

8.8

$$v(t) = -8e^{-250t} + 32e^{-1000t} \quad \text{for } t > 0$$

$$s_1 = -250, \quad s_2 = -1000$$

$$s_1 = -\alpha + \sqrt{\alpha^2 - \omega_0^2} = -250$$

$$s_2 = -\alpha - \sqrt{\alpha^2 - \omega_0^2} = -1000$$

$$\therefore s_1 + s_2 = -1250 = -2\alpha$$

$$\therefore \alpha = 625 \text{ V/s}$$

$$\text{But } \alpha = \frac{1}{2RC} = 625$$

$$C = 0.1 \text{ } \mu\text{F}$$

$$\therefore R = 8 \text{ k}\Omega$$

$$s_1 - s_2 = 1000 - 250 = 750 = 2\sqrt{\alpha^2 - \omega_0^2}$$

$$\therefore \omega_0 = 500 \text{ V/s}$$

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

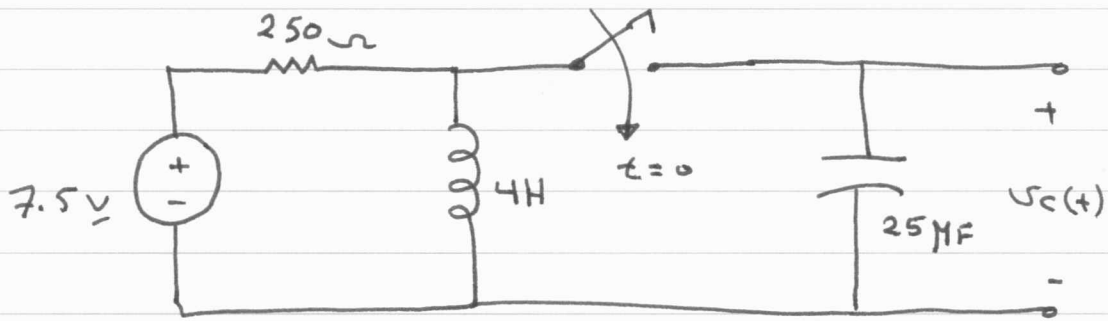
$$\therefore L = 40 \text{ H}$$

$$b) \quad i_R(t) = \frac{v(t)}{R} = -e^{-250t} + 4e^{-1000t} \quad \text{mA for } t > 0$$

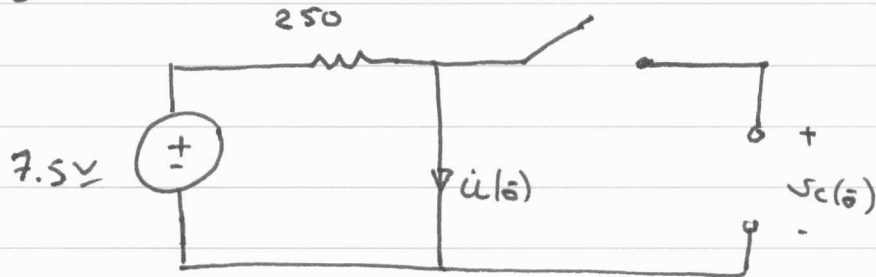
$$i_C(t) = C \frac{dv(t)}{dt} = 0.2e^{-250t} - 3.2e^{-1000t} \quad \text{mA for } t > 0$$

$$i_L(t) = -(i_R(t) + i_C(t)) = 0.8e^{-250t} - 0.8e^{-1000t} \quad \text{mA for } t > 0$$

8.36

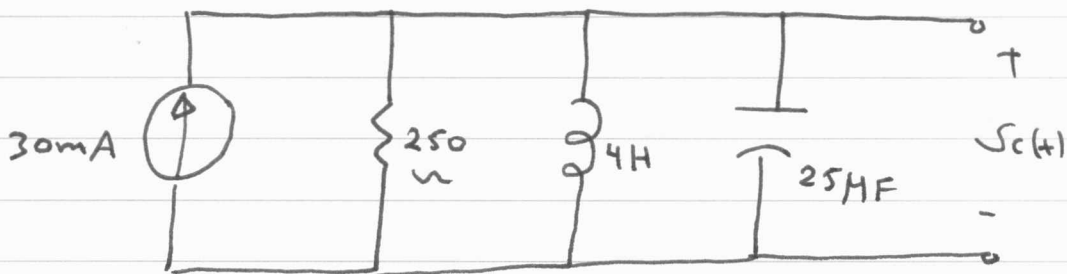
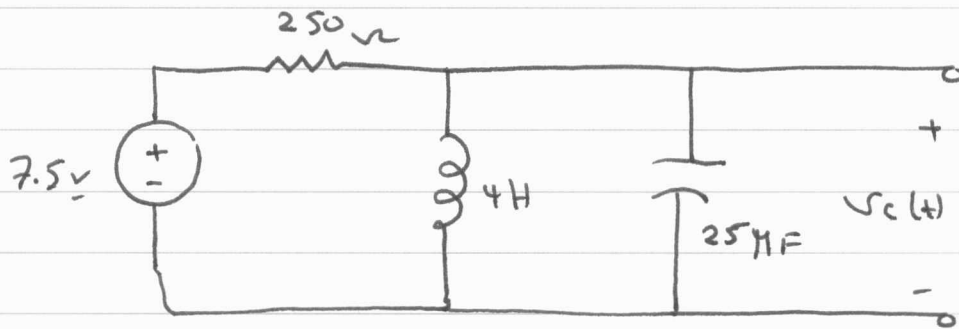


at $t = 0^-$



$$i_L(0^-) = \frac{7.5}{250} = 30 \text{ mA} ; v_c(0^-) = 0$$

for $t > 0$



$$\alpha = \frac{1}{2RC} = 80 \text{ v/s} ; \omega_0 = \frac{1}{\sqrt{LC}} = 100 \text{ v/s}$$

Since $\omega_0 > \alpha$

\therefore underdamped Case

$$\omega_d = \sqrt{\omega_0^2 - \alpha^2} = 60 \text{ rad/s}$$

$$V_c(t) = V_n(t) + V_f(t)$$

$$V_f(t) = V_c(\infty) = 0$$

$$\therefore V_c(t) = e^{-\alpha t} (\beta_1 \cos \omega_d t + \beta_2 \sin \omega_d t)$$

$$V_c(t) = e^{-80t} (\beta_1 \cos 60t + \beta_2 \sin 60t) \quad \underline{\quad} \text{ for } t > 0$$

$$V_c(0^+) = V_c(0^-) = 0$$

To find $\frac{dV_c(0^+)}{dt}$:

for $t > 0$

$$30 \text{ mA} = \frac{V_c(t)}{R} + i_L(0^-) + \frac{1}{L} \int_0^t V_c(t) dt + c \frac{dV_c(t)}{dt}$$

at $t = 0$

$$30 \text{ mA} = \frac{V_c(0^+)}{R} + i_L(0^-) + 0 + c \frac{dV_c(0^+)}{dt}$$

$$\therefore \frac{dV_c(0^+)}{dt} = 0$$

$$V_c(t) = e^{-80t} (\beta_1 \cos 60t + \beta_2 \sin 60t) \quad \underline{\quad} \text{ for } t > 0$$

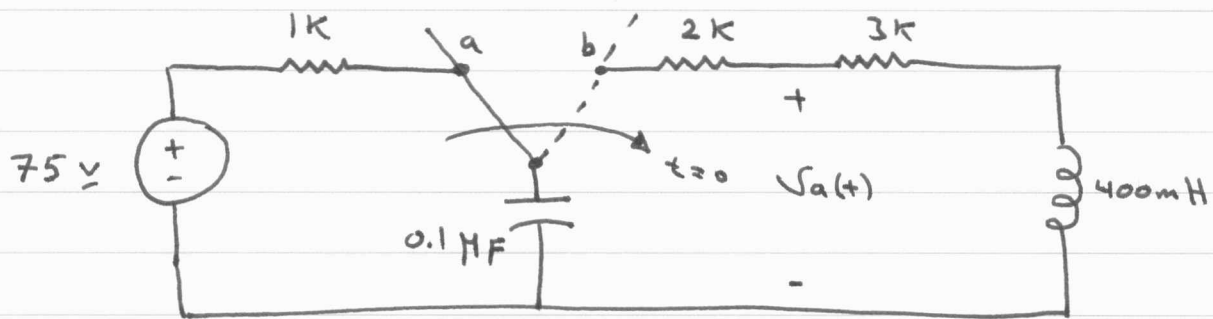
$$V_c(0^+) = \beta_1 = 0 \quad ; \quad \therefore \beta_1 = 0$$

$$\frac{dV_c(t)}{dt} = -\alpha \beta_1 + \omega_d \beta_2 = 0$$

$$\therefore \beta_2 = 0$$

$$\therefore V_c(t) = 0 \quad \text{for } t \geq 0$$

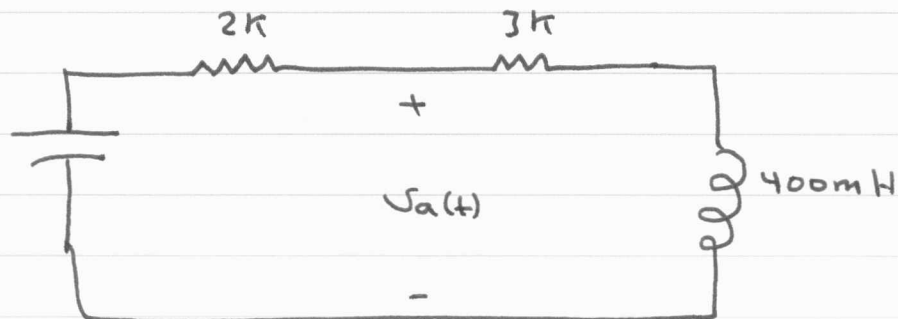
8.46



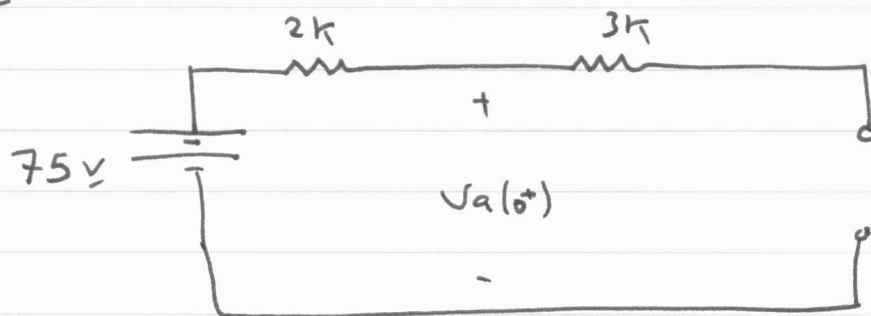
at = 0

$$V_c(0^-) = 75 \text{ V} , \quad i_L(0^-) = 0$$

for t > 0



at = 0+



$$\therefore V_a(0^+) = 75 \text{ V}$$

for t > 0

$$b) \quad V_a(t) = 3k i_L(t) + L \frac{di_L(t)}{dt}$$

$$V_a(0^+) = 3k i_L(0^+) + 0.4 \frac{di_L(0^+)}{dt}$$

$$\therefore \frac{di_L(0^+)}{dt} = 187.5$$

for $t > 0$

$$\text{KVL : } 5k i_L(t) + 0.4 \frac{di_L(t)}{dt} + 10 \times 10^6 \int_{0^-}^t i_L(t) dt = 75$$

$$\text{Diff : } 5k \frac{di_L(t)}{dt} + 0.4 \frac{d^2 i_L(t)}{dt^2} + 10 \times 10^6 i_L(t) = 0$$

at $t = 0^+$

$$5k \frac{di_L(0^+)}{dt} + 0.4 \frac{d^2 i_L(0^+)}{dt^2} + 10 \times 10^6 i_L(0^+) = 0$$

$$\therefore \frac{d^2 i_L(0^+)}{dt^2} = -2343750$$

$$\frac{dV_a(0^+)}{dt} = 3k \frac{di_L(0^+)}{dt} + 0.4 \frac{d^2 i_L(0^+)}{dt^2}$$

$$\frac{dV_a(0^+)}{dt} = -375000$$

$$c) \alpha = \frac{R}{2L} = \frac{5k}{0.8} = 6250 \text{ v/s}$$

$$\omega_0 = \frac{1}{\sqrt{LC}} = 5000 \text{ v/s}$$

\therefore overdamped case

$$s_1 = -\alpha + \sqrt{\alpha^2 - \omega_0^2} = -2500 \text{ v/s}$$

$$s_2 = -\alpha - \sqrt{\alpha^2 - \omega_0^2} = -10000 \text{ v/s}$$

$$v_a(t) = A_1 e^{-2500t} + A_2 e^{-10000t} \quad \text{for } t > 0$$

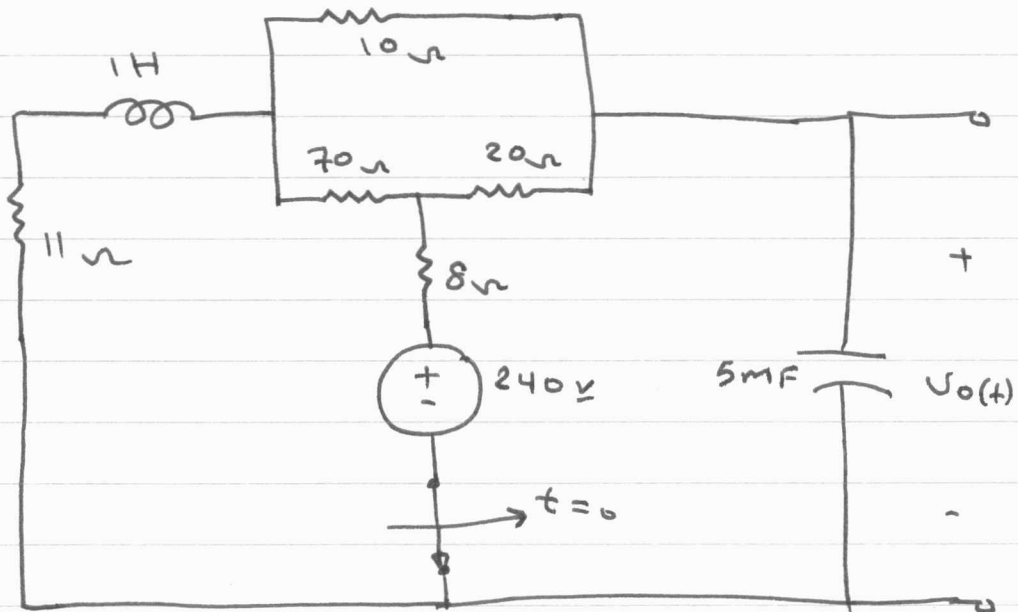
$$v_a(0^+) = A_1 + A_2 = 75$$

$$\frac{dv_a(t)}{dt} = -2500 A_1 - 10000 A_2 = -375000$$

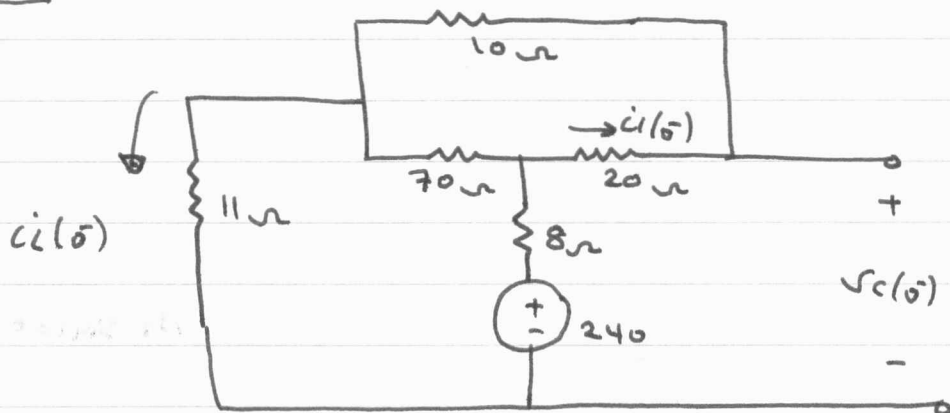
$$\therefore A_1 = 50, \quad A_2 = 25$$

$$\therefore v_a(t) = 50 e^{-2500t} + 25 e^{-10000t} \quad \text{for } t > 0$$

8.49



at $t = 0^-$

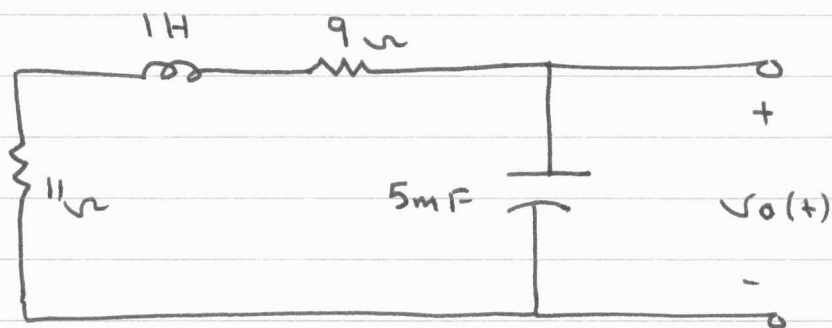


$$i_2(t) = \frac{240}{8 + (20+10) \parallel 70 + 11} = 6 \text{ A}$$

$$i_1(t) = \frac{70}{70+20+10} \cdot 6 = 4.2 \text{ A}$$

$$v_c(t) = -20 i_1(t) - 8 i_2(t) + 240 = 108 \text{ V}$$

For $t > 0$



$$\alpha = \frac{R}{2L} = \frac{20}{2} = 10 \text{ v/s}$$

$$\omega_0 = \frac{1}{\sqrt{LC}} = 14.142 \text{ v/s}$$

Since $\omega_0 > \alpha$

underdamped ; $\omega_d = \sqrt{\omega_0^2 - \alpha^2} = 10$

$$s_1 = -10 + j10 \text{ v/s}$$

$$s_2 = -10 - j10 \text{ v/s}$$

$$v_o(t) = e^{-10t} (\beta_1 \cos 10t + \beta_2 \sin 10t)$$

$$v_o(0^+) = v_c(0) = 108 \text{ V}$$

$$\frac{dv_o(0^+)}{dt} = -\frac{i(0^+)}{C} = -1200$$

$$v_o(0^+) = \beta_1 = 108$$

$$\frac{dv_o(0^+)}{dt} = -\alpha \beta_1 + \omega_d \beta_2$$

$$\therefore \beta_2 = -12$$

$$\therefore v_o(t) = 108 e^{-10t} \cos 10t - 12 e^{-10t} \sin 10t \text{ for } t > 0$$