Phasors

Given the sinusoids $i(t) = I_m \cos(\omega t + \phi_i)$ and $v(t) = V_m \cos(\omega t + \theta_v)$

We can obtain the phasor forms as:

$i(t) = I_m \cos(\omega t + \phi_i)$, then $\vec{I} = I_m \angle \phi_i$

$v(t) = V_m \cos(\omega t + \theta_v)$, then $\vec{V} = V_m \angle \theta_v$

$$i(t) = 6 \cos(50t - 40^\circ) \text{ A}$$

$$\therefore \vec{I} = 6 \angle -40^\circ \quad \text{A}$$

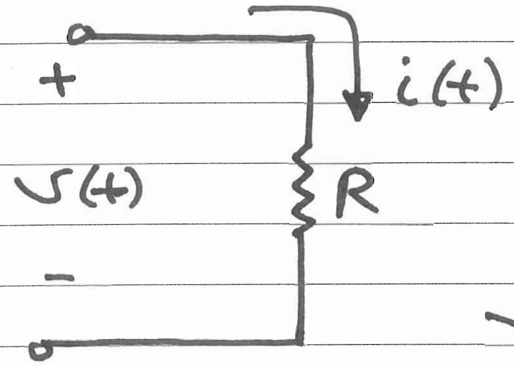
$$v(t) = -4 \sin(30t + 50^\circ) \text{ V}$$

$$v(t) = 4 \cos(30t + 140^\circ) \text{ V}$$

$$\therefore \vec{V} = 4 \angle 140^\circ \quad \text{V}$$

Phasor Relationships for Circuit Elements

Resistor :



$$v(t) = R\,i(t)$$

$$V_m e^{j(\omega t + \theta_v)} = R\,I_m e^{j(\omega t + \phi)}$$

$$V_m e^{j\theta_v} = R\,I_m e^{j\phi_i}$$

$$V_m \angle \theta_v = R\,I_m \angle \phi_i$$

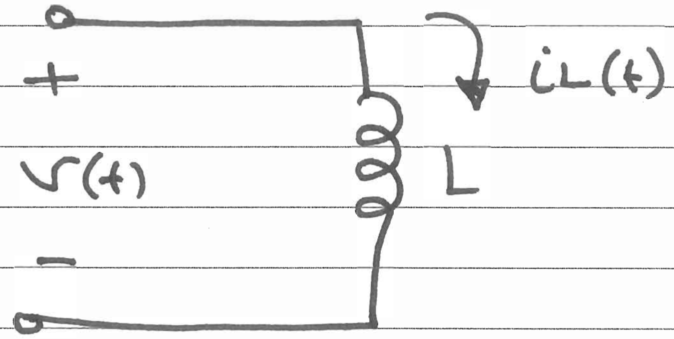
$$\boxed{\vec{V} = R\,\vec{I}}$$

$$V_m = R I_m$$

$$\theta_v = \phi_i$$

* Voltage and Current of a resistor are in phase.

Inductor :



$$v(t) = L \frac{di(t)}{dt}$$

$$V_m e^{j(\omega t + \theta_v)} = L \frac{d}{dt}\left(I_m e^{j(\omega t + \phi_i)}\right)$$

$$V_m e^{j(\omega t + \theta_v)} = j\omega L \, I_m e^{j(\omega t + \phi_i)}$$

$$V_m e^{j\theta_v} = j\omega L \, I_m e^{j\phi_i}$$

$$V_m \angle \theta_v = j\omega L \, I_m \angle \phi_i$$

$$\boxed{\vec{V} = j\omega L \, \vec{I}}$$

$*$ $\quad V_m = \omega L I$

$$V_m \angle \theta_v = \omega L \angle 90^\circ \cdot I_m \angle \phi_i$$

$$V_m \angle \theta_v = \omega L I_m \angle \phi_i + 90^\circ$$

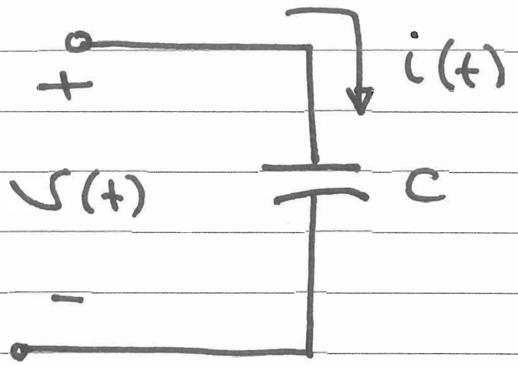
$$V_m \angle \theta_v = \omega L\, I_m \angle \phi_i + 90^\circ$$

$$\therefore \quad V_m = \omega L\, I_m$$

$$\theta_v = \phi_i + 90^\circ$$

The voltage leads the current by 90°

Capacitor :



$$i(t) = C\,\frac{dv(t)}{dt}$$

$$I_m e^{j(\omega t+\phi_i)} = C\,\frac{d}{dt}\left(V_m e^{j(\omega t+\theta_v)}\right)$$

$$I_m e^{j(\omega t+\phi_i)} = j\omega C V_m e^{j(\omega t+\theta_v)}$$

$$I_m e^{j\phi_i} = j\omega C V_m e^{j\theta_v}$$

$$I_m \angle \phi_i = j\omega C V_m \angle \theta_v$$

$$\boxed{\vec{I} = j\omega C \vec{V}}$$

$$I_m \angle \phi_i = \omega C \angle 90^\circ \; V_m \angle \theta_v$$

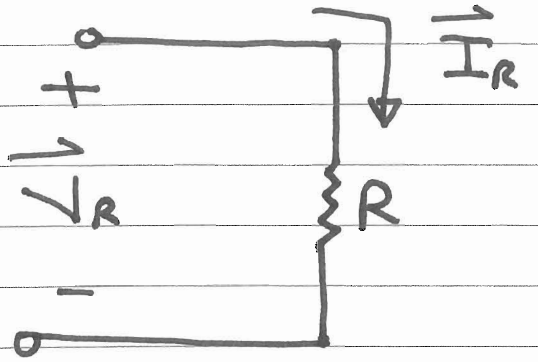
$$I_m \angle \phi_i = \omega C V_m \angle \theta_v + 90^\circ$$

$$\therefore \quad I_m = \omega C V_m$$

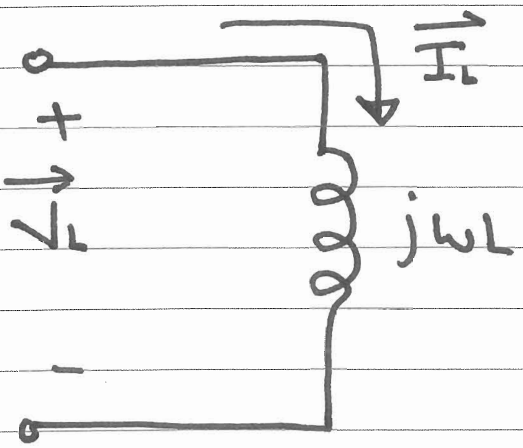
$$\phi_i = \theta_v + 90^\circ$$

The Current Leads the Voltage by 90°.

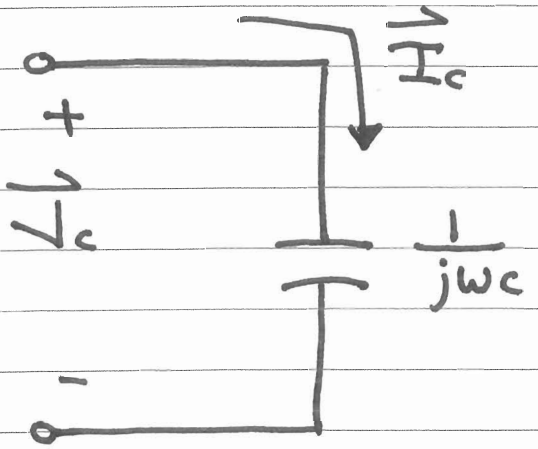
Phasor Relationships For Circuit Elements



$$\vec{V}_R = R\,\vec{I}_R$$



$$\vec{V}_L = j\omega L\,\vec{I}_L$$



$$\vec{V}_c = \frac{1}{j\omega c}\,\vec{I}_c$$

$$\vec{V} = Z(j\omega)\,\vec{I}$$

Impedance and Admittance

$$Z(j\omega) = \frac{\vec{V}}{\vec{I}} \qquad \text{Impedance}, \Omega$$

or $\vec{V} = Z(j\omega)\,\vec{I}$

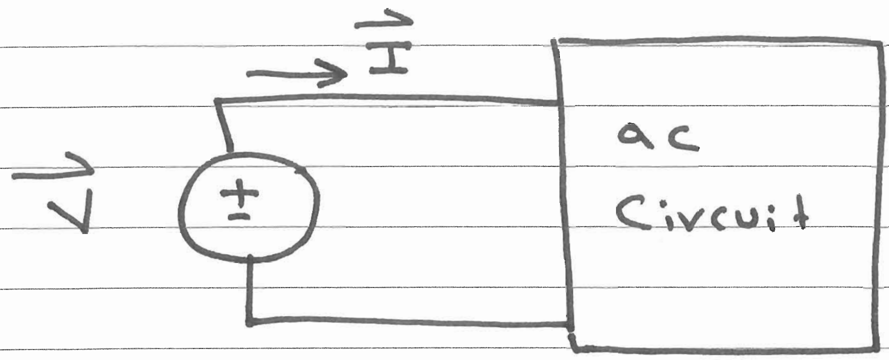
$$Y(j\omega) = \frac{\vec{I}}{\vec{V}} \qquad \text{Admittance}, \mho$$

or $\vec{I} = Y(j\omega)\,\vec{V}$

$$\therefore \quad Z(j\omega) = \frac{1}{Y(j\omega)}$$

Element	Impedance	Admittance
R	$Z(j\omega) = R$	$Y(j\omega) = \frac{1}{R}$
C	$Z(j\omega) = \frac{1}{j\omega C}$	$Y(j\omega) = j\omega C$
L	$Z(j\omega) = j\omega L$	$Y(j\omega) = \frac{1}{j\omega L}$

Impedance: $Z(j\omega)$



$$Z(j\omega) = \frac{\vec{V}}{\vec{I}} = \frac{V_m \angle \theta_v}{I_m \angle \phi_i}$$

$$Z(j\omega) = \frac{V_m}{I_m} \angle \theta_v - \phi_i$$

$$Z(j\omega) = |Z| \angle \theta_z$$

The unit of impedance is Ohm

Impedance is not aphasor but a Complex number that Can be Written in polar or Cartesian forms

$$\vec{Z} = R + jX$$

$R \equiv$ Resistive part

$X \equiv$ Reactive part

$$Z = |Z| \angle \theta_z$$

$$Z = R + jX$$

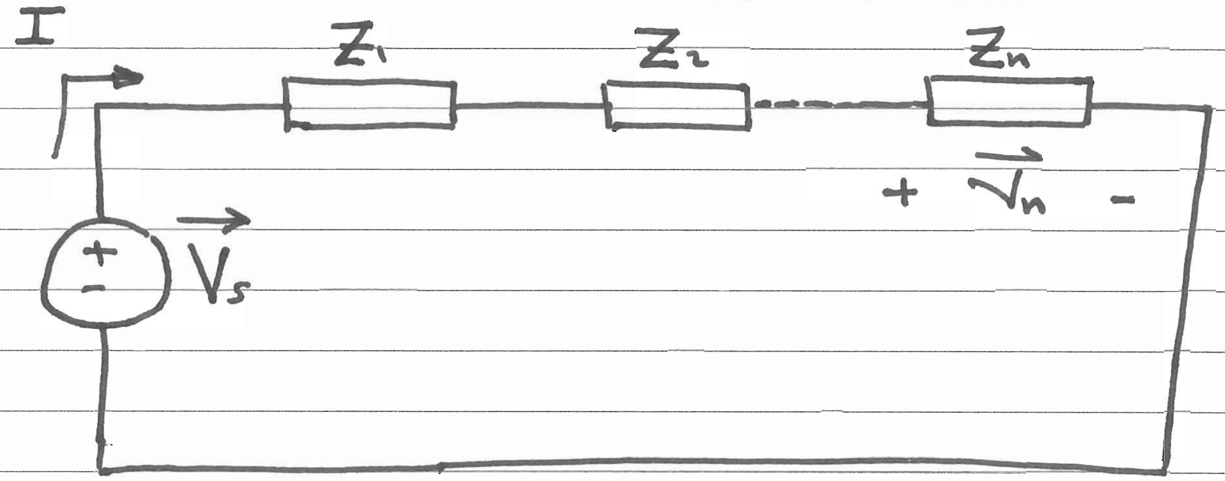
$$|Z| = \sqrt{R^2 + X^2}$$

$$\theta_z = \tan^{-1} \frac{X}{R}$$

$$X = |Z| \sin \theta_z$$

$$R = |Z| \cos \theta_z$$

Application of KVL for phasors



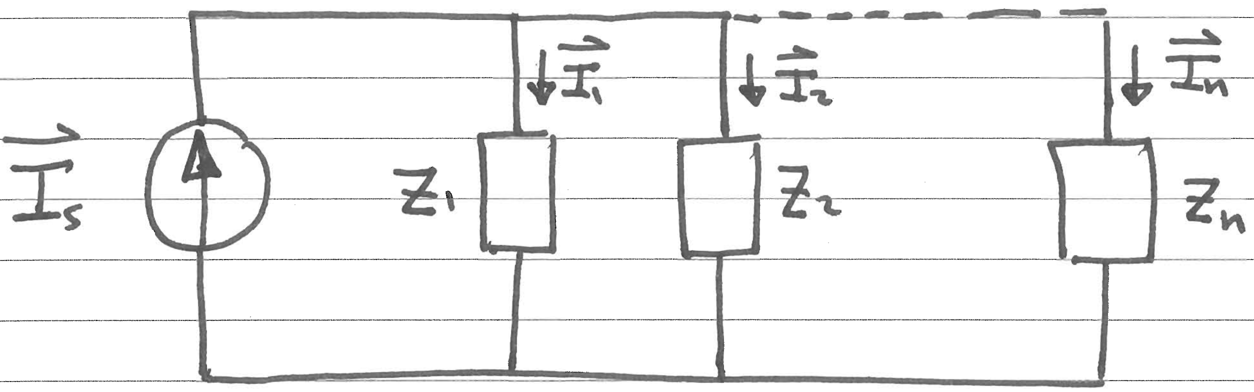
KVL : $v_s(t) = v_1(t) + v_2(t) + \cdots\cdots v_n(t)$

$$\vec{V}_s = \vec{V}_1 + \vec{V}_2 + \cdots\cdots \vec{V}_n$$

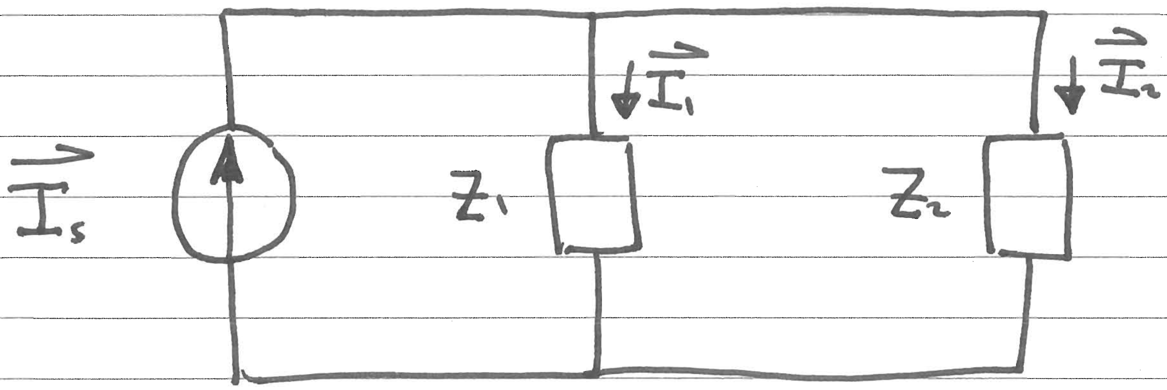
$$Z_{eq} = Z_1 + Z_2 + Z_3 + \cdots\cdots Z_n$$

$$\vec{V}_n = \frac{Z_n}{Z_1 + Z_2 + \cdots\cdots Z_n} \cdot \vec{V}_s$$

Application of KCL for phasors



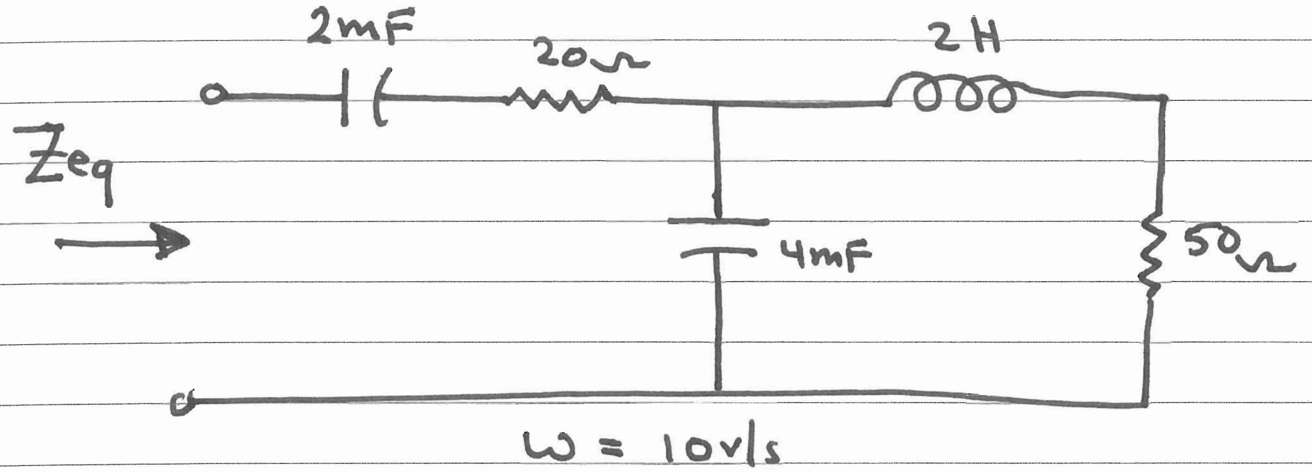
KCL : $\vec{I_s} = \vec{I_1} + \vec{I_2} + \text{-----} \vec{I_n}$



$$\vec{I_1} = \frac{Z_2}{Z_1 + Z_2} \vec{I_s}$$

$$\vec{I_2} = \frac{Z_1}{Z_1 + Z_2} \vec{I_s}$$

Find Zeq



$$Z_1 = 20 + \frac{1}{j(10)(2)(10^{-3})} = 20 - j50$$

$$Z_2 = 50 + j(10)(2) = 50 + j20$$

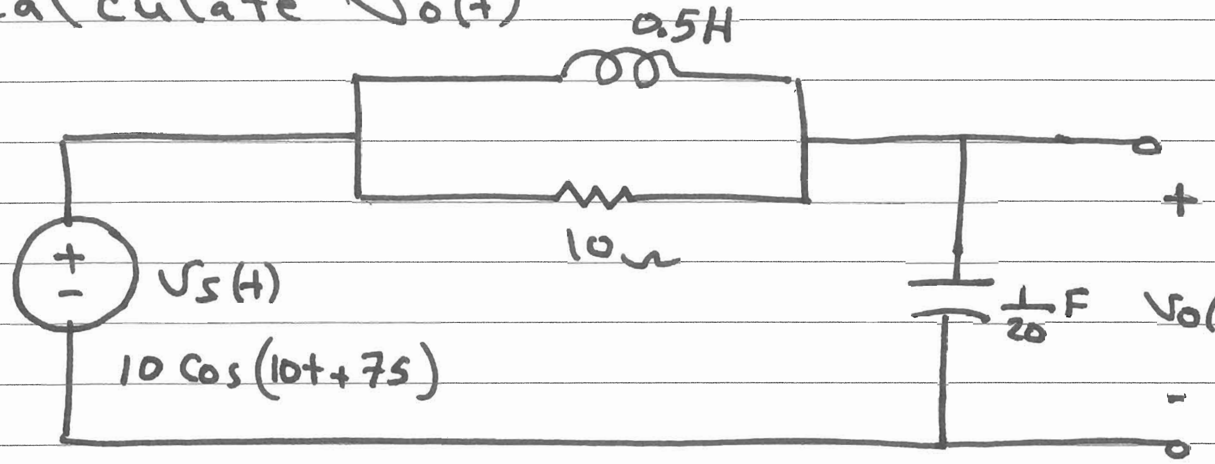
$$Z_3 = (50 + j20) \parallel \frac{1}{j(10)(4)(10^{-3})}$$

$$Z_3 = (50 + j20) \parallel -j25$$

$$Z_3 = \frac{(50 + j20)(-j25)}{50 + j20 - j25} = 12.38 - j23.76$$

$$Zeq = Z_1 + Z_3 = 32.38 - j73.76 \ \Omega$$

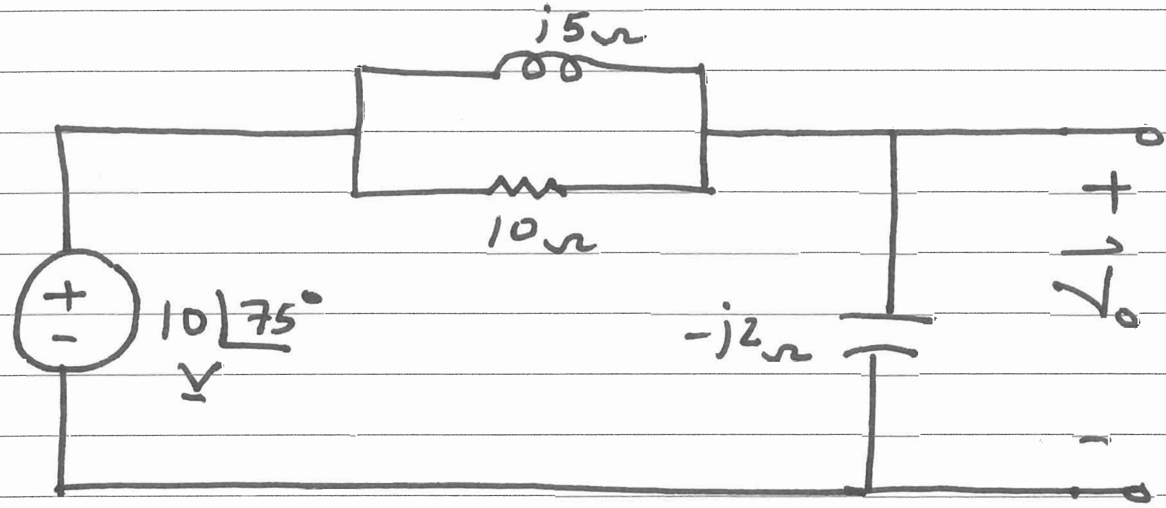
Calculate $v_o(t)$



$$Z_L(j\omega) = j\omega L = j5\ \Omega$$

$$Z_C(j\omega) = -j\frac{1}{\omega C} = -j2\ \Omega$$

$$\vec{V}_s = 10\angle 75^\circ\ \text{V}$$

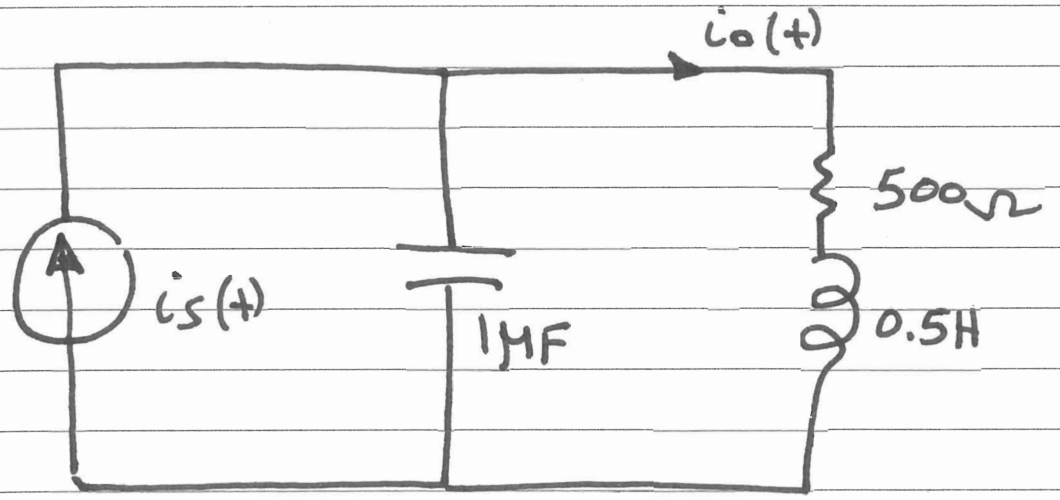


$$\vec{V}_o = \frac{-j2}{-j2 + 10 \| j5} \cdot 10\angle 75^\circ$$

$$\vec{V}_o = 7.071\angle -60^\circ\ \text{V}$$

$$\therefore v_o(t) = 7.071\ \text{Cos}\left(10t - 60^\circ\right)\ \text{V}$$

Calculate $i_o(t)$

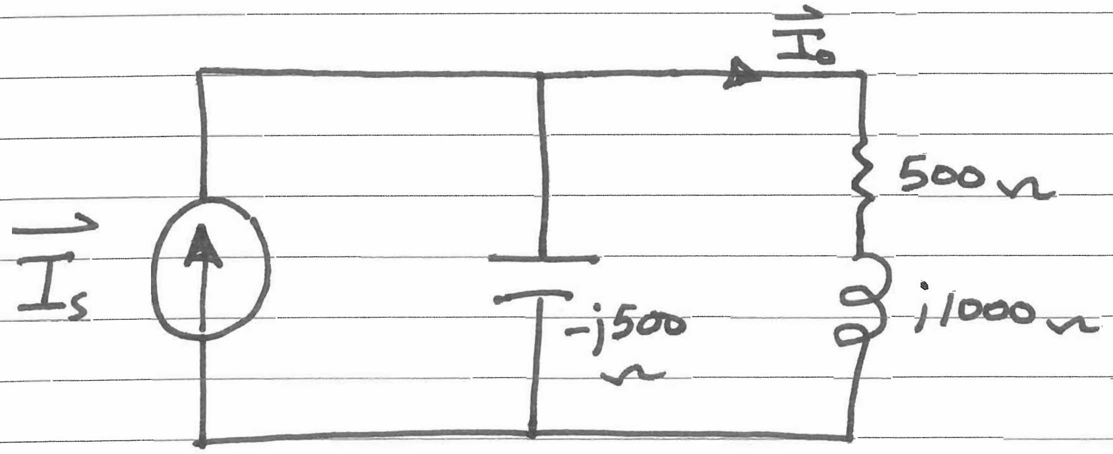


$$i_s(t) = 0.05 \cos 2000t \quad A$$

$$Z_C(j\omega) = -j\,\frac{1}{\omega C} = -j\,500\ \Omega$$

$$Z_L(j\omega) = j\omega L = j1000\ \Omega$$

$$\vec{I_s} = 0.05\angle 0^\circ \quad A$$

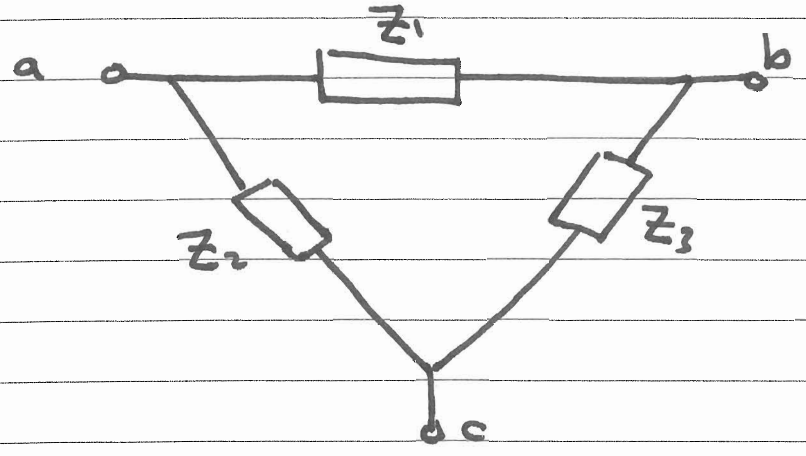


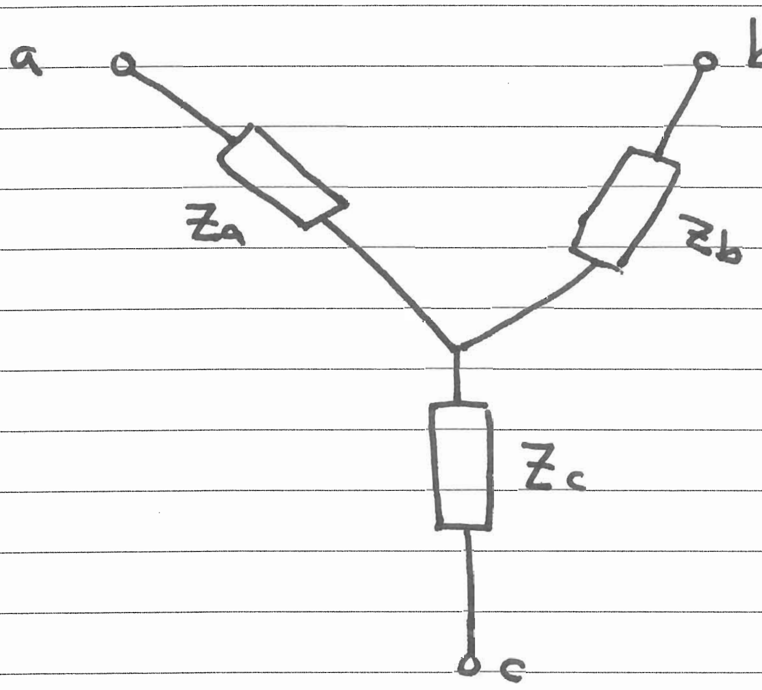
$$\vec{I_o} = \frac{-j\,500}{-j\,500 + 500 + j\,1000}\left(0.05\angle 0^\circ\right)$$

$$\vec{I_o} = 0.03535 \angle -45^\circ \text{ A}$$

$$\therefore i_o(t) = 0.03535 \cos(2000t - 45^\circ) \text{ A}$$

Y–Δ Transformation





$$Z_a = \frac{Z_1 Z_2}{Z_1 + Z_2 + Z_3}$$

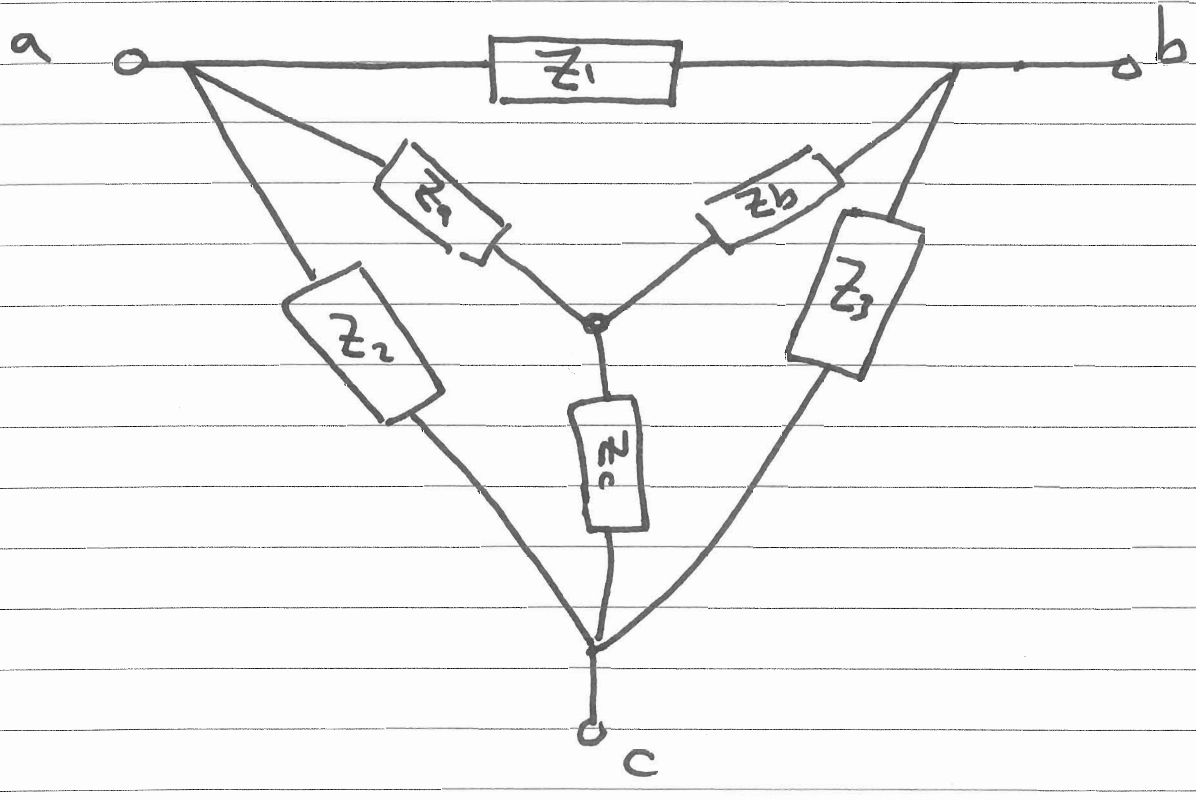
$$Z_b = \frac{Z_1 Z_3}{Z_1 + Z_2 + Z_3}$$

$$Z_c = \frac{Z_2 Z_3}{Z_1 + Z_2 + Z_3}$$

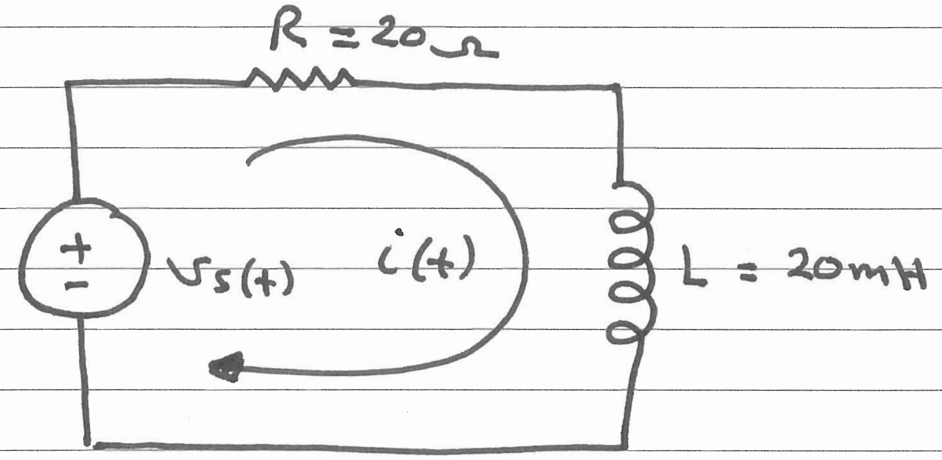
$$Z_1 = \frac{Z_a Z_b + Z_b Z_c + Z_c Z_a}{Z_c}$$

$$Z_2 = \frac{Z_a Z_b + Z_b Z_c + Z_c Z_a}{Z_b}$$

$$Z_3 = \frac{Z_a Z_b + Z_b Z_c + Z_c Z_a}{Z_a}$$



Series RL circuit

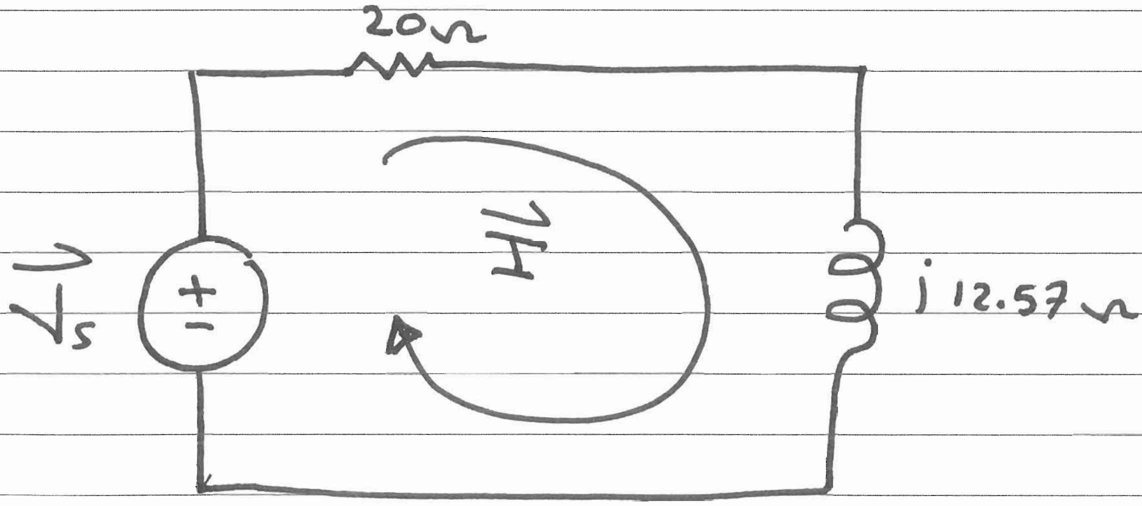


$$v_s(t) = 60 \cos(200\pi t) \text{ V}$$

Find $i(t)$

$$Z_R(j\omega) = 20 \ \Omega$$
$$Z_L(j\omega) = j12.57 \ \Omega$$

$$\vec{V}_s = 60\angle 0^\circ \text{ V}$$



KVL : $\vec{V}_s = \vec{V}_R + \vec{V}_L$

$$60\angle 0^\circ = 20\vec{I} + j12.57\vec{I}$$

$$\vec{I} = \frac{60\angle 0^\circ}{20 + j12.57} = \frac{60\angle 0^\circ}{23.6\angle 32.1^\circ}$$

$$\therefore \vec{I} = 2.54\angle -32.1^\circ \text{ A}$$

$$\vec{V_R} = 20\vec{I} = 50.8\angle -32.1^\circ \text{ V}$$

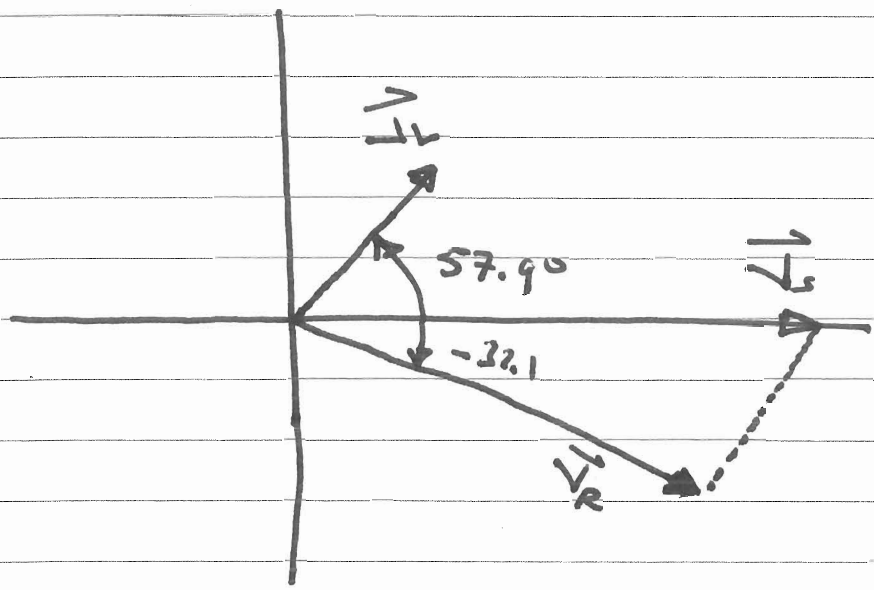
$$\vec{V_L} = j12.57\vec{I} = 31.9\angle +57.9^\circ \text{ V}$$

$\vec{V_L}$ Leads $\vec{V_R}$ by 90°

$\vec{I_L}$ Lags $\vec{V_s}$ by 32.1°

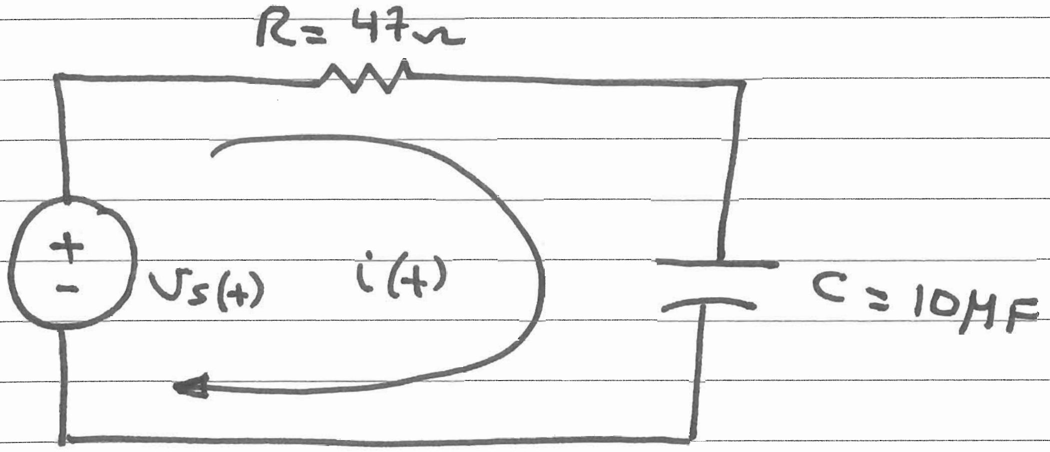
$$Z_{eq} = 20 + j12.57 \ \Omega \text{ inductive}$$

$$= 23.6\angle 32.1^\circ \ \Omega \text{ inductive}$$



phasor diagram

Series RC Circuit

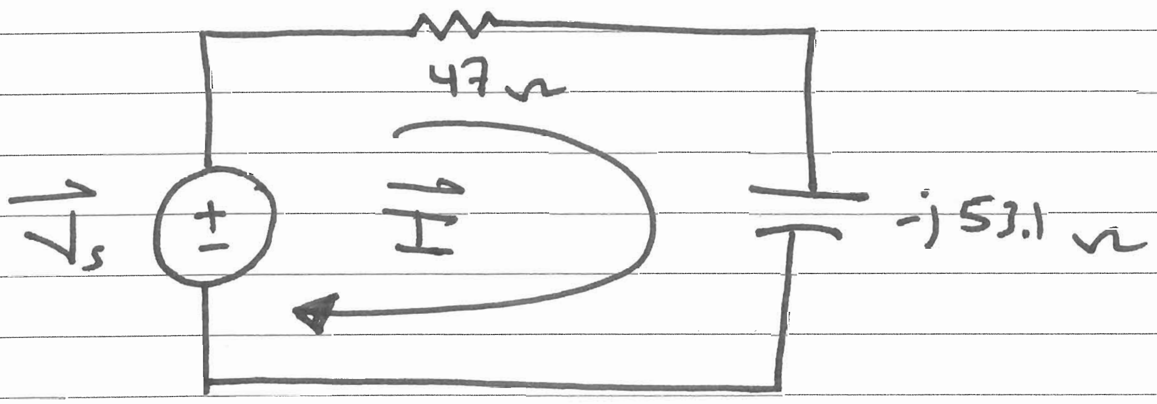


$$v_s(t) = 100 \cos 600\pi t \ \text{V}$$

$$Z_R(j\omega) = 47\ \Omega$$

$$Z_C(j\omega) = -j\frac{1}{\omega C} = -j\,53.1\ \Omega$$

$$\vec{V}_s = 100\angle 0^\circ \ \text{V}$$



KVL:

$$\vec{V}_s = 47\vec{I} - j\,53.1\,\vec{I}$$

$$\vec{I} = \frac{\vec{V}_s}{47 - j\,53.1} = \frac{100\angle 0^\circ}{47 - j\,53.1}$$

$$\vec{I} = \frac{100\angle 0^\circ}{70.9\angle -48.5^\circ}$$

$$\vec{I} = 1.41\angle 48.5^\circ \ A$$

$\vec{I}$ Leads $\vec{V_s}$ by 48.5°

→ Capacitive Circuit

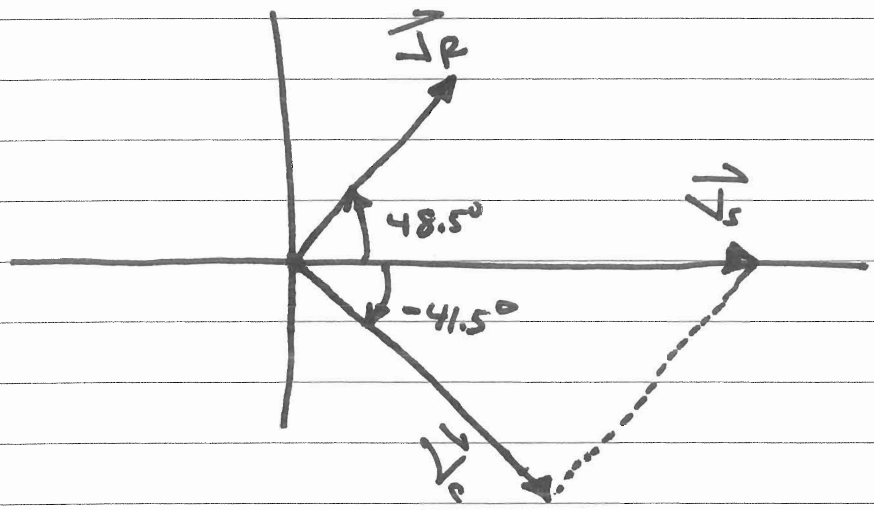
$Z(j\omega) = 47 - j\,53.1^\circ$ Capacitive

$Z(j\omega) = 70.9\angle -48.5^\circ$ Capacitive

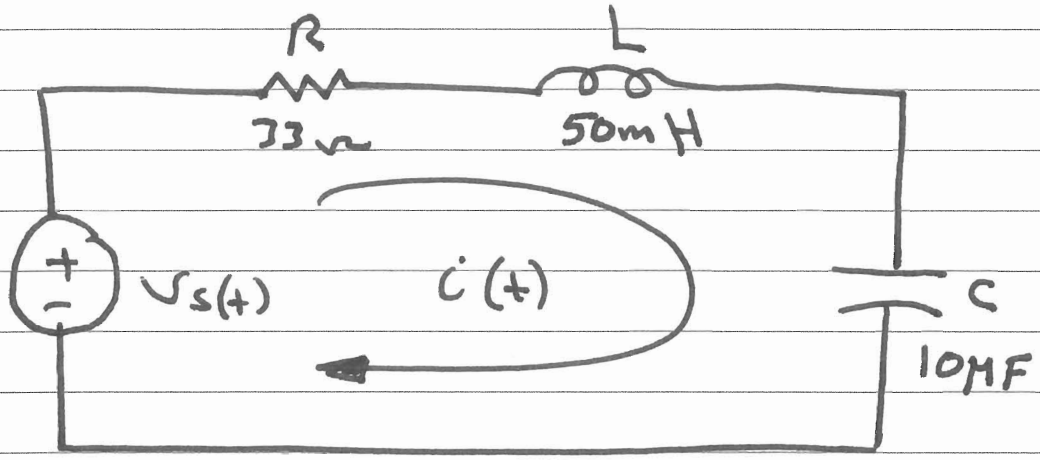
$$\vec{V_R} = 47\vec{I} = 66.3\angle 48.5^\circ \ \underline{V}$$

$$\vec{V_c} = -j\,53.1\,\vec{I} = 74.9\angle -41.5^\circ \ \underline{V}$$

$\vec{V_c}$ Lags $\vec{I}$ by 90°



Series RLC

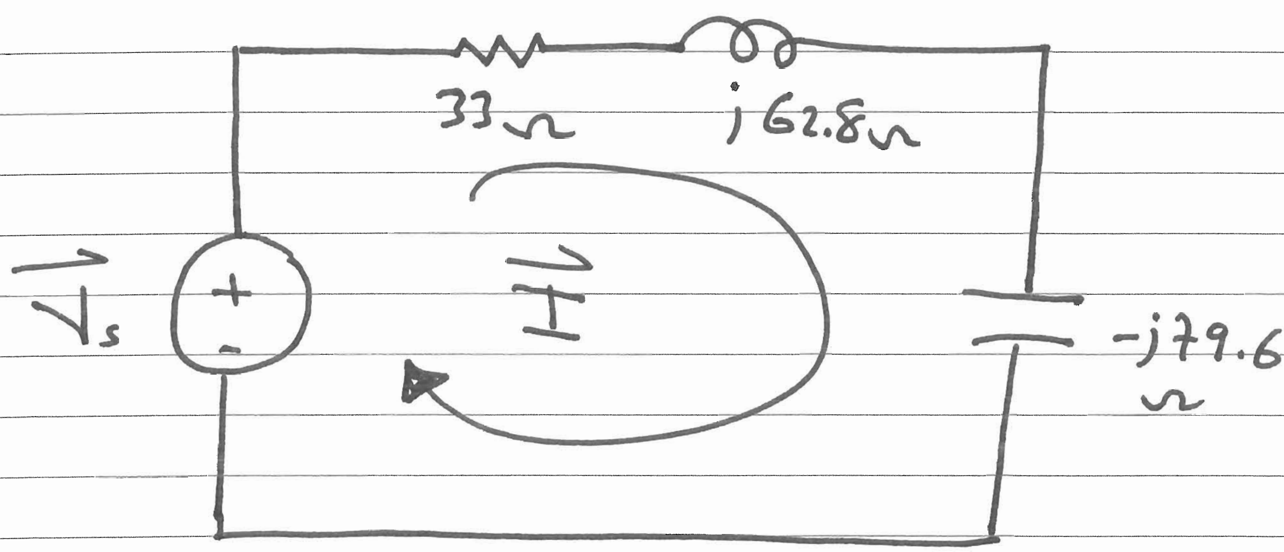


$$v_s(t) = 75 \cos 400\pi t \ \underline{V}$$

$$Z_R(j\omega) = 33\ \Omega$$

$$Z_L(j\omega) = j\omega L = j\,62.8\ \Omega$$

$$Z_C(j\omega) = -j\frac{1}{\omega C} = -j\,79.6\ \Omega$$



KVL: $\vec{V}_s = \vec{V}_R + \vec{V}_L + \vec{V}_C$

$$\vec{V_s} = 33\vec{I} + j62.8\vec{I} - j79.6\vec{I}$$

$$\vec{I} = \frac{\vec{V_s}}{33 - j16.8} = \frac{75\angle 0^\circ}{37\angle -27^\circ}$$

$$\vec{I} = 2.03\angle 27^\circ \quad A$$

$\vec{I}$ Leads $\vec{V_s}$ by 27°

$\therefore$ Capacitive Circuit

$$Z_{eq} = R + j\omega L - j\frac{1}{\omega C}$$

$$Z_{eq} = 33 + j62.8 - 79.6$$

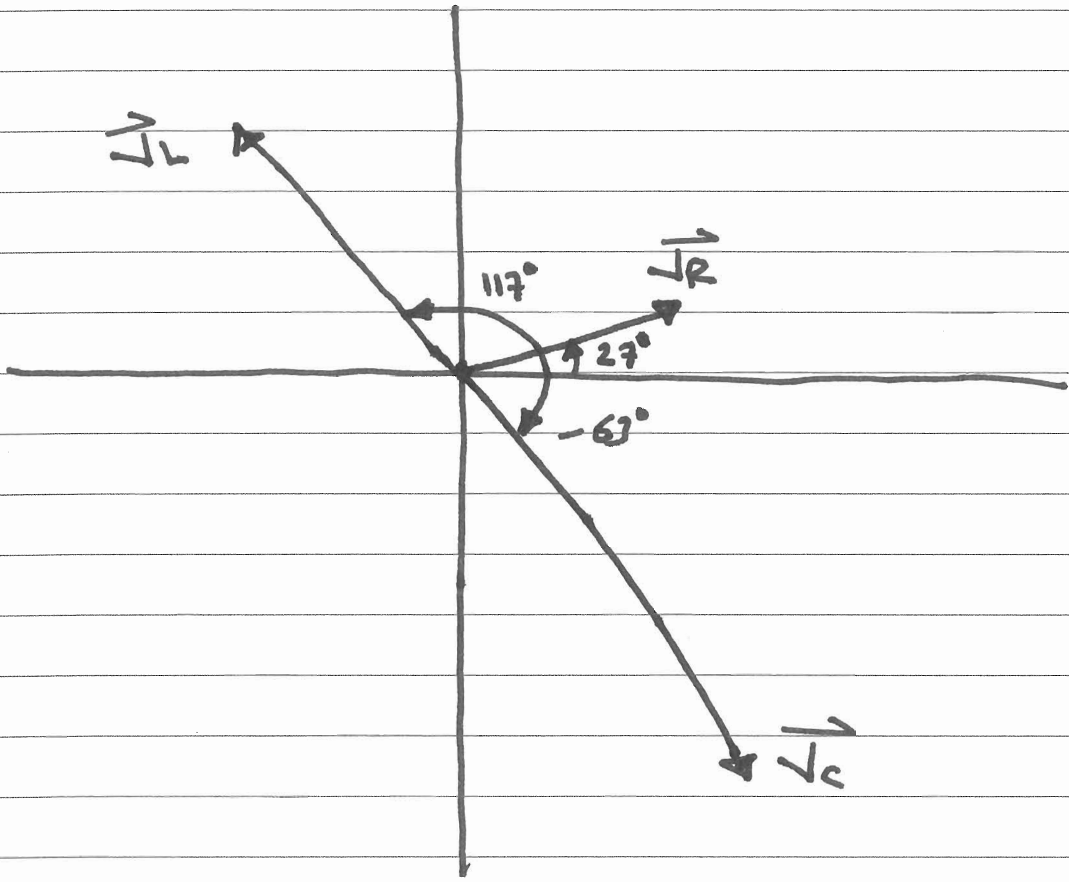
$$Z_{eq} = 33 - j16.8 \quad \Omega$$ Capacitive

$$Z_{eq} = 37\angle -27^\circ \quad \Omega$$ Capacitive

$$\vec{V_R} = R\vec{I} = 67\angle 27^\circ \quad V$$

$$\vec{V_L} = j\omega L\vec{I} = 127\angle 117^\circ \quad V$$

$$\vec{V_C} = -j\frac{1}{\omega C}\vec{I} = 162\angle -63^\circ \quad V$$

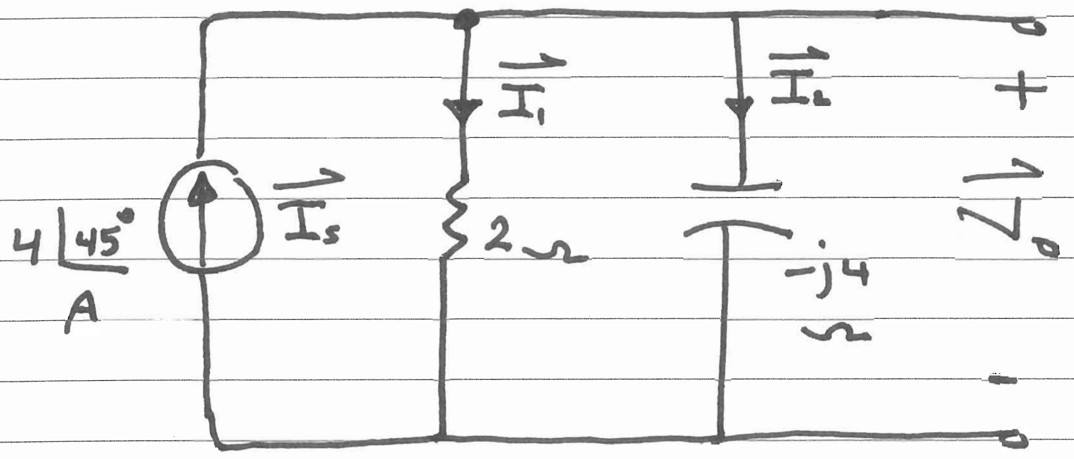


If $j\omega L - j\frac{1}{\omega C} = 0$

$$\omega = \frac{1}{\sqrt{LC}}$$

resonant frequency

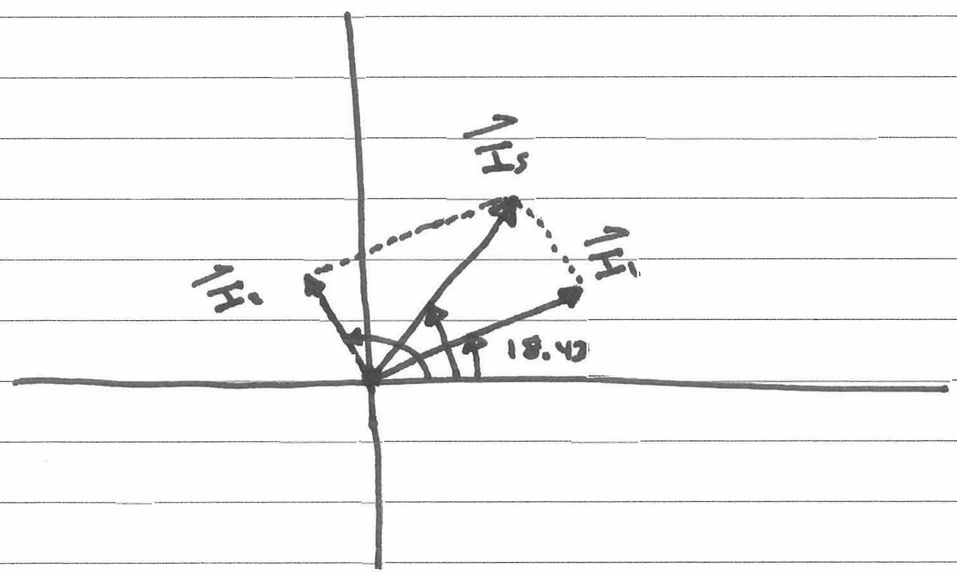
$Z_{eq} = R$ resistive



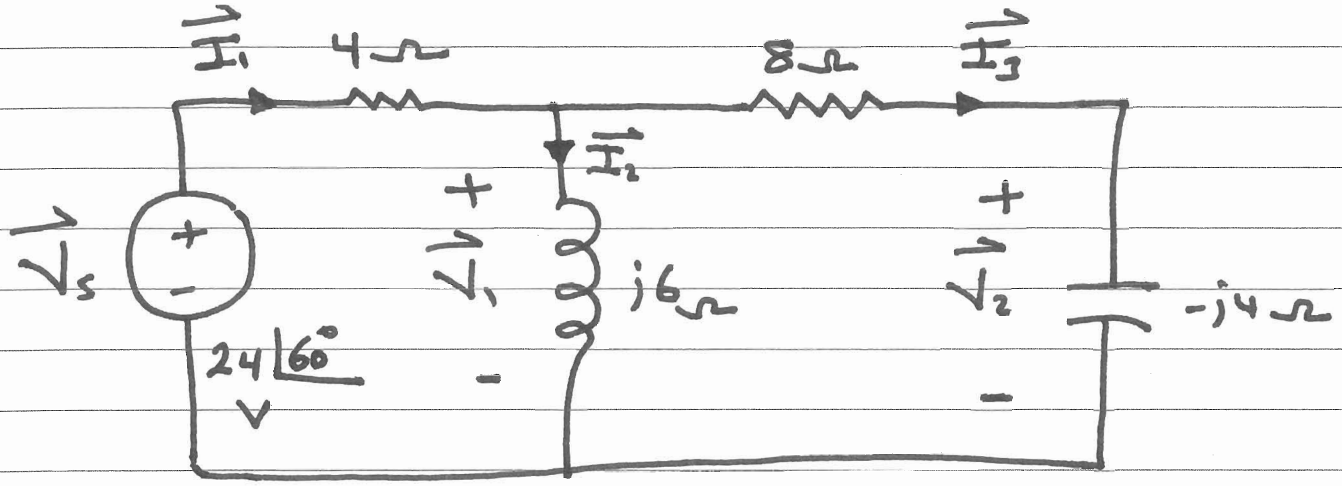
$$\vec{I}_1 = \frac{-j4}{-j4+2}\vec{I}_s = 3.578\angle 18.435^\circ \text{ A}$$

$$\vec{I}_2 = \frac{2}{2-j4}\vec{I}_s = 1.789\angle 108.435^\circ \text{ A}$$

$$\vec{V}_o = 2\vec{I}_1 = 7.156\angle 18.435^\circ \text{ V}$$



phasor diagram



Calculate all the voltages and currents

$$Z_{eq} = 4 + j6 \,\|\, (8 - j4)$$

$$Z_{eq} = 9.604 \angle 30.964^\circ \ \Omega$$

$$\vec{I_1} = \frac{\vec{V_s}}{Z_{eq}} = \frac{24\angle 60^\circ}{9.604\angle 30.964^\circ} = 2.498\angle 29.036^\circ \text{ A}$$

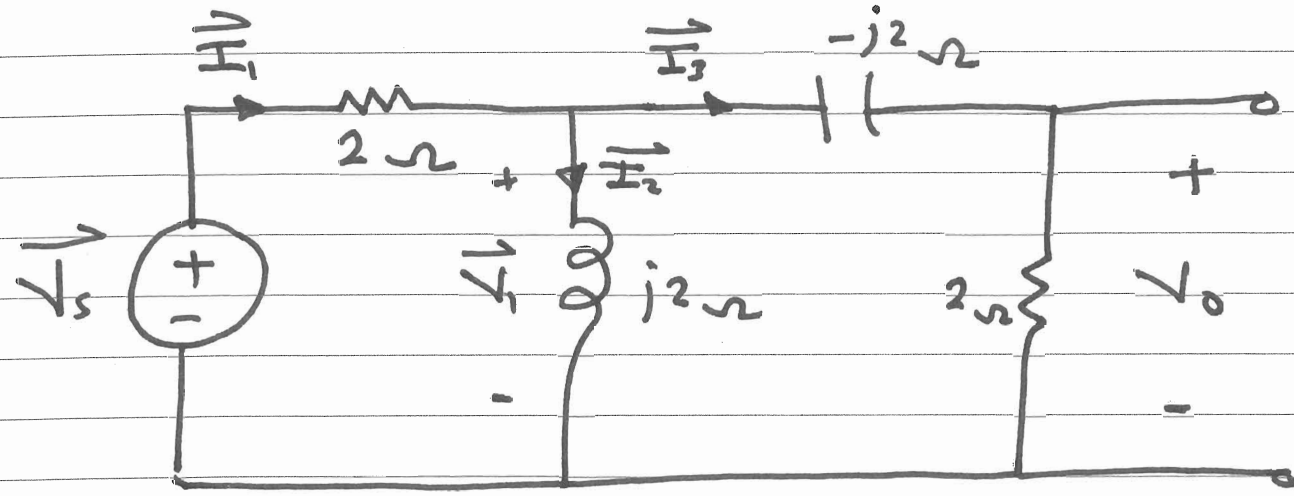
$$\vec{I_3} = \frac{j6}{j6+8-j4}\,\vec{I_1} = 1.82\angle 105^\circ \quad \text{A}$$

$$\vec{I_2} = \frac{8-j4}{8-j4+j6}\,\vec{I_1} = 2.71\angle -11.58 \quad \text{A}$$

$$\vec{V_1} = j6\,\vec{I_2} = 16.26\angle 78.42^\circ \ \text{V}$$

$$\vec{V_2} = -j4\,I_3 = 7.28\angle 15^\circ \quad \text{V}$$

If $\vec{V}_o = 8\angle 45^\circ$ V, find $\vec{V}_s$



$$I_3 = \frac{\vec{V}_o}{2} = 4\angle 45^\circ \text{ A}$$

$$\vec{V}_1 = (2 - j2)\vec{I}_3 = 11.314\angle 0^\circ$$

$$\vec{I}_2 = \frac{\vec{V}_1}{j2} = 5.657\angle -90^\circ \text{ A}$$

$$\vec{I}_1 = \vec{I}_2 + \vec{I}_3 = (2.828 - j2.829) \text{ A}$$

$$\vec{V}_s = 2\vec{I}_1 + \vec{V}_1$$

$$\vec{V}_s = 17.888\angle -18.439^\circ \quad \underline{\text{V}}$$