \quad 0 \le t \le T$$

$$b_k = \frac{4}{T}\int_0^{T/2} f(t)\sin k\omega_o t\,dt$$

$$= \frac{4}{T}\int_0^{T/2}\left(I_m - \frac{2I_m}{T}t\right)\sin k\omega_o t\,dt$$

$$= \frac{4I_m}{T}\left[\int_0^{T/2}\sin k\omega_0 t\,dt - \frac{2}{T}\int_0^{T/2} t\sin k\omega_0 t\,dt\right]$$

$$= \frac{4I_m}{T}\left[\frac{-\cos k\omega_0 t}{k\omega_0}\Big|_0^{T/2} - \frac{2}{T}\left(\frac{\sin k\omega_0 t}{k^2\omega_0^2} - \frac{t\cos k\omega_0 t}{k\omega_0}\right)\Big|_0^{T/2}\right]$$

$$= \frac{4I_m}{T}\left[\frac{1-\cos k\pi}{k\omega_0} + \frac{\cos k\pi}{k\omega_0}\right]$$

$$= \frac{4I_m}{k\omega_0 T} = \frac{2I_m}{k\pi}$$

$$\therefore \quad i(t) = \frac{2I_m}{\pi}\sum_{n=1}^{\infty}\frac{1}{n}\sin n\omega_o t$$

[b] $i(t) = \frac{2I_m}{\pi}\sum_{n=1}^{\infty}\frac{1}{n}\sin n\omega_o(t + T/2)$

$$= \frac{2I_m}{\pi}\sum_{n=1}^{\infty}\frac{\cos n\pi}{n}\sin n\omega_o t$$

P 16.18 $v_2(t + T/8)$ is even, so $b_k = 0$ for all k.

$$a_v = \frac{(V_m/2)(T/4)}{T} = \frac{V_m}{8}$$

$$a_k = \frac{4}{T}\int_0^{T/8}\frac{V_m}{2}\cos k\omega_0 t\,dt = \frac{V_m}{k\pi}\sin\frac{k\pi}{4}$$

Therefore, $\quad v_2(t + T/8) = \frac{V_m}{8} + \frac{V_m}{\pi}\sum_{n=1}^{\infty}\frac{1}{n}\sin\frac{n\pi}{4}\cos n\omega_0 t$

so $\quad v_2(t) = \frac{V_m}{8} + \frac{V_m}{\pi}\sum_{n=1}^{\infty}\frac{1}{n}\sin\frac{n\pi}{4}\cos n\omega_0(t - T/8)$

$$\therefore \qquad v(t) = \frac{V_m}{2} + \frac{V_m}{8} + \frac{V_m}{\pi}\sum_{n=1}^{\infty}\left(\frac{1}{n}\sin\frac{n\pi}{4}\cos\frac{n\pi}{4}\right)\cos n\omega_0 t + \left(\frac{1}{n}\sin^2\frac{n\pi}{4}\right)\sin n\omega_0 t$$

$$= \frac{5V_m}{8} + \frac{V_m}{2\pi}\sum_{n=1}^{\infty}\left(\frac{1}{n}\sin\frac{n\pi}{2}\right)\cos n\omega_0 t + \left(1 - \cos\frac{n\pi}{2}\right)\sin n\omega_0 t\ \mathrm{V}$$

Thus, since $a_v = 5V_m/8 = 37.5\pi\,\mathrm{V}$,

$$a_k = \frac{V_m}{2\pi k}\sin\frac{k\pi}{2} = \frac{30}{k}\sin\frac{k\pi}{2}$$

and

$$b_k = \frac{V_m}{2\pi k}\left[1 - \cos\frac{k\pi}{2}\right] = \frac{30}{k}\left[1 - \cos\frac{k\pi}{2}\right]$$

These equations match the equations for a_v, a_k, and b_k derived in Problem 16.4.

P 16.19 The periodic function in Fig. P16.3(a) has half-wave symmetry so $a_v = 0$ and

$$a_n = -\frac{80}{\pi n}\sin\frac{\pi n}{2} \qquad \text{and} \qquad b_n = \frac{240}{\pi n} \quad \text{for } n \text{ odd.}$$

$$A_n\underline{/-\theta_n} = a_n - jb_n = -\frac{80}{\pi n}\sin\frac{\pi n}{2} - j\frac{240}{\pi n}, \qquad n \text{ odd}$$

Therefore,

$$A_n = \frac{\sqrt{80^2 + 240^2}}{n\pi} = \frac{252.98}{n\pi}, \qquad n \text{ odd}$$

and

$$\theta_n = \tan^{-1}(-240/-80) = -108.43^\circ, \quad n = 1, 5, 9, ...$$

and

$$\theta_n = \tan^{-1}(-240/80) = -71.565^\circ, \quad n = 3, 7, 11, ...$$

Thus, $v(t) = \dfrac{252.98}{\pi}\displaystyle\sum_{n=1,5,9,...}^{\infty}\frac{1}{n}\cos(n\omega_o t - 108.43^\circ) + \frac{252.98}{\pi}\sum_{n=3,7,11,...}^{\infty}\frac{1}{n}\cos(n\omega_o t - 71.565^\circ)\,\text{V}$

The periodic function in Fig. P16.3(b) is even, so $b_k = 0$ for all k. Thus,

$$A_n\underline{/-\theta_n} = a_n - jb_n = a_n = a_n\underline{/0^\circ}$$

From Problem 16.3(b),

$$a_v = 25\,\text{V} = A_0$$

$$a_n = \frac{200}{n\pi}\sin\frac{n\pi}{4}$$

Therefore,

$$A_n = \frac{200}{n\pi}\sin\frac{n\pi}{4}$$

and

$$-\theta_n = 0^\circ$$

Thus, $v(t) = 25 + \dfrac{200}{\pi}\displaystyle\sum_{n=1}^{\infty}\frac{1}{n}\sin\frac{n\pi}{4}\cos n\omega_o t\,\text{V}$

P 16.20 The periodic function in Problem 16.12 is odd, so $a_v = 0$ and $a_k = 0$ for all k. Thus,

$$A_n \underline{/-\theta_n} = a_n - jb_n = 0 - jb_n = b_n \underline{/-90^\circ}$$

From Problem 16.12,

$$b_k = \frac{0.16}{\pi^2 k^2}\sin\frac{k\pi}{4}, \qquad k \text{ odd}$$

Therefore,

$$A_n = \frac{0.16}{\pi^2 k^2}\sin\frac{k\pi}{4}, \qquad k \text{ odd}$$

and

$$-\theta_n = -90^\circ, \qquad n \text{ odd}$$

Thus, $i(t) = \dfrac{0.16}{\pi^2}\displaystyle\sum_{n=1,3,5,\ldots}^{\infty}\frac{\sin(n\pi/4)}{n^2}\cos(n\omega_o t - 90^\circ)\,\text{A}$

P 16.21 The periodic function in Problem 16.15 is even, so $b_k = 0$ for all k. Thus,

$$A_n \underline{/-\theta_n} = a_n - jb_n = a_n = a_n \underline{/0^\circ}$$

From Problem 16.15,

$$a_v = 0 = A_0$$

$$a_n = \frac{64}{\pi^3 n^3}(n^2\pi^2 - 8)\sin\frac{n\pi}{2}$$

Therefore,

$$A_n = \frac{64}{\pi^3 n^3}(n^2\pi^2 - 8)\sin\frac{n\pi}{2}$$

and

$$-\theta_n = 0^\circ$$

Thus, $f(t) = \dfrac{64}{\pi^3}\displaystyle\sum_{n=1,3,5,\ldots}^{\infty}\left(\frac{n^2\pi^2 - 8}{n^3}\right)\sin\frac{n\pi}{2}\cos n\omega_o t$

P 16.22 [**a**] The current has half-wave symmetry. Therefore,

$$a_v = 0; \qquad a_k = b_k = 0, \quad k \text{ even}$$

For k odd,

$$a_k = \frac{4}{T}\int_0^{T/2}\left(I_m - \frac{2I_m}{T}t\right)\cos k\omega_o t\,dt$$

$$= \frac{4}{T}\int_0^{T/2} I_m\cos k\omega_0 t\,dt - \frac{8I_m}{T^2}\int_0^{T/2} t\cos k\omega_0 t\,dt$$

$$= \frac{4I_m}{T}\frac{\sin k\omega_0 t}{k\omega_0}\Big|_0^{T/2} - \frac{8I_m}{T^2}\left[\frac{\cos k\omega_o t}{k^2\omega_0^2} + \frac{t}{k\omega_0}\sin k\omega_0 T\right]_0^{T/2}$$

$$= 0 - \frac{8I_m}{T^2}\left[\frac{\cos k\pi}{k^2\omega_0^2} - \frac{1}{k^2\omega_0^2}\right]$$

$$= \left(\frac{8I_m}{T^2}\right)\left(\frac{1}{k^2\omega_0^2}\right)(1 - \cos k\pi)$$

$$= \frac{4I_m}{\pi^2 k^2} = \frac{20}{k^2}, \quad \text{for } k \text{ odd}$$

$$b_k = \frac{4}{T}\int_0^{T/2}\left(I_m - \frac{2I_m}{T}t\right)\sin k\omega_o t\,dt$$

$$= \frac{4I_m}{T}\int_0^{T/2}\sin k\omega_0 t\,dt - \frac{8I_m}{T^2}\int_0^{T/2} t\sin k\omega_0 t\,dt$$

$$= \frac{4I_m}{T}\left[\frac{-\cos k\omega_0 t}{k\omega_0}\right]_0^{T/2} - \frac{8I_m}{T^2}\left[\frac{\sin k\omega_o t}{k^2\omega_0^2} - \frac{t}{k\omega_0}\cos k\omega_0 T\right]_0^{T/2}$$

$$= \frac{4I_m}{T}\left[\frac{1 - \cos k\pi}{k\omega_0}\right] - \frac{8I_m}{T^2}\left[\frac{-T\cos k\pi}{2k\omega_0}\right]$$

$$= \frac{8I_m}{k\omega_0 T}\left[1 + \frac{1}{2}\cos k\pi\right]$$

$$= \frac{2I_m}{\pi k} = \frac{10\pi}{k}, \quad \text{for } k \text{ odd}$$

$$a_k - jb_k = \frac{20}{k^2} - j\frac{10\pi}{k} = \frac{10}{k}\left(\frac{2}{k} - j\pi\right) = \frac{10}{k^2}\sqrt{\pi^2k^2+4}\underline{/-\theta_k}$$

$$\text{where} \qquad \tan\theta_k = \frac{\pi k}{2}$$

$$i(t) = 10\sum_{n=1,3,5,\ldots}^{\infty}\frac{\sqrt{(n\pi)^2+4}}{n^2}\cos(n\omega_0 t - \theta_n), \qquad \theta_n = \tan^{-1}\frac{n\pi}{2}$$

[b] $A_1 = 10\sqrt{4+\pi^2} \cong 37.24\,\text{A} \qquad \tan\theta_1 = \frac{\pi}{2} \qquad \theta_1 \cong 57.52^\circ$

$$A_3 = \frac{10}{9}\sqrt{4+9\pi^2} \cong 10.71\,\text{A} \qquad \tan\theta_3 = \frac{3\pi}{2} \qquad \theta_3 \cong 78.02^\circ$$

$$A_5 = \frac{10}{25}\sqrt{4+25\pi^2} \cong 6.33\,\text{A} \qquad \tan\theta_5 = \frac{5\pi}{2} \qquad \theta_5 \cong 82.74^\circ$$

$$A_7 = \frac{10}{49}\sqrt{4+49\pi^2} \cong 4.51\,\text{A} \qquad \tan\theta_7 = \frac{7\pi}{2} \qquad \theta_7 \cong 84.80^\circ$$

$$A_9 = \frac{10}{81}\sqrt{4+81\pi^2} \cong 3.50\,\text{A} \qquad \tan\theta_9 = \frac{9\pi}{2} \qquad \theta_9 \cong 85.95^\circ$$

$$i(t) \cong 37.24\cos(\omega_o t - 57.52^\circ) + 10.71\cos(3\omega_o t - 78.02^\circ)$$

$$+6.33\cos(5\omega_o t - 82.74^\circ) + 4.51\cos(7\omega_o t - 84.80^\circ)$$

$$+3.50\cos(9\omega_o t - 85.95^\circ) + \ldots$$

$$i(T/4) \cong 37.24\cos(90 - 57.52^\circ) + 10.71\cos(270 - 78.02^\circ)$$

$$+6.33\cos(450 - 82.74^\circ) + 4.51\cos(630 - 84.80^\circ)$$

$$+3.50\cos(810 - 85.95^\circ) \cong 26.23\,\text{A}$$

Actual value:

$$i\left(\frac{T}{4}\right) = \frac{1}{2}(5\pi^2) \cong 24.67\,\text{A}$$

P 16.23 The function has half-wave symmetry, thus $a_k = b_k = 0$ for k-even, $a_v = 0$; for k-odd

$$a_k = \frac{4}{T}\int_0^{T/2} V_m \cos k\omega_0 t\,dt - \frac{8V_m}{\rho T}\int_0^{T/2} e^{-t/RC}\cos k\omega_0 t\,dt$$

where $\rho = \left[1 + e^{-T/2RC}\right].$

Upon integrating we get

$$a_k = \frac{4V_m}{T}\frac{\sin k\omega_0 t}{k\omega_0}\Big|_0^{T/2}$$

$$-\frac{8V_m}{\rho T}\cdot\left\{\frac{e^{-t/RC}}{(1/RC)^2 + (k\omega_0)^2}\cdot\left[\frac{-\cos k\omega_0 t}{RC} + k\omega_0 \sin k\omega_0 t\right]\Big|_0^{T/2}\right\}$$

$$= \frac{-8V_m RC}{T[1 + (k\omega_0 RC)^2]}$$

$$
\begin{aligned}
b_k &= \frac{4}{T}\int_0^{T/2} V_m \sin k\omega_0 t\,dt - \frac{8V_m}{\rho T}\int_0^{T/2} e^{-t/RC}\sin k\omega_0 t\,dt \\
&= -\frac{4V_m}{T}\frac{\cos k\omega_0 t}{k\omega_0}\Bigg|_0^{T/2} \\
&\quad -\frac{8V_m}{\rho T}\cdot\left\{\frac{-e^{-t/RC}}{(1/RC)^2+(k\omega_0)^2}\cdot\left[\frac{\sin k\omega_0 t}{RC}+k\omega_0\cos k\omega_0 t\right]\Bigg|_0^{T/2}\right\} \\
&= \frac{4V_m}{\pi k} - \frac{8k\omega_0 V_m R^2C^2}{T[1+(k\omega_0 RC)^2]}
\end{aligned}
$$

P 16.24 **[a]** $a_k^2 + b_k^2 = a_k^2 + \left(\frac{4V_m}{\pi k} + k\omega_0 RCa_k\right)^2$

$$= a_k^2\left[1+(k\omega_0 RC)^2\right] + \tfrac{8V_m}{\pi k}\left[\tfrac{2V_m}{\pi k} + k\omega_0 RCa_k\right]$$

But $a_k = \left\{\frac{-8V_m RC}{T\left[1+(k\omega_0 RC)^2\right]}\right\}$

Therefore $a_k^2 = \left\{\frac{64V_m^2R^2C^2}{T^2[1+(k\omega_0 RC)^2]^2}\right\}$, thus we have

$$a_k^2 + b_k^2 = \frac{64V_m^2R^2C^2}{T^2[1+(k\omega_0 RC)^2]} + \frac{16V_m^2}{\pi^2k^2} - \frac{64V_m^2k\omega_0R^2C^2}{\pi kT[1+(k\omega_0 RC)^2]}$$

Now let $\alpha = k\omega_0 RC$ and note that $T = 2\pi/\omega_0$, thus the expression for $a_k^2+b_k^2$ reduces to $a_k^2+b_k^2 = 16V_m^2/\pi^2k^2(1+\alpha^2)$. It follows that

$$\sqrt{a_k^2+b_k^2} = \frac{4V_m}{\pi k\sqrt{1+(k\omega_0 RC)^2}}$$

[b] $b_k = k\omega_0 RCa_k + \frac{4V_m}{\pi k}$

Thus $\frac{b_k}{a_k} = k\omega_0 RC + \frac{4V_m}{\pi ka_k} = \alpha - \frac{1+\alpha^2}{\alpha} = -\frac{1}{\alpha}$

Therefore $\frac{a_k}{b_k} = -\alpha = -k\omega_0 RC$

P 16.25 Since $a_v = 0$ (half-wave symmetry), Eq. 16.38 gives us

$$v_o(t) = \sum_{1,3,5,\ldots}^{\infty} \frac{4V_m}{n\pi}\frac{1}{\sqrt{1+(n\omega_0 RC)^2}}\cos(n\omega_0 t - \theta_n) \quad \text{where} \quad \tan\theta_n = \frac{b_n}{a_n}$$

But from Eq. 16.57, we have $\tan\beta_k = k\omega_0 RC$. It follows from Eq. 16.72 that $\tan\beta_k = -a_k/b_k$ or $\tan\theta_n = -\cot\beta_n$. Therefore $\theta_n = 90° + \beta_n$ and

$\cos(n\omega_0 t - \theta_n) = \cos(n\omega_0 t - \beta_n - 90^\circ) = \sin(n\omega_0 t - \beta_n)$, thus our expression for v_o becomes

$$v_o = \frac{4V_m}{\pi} \sum_{n=1,3,5,\ldots}^{\infty} \frac{\sin(n\omega_0 t - \beta_n)}{n\sqrt{1 + (n\omega_0 RC)^2}}$$

P 16.26 **[a]** $e^{-x} \cong 1 - x$ for small x; therefore

$$e^{-t/RC} \cong \left(1 - \frac{t}{RC}\right) \quad \text{and} \quad e^{-T/2RC} \cong \left(1 - \frac{T}{2RC}\right)$$

$$v_o \cong V_m - \frac{2V_m[1 - (t/RC)]}{2 - (T/2RC)} = \left(\frac{V_m}{RC}\right)\left[\frac{2t - (T/2)}{2 - (T/2RC)}\right]$$

$$\cong \left(\frac{V_m}{RC}\right)\left(t - \frac{T}{4}\right) = \left(\frac{V_m}{RC}\right)t - \frac{V_m T}{4RC} \qquad \text{for} \quad 0 \le t \le \frac{T}{2}$$

[b] $a_k = \left(\frac{-8}{\pi^2 k^2}\right) V_p = \left(\frac{-8}{\pi^2 k^2}\right)\left(\frac{V_m T}{4RC}\right) = \frac{-4V_m}{\pi\omega_0 RCk^2}$

P 16.27 **[a]** From the solution to Problem 16.13(a) the Fourier series for the input voltage is

$$v_g = \frac{4V_m}{\pi} \sum_{n=1,3,5,\ldots}^{\infty} \left[\frac{1}{n}\sin\frac{n\pi}{2}\right]\cos n\omega_0 t \text{ V}$$

Since $V_m = 10.5\pi$ V and $t = \pi$ ms, we can write the input voltage as

$$v_g = 42 \sum_{n=1,3,5,\ldots}^{\infty} \left[\frac{1}{n}\sin\left(\frac{n\pi}{2}\right)\right]\cos 2000nt \text{ V}$$

$$= 42\cos 2000t - \frac{42}{3}\cos 6000t + \frac{42}{5}\cos 10{,}000t - \frac{42}{7}\cos 14{,}000t + \cdots$$

We can phasor transform this Fourier series to get

$\mathbf{V}_{g1} = 42\underline{/0^\circ}$ $\quad \omega_0 = 2000$ rad/s

$\mathbf{V}_{g3} = 14\underline{/180^\circ}$ $\quad 3\omega_0 = 6000$ rad/s

$\mathbf{V}_{g5} = 8.4\underline{/0^\circ}$ $\quad 5\omega_0 = 10{,}000$ rad/s

$\mathbf{V}_{g7} = 6\underline{/180^\circ}$ $\quad 7\omega_0 = 14{,}000$ rad/s

From the circuit in Fig. P16.27 we have

$$\frac{V_o}{R} + \frac{V_o - V_g}{sL} + (V_o - V_g)sC = 0$$

$$\therefore \quad \frac{V_o}{V_g} = H(s) = \frac{s^2 + 1/LC}{s^2 + (s/RC) + (1/LC)}$$

Substituting in the numerical values gives

$$H(s) = \frac{s^2 + 10^8}{s^2 + 500s + 10^8}$$

$$H(j2000) = \frac{96}{96 + j1} = 0.9999\underline{/-0.60^\circ}$$

$$H(j6000) = \frac{64}{64 + j3} = 0.9989\underline{/-2.68^\circ}$$

$$H(j10{,}000) = 0$$

$$H(j14{,}000) = \frac{96}{96 + j7} = 0.9974\underline{/4.17^\circ}$$

$$\mathbf{V}_{o1} = (42\underline{/0^\circ})(0.9999\underline{/-0.60^\circ}) = 41.998\underline{/-0.60^\circ}\,\text{V}$$

$$\mathbf{V}_{o3} = (14\underline{/180^\circ})(0.9989\underline{/-2.68^\circ}) = 13.985\underline{/177.32^\circ}\,\text{V}$$

$$\mathbf{V}_{o5} = 0\,\text{V}$$

$$\mathbf{V}_{o7} = (6\underline{/180^\circ})(0.9974\underline{/4.17^\circ}) = 5.984\underline{/-175.83^\circ}\,\text{V}$$

$$v_o = 41.998\cos(2000t - 0.60^\circ) + 13.985\cos(6000t + 177.32^\circ)$$

$$+5.984\cos(14{,}000t - 175.83^\circ) + \ldots\ \text{V}$$

[b] The 5th harmonic at the frequency $\sqrt{1/LC} = 10{,}000$ rad/s has been eliminated from the output voltage by the circuit, which is a band reject filter with a center frequency of 10,000 rad/s.

P 16.28 $$v_i = \frac{4A}{\pi}\sum_{n=1,3,5,\ldots}^{\infty} \frac{1}{n}\sin n\omega_0(t + T/4)$$

$$= \frac{4A}{\pi}\sum_{n=1,3,5,\ldots}^{\infty}\left(\frac{1}{n}\sin\frac{n\pi}{2}\right)\cos n\omega_0 t$$

$$\omega_0 = \frac{2\pi}{4\pi}\times 10^3 = 500\ \text{rad/s}; \qquad \frac{4A}{\pi} = 60$$

$$v_i = 60\sum_{n=1,3,5,\ldots}^{\infty}\left(\frac{1}{n}\sin\frac{n\pi}{2}\right)\cos 500nt\,\text{V}$$

From the circuit

$$\mathbf{V}_o = \frac{\mathbf{V}_i}{R + j\omega L}\cdot j\omega L = \frac{j\omega}{R/L + j\omega}\mathbf{V}_i = \frac{j\omega}{1000 + j\omega}\mathbf{V}_i$$

$$\mathbf{V}_{i1} = 60\underline{/0^\circ}\,\text{V}; \qquad \omega = 500\ \text{rad/s}$$

$$\mathbf{V}_{i3} = -20\underline{/0^\circ} = 20\underline{/180^\circ}\,\text{V}; \qquad 3\omega = 1500\ \text{rad/s}$$

$$\mathbf{V}_{i5} = 12\underline{/0^\circ}\,\text{V}; \qquad 5\omega = 2500\ \text{rad/s}$$

$$\mathbf{V}_{o1} = \frac{j500}{1000 + j500}(60\underline{/0^\circ}) = 26.83\underline{/63.43^\circ}\,\text{V}$$

$$\mathbf{V}_{o3} = \frac{j1500}{1000 + j1500}(20\underline{/180^\circ}) = 16.64\underline{/-146.31^\circ}\,\text{V}$$

$$\mathbf{V}_{o5} = \frac{j2500}{1000 + j2500}(12\underline{/0^\circ}) = 11.14\underline{/21.80^\circ}\,\text{V}$$

$$\therefore \qquad v_o = 26.83\cos(500t + 63.43^\circ) + 16.64\cos(1500t - 146.31^\circ)$$

$$+11.14\cos(2500t + 21.80^\circ) + \ldots\ \text{V}$$

P 16.29 [a] $\dfrac{V_0 - V_g}{16s} + V_0(12.5 \times 10^{-6}s) + \dfrac{V_0}{1000} = 0$

$$V_0\left[\frac{1}{16s} + 12.6 \times 10^{-6}s + \frac{1}{1000}\right] = \frac{V_g}{16s}$$

$$V_0(1000 + 0.2s^2 + 16s) = 1000V_g$$

$$V_0 = \frac{5000V_g}{s^2 + 80s + 5000}$$

$$I_0 = \frac{V_0}{1000} = \frac{5V_g}{s^2 + 80s + 5000}$$

$$H(s) = \frac{I_0}{V_g} = \frac{5}{s^2 + 80s + 5000}$$

$$H(nj\omega_0) = \frac{5}{(5000 - n^2\omega_0^2) + j80n\omega_0}$$

$$\omega_0 = \frac{2\pi}{T} = 240\pi; \qquad \omega_0^2 = 57{,}600\pi^2; \qquad 80\omega_0 = 19{,}200\pi$$

$$H(jn\omega_0) = \frac{5}{(5000 - 57{,}600\pi^2 n^2) + j19{,}200\pi n}$$

$$H(0) = 10^{-3}$$

$$H(j\omega_0) = 8.82 \times 10^{-6}\underline{/-173.89^\circ}$$

$$H(j2\omega_0) = 2.20 \times 10^{-6} \underline{/-176.96^\circ}$$

$$H(j3\omega_0) = 9.78 \times 10^{-7} \underline{/-177.97^\circ}$$

$$H(j4\omega_0) = 5.5 \times 10^{-7} \underline{/-178.48^\circ}$$

$$v_g = \frac{680}{\pi} - \frac{1360}{\pi}\left[\frac{1}{3}\cos\omega_0 t + \frac{1}{15}\cos 2\omega_0 t + \frac{1}{35}\cos 3\omega_0 t + \frac{1}{63}\cos 4\omega_0 t + \ldots\right]$$

$$i_0 = \frac{680}{\pi} \times 10^{-3} - \frac{1360}{3\pi}(8.82 \times 10^{-6})\cos(\omega_0 t - 173.89^\circ)$$

$$-\frac{1360}{15\pi}(2.20 \times 10^{-6})\cos(2\omega_0 t - 176.96^\circ)$$

$$-\frac{1360}{35\pi}(9.78 \times 10^{-7})\cos(3\omega_0 t - 177.97^\circ)$$

$$-\frac{1360}{63\pi}(5.5 \times 10^{-7})\cos(4\omega_0 t - 178.48^\circ) - \ldots$$

$$= 216.45 \times 10^{-3} + 1.27 \times 10^{-3}\cos(240\pi t + 6.11^\circ)$$

$$+6.35 \times 10^{-5}\cos(480\pi t + 3.04^\circ)$$

$$+1.21 \times 10^{-5}\cos(720\pi t + 2.03^\circ)$$

$$+3.8 \times 10^{-6}\cos(960\pi t + 1.11^\circ) - \ldots$$

$$i_0 \cong 216.45 + 1.27\cos(240\pi t + 6.11^\circ)\,\mathrm{mA}$$

Note that the sinusoidal component is very small compared to the dc component, so

$$i_0 \cong 216.45\,\mathrm{mA} \quad \text{(a dc current)}$$

[b] The circuit is a low pass filter, so the harmonic terms are greatly reduced in the output.

P 16.30 **[a]** Express v_g as a constant plus a symmetrical square wave. The constant is $V_m/2$ and the square wave has an amplitude of $V_m/2$, is odd, and has half- and quarter-wave symmetry. Therefore the Fourier series for v_g is

$$v_g = \frac{V_m}{2} + \frac{2V_m}{\pi}\sum_{n=1,3,5,\ldots}^{\infty}\frac{1}{n}\sin n\omega_0 t$$

The dc component of the current is $V_m/2R$ and the kth harmonic phase current is

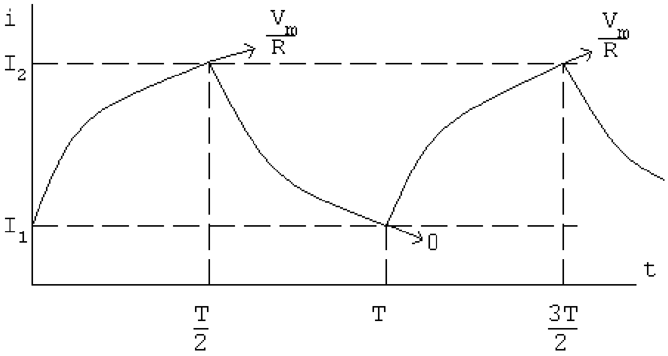
$$\mathbf{I}_k = \frac{2V_m/k\pi}{R + jk\omega_0 L} = \frac{2V_m}{k\pi\sqrt{R^2 + (k\omega_0 L)^2}}\underline{/-\theta_k}$$

where $\theta_k = \tan^{-1}\left(\frac{k\omega_0 L}{R}\right)$

Thus the Fourier series for the steady-state current is

$$i = \frac{V_m}{2R} + \frac{2V_m}{\pi}\sum_{n=1,3,5,\ldots}^{\infty}\frac{\sin(n\omega_0 t - \theta_n)}{n\sqrt{R^2 + (n\omega_0 L)^2}}\text{ A}$$

[b]



The steady-state current will alternate between I_1 and I_2 in exponential traces as shown. Assuming $t = 0$ at the instant i increases toward (V_m/R), we have

$$i = \frac{V_m}{R} + \left(I_1 - \frac{V_m}{R}\right)e^{-t/\tau} \quad \text{for} \quad 0 \le t \le \frac{T}{2}$$

and $i = I_2 e^{-[t-(T/2)]/\tau}$ for $T/2 \le t \le T$, where $\tau = L/R$. Now we solve for I_1 and I_2 by noting that

$$I_1 = I_2 e^{-T/2\tau} \quad \text{and} \quad I_2 = \frac{V_m}{R} + \left(I_1 - \frac{V_m}{R}\right)e^{-T/2\tau}$$

These two equations are now solved for I_1. Letting $x = T/2\tau$, we get

$$I_1 = \frac{(V_m/R)e^{-x}}{1 + e^{-x}}$$

Therefore the equations for i become

$$i = \frac{V_m}{R} - \left[\frac{V_m}{R(1 + e^{-x})}\right]e^{-t/\tau} \qquad \text{for} \quad 0 \le t \le \frac{T}{2} \quad \text{and}$$

$$i = \left[\frac{V_m}{R(1 + e^{-x})}\right]e^{-[t-(T/2)]/\tau} \qquad \text{for} \quad \frac{T}{2} \le t \le T$$

A check on the validity of these expressions shows they yield an average value of $(V_m/2R)$:

$$I_{\rm avg} = \frac{1}{T}\left\{\int_0^{T/2}\left[\frac{V_m}{R} + \left(I_1 - \frac{V_m}{R}\right)e^{-t/\tau}\right]dt + \int_{T/2}^{T} I_2 e^{-[t-(T/2)]/\tau}\,dt\right\}$$

$$= \frac{1}{T}\left\{\frac{V_m T}{2R} + \tau(1-e^{-x})\left(I_1 - \frac{V_m}{R} + I_2\right)\right\}$$

$$= \frac{V_m}{2R} \quad \text{since} \quad I_1 + I_2 = \frac{V_m}{R}$$

P 16.31 $\omega_o = \dfrac{2\pi}{T} = \dfrac{2\pi}{10\pi}\times 10^6 = 200$ krad/s

$$\therefore \quad n = \frac{3\times 10^6}{0.2\times 10^6} = 15; \qquad n = \frac{5\times 10^6}{0.2\times 10^6} = 25$$

$$H(s) = \frac{V_o}{V_g} = \frac{(1/RC)s}{s^2 + (1/RC)s + (1/LC)}$$

$$\frac{1}{RC} = \frac{10^{12}}{(250\times 10^3)(4)} = 10^6; \qquad \frac{1}{LC} = \frac{(10^3)(10^{12})}{10)(4)} = 25\times 10^{12}$$

$$H(s) = \frac{10^6 s}{s^2 + 10^6 s + 25\times 10^{12}}$$

$$H(j\omega) = \frac{j\omega\times 10^6}{(25\times 10^{12} - \omega^2) + j10^6\omega}$$

15th harmonic input:

$$v_{g15} = (150)(1/15)\sin(15\pi/2)\cos 15\omega_o t = -10\cos 3\times 10^6 t\,\text{V}$$

$$\therefore \quad \mathbf{V}_{g15} = 10\underline{/-180^\circ}\,\text{V}$$

$$H(j3\times 10^6) = \frac{j3}{16+j3} = 0.1843\underline{/79.38^\circ}$$

$$\mathbf{V}_{o15} = (10)(0.1843)\underline{/-100.62^\circ}\,\text{V}$$

$$v_{o15} = 1.84\cos(3\times 10^6 t - 100.62^\circ)\,\text{V}$$

25th harmonic input:

$$v_{g25} = (150)(1/25)\sin(25\pi/2)\cos 5\times 10^6 t = 6\cos 5\times 10^6 t\,\text{V}$$

$$\therefore \quad \mathbf{V}_{g25} = 6\underline{/0^\circ}\,\text{V}$$

$$H(j5 \times 10^6) = \frac{j5}{0 + j5} = 1\underline{/0^\circ}$$

$$\mathbf{V}_{o25} = 6\underline{/0^\circ}\,\text{V}$$

$$v_{o25} = 6\cos 5 \times 10^6 t\,\text{V}$$

P 16.32 The function is odd with half-wave and quarter-wave symmetry. Therefore,

$a_k = 0,$ for all k; the function is odd

$b_k = 0,$ for k even, the function has half-wave symmetry

$$b_k = \frac{8}{T}\int_0^{T/4} f(t)\sin k\omega_o t, \qquad k \text{ odd}$$

$$= \frac{8}{T}\left\{\int_0^{T/10} 500t\sin k\omega_o t\,dt + \int_{T/10}^{T/4}\sin k\omega_o t\,dt\right\}$$

$$= \frac{8}{T}\{\text{Int1} + \text{Int2}\}$$

$$\text{Int1} \;= 500\int_0^{T/10} t\sin k\omega_o t\,dt$$

$$= 500\left[\frac{1}{k^2\omega_o^2}\sin k\omega_o t - \frac{t}{k\omega_o}\cos k\omega_o t\,\Bigg|_0^{T/10}\right]$$

$$= \frac{500}{k^2\omega_o^2}\sin\frac{k\pi}{5} - \frac{50T}{k\omega_o}\cos\frac{k\pi}{5}$$

$$\text{Int2} \;= \int_{T/10}^{T/4}\sin k\omega_o t\,dt = \frac{-1}{k\omega_o}\cos k\omega_o t\,\Bigg|_{T/10}^{T/4} = \frac{1}{k\omega_o}\cos\frac{k\pi}{5}$$

$$\text{Int1} \;+\; \text{Int2} \;= \frac{500}{k^2\omega_o^2}\sin\frac{k\pi}{5} + \left(\frac{1}{k\omega_o} - \frac{50T}{k\omega_o}\right)\cos\frac{k\pi}{5}$$

$$50T = 50(20 \times 10^{-3}) = 1$$

$$\therefore \quad \text{Int1} \;+\; \text{Int2} \;= \frac{500}{k^2\omega_o^2}\sin\frac{k\pi}{5}$$

$$b_k = \left[\frac{8}{T}\cdot\frac{500}{4\pi^2k^2}\cdot T^2\right]\sin\frac{k\pi}{5} = \frac{20}{\pi^2k^2}\sin\frac{k\pi}{5}, \qquad k \text{ odd}$$

$$i(t) = \frac{20}{\pi^2}\sum_{n=1,3,5,\ldots}^{\infty} \frac{\sin(n\pi/5)}{n^2}\sin n\omega_o t\,\text{A}$$

From the circuit,

$$H(s) = \frac{V_o}{I_g} = Z_{\text{eq}}$$

$$Y_{\text{eq}} = \frac{1}{R_1} + \frac{1}{R_2 + sL} + sC$$

$$Z_{\text{eq}} = \frac{1/C(s + R_2/L)}{s^2 + s(R_1R_2C + L)/R_1LC + (R_1 + R_2)/R_1LC}$$

Therefore,

$$H(s) = \frac{320 \times 10^4(s + 32 \times 10^4)}{s^2 + 32.8 \times 10^4 s + 28.8 \times 10^8}$$

We want the output for the third harmonic:

$$\omega_0 = \frac{2\pi}{T} = \frac{2\pi}{20 \times 10^{-3}} = 100\pi; \qquad 3\omega_0 = 300\pi$$

$$I_{g3} = \frac{20}{\pi^2}\frac{1}{9}\sin\frac{3\pi}{5} = 0.214\underline{/0^\circ}$$

$$H(j300\pi) = \frac{320 \times 10^4(j300\pi + 32 \times 10^4)}{(j300\pi)^2 + 32.8 \times 10^4(j300\pi) + 28.8 \times 10^8} = 353.6\underline{/-5.96^\circ}$$

Therefore,

$$V_{o3} = H(j300\pi)I_{g3} = (353.6\underline{/-5.96^\circ})(0.214\underline{/0^\circ}) = 75.7\underline{/-5.96^\circ}\,\text{V}$$

$$v_{o3} = 75.7\sin(300\pi t - 5.96^\circ)\,\text{V}$$

P 16.33 **[a]** $a_v = \frac{1}{T}\left[\frac{1}{2}\left(\frac{T}{2}\right)I_m + \frac{T}{2}I_m\right] = \frac{3V_m}{4}$

$$i(t) = \frac{2I_m}{T}t, \qquad 0 \le t \le T/2$$

$$i(t) = I_m, \qquad T/2 \le t \le T$$

$$a_k = \frac{2}{T}\int_0^{T/2}\frac{2I_m}{T}t\cos k\omega_o t\,dt + \frac{2}{T}\int_{T/2}^{T} I_m\cos k\omega_o t\,dt$$

$$= \frac{I_m}{\pi^2 k^2}(\cos k\pi - 1)$$

$$b_k = \frac{2}{T}\int_0^{T/2} \frac{2I_m}{T} t \sin k\omega_o t\, dt + \frac{2}{T}\int_{T/2}^{T} I_m \sin k\omega_o t\, dt$$

$$= \frac{I_m}{\pi k}$$

$$a_1 = \frac{-2I_m}{\pi^2}, \quad a_2 = 0, \quad a_v = \frac{3I_m}{4}$$

$$a_3 = \frac{-2I_m}{9\pi^2}$$

$$b_1 = \frac{I_m}{\pi}, \quad b_2 = \frac{I_m}{2\pi}$$

$$\therefore \quad I_{\text{rms}} = I_m\sqrt{\frac{9}{16} + \frac{2}{\pi^4} + \frac{1}{2\pi^2} + \frac{1}{8\pi^2}} = 0.8040 I_m$$

$$I_{\text{rms}} = 192.95\,\text{mA}$$

$$P = (0.19295)^2(1000) = 37.23\,\text{W}$$

[b] Area under i^2:

$$A = \int_0^{T/2} \frac{4I_m^2}{T^2} t^2\, dt + I_m^2 \frac{T}{2}$$

$$= \frac{4I_m^2}{T^2}\frac{t^3}{3}\bigg|_0^{T/2} + I_m^2\frac{T}{2}$$

$$= I_m^2 T\left[\frac{1}{6} + \frac{3}{6}\right] = \frac{2}{3} T I_m^2$$

$$I_{\text{rms}} = \sqrt{\frac{1}{T}\cdot\frac{2}{3}TI_m^2} = \sqrt{\frac{2}{3}} I_m = 195.96\,\text{mA}$$

$$P = (0.19596)^2(1000) = 38.4\,\text{W}$$

[c] Error $= \left(\frac{37.23}{38.40} - 1\right)100 = -3.05\%$

P 16.34 **[a]** $a_v = \frac{2\left(\frac{1}{2}\frac{T}{4}V_m\right)}{T} = \frac{V_m}{4}$

$$a_k = \frac{4}{T}\int_0^{T/4}\left[V_m - \frac{4V_m}{T}t\right]\cos k\omega_o t\, dt$$

$$= \frac{4V_m}{\pi^2 k^2}\left[1 - \cos\frac{k\pi}{2}\right]$$

$$b_k = 0, \quad \text{all } k$$

$$a_v = \frac{60}{4} = 15\,\text{V}$$

$$a_1 = \frac{240}{\pi^2}$$

$$a_2 = \frac{240}{4\pi^2}(1 - \cos\pi) = \frac{120}{\pi^2}$$

$$V_{\text{rms}} = \sqrt{(15)^2 + \frac{1}{2}\left[\left(\frac{240}{\pi^2}\right)^2 + \left(\frac{120}{\pi^2}\right)^2\right]} = 24.38\,\text{V}$$

$$P = \frac{(24.38)^2}{10} = 59.46\,\text{W}$$

[b] Area under v^2; $\qquad 0 \leq t \leq T/4$

$$v^2 = 3600 - \frac{28{,}800}{T}t + \frac{57{,}600}{T^2}t^2$$

$$A = 2\int_0^{T/4}\left[3600 - \frac{28{,}800}{T}t + \frac{57{,}600}{T^2}t^2\right]\,dt = 600T$$

$$V_{\text{rms}} = \sqrt{\frac{1}{T}600T} = \sqrt{600} = 24.49\,\text{V}$$

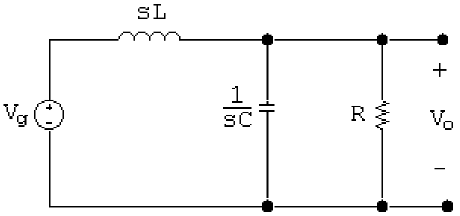
$$P = \sqrt{600}^2/10 = 60\,\text{W}$$

[c] Error $= \left(\frac{59.46}{60.00} - 1\right)100 = -0.9041\%$

P 16.35 $v_g = 10 - \frac{80}{\pi^2}\sum_{n=1,3,5,\ldots}^{\infty}\frac{1}{n^2}\cos n\omega_o t\,\text{V}$

$$\omega_o = \frac{2\pi}{T} = \frac{2\pi}{4\pi} \times 10^3 = 500 \text{ rad/s}$$

$$v_g = 10 - \frac{80}{\pi^2}\cos 500t - \frac{80}{9\pi^2}\cos 1500t + \ldots$$



$$\frac{V_o - V_g}{sL} + sCV_o + \frac{V_o}{R} = 0$$

$$V_o(RLCs^2 + Ls + R) = RV_g$$

$$H(s) = \frac{V_o}{V_g} = \frac{1/LC}{s^2 + s/RC + 1/LC}$$

$$\frac{1}{LC} = \frac{10^6}{(0.1)(10)} = 10^6$$

$$\frac{1}{RC} = \frac{10^6}{(50\sqrt{2})(10)} = 1000\sqrt{2}$$

$$H(s) = \frac{10^6}{s^2 + 1000\sqrt{2}s + 10^6}$$

$$H(j\omega) = \frac{10^6}{10^6 - \omega^2 + j1000\omega\sqrt{2}}$$

$$H(j0) = 1$$

$$H(j500) = 0.9701\underline{/-43.31^\circ}$$

$$H(j1500) = 0.4061\underline{/-120.51^\circ}$$

$$v_o = 10(1) + \frac{80}{\pi^2}(0.9701)\cos(500t - 43.31^\circ)$$

$$+\frac{80}{9\pi^2}(0.4061)\cos(1500t - 120.51^\circ) + \ldots$$

$$v_o = 10 + 7.86\cos(500t - 43.31^\circ) + 0.3658\cos(1500t - 120.51^\circ) + \ldots$$

$$V_{\text{rms}} \cong \sqrt{10^2 + \left(\frac{7.86}{\sqrt{2}}\right)^2 + \left(\frac{0.3658}{\sqrt{2}}\right)^2} = 11.44\,\text{V}$$

$$P \cong \frac{V_{\text{rms}}^2}{50\sqrt{2}} = 1.85\,\text{W}$$

Note – the higher harmonics are severely attenuated and can be ignored. For example, the 5th harmonic component of v_o is

$$v_{o5} = (0.1580)\left(\frac{80}{25\pi^2}\right)\cos(2500t - 146.04^\circ) = 0.0512\cos(2500t - 146.04^\circ)\,\text{V}$$

P 16.36 **[a]** Area under $v^2 = A = 4\int_0^{T/6} \frac{36V_m^2}{T^2}t^2\,dt + 2V_m^2\left(\frac{T}{3} - \frac{T}{6}\right)$

$$= \frac{2V_m^2T}{9} + \frac{V_m^2T}{3}$$

Therefore $V_{\text{rms}} = \sqrt{\frac{1}{T}\left(\frac{2V_m^2T}{9} + \frac{V_m^2T}{3}\right)} = V_m\sqrt{\frac{2}{9} + \frac{1}{3}} = 74.5356\,\text{V}$

[b] From Asssessment Problem 16.3,

$$v_g = 105.30\sin\omega_0 t - 4.21\sin 5\omega_0 t + 2.15\sin 7\omega_0 t + \cdots\,\text{V}$$

Therefore $V_{\text{rms}} \cong \sqrt{\frac{(105.30)^2 + (4.21)^2 + (2.15)^2}{2}} = 74.5306\,\text{V}$

P 16.37 **[a]** $v = 15 + 400\cos 500t + 100\cos(1500t - 90^\circ)\,\text{V}$

$$i = 2 + 5\cos(500t - 30^\circ) + 3\cos(1500t - 15^\circ)\,\text{A}$$

$$P = (15)(2) + \frac{1}{2}(400)(5)\cos(30^\circ) + \frac{1}{2}(100)(3)\cos(-75^\circ) = 934.85\,\text{W}$$

[b] $V_{\text{rms}} = \sqrt{(15)^2 + \left(\frac{400}{\sqrt{2}}\right)^2 + \left(\frac{100}{\sqrt{2}}\right)^2} = 291.93\,\text{V}$

[c] $I_{\text{rms}} = \sqrt{(2)^2 + \left(\frac{5}{\sqrt{2}}\right)^2 + \left(\frac{3}{\sqrt{2}}\right)^2} = 4.58\,\text{A}$

P 16.38 **[a]** $v(t) \approx \frac{340}{\pi} - \frac{680}{\pi}\left\{\frac{1}{3}\cos\omega_o t + \frac{1}{15}\cos 2\omega_o t + \cdots\right\}$

$$V_{\text{rms}} \approx \sqrt{\left(\frac{340}{\pi}\right)^2 + \left(\frac{680}{\pi}\right)^2\left[\left(\frac{1}{3\sqrt{2}}\right)^2 + \left(\frac{1}{15\sqrt{2}}\right)^2\right]}$$

$$= \frac{340}{\pi}\sqrt{1 + 4\left(\frac{1}{18} + \frac{1}{450}\right)} = 120.0819\,\text{V}$$

[b] $$V_{\rm rms} = \frac{170}{\sqrt{2}} = 120.2082$$

$$\%\ \text{error} = \left(\frac{120.0819}{120.2082} - 1\right)(100) = -0.11\%$$

[c] $$v(t) \approx \frac{170}{\pi} + 85\sin\omega_o t - \frac{340}{3\pi}\cos 2\omega_o t$$

$$V_{\rm rms} \approx \sqrt{\left(\frac{170}{\pi}\right)^2 + \left(\frac{85}{\sqrt{2}}\right)^2 + \left(\frac{340}{3\sqrt{2}\pi}\right)^2} \approx 84.8021\,\text{V}$$

$$V_{\rm rms} = \frac{170}{2} = 85\,\text{V}$$

$$\%\ \text{error} = -0.23\%$$

P 16.39 **[a]** $$v(t) = \frac{480}{\pi}\{\sin\omega_o t + \frac{1}{3}\sin 3\omega_o t + \frac{1}{5}\sin 5\omega_o t + \frac{1}{7}\sin 7\omega_o t + \frac{1}{9}\sin 9\omega_o t + \cdots$$

$$\begin{aligned} V_{\rm rms} &= \frac{480}{\pi}\sqrt{\left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{1}{3\sqrt{2}}\right)^2 + \left(\frac{1}{5\sqrt{2}}\right)^2 + \left(\frac{1}{7\sqrt{2}}\right)^2 + \left(\frac{1}{9\sqrt{2}}\right)^2} \\ &= \frac{480}{\pi\sqrt{2}}\sqrt{1 + \frac{1}{9} + \frac{1}{25} + \frac{1}{49} + \frac{1}{81}} \\ &= 117.55\,\text{V} \end{aligned}$$

[b] $$\%\ \text{error} = \left(\frac{117.55}{120} - 1\right)(100) = -2.04\%$$

[c] $$v(t) = \frac{960}{\pi^2}\left\{\sin\omega_o t + \frac{1}{9}\sin 3\omega_o t + \frac{1}{25}\sin 5\omega_o t \right.$$
$$\left. - \frac{1}{49}\sin 7\omega_o t + \frac{1}{81}\sin 9\omega_o t - \cdots\right\}$$

$$V_{\rm rms} \cong \frac{960}{\pi^2\sqrt{2}}\sqrt{1 + \frac{1}{81} + \frac{1}{625} + \frac{1}{2401} + \frac{1}{6561}}$$

$$\cong 69.2765\,\text{V}$$

$$V_{\rm rms} = \frac{120}{\sqrt{3}} = 69.2820\,\text{V}$$

$$\%\ \text{error} = \left(\frac{69.2765}{69.2820} - 1\right)(100) = -0.0081\%$$

P 16.40 **[a]** v_g has half-wave symmetry, quarter-wave symmetry, and is odd

$$\therefore\quad a_v = 0,\ a_k = 0 \text{ all } k,\ b_k = 0\ k\text{-even}$$

$$\begin{aligned} b_k &= \frac{8}{T}\int_0^{T/4} f(t)\sin k\omega_o t\,dt, \quad k\text{-odd} \\ &= \frac{8}{T}\left\{\int_0^{T/8}\frac{V_m}{4}\sin k\omega_o t\,dt + \int_{T/8}^{T/4} V_m \sin k\omega_o t\,dt\right\} \\ &= \frac{8V_m}{4T}\left[-\frac{\cos k\omega_o t}{k\omega_o}\Big|_0^{T/8} + \frac{8V_m}{T}\left[-\frac{\cos k\omega_o t}{k\omega_o}\Big|_{T/8}^{T/4} \\ &= \frac{8V_m}{4k\omega_o T}\left[1-\cos\frac{k\pi}{4}\right] + \frac{8V_m}{Tk\omega_o}\left[\cos\frac{k\pi}{4} - 0\right] \\ &= \frac{8V_m}{k\omega_o T}\left\{\frac{1}{4} - \frac{1}{4}\cos\frac{k\pi}{4} + \cos\frac{k\pi}{4}\right\} \\ &= \frac{4V_m}{\pi k}\left\{\frac{1}{4} + 0.75\cos\frac{k\pi}{4}\right\} = \frac{1}{k}(10 + 30\cos(k\pi/4) \end{aligned}$$

$$\begin{aligned} b_1 &= 10 + 30\cos(\pi/4) = 31.21 \\ b_3 &= \frac{1}{3}[10 + 30\cos(3\pi/4)] = -3.74 \\ b_5 &= \frac{1}{5}[10 + 30\cos(5\pi/4)] = -2.24 \\ b_7 &= \frac{1}{7}[10 + 30\cos(7\pi/4)] = 4.46 \end{aligned}$$

$$V_g(\text{rms}) \approx \mathbf{V}_m\sqrt{\frac{31.21^2 + 3.74^2 + 2.24^2 + 4.46^2}{2}} = 22.51$$

[b] $\text{Area} = 2\left[2(6.25\pi)^2\left(\frac{T}{8}\right) + 100\pi^2\left(\frac{T}{4}\right)\right] = 53.125\pi^2 T$

$$V_g(\text{rms}) = \sqrt{\frac{1}{T}(53.125\pi^2)t} = \sqrt{53.125}\pi = 22.90$$

[c] $\%\ \text{Error} = \left(\frac{22.51}{22.90} - 1\right)(100) = -1.7\%$

P 16.41 **[a]** Half-wave symmetry $a_v = 0$, $a_k = b_k = 0$, even k

$$a_k = \frac{4}{T}\int_0^{T/4} \frac{4I_m}{T} t\cos k\omega_0 t\,dt = \frac{16I_m}{T^2}\int_0^{T/4} t\cos k\omega_0 t\,dt$$

$$= \frac{16I_m}{T^2}\left\{\frac{\cos k\omega_0 t}{k^2\omega_0^2} + \frac{t}{k\omega_0}\sin k\omega_0 t \,\Big|_0^{T/4}\right\}$$

$$= \frac{16I_m}{T^2}\left\{0 + \frac{T}{4k\omega_0}\sin\frac{k\pi}{2} - \frac{1}{k^2\omega_0^2}\right\}$$

$$a_k = \frac{2I_m}{\pi k}\left[\sin\left(\frac{k\pi}{2}\right) - \frac{2}{\pi k}\right], \qquad k\text{—odd}$$

$$b_k = \frac{4}{T}\int_0^{T/4} \frac{4I_m}{T} t\sin k\omega_0 t\,dt = \frac{16I_m}{T^2}\int_0^{T/4} t\sin k\omega_0 t\,dt$$

$$= \frac{16I_m}{T^2}\left\{\frac{\sin k\omega_0 t}{k^2\omega_0^2} - \frac{t}{k\omega_0}\cos k\omega_0 t \,\Big|_0^{T/4}\right\} = \frac{4I_m}{\pi^2 k^2}\sin\left(\frac{k\pi}{2}\right)$$

[b] $$a_k - jb_k = \frac{2I_m}{\pi k}\left\{\left[\sin\left(\frac{k\pi}{2}\right) - \frac{2}{\pi k}\right] - \left[j\frac{2}{\pi k}\sin\left(\frac{k\pi}{2}\right)\right]\right\}$$

$$a_1 - jb_1 = \frac{2I_m}{\pi}\left\{\left(1 - \frac{2}{\pi}\right) - j\frac{2}{\pi}\right\} = 0.47I_m\underline{/-60.28^\circ}$$

$$a_3 - jb_3 = \frac{2I_m}{3\pi}\left\{\left(-1 - \frac{2}{3\pi}\right) + j\left(\frac{2}{3\pi}\right)\right\} = 0.26I_m\underline{/170.07^\circ}$$

$$a_5 - jb_5 = \frac{2I_m}{5\pi}\left\{\left(1 - \frac{2}{5\pi}\right) - j\left(\frac{2}{5\pi}\right)\right\} = 0.11I_m\underline{/-8.30^\circ}$$

$$a_7 - jb_7 = \frac{2I_m}{7\pi}\left\{\left(-1 - \frac{2}{7\pi}\right) + j\left(\frac{2}{7\pi}\right)\right\} = 0.10I_m\underline{/175.23^\circ}$$

$$i_g = 0.47I_m\cos(\omega_0 t - 60.28^\circ) + 0.26I_m\cos(3\omega_0 t + 170.07^\circ)$$
$$+0.11I_m\cos(5\omega_0 t - 8.30^\circ) + 0.10I_m\cos(7\omega_0 t + 175.23^\circ) + \cdots$$

[c] $$I_g = \sqrt{\sum_{n=1,3,5,\ldots}^{\infty}\left(\frac{A_n^2}{2}\right)}$$

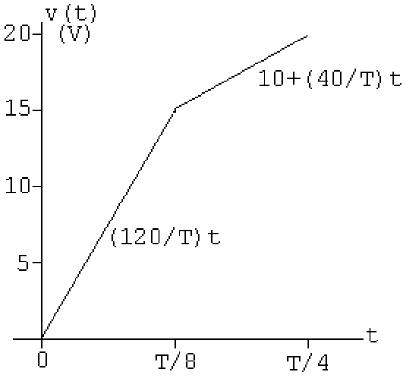
$$\cong I_m\sqrt{\frac{(0.47)^2 + (0.26)^2 + (0.11)^2 + (0.10)^2}{2}} = 0.39I_m$$

[d] $$\text{Area} = 2\int_0^{T/4}\left(\frac{4I_m}{T}t\right)^2 dt = \left(\frac{32I_m^2}{T^2}\right)\left(\frac{t^3}{3}\right)\Big|_0^{T/4} = \frac{I_m^2 T}{6}$$

$$I_g = \sqrt{\frac{1}{T}\left(\frac{I_m^2 T}{6}\right)} = \frac{I_m}{\sqrt{6}} = 0.41I_m$$

[e] $$\% \text{ error} = \left(\frac{\text{estimated}}{\text{exact}} - 1\right)100 = \left(\frac{0.3927 I_m}{(I_m/\sqrt{6})} - 1\right)100 = -3.8\%$$

P 16.42 **[a]** From Problem 16.14,



The area under v^2:

$$A = 4\left[\int_0^{T/8} \frac{14{,}400}{T^2}t^2\,dt + \int_{T/8}^{T/4}\left(10 + \frac{40t}{T}\right)^2\,dt\right]$$

$$= \frac{57{,}600}{T^2}\frac{t^3}{3}\Big|_0^{T/8} + 400t\Big|_{T/8}^{T/4} + \frac{3200}{T}\frac{t^2}{2}\Big|_{T/8}^{T/4} + \frac{6400}{T^2}\frac{t^3}{3}\Big|_{T/8}^{T/4}$$

$$= \frac{57{,}600}{1536}T + 400\frac{T}{8} + 1600\frac{3T}{64} + 6400\frac{7T}{1536} = \frac{575}{3}T$$

$$V_{\text{rms}} = \sqrt{\frac{1}{T}\left(\frac{575}{3}T\right)} = \sqrt{\frac{575}{3}} = 13.84\,\text{V}$$

[b] $$P = \frac{V_{\text{rms}}^2}{15} = 12.78\,\text{W}$$

[c] From Problem 16.14,

$$b_1 = \frac{80}{\pi^2}(2\sin 45^\circ + \sin 90^\circ) = 18.57\,\text{V}$$

$$v_g \cong 19.57 \sin \omega_0 t\,\text{V}$$

$$P = \frac{(19.57/\sqrt{2})^2}{15} = 12.76\,\text{W}$$

[d] $$\% \text{ error } = \left(\frac{12.76}{12.78} - 1\right)(100) = -0.1024\%$$

P 16.43 Figure P16.43(b): $t_a = 0.2s$; $t_b = 0.6s$

$$v = 50t \quad 0 \le t \le 0.2$$

$$v = -50t + 20 \quad 0.2 \le t \le 0.6$$

$v = 25t - 25 \quad 0.6 \leq t \leq 1.0$

$$\text{Area } 1 = A_1 = \int_0^{0.2} 2500t^2\,dt = \frac{20}{3}$$

$$\text{Area } 2 = A_2 = \int_{0.2}^{0.6} 100(4 - 20t + 25t^2)\,dt = \frac{40}{3}$$

$$\text{Area } 3 = A_3 = \int_{0.6}^{1.0} 625(t^2 - 2t + 1)\,dt = \frac{40}{3}$$

$$A_1 + A_2 + A_3 = \frac{100}{3}$$

$$V_{\text{rms}} = \sqrt{\frac{1}{1}\left(\frac{100}{3}\right)} = \frac{10}{\sqrt{3}}\,\text{V.}$$

Figure P16.43(c): $t_a = t_b = 0.4s$

$v(t) = 25t \quad 0 \leq t \leq 0.4$

$$v(t) = \frac{50}{3}(t - 1) \quad 0.4 \leq t \leq 1$$

$$A_1 = \int_0^{0.4} 625t^2\,dt = \frac{40}{3}$$

$$A_2 = \int_{0.4}^{1.0} \frac{2500}{9}(t^2 - 2t + 1)\,dt = \frac{60}{3}$$

$$A_1 + A_2 = \frac{100}{3}$$

$$V_{\text{rms}} = \sqrt{\frac{1}{T}(A_1 + A_2)} = \sqrt{\frac{1}{1}\left(\frac{100}{3}\right)} = \frac{10}{\sqrt{3}}\,\text{V.}$$

Figure P16.43(d): $t_a = t_b = 1$

$v = 10t \quad 0 \leq t \leq 1$

$$A_1 = \int_0^1 100t^2\,dt = \frac{100}{3}$$

$$V_{\text{rms}} = \sqrt{\frac{1}{1}\left(\frac{100}{3}\right)} = \frac{10}{\sqrt{3}}\,\text{V.}$$

P 16.44 $$c_n = \frac{1}{T}\int_0^{T/4} V_m e^{-jn\omega_o t}\,dt = \frac{V_m}{T}\left[\frac{e^{-jn\omega_o t}}{-jn\omega_o}\Big|_0^{T/4}\right]$$

$$= \frac{V_m}{Tn\omega_o}[j(e^{-jn\pi/2}-1)] = \frac{V_m}{2\pi n}\sin\frac{n\pi}{2} + j\frac{V_m}{2\pi n}\left(\cos\frac{n\pi}{2}-1\right)$$

$$= \frac{V_m}{2\pi n}\left[\sin\frac{n\pi}{2} - j\left(1-\cos\frac{n\pi}{2}\right)\right]$$

$$v(t) = \sum_{n=-\infty}^{\infty} c_n e^{jn\omega_o t}$$

$$c_o = a_v = \frac{1}{T}\int_0^{T/4} V_m\,dt = \frac{V_m}{4}$$

or

$$c_o = \frac{V_m}{2\pi}\lim_{n\to 0}\left[\frac{\sin(n\pi/2)}{n} - j\frac{1-\cos(n\pi/2)}{n}\right]$$

$$= \frac{V_m}{2\pi}\lim_{n\to 0}\left[\frac{(\pi/2)\cos(n\pi/2)}{1} - j\frac{(\pi/2)\sin(n\pi/2)}{1}\right]$$

$$= \frac{V_m}{2\pi}\left[\frac{\pi}{2} - j0\right] = \frac{V_m}{4}$$

Note it is much easier to use $c_o = a_v$ than to use L'Hopital's rule to find the limit of 0/0.

P 16.45 $$c_o = a_v = \frac{V_m T}{2}\cdot\frac{1}{T} = \frac{V_m}{2}$$

$$c_n = \frac{1}{T}\int_0^T \frac{V_m}{T} t e^{-jn\omega_o t}\,dt$$

$$= \frac{V_m}{T^2}\left[\frac{e^{-jn\omega_0 t}}{-n^2\omega_0^2}(-jn\omega_0 t - 1)\right]_0^T$$

$$= \frac{V_m}{T^2}\left[\frac{e^{-jn2\pi T/T}}{-n^2\omega_0^2}\left(-jn\frac{2\pi}{T}T-1\right) - \frac{1}{-n^2\omega_0^2}(-1)\right]$$

$$= \frac{V_m}{T^2}\left[\frac{1}{n^2\omega_0^2}(1+jn2\pi) - \frac{1}{n^2\omega_0^2}\right]$$

$$= j\frac{V_m}{2n\pi},\quad n = \pm 1, \pm 2, \pm 3, \ldots$$

P 16.46 **[a]** $V_{\rm rms} = \sqrt{\frac{1}{T}\int_0^T v^2\,dt} = \sqrt{\frac{1}{T}\int_0^T \left(\frac{V_m}{T}\right)^2 t^2\,dt}$

$$= \sqrt{\frac{V_m^2}{T^3}\frac{t^3}{3}\bigg|_0^T}$$

$$= \sqrt{\frac{V_m^2}{3}} = \frac{V_m}{\sqrt{3}}$$

$$P = \frac{(120/\sqrt{3})^2}{10} = 480\,\text{W}$$

[b] From the solution to Problem 16.45

$c_0 = \frac{120}{2} = 60\,\text{V};$ $\qquad c_4 = j\frac{120}{8\pi} = j\frac{15}{\pi}$

$c_1 = j\frac{120}{2\pi} = j\frac{60}{\pi};$ $\qquad c_5 = j\frac{120}{10\pi} = j\frac{12}{\pi}$

$c_2 = j\frac{120}{4\pi} = j\frac{30}{\pi};$ $\qquad c_6 = j\frac{120}{12\pi} = j\frac{10}{\pi}$

$c_3 = j\frac{120}{6\pi} = j\frac{20}{\pi};$ $\qquad c_7 = j\frac{120}{14\pi} = j\frac{8.57}{\pi}$

$$V_{\rm rms} = \sqrt{c_o^2 + 2\sum_{n=1}^{\infty}|c_n|^2}$$

$$= \sqrt{60^2 + \tfrac{2}{\pi^2}(60^2 + 30^2 + 20^2 + 15^2 + 12^2 + 10^2 + 8.57^2)}$$

$$= 68.58\,\text{V}$$

[c] $P = \frac{(68.58)^2}{10} = 470.32\,\text{W}$

$$\%\ \text{error} = \left(\frac{470.32}{480} - 1\right)(100) = -2.02\%$$

P 16.47 **[a]** $C_o = a_v = \frac{(1/2)(T/2)V_m}{T} = \frac{V_m}{4}$

$$C_n = \frac{1}{T}\int_0^{T/2} \frac{2V_m}{T} t e^{-jn\omega_o t}\,dt$$

$$= \frac{2V_m}{T^2}\left[\frac{e^{-jn\omega_o t}}{-n^2\omega_o^t}(-jn\omega_o t - 1)\right]_0^{T/2}$$

$$= \frac{V_m}{2n^2\pi^2}[e^{-jn\pi}(-jn\pi + 1) - 1]$$

Since $e^{-jn\pi} = \cos n\pi$ we can write

$$C_n = \frac{V_m}{2\pi^2 n^2}(\cos n\pi - 1) + j\frac{V_m}{2n\pi}\cos n\pi$$

[b] $C_o = \dfrac{54}{4} = 13.5\,\text{V}$

$$C_{-1} = \frac{-54}{\pi^2} + j\frac{27}{\pi} = 10.19\underline{/122.48^\circ}\,\text{V}$$

$$C_1 = 10.19\underline{/-122.48^\circ}\,\text{V}$$

$$C_{-2} = -j\frac{13.5}{\pi} = 4.30\underline{/-90^\circ}\,\text{V}$$

$$C_2 = 4.30\underline{/90^\circ}\,\text{V}$$

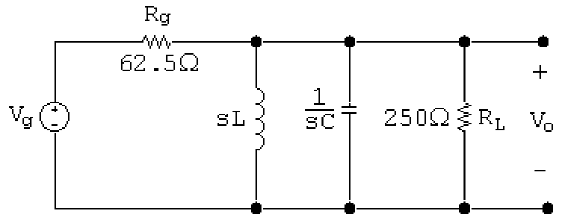
$$C_{-3} = \frac{-6}{\pi^2} + j\frac{9}{\pi} = 2.93\underline{/101.98^\circ}\,\text{V}$$

$$C_3 = 2.93\underline{/-101.98^\circ}\,\text{V}$$

$$C_{-4} = -j\frac{6.75}{\pi} = 2.15\underline{/-90^\circ}\,\text{V}$$

$$C_4 = 2.15\underline{/90^\circ}\,\text{V}$$

[c]



$$\frac{V_o}{250} + \frac{V_o}{sL} + V_o sC + \frac{V_o - V_g}{62.5} = 0$$

$$\therefore \qquad (250LCs^2 + 5sL + 250)V_o = 4sLV_g$$

$$\frac{V_o}{V_g} = H(s) = \frac{(4/250C)s}{s^2 + 1/50C + 1/LC}$$

$$H(s) = \frac{16{,}000s}{s^2 + 2 \times 10^4 s + 4 \times 10^{10}}$$

$$\omega_o = \frac{2\pi}{T} = \frac{2\pi}{10\pi} \times 10^6 = 2 \times 10^5 \text{ rad/s}$$

$$H(j0) = 0$$

$$H(j2 \times 10^5 k) = \frac{j8k}{100(1 - k^2) + j10k}$$

Therefore,

$H_{-1} = 0.8\underline{/0^\circ}$; $\qquad H_1 = 0.8\underline{/0^\circ}$

$H_{-2} = \dfrac{-j16}{-300 - j20} = 0.0532\underline{/86.19^\circ}$; $\qquad H_2 = 0.0532\underline{/-86.19^\circ}$

$H_{-3} = \dfrac{-j24}{-800 - j30} = 0.0300\underline{/87.85^\circ}$; $\qquad H_2 = 0.0300\underline{/-87.85^\circ}$

$H_{-4} = \dfrac{-j32}{-1500 - j40} = 0.0213\underline{/88.47^\circ}$; $\qquad H_2 = 0.0213\underline{/-88.47^\circ}$

The output voltage coefficients:

$C_0 = 0$

$C_{-1} = (10.19\underline{/122.48^\circ})(0.8\underline{/0^\circ}) = 8.15\underline{/122.48^\circ}\,\mathrm{V}$

$C_1 = 8.15\underline{/-122.48^\circ}\,\mathrm{V}$

$C_{-2} = (4.30\underline{/-90^\circ})(0.05\underline{/86.19^\circ}) = 0.2287\underline{/-3.81^\circ}\,\mathrm{V}$

$C_2 = 0.2287\underline{/3.81^\circ}\,\mathrm{V}$

$C_{-3} = (2.93\underline{/101.98^\circ})(0.03\underline{/87.85^\circ}) = 0.0878\underline{/-170.17^\circ}\,\mathrm{V}$

$C_3 = 0.0878\underline{/170.17^\circ}\,\mathrm{V}$

$C_{-4} = (2.15\underline{/-90^\circ})(0.02\underline{/88.47^\circ}) = 0.0458\underline{/-1.53^\circ}\,\mathrm{V}$

$C_4 = 0.0458\underline{/1.53^\circ}\,\mathrm{V}$

[d] $V_{\text{rms}} \cong \sqrt{C_o^2 + 2\sum_{n=1}^{4}|C_n|^2} \cong \sqrt{2\sum_{n=1}^{4}|C_n|^2}$

$$\cong \sqrt{2(8.15^2 + 0.2287^2 + 0.0878^2 + 0.0458^2} \cong 11.53\,\mathrm{V}$$

$$P = \frac{(11.53)^2}{250} = 531.95\,\mathrm{mW}$$

P 16.48 **[a]** $V_{\text{rms}} = \sqrt{\dfrac{1}{T}\int_0^{T/2}\left(\dfrac{2V_m}{T}t\right)^2\,dt}$

$$= \sqrt{\frac{1}{T}\left[\frac{4V_m^2}{T^2}\frac{t^3}{3}\right]_0^{T/2}}$$

$$= \sqrt{\frac{4V_m^2}{(3)(8)}} = \frac{V_m}{\sqrt{6}}$$

$$V_{\text{rms}} = \frac{54}{\sqrt{6}} = 22.05\,\mathrm{V}$$

[b] From the solution to Problem 16.47

$C_0 = 13.5;$ $\qquad |C_3| = 2.93$

$|C_1| = 10.19;$ $\qquad |C_4| = 2.15$

$|C_2| = 4.30$

$$V_g(\text{rms}) \cong \sqrt{13.5^2 + 2(10.19^2 + 4.30^2 + 2.93^2 + 2.15^2)} \cong 21.29\,\text{V}$$

[c] $\%\text{ Error } = \left(\dfrac{21.29}{22.05} - 1\right)(100) = -3.44\%$

P 16.49 **[a]** From Example 16.3 we have:

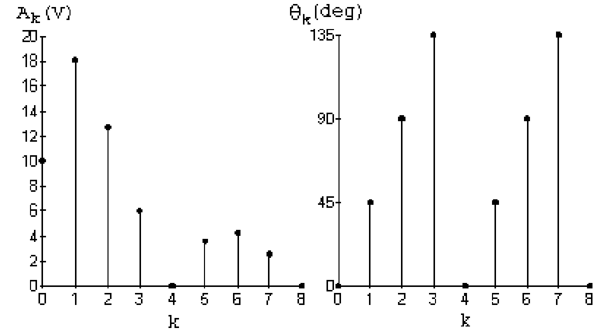
$$a_v = \frac{40}{4} = 10\,\text{V}, \qquad a_k = \frac{40}{\pi k}\sin\left(\frac{k\pi}{2}\right)$$

$$b_k = \frac{40}{\pi k}\left[1 - \cos\left(\frac{k\pi}{2}\right)\right], \qquad A_k\underline{/-\theta_k^\circ} = a_k - jb_k$$

$A_1 = 18.01\,\text{V} \quad \theta_1 = 45^\circ, \quad A_2 = 12.73\,\text{V}, \quad \theta_2 = 90^\circ$

$A_3 = 6\,\text{V}, \quad \theta_3 = 135^\circ, \quad A_4 = 0, \quad A_5 = 3.6\,\text{V}, \quad \theta_5 = 45^\circ$

$A_6 = 4.24\,\text{V}, \quad \theta_6 = 90^\circ, \quad A_7 = 2.57\,\text{V}, \quad \theta_7 = 135^\circ; \quad A_8 = 0$



[b] $C_n = \dfrac{a_n - jb_n}{2}, \qquad C_{-n} = \dfrac{a_n + jb_n}{2} = C_n^*$

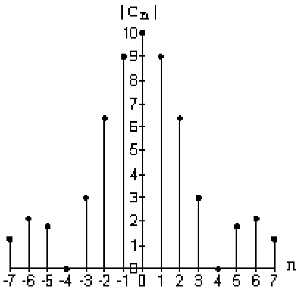
$C_0 = a_v = 10\,\text{V}$ $\qquad C_3 = 3\underline{/135^\circ}\,\text{V}$ $\qquad C_6 = 2.12\underline{/90^\circ}\,\text{V}$

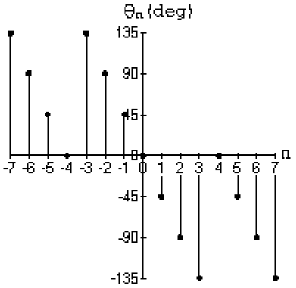
$C_1 = 9\underline{/45^\circ}\,\text{V}$ $\qquad C_{-3} = 3\underline{/-135^\circ}\,\text{V}$ $\qquad C_{-6} = 2.12\underline{/-90^\circ}\,\text{V}$

$C_{-1} = 9\underline{/-45^\circ}\,\text{V}$ $\qquad C_4 = C_{-4} = 0$ $\qquad C_7 = 1.29\underline{/135^\circ}\,\text{V}$

$C_2 = 6.37\underline{/90^\circ}\,\text{V}$ $\qquad C_5 = 1.8\underline{/45^\circ}\,\text{V}$ $\qquad C_{-7} = 1.29\underline{/-135^\circ}\,\text{V}$

$C_{-2} = 6.37\underline{/-90^\circ}\,\text{V}$ $\qquad C_{-5} = 1.8\underline{/-45^\circ}\,\text{V}$





P 16.50 [**a**] From the solution to Problem 16.33 we have

$$A_k = a_k - jb_k = \frac{I_m}{\pi^2 k^2}(\cos k\pi - 1) + j\frac{I_m}{\pi k}$$

$$A_0 = 0.75I_m = 180\,\text{mA}$$

$$A_1 = \frac{240}{\pi^2}(-2) + j\frac{240}{\pi} = 90.56\underline{/122.48^\circ}\,\text{mA}$$

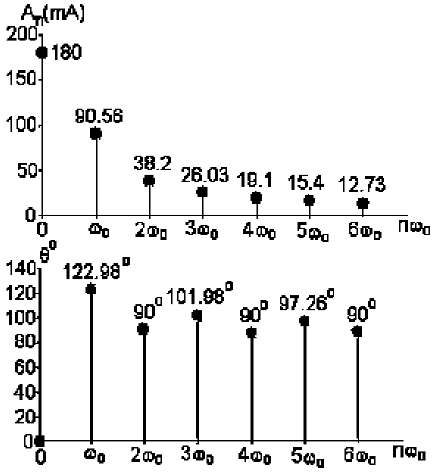
$$A_2 = j\frac{240}{2\pi} = 38.20\underline{/90^\circ}\,\text{mA}$$

$$A_3 = \frac{240}{9\pi^2}(-2) + j\frac{240}{3\pi} = 26.03\underline{/101.98^\circ}\,\text{mA}$$

$$A_4 = j\frac{240}{4\pi} = 19.10\underline{/90^\circ}\,\text{mA}$$

$$A_5 = \frac{240}{25\pi^2}(-2) + j\frac{240}{5\pi} = 15.40\underline{/97.26^\circ}\,\text{mA}$$

$$A_6 = j\frac{240}{6\pi} = 12.73\underline{/90^\circ}\,\text{mA}$$



[b] $C_0 = A_0 = 180\,\text{mA}$

$$C_1 = \frac{1}{2}A_1\underline{/\theta_1} = 45.28\underline{/122.48^\circ}\,\text{mA}$$

$$C_{-1} = 45.28\underline{/-122.48^\circ}\,\text{mA}$$

$$C_2 = \frac{1}{2}A_2\underline{/\theta_2} = 19.1\underline{/90^\circ}\,\text{mA}$$

$$C_{-2} = 19.1\underline{/-90^\circ}\,\text{mA}$$

$$C_3 = \frac{1}{2}A_3\underline{/\theta_3} = 13.02\underline{/101.98^\circ}\,\text{mA}$$

$$C_{-3} = 13.02\underline{/-101.98^\circ}\,\text{mA}$$

$$C_4 = \frac{1}{2}A_4\underline{/\theta_4} = 9.55\underline{/90^\circ}\,\text{mA}$$

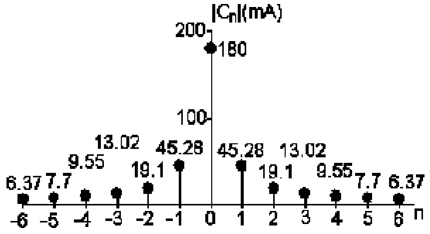
$$C_{-4} = 9.55\underline{/-90^\circ}\,\text{mA}$$

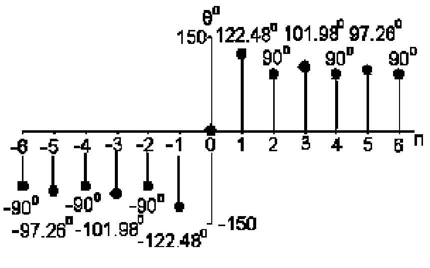
$$C_5 = \frac{1}{2}A_5\underline{/\theta_5} = 7.70\underline{/97.26^\circ}\,\text{mA}$$

$$C_{-5} = 7.70\underline{/-97.26^\circ}\,\text{mA}$$

$$C_6 = \frac{1}{2}A_6\underline{/\theta_6} = 6.37\underline{/90^\circ}\,\text{mA}$$

$$C_{-6} = 6.37\underline{/-90^\circ}\,\text{mA}$$





P 16.51 **[a]** $i = 11{,}025\cos 10{,}000t + 1225\cos(30{,}000t - 180^\circ) + 441\cos(50{,}000t - 180^\circ)$

$+225\cos 70{,}000t\,\mu\text{A}$

$= 11{,}025\cos 10{,}000t - 1225\cos 30{,}000t - 441\cos 50{,}000t$

$+225\cos 70{,}000t\,\mu\text{A}$

[b] $i(t) = i(-t)$, Function is even

[c] Yes, $A_0 = 0$, $A_n = 0$ for n even

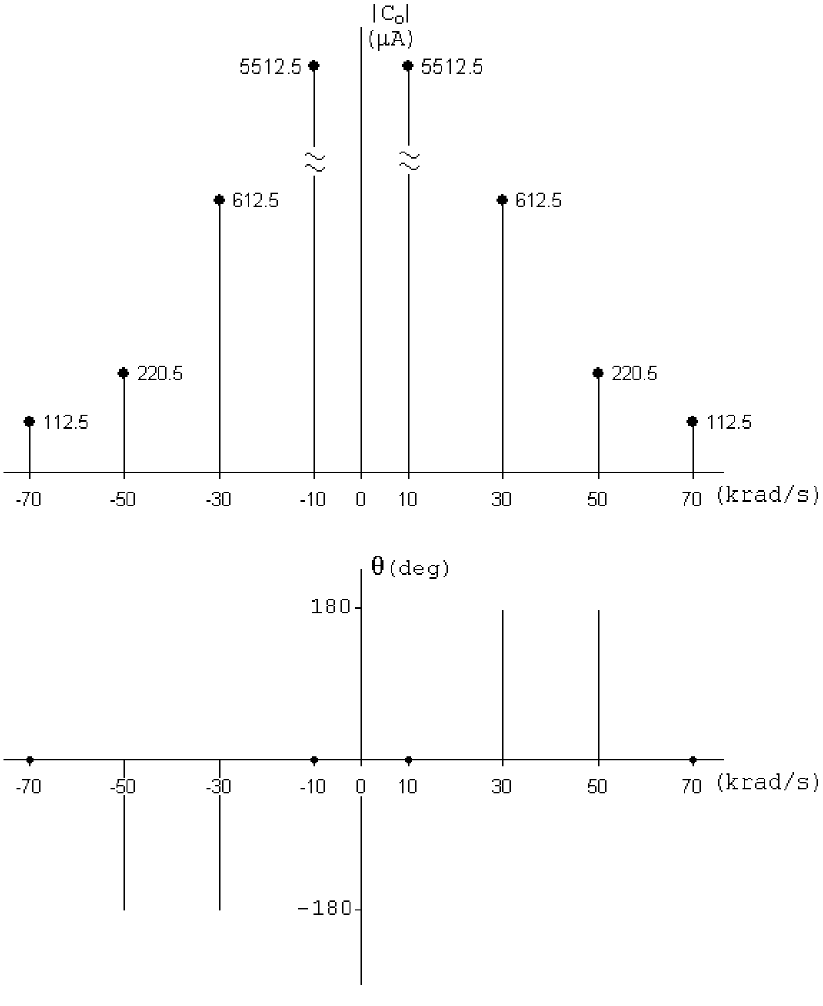
[d] $$I_{\text{rms}} = \sqrt{\frac{11{,}025^2 + 1225^2 + 441^2 + 225^2}{2}} = 7.85\,\text{mA}$$

[e] $A_1 = 11{,}025\underline{/0^\circ}\,\mu\text{A}$; $C_1 = 5512.50\underline{/0^\circ}\,\mu\text{A}$

$A_3 = 1225\underline{/180^\circ}\,\mu\text{A}$; $C_3 = 612.5\underline{/180^\circ}\,\mu\text{A}$

$A_5 = 441\underline{/180^\circ}\,\mu\text{A}$; $C_5 = 220.5\underline{/180^\circ}\,\mu\text{A}$

[f]



P 16.52 **[a]** $v = A_1\cos(\omega_o t - 90^\circ) + A_3\cos(3\omega_o t + 90^\circ)$

$$+A_5\cos(5\omega_o t - 90^\circ) + A_7\cos(7\omega_o t + 90^\circ)$$

$$v = -A_1\sin\omega_o t + A_3\sin 3\omega_o t - A_5\sin 5\omega_o t + A_7\sin 7\omega_o t$$

[b] $v(-t) = -A_1\sin\omega_o t + A_3\sin 3\omega_o t - A_5\sin 5\omega_o t + A_7\sin 7\omega_o t$

$\therefore\;\; v(-t) = -v(t);\qquad$ odd function

[c] $v(t - T/2) = A_1\sin(\omega_o t - \pi) - A_3\sin(3\omega_o t - 3\pi)$

$$+A_5\sin(5\omega_o t - 5\pi) - A_7\sin(7\omega_o t - 7\pi)$$

$$= -A_1\sin\omega_o t + A_3\sin 3\omega_o t - A_5\sin 5\omega_o t + A_7\sin 7\omega_o t$$

$\therefore\;\; v(t - T/2) = -v(t)$, yes, the function has half-wave symmetry

[d] Since the function is odd, with hws, we test to see if

$$f(T/2 - t) = f(t)$$

$$f(T/2 - t) = A_1\sin(\pi - \omega_o t) - A_3\sin(3\pi - 3\omega_o t)$$

$$+A_5\sin(5\pi - 5\omega_o t) - A_7\sin(7\pi - 7\omega_o t)$$

$$= A_1\sin\omega_o t - A_3\sin 3\omega_o t + A_5\sin 5\omega_o t - A_7\sin 7\omega_o t$$

$\therefore\;\; f(T/2 - t) = f(t)$ and the voltage has quarter-wave symmetry

P 16.53 From Table 15.1 we have

$$H(s) = \frac{1}{(s+1)(s^2+s+1)}$$

After scaling we get

$$H'(s) = \frac{10^6}{(s+100)(s^2+100s+10^4)}$$

$$\omega_o = \frac{2\pi}{T} = \frac{2\pi}{5\pi}\times 10^3 = 400 \text{ rad/s}$$

$$\therefore\;\; H'(jn\omega_o) = \frac{1}{(1+j4n)[(1-16n^2)+j4n]}$$

It follows that

$$H(j0) = 1\underline{/0^\circ}$$

$$H(j\omega_o) = \frac{1}{(1+j4)(-15+j4)} = 0.0156\underline{/-241.03^\circ}$$

$$H(j2\omega_o) = \frac{1}{(1+j8)(-63+j8)} = 0.00195\underline{/-255.64^\circ}$$

$$v_g(t) = \frac{A}{\pi} + \frac{A}{2}\sin\omega_o t - \frac{2A}{\pi}\sum_{n=2,4,6,}^{\infty}\frac{\cos n\omega_o t}{n^2-1}$$

$$= 54 + 27\pi\sin\omega_o t - 36\cos 2\omega_o t - \cdots \text{ V}$$

$$\therefore \quad v_o = 54 + 1.33\sin(400t + 118.97^\circ) + 0.07\cos(800t - 75.64^\circ) - \cdots \text{ V}$$

P 16.54 Using the technique outlined in Problem 16.18 we can derive the Fourier series for $v_g(t)$. We get

$$v_g(t) = 100 + \frac{800}{\pi^2}\sum_{n=1,3,5,}^{\infty}\frac{1}{n^2}\cos n\omega_o t$$

The transfer function of the prototype second-order low pass Butterworth filter is

$$H(s) = \frac{1}{s^2+\sqrt{2}s+1}, \qquad \text{where } \omega_c = 1 \text{ rad/s}$$

Now frequency scale using $k_f = 2000$ to get $\omega_c = 2$ krad/s:

$$H(s) = \frac{4\times 10^6}{s^2 + 2000\sqrt{2}s + 4\times 10^6}$$

$$H(j0) = 1$$

$$H(j5000) = \frac{4\times 10^6}{(j5000)^2 + 2000\sqrt{2}(j5000)^2 + 4\times 10^6} = 0.1580\underline{/-146.04^\circ}$$

$$H(j15{,}000) = \frac{4\times 10^6}{(j15{,}000)^2 + 2000\sqrt{2}(j15{,}000)^2 + 4\times 10^6} = 0.0178\underline{/-169.13^\circ}$$

$$\mathbf{V}_{\text{dc}} = 100\,\text{V}$$

$$\mathbf{V}_{g1} = \frac{800}{n^2}\underline{/0^\circ}\,\text{V}$$

$$\mathbf{V}_{g3} = \frac{800}{9\pi^2}\underline{/0^\circ}\,\text{V}$$

$$V_{odc} = 100(1) = 100\,\text{V}$$

$$\mathbf{V}_{o1} = \frac{800}{\pi^2}(0.1580\underline{/-146.04^\circ}) = 12.81\underline{/-146.04^\circ}\,\text{V}$$

$$\mathbf{V}_{o3} = \frac{800}{9\pi^2}(0.0178\underline{/-169.13^\circ}) = 0.16\underline{/-169.13^\circ}\,\text{V}$$

$$v_o(t) = 100 + 12.81\cos(5000t - 146.04^\circ)$$
$$+\, 0.16\cos(15{,}000t - 169.13^\circ) + \cdots\ \text{V}$$

P 16.55 $v_g = \dfrac{2(2.5\pi)}{\pi} - \dfrac{4(2.5\pi)}{\pi}\dfrac{\cos 5000t}{4-1} = 5 - (10/3)\cos 5000t - \cdots\ \text{V}$

$$H(j0) = 1$$

$$H(j5000) = \frac{10^6}{(10^6 - 25\times 10^6) + j5\sqrt{2}\times 10^6} = 0.04\underline{/-163.58^\circ}$$

$$\therefore\quad v_o(t) = 5 - 0.1332\cos(5000t - 163.58^\circ) - \cdots\ \text{V}$$

P 16.56 **[a]** Let V_a represent the node voltage across R_2, then the node-voltage equations are

$$\frac{V_a - V_g}{R_1} + \frac{V_a}{R_2} + V_a sC_2 + (V_a - V_o)sC_1 = 0$$

$$(0 - V_a)sC_2 + \frac{0 - V_o}{R_3} = 0$$

Solving for V_o in terms of V_g yields

$$\frac{V_o}{V_g} = H(s) = \frac{\frac{-1}{R_1C_1}s}{s^2 + \frac{1}{R_3}\left(\frac{1}{C_1} + \frac{1}{C_2}\right)s + \frac{R_1+R_2}{R_1R_2R_3C_1C_2}}$$

It follows that

$$\omega_o^2 = \frac{R_1 + R_2}{R_1R_2R_3C_1C_2}$$

$$\beta = \frac{1}{R_3}\left(\frac{1}{C_1} + \frac{1}{C_2}\right)$$

$$K_o = \frac{R_3}{R_1}\left(\frac{C_2}{C_1 + C_2}\right)$$

Note that

$$H(s) = \frac{-\frac{R_3}{R_1}\left(\frac{C_2}{C_1+C_2}\right)\frac{1}{R_3}\left(\frac{1}{C_1} + \frac{1}{C_2}\right)s}{s^2 + \frac{1}{R_3}\left(\frac{1}{C_1} + \frac{1}{C_2}\right)s + \left(\frac{R_1+R_2}{R_1R_2R_3C_1C_2}\right)}$$

[b] For the given values of R_1, R_2, R_3, C_1, and C_2 we have

$$H(s) = \frac{-400s}{s^2 + 400s + 10^8}$$

$$v_g = \frac{(8)(2.25\pi^2)}{\pi^2} \sum_{n=1,3,5,}^{\infty} \frac{1}{n^2} \cos n\omega_o t$$

$$= 18\left[\cos\omega_o t + \frac{1}{9}\cos 3\omega_o t + \frac{1}{25}\cos 5\omega_o t + \cdots\right] \text{mV}$$

$$= [18\cos\omega_o t + 2\cos 3\omega_o t + 0.72\cos 5\omega_o t + \cdots]\,\text{mV}$$

$$\omega_o = \frac{2\pi}{0.2\pi} \times 10^3 = 10^4 \text{ rad/s}$$

$$H(jk10^4) = \frac{-400jk10^4}{10^8 - k^2 10^8 + j400k10^4} = \frac{-jk}{25(1-k^2) + jk}$$

$$H_1 = -1 = 1\underline{/180^\circ}$$

$$H_3 = \frac{-j3}{-200 + j3} = 0.015\underline{/90.86^\circ}$$

$$H_5 = \frac{-j5}{-600 + j5} = 0.0083\underline{/90.48^\circ}$$

$$v_o = -18\cos\omega_o t + 0.03\cos(3\omega_o t + 90.86^\circ)$$

$$+\, 0.006\cos(5\omega_o t + 90.48^\circ) + \cdots \text{ mV}$$

[c] The fundamental frequency component dominates the output, so we expect the quality factor Q to be quite high.

[d] $\omega_o = 10^4$ rad/s and $\beta = 400$ rad/s. Therefore, $Q = 10{,}000/400 = 25$. We expect the output voltage to be dominated by the fundamental frequency component since the bandpass filter is tuned to this frequency!

P 16.57 **[a]** Using the equations derived in Problem 16.56(a),

$$K_o = \frac{R_3}{R_1}\left(\frac{C_2}{C_1 + C_2}\right) = \frac{400}{313}$$

$$\beta = \frac{1}{R_3}\left(\frac{1}{C_1} + \frac{1}{C_2}\right) = 2000 \text{ rad/s}$$

$$\omega_o^2 = \frac{R_1 + R_2}{R_1 R_2 R_3 C_1 C_2} = 16 \times 10^8$$

[b] $$H(jn\omega_o) = \frac{-(400/313)(2000)jn\omega_o}{16 \times 10^8 - n^2\omega_o^2 + j2000n\omega_o}$$

$$= \frac{-j(20/313)n}{(1 - n^2) + j0.05n}$$

$$H(j\omega_o) = \frac{-j(20/313)}{j(0.050)} = -\frac{400}{313} = -1.28$$

$$H(j3\omega_o) = \frac{-j(20/313)(3)}{-8 + j0.15} = 0.0240\underline{/91.07^\circ}$$

$$H(j5\omega_o) = \frac{-j(100/313)}{-24 + j0.25} = 0.0133\underline{/90.60^\circ}$$

$$v_g(t) = \frac{4A}{\pi} \sum_{n=1,3,5,\ldots}^{\infty} \frac{1}{n} \sin(n\pi/2) \cos n\omega_o t$$

$$A = 15.65\pi \text{ V}$$

$$v_g(t) = 62.60 \cos \omega_o t - 20.87 \cos 3\omega_o t + 12.52 \cos 5\omega_o t - \cdots$$

$$v_o(t) = -80 \cos \omega_o t - 0.50 \cos(3\omega_o t + 91.07^\circ)$$
$$+ 0.17 \cos(5\omega_o t + 90.60^\circ) - \cdots \text{ V}$$

17

The Fourier Transform

Assessment Problems

AP 17.1 **[a]** $$F(\omega) = \int_{-\tau/2}^{0} (-Ae^{-j\omega t})\,dt + \int_{0}^{\tau/2} Ae^{-j\omega t}\,dt$$

$$= \frac{A}{j\omega}[2 - e^{j\omega\tau/2} - e^{-j\omega\tau/2}]$$

$$= \frac{2A}{j\omega}\left[1 - \frac{e^{j\omega\tau/2} + e^{-j\omega\tau/2}}{2}\right]$$

$$= \frac{-j2A}{\omega}[1 - \cos(\omega\tau/2)]$$

[b] $$F(\omega) = \int_{0}^{\infty} te^{-at}e^{-j\omega t}\,dt = \int_{0}^{\infty} te^{-(a+j\omega)t}\,dt = \frac{1}{(a+j\omega)^2}$$

AP 17.2

$$f(t) = \frac{1}{2\pi}\left\{\int_{-3}^{-2} 4e^{jt\omega}\,d\omega + \int_{-2}^{2} e^{jt\omega}\,d\omega + \int_{2}^{3} 4e^{jt\omega}\,d\omega\right\}$$

$$= \frac{1}{j2\pi t}\{4e^{-j2t} - 4e^{-j3t} + e^{j2t} - e^{-j2t} + 4e^{j3t} - 4e^{j2t}\}$$

$$= \frac{1}{\pi t}\left[\frac{3e^{-j2t} - 3e^{j2t}}{j2} + \frac{4e^{j3t} - 4e^{-j3t}}{j2}\right]$$

$$= \frac{1}{\pi t}(4\sin 3t - 3\sin 2t)$$

AP 17.3 **[a]** $$F(\omega) = F(s)\,|_{s=j\omega} = \mathcal{L}\{e^{-at} sin\omega_0 t\}_{s=j\omega}$$

$$= \frac{\omega_0}{(s+a)^2 + \omega_0^2}\bigg|_{s=j\omega} = \frac{\omega_0}{(a+j\omega)^2 + \omega_0^2}$$

[b] $$F(\omega) = \mathcal{L}\{f^{-}(t)\}_{s=-j\omega} = \left[\frac{1}{(s+a)^2}\right]_{s=-j\omega} = \frac{1}{(a-j\omega)^2}$$

[c] $f^+(t) = te^{-at}, \qquad f^-(t) = -te^{-at}$

$$\mathcal{L}\{f^+(t)\} = \frac{1}{(s+a)^2}, \quad \mathcal{L}\{f^-(t)\} = \frac{-1}{(s+a)^2}$$

Therefore $F(\omega) = \dfrac{1}{(a+j\omega)^2} - \dfrac{1}{(a-j\omega)^2} = \dfrac{-j4a\omega}{(a^2+\omega^2)^2}$

AP 17.4 **[a]** $f'(t) = \dfrac{2A}{\tau}, \quad \dfrac{-\tau}{2} < t < 0; \qquad f'(t) = \dfrac{-2A}{\tau}, \quad 0 < t < \dfrac{\tau}{2}$

$$\therefore \quad f'(t) = \frac{2A}{\tau}[u(t+\tau/2) - u(t)] - \frac{2A}{\tau}[u(t) - u(t-\tau/2)]$$

$$= \frac{2A}{\tau}u(t+\tau/2) - \frac{4A}{\tau}u(t) + \frac{2A}{\tau}u(t-\tau/2)$$

$$\therefore \quad f''(t) = \frac{2A}{\tau}\delta\left(t+\frac{\tau}{2}\right) - \frac{4A}{\tau} + \frac{2A}{\tau}\delta\left(t-\frac{\tau}{2}\right)$$

[b] $\mathcal{F}\{f''(t)\} = \left[\dfrac{2A}{\tau}e^{j\omega\tau/2} - \dfrac{4A}{\tau} + \dfrac{2A}{\tau}e^{-j\omega\tau/2}\right]$

$$= \frac{4A}{\tau}\left[\frac{e^{j\omega\tau/2} + e^{-j\omega\tau/2}}{2} - 1\right] = \frac{4A}{\tau}\left[\cos\left(\frac{\omega\tau}{2}\right) - 1\right]$$

[c] $\mathcal{F}\{f''(t)\} = (j\omega)^2F(\omega) = -\omega^2F(\omega); \qquad$ therefore $F(\omega) = -\dfrac{1}{\omega^2}\mathcal{F}\{f''(t)\}$

Thus we have $F(\omega) = -\dfrac{1}{\omega^2}\left\{\dfrac{4A}{\tau}\left[\cos\left(\dfrac{\omega\tau}{2}\right) - 1\right]\right\}$

AP 17.5

$$v(t) = V_m\left[u\left(t+\frac{\tau}{2}\right) - u\left(t-\frac{\tau}{2}\right)\right]$$

$$\mathcal{F}\left\{u\left(t+\frac{\tau}{2}\right)\right\} = \left[\pi\delta(\omega) + \frac{1}{j\omega}\right]e^{j\omega\tau/2}$$

$$\mathcal{F}\left\{u\left(t-\frac{\tau}{2}\right)\right\} = \left[\pi\delta(\omega) + \frac{1}{j\omega}\right]e^{-j\omega\tau/2}$$

Therefore $V(\omega) = V_m\left[\pi\delta(\omega) + \dfrac{1}{j\omega}\right]\left[e^{j\omega\tau/2} - e^{-j\omega\tau/2}\right]$

$$= j2V_m\pi\delta(\omega)\sin\left(\frac{\omega\tau}{2}\right) + \frac{2V_m}{\omega}\sin\left(\frac{\omega\tau}{2}\right)$$

$$= \frac{(V_m\tau)\sin(\omega\tau/2)}{\omega\tau/2}$$

AP 17.6 **[a]** $I_g(\omega) = \mathcal{F}\{10\text{sgn}\, t\} = \dfrac{20}{j\omega}$

[b] $H(s) = \dfrac{V_o}{I_g}$

Using current division and Ohm's law,

$$V_o = -I_2 s = -\left[\frac{4}{4+1+s}\right](-I_g)s = \frac{4s}{5+s}I_g$$

$$H(s) = \frac{4s}{s+5}, \qquad H(j\omega) = \frac{j4\omega}{5+j\omega}$$

[c] $V_o(\omega) = H(j\omega)\cdot I_g(\omega) = \left(\dfrac{j4\omega}{5+j\omega}\right)\left(\dfrac{20}{j\omega}\right) = \dfrac{80}{5+j\omega}$

[d] $v_o(t) = 80e^{-5t}u(t)\,\text{V}$

[e] Using current division,

$$i_1(0^-) = \frac{1}{5}i_g = \frac{1}{5}(-10) = -2\,\text{A}$$

[f] $i_1(0^+) = i_g + i_2(0^+) = 10 + i_2(0^-) = 10 + 8 = 18\,\text{A}$

[g] Using current division,

$$i_2(0^-) = \frac{4}{5}(10) = 8\,\text{A}$$

[h] Since the current in an inductor must be continuous,

$$i_2(0^+) = i_2(0^-) = 8\,\text{A}$$

[i] Since the inductor behaves as a short circuit for $t < 0$,

$$v_o(0^-) = 0\,\text{V}$$

[j] $v_o(0^+) = 1i_2(0^+) + 4i_1(0^+) = 80\,\text{V}$

AP 17.7 **[a]** $V_g(\omega) = \dfrac{1}{1-j\omega} + \pi\delta(\omega) + \dfrac{1}{j\omega}$

$$H(s) = \frac{V_a}{V_g} = \frac{0.5\|(1/s)}{1+0.5\|(1/s)} = \frac{1}{s+3}, \qquad H(j\omega) = \frac{1}{3+j\omega}$$

$$\begin{aligned} V_a(\omega) &= H(j\omega)V_g(j\omega) \\ &= \frac{1}{(1-j\omega)(3+j\omega)} + \frac{1}{j\omega(3+j\omega)} + \frac{\pi\delta(\omega)}{3+j\omega} \\ &= \frac{1/4}{1-j\omega} + \frac{1/4}{3+j\omega} + \frac{1/3}{j\omega} - \frac{1/3}{3+j\omega} + \frac{\pi\delta(\omega)}{3+j\omega} \\ &= \frac{1/4}{1-j\omega} + \frac{1/3}{j\omega} - \frac{1/12}{3+j\omega} + \frac{\pi\delta(\omega)}{3+j\omega} \end{aligned}$$

Therefore $\quad v_a(t) = \left[\dfrac{1}{4}e^{t}u(-t) + \dfrac{1}{6}\text{sgn}\, t - \dfrac{1}{12}e^{-3t}u(t) + \dfrac{1}{6}\right]\,\text{V}$

[b] $$v_a(0^-) = \frac{1}{4} - \frac{1}{6} + 0 + \frac{1}{6} = \frac{1}{4}\,\mathrm{V}$$

$$v_a(0^+) = 0 + \frac{1}{6} - \frac{1}{12} + \frac{1}{6} = \frac{1}{4}\,\mathrm{V}$$

$$v_a(\infty) = 0 + \frac{1}{6} + 0 + \frac{1}{6} = \frac{1}{3}\,\mathrm{V}$$

AP 17.8

$$v(t) = 4te^{-t}u(t); \qquad V(\omega) = \frac{4}{(1+j\omega)^2}$$

Therefore $|V(\omega)| = \dfrac{4}{1+\omega^2}$

$$W_{1\Omega} = \frac{1}{\pi}\int_0^{\sqrt{3}} \left[\frac{4}{(1+\omega^2)}\right]^2 d\omega$$

$$= \frac{16}{\pi}\left\{\frac{1}{2}\left[\frac{\omega}{\omega^2+1} + \tan^{-1}\frac{\omega}{1}\right]_0^{\sqrt{3}}\right\}$$

$$= 16\left[\frac{\sqrt{3}}{8\pi} + \frac{1}{6}\right] = 3.769\,\mathrm{J}$$

$$W_{1\Omega}(\text{total}) = \frac{8}{\pi}\left[\frac{\omega}{\omega^2+1} + \tan^{-1}\frac{\omega}{1}\right]_0^{\infty} = \frac{8}{\pi}\left[0 + \frac{\pi}{2}\right] = 4\,\mathrm{J}$$

Therefore $\% = \dfrac{3.769}{4}(100) = 94.23\%$

AP 17.9

$$|V(\omega)| = 6 - \left(\frac{6}{2000\pi}\right)\omega, \qquad 0 \leq \omega \leq 2000\pi$$

$$|V(\omega)|^2 = 36 - \left(\frac{72}{2000\pi}\right)\omega + \left(\frac{36}{4\pi^2 \times 10^6}\right)\omega^2$$

$$W_{1\Omega} = \frac{1}{\pi}\int_0^{2000\pi}\left[36 - \frac{72\omega}{2000\pi} + \frac{36 \times 10^{-6}}{4\pi^2}\omega^2\right] d\omega$$

$$= \frac{1}{\pi}\left[36\omega - \frac{72\omega^2}{4000\pi} + \frac{36 \times 10^{-6}\omega^3}{12\pi^2}\right]_0^{2000\pi}$$

$$= \frac{1}{\pi}\left[36(2000\pi) - \frac{72}{4000\pi}(2000\pi)^2 + \frac{36 \times 10^{-6}(2000\pi)^3}{12\pi^2}\right]$$

$$= 36(2000) - \frac{72(2000)^2}{4000} + \frac{36 \times 10^{-6}(2000)^3}{12}$$

$$= 24\,\mathrm{kJ}$$

$$W_{6\mathrm{k}\Omega} = \frac{24 \times 10^3}{6 \times 10^3} = 4\,\mathrm{J}$$

Problems

P 17.1 **[a]** $F(\omega) = \int_{-\tau/2}^{\tau/2} \frac{2A}{\tau} te^{-j\omega t}\,dt$

$$= \frac{2A}{\tau}\left[\frac{e^{-j\omega t}}{-\omega^2}(-j\omega t - 1)\right]_{-\tau/2}^{\tau/2}$$

$$= \frac{2A}{\omega^2\tau}\left[e^{-j\omega\tau/2}\left(\frac{j\omega\tau}{2}+1\right) - e^{j\omega\tau/2}\left(\frac{-j\omega\tau}{2}+1\right)\right]$$

$$F(\omega) = \frac{2A}{\omega^2\tau}\left[e^{-j\omega\tau/2} - e^{j\omega\tau/2} + j\frac{\omega\tau}{2}\left(e^{-j\omega\tau/2}+e^{j\omega\tau/2}\right)\right]$$

$$F(\omega) = j\frac{2A}{\tau}\left[\frac{\omega\tau\cos(\omega\tau/2) - 2\sin(\omega\tau/2)}{\omega^2}\right]$$

[b] Using L'Hopital's rule,

$$F(0) = \lim_{\omega\to 0} 2A\left[\frac{\omega\tau(\tau/2)[-\sin(\omega\tau/2)] + \tau\cos(\omega\tau/2) - 2(\tau/2)\cos(\omega\tau/2)}{2\omega\tau}\right]$$

$$= \lim_{\omega\to 0} 2A\left[\frac{-\omega\tau(\tau/2)\sin(\omega\tau/2)}{2\omega\tau}\right]$$

$$= \lim_{\omega\to 0} 2A\left[\frac{-\tau\sin(\omega\tau/2)}{4}\right] = 0$$

$$\therefore \qquad F(0) = 0$$

[c] When $A = 1$ and $\tau = 1$

$$F(\omega) = j2\left[\frac{\omega\cos(\omega/2) - 2\sin(\omega/2)}{\omega^2}\right]$$

$$|F(\omega)| = \left|\frac{2\omega\cos(\omega/2) - 4\sin(\omega/2)}{\omega^2}\right|$$

$$F(0) = 0$$

$$|F(2)| = \left|\frac{4\cos 1 - 4\sin 1}{4}\right| = 0.30$$

$$|F(4)| = \left|\frac{8\cos 2 - 4\sin 2}{16}\right| = 0.44$$

$$|F(6)| = \left|\frac{12\cos 3 - 4\sin 3}{36}\right| = 0.35$$

$$|F(8)| = \left|\frac{16\cos 4 - 4\sin 4}{64}\right| = 0.12$$

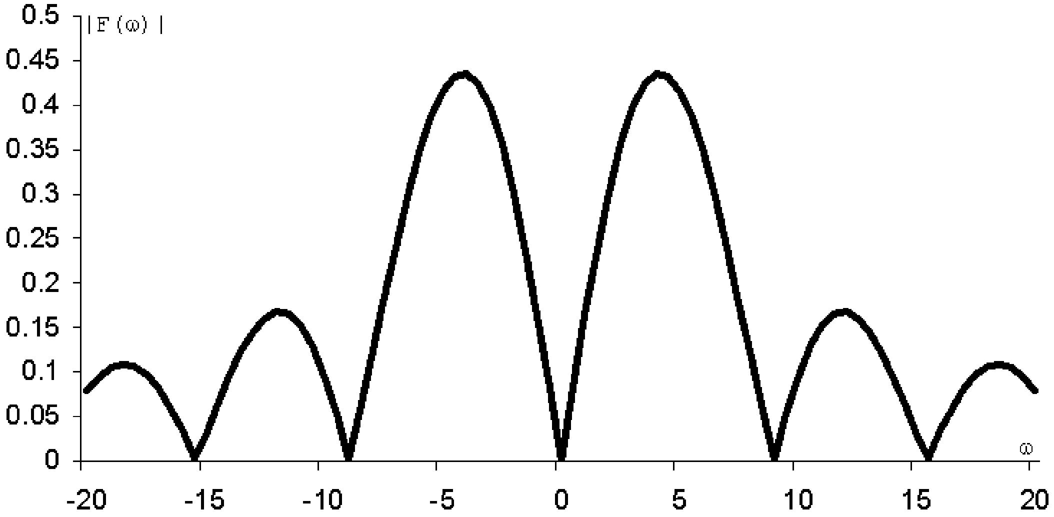
$$|F(9)| = \left|\frac{18\cos 4.5 - 4\sin 4.5}{81}\right| \cong 0$$

$$|F(10)| = \left|\frac{20\cos 5 - 4\sin 5}{100}\right| = 0.10$$

$$|F(12)| = \left|\frac{24\cos 6 - 4\sin 6}{144}\right| = 0.17$$

$$|F(14)| = \left|\frac{28\cos 7 - 4\sin 7}{196}\right| = 0.09$$

$$|F(15.5)| = \left|\frac{31\cos 7.75 - 4\sin 7.75}{240.25}\right| \cong 0$$



P 17.2 **[a]** $F(\omega) = A + \frac{2A}{\omega_o}\omega, \quad -\omega_o/2 \le \omega \le 0$

$$F(\omega) = A - \frac{2A}{\omega_o}\omega, \quad 0 \le \omega \le \omega_o/2$$

$$F(\omega) = 0 \qquad \text{elsewhere}$$

$$f(t) = \frac{1}{2\pi}\int_{-\omega_o/2}^{0}\left(A + \frac{2A}{\omega_o}\omega\right)e^{jt\omega}\,d\omega$$

$$+\frac{1}{2\pi}\int_{0}^{\omega_o/2}\left(A - \frac{2A}{\omega_o}\omega\right)e^{jt\omega}\,d\omega$$

$$f(t) = \frac{1}{2\pi}\left[\int_{-\omega_o/2}^{0} Ae^{jt\omega}\,d\omega + \int_{-\omega_o/2}^{0}\frac{2A}{\omega_o}\omega e^{jt\omega}\,d\omega\right.$$

$$+\int_0^{\omega_o/2} Ae^{jt\omega}\,d\omega - \int_0^{\omega_o/2}\frac{2A}{\omega_o}\omega e^{jt\omega}\,d\omega\ \Big]$$

$$= \frac{1}{2\pi}[\ \text{Int1}\ +\ \text{Int2}\ +\ \text{Int3}\ -\ \text{Int4}\ ]$$

$$\text{Int1}\ = \int_{-\omega_o/2}^{0} Ae^{jt\omega}\,d\omega = \frac{A}{jt}(1-e^{-jt\omega_o/2})$$

$$\text{Int2}\ = \int_{-\omega_o/2}^{0}\frac{2A}{\omega_o}\omega e^{jt\omega}\,d\omega = \frac{2A}{\omega_o t^2}(1 - j\frac{t\omega_o}{2}e^{-jt\omega_o/2} - e^{-jt\omega_o/2})$$

$$\text{Int3}\ = \int_0^{\omega_o/2} Ae^{jt\omega}\,d\omega = \frac{A}{jt}(e^{jt\omega_o/2}-1)$$

$$\text{Int4}\ = \int_0^{\omega_o/2}\frac{2A}{\omega_o}\omega e^{jt\omega}\,d\omega = \frac{2A}{\omega_o t^2}(-j\frac{t\omega_o}{2}e^{jt\omega_o/2} + e^{jt\omega_o/2} - 1)$$

$$\text{Int1}\ +\ \text{Int3}\ = \frac{2A}{t}\sin(\omega_o t/2)$$

$$\text{Int2}\ -\ \text{Int4}\ = \frac{4A}{\omega_o t^2}[1-\cos(\omega_o t/2)] - \frac{2A}{t}\sin(\omega_o t/2)$$

$$\therefore\qquad f(t) = \frac{1}{2\pi}\left[\frac{4A}{\omega_o t^2}\left(1-\cos(\omega_o t/2)\right)\right]$$

$$= \frac{2A}{\pi\omega_o t^2}\left[2\sin^2(\omega_o t/4)\right]$$

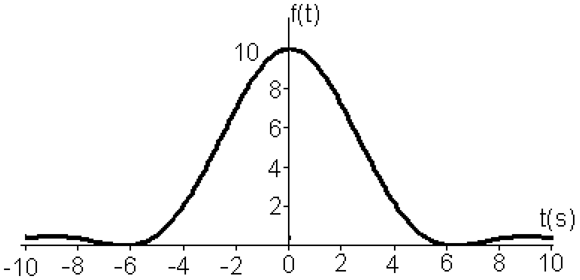
$$= \frac{4\omega_o A}{\pi\omega_o^2 t^2}\sin^2(\omega_o t/4)$$

$$= \frac{\omega_o A}{4\pi}\left[\frac{\sin(\omega_o t/4)}{(\omega_o t/4)}\right]^2$$

[b] $f(0) = \frac{\omega_o A}{4\pi}(1)^2 = 79.58\times 10^{-3}\omega_o A$

[c] $A = 20\pi;\qquad \omega_o = 2$ rad/s

$$f(t) = 10\left[\frac{\sin(t/2)}{(t/2)}\right]^2$$



P 17.3 **[a]** $F(\omega) = \int_{-2}^{2}\left[A\sin\left(\frac{\pi}{2}\right)t\right]e^{-j\omega t}\,dt = \frac{-j4\pi A}{\pi^2 - 4\omega^2}\sin 2\omega$

[b] $F(\omega) = \int_{-\tau/2}^{0}\left(\frac{2A}{\tau}t + A\right)e^{-j\omega t}\,dt + \int_{0}^{\tau/2}\left(\frac{-2A}{\tau}t + A\right)e^{-j\omega t}\,dt$

$$= \frac{4A}{\omega^2\tau}\left[1 - \cos\left(\frac{\omega\tau}{2}\right)\right]$$

P 17.4 $\mathcal{F}\{\sin\omega_0 t\} = \mathcal{F}\left\{\frac{e^{j\omega_0 t}}{2j}\right\} - \mathcal{F}\left\{\frac{e^{-j\omega_0 t}}{2j}\right\}$

$$= \frac{1}{2j}[2\pi\delta(\omega - \omega_0) - 2\pi\delta(\omega + \omega_0)]$$

$$= j\pi[\delta(\omega + \omega_0) - \delta(\omega - \omega_0)]$$

P 17.5 **[a]** $F(s) = \mathcal{L}\{te^{-at}\} = \frac{1}{(s+a)^2}$

$$F(\omega) = F(s)\Big|_{s=j\omega} + F(s)\Big|_{s=-j\omega}$$

$$F(\omega) = \left[\frac{1}{(a + j\omega)^2}\right] + \left[\frac{1}{(a - j\omega)^2}\right]$$

$$= \frac{2(a^2 - \omega^2)}{(a^2 - \omega^2)^2 + 4a^2\omega^2} = \frac{2(a^2 - \omega^2)}{(a^2 + \omega^2)^2}$$

[b] $F(s) = \mathcal{L}\{t^3 e^{-at}\} = \frac{6}{(s+a)^4}$

$$F(\omega) = F(s)\Big|_{s=j\omega} - F(s)\Big|_{s=-j\omega}$$

$$F(\omega) = \frac{6}{(a + j\omega)^4} + \frac{6}{(a - j\omega)^4} = -j48a\omega\frac{a^2 - \omega^2}{(a^2 + \omega^2)^4}$$

[c] $F(s) = \mathcal{L}\{e^{-at}\cos\omega_0 t\} = \frac{s + a}{(s+a)^2 + \omega_0^2} = \frac{0.5}{(s+a) - j\omega_0} + \frac{0.5}{(s+a) + j\omega_0}$

$$F(\omega) = F(s)\Big|_{s=j\omega} + F(s)\Big|_{s=-j\omega}$$

$$F(\omega) = \frac{0.5}{(a + j\omega) - j\omega_0} + \frac{0.5}{(a + j\omega) + j\omega_0}$$

$$+ \frac{0.5}{(a - j\omega) - j\omega_0} + \frac{0.5}{(a - j\omega) + j\omega_0}$$

$$= \frac{a}{a^2 + (\omega - \omega_0)^2} + \frac{a}{a^2 + (\omega + \omega_0)^2}$$

[d] $F(s) = \mathcal{L}\{e^{-at}\sin\omega_0 t\} = \dfrac{\omega_0}{(s+a)^2+\omega_0^2} = \dfrac{-j0.5}{(s+a)-j\omega_0} + \dfrac{j0.5}{(s+a)+j\omega_0}$

$$F(\omega) = F(s)\Big|_{s=j\omega} - F(s)\Big|_{s=-j\omega}$$

$$F(\omega) = \frac{-ja}{a^2+(\omega-\omega_0)^2} + \frac{ja}{a^2+(\omega+\omega_0)^2}$$

[e] $F(\omega) = \displaystyle\int_{-\infty}^{\infty} \delta(t-t_o)e^{-j\omega t}\,dt = e^{-j\omega t_o}$

(Use the sifting property of the Dirac delta function.)

P 17.6 $f(t) = \dfrac{1}{2\pi}\displaystyle\int_{-\infty}^{\infty}[A(\omega)+jB(\omega)][\cos t\omega + j\sin t\omega]\,d\omega$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty}[A(\omega)\cos t\omega - B(\omega)\sin t\omega]\,d\omega$$

$$+ \frac{j}{2\pi}\int_{-\infty}^{\infty}[A(\omega)\sin t\omega + B(\omega)\cos t\omega]\,d\omega$$

But $f(t)$ is real, therefore the second integral in the sum is zero.

P 17.7 By hypothesis, $f(t) = -f(-t)$. From Problem 17.6, we have

$$f(-t) = \frac{1}{2\pi}\int_{-\infty}^{\infty}[A(\omega)\cos t\omega + B(\omega)\sin t\omega]\,d\omega$$

For $f(t) = -f(-t)$, the integral $\int_{-\infty}^{\infty} A(\omega)\cos t\omega\,d\omega$ must be zero. Therefore, if $f(t)$ is real and odd, we have

$$f(t) = \frac{-1}{2\pi}\int_{-\infty}^{\infty} B(\omega)\sin t\omega\,d\omega$$

P 17.8 $F(\omega) = \dfrac{-j2}{\omega}$; therefore $B(\omega) = \dfrac{-2}{\omega}$; thus we have

$$f(t) = -\frac{1}{2\pi}\int_{-\infty}^{\infty}\left(\frac{-2}{\omega}\right)\sin t\omega\,d\omega = \frac{1}{\pi}\int_{-\infty}^{\infty}\frac{\sin t\omega}{\omega}\,d\omega$$

But $\dfrac{\sin t\omega}{\omega}$ is even; therefore $f(t) = \dfrac{2}{\pi}\displaystyle\int_{0}^{\infty}\frac{\sin t\omega}{\omega}\,d\omega$

Therefore,

$$\left.\begin{array}{ll} f(t) = \dfrac{2}{\pi}\cdot\dfrac{\pi}{2} = 1 & t > 0 \\ f(t) = \dfrac{2}{\pi}\cdot\left(\dfrac{-\pi}{2}\right) = -1 & t < 0 \end{array}\right\}\text{from a table of definite integrals}$$

Therefore $f(t) = \operatorname{sgn} t$

P 17.9 From Problem 17.5[c] we have

$$F(\omega) = \frac{\epsilon}{\epsilon^2 + (\omega - \omega_0)^2} + \frac{\epsilon}{\epsilon^2 + (\omega + \omega_0)^2}$$

Note that as $\epsilon \to 0$, $F(\omega) \to 0$ everywhere except at $\omega = \pm\omega_0$. At $\omega = \pm\omega_0$, $F(\omega) = 1/\epsilon$, therefore $F(\omega) \to \infty$ at $\omega = \pm\omega_0$ as $\epsilon \to 0$. The area under each bell-shaped curve is independent of ϵ, that is

$$\int_{-\infty}^{\infty} \frac{\epsilon d\omega}{\epsilon^2 + (\omega - \omega_0)^2} = \int_{-\infty}^{\infty} \frac{\epsilon d\omega}{\epsilon^2 + (\omega + \omega_0)^2} = \pi$$

Therefore as $\epsilon \to 0$, $\quad F(\omega) \to \pi\delta(\omega - \omega_0) + \pi\delta(\omega + \omega_0)$

P 17.10 $A(\omega) = \int_{-\infty}^{0} f(t)\cos\omega t\,dt + \int_{0}^{\infty} f(t)\cos\omega t\,dt = 0$

since $f(t)\cos\omega t$ is an odd function.

$B(\omega) = -2\int_0^\infty f(t)\sin\omega t\,dt$, since $f(t)\sin\omega t$ is an even function.

P 17.11

$$\begin{aligned} A(\omega) &= \int_{-\infty}^{\infty} f(t)\cos\omega t\,dt \\ &= \int_{-\infty}^{0} f(t)\cos\omega t\,dt + \int_{0}^{\infty} f(t)\cos\omega t\,dt \\ &= 2\int_0^\infty f(t)\cos\omega t\,dt, \quad \text{since } f(t)\cos\omega t \text{ is also even.} \end{aligned}$$

$B(\omega) = 0$, since $f(t)\sin\omega t$ is an odd function and

$$\int_{-\infty}^{0} f(t)\sin\omega t\,dt = -\int_0^\infty f(t)\sin\omega t\,dt$$

P 17.12 **[a]** $\mathcal{F}\left\{\frac{df(t)}{dt}\right\} = \int_{-\infty}^{\infty} \frac{df(t)}{dt} e^{-j\omega t}\,dt$

Let $u = e^{-j\omega t}$, then $du = -j\omega e^{-j\omega t}\,dt$; let $dv = [df(t)/dt]\,dt$, then $v = f(t)$.

$$\text{Therefore} \quad \mathcal{F}\left\{\frac{df(t)}{dt}\right\} = f(t)e^{-j\omega t}\Big|_{-\infty}^{\infty} - \int_{-\infty}^{\infty} f(t)[-j\omega e^{-j\omega t}\,dt]$$

$$= 0 + j\omega F(\omega)$$

[b] Fourier transform of $f(t)$ exists, i.e., $f(\infty) = f(-\infty) = 0$.

[c] To find $\mathcal{F}\left\{\frac{d^2f(t)}{dt^2}\right\}$, let $g(t) = \frac{df(t)}{dt}$

Then $\mathcal{F}\left\{\frac{d^2f(t)}{dt^2}\right\} = \mathcal{F}\left\{\frac{dg(t)}{dt}\right\} = j\omega G(\omega)$

But $G(\omega) = \mathcal{F}\left\{\frac{df(t)}{dt}\right\} = j\omega F(\omega)$

Therefore we have $\mathcal{F}\left\{\frac{d^2f(t)}{dt^2}\right\} = (j\omega)^2F(\omega)$

Repeated application of this thought process gives

$$\mathcal{F}\left\{\frac{d^nf(t)}{dt^n}\right\} = (j\omega)^nF(\omega)$$

P 17.13 **[a]** $\mathcal{F}\left\{\int_{-\infty}^{t} f(x)\,dx\right\} = \int_{-\infty}^{\infty}\left[\int_{-\infty}^{t} f(x)\,dx\right]e^{-j\omega t}\,dt$

Now let $u = \int_{-\infty}^{t} f(x)\,dx,$ then $du = f(t)dt$

Let $dv = e^{-j\omega t}\,dt,$ then $v = \frac{e^{-j\omega t}}{-j\omega}$

Therefore,

$$\mathcal{F}\left\{\int_{-\infty}^{t} f(x)\,dx\right\} = \frac{e^{-j\omega t}}{-j\omega}\int_{-\infty}^{t} f(x)\,dx\,\Big|_{-\infty}^{\infty} - \int_{-\infty}^{\infty}\left[\frac{e^{-j\omega t}}{-j\omega}\right]f(t)\,dt$$

$$= 0 + \frac{F(\omega)}{j\omega}$$

[b] We require $\int_{-\infty}^{\infty} f(x)\,dx = 0$

[c] No, because $\int_{-\infty}^{\infty} e^{-ax}u(x)\,dx = \frac{1}{a} \neq 0$

P 17.14 **[a]** $\mathcal{F}\{f(at)\} = \int_{-\infty}^{\infty} f(at)e^{-j\omega t}\,dt$

Let $u = at,$ $du = adt,$ $u = \pm\infty$ when $t = \pm\infty$

Therefore,

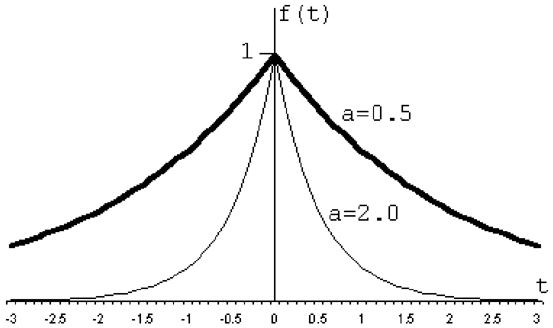
$$\mathcal{F}\{f(at)\} = \int_{-\infty}^{\infty} f(u)e^{-j\omega u/a}\left(\frac{du}{a}\right) = \frac{1}{a}F\left(\frac{\omega}{a}\right), \qquad a > 0$$

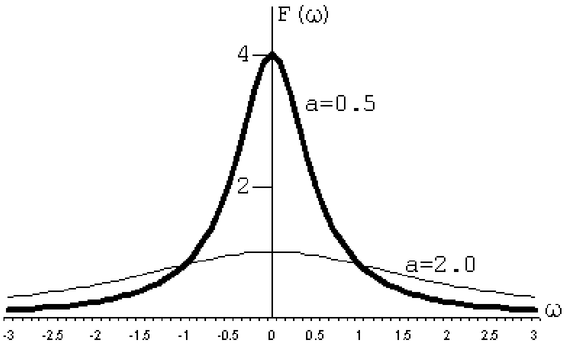
[b] $\mathcal{F}\{e^{-|t|}\} = \dfrac{1}{1+j\omega} + \dfrac{1}{1-j\omega} = \dfrac{2}{1+\omega^2}$

Therefore $\mathcal{F}\{e^{-a|t|}\} = \dfrac{(1/a)2}{(\omega/a)^2+1}$

Therefore $\mathcal{F}\{e^{-0.5|t|}\} = \dfrac{4}{4\omega^2+1}$, $\qquad \mathcal{F}\{e^{-|t|}\} = \dfrac{2}{\omega^2+1}$

$\mathcal{F}\{e^{-2|t|}\} = 1/[0.25\omega^2+1]$, yes as "$a$" increases, the sketches show that $f(t)$ approaches zero faster and $F(\omega)$ flattens out over the frequency spectrum.





P 17.15 [a] $\mathcal{F}\{f(t-a)\} = \int_{-\infty}^{\infty} f(t-a)e^{-j\omega t}\,dt$

Let $u = t - a$, then $du = dt$, $t = u + a$, and $u = \pm\infty$ when $t = \pm\infty$. Therefore,

$$\mathcal{F}\{f(t-a)\} = \int_{-\infty}^{\infty} f(u)e^{-j\omega(u+a)}\,du$$

$$= e^{-j\omega a}\int_{-\infty}^{\infty} f(u)e^{-j\omega u}\,du = e^{-j\omega a}F(\omega)$$

[b] $\mathcal{F}\{e^{j\omega_0 t}f(t)\} = \int_{-\infty}^{\infty} f(t)e^{-j(\omega-\omega_0)t}\,dt = F(\omega-\omega_0)$

[c] $\mathcal{F}\{f(t)\cos\omega_0 t\} = \mathcal{F}\left\{f(t)\left[\dfrac{e^{j\omega_0 t}+e^{-j\omega_0 t}}{2}\right]\right\}$

$$= \frac{1}{2}F(\omega-\omega_0) + \frac{1}{2}F(\omega+\omega_0)$$

P 17.16 $Y(\omega) = \int_{-\infty}^{\infty}\left[\int_{-\infty}^{\infty} x(\lambda)h(t-\lambda)\,d\lambda\right]e^{-j\omega t}\,dt$

$$= \int_{-\infty}^{\infty} x(\lambda)\left[\int_{-\infty}^{\infty} h(t-\lambda)e^{-j\omega t}\,dt\right]d\lambda$$

Let $u = t - \lambda$, $du = dt$, and $u = \pm\infty$, when $t = \pm\infty$.

$$\text{Therefore} \quad Y(\omega) = \int_{-\infty}^{\infty} x(\lambda)\left[\int_{-\infty}^{\infty} h(u)e^{-j\omega(u+\lambda)}\,du\right]d\lambda$$

$$= \int_{-\infty}^{\infty} x(\lambda)\left[e^{-j\omega\lambda}\int_{-\infty}^{\infty} h(u)e^{-j\omega u}\,du\right]d\lambda$$

$$= \int_{-\infty}^{\infty} x(\lambda)e^{-j\omega\lambda}H(\omega)\,d\lambda = H(\omega)X(\omega)$$

P 17.17 $\mathcal{F}\{f_1(t)f_2(t)\} = \int_{-\infty}^{\infty}\left[\frac{1}{2\pi}\int_{-\infty}^{\infty} F_1(u)e^{jtu}du\right] f_2(t)e^{-j\omega t}\,dt$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty}\left[\int_{-\infty}^{\infty} F_1(u)f_2(t)e^{-j\omega t}e^{jtu}\,du\right]dt$$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty}\left[F_1(u)\int_{-\infty}^{\infty} f_2(t)e^{-j(\omega-u)t}\,dt\right]du$$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty} F_1(u)F_2(\omega-u)\,du$$

P 17.18 **[a]** $F(\omega) = \int_{-\infty}^{\infty} f(t)e^{-j\omega t}\,dt$

$$\frac{dF}{d\omega} = \int_{-\infty}^{\infty}\frac{d}{d\omega}\left[f(t)e^{-j\omega t}\,dt\right] = -j\int_{-\infty}^{\infty} tf(t)e^{-j\omega t}\,dt = -j\mathcal{F}\{tf(t)\}$$

$$\text{Therefore} \quad j\frac{dF(\omega)}{d\omega} = \mathcal{F}\{tf(t)\}$$

$$\frac{d^2F(\omega)}{d\omega^2} = \int_{-\infty}^{\infty}(-jt)(-jt)f(t)e^{-j\omega t}\,dt = (-j)^2\mathcal{F}\{t^2f(t)\}$$

$$\text{Note that} \quad (-j)^n = \frac{1}{j^n}$$

$$\text{Thus we have} \quad j^n\left[\frac{d^nF(\omega)}{d\omega^n}\right] = \mathcal{F}\{t^nf(t)\}$$

[b] (i) $\mathcal{F}\{e^{-at}u(t)\} = \frac{1}{a+j\omega} = F(\omega);\qquad \frac{dF(\omega)}{d\omega} = \frac{-j}{(a+j\omega)^2}$

$$\text{Therefore} \quad j\left[\frac{dF(\omega)}{d\omega}\right] = \frac{1}{(a+j\omega)^2}$$

$$\text{Therefore} \quad \mathcal{F}\{te^{-at}u(t)\} = \frac{1}{(a+j\omega)^2}$$

(ii) $\mathcal{F}\{|t|e^{-a|t|}\} = \mathcal{F}\{te^{-at}u(t)\} - \mathcal{F}\{te^{at}u(-t)\}$

$$= \frac{1}{(a+j\omega)^2} - j\frac{d}{d\omega}\left(\frac{1}{a-j\omega}\right)$$

$$= \frac{1}{(a+j\omega)^2} + \frac{1}{(a-j\omega)^2}$$

(iii) $\mathcal{F}\{te^{-a|t|}\} = \mathcal{F}\{te^{-at}u(t)\} + \mathcal{F}\{te^{at}u(-t)\}$

$$= \frac{1}{(a+j\omega)^2} + j\frac{d}{d\omega}\left(\frac{1}{a-j\omega}\right)$$

$$= \frac{1}{(a+j\omega)^2} - \frac{1}{(a-j\omega)^2}$$

P 17.19 **[a]** $f_1(t) = \cos\omega_0 t, \qquad F_1(u) = \pi[\delta(u+\omega_0) + \delta(u-\omega_0)]$

$f_2(t) = 1, \quad -\tau/2 < t < \tau/2, \qquad$ and $f_2(t) = 0$ elsewhere

Thus $F_2(u) = \dfrac{\tau\sin(u\tau/2)}{u\tau/2}$

Using convolution,

$$F(\omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty} F_1(u)F_2(\omega-u)\,du$$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty} \pi[\delta(u+\omega_0) + \delta(u-\omega_0)]\tau\frac{\sin[(\omega-u)\tau/2]}{(\omega-u)(\tau/2)}\,du$$

$$= \frac{\tau}{2}\int_{-\infty}^{\infty} \delta(u+\omega_0)\frac{\sin[(\omega-u)\tau/2]}{(\omega-u)(\tau/2)}\,du$$

$$+ \frac{\tau}{2}\int_{-\infty}^{\infty} \delta(u-\omega_0)\frac{\sin[(\omega-u)\tau/2]}{(\omega-u)(\tau/2)}\,du$$

$$= \frac{\tau}{2}\cdot\frac{\sin[(\omega+\omega_0)\tau/2]}{(\omega+\omega_0)(\tau/2)} + \frac{\tau}{2}\cdot\frac{\sin[(\omega-\omega_0)\tau/2]}{(\omega-\omega_0)\tau/2}$$

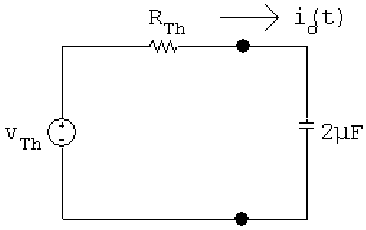
[b] As τ increases, the amplitude of $F(\omega)$ increases at $\omega = \pm\omega_0$ and at the same time the width of the frequency band of $F(\omega)$ approaches zero as ω deviates from $\pm\omega_0$.
The area under the $[\sin x]/x$ function is independent of τ, that is

$$\frac{\tau}{2}\int_{-\infty}^{\infty} \frac{\sin[(\omega-\omega_0)(\tau/2)]}{(\omega-\omega_0)(\tau/2)}\,d\omega = \int_{-\infty}^{\infty} \frac{\sin[(\omega-\omega_0)(\tau/2)]}{(\omega-\omega_0)(\tau/2)}[(\tau/2)\,d\omega] = \pi$$

Therefore as $t \to \infty$,

$$f_1(t)f_2(t) \to \cos\omega_0 t \quad \text{and} \quad F(\omega) \to \pi[\delta(\omega-\omega_0) + \delta(\omega+\omega_0)]$$

P 17.20 **[a]** Find the Thévenin equivalent with respect to the terminals of the capacitor:



$$v_{\text{Th}} = \frac{5}{6}v_g; \qquad R_{\text{Th}} = 60\|12 = 10\,\text{k}\Omega$$

$$I_o = \frac{V_{\text{Th}}}{10{,}000 + 10^6/2s} = \frac{2sV_{\text{Th}}}{20{,}000s + 10^6}$$

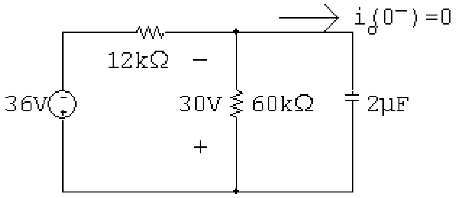
$$H(s) = \frac{I_o}{V_{\text{Th}}} = \frac{10^{-4}s}{s+50}; \qquad H(j\omega) = \frac{j\omega \times 10^{-4}}{j\omega + 50}$$

$$v_{\text{Th}} = \frac{5}{6}v_g = 30\text{sgn}(t); \qquad V_{\text{Th}} = \frac{60}{j\omega}$$

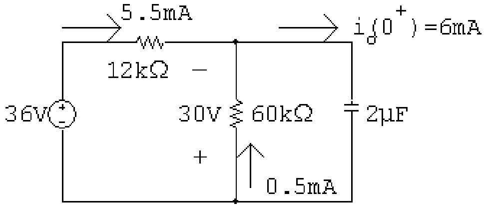
$$I_o = H(j\omega)V_{\text{Th}}(j\omega) = \left(\frac{60}{j\omega}\right)\left(\frac{j\omega \times 10^{-4}}{j\omega + 50}\right) = \frac{6 \times 10^{-3}}{j\omega + 50}$$

$$i_o(t) = 6e^{-50t}u(t)\,\text{mA}$$

[b] At $t = 0^-$ the circuit is



At $t = 0^+$ the circuit is



$$i_g(0^+) = \frac{30+36}{12} = 5.5\,\text{mA}$$

$$i_{60k}(0^+) = \frac{30}{60} = 0.5\,\text{mA}$$

$$i_o(0^+) = 5.5 + 0.5 = 6\,\text{mA}$$

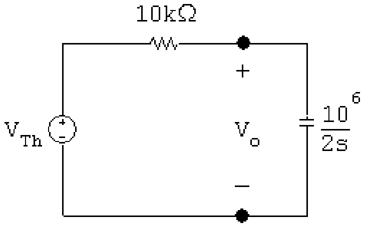
which agrees with our solution.

We also know $i_o(\infty) = 0$, which agrees with our solution.
The time constant with respect to the terminals of the capacitor is $R_{\text{Th}}C$ Thus,

$$\tau = (10{,}000)(2 \times 10^{-6}) = 20\,\text{ms}; \qquad \therefore \quad \frac{1}{\tau} = 50,$$

which also agrees with our solution.
Thus our solution makes sense in terms of known circuit behavior.

P 17.21 **[a]** From the solution of Problem 17.20 we have



$$V_o = \frac{V_{\text{Th}}}{10^4 + (10^6/2s)} \cdot \frac{10^6}{2s}$$

$$H(s) = \frac{V_o}{V_{\text{Th}}} = \frac{50}{s+50}$$

$$H(j\omega) = \frac{50}{j\omega + 50}$$

$$V_{\text{Th}}(\omega) = \frac{60}{j\omega}$$

$$V_o(\omega) = H(j\omega)V_{\text{Th}}(\omega) = \left(\frac{60}{j\omega}\right)\frac{50}{j\omega+50}$$

$$= \frac{3000}{(j\omega)(j\omega+50)} = \frac{60}{j\omega} - \frac{60}{j\omega+50}$$

$$v_o(t) = 30\text{sgn}(t) - 60e^{-50t}u(t)\,\text{V}$$

[b] $v_o(0^-) = -30\,\text{V}$

$$v_o(0^+) = 30 - 60 = -30\,\text{V}$$

This makes sense because there cannot be an instantaneous change in the voltage across a capacitor.

$$v_o(\infty) = 30\,\text{V}$$

This agrees with $v_{\text{Th}}(\infty) = 30$ V.
As in Problem 17.22 we know the time constant is 20 ms.

P 17.22 **[a]** $v_g = 100u(t)$

$$V_g(\omega) = 100\left[\pi\delta(\omega) + \frac{1}{j\omega}\right]$$

$$H(s) = \frac{10}{5s+10} = \frac{2}{s+2}$$

$$H(\omega) = \frac{2}{j\omega+2}$$

$$V_o(\omega) = H(\omega)V_g(\omega) = \frac{200\pi\delta(\omega)}{j\omega+2} + \frac{200}{j\omega(j\omega+2)}$$

$$= V_1(\omega) + V_2(\omega)$$

$$v_1(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \frac{200\pi e^{jt\omega}}{j\omega+2}\delta(\omega)\,d\omega = \frac{1}{2\pi}\left(\frac{200\pi}{2}\right) = 50 \text{ (sifting property)}$$

$$V_2(\omega) = \frac{K_1}{j\omega} + \frac{K_2}{j\omega+2} = \frac{100}{j\omega} - \frac{100}{j\omega+2}$$

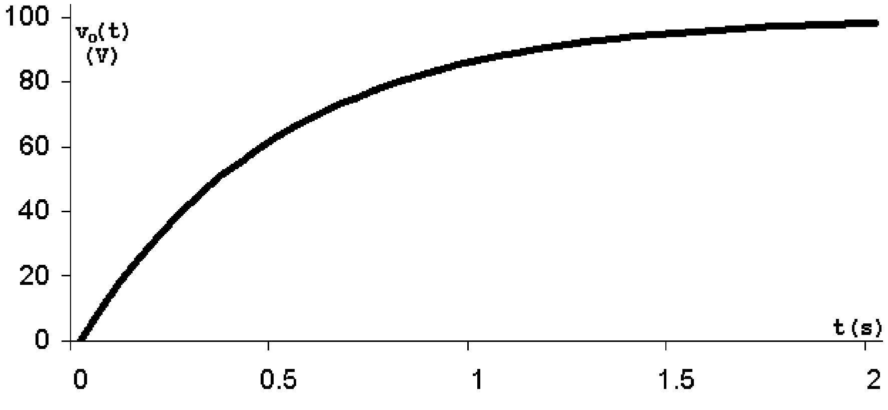
$$v_2(t) = 50\text{sgn}(t) - 100e^{-2t}u(t)$$

$$v_o(t) = v_1(t) + v_2(t) = 50 + 50\text{sgn}(t) - 100e^{-2t}u(t)$$

$$= 100u(t) - 100e^{-2t}u(t)$$

$$v_o(t) = 100(1 - e^{-2t})u(t)\,\text{V}$$

[b]



P 17.23 **[a]** From the solution to Problem 17.22

$$H(\omega) = \frac{2}{j\omega+2}$$

Now,

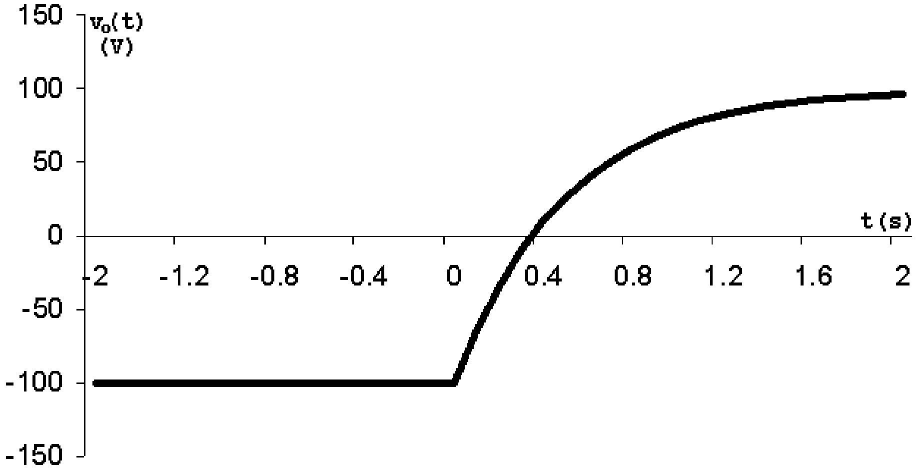
$$V_g(\omega) = \frac{200}{j\omega}$$

Then,

$$V_o(\omega) = H(\omega)V_g(\omega) = \frac{400}{j\omega(j\omega+2)} = \frac{K_1}{j\omega} + \frac{K_2}{j\omega+2} = \frac{200}{j\omega} - \frac{200}{j\omega+2}$$

$$\therefore \quad v_o(t) = 100\text{sgn}(t) - 200e^{-2t}u(t)\,\text{V}$$

[**b**]



P 17.24 [**a**] $I_o = \dfrac{I_gR}{R+1/sC} = \dfrac{RCsI_g}{RCs+1}; \qquad H(s) = \dfrac{I_o}{I_g} = \dfrac{s}{s+1/RC}$

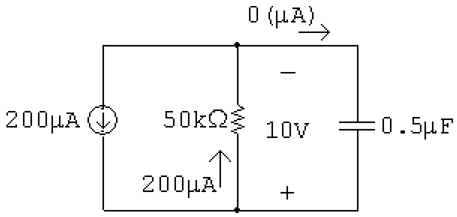
$$\frac{1}{RC} = \frac{10^6}{25\times 10^3} = 40; \qquad H(j\omega) = \frac{j\omega}{j\omega+40}$$

$$i_g = 200\text{sgn}(t)\,\mu\text{A}; \qquad I_g = (200\times 10^{-6})\left(\frac{2}{j\omega}\right) = \frac{400\times 10^{-6}}{j\omega}$$

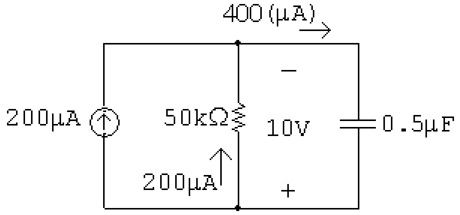
$$I_o = I_g[H(j\omega)] = \frac{400\times 10^{-6}}{j\omega}\cdot\frac{j\omega}{j\omega+40} = \frac{400\times 10^{-6}}{j\omega+40}$$

$$i_o(t) = 400e^{-40t}u(t)\,\mu\text{A}$$

[**b**] Yes, at the time the source current jumps from $-200\,\mu$A to $+200\,\mu$A the capacitor is charged to $(200)(50)\times 10^{-3} = 10$ V, positive at the lower terminal. The circuit at $t = 0^-$ is



At $t = 0^+$ the circuit is



The time constant is $(50 \times 10^3)(0.5 \times 10^{-6}) = 25$ ms.

$$\therefore \quad \frac{1}{\tau} = 40 \quad \therefore \qquad \text{for } t > 0, \quad i_o = 400e^{-40t}\,\mu\text{A}$$

P 17.25 **[a]** $V_o = \dfrac{I_g R(1/sC)}{R + (1/sC)} = \dfrac{I_g R}{RCs + 1}$

$$H(s) = \frac{V_o}{I_g} = \frac{1/C}{s + (1/RC)} = \frac{2 \times 10^6}{s + 40}$$

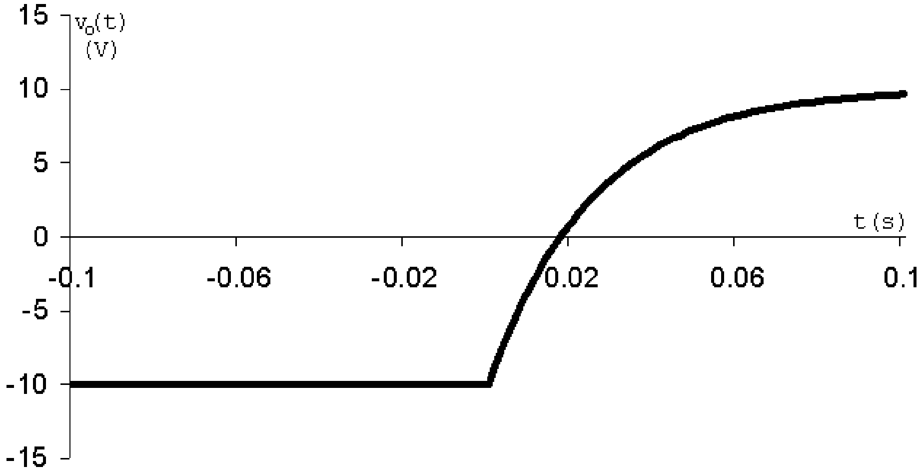
$$H(j\omega) = \frac{2 \times 10^6}{40 + j\omega}; \qquad I_g(\omega) = \frac{400 \times 10^{-6}}{j\omega}$$

$$V_o(\omega) = H(j\omega)I_g(\omega) = \left(\frac{400 \times 10^{-6}}{j\omega}\right)\left(\frac{2 \times 10^6}{40 + j\omega}\right)$$

$$= \frac{800}{j\omega(40 + j\omega)} = \frac{20}{j\omega} - \frac{20}{40 + j\omega}$$

$$v_o(t) = 10\text{sgn}(t) - 20e^{-40t}u(t)\,\text{V}$$

[b] Yes, at the time the current source jumps from −200 to +200 μA the capacitor is charged to −10 V. That is, at $t = 0^-$,
$v_o(0^-) = (50 \times 10^3)(-200 \times 10^{-6}) = -10$ V.
At $t = \infty$ the capacitor will be charged to +10 V. That is,
$v_o(\infty) = (50 \times 10^3)(200 \times 10^{-6}) = 10$ V
The time constant of the circuit is $(50 \times 10^3)(0.5 \times 10^{-6}) = 25$ ms, so $1/\tau = 40$. The function $v_o(t)$ is plotted below:



P 17.26 **[a]** $\frac{V_o}{V_g} = H(s) = \frac{4/s}{0.5 + 0.01s + 4/s}$

$$H(s) = \frac{400}{s^2 + 50s + 400} = \frac{400}{(s + 10)(s + 40)}$$

$$H(j\omega) = \frac{400}{(j\omega + 10)(j\omega + 40)}$$

$$V_g(\omega) = \frac{6}{j\omega}$$

$$V_o(\omega) = V_g(\omega)H(j\omega) = \frac{2400}{j\omega(j\omega + 10)(j\omega + 40)}$$

$$V_o(\omega) = \frac{K_1}{j\omega} + \frac{K_2}{j\omega + 10} + \frac{K_3}{j\omega + 40}$$

$$K_1 = \frac{2400}{400} = 6; \qquad K_2 = \frac{2400}{(-10)(30)} = -8$$

$$K_3 = \frac{2400}{(-40)(-30)} = 2$$

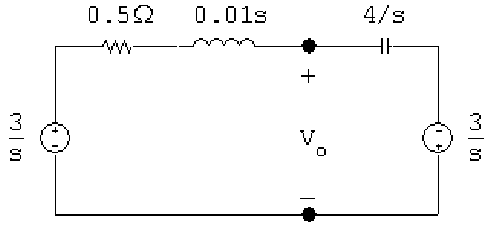
$$V_o(\omega) = \frac{6}{j\omega} - \frac{8}{j\omega + 10} + \frac{2}{j\omega + 40}$$

$$v_o(t) = 3\text{sgn}(t) - 8e^{-10t}u(t) + 2e^{-40t}u(t)\,\text{V}$$

[b] $v_o(0^-) = -3\,\text{V}$

[c] $v_o(0^+) = 3 - 8 + 2 = -3\,\text{V}$

[d] For $t \geq 0^+$:



$$\frac{V_o - 3/s}{0.5 + 0.01s} + \frac{(V_o + 3/s)s}{4} = 0$$

$$V_o\left[\frac{100}{s + 50} + \frac{s}{4}\right] = \frac{300}{s(s + 50)} - 0.75$$

$$V_o = \frac{1200 - 3s^2 - 150s}{s(s + 10)(s + 40)} = \frac{K_1}{s} + \frac{K_2}{s + 10} + \frac{K_3}{s + 40}$$

$$K_1 = \frac{1200}{400} = 3; \qquad K_2 = \frac{1200 - 300 + 1500}{(-10)(30)} = -8$$

$$K_3 = \frac{1200 - 4800 + 6000}{(-40)(-30)} = 2$$

$$v_o(t) = (3 - 8e^{-10t} + 2e^{-40t})u(t)\,\text{V}$$

[e] Yes.

P 17.27 **[a]** $I_o = \dfrac{V_g}{0.5 + 0.01s + 4/s}$

$$H(s) = \frac{I_o}{V_g} = \frac{100s}{s^2 + 50s + 400} = \frac{100s}{(s+10)(s+40)}$$

$$H(j\omega) = \frac{100(j\omega)}{(j\omega + 10)(j\omega + 40)}$$

$$V_g(\omega) = \frac{6}{j\omega}$$

$$I_o(\omega) = H(j\omega)V_g(\omega) = \frac{600}{(j\omega + 10)(j\omega + 40)}$$

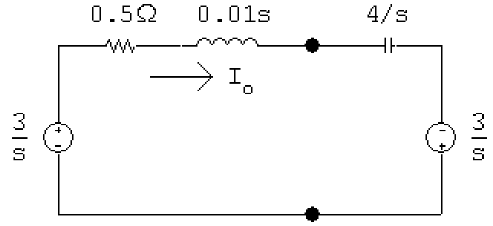
$$= \frac{20}{j\omega + 10} - \frac{20}{j\omega + 40}$$

$$i_o(t) = (20e^{-10t} - 20e^{-40t})u(t)\,\text{A}$$

[b] $i_o(0^-) = 0$

[c] $i_o(0^+) = 0$

[d]



$$I_o = \frac{6/s}{0.5 + 0.01s + 4/s} = \frac{600}{s^2 + 50s + 400}$$

$$= \frac{600}{(s+10)(s+40)} = \frac{20}{s+10} - \frac{20}{s+40}$$

$$i_o(t) = (20e^{-10t} - 20e^{-40t})u(t)\,\text{A}$$

[e] Yes.

P 17.28 **[a]** $i_g = 3e^{-5|t|}$

$$\therefore \qquad I_g(\omega) = \frac{3}{j\omega + 5} + \frac{3}{-j\omega + 5} = \frac{30}{(j\omega + 5)(-j\omega + 5)}$$

$$\frac{V_o}{10} + \frac{V_o s}{10} = I_g$$

$$\therefore \qquad \frac{V_o}{I_g} = H(s) = \frac{10}{s+1}; \qquad H(\omega) = \frac{10}{j\omega + 1}$$

$$V_o(\omega) = I_g(\omega)H(\omega) = \frac{300}{(j\omega + 1)(j\omega + 5)(-j\omega + 5)}$$

$$= \frac{K_1}{j\omega + 1} + \frac{K_2}{j\omega + 5} + \frac{K_3}{-j\omega + 5}$$

$$K_1 = \frac{300}{(4)(6)} = 12.5$$

$$K_2 = \frac{300}{(-4)(10)} = -7.5$$

$$K_3 = \frac{300}{(6)(10)} = 5$$

$$V_o(\omega) = \frac{12.5}{j\omega + 1} - \frac{7.5}{j\omega + 5} + \frac{5}{-j\omega + 5}$$

$$v_o(t) = [12.5e^{-t} - 7.5e^{-5t}]u(t) + 5e^{5t}u(-t)\,\text{V}$$

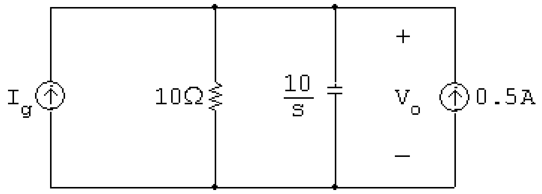
[b] $v_o(0^-) = 5\,\text{V}$

[c] $v_o(0^+) = 12.5 - 7.5 = 5\,\text{V}$

[d] $i_g = 3e^{-5t}u(t), \quad t \geq 0^+$

$$I_g = \frac{3}{s+5}; \qquad H(s) = \frac{10}{s+1}$$

$$v_o(0^+) = 5\,\text{V}; \qquad \gamma C = 0.5$$



$$\frac{V_o}{10} + \frac{V_o s}{10} = I_g + 0.5$$

$$V_o(s+1) = \frac{30}{s+5} + 5$$

$$V_o = \frac{30}{(s+5)(s+1)} + \frac{5}{s+1}$$

$$= \frac{-7.5}{s+5} + \frac{7.5}{s+1} + \frac{5}{s+1} = \frac{12.5}{s+1} - \frac{7.5}{s+5}$$

$$\therefore \quad v_o(t) = (12.5e^{-t} - 7.5e^{-5t})u(t)\,\mathrm{V}$$

[e] Yes, for $t \geq 0^+$ the solution in part (a) is also

$$v_o(t) = (12.5e^{-t} - 7.5e^{-5t})u(t)\,\mathrm{V}$$

P 17.29 **[a]** $I_o = \dfrac{V_g}{10 + 10/s} = \dfrac{V_g s}{10s + 10}$

$$H(s) = \frac{I_o}{V_g} = \frac{0.1}{s+1}$$

$$H(j\omega) = \frac{0.1}{j\omega + 1}$$

$$V_g(\omega) = \frac{30}{-j\omega + 5} + \frac{30}{j\omega + 5}$$

$$I_o(\omega) = H(j\omega)V_g(j\omega) = \frac{0.1j\omega}{j\omega + 1}\left[\frac{30}{-j\omega + 5} + \frac{30}{j\omega + 5}\right]$$

$$= \frac{3j\omega}{(j\omega + 1)(-j\omega + 5)} + \frac{3j\omega}{(j\omega + 1)(j\omega + 5)}$$

$$= \frac{K_1}{j\omega + 1} + \frac{K_2}{-j\omega + 5} + \frac{K_3}{j\omega + 1} + \frac{K_4}{j\omega + 5}$$

$$K_1 = \frac{3(-1)}{6} = -0.5; \qquad K_2 = \frac{3(5)}{6} = 2.5$$

$$K_3 = \frac{3(-1)}{4} = -0.75; \qquad K_4 = \frac{3(-5)}{-4} = 3.75$$

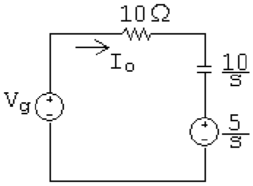
$$\therefore \quad I_o(\omega) = \frac{-1.25}{j\omega + 1} + \frac{2.5}{-j\omega + 5} + \frac{3.75}{j\omega + 5}$$

$$i_o(t) = 2.5e^{5t}u(-t) + [-1.25e^{-t} + 3.75e^{-5t}]u(t)\,\mathrm{A}$$

[b] $i_o(0^-) = 2.5\,\mathrm{V}$

[c] $i_o(0^+) = 2.5\,\mathrm{V}$

[**d**] Note – since $i_o(0^+) = 2.5$ A, $v_o(0^+) = 30 - 25 = 5$ V.



$$I_o = \frac{V_g - (5/s)}{10 + (10/s)} = \frac{sV_g - 5}{10s + 10}; \qquad V_g = \frac{30}{s+5}$$

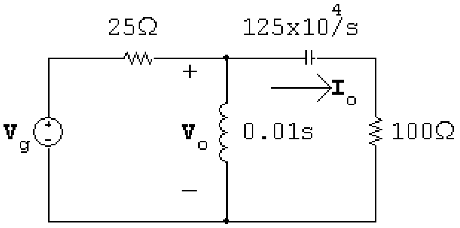
$$\therefore \quad I_o = \frac{25s - 25}{10(s+1)(s+5)} = \frac{2.5(s-1)}{(s+1)(s+5)} = \frac{-1.25}{s+1} + \frac{3.75}{s+5}$$

$$i_o(t) = (-1.25e^{-t} + 3.75e^{-5t})u(t)\,\mathrm{A}$$

[**e**] Yes, for $t \geq 0^+$ the solution in part (a) is also

$$i_o(t) = (-1.25e^{-t} + 3.75e^{-5t})u(t)\,\mathrm{A}$$

P 17.30



$$\frac{V_o - V_g}{2s} + \frac{100V_o}{s} + \frac{V_o s}{100s + 125 \times 10^4} = 0$$

$$\therefore \quad V_o = \frac{s(100s + 125 \times 10^4)V_g}{125(s^2 + 12{,}000s + 25 \times 10^6)}$$

$$I_o = \frac{sV_o}{100s + 125 \times 10^4}$$

$$H(s) = \frac{I_o}{V_g} = \frac{s^2}{125(s^2 + 12{,}000s + 25 \times 10^6)}$$

$$H(j\omega) = \frac{-8 \times 10^{-3}\omega^2}{(25 \times 10^6 - \omega^2) + j12{,}000\omega}$$

$$V_g(\omega) = 300\pi[\delta(\omega + 5000) + \delta(\omega - 5000)]$$

$$I_o(\omega) = H(j\omega)V_g(\omega) = \frac{-2.4\pi\omega^2[\delta(\omega + 5000) + \delta(\omega - 5000)]}{(25 \times 10^6 - \omega^2) + j12{,}000\omega}$$

$$i_o(t) = \frac{-2.4\pi}{2\pi}\int_{-\infty}^{\infty} \frac{\omega^2[\delta(\omega+5000)+\delta(\omega-5000)]}{(25\times 10^6-\omega^2)+j12{,}000\omega}e^{jt\omega}\,d\omega$$

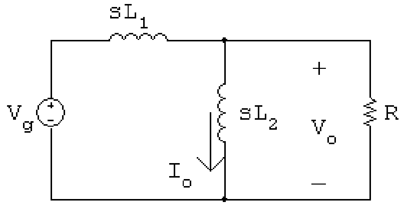
$$= -1.2\left\{\frac{25\times 10^6 e^{-j5000t}}{-j(12{,}000)(5000)} + \frac{25\times 10^6 e^{j5000t}}{j(12{,}000)(5000)}\right\}$$

$$= \frac{6}{12}\left\{\frac{e^{-j5000t}}{-j} + \frac{e^{j5000t}}{j}\right\}$$

$$= 0.5[e^{-j(5000t+90^\circ)} + e^{j(5000t+90^\circ)}]$$

$$i_o(t) = 1\cos(5000t+90^\circ)\,\text{A}$$

P 17.31 **[a]**



$$\frac{V_o-V_g}{sL_1}+\frac{V_o}{sL_2}+\frac{V_o}{R}=0$$

$$\therefore\quad V_o = \frac{RV_g}{L_1\left[s+R\left(\frac{1}{L1}+\frac{1}{L_2}\right)\right]}$$

$$I_o = \frac{V_o}{sL_2}$$

$$\therefore\quad \frac{I_o}{V_g} = H(s) = \frac{R/L_1L_2}{s(s+R[(1/L_1)+(1/L_2)])}$$

$$\frac{R}{L_1L_2} = 12\times 10^5$$

$$R\left(\frac{1}{L_1}+\frac{1}{L_2}\right) = 3\times 10^4$$

$$\therefore\quad H(s) = \frac{12\times 10^5}{s(s+3\times 10^4)}$$

$$H(j\omega) = \frac{12\times 10^5}{j\omega(j\omega+3\times 10^4)}$$

$$V_g(\omega) = 125\pi[\delta(\omega+4\times 10^4)+\delta(\omega-4\times 10^4)]$$

$$I_o(\omega) = H(j\omega)V_g(\omega) = \frac{1500\pi\times 10^5[\delta(\omega+4\times 10^4)+\delta(\omega-4\times 10^4)]}{j\omega(j\omega+3\times 10^4)}$$

$$i_o(t) = \frac{1500\pi \times 10^5}{2\pi}\int_{-\infty}^{\infty}\frac{[\delta(\omega + 4\times 10^4) + \delta(\omega - 4\times 10^4)]e^{jt\omega}}{j\omega(j\omega + 3\times 10^4)}\,d\omega$$

$$i_o(t) = 750\times 10^5\left\{\frac{e^{-j40{,}000t}}{-j40{,}000(30{,}000 - j40{,}000)}\right.$$

$$\left. + \frac{e^{j40{,}000t}}{j40{,}000(30{,}000 + j40{,}000)}\right\}$$

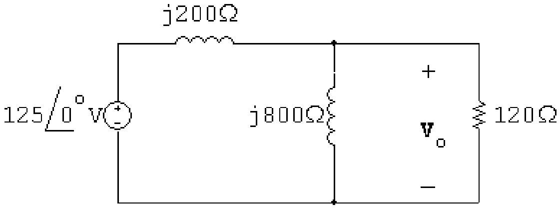
$$= \frac{75\times 10^6}{4\times 10^8}\left\{\frac{e^{-j40{,}000t}}{-j(3+j4)} + \frac{e^{j40{,}000t}}{j(3+j4)}\right\}$$

$$= \frac{75}{400}\left\{\frac{e^{-j40{,}000t}}{5\underline{/-143.13^\circ}} + \frac{e^{j40{,}000t}}{5\underline{/143.13^\circ}}\right\}$$

$$= 0.075\cos(40{,}000t - 143.13^\circ)\,\text{A}$$

$$i_o(t) = 75\cos(40{,}000t - 143.13^\circ)\,\text{mA}$$

[b] In the phasor domain:



$$\frac{\mathbf{V}_o - 125}{j200} + \frac{\mathbf{V}_o}{j800} + \frac{\mathbf{V}_o}{120} = 0$$

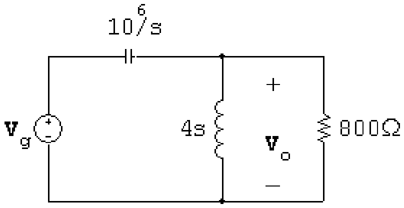
$$12\mathbf{V}_o - 1500 + 3\mathbf{V}_o + j20\mathbf{V}_o = 0$$

$$\mathbf{V}_o = \frac{1500}{15 + j20} = 60\underline{/-53.13^\circ}\,\text{V}$$

$$\mathbf{I}_o = \frac{\mathbf{V}_o}{j800} = 75\times 10^{-3}\underline{/-143.13^\circ}\,\text{A}$$

$$i_o(t) = 75\cos(40{,}000t - 143.13^\circ)\,\text{mA}$$

P 17.32 **[a]**



$$\frac{(V_o - V_g)s}{10^6} + \frac{V_o}{4s} + \frac{V_o}{800} = 0$$

$$\therefore \quad V_o = \frac{s^2 V_g}{s^2 + 1250s + 25 \times 10^4}$$

$$\frac{V_o}{V_g} = H(s) = \frac{s^2}{(s+250)(s+1000)}$$

$$H(j\omega) = \frac{(j\omega)^2}{(j\omega+250)(j\omega+1000)}$$

$$v_g = 45e^{-500|t|}; \qquad V_g(\omega) = \frac{45{,}000}{(j\omega+500)(-j\omega+500)}$$

$$\therefore \quad V_o(\omega) = H(j\omega)V_g(\omega) = \frac{45{,}000(j\omega)^2}{(j\omega+250)(j\omega+500)(j\omega+1000)(-j\omega+500)}$$

$$= \frac{K_1}{j\omega+250} + \frac{K_2}{j\omega+500} + \frac{K_3}{j\omega+1000} + \frac{K_4}{-j\omega+500}$$

$$K_1 = \frac{45{,}000(-250)^2}{(250)(750)(750)} = 20$$

$$K_2 = \frac{45{,}000(-500)^2}{(-250)(500)(1000)} = -90$$

$$K_3 = \frac{45{,}000(-1000)^2}{(-750)(-500)(1500)} = 80$$

$$K_4 = \frac{45{,}000(500)^2}{(750)(1000)(1500)} = 10$$

$$\therefore \quad v_o(t) = [20e^{-250t} - 90e^{-500t} + 80e^{-1000t}]u(t) + 10e^{500t}u(-t)\,\text{V}$$

[b] $v_o(0^-) = 10\,\text{V}; \qquad V_o(0^+) = 20 - 90 + 80 = 10\,\text{V}$

$$v_o(\infty) = 0\,\text{V}$$

[c] $I_L = \dfrac{V_o}{4s} = \dfrac{0.25sV_g}{(s+250)(s+1000)}$

$$H(s) = \frac{I_L}{V_o} = \frac{0.25s}{(s+250)(s+1000)}$$

$$H(j\omega) = \frac{0.25(j\omega)}{(j\omega+250)(j\omega+1000)}$$

$$I_L(\omega) = \frac{0.25(j\omega)(45{,}000)}{(j\omega+250)(j\omega+500)(j\omega+1000)(-j\omega+500)}$$

$$= \frac{K_1}{j\omega+250} + \frac{K_2}{j\omega+500} + \frac{K_3}{j\omega+1000} + \frac{K_4}{-j\omega+500}$$

$$K_4 = \frac{(0.25)(500)(45{,}000)}{(750)(1000)(1500)} = 5\,\mathrm{mA}$$

$$i_L(t) = 5e^{500t}u(-t); \qquad \therefore \quad i_L(0^-) = 5\,\mathrm{mA}$$

$$K_1 = \frac{(0.25)(-250)(45{,}000)}{(250)(750)(750)} = -20\,\mathrm{mA}$$

$$K_2 = \frac{(0.25)(-500)(45{,}000)}{(-250)(500)(1000)} = 45\,\mathrm{mA}$$

$$K_3 = \frac{(0.25)(-1000)(45{,}000)}{(-750)(-500)(1500)} = -20\,\mathrm{mA}$$

$$\therefore \quad i_L(0^+) = K_1 + K_2 + K_3 = -20 + 45 - 20 = 5\,\mathrm{mA}$$

Checks, i.e., $i_L(0^+) = i_L(0^-) = 5\,\mathrm{mA}$

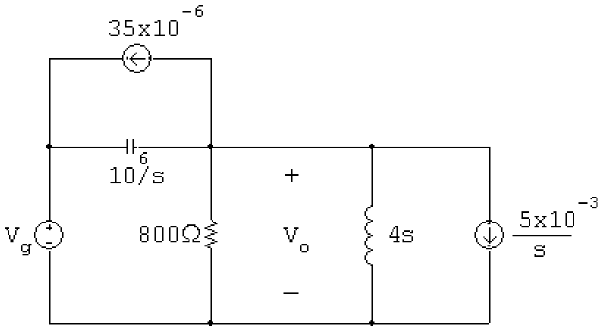
At $t = 0^-$:

$$v_C(0^-) = 45 - 10 = 35\,\mathrm{V}$$

At $t = 0^+$:

$$v_C(0^+) = 45 - 10 = 35\,\mathrm{V}$$

[d] We can check the correctness of out solution for $t \geq 0^+$ by using the Laplace transform. Our circuit becomes



$$\frac{V_o}{800} + \frac{V_o}{4s} + \frac{(V_o - V_g)s}{10^6} + 35 \times 10^{-6} + \frac{5 \times 10^{-3}}{s} = 0$$

$$\therefore \quad (s^2 + 1250s + 24 \times 10^4)V_o = s^2V_g - (35s + 5000)$$

$$v_g(t) = 45e^{-500t}u(t)\,\mathrm{V}; \qquad V_g = \frac{45}{s + 500}$$

$$\therefore \quad (s + 250)(s + 1000)V_o = \frac{45s^2 - (35s + 5000)(s + 500)}{(s + 500)}$$

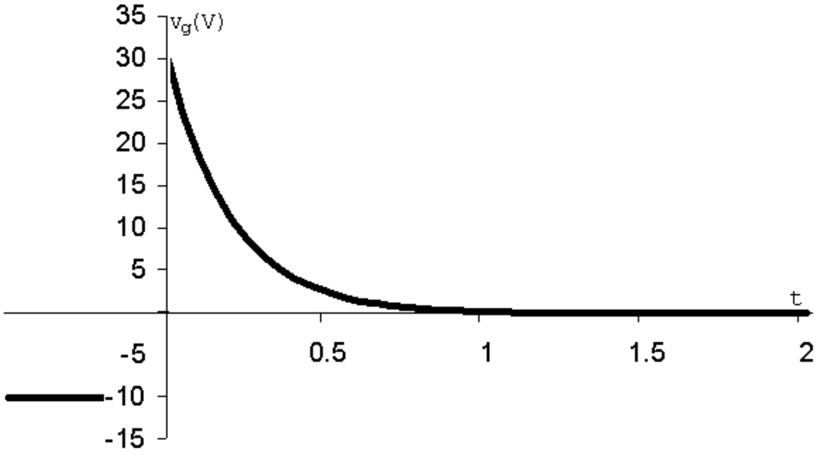
$$\therefore \quad V_o = \frac{10s^2 - 22{,}500s - 250 \times 10^4}{(s + 250)(s + 500)(s + 1000)}$$

$$= \frac{20}{s + 250} - \frac{90}{s + 500} + \frac{80}{s + 1000}$$

$$\therefore \quad v_o(t) = [20e^{-250t} - 90e^{-500t} + 80e^{-1000t}]u(t)\,\text{V}$$

This agrees with our solution for $v_o(t)$ for $t \geq 0^+$.

P 17.33 **[a]**



From the plot of v_g note that v_g is -10 V for an infinitely long time before $t = 0$. Therefore

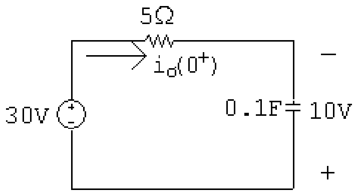
$$\therefore \qquad v_o(0^-) = -10\,\text{V}$$

There cannot be an instantaneous change in the voltage across a capacitor, so

$$v_o(0^+) = -10\,\text{V}$$

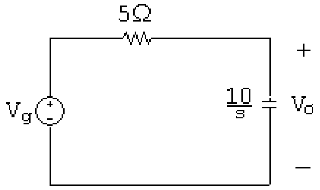
[b] $i_o(0^-) = 0\,\text{A}$

At $t = 0^+$ the circuit is



$$i_o(0^+) = \frac{30 - (-10)}{5} = \frac{40}{5} = 8\,\text{A}$$

[c] The s-domain circuit is



$$V_o = \left[\frac{V_g}{5 + (10/s)}\right]\left(\frac{10}{s}\right) = \frac{2V_g}{s+2}$$

$$\frac{V_o}{V_g} = H(s) = \frac{2}{s+2}$$

$$H(j\omega) = \frac{2}{j\omega+2}$$

$$V_g(\omega) = 5\left(\frac{2}{j\omega}\right) - 5[2\pi\delta(\omega)] + \frac{30}{j\omega+5} = \frac{10}{j\omega} - 10\pi\delta(\omega) + \frac{30}{j\omega+5}$$

$$V_o(\omega) = H(\omega)V_g(\omega) = \frac{2}{j\omega+2}\left[\frac{10}{j\omega} - 10\pi\delta(\omega) + \frac{30}{j\omega+5}\right]$$

$$= \frac{20}{j\omega(j\omega+2)} - \frac{20\pi\delta(\omega)}{j\omega+2} + \frac{60}{(j\omega+2)(j\omega+5)}$$

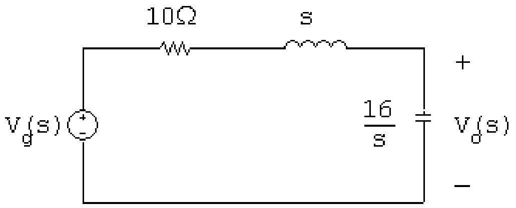
$$= \frac{K_0}{j\omega} + \frac{K_1}{j\omega+2} + \frac{K_2}{j\omega+2} + \frac{K_3}{j\omega+5} - \frac{20\pi\delta(\omega)}{j\omega+2}$$

$$K_0 = \frac{20}{2} = 10; \quad K_1 = \frac{20}{-2} = -10; \quad K_2 = \frac{60}{3} = 20; \quad K_3 = \frac{60}{-3} = -20$$

$$V_o(\omega) = \frac{10}{j\omega} + \frac{10}{j\omega+2} - \frac{20}{j\omega+5} - \frac{20\pi\delta(\omega)}{j\omega+2}$$

$$v_o(t) = 5\text{sgn}(t) + [10e^{-2t} - 20e^{-5t}]u(t) - 5\,\text{V}$$

P 17.34 **[a]**



$$V_g(\omega) = \frac{36}{4-j\omega} - \frac{36}{4+j\omega} = \frac{72j\omega}{(4-j\omega)(4+j\omega)}$$

$$V_o(s) = \frac{(16/s)}{10+s+(16/s)}V_g(s)$$

$$H(s) = \frac{V_o(s)}{V_g(s)} = \frac{16}{s^2+10s+16} = \frac{16}{(s+2)(s+8)}$$

$$H(j\omega) = \frac{16}{(j\omega+2)(j\omega+8)}$$

$$V_o(j\omega) = H(j\omega)\cdot V_g(\omega) = \frac{1152j\omega}{(4-j\omega)(4+j\omega)(2+j\omega)(8+j\omega)}$$

$$= \frac{K_1}{4-j\omega} + \frac{K_2}{4+j\omega} + \frac{K_3}{2+j\omega} + \frac{K_4}{8+j\omega}$$

$$K_1 = \frac{1152(4)}{(8)(6)(12)} = 8$$

$$K_2 = \frac{1152(-4)}{(8)(-2)(4)} = 72$$

$$K_3 = \frac{1152(-2)}{(6)(2)(6)} = -32$$

$$K_4 = \frac{1152(-8)}{(12)(-4)(-6)} = -32$$

$$\therefore \quad V_o(j\omega) = \frac{8}{4-j\omega} + \frac{72}{4+j\omega} - \frac{32}{2+j\omega} - \frac{32}{8+j\omega}$$

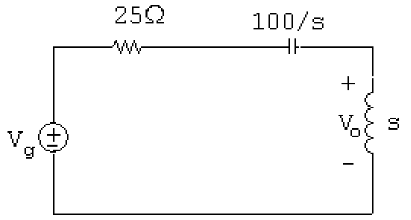
$$\therefore \quad v_o(t) = 8e^{4t}u(-t) + [72e^{-4t} - 32e^{-2t} - 32e^{-8t}]u(t)\text{V}$$

[b] $v_o(0^-) = 8\text{V}$

[c] $v_o(0^+) = 72 - 32 - 32 = 8\text{V}$

The voltages at 0^- and 0^+ must be the same since the voltage cannot change instantaneously across a capacitor.

P 17.35 **[a]**



$$V_o = \frac{V_g s}{25 + (100/s) + s} = \frac{V_g s^2}{s^2 + 25s + 100}$$

$$H(s) = \frac{V_o}{V_g} = \frac{s^2}{(s+5)(s+20)}; \qquad H(j\omega) = \frac{(j\omega)^2}{(j\omega+5)(j\omega+20)}$$

$$v_g = 25i_g = 450e^{10t}u(-t) - 450e^{-10t}u(t)\,\text{V}$$

$$V_g = \frac{450}{-j\omega+10} - \frac{450}{j\omega+10}$$

$$V_o(\omega) = H(j\omega)V_g = \frac{450(j\omega)^2}{(-j\omega+10)(j\omega+5)(j\omega+20)}$$

$$+\frac{-450(j\omega)^2}{(j\omega+10)(j\omega+5)(j\omega+20)}$$

$$= \frac{K_1}{-j\omega + 10} + \frac{K_2}{j\omega + 5} + \frac{K_3}{j\omega + 20} + \frac{K_4}{j\omega + 5} + \frac{K_5}{j\omega + 10} + \frac{K_6}{j\omega + 20}$$

$$K_1 = \frac{450(100)}{(15)(30)} = 100 \qquad K_4 = \frac{-450(25)}{(5)(15)} = -150$$

$$K_2 = \frac{450(25)}{(15)(15)} = 50 \qquad K_5 = \frac{-450(100)}{(-5)(10)} = 900$$

$$K_3 = \frac{450(400)}{(30)(-15)} = -400 \qquad K_6 = \frac{-450(400)}{(-15)(-10)} = -1200$$

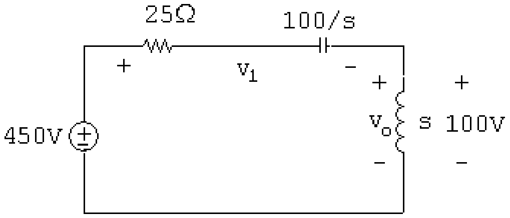
$$V_o(\omega) = \frac{100}{-j\omega + 10} + \frac{-100}{j\omega + 5} + \frac{-1600}{j\omega + 20} + \frac{900}{j\omega + 10}$$

$$v_o = 100e^{10t}u(-t) + [900e^{-10t} - 100e^{-5t} - 1600e^{-20t}]u(t)\,\mathrm{V}$$

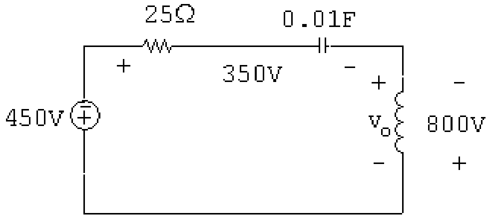
[b] $v_o(0^-) = 100\,\mathrm{V}$

[c] $v_o(0^+) = 900 - 100 - 1600 = -800\,\mathrm{V}$

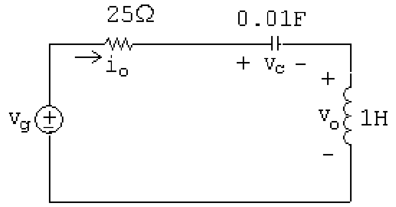
[d] At $t = 0^-$ the circuit is



Therefore, the solution predicts $v_1(0^-)$ will be 350 V.
Now $v_1(0^+) = v_1(0^-)$ because the inductor will not let the current in the 25 Ω resistor change instantaneously, and the capacitor will not let the voltage across the 0.01 F capacitor change instantaneously.
At $t = 0^+$ the circuit is



From the circuit at $t = 0^+$ we see that v_o must be −800 V, which is consistent with the solution for v_o obtained in part (a).
It is informative to solve for either the current in the circuit or the voltage across the capacitor and note the solutions for i_o and v_C are consistent with the solution for v_o



The solutions are

$$i_o = 10e^{10t}u(-t) + [20e^{-5t} + 80e^{-20t} - 90e^{-10t}]u(t)\,\text{A}$$

$$v_C = 100e^{10t}u(-t) + [900e^{-10t} - 400e^{-5t} - 400e^{-20t}]u(t)\,\text{V}$$

P 17.36 $V_o(s) = \dfrac{10}{s} + \dfrac{30}{s+20} - \dfrac{40}{s+30} = \dfrac{600(s+10)}{s(s+20)(s+30)}$

$$V_o(s) = H(s)\cdot\frac{15}{s}$$

$$\therefore \qquad H(s) = \frac{40(s+10)}{(s+20)(s+30)}$$

$$\therefore \qquad H(\omega) = \frac{40(j\omega+10)}{(j\omega+20)(j\omega+30)}$$

$$\therefore \qquad V_o(\omega) = \frac{30}{j\omega}\cdot\frac{40(j\omega+10)}{(j\omega+20)(j\omega+30)} = \frac{1200(j\omega+10)}{j\omega(j\omega+20)(j\omega+30)}$$

$$v_o(\omega) = \frac{20}{j\omega} + \frac{60}{j\omega+20} - \frac{80}{j\omega+30}$$

$$v_o(t) = 10\text{sgn}(t) + [60e^{-20t} - 80e^{-30t}]u(t)\,\text{V}$$

P 17.37 **[a]** $f(t) = \dfrac{1}{2\pi}\left\{\displaystyle\int_{-\infty}^{0} e^{\omega}e^{jt\omega}\,d\omega + \int_{0}^{\infty} e^{-\omega}e^{jt\omega}\,d\omega\right\} = \dfrac{1/\pi}{1+t^2}$

[b] $W = 2\displaystyle\int_0^\infty \frac{(1/\pi)^2}{(1+t^2)^2}\,dt = \frac{2}{\pi^2}\int_0^\infty \frac{dt}{(1+t^2)^2} = \frac{1}{2\pi}\,\text{J}$

[c] $W = \dfrac{1}{\pi}\displaystyle\int_0^\infty e^{-2\omega}\,d\omega = \frac{1}{\pi}\frac{e^{-2\omega}}{-2}\Big|_0^\infty = \frac{1}{2\pi}\,\text{J}$

[d] $\dfrac{1}{\pi}\displaystyle\int_0^{\omega_1} e^{-2\omega}\,d\omega = \frac{0.9}{2\pi}, \qquad 1 - e^{-2\omega_1} = 0.9, \qquad e^{2\omega_1} = 10$

$$\omega_1 = (1/2)\ln 10 \cong 1.15\,\text{rad/s}$$

P 17.38 $I_o = \dfrac{0.5sI_g}{0.5s+25} = \dfrac{sI_g}{s+50}$

$$H(s) = \frac{I_o}{I_g} = \frac{s}{s+50}$$

$$H(j\omega) = \frac{j\omega}{j\omega+50}$$

$$I(\omega) = \frac{12}{j\omega+10}$$

$$I_o(\omega) = H(j\omega)I(\omega) = \frac{12(j\omega)}{(j\omega+10)(j\omega+50)}$$

$$|I_o(\omega)| = \frac{12\omega}{\sqrt{(\omega^2+100)(\omega^2+2500)}}$$

$$|I_o(\omega)|^2 = \frac{144\omega^2}{(\omega^2+100)(\omega^2+2500)}$$

$$= \frac{-6}{\omega^2+100} + \frac{150}{\omega^2+2500}$$

$$W_o(\text{total}) = \frac{1}{\pi}\int_0^\infty \frac{150d\omega}{\omega^2+2500} - \frac{1}{\pi}\int_0^\infty \frac{6d\omega}{\omega^2+100}$$

$$= \frac{3}{\pi}\tan^{-1}\left(\frac{\omega}{50}\Big|_0^\infty\right) - \frac{0.6}{\pi}\tan^{-1}\left(\frac{\omega}{10}\Big|_0^\infty\right)$$

$$= 1.5 - 0.3 = 1.2\,\text{J}$$

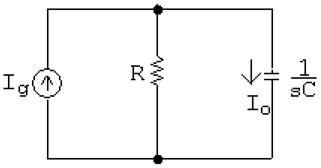
$$W_o(0 \text{ — } 100 \text{ rad/s}) = \frac{3}{\pi}\tan^{-1}(2) - \frac{0.6}{\pi}\tan^{-1}(10)$$

$$= 1.06 - 0.28 = 0.78\,\text{J}$$

Therefore, the percent between 0 and 100 rad/s is

$$\frac{0.78}{1.2}(100) = 64.69\%$$

P 17.39



$$I_o = \frac{I_g R}{R + (1/sC)} = \frac{RCsI_g}{RCs + 1}$$

$$H(s) = \frac{I_o}{I_g} = \frac{s}{s + (1/RC)}$$

$$RC = (100 \times 10^3)(1.25 \times 10^{-6}) = 125 \times 10^{-3}; \qquad \frac{1}{RC} = \frac{1}{0.125} = 8$$

$$H(s) = \frac{s}{s+8}; \qquad H(j\omega) = \frac{j\omega}{j\omega + 8}$$

$$I_g(\omega) = \frac{30 \times 10^{-6}}{j\omega + 2}$$

$$I_o(\omega) = H(j\omega)I_g(\omega) = \frac{30 \times 10^{-6} j\omega}{(j\omega + 2)(j\omega + 8)}$$

$$|I_o(\omega)| = \frac{\omega(30 \times 10^{-6})}{(\sqrt{\omega^2 + 4})(\sqrt{\omega^2 + 64})}$$

$$|I_o(\omega)|^2 = \frac{900 \times 10^{-12}\omega^2}{(\omega^2 + 4)(\omega^2 + 64)} = \frac{K_1}{\omega^2 + 4} + \frac{K_2}{\omega^2 + 64}$$

$$K_1 = \frac{(900 \times 10^{-12})(-4)}{(60)} = -60 \times 10^{-12}$$

$$K_2 = \frac{(900 \times 10^{-12})(-64)}{(-60)} = 960 \times 10^{-12}$$

$$|I_o(\omega)|^2 = \frac{960 \times 10^{-12}}{\omega^2 + 64} - \frac{60 \times 10^{-12}}{\omega^2 + 4}$$

$$W_{1\Omega} = \frac{1}{\pi}\int_0^\infty |I_o(\omega)|^2\, d\omega = \frac{960 \times 10^{-12}}{\pi}\int_0^\infty \frac{d\omega}{\omega^2 + 64} - \frac{60 \times 10^{-12}}{\pi}\int_0^\infty \frac{d\omega}{\omega^2 + 4}$$

$$= \frac{120 \times 10^{-12}}{\pi} \tan^{-1} \frac{\omega}{8} \Big|_0^{\infty} - \frac{30 \times 10^{-12}}{\pi} \tan^{-1} \frac{\omega}{2} \Big|_0^{\infty}$$

$$= \left(\frac{120}{\pi} \cdot \frac{\pi}{2} - \frac{30}{\pi} \cdot \frac{\pi}{2}\right) \times 10^{-12} = (60 - 15) \times 10^{-12} = 45\,\text{pJ}$$

Between 0 and 4 rad/s

$$W_{1\Omega} = \left[\frac{120}{\pi} \tan^{-1} \frac{1}{2} - \frac{30}{\pi} \tan^{-1} 2\right] \times 10^{-12} = 7.14\,\text{pJ}$$

$$\% = \frac{7.14}{45}(100) = 15.86\%$$

P 17.40 **[a]** $V_g(\omega) = \dfrac{60}{(j\omega + 1)(-j\omega + 1)}$

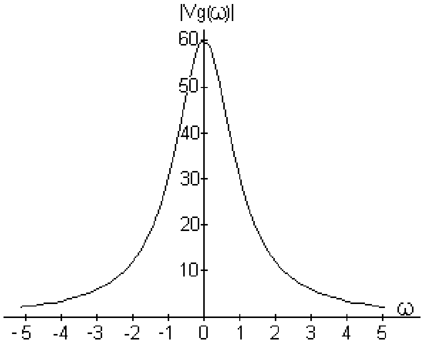
$$H(s) = \frac{V_o}{V_g} = \frac{0.4}{s + 0.5}; \qquad H(\omega) = \frac{0.4}{(j\omega + 0.5)}$$

$$V_o(\omega) = \frac{24}{(j\omega + 1)(j\omega + 0.5)(-j\omega + 1)}$$

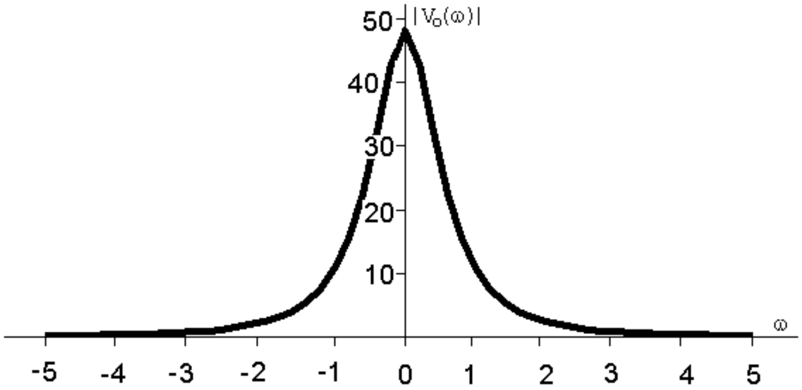
$$V_o(\omega) = \frac{-24}{j\omega + 1} + \frac{32}{j\omega + 0.5} + \frac{8}{-j\omega + 1}$$

$$v_o(t) = [-24e^{-t} + 32e^{-t/2}]u(t) + 8e^{t}u(-t)\,\text{V}$$

[b] $|V_g(\omega)| = \dfrac{60}{(\omega^2 + 1)}$



[c] $|V_o(\omega)| = \dfrac{24}{(\omega^2+1)\sqrt{\omega^2+0.25}}$



[d] $W_i = 2\int_0^{\infty} 900e^{-2t}\,dt = 1800\left.\dfrac{e^{-2t}}{-2}\right|_0^{\infty} = 900\,\mathrm{J}$

[e] $W_o = \int_{-\infty}^{0} 64e^{2t}\,dt + \int_0^{\infty}(-24e^{-t}+32e^{-t/2})^2\,dt$

$$= 32 + \textstyle\int_0^{\infty}[576e^{-2t} - 1536e^{-3t/2} + 1024e^{-t}]\,dt$$

$$= 32 + 288 - 1024 + 1024 = 320\,\mathrm{J}$$

[f] $|V_g(\omega)| = \dfrac{60}{\omega^2+1}, \qquad |V_g^2(\omega)| = \dfrac{3600}{(\omega^2+1)^2}$

$$W_g = \frac{3600}{\pi}\int_0^2 \frac{d\omega}{(\omega^2+1)^2}$$

$$= \frac{3600}{\pi}\left\{\frac{1}{2}\left(\frac{\omega}{\omega^2+1} + \tan^{-1}\omega\right)\Big|_0^2\right\}$$

$$= \frac{1800}{\pi}\left(\frac{2}{5} + \tan^{-1}2\right) = 863.53\,\mathrm{J}$$

$$\therefore\quad \% = \left(\frac{863.53}{900}\right)\times 100 = 95.95\%$$

[g] $$|V_o(\omega)|^2 = \frac{576}{(\omega^2+1)^2(\omega^2+0.25)}$$

$$= \frac{1024}{\omega^2+0.25} - \frac{768}{(\omega^2+1)^2} - \frac{1024}{(\omega^2+1)}$$

$$W_o = \frac{1}{\pi}\left\{1024\cdot 2\cdot\tan^{-1}2\omega\,\Big|_0^2 - 768\left(\frac{1}{2}\right)\left(\frac{\omega}{\omega^2+1}+\tan^{-1}\omega\right)_0^2\right.$$

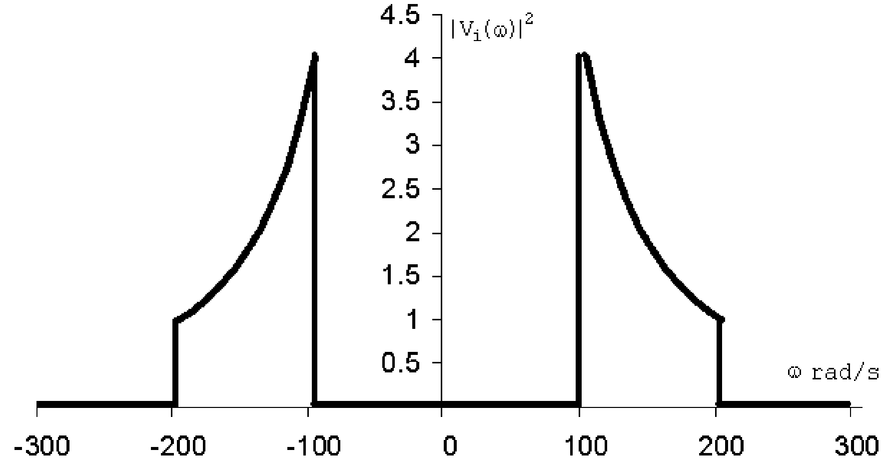
$$\left. -1024\tan^{-1}\omega\,\Big|_0^2\right\}$$

$$= \frac{2048}{\pi}\tan^{-1}4 - \frac{384}{\pi}\left(\frac{2}{5}+\tan^{-1}2\right) - \frac{1024}{\pi}\tan^{-1}2$$

$$= 319.2\,\text{J}$$

$$\% = \frac{319.2}{320}\times 100 = 99.75\%$$

P 17.41 **[a]** $|V_i(\omega)|^2 = \dfrac{4\times 10^4}{\omega^2}$; $\quad |V_i(100)|^2 = \dfrac{4\times 10^4}{100^2} = 4$; $\quad |V_i(200)|^2 = \dfrac{4\times 10^4}{200^2} = 1$



[b] $$V_o = \frac{V_iR}{R+(1/sC)} = \frac{sRCV_i}{RCs+1}$$

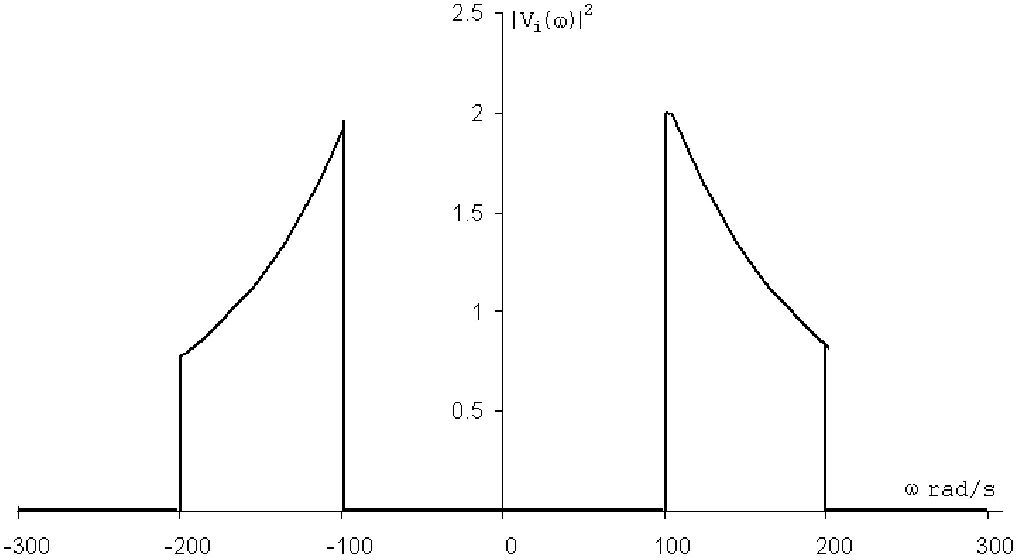
$$H(s) = \frac{V_o}{V_i} = \frac{s}{s+(1/RC)}; \qquad \frac{1}{RC} = \frac{10^6 10^{-3}}{(0.5)(20)} = \frac{1000}{10} = 100$$

$$H(j\omega) = \frac{j\omega}{j\omega+100}$$

$$|V_o(\omega)| = \frac{200}{|\omega|}\cdot\frac{|\omega|}{\sqrt{\omega^2+10^4}} = \frac{200}{\sqrt{\omega^2+10^4}}$$

$$|V_o(\omega)|^2 = \frac{4\times 10^4}{\omega^2+10^4}, \quad 100\le\omega\le 200\ \text{rad/s}; \quad |V_o(\omega)|^2 = 0, \quad \text{elsewhere}$$

$$|V_o(100)|^2 = \frac{4\times10^4}{10^4+10^4} = 2; \qquad |V_o(200)|^2 = \frac{4\times10^4}{5\times10^4} = 0.8$$



[**c**] $W_{1\Omega} = \dfrac{1}{\pi}\displaystyle\int_{100}^{200} \frac{4\times10^4}{\omega^2}\,d\omega = \frac{4\times10^4}{\pi}\left[-\frac{1}{\omega}\right]_{100}^{200}$

$$= \frac{4\times10^4}{\pi}\left[\frac{1}{100} - \frac{1}{200}\right] = \frac{200}{\pi} \cong 63.66\,\text{J}$$

[**d**] $W_{1\Omega} = \dfrac{1}{\pi}\displaystyle\int_{100}^{200} \frac{4\times10^4}{\omega^2+10^4}\,d\omega = \frac{4\times10^4}{\pi}\cdot\tan^{-1}\frac{\omega}{100}\Big|_{100}^{200}$

$$= \frac{400}{\pi}[\tan^{-1}2 - \tan^{-1}1] \cong 40.97\,\text{J}$$

P 17.42 [**a**] $V_i(\omega) = \dfrac{A}{a+j\omega}; \qquad |V_i(\omega)| = \dfrac{A}{\sqrt{a^2+\omega^2}}$

$$H(s) = \frac{s}{s+\alpha}; \qquad H(j\omega) = \frac{j\omega}{\alpha+j\omega}; \qquad |H(\omega)| = \frac{\omega}{\sqrt{\alpha^2+\omega^2}}$$

Therefore $|V_o(\omega)| = \dfrac{\omega A}{\sqrt{(a^2+\omega^2)(\alpha^2+\omega^2)}}$

Therefore $|V_o(\omega)|^2 = \dfrac{\omega^2 A^2}{(a^2+\omega^2)(\alpha^2+\omega^2)}$

$W_{\text{IN}} = \displaystyle\int_0^\infty A^2 e^{-2at}\,dt = \frac{A^2}{2a};$ when $\alpha = a$ we have

$$W_{\text{OUT}} = \frac{A^2}{\pi}\textstyle\int_0^a \dfrac{\omega^2\,d\omega}{(\omega^2+a^2)^2} = \dfrac{A^2}{\pi}\left\{\displaystyle\int_0^a \frac{d\omega}{a^2+\omega^2} - \int_0^a \frac{a^2\,d\omega}{(a^2+\omega^2)^2}\right\}$$

$$= \frac{A^2}{4a\pi}\left(\frac{\pi}{2} - 1\right)$$

$$W_{\rm OUT}(\text{total}) = \frac{A^2}{\pi}\int_0^\infty \left[\frac{\omega^2}{(a^2+\omega^2)^2}\right] d\omega = \frac{A^2}{4a}$$

$$\text{Therefore} \quad \frac{W_{\rm OUT}(a)}{W_{\rm OUT}(\text{total})} = 0.5 - \frac{1}{\pi} = 0.1817 \quad \text{or} \quad 18.17\%$$

[**b**] When $\alpha \neq a$ we have

$$W_{\rm OUT}(\alpha) = \frac{1}{\pi}\int_0^\alpha \frac{\omega^2 A^2 d\omega}{(a^2+\omega^2)(\alpha^2+\omega^2)}$$

$$= \frac{A^2}{\pi}\left\{\int_0^\alpha \left[\frac{K_1}{a^2+\omega^2} + \frac{K_2}{\alpha^2+\omega^2}\right] d\omega\right\}$$

$$\text{where} \quad K_1 = \frac{a^2}{a^2-\alpha^2} \quad \text{and} \quad K_2 = \frac{-\alpha^2}{a^2-\alpha^2}$$

Therefore

$$W_{\rm OUT}(\alpha) = \frac{A^2}{\pi(a^2-\alpha^2)}\left[a\tan^{-1}\left(\frac{\alpha}{a}\right) - \frac{\alpha\pi}{4}\right]$$

$$W_{\rm OUT}(\text{total}) = \frac{A^2}{\pi(a^2-\alpha^2)}\left[a\frac{\pi}{2} - \alpha\frac{\pi}{2}\right] = \frac{A^2}{2(a+\alpha)}$$

$$\text{Therefore} \quad \frac{W_{\rm OUT}(\alpha)}{W_{\rm OUT}(\text{total})} = \frac{2}{\pi(a-\alpha)} \cdot \left[a\tan^{-1}\left(\frac{\alpha}{a}\right) - \frac{\alpha\pi}{4}\right]$$

For $\alpha = a\sqrt{3}$, this ratio is 0.2723, or 27.23% of the output energy lies in the frequency band between 0 and $a\sqrt{3}$.

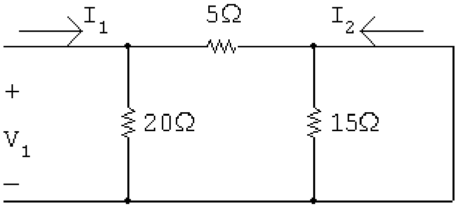
[**c**] For $\alpha = a/\sqrt{3}$, the ratio is 0.1057, or 10.57% of the output energy lies in the frequency band between 0 and $a/\sqrt{3}$.

18

Two-Port Circuits

Assessment Problems

AP 18.1 With port 2 short-circuited, we have



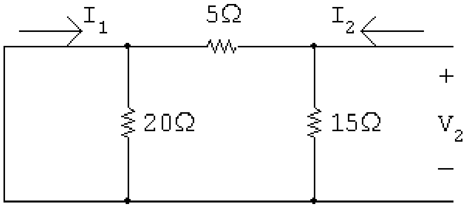
$$I_1 = \frac{V_1}{20} + \frac{V_1}{5}; \qquad \frac{I_1}{V_1} = y_{11} = 0.25\,\text{S}; \qquad I_2 = \left(\frac{-20}{25}\right) I_1 = -0.8I_1$$

When $V_2 = 0$, we have $I_1 = y_{11}V_1$ and $I_2 = y_{21}V_1$

Therefore $I_2 = -0.8(y_{11}V_1) = -0.8y_{11}V_1$

Thus $y_{21} = -0.8y_{11} = -0.2\,\text{S}$

With port 1 short-circuited, we have



$$I_2 = \frac{V_2}{15} + \frac{V_2}{5}; \qquad \frac{I_2}{V_2} = y_{22} = \left(\frac{4}{15}\right)\,\text{S}$$

$$I_1 = \left(\frac{-15}{20}\right) I_2 = -0.75I_2 = -0.75y_{22}V_2$$

Therefore $y_{12} = (-0.75)\dfrac{4}{15} = -0.2\,\mathrm{S}$

AP 18.2

$$h_{11} = \left(\frac{V_1}{I_1}\right)_{V_2=0} = 20\|5 = 4\,\Omega$$

$$h_{21} = \left(\frac{I_2}{I_1}\right)_{V_2=0} = \frac{(-20/25)I_1}{I_1} = -0.8$$

$$h_{12} = \left(\frac{V_1}{V_2}\right)_{I_1=0} = \frac{(20/25)V_2}{V_2} = 0.8$$

$$h_{22} = \left(\frac{I_2}{V_2}\right)_{I_1=0} = \frac{1}{15} + \frac{1}{25} = \frac{8}{75}\,\mathrm{S}$$

$$g_{11} = \left(\frac{I_1}{V_1}\right)_{I_2=0} = \frac{1}{20} + \frac{1}{20} = 0.1\,\mathrm{S}$$

$$g_{21} = \left(\frac{V_2}{V_1}\right)_{I_2=0} = \frac{(15/20)V_1}{V_1} = 0.75$$

$$g_{12} = \left(\frac{I_1}{I_2}\right)_{V_1=0} = \frac{(-15/20)I_2}{I_2} = -0.75$$

$$g_{22} = \left(\frac{V_2}{I_2}\right)_{V_1=0} = 15\|5 = \frac{75}{20} = 3.75\,\Omega$$

AP 18.3

$$g_{11} = \frac{I_1}{V_1}\bigg|_{I_2=0} = \frac{5\times10^{-6}}{50\times10^{-3}} = 0.1\,\mathrm{mS}$$

$$g_{21} = \frac{V_2}{V_1}\bigg|_{I_2=0} = \frac{200\times10^{-3}}{50\times10^{-3}} = 4$$

$$g_{12} = \frac{I_1}{I_2}\bigg|_{V_1=0} = \frac{2\times10^{-6}}{0.5\times10^{-6}} = 4$$

$$g_{22} = \frac{V_2}{I_2}\bigg|_{V_1=0} = \frac{10\times10^{-3}}{0.5\times10^{-6}} = 20\,\mathrm{k}\Omega$$

AP 18.4 First calculate the b-parameters:

$$b_{11} = \left.\frac{V_2}{V_1}\right|_{I_1=0} = \frac{15}{10} = 1.5\,\Omega; \qquad b_{21} = \left.\frac{I_2}{V_1}\right|_{I_1=0} = \frac{30}{10} = 3\,\text{S}$$

$$b_{12} = \left.\frac{-V_2}{I_1}\right|_{V_1=0} = \frac{-10}{-5} = 2\,\Omega; \qquad b_{22} = \left.\frac{-I_2}{I_1}\right|_{V_1=0} = \frac{-4}{-5} = 0.8$$

Now the z-parameters are calculated:

$$z_{11} = \frac{b_{22}}{b_{21}} = \frac{0.8}{3} = \frac{4}{15}\,\Omega; \qquad z_{12} = \frac{1}{b_{21}} = \frac{1}{3}\,\Omega$$

$$z_{21} = \frac{\Delta b}{b_{21}} = \frac{(1.5)(0.8) - 6}{3} = -1.6\,\Omega; \qquad z_{22} = \frac{b_{11}}{b_{21}} = \frac{1.5}{3} = \frac{1}{2}\,\Omega$$

AP 18.5

$z_{11} = z_{22}, \quad z_{12} = z_{21}, \quad 95 = z_{11}(5) + z_{12}(0)$

Therefore, $\quad z_{11} = z_{22} = 95/5 = 19\,\Omega$

$$11.52 = 19I_1 - z_{12}(2.72)$$

$$0 = z_{12}I_1 - 19(2.72)$$

Solving these simultaneous equations for z_{12} yields the quadratic equation

$$z_{12}^2 + \left(\frac{72}{17}\right) z_{12} - \frac{6137}{17} = 0$$

For a purely resistive network, it follows that $z_{12} = z_{21} = 17\,\Omega$.

AP 18.6 **[a]** $I_2 = \dfrac{-V_g}{a_{11}Z_L + a_{12} + a_{21}Z_gZ_L + a_{22}Z_g}$

$$= \frac{-50 \times 10^{-3}}{(5 \times 10^{-4})(5 \times 10^3) + 10 + (10^{-6})(100)(5 \times 10^3) + (-3 \times 10^{-2})(100)}$$

$$= \frac{-50 \times 10^{-3}}{10} = -5\,\text{mA}$$

$$P_L = \frac{1}{2}(5 \times 10^{-3})^2(5 \times 10^3) = 62.5\,\text{mW}$$

[b] $Z_{\text{Th}} = \dfrac{a_{12} + a_{22}Z_g}{a_{11} + a_{21}Z_g} = \dfrac{10 + (-3 \times 10^{-2})(100)}{5 \times 10^{-4} + (10^{-6})(100)}$

$$= \frac{7}{6 \times 10^{-4}} = \frac{70}{6}\,\text{k}\Omega$$

[c] $V_{\text{Th}} = \dfrac{V_g}{a_{11} + a_{21}Z_g} = \dfrac{50 \times 10^{-3}}{6 \times 10^{-4}} = \dfrac{500}{6}\,\text{V}$

Therefore $V_2 = \dfrac{250}{6}\,\text{V}; \qquad P_{\text{max}} = \dfrac{(1/2)(250/6)^2}{(70/6) \times 10^3} = 74.4\,\text{mW}$

AP 18.7 **[a]** For the given bridged-tee circuit, we have

$$a'_{11} = a'_{22} = 1.25, \qquad a'_{21} = \frac{1}{20}\,\text{S}, \qquad a'_{12} = 11.25\,\Omega$$

The a-parameters of the cascaded networks are

$$a_{11} = (1.25)^2 + (11.25)(0.05) = 2.125$$

$$a_{12} = (1.25)(11.25) + (11.25)(1.25) = 28.125\,\Omega$$

$$a_{21} = (0.05)(1.25) + (1.25)(0.05) = 0.125\,\text{S}$$

$$a_{22} = a_{11} = 2.125, \qquad R_{\text{Th}} = (45.125/3.125) = 14.44\,\Omega$$

[b] $V_t = \dfrac{100}{3.125} = 32\,\text{V}; \qquad$ therefore $V_2 = 16\,\text{V}$

[c] $P = \dfrac{16^2}{14.44} = 17.73\,\text{W}$

Problems

P 18.1 $$h_{11} = \left(\frac{V_1}{I_1}\right)_{V_2=0} = 20\|5 = 4\,\Omega$$

$$h_{21} = \left(\frac{I_2}{I_1}\right)_{V_2=0} = \frac{(-20/25)I_1}{I_1} = -0.8$$

$$h_{12} = \left(\frac{V_1}{V_2}\right)_{I_1=0} = \frac{(20/25)V_2}{V_2} = 0.8$$

$$h_{22} = \left(\frac{I_2}{V_2}\right)_{I_1=0} = \frac{1}{15} + \frac{1}{25} = \frac{8}{75}\,\mathrm{S}$$

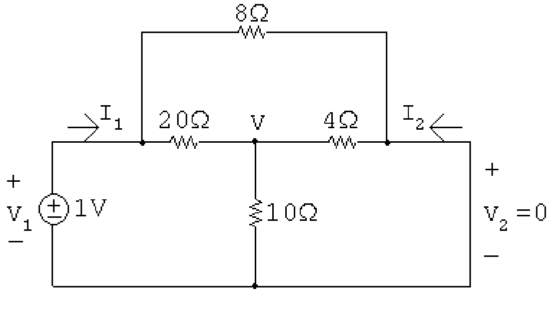
$$g_{11} = \left(\frac{I_1}{V_1}\right)_{I_2=0} = \frac{1}{20} + \frac{1}{20} = 0.1\,\mathrm{S}$$

$$g_{21} = \left(\frac{V_2}{V_1}\right)_{I_2=0} = \frac{(15/20)V_1}{V_1} = 0.75$$

$$g_{12} = \left(\frac{I_1}{I_2}\right)_{V_1=0} = \frac{(-15/20)I_2}{I_2} = -0.75$$

$$g_{22} = \left(\frac{V_2}{I_2}\right)_{V_1=0} = 15\|5 = \frac{75}{20} = 3.75\,\Omega$$

P 18.2 $$y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0}; \qquad y_{21} = \left.\frac{I_2}{V_1}\right|_{V_2=0}$$

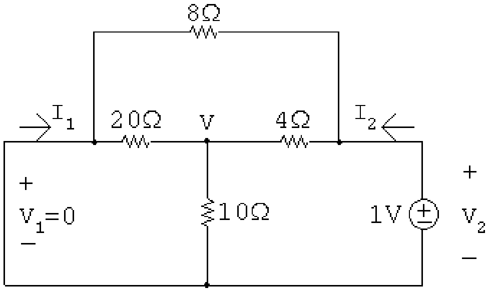


$$\frac{V-1}{20} + \frac{V}{10} + \frac{V}{4} = 0; \quad \text{so} \quad V = 0.125\,\mathrm{V}$$

$$\therefore \quad I_1 = \frac{1-0.125}{20} + \frac{1-0}{8} = 168.75\,\mathrm{mA}; \qquad I_2 = \frac{0-0.125}{4} + \frac{0-1}{8} = -156.25\,\mathrm{mA}$$

$$y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0} = 168.75\,\text{mS}; \qquad y_{21} = \left.\frac{I_2}{V_1}\right|_{V_2=0} = -156.25\,\text{mS}$$

$$y_{12} = \left.\frac{I_1}{V_2}\right|_{V_1=0}; \qquad y_{22} = \left.\frac{I_2}{V_2}\right|_{V_1=0}$$



$$\frac{V}{20} + \frac{V}{10} + \frac{V-1}{4} = 0; \quad \text{so} \quad V = 0.625\,\text{V}$$

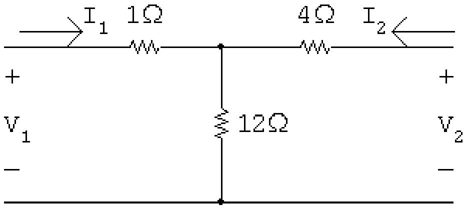
$$\therefore \quad I_1 = \frac{0 - 0.625}{20} + \frac{0-1}{8} = -156.25\,\text{mA}; \qquad I_2 = \frac{1 - 0.625}{4} + \frac{1-0}{8} = 218.75\,\text{mA}$$

$$y_{12} = \left.\frac{I_1}{V_2}\right|_{V_1=0} = -156.25\,\text{mS}; \qquad y_{22} = \left.\frac{I_2}{V_2}\right|_{V_1=0} = 218.75\,\text{mS}$$

Summary:

$y_{11} = 168.75\,\text{mS}$ $y_{12} = -156.25\,\text{mS}$ $y_{21} = -156.25\,\text{mS}$ $y_{22} = 218.75\,\text{mS}$

P 18.3



$$z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0} = 1 + 12 = 13\,\Omega$$

$$z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0} = 12\,\Omega$$

$$z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0} = 4 + 12 = 16\,\Omega$$

$$z_{21} = \left.\frac{V_1}{I_2}\right|_{I_1=0} = 12\,\Omega$$

P 18.4 $\Delta z = (13)(16) - (12)(12) = 64$

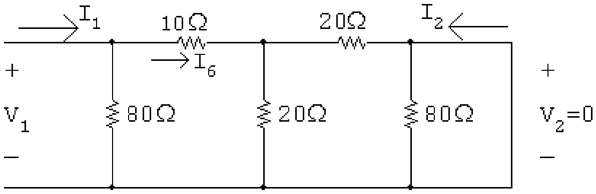
$$y_{11} = \frac{z_{22}}{\Delta z} = \frac{16}{64} = 0.25\,\mathrm{S}$$

$$y_{12} = \frac{-z_{12}}{\Delta z} = \frac{-12}{64} = -0.1875\,\mathrm{S}$$

$$y_{21} = \frac{-z_{21}}{\Delta z} = \frac{-12}{64} = -0.1875\,\mathrm{S}$$

$$y_{22} = \frac{-z_{11}}{\Delta z} = \frac{13}{64} = 0.203125\,\mathrm{S}$$

P 18.5 $h_{11} = \left.\frac{V_1}{I_1}\right|_{V_2=0}; \qquad h_{21} = \left.\frac{I_2}{I_1}\right|_{V_2=0}$

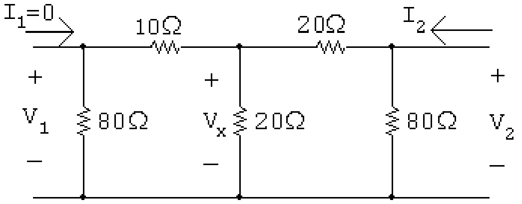


$$\frac{V_1}{I_1} = 80\|[10 + 20\|20] = 80\|20 = 16\,\Omega \qquad \therefore \quad h_{11} = 16\,\Omega$$

$$I_6 = \frac{80}{80 + 20} I_1 = 0.8 I_1$$

$$I_2 = \frac{-20}{20 + 20} I_6 = -0.5 I_6 = -0.5(0.8) I_1 = -0.4 I_1 \qquad \therefore \quad h_{21} = -0.4$$

$$h_{12} = \left.\frac{V_1}{V_2}\right|_{I_1=0}; \qquad h_{22} = \left.\frac{I_2}{V_2}\right|_{I_1=0}$$



$$\frac{V_2}{I_2} = 80\|[20 + 20\|90] = 25\,\Omega \qquad \therefore \quad h_{22} = \frac{1}{25} = 40 \text{ mS}$$

$$V_x = \frac{20\|90}{20 + 20\|90}V_2$$

$$V_1 = \frac{80}{80 + 10}V_x = \frac{80(20\|90)}{90(20 + 20\|90)}V_2 = 0.4V_2$$

$$\therefore \quad h_{12} = 0.4$$

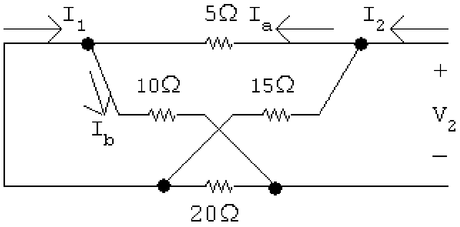
Summary:

$$h_{11} = 16\,\Omega; \quad h_{12} = 0.4; \quad h_{21} = -0.4; \quad h_{22} = 40 \text{ mS}$$

P 18.6 $V_2 = b_{11}V_1 - b_{12}I_1$

$$I_2 = b_{21}V_1 - b_{22}I_1$$

$$b_{12} = \left.\frac{-V_2}{I_1}\right|_{V_1=0}; \qquad b_{22} = \left.\frac{-I_2}{I_1}\right|_{V_1=0}$$



$$5\|15 = (15/4)\,\Omega; \qquad 10\|20 = (20/3)\,\Omega$$

$$I_2 = \frac{V_2}{(15/4) + (20/3)} = \frac{12V_2}{125}; \qquad I_1 = I_\text{b} - I_\text{a}$$

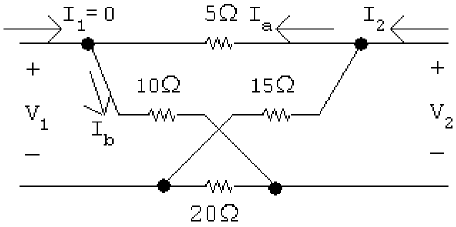
$$I_a = \frac{15}{20}I_2; \qquad I_b = \frac{20}{30}I_2$$

$$I_1 = \left(\frac{20}{30} - \frac{15}{20}\right) I_2 = \frac{-5}{60}I_2 = \frac{-1}{12}I_2$$

$$b_{22} = \frac{-I_2}{I_1} = 12$$

$$b_{12} = \frac{-V_2}{I_1} = \frac{-V_2}{I_2}\left(\frac{I_2}{I_1}\right) = \frac{125}{12}(12) = 125\,\Omega$$

$$b_{11} = \left.\frac{V_2}{V_1}\right|_{I_1=0}; \qquad b_{21} = \left.\frac{I_2}{V_1}\right|_{I_1=0}$$



$$V_1 = V_a - V_b; \quad V_a = \frac{10}{15}V_2; \quad V_b = \frac{20}{35}V_2$$

$$V_1 = \frac{10}{15}V_2 - \frac{20}{35}V_2 = \frac{2}{21}V_2$$

$$b_{11} = \frac{V_2}{V_1} = \frac{21}{2} = 10.5$$

$$V_2 = (10+5)\|(20+15)I_2 = 10.5I_2$$

$$b_{21} = \frac{I_2}{V_1} = \left(\frac{I_2}{V_2}\right)\left(\frac{V_2}{V_1}\right) = \left(\frac{1}{10.5}\right)(10.5) = 1\,\text{S}$$

P 18.7 $$h_{11} = \left.\frac{V_1}{I_1}\right|_{V_2=0} = R_1\|R_2 = 4 \quad \therefore \quad \frac{R_1R_2}{R_1+R_2} = 4$$

$$h_{21} = \left.\frac{I_2}{I_1}\right|_{V_2=0} = \frac{-R_2}{R_1+R_2} = -0.8$$

$$\therefore \qquad R_2 = 0.8R_1 + 0.8R_2 \quad \text{so} \quad R_1 = \frac{R_2}{4}$$

Substituting,

$$\frac{(R_2/4)R_2}{(R_2/4)+R_2} = 4 \quad \text{so} \quad R_2 = 20\,\Omega \quad \text{and} \quad R_1 = 5\,\Omega$$

$$h_{22} = \frac{I_2}{V_2}\bigg|_{I_1=0} = \frac{1}{R_3\|(R_1+R_2)} = \frac{1}{R_3\|25} = 0.14$$

$$\therefore \qquad R_3 = 10$$

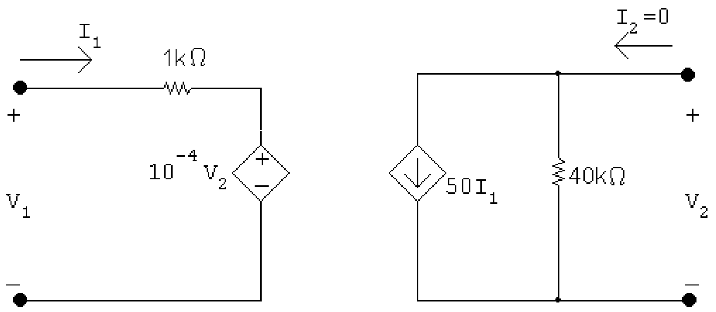
Summary:

$R_1 = 5\,\Omega; \qquad R_2 = 20\,\Omega; \qquad R_3 = 10\,\Omega$

P 18.8 $V_1 = a_{11}V_2 - a_{12}I_2$

$$I_1 = a_{21}V_2 - a_{22}I_2$$

$$a_{11} = \frac{V_1}{V_2}\bigg|_{I_2=0}; \qquad a_{21} = \frac{I_1}{V_2}\bigg|_{I_2=0}$$

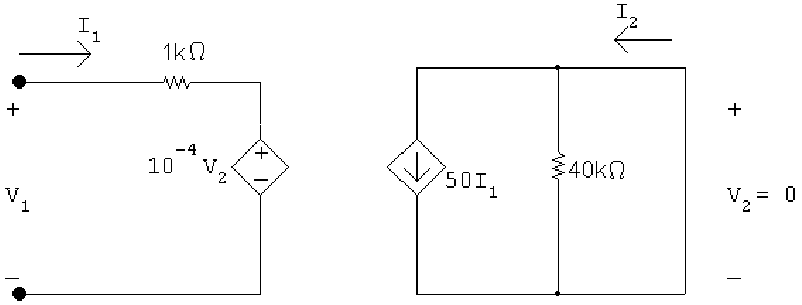


$$V_1 = 10^3I_1 + 10^{-4}V_2 = 10^3(-0.5\times10^{-6})V_2 + 10^{-4}V_2$$

$$\therefore \quad a_{11} = -5\times10^{-4} + 10^{-4} = -4\times10^{-4}$$

$$V_2 = -(50I_1)(40\times10^3); \qquad \therefore \quad a_{21} = -\frac{1}{2\times10^6} = -0.5\,\mu\text{S}$$

$$a_{12} = \left.\frac{-V_1}{I_2}\right|_{V_2=0}; \qquad a_{22} = \left.\frac{-I_1}{I_2}\right|_{V_2=0}$$



$$I_2 = 50I_1; \qquad \therefore \quad a_{22} = -\frac{I_1}{I_2} = -\frac{1}{50}$$

$$V_1 = 1000I_1; \qquad \therefore \quad a_{12} = -\frac{V_1}{I_2} = -\frac{V_1}{I_1}\frac{I_1}{I_2} = -(1000)(1/50) = -20\,\Omega$$

Summary

$$a_{11} = -4 \times 10^{-4}; \quad a_{12} = -20\,\Omega; \quad a_{21} = -0.5\,\mu\text{S}; \quad a_{22} = -0.02$$

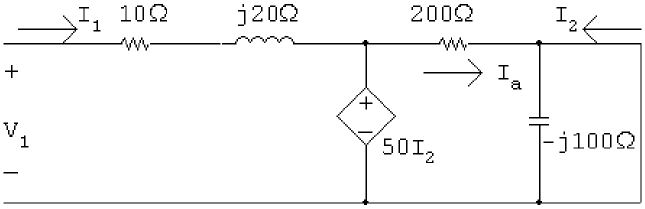
P 18.9 $g_{11} = \frac{a_{21}}{a_{11}} = \frac{-0.5 \times 10^{-6}}{-4 \times 10^{-4}} = 1.25\,\text{mS}$

$$g_{12} = \frac{-\Delta a}{a_{11}} = \frac{-(-4 \times 10^{-4})(-1/50) - (-0.5 \times 10^{-6})(-20)}{-4 \times 10^{-4}} = -0.005$$

$$g_{21} = \frac{1}{a_{11}} = \frac{1}{-4 \times 10^{-4}} = -2500$$

$$g_{22} = \frac{a_{12}}{a_{11}} = \frac{(-20)}{-400 \times 10^{-6}} = 5 \times 10^4\,\Omega$$

P 18.10 For $V_2 = 0$:

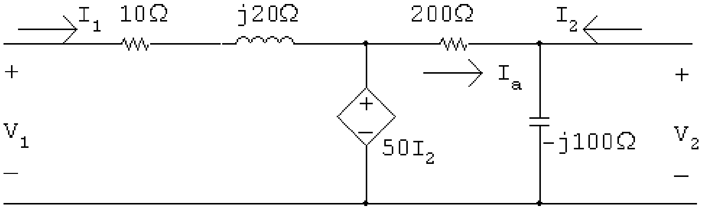


$$I_a = \frac{50I_2}{200} = \frac{1}{4}I_2 = -I_2; \quad \therefore \qquad I_2 = 0$$

$$h_{21} = \frac{I_2}{I_1}\bigg|_{V_2=0} = 0$$

$$V_1 = (10 + j20)I_1 \qquad \therefore \qquad h_{11} = \frac{V_1}{I_1}\bigg|_{V_2=0} = 10 + j20\,\Omega$$

For $I_1 = 0$:



$$V_1 = 50I_2; \qquad I_2 = \frac{V_2}{-j100} + \frac{V_2 - 50I_2}{200}$$

$$200I_2 = j2V_2 + V_2 - 50i_2$$

$$250I_2 = V_2(1 + j2)$$

$$50I_2 = V_2\left(\frac{1 + j2}{5}\right) = (0.2 + j0.4)V_2$$

$$\therefore \qquad V_1 = (0.2 + j0.4)V_2$$

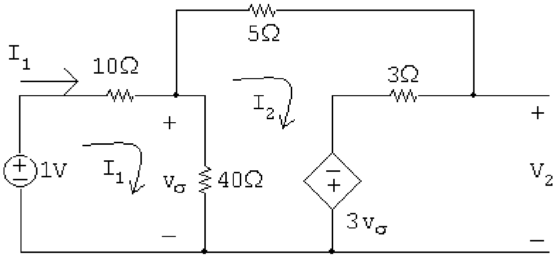
$$h_{12} = \frac{V_1}{V_2}\bigg|_{I_1=0} = 0.2 + j0.4$$

$$h_{22} = \frac{I_2}{V_2}\bigg|_{I_1=0} = \frac{1 + j2}{250} = 4 + j8 \text{ mS}$$

Summary:

$$h_{11} = 10 + j20\,\Omega; \quad h_{12} = 0.2 + j0.4; \quad h_{21} = 0; \quad h_{22} = 4 + j8\,\text{mS}$$

P 18.11 For $I_2 = 0$:



$$50I_1 - 40I_2 = 1$$

$$-40I_1 + 48I_2 - 3(40)(I_1 - I_2) = 0 \quad \text{so} \quad -160I_1 + 168I_2 = 0$$

Solving,

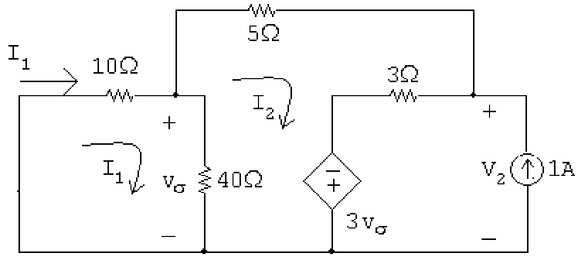
$$I_1 = 84\,\text{mA}; \qquad I_2 = 80\,\text{mA}$$

$$V_2 = 3I_2 - 3(40)(I_1 - I_2) = -0.24\,\text{V}$$

$$g_{11} = \frac{I_1}{V_1}\bigg|_{I_2=0} = \frac{84\,\text{m}}{1} = 84\,\text{mS}$$

$$g_{21} = \frac{V_2}{V_1}\bigg|_{I_2=0} = \frac{-0.24}{1} = -0.24$$

For $V_1 = 0$:



$$50I_1 - 40I_2 = 0$$

$$-40I_1 + 48I_2 + 3 - 3(40)(I_1 - I_2) = 0 \quad \text{so} \quad -160I_1 + 168I_2 = -3$$

Solving,

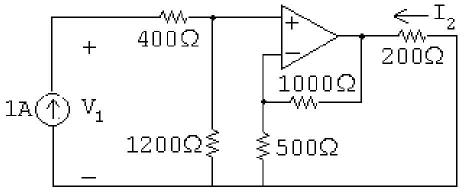
$$I_1 = -60\,\text{mA}; \qquad I_2 = -75\,\text{mA}$$

$$V_2 = 3(I_2 + 1) - 3(40)(I_1 - I_2) = 0.975\,\text{V}$$

$$g_{12} = \frac{I_1}{I_2}\bigg|_{V_1=0} = \frac{-60\,\text{m}}{1} = -0.06$$

$$g_{22} = \frac{V_2}{I_2}\bigg|_{V_1=0} = \frac{0.975}{1} = 0.975\,\Omega$$

P 18.12 For $V_2 = 0$:



$$V_1 = (400 + 1200)I_1$$

$$h_{11} = \frac{V_1}{I_1}\bigg|_{V_2=0} = \frac{1600}{1} = 1600\,\Omega$$

$$V_p = 1200(1\,\text{A}) = 1200\,\text{V} = V_n$$

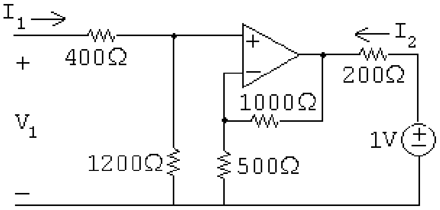
At V_n,

$$\frac{1200}{500} + \frac{1200 - V_o}{1000} = 0 \quad \text{so} \quad V_o = 3600\,\text{V}$$

$$\therefore \qquad I_2 = -\frac{3600}{200} = -18\,\text{A}$$

$$h_{21} = \frac{I_2}{I_1}\bigg|_{I_1=0} = \frac{-18}{1} = -18$$

For $I_1 = 0$:



$$V_1 = 0$$

$$h_{12} = \frac{V_1}{V_2}\bigg|_{I_1=0} = \frac{0}{1} = 0$$

At V_n,

$$\frac{V_n}{500} + \frac{V_n - V_o}{100} = 0$$

But $V_n = V_p = 0$ so $V_o = 0$; therefore,

$$I_2 = \frac{1\,\mathrm{V}}{200\,\Omega} = 5\,\mathrm{mS}$$

$$h_{22} = \frac{I_2}{V_2}\bigg|_{I_1=0} = \frac{5\,\mathrm{m}}{1} = 5\,\mathrm{mS}$$

P 18.13 $I_1 = g_{11}V_1 + g_{12}I_2; \qquad V_2 = g_{21}V_1 + g_{22}I_2$

$$g_{11} = \frac{I_1}{V_1}\bigg|_{I_2=0} = \frac{0.25 \times 10^{-6}}{20 \times 10^{-3}} = 12.5 \times 10^{-6} = 12.5\,\mu\mathrm{S}$$

$$g_{21} = \frac{V_2}{V_1}\bigg|_{I_2=0} = \frac{-5}{20} \times 10^3 = -250$$

$$0 = -250(10) + g_{22}(50 \times 10^{-6})$$

$$g_{22} = \frac{2500}{50 \times 10^{-6}} = 50\,\mathrm{M}\Omega$$

$$200 \times 10^{-6} = 12.5 \times 10^{-6}(10) + g_{12}(50 \times 10^{-6})$$

$$(200 - 125)10^{-6} = g_{12}(50 \times 10^{-6})$$

$$g_{12} = \frac{75}{50} = 1.5$$

P 18.14 **[a]** $I_1 = y_{11}V_1 + y_{12}V_2; \qquad I_2 = y_{21}V_1 + y_{22}V_2$

$$y_{21} = \frac{I_2}{V_1}\bigg|_{V_2=0} = \frac{50 \times 10^{-6}}{10} = 5\,\mu\text{S}$$

$$0 = y_{21}(20 \times 10^{-3}) + y_{22}(-5)$$

$$\therefore \qquad y_{22} = \frac{1}{5}y_{21}(20 \times 10^{-3}) = 20\,\text{nS}$$

$$200 \times 10^{-6} = y_{11}(10) \quad \text{so} \quad y_{11} = 20\,\mu\text{S}$$

$$0.25 \times 10^{-6} = 20 \times 10^{-6}(20 \times 10^{-3}) + y_{12}(-5)$$

$$y_{12} = \frac{0.25 \times 10^{-6} - 0.4 \times 10^{-6}}{-5} = 30\,\text{nS}$$

Summary:

$$y_{11} = 20\,\mu\text{S}; \quad y_{12} = 30\,\text{nS}; \quad y_{21} = 5\,\mu\text{S}; \quad y_{22} = 20\,\text{nS}$$

[b] $y_{11} = \dfrac{\Delta g}{g_{22}}; \quad y_{12} = \dfrac{g_{12}}{g_{22}}; \quad y_{21} = \dfrac{-g_{21}}{g_{22}}; \quad y_{22} = \dfrac{1}{g_{22}}$

$$\Delta g = g_{11}g_{22} - g_{12}g_{21} = (12.5 \times 10^{-6})(50 \times 10^{6}) - 1.5(-250)$$

$$= 625 + 375 = 1000$$

$$y_{11} = \frac{1000}{50 \times 10^6} = 20\,\mu\text{S}; \qquad y_{21} = \frac{250}{5 \times 10^6} = 5\,\mu\text{S}$$

$$y_{12} = \frac{1.5}{50 \times 10^6} = 30\,\text{nS}; \qquad y_{22} = \frac{1}{5 \times 10^6} = 20\,\text{nS}$$

These values are the same as those in part (a).

P 18.15 $I_1 = g_{11}V_1 + g_{12}I_2$

$$V_2 = g_{21}V_1 + g_{22}I_2$$

$$V_1 = \frac{I_1}{g_{11}} - \frac{g_{12}}{g_{11}}I_2 \quad \text{and} \quad I_2 = \frac{V_2}{g_{22}} - \frac{g_{21}}{g_{22}}V_1$$

Substituting,

$$V_1 = \frac{I_1}{g_{11}} - \frac{g_{12}}{g_{11}}\left[\frac{V_2}{g_{22}} - \frac{g_{21}}{g_{22}}V_1\right]$$

$$V_1 = \left(1 - \frac{g_{12}g_{21}}{g_{11}g_{22}}\right) = \frac{I_1}{g_{11}} - \frac{g_{12}}{g_{11}g_{22}}V_2$$

$$V_1 = \frac{g_{22}}{g_{11}g_{22} - g_{12}g_{21}} I_1 - \frac{g_{12}}{g_{11}g_{22} - g_{12}g_{21}} V_2$$

$$V_1 = h_{11}I_1 + h_{12}V_2$$

Therefore,

$$h_{11} = \frac{g_{22}}{\Delta g}; \qquad h_{12} = \frac{-g_{12}}{\Delta g} \quad \text{where} \quad \Delta g = g_{11}g_{22} - g_{12}g_{21}$$

$$I_2 = \frac{V_2}{g_{22}} - \frac{g_{21}}{g_{22}} \left[\frac{I_1}{g_{11}} - \frac{g_{12}}{g_{11}} I_2 \right]$$

$$I_2 = \left(1 - \frac{g_{12}g_{21}}{g_{11}g_{22}} \right) = \frac{V_2}{g_{22}} - \frac{g_{21}}{g_{11}g_{22}} I_1$$

$$I_2 = \frac{g_{11}}{\Delta g} V_2 - \frac{g_{21}}{\Delta g} I_1$$

$$I_2 = h_{21}I_1 + h_{22}V_2$$

Therefore,

$$h_{21} = \frac{-g_{21}}{\Delta g}; \quad h_{22} = \frac{g_{11}}{\Delta g}$$

P 18.16 $V_1 = h_{11}I_1 + h_{12}V_2; \qquad I_2 = h_{21}I_1 + h_{22}V_2$

Rearranging the first equation,

$$V_2 = \frac{1}{h_{12}} V_1 - \frac{h_{11}}{h_{12}} I_1$$

$$V_2 = b_{11}V_1 - b_{12}I_1$$

Therefore,

$$b_{11} = \frac{1}{h_{12}}; \qquad b_{12} = \frac{h_{11}}{h_{12}}$$

Solving the second h-parameter equation for I_2:

$$I_2 = h_{21}I_1 + h_{22} \left(\frac{1}{h_{12}} V_1 - \frac{h_{11}}{h_{12}} I_1 \right)$$

$$= I_1\left(h_{21} - \frac{h_{22}h_{11}}{h_{12}}\right) + \frac{h_{22}}{h_{12}}V_1$$

$$= \frac{-\Delta h}{h_{12}}I_1 + \frac{h_{22}}{h_{12}}V_1$$

$$I_2 = b_{21}V_1 - b_{22}I_1$$

Therefore,

$$b_{21} = \frac{h_{22}}{h_{12}}; \qquad b_{22} = \frac{\Delta h}{h_{12}} \quad \text{where} \quad \Delta h = h_{11}h_{22} - h_{12}h_{21}$$

P 18.17 $I_1 = g_{11}V_1 + g_{12}I_2; \qquad V_2 = g_{21}V_1 + g_{22}I_2$

$$V_1 = z_{11}I_1 + z_{12}I_2; \qquad V_2 = z_{21}I_1 + z_{22}I_2$$

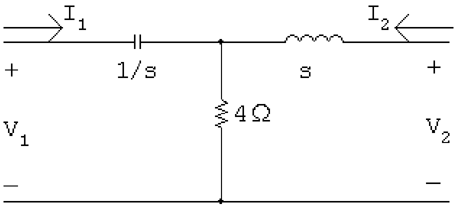
$$I_1 = \frac{V_1}{z_{11}} - \frac{z_{12}}{z_{11}}I_2$$

$$\therefore \quad g_{11} = \frac{1}{z_{11}}; \qquad g_{12} = \frac{-z_{12}}{z_{11}}$$

$$V_2 = z_{21}\left(\frac{V_1}{z_{11}} - \frac{z_{12}}{z_{11}}I_2\right) + z_{22}I_2 = \frac{z_{21}}{z_{11}}V_1 + \left(\frac{z_{11}z_{22} - z_{12}z_{21}}{z_{11}}\right)I_2$$

$$\therefore \quad g_{21} = \frac{z_{21}}{z_{11}}; \qquad g_{22} = \frac{\Delta z}{z_{11}} \qquad \text{where} \qquad \Delta z = z_{11}z_{22} - z_{12}z_{21}$$

P 18.18 For $I_2 = 0$:



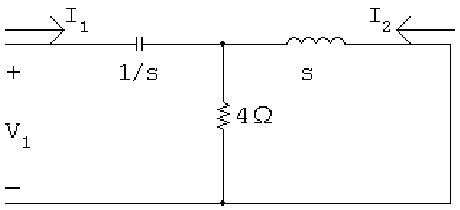
$$V_2 = \frac{V_1}{4 + (1/s)}(4) = \frac{4sV_1}{4s + 1}$$

$$a_{11} = \frac{V_1}{V_2}\bigg|_{I_2=0} = \frac{4s + 1}{4s} = \frac{s + 0.25}{s}$$

$$I_1 = \frac{V_1}{4 + (1/s)} = \frac{sV_1}{4s + 1} \quad \text{so} \quad V_2 = 4I_1 = \frac{4sV_1}{4s + 1}$$

$$a_{21} = \frac{I_1}{V_2}\bigg|_{I_2=0} = \frac{1}{4} = 0.25$$

For $V_2 = 0$:



$$I_2 = \frac{-I_1(4)}{s+4}$$

$$a_{22} = -\frac{I_1}{I_2}\bigg|_{V_2=0} = \frac{s+4}{4}$$

$$V_1 = \frac{1}{s}I_1 + \frac{4s}{s+4}I_1 = \left(\frac{1}{s} + \frac{4s}{s+4}\right)I_1 = \left(\frac{1}{s} + \frac{4s}{s+4}\right)\left(\frac{-(s+4)}{4}I_2\right)$$

$$= -\left(\frac{s+4}{4s} + s\right)I_2 = -\frac{4s^2+s+4}{4s}I_2$$

$$a_{12} = -\frac{V_1}{I_2}\bigg|_{V_2=0} = \frac{s^2+0.25s+1}{s}$$

P 18.19 $z_{11} = \frac{V_1}{I_1}\bigg|_{I_2=0} = \frac{(1+1/s)(1)}{2+1/s} + s = \frac{s+1}{2s+1} + s$

$$= \frac{s+1+2s^2+2}{2s+1} = \frac{2s^2+2s+1}{2s+1}$$

$z_{22} = z_{11}$ (the circuit is reciprocal and symmetrical)

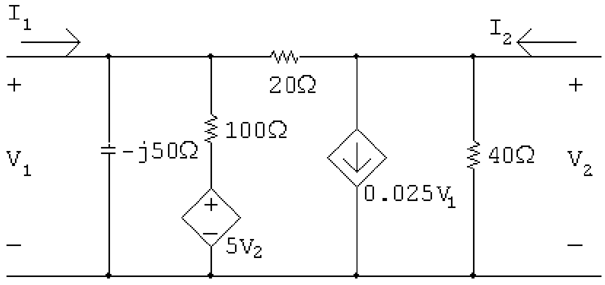
$$z_{21} = \frac{V_2}{I_1}\bigg|_{I_2=0}$$

$V_2 = I_1\frac{1}{2+1/s}(1) + sI_1;$ $\qquad \frac{V_2}{I_1} = \frac{s}{2s+1} + s = \frac{s+2s^2+s}{2s+1}$

$$z_{21} = \frac{2s^2+2s}{2s+1} = \frac{2s(s+1)}{2s+1}$$

$z_{12} = z_{21}$ (the circuit is reciprocal and symmetrical)

P 18.20 For $I_2 = 0$:



$$\frac{V_2 - V_1}{20} + 0.025V_1 + \frac{V_2}{40} = 0$$

$$2V_2 - 2V_1 + V_1 + V_2 = 0 \quad \text{so} \quad 3V_2 = V_1$$

$$\therefore \qquad a_{11} = \frac{V_1}{V_2}\bigg|_{I_2=0} = 3$$

$$I_1 = \frac{V_1}{-j50} + \frac{V_1 - 5V_2}{100} + \frac{V_1 - V_2}{20}$$

$$= V_1\left[\frac{j}{50} + \frac{1}{100} + \frac{1}{20}\right] - V_2\left[\frac{5}{100} + \frac{1}{20}\right]$$

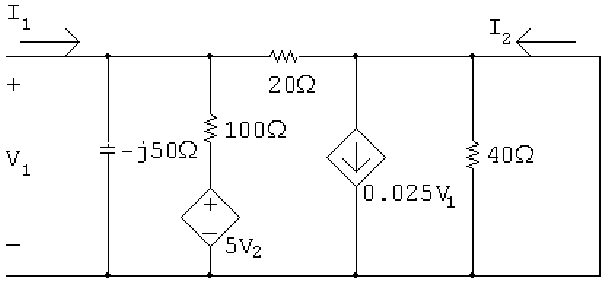
$$= V_1\left[\frac{6 + j2}{100}\right] - V_2\left[\frac{1}{10}\right]$$

But $V_1 = 3V_2$ so

$$I_1 = \left[\frac{18 + j6 - 10}{100}\right]V_2 = (0.08 + j0.06)V_2$$

$$a_{21} = \frac{I_1}{V_2}\bigg|_{I_2=0} = 0.08 + j0.06\,\text{S} = 80 + j60\,\text{mS}$$

For $V_2 = 0$:



$$I_1 = \frac{V_1}{-j50} + \frac{V_1}{100} + \frac{V_1}{20} = V_1\frac{(6+j2)}{100}$$

$$I_2 = 0.025V_1 - \frac{V_1}{20} = -0.025V_1$$

$$a_{12} = -\frac{V_1}{I_2}\bigg|_{V_2=0} = \frac{1}{0.025} = 40\,\Omega$$

$$a_{22} = -\frac{I_1}{I_2}\bigg|_{V_2=0} = \frac{-2V_1(3+j1)}{100(-0.025)V_1} = 2.4 + j0.8$$

Summary:

$a_{11} = 3$; $a_{12} = 40\,\Omega$; $a_{21} = 80 + j60\,\text{mS}$; $a_{22} = 2.4 + j0.8$

P 18.21 $h_{11} = \frac{a_{12}}{a_{22}} = \frac{40}{(0.8)(3+j1)} = 15 - j5\,\Omega$

$$h_{12} = \frac{\Delta a}{a_{22}}$$

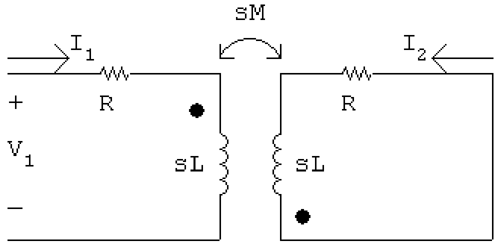
$$\Delta a = 3(2.4 + j0.8) - 40(0.08 + j0.06) = 7.2 + j2.4 - 3.2 - j2.4 = 4$$

$$h_{12} = \frac{4}{(0.8)(3+j1)} = 1.5 - j0.50$$

$$h_{21} = -\frac{1}{a_{22}} = \frac{-1}{(0.8)(3+j1)} = -0.375 + j0.125$$

$$h_{22} = \frac{a_{21}}{a_{22}} = \frac{0.08 + j0.06}{(0.8)(3+j1)} = 0.0375 + j0.0125 = 37.5 + j12.5\,\text{mS}$$

P 18.22 **[a]** $h_{11} = \left.\frac{V_1}{I_1}\right|_{V_2=0}$; $\qquad h_{21} = \left.\frac{I_2}{I_1}\right|_{V_2=0}$



$$V_1 = (R + sL)I_1 - sMI_2$$

$$0 = -sMI_1 + (R + sL)I_2$$

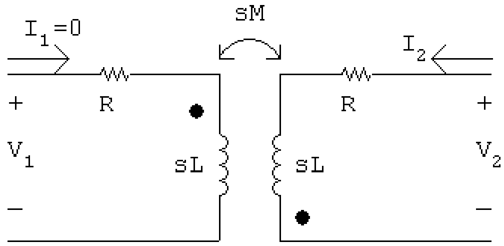
$$\Delta = \begin{vmatrix} (R+sL) & -sM \\ -sM & (R+sL) \end{vmatrix} = (R+sL)^2 - s^2M^2$$

$$N_1 = \begin{vmatrix} V_1 & -sM \\ 0 & (R+sL) \end{vmatrix} = (R+sL)V_1$$

$$I_1 = \frac{N_1}{\Delta} = \frac{(R+sL)V_1}{(R+sL)^2 - s^2M^2}; \qquad h_{11} = \frac{V_1}{I_1} = \frac{(R+sL)^2 - s^2M^2}{R+sL}$$

$$0 = -sMI_1 + (R+sL)I_2; \qquad \therefore \quad h_{21} = \frac{I_2}{I_1} = \frac{sM}{R+sL}$$

$$h_{12} = \left.\frac{V_1}{V_2}\right|_{I_1=0}; \qquad h_{22} = \left.\frac{I_2}{V_2}\right|_{I_1=0}$$



$$V_1 = -sMI_2; \qquad I_2 = \frac{V_2}{R+sL}$$

$$V_1 = \frac{-sMV_2}{R+sL}; \qquad h_{12} = \frac{V_1}{V_2} = \frac{-sM}{R+sL}$$

$$h_{22} = \frac{I_2}{V_2} = \frac{1}{R+sL}$$

[b] $h_{12} = -h_{21}$ (reciprocal)

$$h_{11}h_{22} - h_{12}h_{21} = 1 \quad \text{(symmetrical, reciprocal)}$$

$$h_{12} = \frac{-sM}{R+sL}; \qquad h_{21} = \frac{sM}{R+sL} \quad \text{(checks)}$$

$$h_{11}h_{22} - h_{12}h_{21} = \frac{(R+sL)^2 - s^2M^2}{R+sL} \cdot \frac{1}{R+sL} - \frac{(sM)(-sM)}{(R+sL)^2}$$

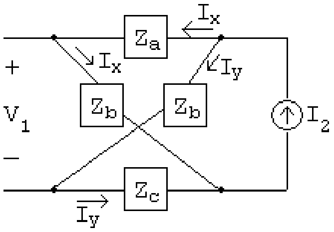
$$= \frac{(R+sL)^2 - s^2M^2 + s^2M^2}{(R+sL)^2} = 1 \quad \text{(checks)}$$

P 18.23 First we note that

$$z_{11} = \frac{(Z_\mathrm{b}+Z_\mathrm{c})(Z_\mathrm{a}+Z_\mathrm{b})}{Z_\mathrm{a}+2Z_\mathrm{b}+Z_\mathrm{c}} \quad \text{and} \quad z_{22} = \frac{(Z_\mathrm{a}+Z_\mathrm{b})(Z_\mathrm{b}+Z_\mathrm{c})}{Z_\mathrm{a}+2Z_\mathrm{b}+Z_\mathrm{c}}$$

Therefore $z_{11} = z_{22}$.

$$z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0}; \qquad \text{Use the circuit below:}$$

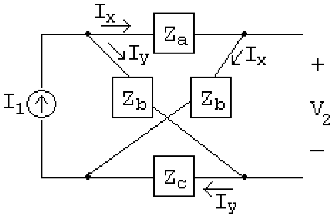


$$V_1 = Z_bI_x - Z_cI_y = Z_bI_x - Z_c(I_2 - I_x) = (Z_b + Z_c)I_x - Z_cI_2$$

$$I_x = \frac{Z_b+Z_c}{Z_a+2Z_b+Z_c}I_2 \qquad \text{so} \qquad V_1 = \frac{(Z_b+Z_c)^2}{Z_a+2Z_b+Z_c}I_2 - Z_cI_2$$

$$\therefore \quad Z_{12} = \frac{V_1}{I_2} = \frac{(Z_b+Z_c)^2}{Z_a+2Z_b+Z_c} - Z_c = \frac{Z_b^2 - Z_aZ_c}{Z_a+2Z_b+Z_c}$$

$z_{21} = \left.\dfrac{V_2}{I_1}\right|_{I_2=0};$ Use the circuit below:



$$V_2 = Z_bI_x - Z_cI_y = Z_bI_x - Z_c(I_1 - I_x) = (Z_b + Z_c)I_x - Z_cI_1$$

$$I_x = \frac{Z_b + Z_c}{Z_a + 2Z_b + Z_c}I_1 \qquad \text{so} \qquad V_2 = \frac{(Z_b + Z_c)^2}{Z_a + 2Z_b + Z_c}I_1 - Z_cI_1$$

$$\therefore \quad z_{21} = \frac{V_2}{I_1} = \frac{(Z_b + Z_c)^2}{Z_a + 2Z_b + Z_c} - Z_c = \frac{Z_b^2 - Z_aZ_c}{Z_a + 2Z_b + Z_c} = z_{12}$$

Thus the network is symmetrical and reciprocal.

P 18.24 $I_1 = y_{11}V_1 + y_{12}V_2; \qquad V_1 = V_g - Z_gI_1$

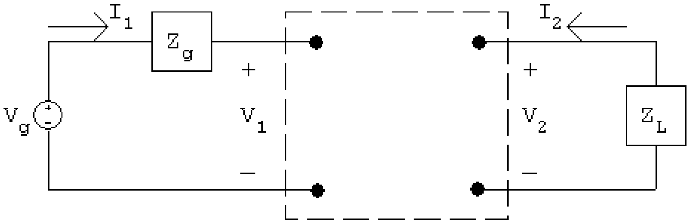
$$I_2 = y_{21}V_1 + y_{22}V_2; \qquad V_2 = -Z_LI_2$$

$$\frac{-V_2}{Z_L} = y_{21}V_1 + y_{22}V_2$$

$$\therefore \quad -y_{21}V_1 = \left(\frac{1}{Z_L} + y_{22}\right)V_2; \qquad -y_{21}Z_LV_1 = (1 + y_{22}Z_L)V_2$$

$$\therefore \quad \frac{V_2}{V_1} = \frac{-y_{21}Z_L}{1 + y_{22}Z_L}$$

P 18.25



$$V_2 = b_{11}V_1 - b_{12}I_1; \qquad V_1 = V_g - I_1Z_g$$

$$I_2 = b_{21}V_1 - b_{22}I_1; \qquad V_2 = -Z_LI_2$$

$$I_2 = -\frac{V_2}{Z_L} = \frac{-b_{11}V_1 + b_{12}I_1}{Z_L}$$

$$\frac{-b_{11}V_1 + b_{12}I_1}{Z_L} = b_{21}V_1 - b_{22}I_1$$

$$\therefore \quad V_1\left(\frac{b_{11}}{Z_L} + b_{21}\right) = \left(b_{22} + \frac{b_{12}}{Z_L}\right) I_1$$

$$\frac{V_1}{I_1} = \frac{b_{22}Z_L + b_{12}}{b_{21}Z_L + b_{11}} = Z_{\text{in}}$$

P 18.26 $V_1 = h_{11}I_1 + h_{12}V_2; \qquad V_1 = V_g - Z_gI_1$

$$I_2 = h_{21}I_1 + h_{22}V_2; \qquad V_2 = -Z_LI_2$$

$$\therefore \quad V_g - Z_gI_1 = h_{11}I_1 + h_{12}V_2; \qquad V_g = (h_{11} + Z_g)I_1 + h_{12}V_2$$

$$\therefore \quad I_1 = \frac{V_g - h_{12}V_2}{h_{11} + Z_g}$$

$$\therefore \quad -\frac{V_2}{Z_L} = h_{21}\left[\frac{V_g - h_{12}V_2}{h_{11} + Z_g}\right] + h_{22}V_2$$

$$\frac{-V_2(h_{11} + Z_g)}{Z_L} = h_{21}V_g - h_{12}h_{21}V_2 + h_{22}(h_{11} + Z_g)V_2$$

$$-V_2(h_{11} + Z_g) = h_{21}Z_LV_g - h_{12}h_{21}Z_LV_2 + h_{22}Z_L(h_{11} + Z_g)V_2$$

$$-h_{21}Z_LV_g = (h_{11} + Z_g)\left[V_2 + h_{22}Z_LV_2\right] - h_{12}h_{21}Z_LV_2$$

$$\therefore \quad \frac{V_2}{V_g} = \frac{-h_{21}Z_L}{(h_{11} + Z_g)(1 + h_{22}Z_L) - h_{12}h_{21}Z_L}$$

P 18.27 $I_1 = g_{11}V_1 + g_{12}I_2; \qquad V_1 = V_g - Z_gI_1$

$$V_2 = g_{21}V_1 + g_{22}I_2; \qquad V_2 = -Z_LI_2$$

$$-Z_LI_2 = g_{21}V_1 + g_{22}I_2; \qquad V_1 = \frac{I_1 - g_{12}I_2}{g_{11}}$$

$$\therefore \quad -Z_LI_2 = \frac{g_{21}}{g_{11}}(I_1 - g_{12}I_2) + g_{22}I_2$$

$$\therefore \quad -Z_LI_2 + \frac{g_{12}g_{21}}{g_{11}}I_2 - g_{22}I_2 = \frac{g_{21}}{g_{11}}I_1$$

$$\therefore \quad (Z_Lg_{11} + \Delta g)I_2 = -g_{21}I_1; \qquad \therefore \quad \frac{I_2}{I_1} = \frac{-g_{21}}{g_{11}Z_L + \Delta g}$$

P 18.28 $V_1 = z_{11}I_1 + z_{12}I_2; \qquad V_1 = V_g - Z_gI_1$

$$V_2 = z_{21}I_1 + z_{22}I_2; \qquad V_2 = -Z_LI_2$$

$$V_{\text{Th}} = V_2\Big|_{I_2=0}; \qquad V_2 = z_{21}I_1; \qquad I_1 = \frac{V_1}{z_{11}} = \frac{V_g - I_1Z_g}{z_{11}}$$

$$\therefore \quad I_1 = \frac{V_g}{z_{11} + Z_g}; \qquad \therefore \quad V_2 = \frac{z_{21}V_g}{z_{11} + Z_g} = V_{\text{Th}}$$

$$Z_{\text{Th}} = \frac{V_2}{I_2}\Big|_{V_g=0}; \qquad V_2 = z_{21}I_1 + z_{22}I_2$$

$$-I_1Z_g = z_{11}I_1 + z_{12}I_2; \qquad I_1 = \frac{-z_{12}I_2}{z_{11} + Z_g}$$

$$\therefore \quad V_2 = z_{21}\left[\frac{-z_{12}I_2}{z_{11} + Z_g}\right] + z_{22}I_2$$

$$\therefore \quad \frac{V_2}{I_2} = z_{22} - \frac{z_{12}z_{21}}{z_{11} + Z_g} = Z_{\text{Th}}$$

P 18.29 $\dfrac{V_2}{V_g} = \dfrac{\Delta b Z_{\text{L}}}{b_{12} + b_{11}Z_g + b_{22}Z_{\text{L}} + b_{21}Z_gZ_{\text{L}}}$

$$\Delta b = b_{11}b_{22} - b_{12}b_{21} = (25)(-40) - (1000)(-1.25) = 250$$

$$\therefore \qquad \frac{V_2}{V_g} = \frac{250(100)}{1000 + 25(20) - 40(100) - 1.25(2000)} = -5$$

$$V_2 = -5(120\underline{/0^\circ}) = 600\underline{/180^\circ}\,\text{V(rms)}$$

$$I_2 = \frac{-V_2}{100} = \frac{-600\underline{/180^\circ}}{100} = 6\,\text{A(rms)}$$

$$\frac{I_2}{I_1} = \frac{-\Delta b}{b_{11} + b_{21}Z_{\text{L}}} = \frac{-250}{25 - 1.25(100)} = 2.5$$

$$\therefore \qquad I_1 = \frac{I_2}{2.5} = \frac{6}{2.5} = 2.4\,\text{A(rms)}$$

$$\therefore \qquad P_g = (120)(2.4) = 288\,\text{W}; \qquad P_o = 36(100) = 3600\,\text{W}$$

$$\therefore \qquad \frac{P_o}{P_g} = \frac{3600}{288} = 12.5$$

P 18.30 [**a**] $$\frac{\mathbf{V}_2}{\mathbf{V}_g} = \frac{y_{21}Z_L}{y_{12}y_{21}Z_gZ_L - (1+y_{11}Z_g)(1+y_{22}Z_L)}$$

$$y_{12}y_{21}Z_gZ_L = (-2\times10^{-6})(100\times10^{-3})(2500)(70{,}000) = -35$$

$$1 + y_{11}Z_g = 1 + (2\times10^{-3})(2500) = 6$$

$$1 + y_{22}Z_L = 1 + (-50\times10^{-6})(70\times10^{3}) = -2.5$$

$$y_{21}Z_L = (100\times10^{-3})(70\times10^{3}) = 7000$$

$$\frac{\mathbf{V}_2}{\mathbf{V}_g} = \frac{7000}{-35-(6)(-2.5)} = \frac{7000}{-20} = -350$$

$$\mathbf{V}_2 = -350\mathbf{V}_g = -350(80)\times10^{-3} = -28\,\text{V(rms)}$$

$$V_2 = 28\,\text{V(rms)}$$

[**b**] $$P = \frac{|\mathbf{V}_2|^2}{70{,}000} = 11.2\times10^{-3} = 11.20\,\text{mW}$$

[**c**] $$\mathbf{I}_2 = \frac{-28\underline{/180^\circ}}{70{,}000} = -0.4\times10^{-3}\underline{/180^\circ} = 400\underline{/0^\circ}\,\mu\text{A}$$

$$\frac{\mathbf{I}_2}{\mathbf{I}_1} = \frac{y_{21}}{y_{11}+\Delta y Z_L}$$

$$\Delta y = (2\times10^{-3})(-50\times10^{-6}) - (-2\times10^{-6})(100\times10^{-3})$$
$$= 100\times10^{-9}$$

$$\Delta y Z_L = (100)(70)\times10^{3}\times10^{-9} = 7\times10^{-3}$$

$$y_{11}+\Delta y Z_L = 2\times10^{-3} + 7\times10^{-3} = 9\times10^{-3}$$

$$\frac{\mathbf{I}_2}{\mathbf{I}_1} = \frac{100\times10^{-3}}{9\times10^{-3}} = \frac{100}{9}$$

$$\therefore\quad 100\mathbf{I}_1 = 9\mathbf{I}_2;\qquad \mathbf{I}_1 = \frac{9(400\times10^{-6})}{100} = 36\,\mu\text{A(rms)}$$

$$P_g = (80)10^{-3}(36)\times10^{-6} = 2.88\,\mu\text{W}$$

P 18.31 [**a**] $$Z_{\text{Th}} = \frac{1+y_{11}Z_g}{y_{22}+\Delta y Z_g}$$

From the solution to Problem 18.30

$$1 + y_{11}Z_g = 1 + (2\times10^{-3})(2500) = 6$$

$$y_{22}+\Delta y Z_g = -50\times10^{-6} + 10^{-7}(2500) = 200\times10^{-6}$$

$$Z_{\text{Th}} = \frac{6}{200}\times10^{6} = 30{,}000\,\Omega$$

$$Z_L = Z_{\text{Th}}^{*} = 30{,}000\,\Omega$$

[b] $y_{21}Z_L = (100 \times 10^{-3})(30{,}000) = 3000$

$$y_{12}y_{21}Z_gZ_L = (-2 \times 10^{-6})(100 \times 10^{-3})(2500)(30{,}000) = -15$$

$$1 + y_{11}Z_g = 6$$

$$1 + y_{22}Z_L = 1 + (-50 \times 10^{-6})(30 \times 10^3) = -0.5$$

$$\frac{\mathbf{V}_2}{\mathbf{V}_g} = \frac{3000}{-15 - 6(-0.5)} = \frac{3000}{-12} = -250$$

$$\mathbf{V}_2 = -250(80 \times 10^{-3}) = -20 = 20\underline{/180^\circ}\,\text{V(rms)}$$

$$P = \frac{|\mathbf{V}_2|^2}{30{,}000} = \frac{400}{30} \times 10^{-3} = 13.33\,\text{mW}$$

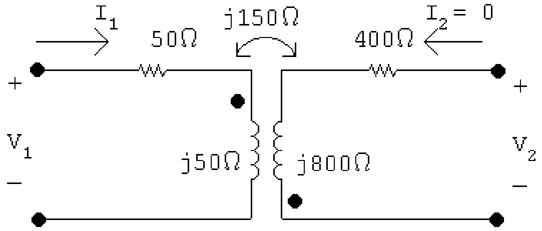
[c] $\mathbf{I}_2 = \dfrac{-\mathbf{V}_2}{30{,}000} = \dfrac{20\underline{/0^\circ}}{30{,}000} = \dfrac{2}{3}\,\text{mA}$

$$\frac{\mathbf{I}_2}{\mathbf{I}_1} = \frac{100 \times 10^{-3}}{2 \times 10^{-3} + 10^{-7}(30{,}000)} = \frac{100 \times 10^{-3}}{5 \times 10^{-3}} = 20$$

$$\mathbf{I}_1 = \frac{\mathbf{I}_2}{20} = \frac{2 \times 10^{-3}}{3(20)} = \frac{1}{30}\,\text{mA(rms)}$$

$$P_g(\text{developed}) = (80 \times 10^{-3})\left(\frac{1}{30} \times 10^{-3}\right) = \frac{8}{3}\,\mu\text{W}$$

P 18.32 **[a]** For $I_2 = 0$:

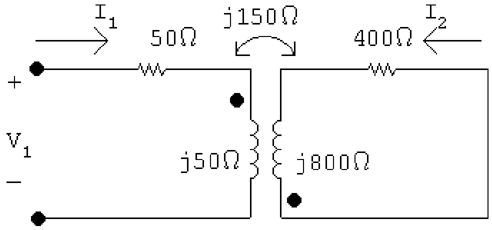


$$V_2 = -j150I_1 = -j150\frac{V_1}{50 + j50} = \frac{-j3V_1}{1 + j1}$$

$$a_{11} = \left.\frac{V_1}{V_2}\right|_{I_2=0} = \frac{1 + j1}{-j3} = \frac{-1 + j1}{3}$$

$$a_{21} = \left.\frac{I_1}{V_2}\right|_{I_2=0} = \frac{1}{-j150} = \frac{j}{150}\,\text{S}$$

For $V_2 = 0$:



$$V_1 = (50 + j50)I_1 - j150I_2$$

$$0 = -j150I_1 + (400 + j800)I_2$$

$$\Delta = \begin{vmatrix} 50 + j50 & -j150 \\ -j150 & 400 + j800 \end{vmatrix} = 2500(1 + j24)$$

$$N_2 = \begin{vmatrix} 50 + j50 & V_1 \\ -j150 & 0 \end{vmatrix} = j150V_1$$

$$I_2 = \frac{N_2}{\Delta} = \frac{j150V_1}{2500(1 + j24)}$$

$$a_{12} = \frac{-V_1}{I_2}\bigg|_{V_2=0} = \frac{-50}{3}(24 - j1)\,\Omega$$

$$j150I_1 = (400 + j800)I_2$$

$$a_{22} = -\frac{I_1}{I_2}\bigg|_{V_2=0} = -\frac{8}{3}(2 - j1)$$

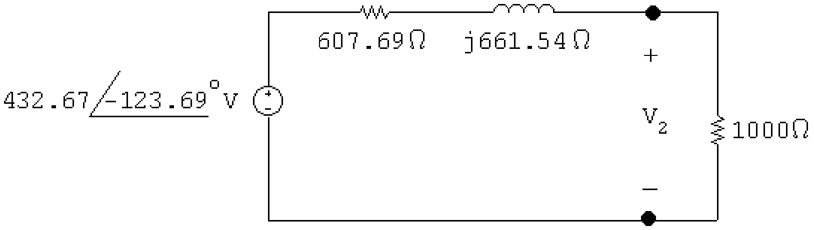
[b] $$V_{\text{Th}} = \frac{V_g}{a_{11} + a_{21}Z_g} = \frac{260\underline{/0^\circ}}{(-1 + j1)/3 + j25/150} = \frac{(260\underline{/0^\circ})6}{-2 + j2 + j1} = \frac{1560\underline{/0^\circ}}{-2 + j3}$$

$$= 120(-2 - j3) = 432.47\underline{/-123.69^\circ}\text{ V}$$

$$Z_{\text{Th}} = \frac{a_{12} + a_{22}Z_g}{a_{11} + a_{21}Z_g} = \frac{[-(50/3)(24 - j1)] + [(-8/3)(2 - j1)(25)]}{[(-1 + j1)/3] + [(j/150)(25)]}$$

$$= \frac{-100(24 - j1) - 16(2 - j1)(25)}{-2 + j2 + j1} = \frac{-3200 + j500}{-2 + j3}$$

$$= 607.69 + j661.54\,\Omega$$



[c] $$V_2 = \frac{1000}{1607.69 + j661.54}(432.67\underline{/-123.69^\circ}) = 248.88\underline{/-146.06^\circ}$$

$$v_2(t) = 248.88\cos(4000t - 146.06^\circ)\,\text{V}$$

P 18.33 [a] $$Z_{\text{Th}} = g_{22} - \frac{g_{12}g_{21}Z_g}{1 + g_{11}Z_g}$$

$$g_{12}g_{21} = \left(-\frac{1}{2} + j\frac{1}{2}\right)\left(\frac{1}{2} - j\frac{1}{2}\right) = j\frac{1}{2}$$

$$1 + g_{11}Z_g = 1 + 1 - j1 = 2 - j1$$

$$\therefore\quad Z_{\text{Th}} = 1.5 + j2.5 - \frac{j3}{2 - j1} = 2.1 + j1.3\,\Omega$$

$$\therefore\quad Z_L = 2.1 - j1.3\,\Omega$$

$$\frac{\mathbf{V}_2}{\mathbf{V}_g} = \frac{g_{21}Z_L}{(1 + g_{11}Z_g)(g_{22} + Z_L) - g_{12}g_{21}Z_g}$$

$$g_{21}Z_L = \left(\frac{1}{2} - j\frac{1}{2}\right)(2.1 - j1.3) = 0.4 - j1.7$$

$$1 + g_{11}Z_g = 1 + 1 - j1 = 2 - j1$$

$$g_{22} + Z_L = 1.5 + j2.5 + 2.1 - j1.3 = 3.6 + j1.2$$

$$g_{12}g_{21}Z_g = j3$$

$$\frac{\mathbf{V}_2}{\mathbf{V}_g} = \frac{0.4 - j1.7}{(2 - j1)(3.6 + j1.2) - j3} = \frac{0.4 - j1.7}{8.4 - j4.2}$$

$$\mathbf{V}_2 = \frac{0.4 - j1.7}{8.4 - j4.2}(42\underline{/0^\circ}) = 5 - j6\,\text{V(rms)} = 7.81\underline{/-50.19^\circ}\,\text{V(rms)}$$

The rms value of V_2 is 7.81 V.

[b] $$\mathbf{I}_2 = \frac{-\mathbf{V}_2}{Z_L} = \frac{-5 + j6}{2.1 - j1.3} = -3 + j1\,\text{A(rms)}$$

$$P = |\mathbf{I}_2|^2(2.1) = 21\,\text{W}$$

[**c**] $\frac{\mathbf{I}_2}{\mathbf{I}_1} = \frac{-g_{21}}{g_{11}Z_L + \Delta g}$

$$\Delta g = \left(\frac{1}{6} - j\frac{1}{6}\right)\left(\frac{3}{2} + j\frac{5}{2}\right) - \left(\frac{1}{2} - j\frac{1}{2}\right)\left(-\frac{1}{2} + j\frac{1}{2}\right)$$

$$= \frac{3}{12} + j\frac{5}{12} - j\frac{3}{12} + \frac{5}{12} - j\frac{1}{2} = \frac{2}{3} - j\frac{1}{3}$$

$$g_{11}Z_L = \left(\frac{1}{6} - j\frac{1}{6}\right)(2.1 - j1.3) = \frac{0.8}{6} - j\frac{3.4}{6}$$

$$\therefore \quad g_{11}Z_L + \Delta g = \frac{0.8}{6} - j\frac{3.4}{6} + \frac{4}{6} - j\frac{2}{6} = 0.8 - j0.9$$

$$\frac{\mathbf{I}_2}{\mathbf{I}_1} = \frac{-[(1/2) - j(1/2)]}{0.8 - j0.9}$$

$$\therefore \quad \mathbf{I}_1 = \frac{(0.8 - j0.9)\mathbf{I}_2}{-0.5 + j0.5} = \left(\frac{1.6 - j1.8}{-1 + j1}\right)\mathbf{I}_2$$

$$= (-1.7 + j0.1)(-3 + j1) = 5 - j2\,\text{A (rms)}$$

$$\therefore \quad P_g(\text{developed}) = (42)(5) = 210\,\text{W}$$

$$\%\ \text{delivered} = \frac{21}{210}(100) = 10\%$$

P 18.34 $V_1 = h_{11}I_1 + h_{12}V_2$

$$I_2 = h_{21}I_1 + h_{22}V_2$$

From the first measurement:

$$h_{11} = \frac{V_1}{I_1} = \frac{4}{5} \times 10^3 = 800\,\Omega$$

$$h_{21} = \frac{-200}{5} = -40$$

$$\therefore \qquad V_1 = 800I_1 + h_{12}V_2; \qquad I_2 = -40I_1 + h_{22}V_2$$

From the second measurement:

$$h_{22}V_2 = 40I_1$$

$$h_{22} = \frac{40(20 \times 10^{-6})}{40} = 20\,\mu\text{S}$$

$$20 \times 10^{-3} = 800(20 \times 10^{-6}) + 40h_{12}$$

$$\therefore \quad h_{12} = \frac{4 \times 10^{-3}}{40} = 10^{-4}$$

Summary:

$$h_{11} = 800\,\Omega; \quad h_{12} = 10^{-4}; \quad h_{21} = -40; \quad h_{22} = 20\,\mu\text{S}$$

From the circuit,

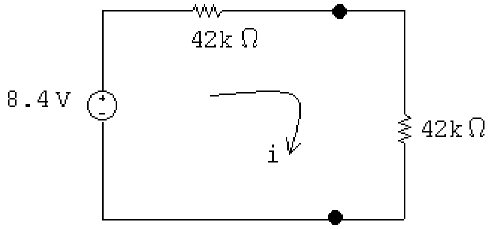
$$Z_g = 250\,\Omega; \qquad V_g = 5.25\,\text{mV}$$

$$Z_{\text{Th}} = \frac{h_{11} + Z_g}{h_{22}Z_g + \Delta h}$$

$$\Delta h = 800(20 \times 10^{-6}) + 40 \times 10^{-4} = 20 \times 10^{-3}$$

$$Z_{\text{Th}} = \frac{800 + 250}{20 \times 10^{-6}(250) + 20 \times 10^{-3}} = 42\,\text{k}\Omega$$

$$V_{\text{Th}} = \frac{-h_{21}V_g}{h_{22}Z_g + \Delta h} = \frac{40(5.25 \times 10^{-3})}{25 \times 10^{-3}} = 8.4\,\text{V}$$



$$i = \frac{8.4}{84{,}000} = 0.10\,\text{mA}$$

$$P = (0.10 \times 10^{-3})^2(42{,}000) = 420\,\mu\text{W}$$

P 18.35 When $V_2 = 0$

$$V_1 = 20\,\text{V}, \qquad I_1 = 1\,\text{A}, \qquad I_2 = -1\,\text{A}$$

When $I_1 = 0$

$$V_2 = 80\,\text{V}, \qquad V_1 = 400\,\text{V}, \qquad I_2 = 3\,\text{A}$$

$$h_{11} = \frac{V_1}{I_1}\bigg|_{V_2=0} = \frac{20}{1} = 20\,\Omega$$

$$h_{12} = \frac{V_1}{V_2}\bigg|_{I_1=0} = \frac{400}{80} = 5$$

$$h_{21} = \frac{I_2}{I_1}\bigg|_{V_2=0} = \frac{-1}{1} = -1$$

$$h_{22} = \frac{I_2}{V_2}\bigg|_{I_1=0} = \frac{3}{80} = 37.5\,\text{mS}$$

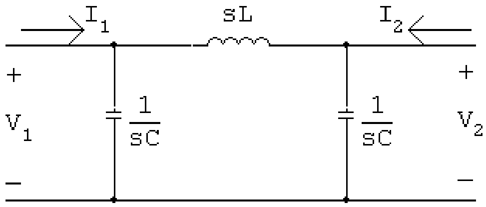
$$Z_{\text{Th}} = \frac{Z_g + h_{11}}{h_{22}Z_g + \Delta h} = 10\,\Omega$$

Source-transform the current source and parallel resistance to get $V_g = 240$ V. Then,

$$I_2 = \frac{h_{21}V_g}{(1 + h_{22}Z_{\text{L}})(h_{11} + Z_g) - h_{12}h_{21}Z_{\text{L}}} = -1.5\,\text{A}$$

$$P = (-1.5)^2(10) = 22.5\,\text{W}$$

P 18.36 **[a]** $g_{11} = \frac{I_1}{V_1}\bigg|_{I_2=0}$; $\qquad g_{21} = \frac{V_2}{V_1}\bigg|_{I_2=0}$



$$\frac{V_1}{I_1} = \frac{1}{sC}\|\left(sL + \frac{1}{sC}\right) = \frac{[sL + (1/sC)](1/sC)}{sL + (2/sC)}$$

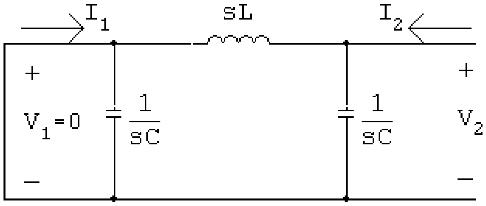
$$= \frac{sL + (1/sC)}{s^2LC + 2} = \frac{s^2LC + 1}{sC(s^2LC + 2)} = \frac{(1/C)[s^2 + (1/LC)]}{s[s^2 + (2/LC)]}$$

$$\therefore \quad g_{11} = \frac{Cs[s^2 + (2/LC)])}{s^2 + (1/LC)}$$

$$V_2 = \frac{(1/sC)}{sL + (1/sC)}V_1 \qquad \text{so} \qquad \frac{V_2}{V_1} = \frac{(1/sC)}{sL + (1/sC)} = \frac{1}{s^2LC + 1} = \frac{(1/LC)}{s^2 + (1/LC)}$$

$$\therefore \quad g_{21} = \frac{(1/LC)}{s^2 + (1/LC)}$$

$$g_{12} = \frac{I_1}{I_2}\bigg|_{V_1=0}; \qquad g_{22} = \frac{V_2}{I_2}\bigg|_{V_1=0}$$



$$I_1 = \frac{-(1/sC)}{sL + (1/sC)} I_2 \qquad \text{so} \qquad g_{12} = \frac{-(1/LC)}{s^2 + (1/LC)}$$

$$g_{22} = sL\|(1/sC) = \frac{sL/sC}{sL + (1/sC)} = \frac{sL}{s^2LC + 1} = \frac{(1/C)s}{s^2 + (1/LC)}$$

Summary:

$$g_{11} = \frac{Cs[s^2 + (2/LC)])}{s^2 + (1/LC)}; \qquad g_{12} = \frac{-(1/LC)}{s^2 + (1/LC)}$$

$$g_{21} = \frac{(1/LC)}{s^2 + (1/LC)}; \qquad g_{22} = \frac{(1/C)s}{s^2 + (1/LC)}$$

[b] $\frac{1}{LC} = \frac{10^9}{(0.2)(200)} = 25 \times 10^6$

$$g_{11} = \frac{2 \times 10^{-7}s(s^2 + 50 \times 10^6)}{s^2 + 25 \times 10^6}$$

$$g_{12} = \frac{-25 \times 10^6}{s^2 + 25 \times 10^6}$$

$$g_{21} = \frac{25 \times 10^6}{s^2 + 25 \times 10^6}$$

$$g_{22} = \frac{5 \times 10^6 s}{s^2 + 25 \times 10^6}$$

$$\frac{V_2}{V_1} = \frac{g_{21}Z_L}{g_{22} + Z_L} = \frac{\left(\frac{25\times10^6}{s^2+25\times10^6}\right)400}{\frac{5\times10^6 s}{(s^2+25\times10^6)} + 400}$$

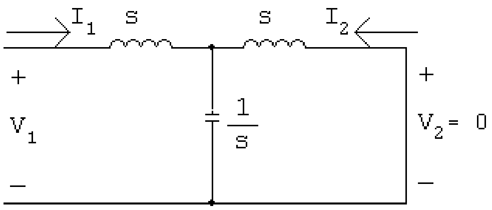
$$\frac{V_2}{V_1} = \frac{25 \times 10^6}{s^2 + 12{,}500s + 25 \times 10^6} = \frac{25 \times 10^6}{(s + 2500)(s + 10{,}000)}$$

$$V_1 = \frac{30}{s}$$

$$V_2 = \frac{750 \times 10^6}{s(s + 2500)(s + 10{,}000)} = \frac{30}{s} - \frac{40}{s + 2500} + \frac{10}{s + 10{,}000}$$

$$v_2 = [30 - 40e^{-2500t} + 10e^{-10{,}000t}]u(t) \quad \text{V}$$

P 18.37 **[a]** $y_{11} = \left.\dfrac{I_1}{V_1}\right|_{V_2=0}$; $\qquad y_{21} = \left.\dfrac{I_2}{V_1}\right|_{V_2=0}$



$$V_1 = \left[s + \left(\frac{1}{s}\|s\right)\right] I_1 = \frac{s(s^2+1)+s}{s^2+1} I_1$$

$$\therefore \quad y_{11} = \frac{I_1}{V_1} = \frac{s^2+1}{s(s^2+2)}$$

$$I_2 = \frac{-(1/s)}{s+(1/s)} I_1 = \frac{-1}{s^2+1} \cdot \frac{s^2+1}{s(s^2+2)} V_1 = \frac{-1}{s(s^2+2)} V_1$$

$$\therefore \quad y_{21} = \frac{-1}{s(s^2+2)}$$

Because the two-port circuit is symmetric,

$$y_{12} = y_{21} = \frac{-1}{s(s^2+2)} \qquad \text{and} \qquad y_{22} = y_{11} = \frac{s^2+1}{s(s^2+2)}$$

[b]
$$\frac{V_2}{V_g} = \frac{y_{21}Z_g}{y_{12}y_{21}Z_gZ_L - (1+y_{11}Z_g)(1+Y_{22}Z_L)}$$

$$= \frac{y_{21}}{y_{12}y_{21} - (1+y_{11})(1+y_{22})}$$

$$= \frac{\dfrac{-1}{s(s^2+2)}}{\dfrac{1}{s^2(s^2+2)^2} - \left(1 + \dfrac{s^2+1}{s(s^2+2)}\right)\left(1 + \dfrac{s^2+1}{s(s^2+2)}\right)}$$

$$= \frac{-s(s^2+2)}{1-(s^3+s^2+2s+1)^2}$$

$$= \frac{1}{s^3+2s^2+3s+2}$$

$$= \frac{1}{(s+1)(s^2+s+2)}$$

$$\therefore \quad V_2 = \frac{50}{s(s+1)(s^2+s+2)}$$

$$s_{1,2} = -\frac{1}{2} \pm j\frac{\sqrt{7}}{2}$$

$$V_2 = \frac{K_1}{s} + \frac{K_2}{s+1} + \frac{K_3}{s+\frac{1}{2}-j\frac{\sqrt{7}}{2}} + \frac{K_3^*}{s+\frac{1}{2}+j\frac{\sqrt{7}}{2}}$$

$$K_1 = 25; \qquad K_2 = -25; \qquad K_3 = 9.45\underline{/90^\circ}$$

$$\therefore \quad v_2(t) = [25 - 25e^{-t} + 18.90e^{-0.5t}\cos(1.32t + 90^\circ)]u(t)\,\text{V}$$

P 18.38 The a parameters of the first two port are

$$a'_{11} = \frac{-\Delta h}{h_{21}} = \frac{-5\times 10^{-3}}{40} = -125\times 10^{-6}$$

$$a'_{12} = \frac{-h_{11}}{h_{21}} = \frac{-1000}{40} = -25\,\Omega$$

$$a'_{21} = \frac{-h_{22}}{h_{21}} = \frac{-25}{40}\times 10^{-6} = -625\times 10^{-9}\,\text{S}$$

$$a'_{22} = \frac{-1}{h_{21}} = \frac{-1}{40} = -25\times 10^{-3}$$

The a parameters of the second two port are

$$a''_{11} = \frac{5}{4}; \quad a''_{12} = \frac{3R}{4}; \quad a''_{21} = \frac{3}{4R}; \quad a''_{22} = \frac{5}{4}$$

or $a''_{11} = 1.25; \quad a''_{12} = 54\,\text{k}\Omega; \quad a''_{21} = \frac{1}{96}\,\text{mS}; \quad a''_{22} = 1.25$

The a parameters of the cascade connection are

$$a_{11} = -125\times 10^{-6}(1.25) + (-25)(10^{-3}/96) = \frac{-10^{-2}}{24}$$

$$a_{12} = -125\times 10^{-6}(54\times 10^3) + (-25)(1.25) = -38\,\Omega$$

$$a_{21} = -625\times 10^{-9}(1.25) + (-25\times 10^{-3})(10^{-3}/96) = \frac{-10^{-4}}{96}\,\text{S}$$

$$a_{22} = -625\times 10^{-9}(54\times 10^3) + (-25\times 10^{-3})(1.25) = -65\times 10^{-3}$$

$$\frac{V_o}{V_g} = \frac{Z_L}{(a_{11} + a_{21}Z_g)Z_L + a_{12} + a_{22}Z_g}$$

$$a_{21}Z_g = \frac{-10^{-4}}{96}(800) = \frac{-10^{-2}}{12}$$

$$a_{11} + a_{21}Z_g = \frac{-10^{-2}}{24} + \frac{-10^{-2}}{12} = \frac{-10^{-2}}{8}$$

$$(a_{11} + a_{21}Z_g)Z_L = \frac{-10^{-2}}{8}(72{,}000) = -90$$

$$a_{22}Z_g = -65 \times 10^{-3}(800) = -52$$

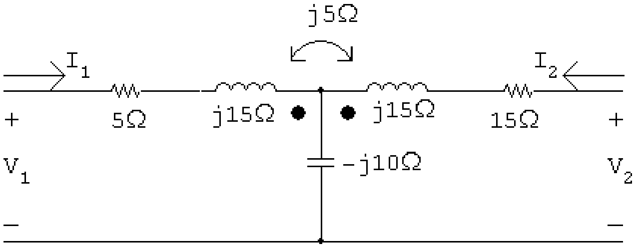
$$\frac{V_o}{V_g} = \frac{72{,}000}{-90 - 38 - 52} = -400$$

$$v_o = V_o = -400V_g = -3.6\,\text{V}$$

P 18.39 [**a**] From reciprocity and symmetry

$$a'_{11} = a'_{22}, \quad \Delta a' = 1; \quad \therefore \quad 5^2 - 24a'_{21} = 1, \quad a'_{21} = 1\,\text{S}$$

For network B



$$a''_{11} = \left.\frac{\mathbf{V}_1}{\mathbf{V}_2}\right|_{I_2=0}$$

$$\mathbf{V}_1 = (5 + j15 - j10)\mathbf{I}_1 = (5 + j5)\mathbf{I}_1$$

$$\mathbf{V}_2 = (-j10 + j5)\mathbf{I}_1 = -j5\mathbf{I}_1$$

$$a''_{11} = \frac{5 + j5}{-j5} = -1 + j1$$

$$a''_{21} = \left.\frac{\mathbf{I}_1}{\mathbf{V}_2}\right|_{I_2=0} = \frac{1}{-j5} = j0.2\,\text{S}$$

$$a''_{22} = a''_{11} = -1 + j1$$

$$\Delta a'' = 1 = (-1 + j1)(-1 + j1) - j0.2a''_{12}$$

$$\therefore \quad a''_{12} = -10 + j5$$

Summary:

$a'_{11} = 5$ $\qquad a''_{11} = -1 + j1$

$a'_{12} = 24\,\Omega$ $\qquad a''_{12} = -10 + j5\,\Omega$

$a'_{21} = 1\,\text{S}$ $\qquad a''_{21} = j0.2\,\text{S}$

$a'_{22} = 5$ $\qquad a''_{22} = -1 + j1$

[b] $a_{11} = a'_{11}a''_{11} + a'_{12}a''_{21} = -5 + j9.8$

$$a_{12} = a'_{11}a''_{12} + a'_{12}a''_{22} = -74 + j49\,\Omega$$

$$a_{21} = a'_{21}a''_{11} + a'_{22}a''_{21} = -1 + j2\,\text{S}$$

$$a_{22} = a'_{21}a''_{12} + a'_{22}a''_{22} = -15 + j10$$

$$\mathbf{I}_2 = \frac{-V_g}{a_{11}Z_\text{L} + a_{12} + a_{21}Z_gZ_\text{L}} = 0.295 + j0.279\,\text{A}$$

$$\mathbf{V}_2 = -10I_2 = -2.95 - j2.79\,\text{V}$$

P 18.40 $a'_{11} = \dfrac{z_{11}}{z_{21}} = \dfrac{35/3}{4000/3} = 8.75 \times 10^{-3}\,\Omega$

$$a'_{12} = \frac{\Delta z}{z_{21}} = \frac{25 \times 10^4/3}{4000/3} = 62.5\,\Omega$$

$$a'_{21} = \frac{1}{z_{21}} = \frac{1}{4000/3} = 0.75 \times 10^{-3}\,\Omega$$

$$a'_{22} = \frac{z_{22}}{z_{21}} = \frac{10{,}000/3}{4000/3} = 2.5\,\Omega$$

$$a''_{11} = \frac{-y_{22}}{y_{21}} = \frac{-40 \times 10^{-6}}{-800 \times 10^{-6}} = 0.05\,\text{S}$$

$$a''_{12} = \frac{-1}{y_{21}} = \frac{-1}{-800 \times 10^{-6}} = 1250\,\text{S}$$

$$a''_{21} = \frac{-\Delta y}{y_{21}} = \frac{-4 \times 10^{-8}}{-800 \times 10^{-6}} = 50 \times 10^{-6}\,\text{S}$$

$$a''_{22} = \frac{-y_{11}}{y_{21}} = \frac{-200 \times 10^{-6}}{-800 \times 10^{-6}} = 0.25\,\text{S}$$

$$a_{11} = a'_{11}a''_{11} + a'_{12}a''_{21} = (8.75 \times 10^{-3})(0.05) + (62.5)(50 \times 10^{-6}) = 3.5625 \times 10^{-3}$$

$$a_{12} = a'_{11}a''_{12} + a'_{12}a''_{22} = (8.75 \times 10^{-3})(1250) + (62.5)(0.25) = 26.5625$$

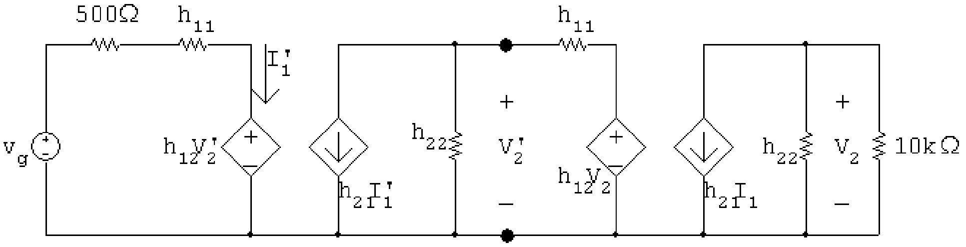
$$a_{21} = a'_{21}a''_{11} + a'_{22}a''_{21} = (0.75 \times 10^{-3})(0.05) + (2.5)(50 \times 10^{-6}) = 162.5 \times 10^{-6}$$

$$a_{22} = a'_{21}a''_{12} + a'_{22}a''_{22} = (0.75 \times 10^{-3})(1250) + (2.5)(0.25) = 1.5625$$

$$V_2 = \frac{Z_LV_g}{(a_{11} + a_{21}Z_g)Z_L + a_{12} + a_{22}Z_g}$$

$$= \frac{(15{,}000)(0.03)}{[3.5625 \times 10^{-3} + (162.5 \times 10^{-6})(10)](15{,}000) + 26.5625 + (1.5625)(10)} = 3.75\,\text{V}$$

P 18.41 **[a]** At the input port: $V_1 = h_{11}I_1 + h_{12}V_2$;
At the output port: $I_2 = h_{21}I_1 + h_{22}V_2$



[b] $$\frac{V_2}{10^4} + (100 \times 10^{-6}V_2) + 100I_1 = 0$$

therefore $I_1 = -2 \times 10^{-6}V_2$

$$V_2' = 1000I_1 + 15 \times 10^{-4}V_2 = -5 \times 10^{-4}V_2$$

$$100I_1' + 10^{-4}V_2' + (-2 \times 10^{-6})V_2 = 0$$

therefore $I_1' = 205 \times 10^{-10}V_2$

$$V_g = 1500I_1' + 15 \times 10^{-4}V_2' = 3000 \times 10^{-8}V_2$$

$$\frac{V_2}{V_g} = \frac{10^5}{3} = 33{,}333$$

P 18.42 **[a]** $$V_1 = I_2(z_{12} - z_{21}) + I_1(z_{11} - z_{21}) + z_{21}(I_1 + I_2)$$

$$= I_2z_{12} - I_2z_{21} + I_1z_{11} - I_1z_{21} + z_{21}I_1 + z_{21}I_2 = z_{11}I_1 + z_{12}I_2$$

$$V_2 = I_2(z_{22} - z_{21}) + z_{21}(I_1 + I_2) = z_{21}I_1 + z_{22}I_2$$

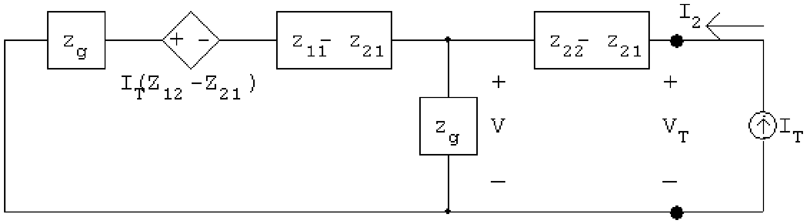
[b] Short circuit V_g and apply a test current source to port 2 as shown. Note that $I_T = I_2$. We have

$$\frac{V}{z_{21}} - I_T + \frac{V + I_T(z_{12} - z_{21})}{Z_g + z_{11} - z_{21}} = 0$$

Therefore

$$V = \left[\frac{z_{21}(Z_g + z_{11} - z_{12})}{Z_g + z_{11}}\right] I_T \quad \text{and} \quad V_T = V + I_T(z_{22} - z_{21})$$

Thus $$\frac{V_T}{I_T} = Z_{\text{Th}} = z_{22} - \left(\frac{z_{12}z_{21}}{Z_g + z_{11}}\right)$$



For V_{Th} note that $V_{\text{oc}} = \dfrac{z_{21}}{Z_g + z_{11}} V_g$ since $I_2 = 0$.

P 18.43 **[a]** $V_1 = (z_{11} - z_{12})I_1 + z_{12}(I_1 + I_2) = z_{11}I_1 + z_{12}I_2$

$$V_2 = (z_{21} - z_{12})I_1 + (z_{22} - z_{12})I_2 + z_{12}(I_2 + I_1) = z_{21}I_1 + z_{22}I_2$$

[b] With port 2 terminated in an impedance Z_L, the two mesh equations are

$$V_1 = (z_{11} - z_{12})I_1 + z_{12}(I_1 + I_2)$$

$$0 = Z_L I_2 + (z_{21} - z_{12})I_1 + (z_{22} - z_{12})I_2 + z_{12}(I_1 + I_2)$$

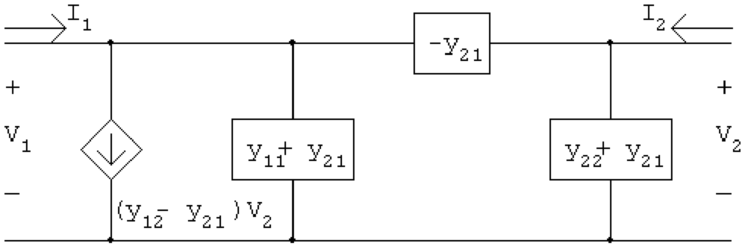
Solving for I_1:

$$I_1 = \frac{V_1(z_{22} + Z_L)}{z_{11}(Z_L + z_{22}) - z_{12}z_{21}}$$

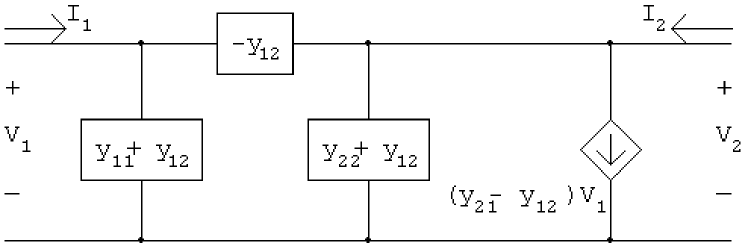
Therefore

$$Z_{\text{in}} = \frac{V_1}{I_1} = z_{11} - \frac{z_{12}z_{21}}{z_{22} + Z_L}$$

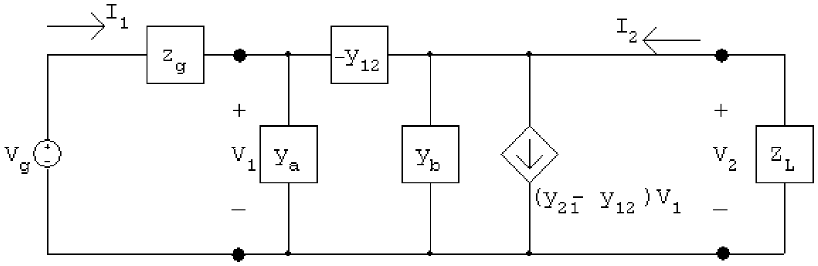
P 18.44 **[a]** $I_1 = y_{11}V_1 + y_{21}V_2 + (y_{12} - y_{21})V_2; \qquad I_2 = y_{21}V_1 + y_{22}V_2$



$I_1 = y_{11}V_1 + y_{12}V_2; \qquad I_2 = y_{12}V_1 + y_{22}V_2 + (y_{21} - y_{12})V_1$



[b] Using the second circuit derived in part [a], we have



where $y_{\mathrm{a}} = (y_{11} + y_{12})$ and $y_{\mathrm{b}} = (y_{22} + y_{12})$

At the input port we have

$$I_1 = y_{\mathrm{a}} V_1 - y_{12}(V_1 - V_2) = y_{11}V_1 + y_{12}V_2$$

At the output port we have

$$\frac{V_2}{Z_L} + (y_{21} - y_{12})V_1 + y_{\mathrm{b}}V_2 - y_{12}(V_2 - V_1) = 0$$

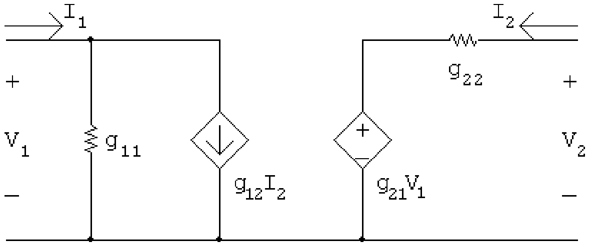
Solving for V_1 gives

$$V_1 = \left(\frac{1 + y_{22}Z_L}{-y_{21}Z_L}\right) V_2$$

Substituting Eq. (18.2) into (18.1) and at the same time using $V_2 = -Z_L I_2$, we get

$$\frac{I_2}{I_1} = \frac{y_{21}}{y_{11} + \Delta y Z_L}$$

P 18.45 **[a]** The *g*-parameter equations are $I_1 = g_{11}V_1 + g_{12}I_2$ and $V_2 = g_{21}V_1 + g_{22}I_2$. These equations are satisfied by the following circuit:



[b] The g parameters for the first two port in Fig P 18.38(a) are

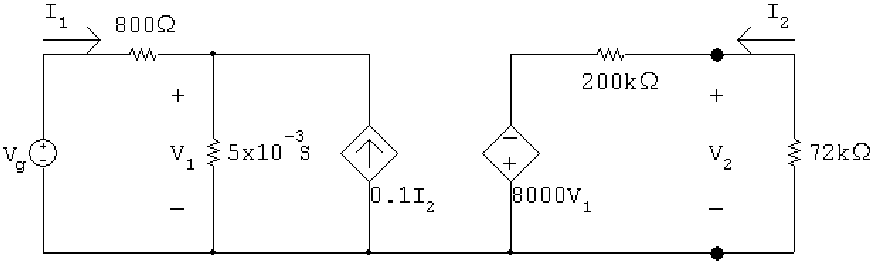
$$g_{11} = \frac{h_{22}}{\Delta h} = \frac{25 \times 10^{-6}}{5 \times 10^{-3}} = 5 \times 10^{-3}\,\mathrm{S}$$

$$g_{12} = \frac{-h_{12}}{\Delta h} = \frac{-5 \times 10^{-4}}{5 \times 10^{-3}} = -0.10$$

$$g_{21} = \frac{-h_{21}}{\Delta h} = \frac{-40}{5 \times 10^{-3}} = -8000$$

$$g_{22} = \frac{h_{11}}{\Delta h} = \frac{1000}{5 \times 10^{-3}} = 200\,\text{k}\Omega$$

From Problem 3.65 $R_{ef} = 72\,\text{k}\Omega$, hence our circuit reduces to



$$V_2 = \frac{-8000V_1(72)}{272}$$

$$I_2 = \frac{-V_2}{72{,}000} = \frac{8V_1}{272}$$

$$v_g = 9\,\text{mV}$$

$$\therefore \quad \frac{V_1 - 9 \times 10^{-3}}{800} + V_1(5 \times 10^{-3}) - 0.1\frac{8V_1}{272} = 0$$

$$V_1 - 9 \times 10^{-3} + 4V_1 - \frac{80V_1}{34} = 0$$

$$\therefore \quad V_1 = 3.4 \times 10^{-3}$$

$$V_2 = \frac{-8000(72)}{272} \times 3.4 \times 10^{-3} = -7.2\,\text{V}$$

From Problem 3.65

$$\frac{V_o}{V_2} = 0.5; \qquad \therefore \quad V_o = -3.6\,\text{V}$$

This result matches the solution to Problem 18.38.

P 18.46 **[a]** To determine b_{11} and b_{21} create an open circuit at port 1. Apply a voltage at port 2 and measure the voltage at port 1 and the current at port 2. To determine b_{12} and b_{22} create a short circuit at port 1. Apply a voltage at port 2 and measure the currents at ports 1 and 2.

[b] The equivalent b-parameters for the black-box amplifier can be calculated as follows:

$$b_{11} = \frac{1}{h_{12}} = \frac{1}{10^{-3}} = 1000$$

$$b_{12} = \frac{h_{11}}{h_{12}} = \frac{500}{10^{-3}} = 500\,\text{k}\Omega$$

$$b_{21} = \frac{h_{22}}{h_{12}} = \frac{0.05}{10^{-3}} = 50\,\text{S}$$

$$b_{22} = \frac{\Delta h}{h_{12}} = \frac{23.5}{10^{-3}} = 23{,}500$$

Create an open circuit a port 1. Apply 1 V at port 2. Then,

$$b_{11} = \frac{V_2}{V_1}\bigg|_{I_1=0} = \frac{1}{V_1} = 1000 \quad \text{so} \quad V_1 = 1\,\text{mV measured}$$

$$b_{21} = \frac{I_2}{V_1}\bigg|_{I_1=0} = \frac{I_2}{10^{-3}} = 50\,\text{S} \quad \text{so} \quad I_2 = 50\,\text{mA measured}$$

Create a short circuit a port 1. Apply 1 V at port 2. Then,

$$b_{12} = -\frac{V_2}{I_1}\bigg|_{V_1=0} = \frac{-1}{I_1} = 500\,\text{k}\Omega \quad \text{so} \quad I_1 = -2\,\mu\text{A measured}$$

$$b_{22} = -\frac{I_2}{I_1}\bigg|_{V_1=0} = \frac{-I_2}{-2\times 10^{-6}} = 23{,}500 \quad \text{so} \quad I_2 = 47\,\text{mA measured}$$

P 18.47 **[a]** To determine z_{11} and z_{21} create an open circuit at port 2. Apply a current at port 1 and measure the voltages at ports 1 and 2. To determine z_{12} and z_{22} create an open circuit at port 1. Apply a current at port 2 and measure the voltages at ports 1 and 2.

[b] The equivalent z-parameters for the black-box amplifier can be calculated as follows:

$$z_{11} = \frac{\Delta h}{h_{22}} = \frac{23.5}{0.05} = 470\,\Omega$$

$$z_{12} = \frac{h_{12}}{h_{22}} = \frac{10^{-3}}{0.05} = 0.02\,\Omega$$

$$z_{21} = -\frac{h_{21}}{h_{22}} = -\frac{1500}{0.05} = -30\,\text{k}\Omega$$

$$z_{22} = \frac{1}{h_{22}} = \frac{1}{0.05} = 20\,\Omega$$

Create an open circuit a port 2. Apply 1 mA at port 1. Then,

$$z_{11} = \frac{V_1}{I_1}\bigg|_{I_2=0} = \frac{V_1}{0.001} = 470\,\Omega \quad \text{so} \quad V_1 = 470\,\text{mV measured}$$

$$z_{21} = \frac{V_2}{I_1}\bigg|_{I_2=0} = \frac{V_2}{0.001} = -30{,}000\,\Omega \quad \text{so} \quad V_2 = -30\,\text{V measured}$$

Create an open circuit a port 1. Apply 1 A at port 2. Then,

$$z_{12} = \frac{V_1}{I_2}\bigg|_{I_1=0} = \frac{V_1}{1} = 0.02\,\Omega \quad \text{so} \quad V_1 = 0.02\,\text{V measured}$$

$$z_{22} = \frac{V_2}{I_2}\bigg|_{I_1=0} = \frac{V_2}{1} = 20\,\Omega \quad \text{so} \quad V_2 = 20\,\text{V measured}$$