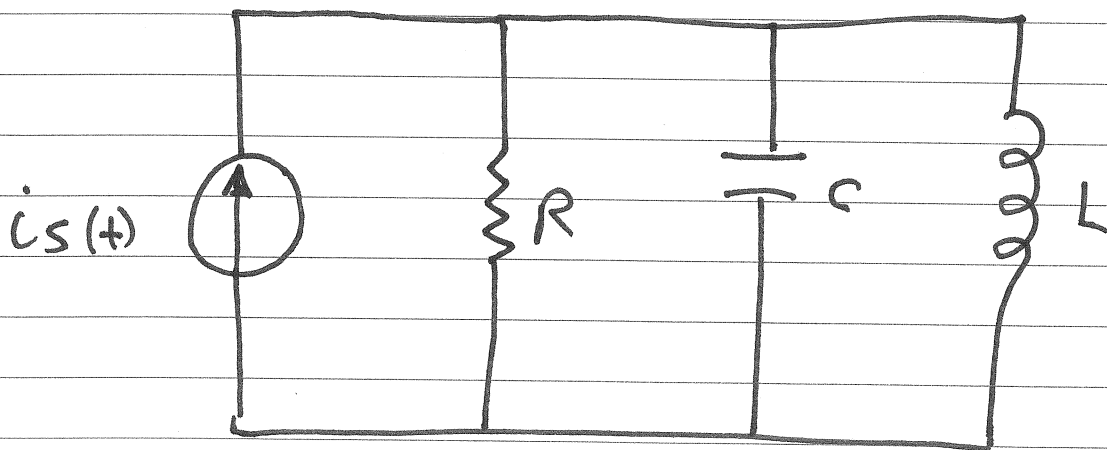
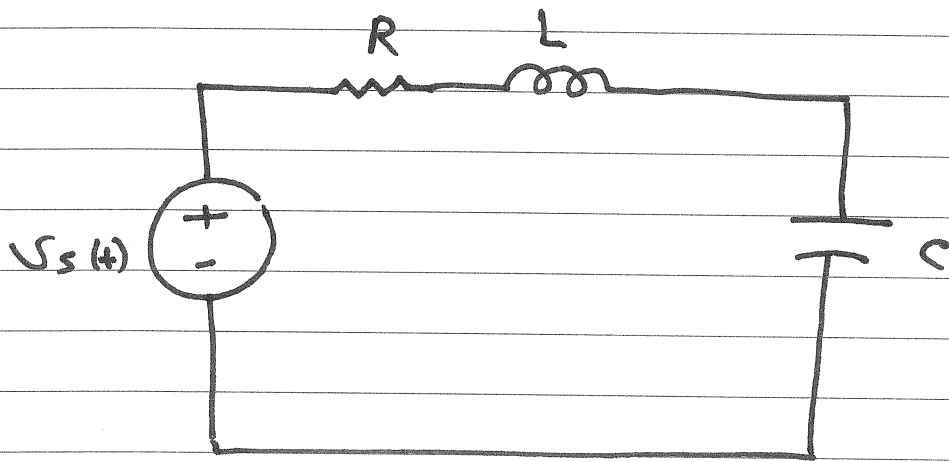


Chapter 8

Natural and Step Response of RLC Circuits

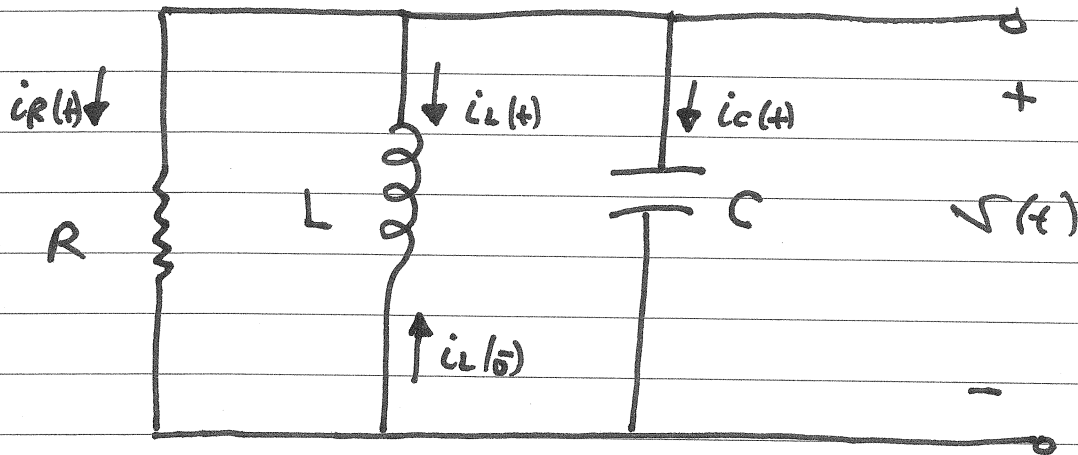
What is a 2nd order Circuit ?

A second-order circuit is characterized by a second-order differential equation.



Natural Response of Parallel RLC Circuit

For $t > 0$



$$v_C(0) = 0 ; i_L(0) = 10A$$

KCL :

$$i_R(t) + i_L(t) + i_C(t) = 0$$

$$\frac{v(t)}{R} + \frac{1}{L} \int_{0^-}^t v(t) dt - i_L(0) + C \frac{dv(t)}{dt} = 0$$

$$\frac{v(t)}{R} + \frac{1}{L} \int_{0^-}^t v(t) dt + C \frac{dv(t)}{dt} = i_L(0) \quad \text{--- (1)}$$

Differentiate (1)

$$C \frac{d^2 v(t)}{dt^2} + \frac{1}{R} \frac{dv(t)}{dt} + \frac{1}{L} v(t) = 0$$

Second-order homogeneous differential equation

$$\therefore v(t) = A e^{st} \quad \text{for } t > 0$$

$$C A s^2 e^{st} + \frac{1}{R} s A e^{st} + \frac{1}{L} A e^{st} = 0$$

$$A e^{st} \left(C s^2 + \frac{1}{R} s + \frac{1}{L} \right) = 0$$

$$\therefore C s^2 + \frac{1}{R} s + \frac{1}{L} = 0$$

Characteristic equation

$$s_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$s_1 = -\frac{1}{2RC} + \sqrt{\left(\frac{1}{2RC}\right)^2 - \frac{1}{LC}}$$

$$s_2 = -\frac{1}{2RC} - \sqrt{\left(\frac{1}{2RC}\right)^2 - \frac{1}{LC}}$$

$$\text{Let } \omega_0 = \frac{1}{\sqrt{LC}}$$

$\omega_0 \equiv$ resonant frequency

and

$$\alpha = \frac{1}{2RC}$$

$\alpha \equiv$ damping coefficient

$$\therefore s_1 = -\alpha + \sqrt{\alpha^2 - \omega_0^2}$$

$$s_2 = -\alpha - \sqrt{\alpha^2 - \omega_0^2}$$

1) If $\alpha > \omega_0$, the solutions are real, unequal and the response is termed overdamped.

2) If $\alpha < \omega_0$, the solutions are complex conjugates and the response is termed underdamped.

3) If $\alpha = \omega_0$, the solutions are real and equal and the response is termed critically damped.

1) The overdamped case

$$s_1 = -\alpha + \sqrt{\alpha^2 - \omega_0^2}$$

$$s_2 = -\alpha - \sqrt{\alpha^2 - \omega_0^2}$$

If $\alpha > \omega_0$

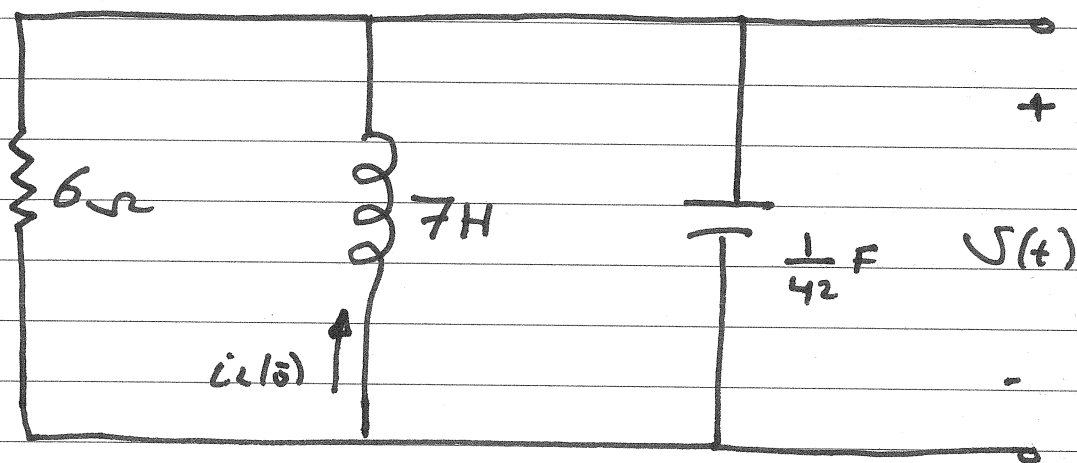
$$\therefore s_1 = -\alpha + \sqrt{\alpha^2 - \omega_0^2} < 0$$

$$s_2 = -\alpha - \sqrt{\alpha^2 - \omega_0^2} < 0$$

s_1, s_2 are real, unequal

$$\therefore v(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t} \quad t > 0$$

Overdamped parallel RLC



$$V_C(0) = 0, \text{ and } i_L(0) = 10\text{A}$$

$$\alpha = \frac{1}{2RC} = 3.5$$

$$\omega_0 = \frac{1}{\sqrt{LC}} = \sqrt{6} = 2.45$$

$\therefore \alpha > \omega_0$ overdamped

$$s_1 = -\alpha + \sqrt{\alpha^2 - \omega_0^2} = -1$$

$$s_2 = -\alpha - \sqrt{\alpha^2 - \omega_0^2} = -6$$

$$\therefore V(t) = A_1 e^{-t} + A_2 e^{-6t} \text{ for } t > 0$$

To find A_1 , and A_2 , we need

$$v(0^+) \text{ and } \frac{dv(0^+)}{dt}$$

For $t > 0$

$$\frac{v(t)}{R} + \frac{1}{L} \int_{0^-}^t v(t) dt + C \frac{dv(t)}{dt} = i_L(0^-)$$

at $t = 0^+$

$$\frac{v(0^+)}{R} + \frac{1}{L} \int_{0^-}^{0^+} v(t) dt + C \frac{dv(0^+)}{dt} = i_L(0^-)$$

$$v(0^+) = v_c(0^+) = v_c(0^-) = 0$$

$$\int_{0^-}^{0^+} v(t) dt = 0$$

$$\therefore i_L(0^-) = C \frac{dv(0^+)}{dt}$$

$$\frac{dv(0^+)}{dt} = \frac{i_L(0^-)}{C} = 420$$

$$\text{also } v(0^+) = 0 = v_c(0^-)$$

$$v(t) = A_1 e^{-t} + A_2 e^{-6t} \quad \text{for } t > 0$$

$$v(0^+) = A_1 + A_2 = v(0) = 0$$

$$\therefore A_1 + A_2 = 0 \quad \text{--- (2)}$$

$$v(t) = A_1 e^{-t} + A_2 e^{-6t}$$

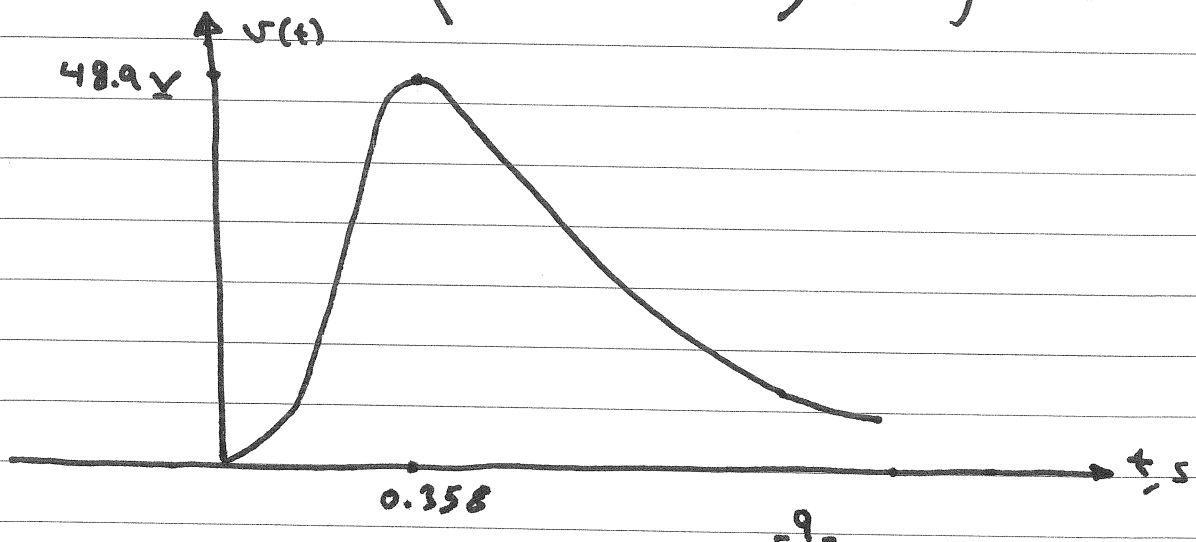
$$\frac{dv(t)}{dt} = -A_1 e^{-t} - 6A_2 e^{-6t}$$

$$\frac{dv(0^+)}{dt} = -A_1 - 6A_2 = 420 \quad \text{--- (3)}$$

Solving (2) and (3), we get

$$A_1 = 84 \quad \text{and} \quad A_2 = -84$$

$$\therefore v(t) = 84 \left(e^{-t} - e^{-6t} \right) \quad \forall \text{ for } t > 0$$



2) Critical Damping Case

$$\alpha = \omega_0$$

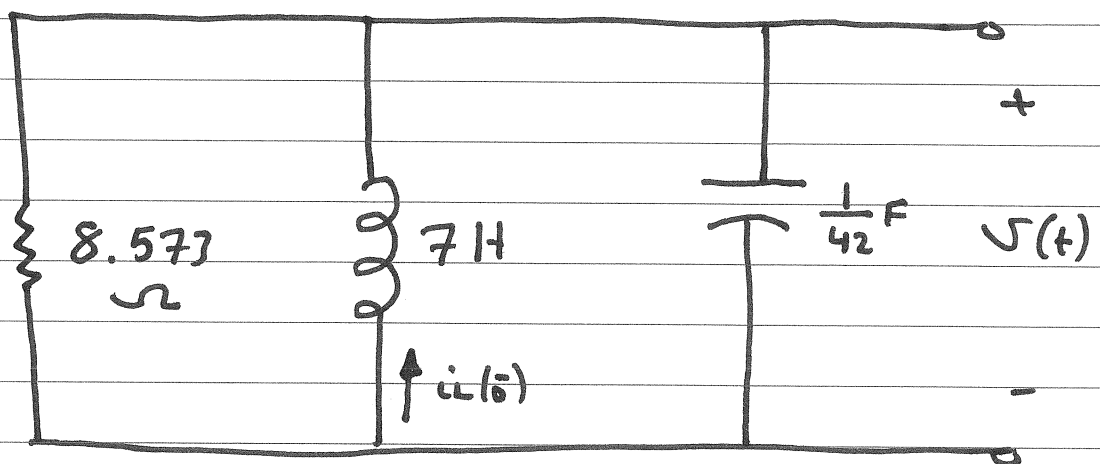
$$s_1 = -\alpha + \sqrt{\alpha^2 - \omega_0^2} = -\alpha$$

$$s_2 = -\alpha - \sqrt{\alpha^2 - \omega_0^2} = -\alpha$$

$$s_1 = s_2 \quad \text{real and equal}$$

$$\therefore v(t) = A_1 t e^{\alpha t} + A_2 e^{-\alpha t} \quad \text{for } t > 0$$

Critical Damped parallel RLC



$$v_C(0) = 0, \text{ and } i_L(0) = 10 \text{ A}$$

$$\alpha = \frac{1}{2RC} = \sqrt{6}$$

$$\omega_0 = \frac{1}{\sqrt{LC}} = \sqrt{6}$$

$$s_1 = -\sqrt{6}$$

$$s_2 = -\sqrt{6}$$

$$\therefore v(t) = A_1 t e^{-\sqrt{6}t} + A_2 e^{-\sqrt{6}t} \text{ for } t > 0$$

$$v(0^+) = 0$$

$$\frac{dv(0^+)}{dt} = 420$$

$$v(t) = A_1 t e^{-\sqrt{6}t} + A_2 e^{-\sqrt{6}t} \quad \text{for } t > 0$$

$$v(0^+) = A_2 = 0$$

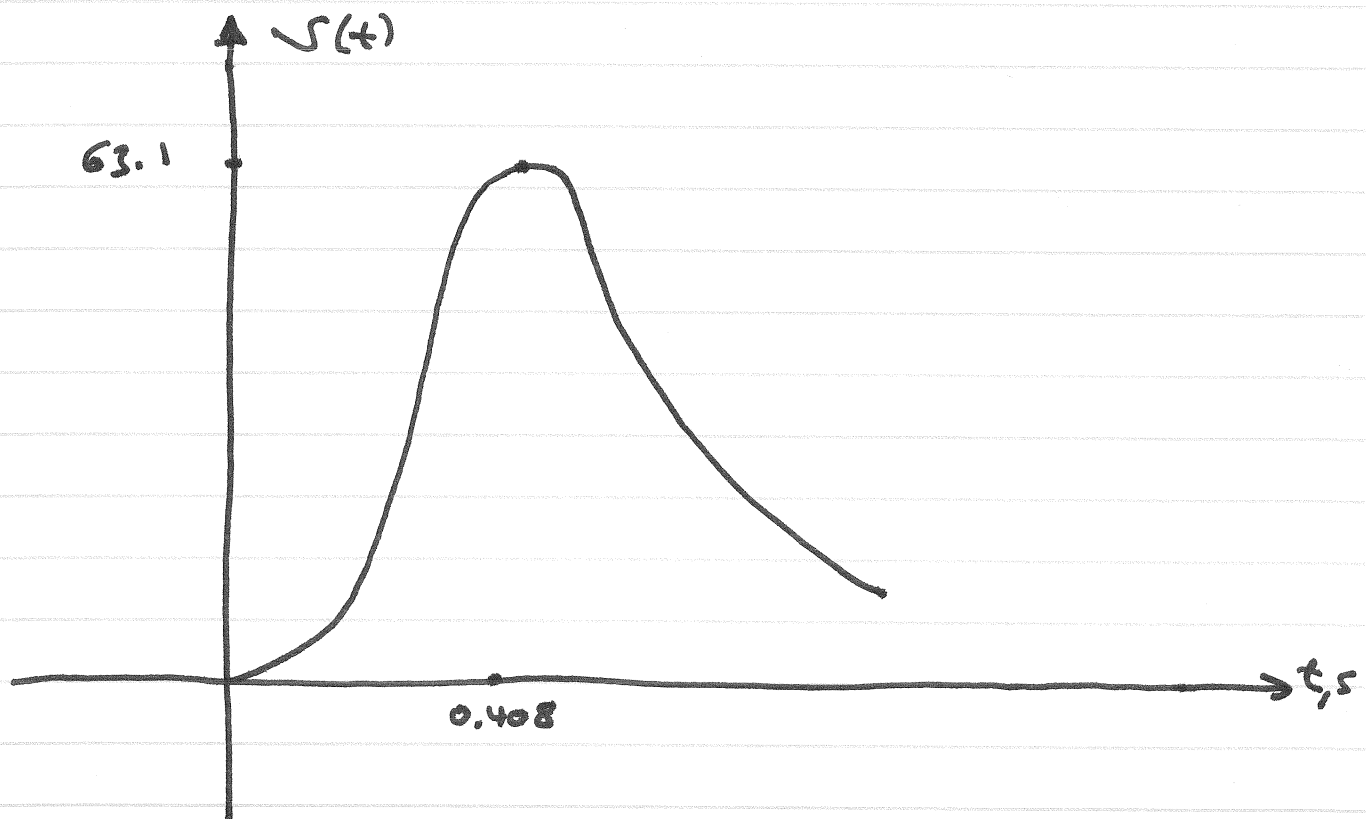
$$\therefore v(t) = A_1 t e^{-\sqrt{6}t} \quad \text{for } t > 0$$

$$\frac{dv(t)}{dt} = (A_1 t)(-\sqrt{6})e^{-\sqrt{6}t} + A_1 e^{-\sqrt{6}t}$$

$$\frac{dv(0^+)}{dt} = 0 + A_1 = 420$$

$$\therefore A_1 = 420$$

$$\therefore v(t) = 420 t e^{-\sqrt{6}t} \quad \text{for } t > 0$$



3) The underdamped Case

$$\alpha < \omega_0$$

$$\therefore \alpha^2 - \omega_0^2 < 0$$

$$\sqrt{\alpha^2 - \omega_0^2} = \sqrt{(-1)(\omega_0^2 - \alpha^2)}$$

$$\sqrt{\alpha^2 - \omega_0^2} = j \sqrt{\omega_0^2 - \alpha^2}$$

$$\sqrt{\alpha^2 - \omega_0^2} = j \omega_d$$

$\omega_d \equiv$ damped radian frequency

$$s_1 = -\alpha + \sqrt{\alpha^2 - \omega_0^2}$$

$$s_1 = -\alpha + j \omega_d$$

$$s_2 = -\alpha - j \omega_d$$

s_1 , and s_2 are Complex Conjugate

$$s_1 = -\alpha + j\omega d$$

$$s_2 = -\alpha - j\omega d$$

$$\therefore v(t) = A_1 e^{(-\alpha + j\omega d)t} + A_2 e^{(-\alpha - j\omega d)t}$$

$$e^{j\omega d t}$$

$$= \cos \omega d t + j \sin \omega d t$$

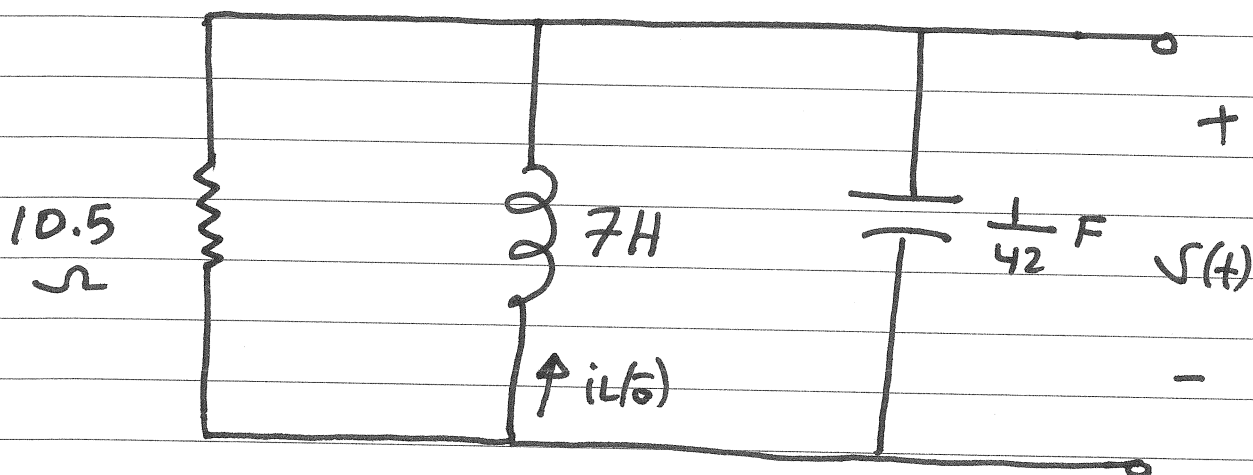
$$e^{-j\omega d t}$$

$$= \cos \omega d t - j \sin \omega d t$$

$$v(t) = e^{-\alpha t} \left[(A_1 + A_2) \cos \omega d t + j (A_1 - A_2) \sin \omega d t \right]$$

$$\therefore v(t) = e^{-\alpha t} \left[\beta_1 \cos \omega d t + \beta_2 \sin \omega d t \right]$$

Underdamped Parallel RLC



$$v_C(0) = 0, \text{ and } i_L(0) = 10 \text{ A}$$

$$\alpha = \frac{1}{2RC} = 2$$

$$\omega_0 = \frac{1}{\sqrt{LC}} = \sqrt{6}$$

$$\therefore \alpha < \omega_0$$

$$\omega_d = \sqrt{\omega_0^2 - \alpha^2} = \sqrt{2}$$

$$\therefore v(t) = e^{-\alpha t} \left(\beta_1 \cos \omega_d t + \beta_2 \sin \omega_d t \right) \text{ for } t > 0$$

$$\therefore v(t) = e^{-2t} \left(\beta_1 \cos \sqrt{2} t + \beta_2 \sin \sqrt{2} t \right) \text{ for } t > 0$$

$$v(0^+) = 0$$

$$\frac{dv(0^+)}{dt} = 420$$

$$v(t) = e^{-2t} (\beta_1 \cos\sqrt{2}t + \beta_2 \sin\sqrt{2}t)$$

$$v(0^+) = \beta_1 = 0$$

$$\therefore \beta_1 = 0$$

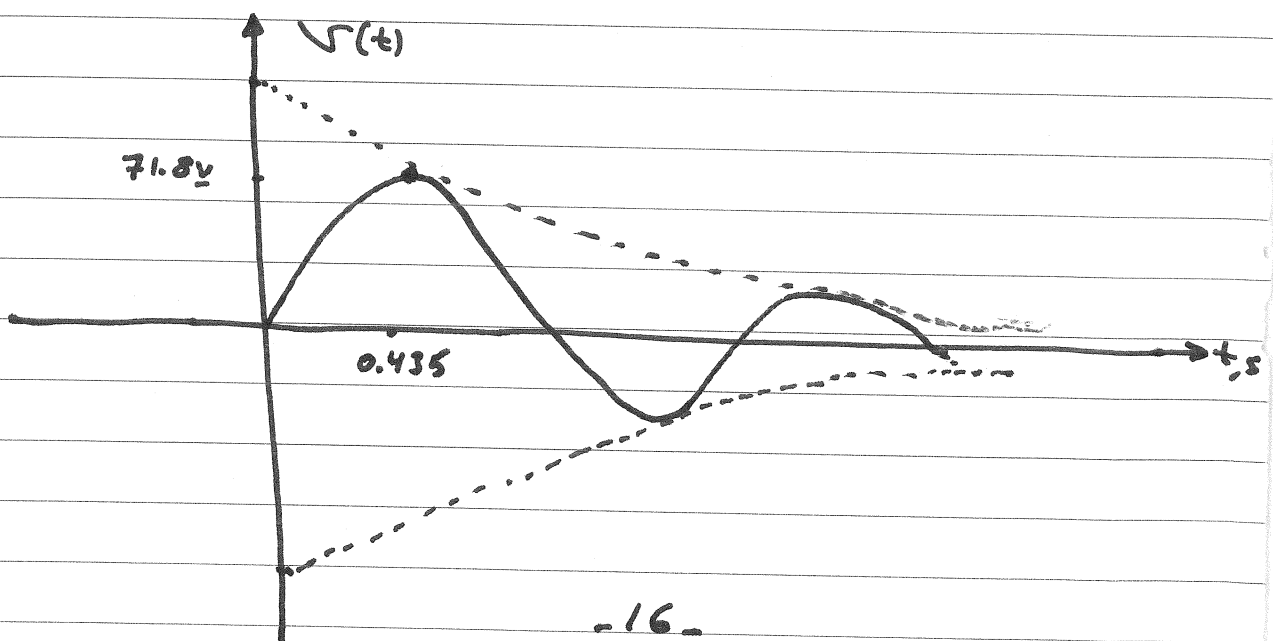
$$\therefore v(t) = e^{-2t} \beta_2 \sin\sqrt{2}t \quad \text{for } t > 0$$

$$\frac{dv(t)}{dt} = (\beta_2 e^{-2t}) (\sqrt{2} \cos\sqrt{2}t) + (\sin\sqrt{2}t) (-2 e^{-2t} \beta_2)$$

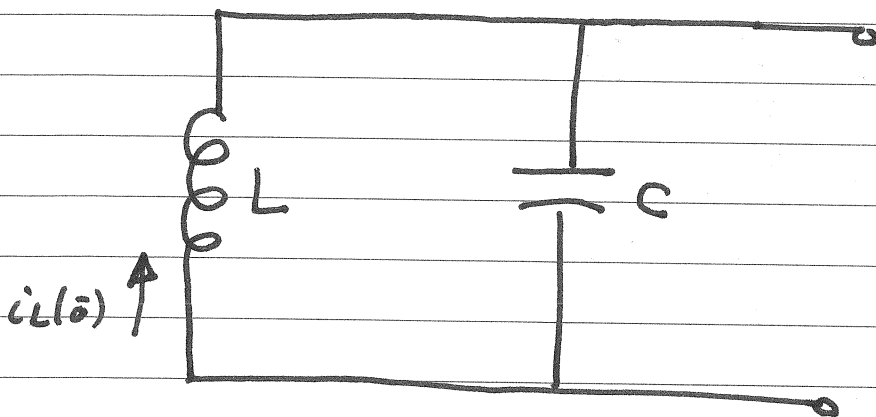
$$\frac{dv(0^+)}{dt} = \sqrt{2} \beta_2 + 0 = 420$$

$$\therefore \beta_2 = \frac{420}{\sqrt{2}}$$

$$\therefore v(t) = \frac{420}{\sqrt{2}} e^{-2t} \sin\sqrt{2}t \quad \text{for } t > 0$$



The Lossless LC Circuit



$$L = 7 \text{ H} , \quad C = \frac{1}{42} \text{ F} , \quad \text{and } R = \infty$$

$$\alpha = \frac{1}{2RC} = 0$$

$$\omega_0 = \frac{1}{\sqrt{LC}} = \sqrt{6}$$

$$\alpha < \omega_0$$

$$\omega_d = \sqrt{\omega_0^2 - \alpha^2} = \sqrt{6}$$

$$\therefore v(t) = \beta_1 \cos \sqrt{6} t + \beta_2 \sin \sqrt{6} t \quad \text{for } t > 0$$

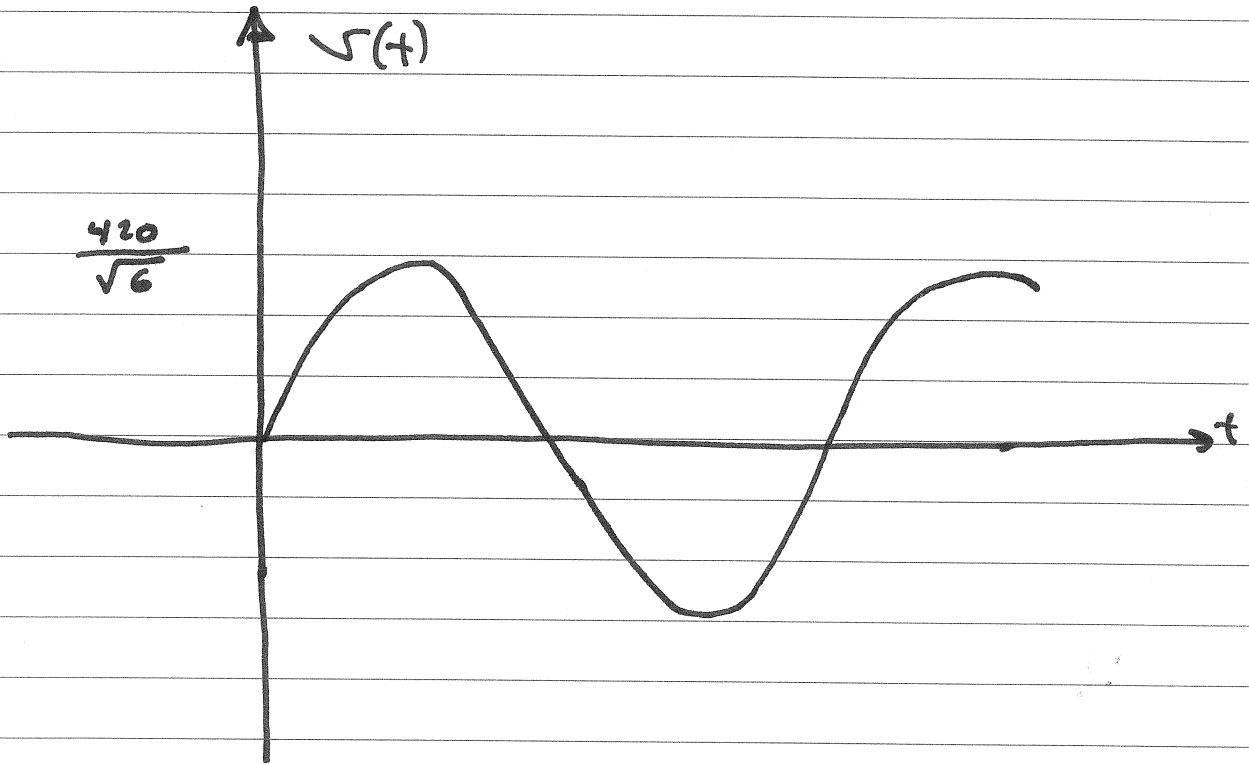
$$v(0^+) = 0 , \quad \text{and } \frac{dv(0^+)}{dt} = 420$$

$$v(0^+) = \beta_1 = 0$$

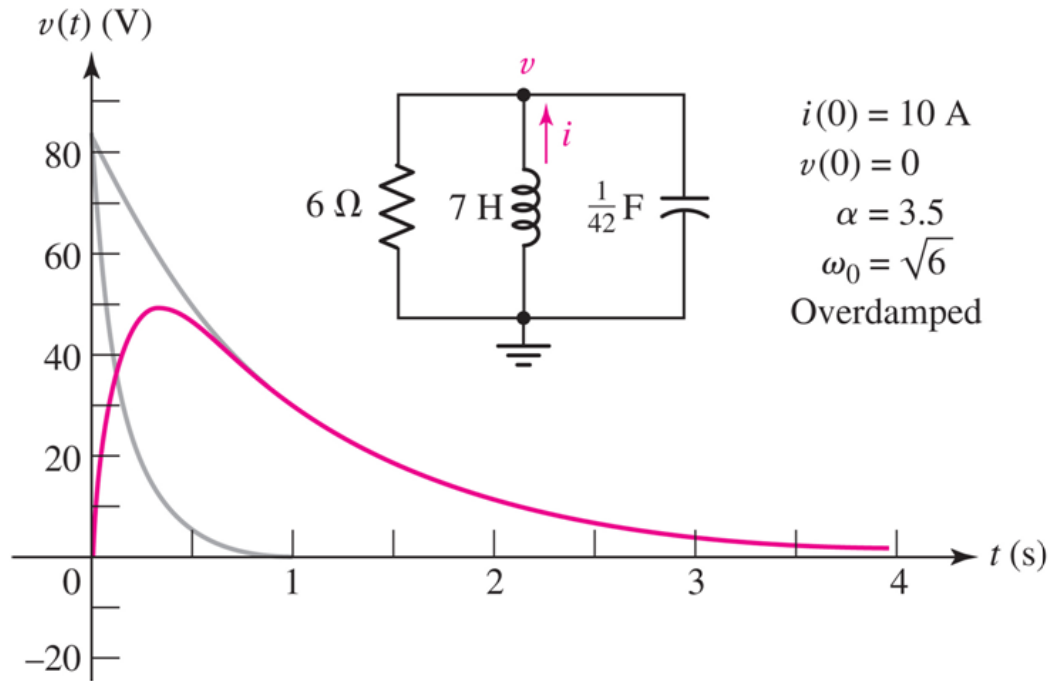
$$\frac{dv(0^+)}{dt} = \sqrt{6} \beta_2 = 420$$

$$\therefore \beta_2 = \frac{420}{\sqrt{6}}$$

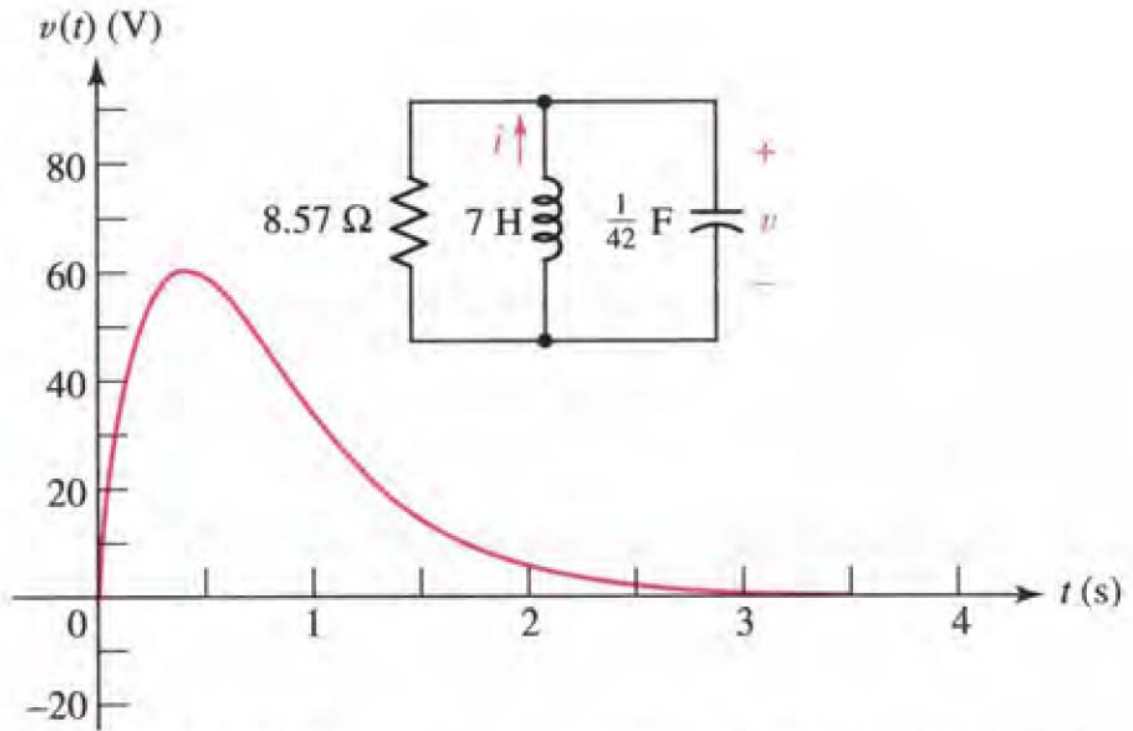
$$v(t) = \frac{420}{\sqrt{6}} \sin \sqrt{6} t \quad \text{for } t > 0$$



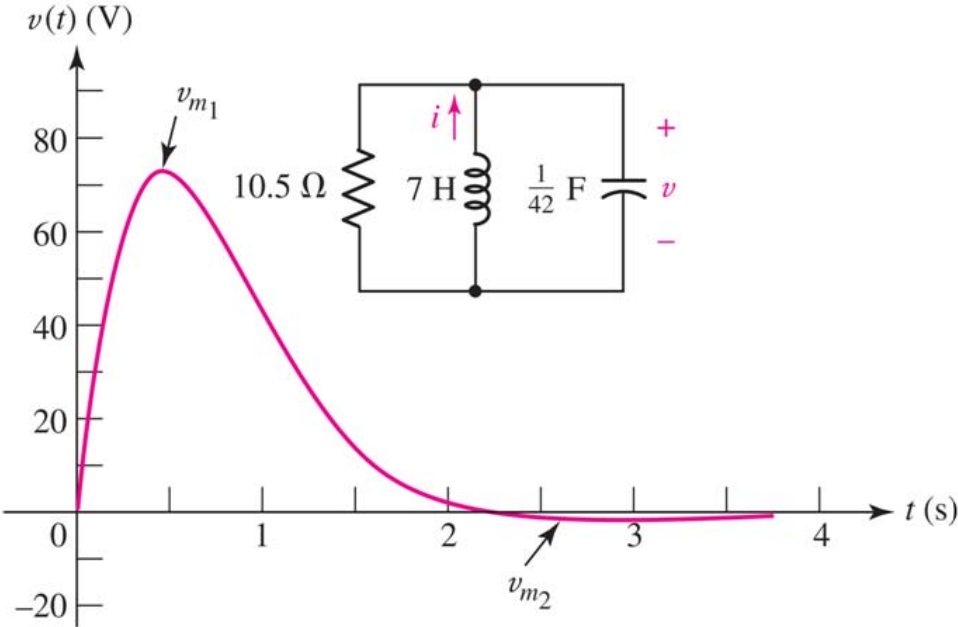
Over damped case



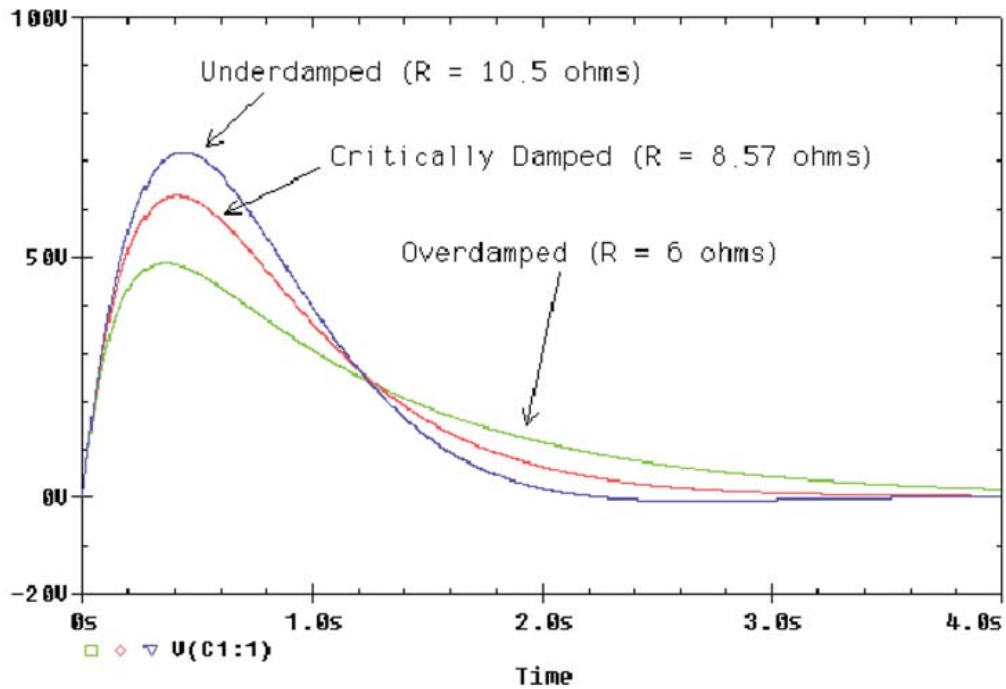
Critical damped case



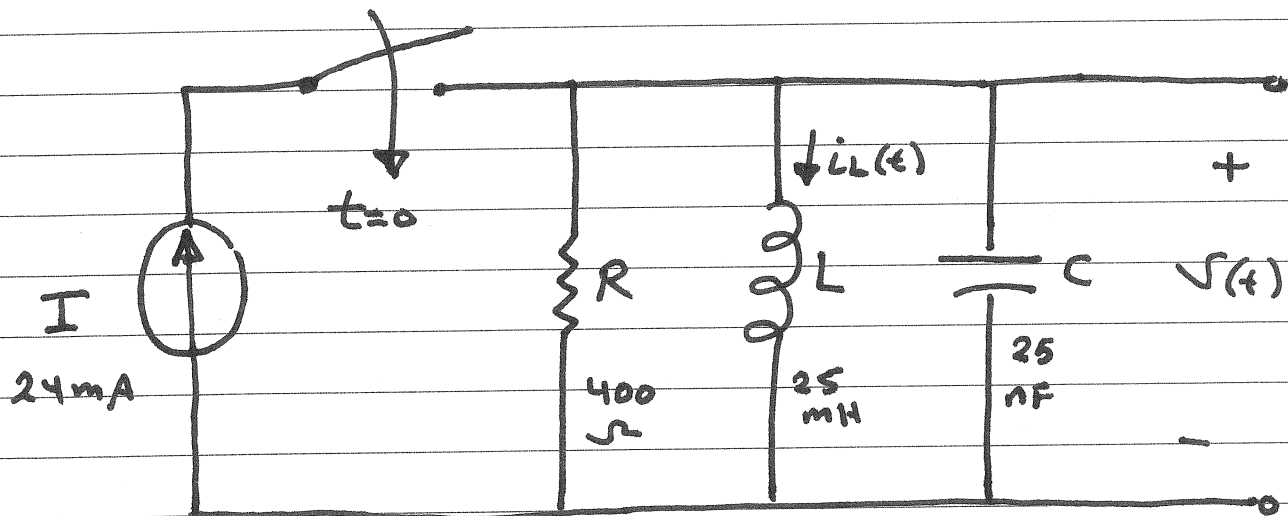
Under damped case



Comparing the Responses



Step response of parallel RLC Circuit

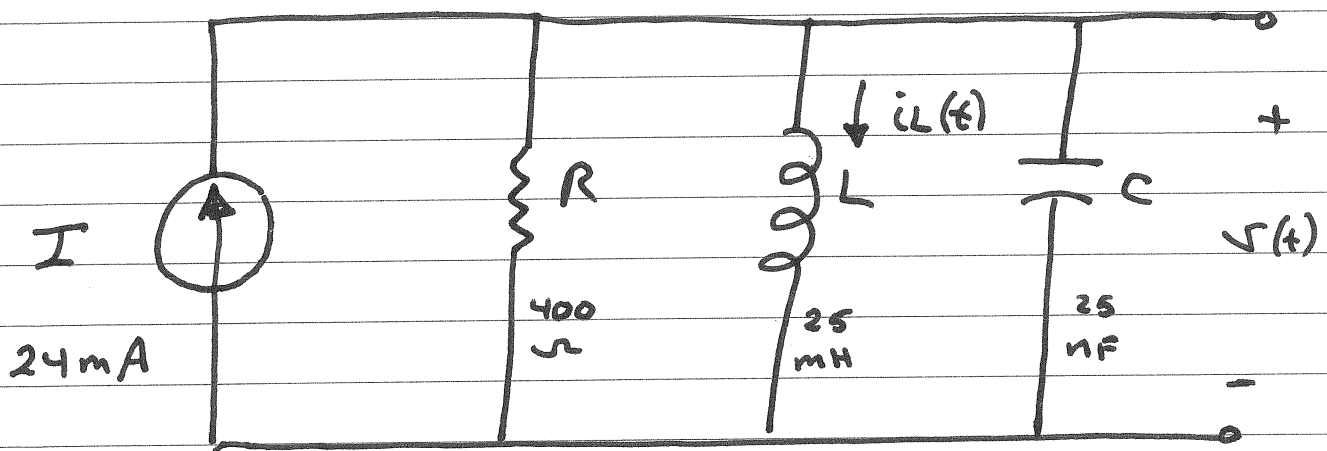


$$i_L(0^-) = 0, \text{ and } v_C(0^-) = 0$$

Find $v(t)$ for $t > 0$

Find $i_L(t)$ for $t > 0$

For $t > 0$



$i_L(0) = 0$, and $v_C(0) = 0$, Find $i_L(t)$

KCL :

$$I = i_R(t) + i_L(t) + i_C(t)$$

$$I = \frac{v(t)}{R} + i_L(t) + C \frac{dv(t)}{dt}$$

$$v(t) = v_L(t) = L \frac{di_L(t)}{dt}$$

$$\therefore I = LC \frac{d^2 i_L(t)}{dt^2} + \frac{L}{R} \frac{di_L(t)}{dt} + i_L(t)$$

$$\frac{d^2 i_L(t)}{dt^2} + \frac{1}{RC} \frac{di_L(t)}{dt} + \frac{1}{LC} i_L(t) = \frac{I}{LC}$$

second order nonhomogeneous diff. equation

$$i_L(t) = i_n(t) + i_f(t)$$

$i_n(t)$ = natural response

$i_f(t)$ = forced response

To find $i_f(t)$

$$\frac{d^2 i_L(t)}{dt^2} + \frac{1}{RC} \frac{d i_L(t)}{dt} + \frac{1}{LC} i_L(t) = \frac{I}{LC}$$

let $i_f(t) = k$

$$0 + 0 + \frac{1}{LC} i_f(t) = \frac{I}{LC}$$

$$\therefore i_f(t) = I = k$$

To find $i_n(t)$

$$\frac{d^2 i_L(t)}{dt^2} + \frac{1}{RC} \frac{d i_L(t)}{dt} + \frac{1}{LC} i_L(t) = 0$$

$$s^2 + \frac{1}{RC} s + \frac{1}{LC} = 0$$

$$s_1 = -\frac{1}{2RC} + \sqrt{\left(\frac{1}{2RC}\right)^2 - \frac{1}{LC}}$$

$$s_2 = -\frac{1}{2RC} - \sqrt{\left(\frac{1}{2RC}\right)^2 - \frac{1}{LC}}$$

$$s_1 = -20000$$

$$s_2 = -80000$$

Since s_1, s_2 are real and unequal

\therefore We have overdamped case

$$\therefore i_L(t) = A_1 e^{-20000t} + A_2 e^{-80000t}$$

$$\therefore i_L(t) = i_L^f(t) + i_L^n(t)$$

$$i_L(t) = 24\text{mA} + A_1 e^{-20000t} + A_2 e^{-80000t} \quad t > 0$$

To find A_1 , and A_2 , we need

$$i_L(0^+) \text{ and } \frac{di_L(0^+)}{dt}$$

$$i_L(0^+) = i_L(0^-) = 0$$

$$v_C(t) = v_L(t) = L \frac{di_L(t)}{dt}$$

$$\therefore v_C(0^+) = L \frac{di_L(0^+)}{dt} = v_C(0^-) = 0$$

$$\therefore \frac{di_L(0^+)}{dt} = 0$$

$$i_L(t) = 24 \text{ mA} + A_1 e^{-20000t} + A_2 e^{-80000t} \quad t > 0$$

$$i_L(0^+) = 24 \text{ mA} + A_1 + A_2$$

$$\therefore A_1 + A_2 = -24 \text{ mA} \quad \text{--- (1)}$$

$$\frac{di_L(0^+)}{dt} = -20000 A_1 - 80000 A_2 = 0 \quad \text{--- (2)}$$

Solving (1) and (2), we get

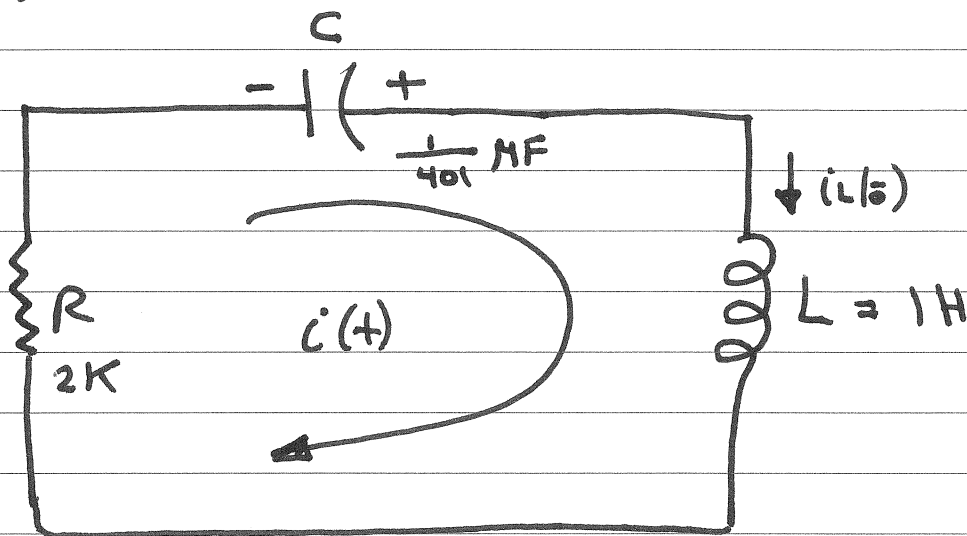
$$A_1 = -32 \text{ mA}$$

$$A_2 = 8 \text{ mA}$$

$$\therefore i_L(t) = \left(24 - 32 e^{-20000t} + 8 e^{-80000t} \right) \text{ mA}, \quad t > 0$$

Natural Response of series RLC circuit

For $t > 0$



$$V_c(s) = V_0, \text{ and } i_L(s) = I_0$$

Find $i(t)$ for $t > 0$

KVL :

$$L \frac{di(t)}{dt} + Ri(t) - V_c(s) + \frac{1}{C} \int_0^t i(t) dt = 0$$

$$L \frac{di(t)}{dt} + Ri(t) + \frac{1}{C} \int_0^t i(t) dt = V_c(s) \quad \text{--- (1)}$$

Differentiation of (1)

$$L \frac{d^2 i(t)}{dt^2} + R \frac{di(t)}{dt} + \frac{1}{C} i(t) = 0$$

second order homogeneous diff. equation.

$$Ls^2 + Rs + \frac{1}{C} = 0$$

$$s_1 = -\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}$$

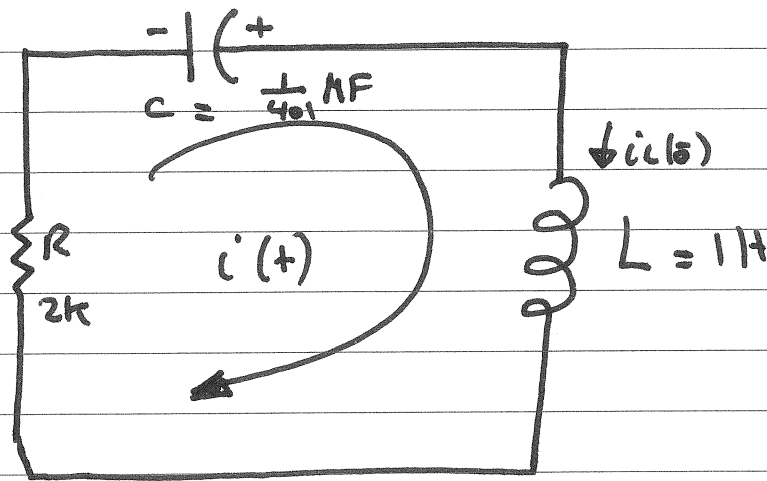
$$s_2 = -\frac{R}{2L} - \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}$$

$$\text{Let } \alpha = \frac{R}{2L}$$

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

$$\therefore s_1 = -\alpha + \sqrt{\alpha^2 - \omega_0^2}$$

$$s_2 = -\alpha - \sqrt{\alpha^2 - \omega_0^2}$$



$$\text{let } V_C(t) = V_0 = 2V$$

$$i_L(t) = I_0 = 2mA$$

$$\alpha = \frac{R}{2L} = 1000$$

$$\omega_0 = \frac{1}{\sqrt{LC}} = 20025$$

Since $\alpha < \omega_0$

\therefore We have underdamped case

$$\therefore \omega_d = \sqrt{\omega_0^2 - \alpha^2} = 20000$$

$$\therefore i(t) = e^{-\alpha t} (\beta_1 \cos \omega_d t + \beta_2 \sin \omega_d t) \text{ for } t > 0$$

$$\therefore i(t) = e^{-1000t} (\beta_1 \cos 20,000t + \beta_2 \sin 20,000t) \text{ for } t > 0$$

To find B_1 and B_2 , we need

to have $i(0^+)$ and $\frac{di(0^+)}{dt}$

$$i(0^+) = i_L(0^+) = i_L(0^-) = 2 \text{ mA}$$

$$L \frac{di(t)}{dt} + Ri(t) + \frac{1}{C} \int_{0^-}^t i(t) dt - V_C(0^-) = 0$$

at $t = 0^+$

$$L \frac{di(0^+)}{dt} + Ri(0^+) + 0 - V_C(0^-) = 0$$

$$\therefore \frac{di(0^+)}{dt} = \frac{V_C(0^-) - Ri(0^+)}{L} = -2$$

$$i(t) = e^{-1000t} \left(\beta_1 \cos 20000t + \beta_2 \sin 20000t \right)$$

$$i(0^+) = \beta_1 = 2 \text{ mA}$$

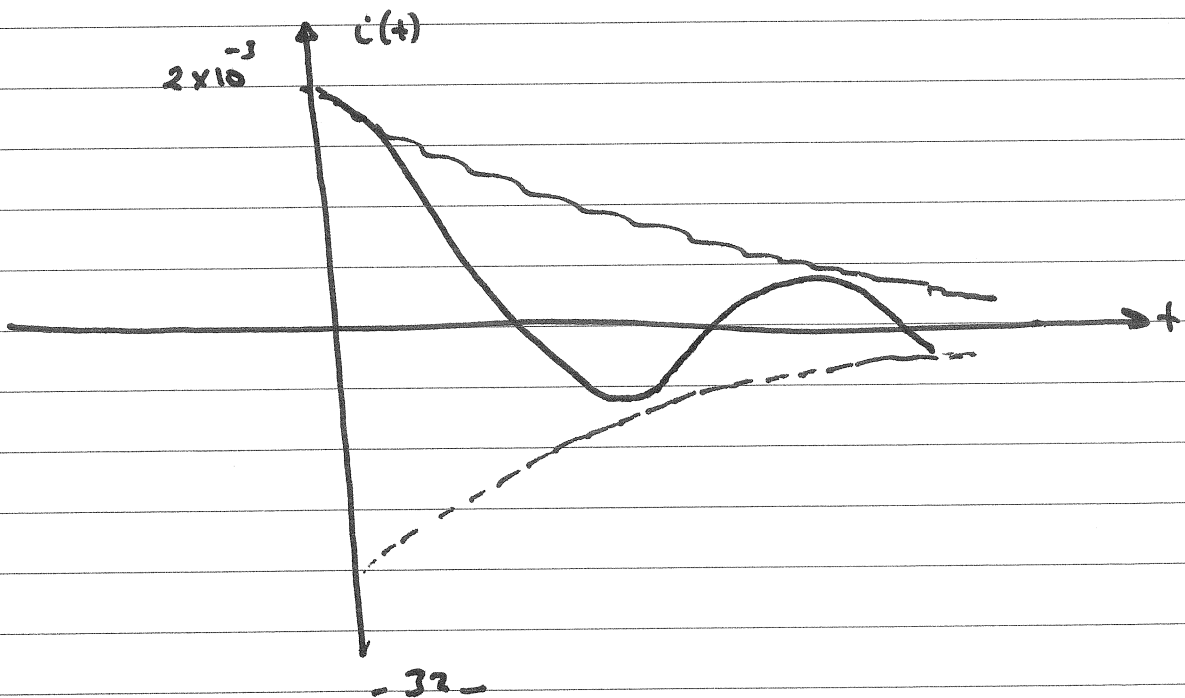
$$\therefore \beta_1 = 2 \text{ mA}$$

$$\frac{di(0^+)}{dt} = 20000 \beta_2 - 2 \times 10^{-3} (1000)$$

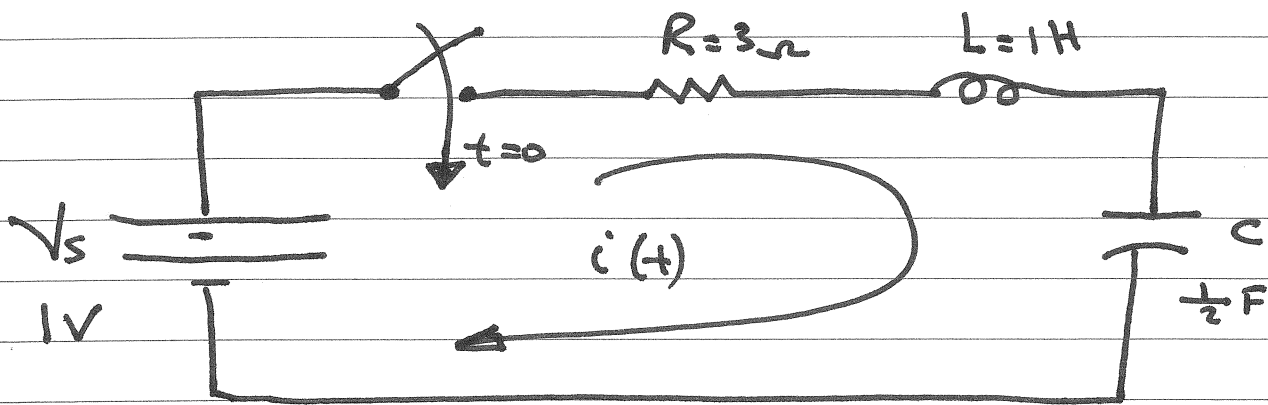
$$\frac{di(0^+)}{dt} = 20000 - 2 = -2$$

$$\therefore \beta_2 = 0$$

$$\therefore i(t) = 2 e^{-1000t} \cos 20000t \text{ mA}, \text{ for } t > 0$$



Step response of series RLC Circuit



$$v_c(0) = 0, \quad i_L(0) = 0$$

Find $i(t)$ for $t > 0$

KVL :

$$V_s = Ri(t) + L \frac{di(t)}{dt} + v_c(0) + \frac{1}{C} \int_0^t i(t) dt$$

$$0 = L \frac{d^2 i(t)}{dt^2} + R \frac{di(t)}{dt} + \frac{1}{C} i(t)$$

$$\therefore i(t) = i_n(t)$$

$$0 = Ls^2 + Rs + \frac{1}{C}$$

$$0 = s^2 + 3s + 2$$

$$\therefore s_1 = -1, \quad s_2 = -2$$

s_1, s_2 are real and unequal

\therefore overdamped case

$$\therefore i(t) = A_1 e^{-t} + A_2 e^{-2t} \quad \text{for } t > 0$$

or

$$\alpha = \frac{R}{2L} = 1.5$$

$$\omega_0 = \frac{1}{\sqrt{LC}} = \sqrt{2}$$

$\alpha > \omega_0 \rightarrow$ overdamped case

To find A_1 and A_2

$$i(0^+) = i(0^-) = 0$$

$$V_s = Ri(t) + L \frac{di(t)}{dt} + \frac{1}{C} \int_{0^-}^t i(t) dt + V_c(0^-)$$

at $t=0^+$

$$V_s = Ri(0^+) + L \frac{di(0^+)}{dt} + 0 + 0$$

$$\therefore \frac{di(0^+)}{dt} = \frac{V_s - Ri(0^+)}{L} = \frac{V_s}{L} = \frac{1}{1} = 1$$

$$i(0^+) = 0$$

$$\frac{di(0^+)}{dt} = 1$$

$$i(t) = A_1 e^{-t} + A_2 e^{-2t} \quad \text{for } t > 0$$

$$i(0^+) = A_1 + A_2 = 0 \quad \text{--- (1)}$$

$$\frac{di(0^+)}{dt} = -A_1 - 2A_2 = 1 \quad \text{--- (2)}$$

Solving (1) and (2)

$$A_1 = 1, \quad A_2 = -1$$

$$\therefore i(t) = (e^{-t} - e^{-2t}) \underline{A}, \quad \text{for } t > 0$$

$$V_c(t) = V_c(0) + \frac{1}{c} \int_{0^-}^t i(t) dt$$

$$V_c(t) = (1 - 2e^{-t} + e^{-2t}) \underline{V}$$

for $t > 0$

To find $V_c(t)$

$$V_s = Ri(t) + L \frac{di(t)}{dt} + V_c(t)$$

$$i(t) = i_c(t) = C \frac{dV_c(t)}{dt}$$

$$V_s = RC \frac{dV_c(t)}{dt} + LC \frac{d^2V_c(t)}{dt^2} + V_c(t)$$

second order nonhomogeneous diff. equation

$$\therefore V_c(t) = V_{cn}(t) + V_{cf}(t)$$

$$V_{cf}(t) = K$$

$$K = V_s$$

$$\therefore V_c(t) = V_s + V_{cn}(t)$$

$$0 = LCs^2 + RCs + 1$$

$$0 = \frac{1}{2}s^2 + \frac{3}{2}s + 1$$

$$\therefore s_1 = -1$$

$$s_2 = -2$$

$$V_c(t) = V_{cn}(t) + V_{cf}(t)$$

$$V_c(t) = A_1 e^{-t} + A_2 e^{-2t} + 1 \quad \text{for } t > 0$$

To find A_1, A_2

$$V_c(0^+) = V_c(0) = 0$$

$$i(t) = i_L(t) = i_C(t) = C \frac{dV_c(t)}{dt}$$

$$\therefore i_L(0^+) = i_C(0^+) = C \frac{dV_c(0^+)}{dt} = 0$$

$$\therefore \frac{dV_c(0^+)}{dt} = 0$$

$$\therefore V_c(0^+) = 0$$

$$\frac{dV_c(0^+)}{dt} = 0$$

$$V_c(t) = 1 + A_1 e^{-t} + A_2 e^{-2t}$$

$$V_c(0^+) = 1 + A_1 + A_2 = 0$$

$$\therefore A_1 + A_2 = -1 \quad \text{--- (1)}$$

$$\frac{dV_c(t)}{dt} = -A_1 e^{-t} - 2A_2 e^{-2t}$$

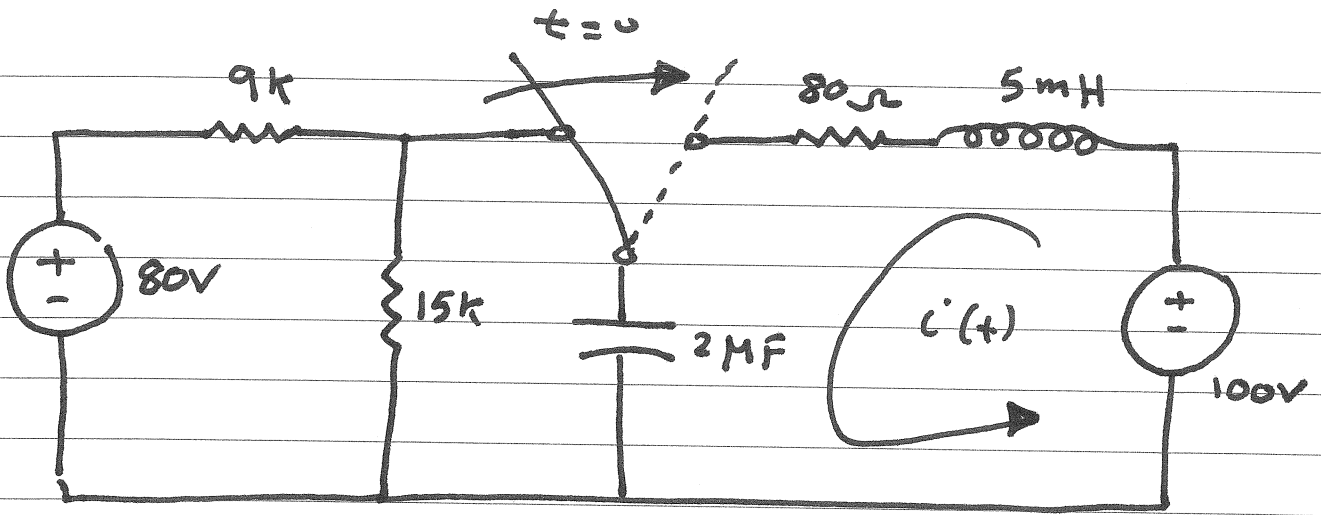
$$\frac{dV_c(0^+)}{dt} = -A_1 - 2A_2 = 0 \quad \text{--- (2)}$$

Solving ① and ②

$$A_1 = -2$$

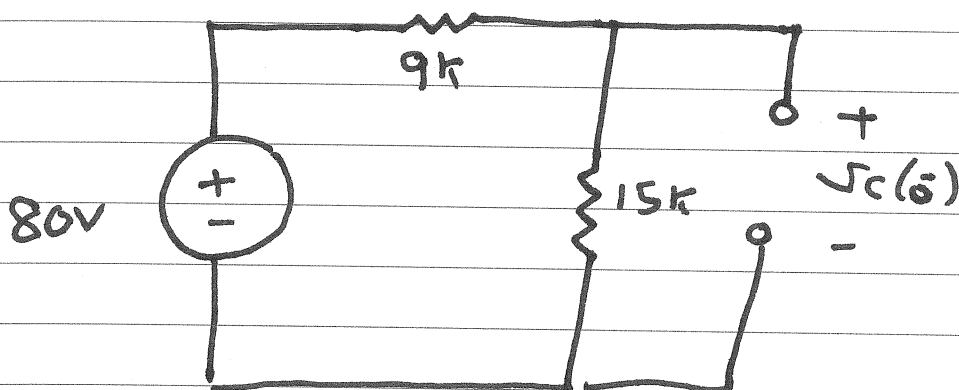
$$A_2 = 1$$

$$\therefore v_c(t) = \left(1 - 2e^{-t} + e^{-2t} \right) \underline{V} \text{ for } t > 0$$



1) Find $i(t)$ for $t > 0$

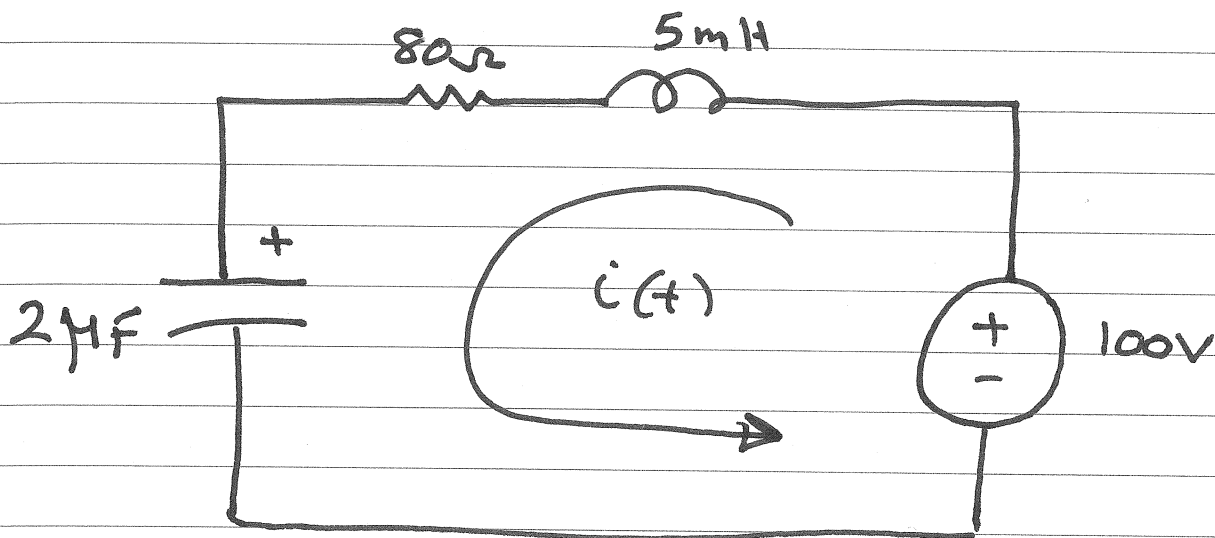
a) For $t < 0$, $t = 0^-$



$$V_c(0^-) = \frac{15k}{15k + 9k} \cdot 80 = 50V$$

$$i_L(0^-) = 0$$

b) For $t > 0$



KVL :

$$100 = L \frac{di(t)}{dt} + Ri(t) + V_c(t) + \frac{1}{C} \int_0^+ i(t) dt$$

$$50 = L \frac{di(t)}{dt} + Ri(t) + \frac{1}{C} \int_0^+ i(t) dt$$

$\frac{d}{dt}$:

$$0 = L \frac{d^2 i(t)}{dt^2} + R \frac{di(t)}{dt} + \frac{1}{C} i(t)$$

second order homogeneous diff. equation

$$\therefore i(t) = i_n(t)$$

$$\text{Let } v_1(t) = 10 \sin(5t - 30^\circ)$$

$$v_2(t) = 15 \sin(5t + 10^\circ)$$

$\therefore v_2(t)$ Leads $v_1(t)$ by 40°

$$\text{Let } i_1(t) = 2 \sin(377t + 45^\circ)$$

$$i_2(t) = 0.5 \cos(377t + 10^\circ)$$

$$\cos \alpha = \sin(\alpha + 90^\circ)$$

$$0.5 \cos(377t + 10^\circ) = 0.5 \sin(377t + 100^\circ)$$

$\therefore i_2(t)$ leads $i_1(t)$ by 55°

$$0 = LC s^2 + RCs + 1$$

$$0 = 10 \times 10^{-9} s^2 + 160 \times 10^{-6} s + 1$$

$$\therefore s_1 = -8000 + j 6000$$

$$s_2 = -8000 - j 6000$$

Underdamped Case

$$i(t) = e^{-8000t} \left(\beta_1 \cos 6000t + \beta_2 \sin 6000t \right)$$

To find β_1 and β_2

$$i(0^+) = i(0^-) = 0$$

$$\frac{di(0^+)}{dt} = \frac{V_s - V_c(0)}{L} = 10,000$$

$$\beta_1 = 0$$

$$\beta_2 = 1.67$$

$$i(t) = 1.67 e^{-8000t} \sin 6000t \quad \underline{A} \quad \text{for } t > 0$$

2) Find $V_c(t)$ for $t > 0$

$$V_s = L \frac{di(t)}{dt} + Ri(t) + V_c(t)$$

$$i(t) = i_c(t) = C \frac{dV_c(t)}{dt}$$

$$\therefore V_s = LC \frac{d^2V_c(t)}{dt^2} + RC \frac{dV_c(t)}{dt} + V_c(t)$$

second order non homogeneous diff. equation

$$\therefore V_c(t) = V_{ch}(t) + V_{cf}(t)$$

$$V_{cf}(t) = K$$

$$V_s = 0 + 0 + K$$

$$\therefore V_{cf} = K$$

To find $V_{ch}(t)$

$$0 = LCs^2 + RCs + 1$$

$$0 = 10 \times 10^{-9} s^2 + 160 \times 10^{-6} s + 1$$

$$s_1 = -8000 + j6000$$

$$s_2 = -8000 - j6000$$

Underdamped Case

$$V_c(t) = 100 + e^{-8000t} \left(\beta_1 \cos 6000t + \beta_2 \sin 6000t \right)$$

To find β_1 , and β_2 , we need

$$V_c(0^+) \text{ and } \frac{dV_c(t)}{dt}$$

$$V_c(0^+) = V_c(0) = 50 \text{ V}$$

$$i_c(0^+) = i_L(0^+) = \frac{C dV_c(0^+)}{dt} = 0$$

$$\therefore \frac{dV_c(0^+)}{dt} = 0$$

$$V_c(0^+) = 100 + \beta_1 = 50$$

$$\therefore \beta_1 = -50$$

$$\frac{dV_c(0^+)}{dt} = -8000\beta_1 + 6000\beta_2$$

$$\therefore \beta_2 = -66.67$$

$$\therefore V_c(t) = \left[100 + e^{-8000t} \left(-50 \cos 6000t - 66.67 \sin 6000t \right) \right] \text{ V}$$

-43- for $t > 0$