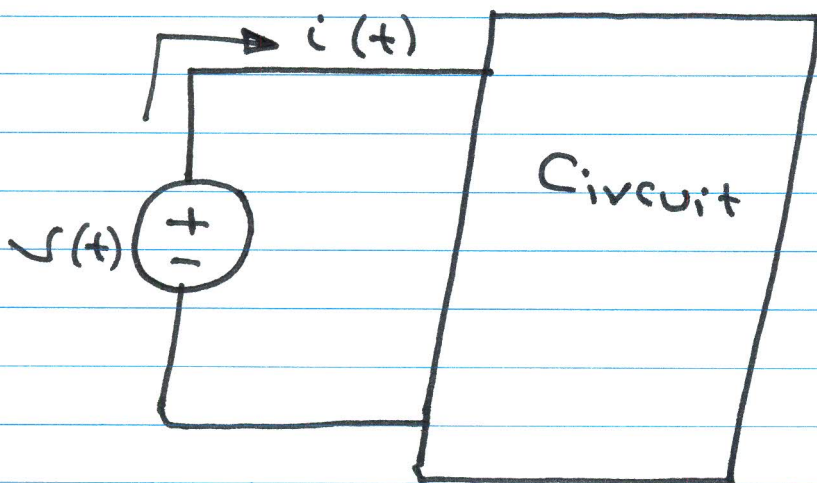


Sinusoidal Steady State

Power Calculation

Instantaneous Power : $P(t)$



$$v(t) = V_m \cos(\omega t + \theta_v)$$

$$i(t) = I_m \cos(\omega t + \phi_i)$$

$$P(t) = v(t) i(t)$$

$$P(t) = V_m I_m \cos(\omega t + \theta_v) \cos(\omega t + \phi_i)$$

$$\cos \alpha \cos \beta = \frac{1}{2} [\cos(\alpha - \beta) + \cos(\alpha + \beta)]$$

$$\therefore P(t) = \frac{V_m I_m}{2} [\cos(\theta_v - \phi_i) + \cos(2\omega t + \theta_v + \phi_i)]$$

Constant

Twice the
excitation frequency

Example

$$v(t) = 4 \cos(\omega t + 60^\circ) \quad \checkmark$$

$$Z(j\omega) = 2 \angle 30^\circ \quad \checkmark$$

Find $p(t)$.

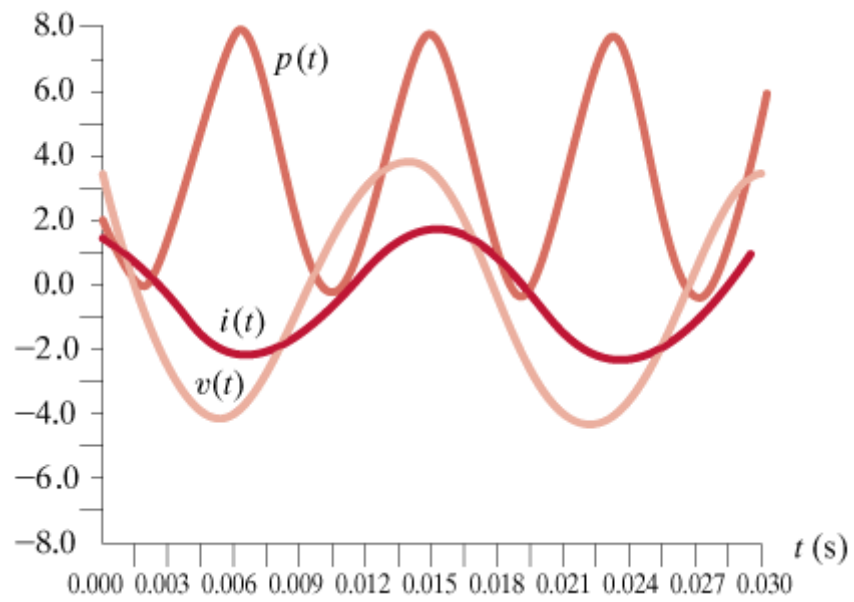
$$\vec{I} = \frac{\vec{V}}{Z} = \frac{4 \angle 60^\circ}{2 \angle 30^\circ} = 2 \angle 30^\circ \text{ A}$$

$$\therefore i(t) = 2 \cos(\omega t + 30^\circ) \text{ A}$$

$$p(t) = v(t) i(t)$$

$$p(t) = 4 \cos 30^\circ + 4 \cos(2\omega t + 90^\circ)$$

$$p(t) = 3.46 + 4 \cos(2\omega t + 90^\circ)$$



Average Power : Real Power

$$P_{av} = \frac{1}{T} \int_0^T P(t) dt$$

$$P_{av} = \frac{1}{2} V_m I_m \cos(\theta_v - \phi_i)$$

$$\theta_v - \phi_i = \theta_z$$

$$\therefore P_{av} = \frac{1}{2} V_m I_m \cos \theta_z$$

1) For Resistor

$$\theta_v - \phi_i = 0 \rightarrow \theta_z = 0$$

$$\therefore P_{av} = \frac{1}{2} V_m I_m = \frac{V_m^2}{2R} = \frac{I_m^2 R}{2}$$

2) For Inductor

$$\theta_v - \phi_i = 90^\circ$$

$$P_{av} = 0$$

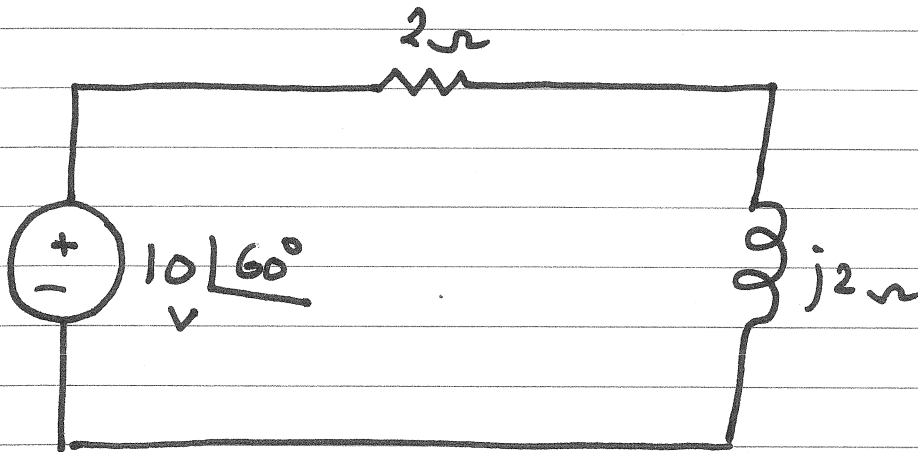
3) For Capacitor

$$\theta_v - \phi_i = -90^\circ$$

$$\therefore P_{av} = 0$$

\therefore Reactive impedances absorb no average power

Example



Find the average power absorbed by each element.

$$\vec{I} = \frac{10 \angle 60^\circ}{2 + j2} = 3.53 \angle 15^\circ \text{ A}$$

$$P_{av} = \frac{V_m I_m}{2} \cos(\theta_v - \theta_i)$$

$$P_{av} = 0$$

$j2\Omega$

$$P_{av} = \frac{I_m^2 R}{2} = \frac{(3.53)^2 \cdot 2}{2} = 12.5 \text{ W}$$

To Calculate the average power
Supplied by the Source

$$P_{av} = \frac{V_m I_m}{2} \cos(\theta_v - \phi_i)$$

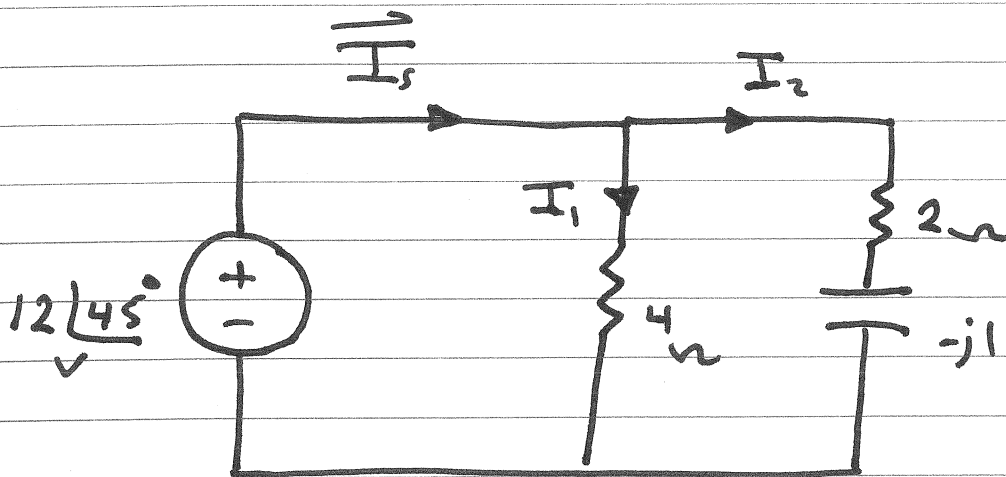
$$I_m = 3.53 \text{ A}$$

$$V_m = 10 \text{ V}$$

$$\theta_v = 60^\circ, \quad \phi_i = 15^\circ$$

$$\begin{aligned} \therefore P_{av} &= \frac{(10)(3.53)}{2} \cos(60 - 15^\circ) \\ &= 12.5 \text{ Watt} \end{aligned}$$

Example



Determine the average power absorbed by each resistor.

Determine the total average power absorbed and the average power supplied by the source.

$$\vec{I}_1 = \frac{12\angle 45^\circ}{4\Omega} = 3\angle 45^\circ \text{ A}$$

$$\vec{I}_2 = \frac{12\angle 45^\circ}{2-j1} = 5.36\angle 71.57^\circ \text{ A}$$

$$\vec{I}_s = \vec{I}_1 + \vec{I}_2 = 8.15\angle 62.1^\circ \text{ A}$$

$$1) P_{4\Omega} = \frac{I_{1m}^2 \cdot 4}{2} = 18 \text{ W}$$

$$2) P_{2\Omega} = \frac{I_{2m}^2 \cdot 2}{2} = 28.7 \text{ W}$$

∴ Total Average power absorbed = 46.7 W

$$P_{V_s} = \frac{V_m I_m}{2} \cos(\theta_v - \phi_i)$$

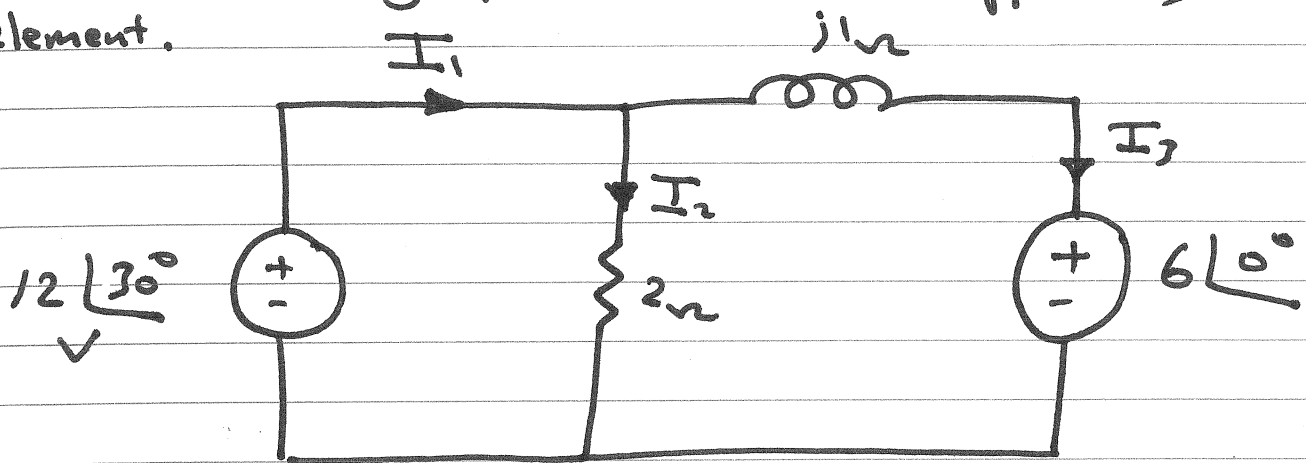
$$P_{V_s} = \frac{(12)(8.16)}{2} \cos(45 - 62.1)$$

$$P_{V_s} = 46.7 \text{ W}$$

$$\therefore P_{V_s} = P_{4\Omega} + P_{2\Omega} + P_{-j1}$$

Example

Determine average power absorbed or supplied by each element.



$$\vec{I}_2 = \frac{12\angle 30^\circ}{2} = 6\angle 30^\circ \text{ A}$$

$$\vec{I}_3 = \frac{12\angle 30^\circ - 6\angle 0^\circ}{j1} = 7.43\angle -36.19^\circ \text{ A}$$

$$\vec{I}_1 = \vec{I}_2 + \vec{I}_3 = 11.29\angle -7.07^\circ \text{ A}$$

$$P_{2\Omega} = \frac{I_{2m}^2 \cdot 2}{2} = 36 \text{ W}$$

$$P_{12\angle 30^\circ} = \frac{V_m I_m \cos(\theta_v - \theta_i)}{2}$$

$$P_{12\angle 30^\circ} = \frac{(12)(11.29)}{2} \cos(30 - (-7.07))$$

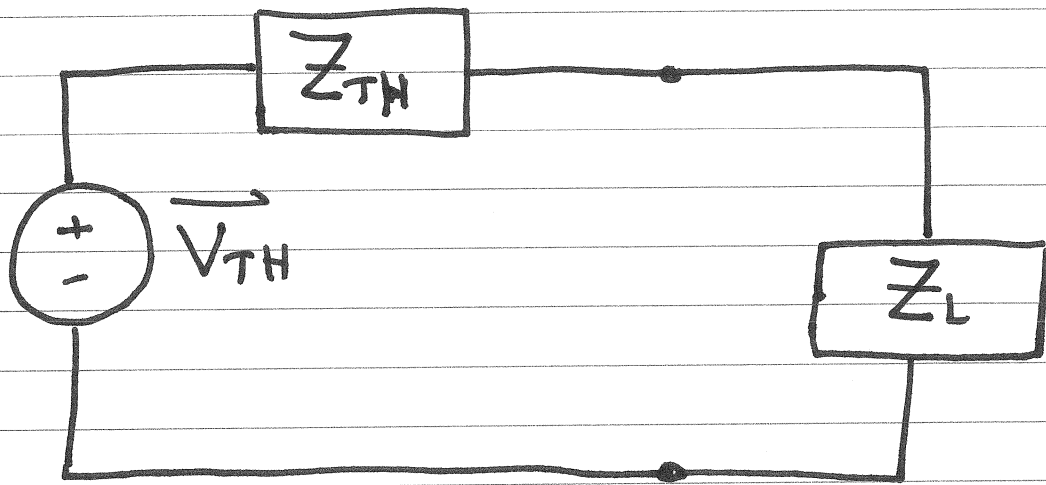
$$P_{12\angle 30^\circ} = 54 \text{ W Supply}$$

$$P_{6\Omega} = \frac{V_m I_m}{2} \cos(\theta_v - \phi_i)$$

$$P_{6\Omega} = \frac{(6)(7.43)}{2} \cos(0 - (-36.19^\circ))$$

$$P_{6\Omega} = 18 \text{ W absorbed}$$

Maximum Average power Transfer



$$Z_{TH} = R_{TH} + j X_{TH}$$

$$Z_L = R_L + j X_L$$

$$P_L = \frac{I_{Lm}^2 \cdot R_L}{2}$$

$$I = \frac{V_{TH}}{Z_{TH} + Z_L}$$

$$I = \frac{V_{TH}}{(R_{TH} + R_L) + j(X_{TH} + X_L)}$$

$$P_L = \frac{I_m^2 R_L}{2}$$

$$P_L = \frac{1}{2} \frac{V_{TH}^2 \cdot R_L}{(R_{TH} + R_L)^2 + (X_{TH} + X_L)^2}$$

$$\frac{\partial P_L}{\partial R_L} = 0 \quad ; \quad \frac{\partial P_L}{\partial X_L} = 0$$

$$\frac{\partial P_L}{\partial X_L} = \frac{-2 V_{TH}^2 R_L (X_L + X_{TH})}{2 \left[(R_L + R_{TH})^2 + (X_L + X_{TH})^2 \right]^2}$$

$$\text{For } \frac{\partial P_L}{\partial X_L} = 0 \rightarrow X_L = -X_{TH}$$

$$\frac{\partial P_L}{\partial R_L} = \frac{V_{TH}^2 \left[(R_L + R_{TH})^2 + (X_L + X_{TH})^2 - 2R_L (R_L + R_{TH}) \right]}{2 \left[(R_L + R_{TH})^2 + (X_L + X_{TH})^2 \right]^2}$$

$$\text{For } \frac{\partial P_L}{\partial R_L} = 0 \rightarrow R_L = \sqrt{R_{TH}^2 + (X_L + X_{TH})^2}$$

$$X_L = -X_{TH}$$

$$\therefore R_L = R_{TH}$$

$$\therefore Z_L = Z_{TH}^*$$

$$P_{L, \max} = \frac{1}{8} \frac{V_{TH}^2}{R_L}$$

∴ For maximum average power Transfer

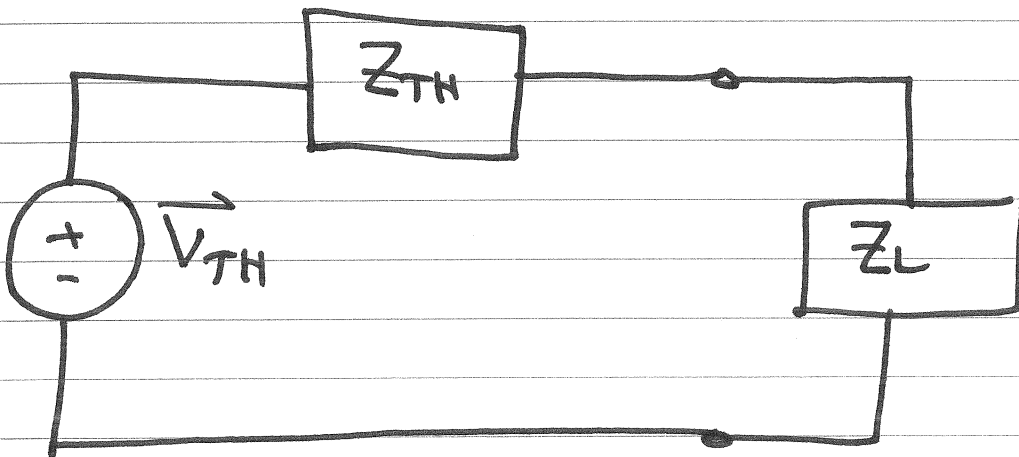
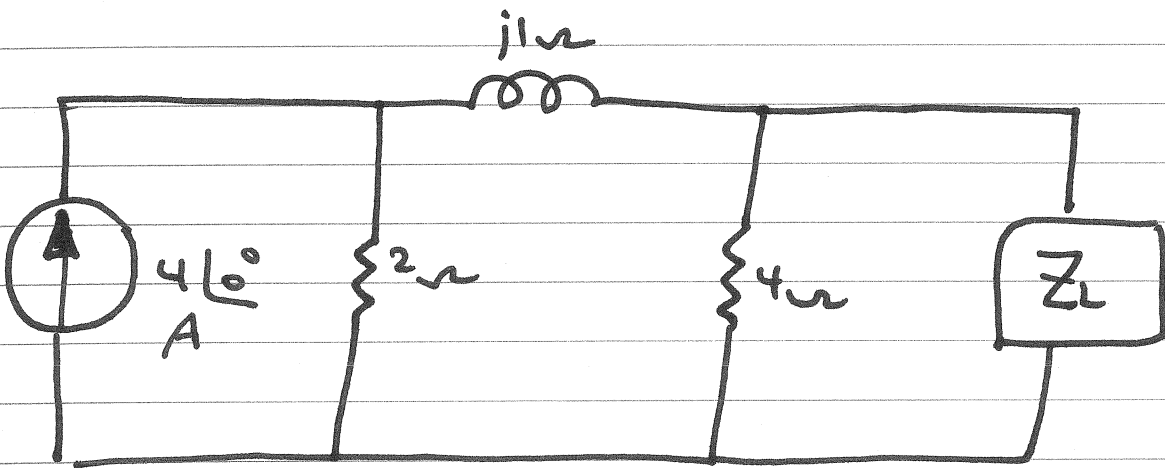
$$\bar{Z}_L = Z_{TH}^*$$

$$P_{L,max} = \frac{1}{8} \frac{V_{TH}^2}{R_{TH}}$$

Example

Find Z_L for maximum average power transfer.

Compute the maximum average power supplied to the load.



$$\vec{V}_{TH} = 4 \angle 0^\circ \frac{2}{2 + j1 + 4} \cdot 4 = 5.28 \angle -9.46^\circ \text{ V}$$

$$Z_{TH} = 4 \Omega \parallel (2 + j1) \Omega$$

$$Z_{TH} = (1.4 + j0.43) \Omega$$

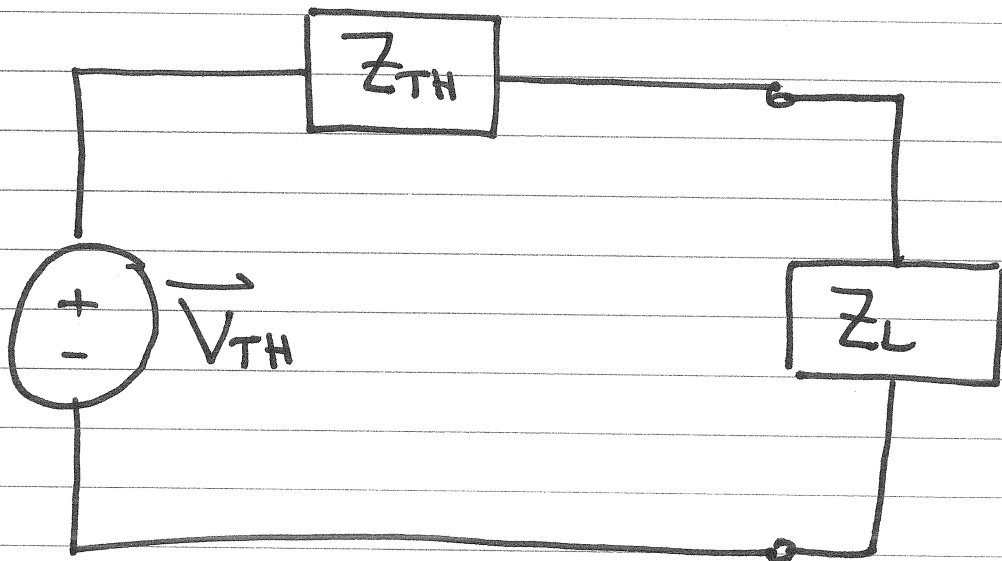
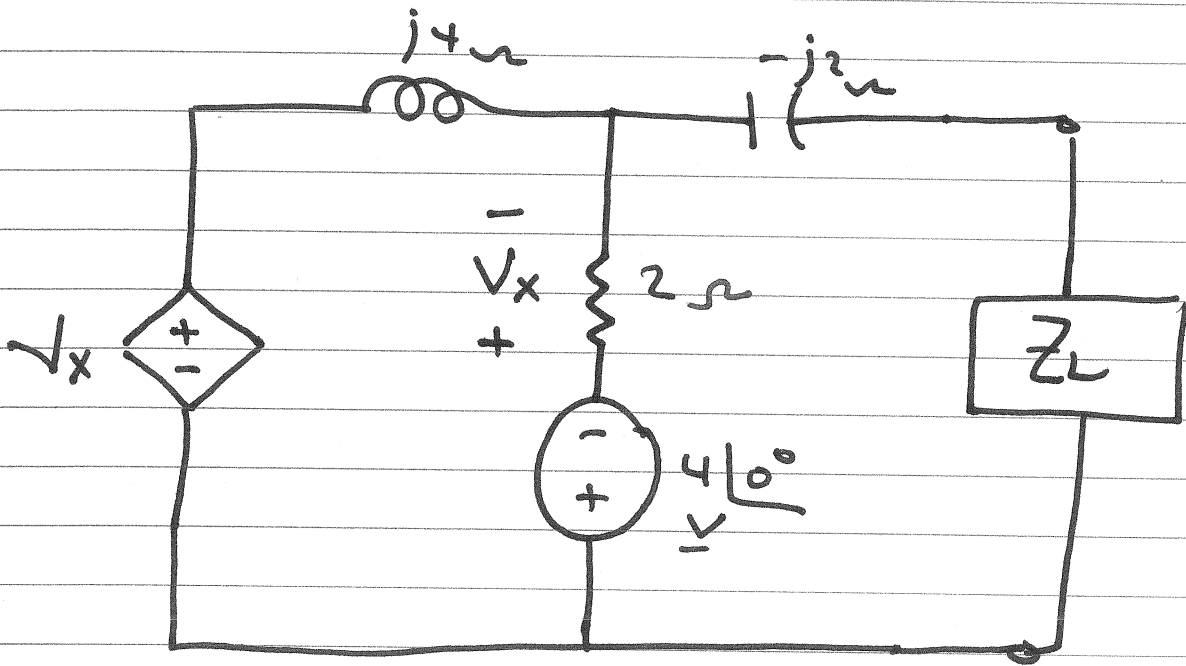
$$\therefore Z_L = (1.4 - j0.43) \Omega$$

$$P_{,max} = \frac{1}{8} \frac{V_{TH}^2}{R_{TH}}$$

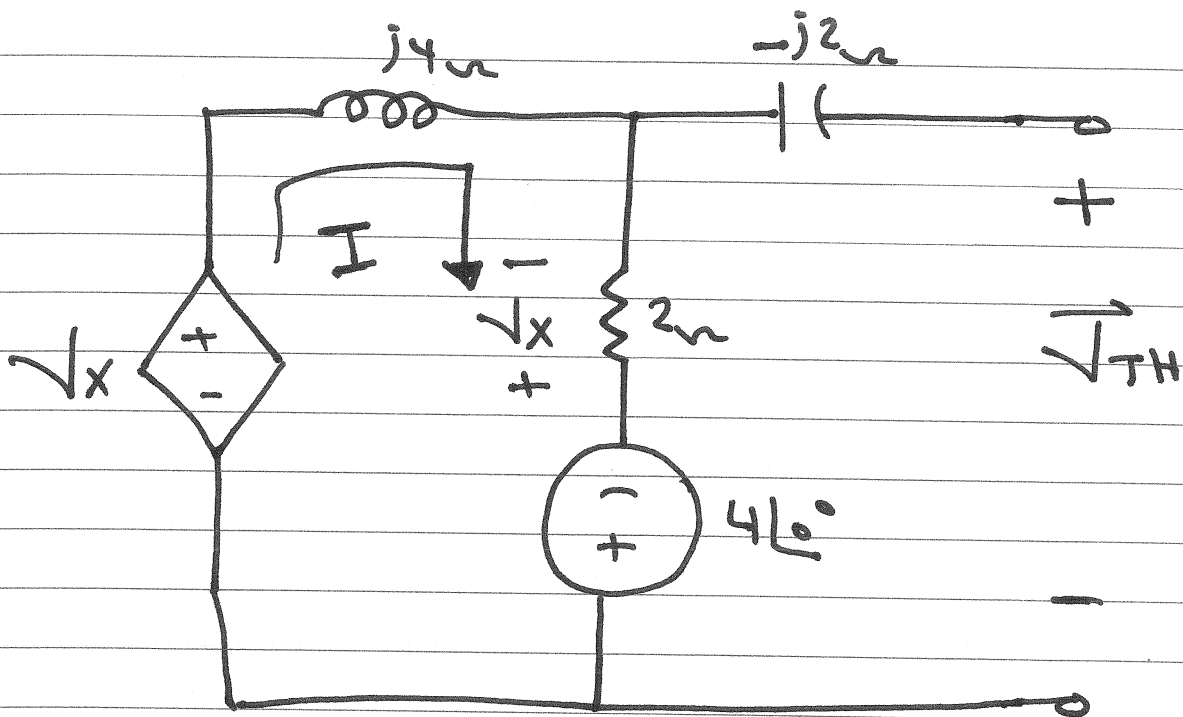
$$P_{,max} = 2.489 \text{ W}$$

Example

Find Z_L for maximum average power transfer
Compute the maximum average power supplied to Z_L



$$Z_L = Z_{TH}^*$$



$$\vec{V}_{TH} = 2\vec{I} - 4\angle 0^\circ$$

$$\vec{I} = \frac{\vec{V}_x + 4\angle 0^\circ}{2 + j4}$$

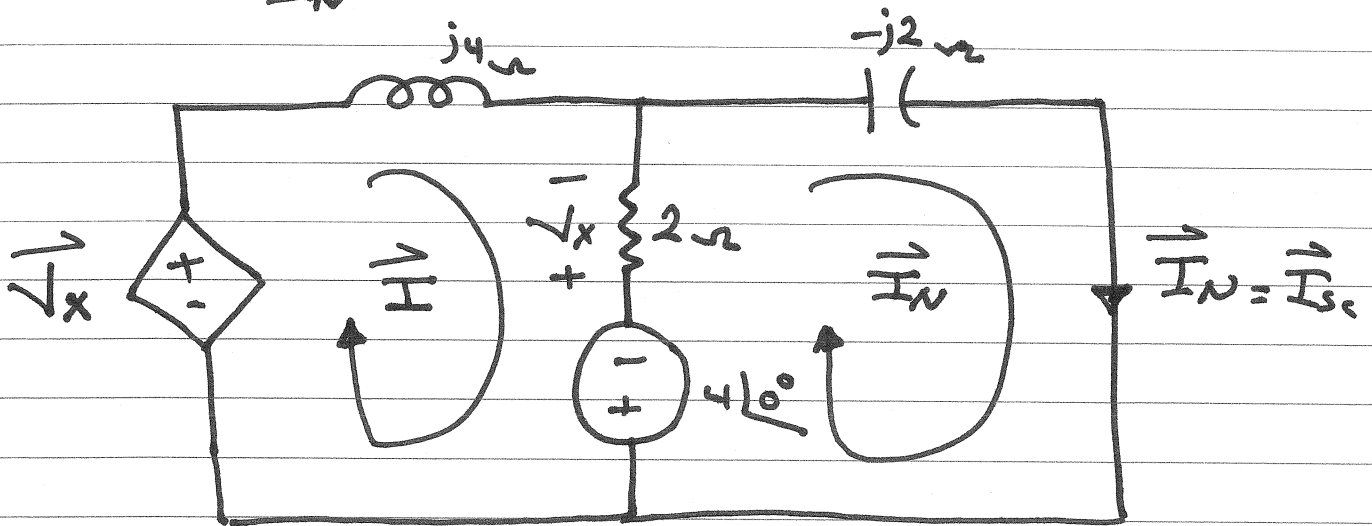
$$\vec{V}_x = -2\vec{I}$$

$$\vec{I} = 0.707\angle -45^\circ \text{ A}$$

$$\therefore \vec{V}_{TH} = (-3 - j1) \text{ V}$$

$$\vec{V}_{TH} = 3.16\angle 198.43^\circ \text{ V}$$

$$Z_{TH} = \frac{\vec{V}_{TH}}{\vec{I}_N}$$



KVL for mesh 1:

$$\vec{V}_x + 4 \angle 0^\circ = (2 + j4) \vec{I} - 2 \vec{I}_N$$

$$\vec{V}_x = 2 (\vec{I}_N - \vec{I})$$

KVL for mesh 2:

$$-4 \angle 0^\circ = -2 \vec{I} + (2 - j2) \vec{I}_N$$

Solving for \vec{I}_N

$$\vec{I}_N = (-1 - j2) \text{ A}$$

$$\vec{I}_N = 2.24 \angle 247.43^\circ \text{ A}$$

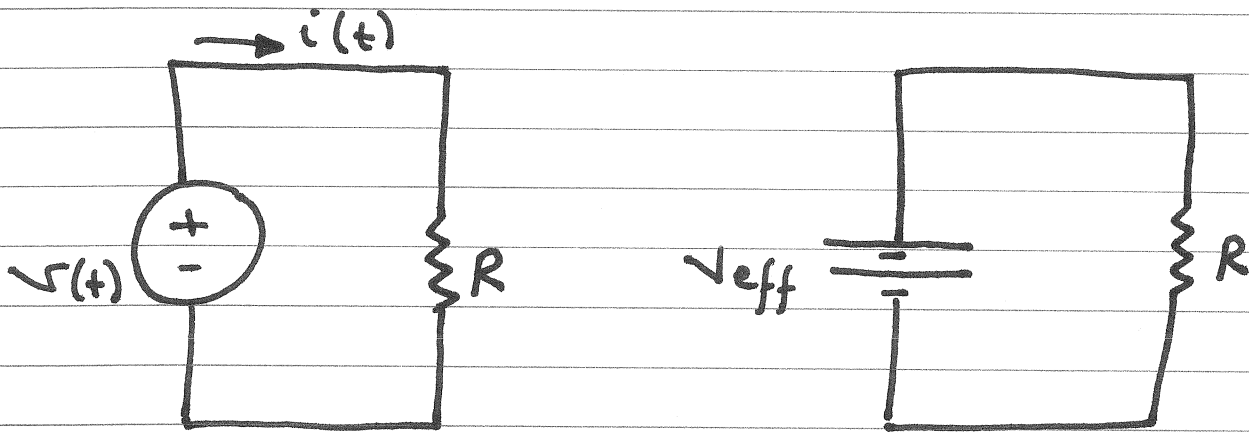
$$\therefore Z_{TH} = \frac{\vec{V}_{TH}}{\vec{I}_N} = 1.41 \angle -45^\circ \Omega = (1 - j1) \Omega$$

$$\therefore Z_L = Z_{TH}^* = 1.41 \angle +45^\circ \Omega = (1 + j1) \Omega$$

$$\therefore P_{L, \max} = \frac{V_{TH}^2}{8 R_{TH}} = 1.25 \text{ W}$$

Effective or RMS Value

The effective value of a periodic voltage (current) is the dc voltage (current) that delivers the same average power to a resistor as the periodic voltage (current).



$$\text{let } v(t) = v_m \cos(\omega t + \phi_v)$$

$$\therefore P_1 = \frac{v_m^2}{2R}$$

$$P_2 = \frac{v_{eff}^2}{R}$$

$$P_1 = P_2$$

$$\therefore \frac{v_m^2}{2R} = \frac{v_{eff}^2}{R}$$

$$\therefore V_{\text{eff}} = \frac{V_m}{\sqrt{2}}$$

RMS : Root Mean Square

$$\text{let } v(t) = V_m \cos(\omega t + \theta_v)$$

$$V_{\text{RMS}} = \sqrt{\frac{1}{T} \int_0^T V_m^2 \cos^2(\omega t + \theta_v) dt}$$

$$V_{\text{RMS}} = V_m \sqrt{\frac{1}{T} \int_0^T \cos^2(\omega t + \theta_v) dt}$$

$$V_{\text{RMS}} = V_m \sqrt{\frac{1}{T} \int_0^T \frac{1}{2} (1 + \cos 2(\omega t + \theta_v)) dt}$$

$$V_{\text{RMS}} = V_m \frac{1}{\sqrt{2}}$$

$$P_{av} = \frac{V_m I_m}{2} \cos(\theta_v - \phi_i)$$

$$P_{av} = V_{rms} I_{rms} \cos(\theta_v - \phi_i)$$

For a resistor

$$P_{av} = V_{rms} I_{rms} \cos(\theta_v - \phi_i)$$

$$V_{rms} = R I_{rms} ; \theta_v - \phi_i = 0$$

$$\therefore P_{av} = \frac{V_{rms}^2}{R}$$

$$\therefore P_{av} = I_{rms}^2 R$$

Apparent power and power factor

$$P_{av} = V_{rms} I_{rms} \cos(\theta_v - \phi_i)$$

$$P_{apparent} = V_{rms} I_{rms}$$

$P_{apparent}$ measured in VA

PF \equiv Power factor

$$PF = \cos(\theta_v - \phi_i)$$

$$\therefore P_{av} = P_a \cdot PF$$

1) For Resistor

$$\Theta_V - \Phi_i = 0$$

$$\therefore PF = 1$$

2) For inductor

$$\Theta_V - \Phi_i = +90^\circ$$

$$\therefore PF = 0$$

3) For Capacitor

$$\Theta_V - \Phi_i = -90^\circ$$

$$\therefore PF = 0$$

4) For Inductive Load

$$90^\circ > \Theta_V - \Phi_i > 0$$

$$1 > PF > 0$$

Lagging Power factor