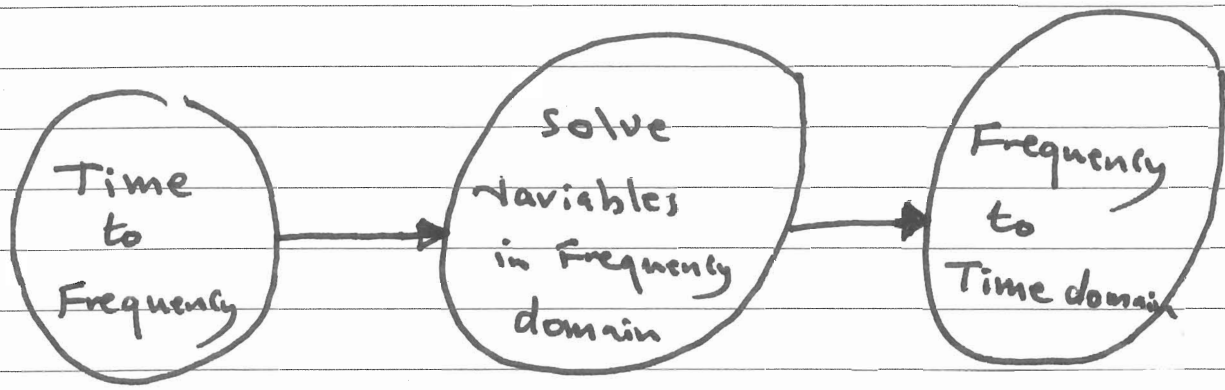
Steps to Analyze Ac Circuits

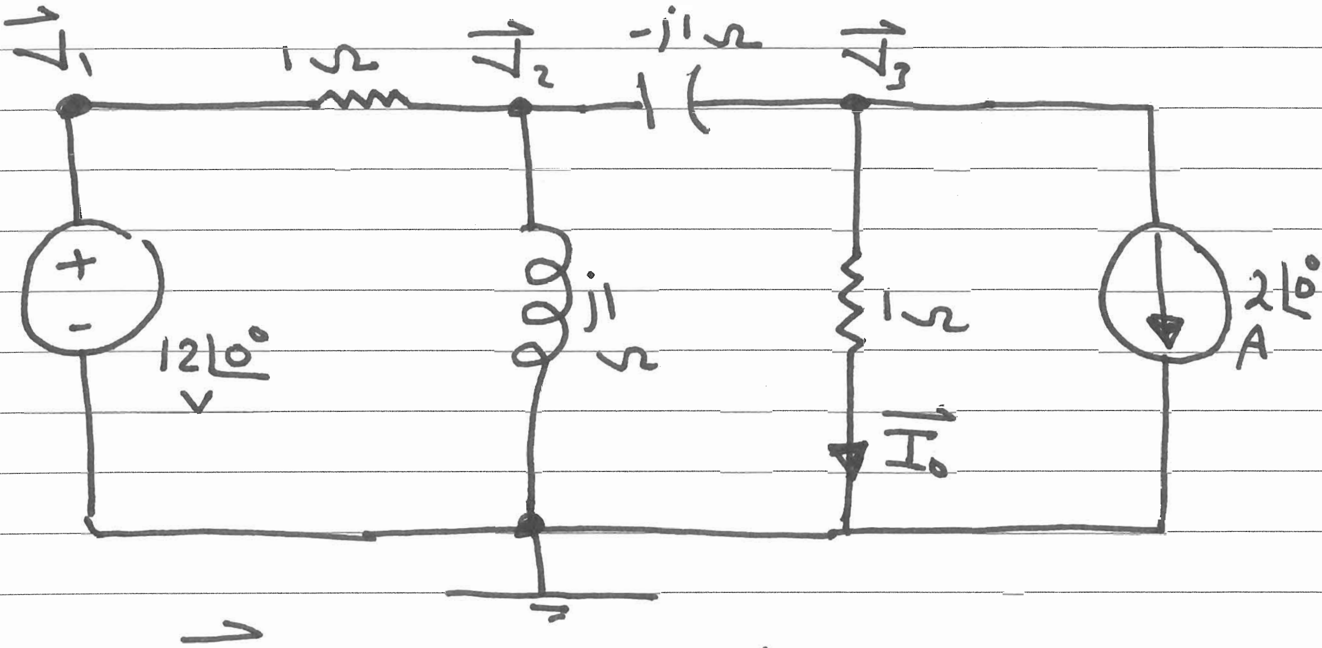
* Transform the Circuit to the phasor or frequency domain.

* Solve the problem using Circuit techniques (nodal analysis, mesh analysis, Superposition, etc.....)

* Transform the resulting phasor to the time domain.



Nodal Analysis



Find $\vec{I}_o$ using Nodal Analysis

$$\vec{I}_o = \frac{\vec{V}_3}{1}$$

$\vec{V}_1 = 12\angle 0^\circ$ Constrain equation

KCL at node 2 :

$$\frac{\vec{V}_2 - \vec{V}_1}{1} + \frac{\vec{V}_2}{j1} + \frac{\vec{V}_2 - \vec{V}_3}{-j1} = 0$$

$$-\vec{V}_1 + \vec{V}_2 - j\vec{V}_3 = 0$$

KCL at node 3:

$$-2\angle 0^\circ = -\frac{1}{-j1}\vec{V_2} + \left(\frac{1}{-j1}+1\right)\vec{V_3}$$

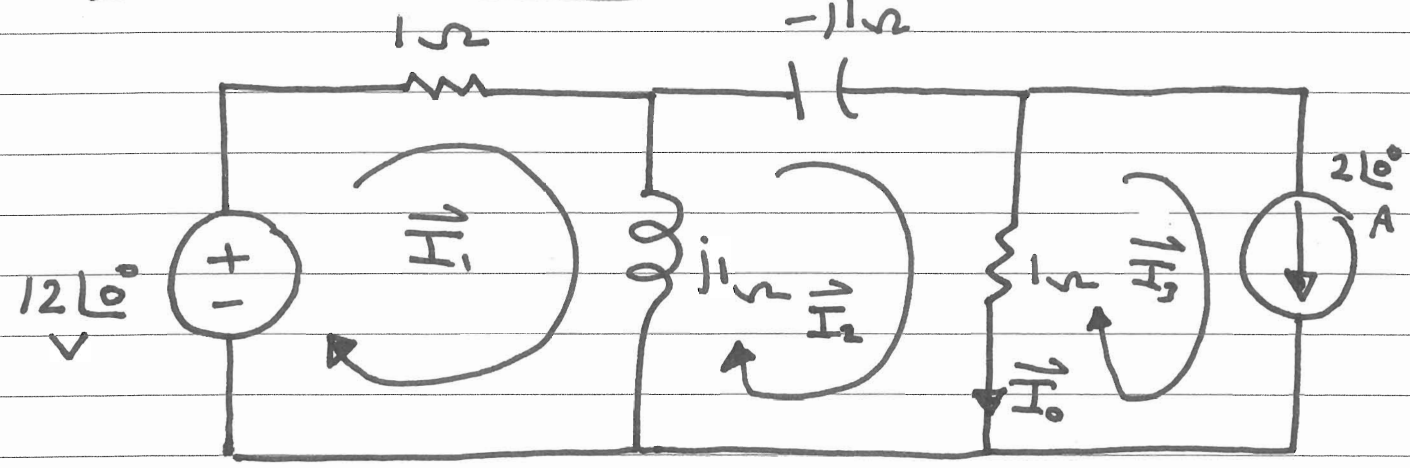
$$-2\angle 0^\circ = -j\vec{V_2} + (1+j)\vec{V_3}$$

Solving for $\vec{V_3}$;

$$\vec{V_3} = \left(\frac{8}{5}+j\frac{26}{5}\right) V$$

$$\therefore \vec{I_0} = \frac{\vec{V_3}}{1} = \left(\frac{8}{5}+j\frac{26}{5}\right) A$$

Mesh Analysis



Find $\vec{I_o}$ using Mesh Analysis

$$\vec{I_o} = \vec{I_2} - \vec{I_3}$$

KVL for mesh 1 :

$$12\angle 0^\circ = (1+j1)\vec{I_1} - j1\,\vec{I_2}$$

KVL for mesh 2 :

$$0 = -j1\,\vec{I_1} + (1+j1-j1)\vec{I_2} - \vec{I_3}$$

$$0 = -j1\,\vec{I_1} + \vec{I_2} - \vec{I_3}$$

$$\vec{I_3} = 2\angle 0^\circ \text{ A} \qquad \text{Constrain equation}$$

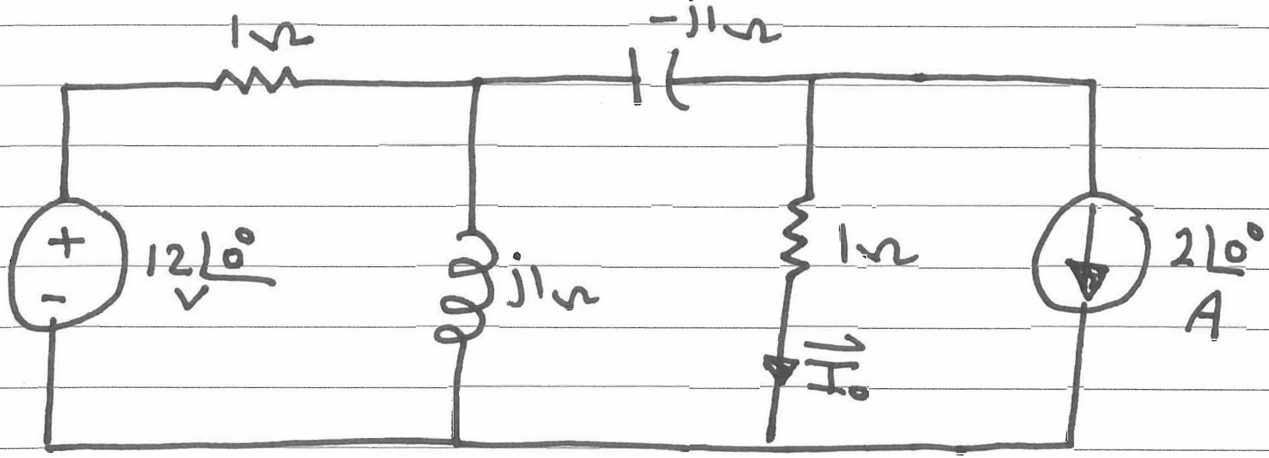
Solving for $\vec{I_2}$ and $\vec{I_3}$

$$\vec{I_2} = \left(\frac{18}{5} + j\,\frac{26}{5}\right) \text{A}$$

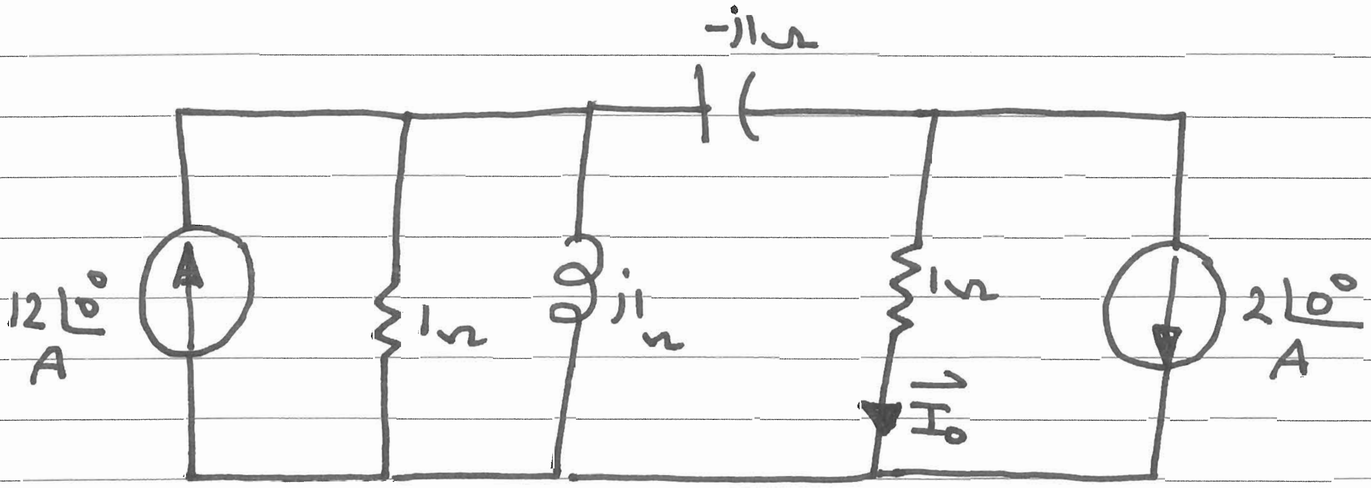
$$\overrightarrow{I_3} = 2\angle 0^\circ \ A$$

$$\therefore \ \overrightarrow{I_0} = \left(\frac{8}{5} + j\frac{26}{5} \right) A$$

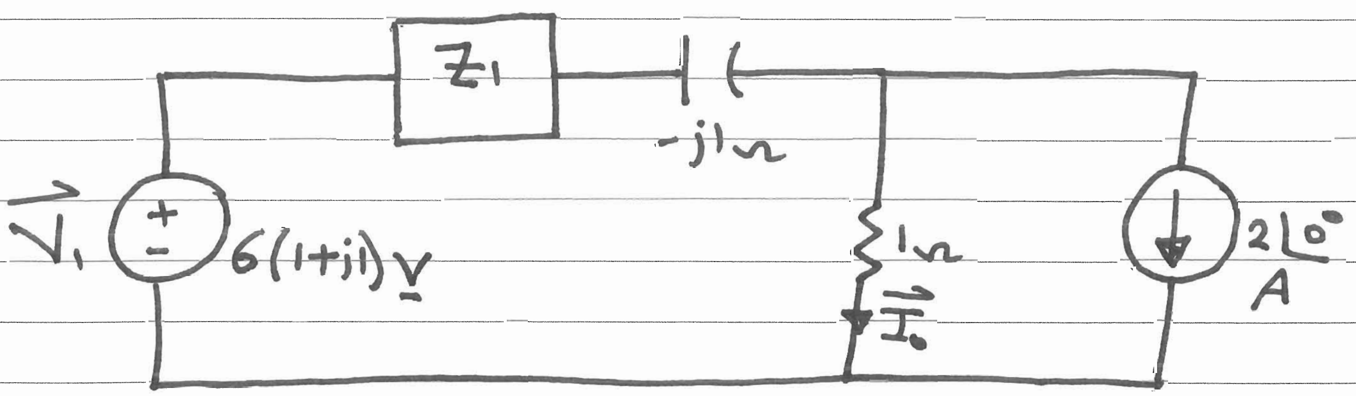
Source Transformation



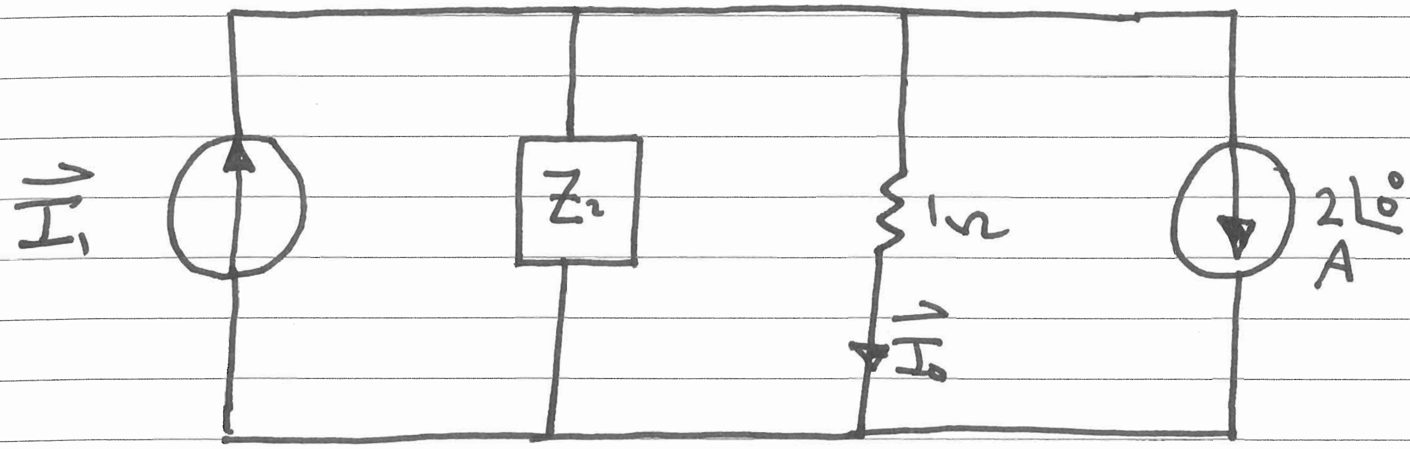
Find $\vec{I_o}$ using Source Transformation



$$Z_1 = 1\Omega \parallel j1\Omega = \left(\frac{1}{2} + j\,\frac{1}{2}\right)\Omega$$

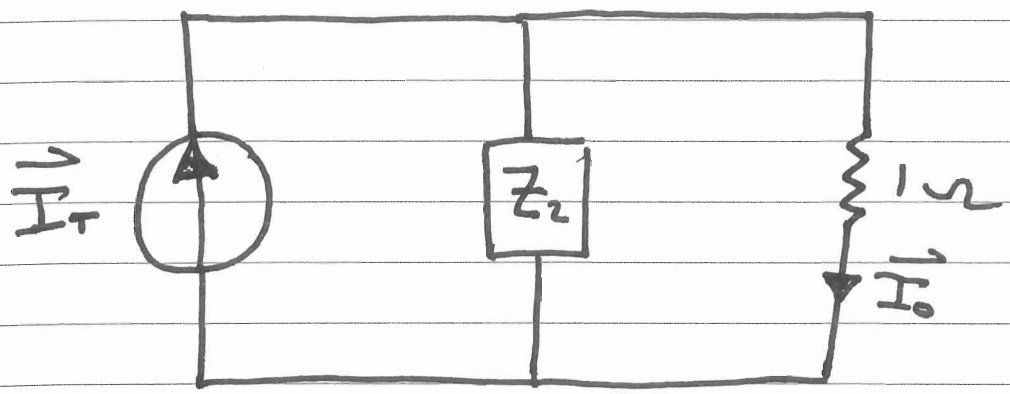


$$\vec{V_1} = 12\angle 0^\circ \cdot Z_1 = 6(1+j1)\ \underline{V}$$



$$\vec{I}_1 = \frac{\vec{V}_1}{Z_2} = \frac{12(1+j1)}{1-j1}$$

$$Z_2 = -j1 + Z_1 = \left(\frac{1}{2} - j\frac{1}{2}\right)\Omega$$

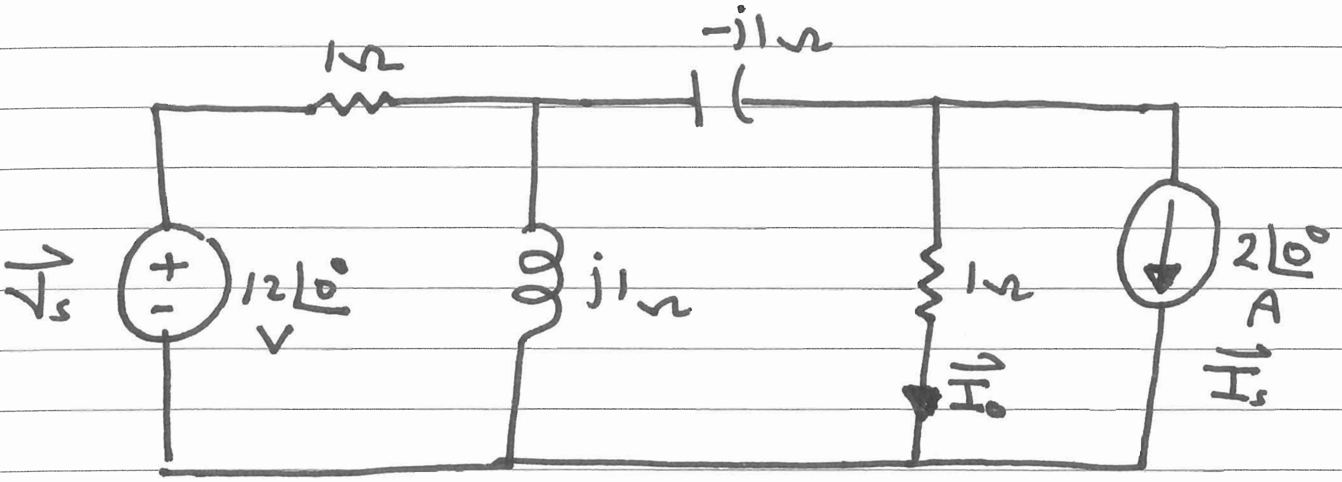


$$\vec{I}_T = \vec{I}_1 - 2\angle 0°$$

$$\vec{I}_T = \left(\frac{10+j14}{1-j1}\right) A$$

$$\vec{I}_o = \frac{Z_2}{Z_2+1}\,\vec{I}_T = \left(\frac{8}{5} + j\,\frac{26}{5}\right) A$$

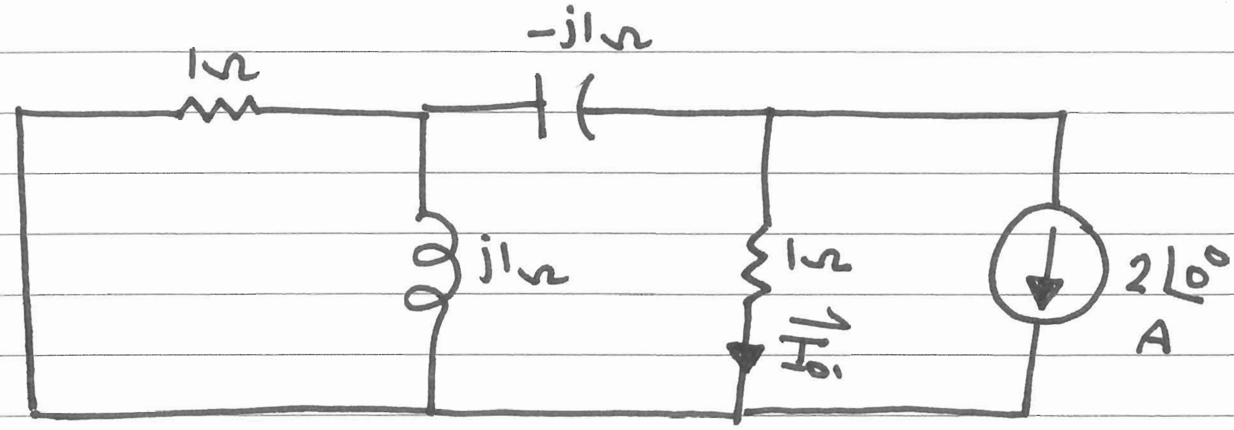
Superposition



Find $\vec{I_o}$ using Superposition

$$\vec{I_o} = \vec{I_{o1}} + \vec{I_{o2}}$$

1) let $\vec{V_s}$ OFF, and $\vec{I_s}$ on

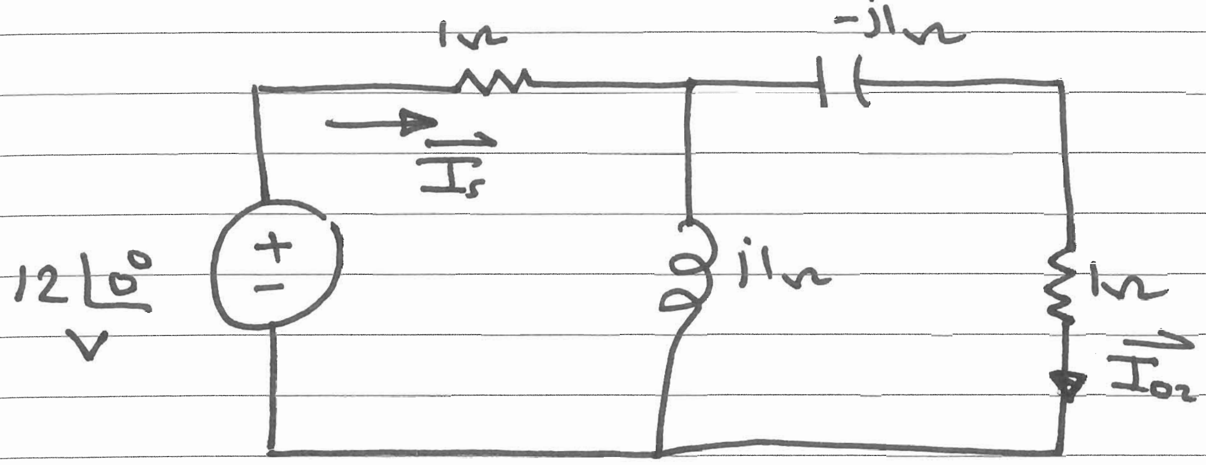


$$\vec{I_{o1}} = -2\angle 0° \; \frac{Z_1}{Z_1 + 1}$$

$$Z_1 = -j1 + 1 \,\|\, j1 = -j1 + \frac{j1}{1+j1}$$

$$\vec{I_{o1}} = \frac{-2}{2+j1} \text{ A}$$

2) let $\vec{I_s}$ off, and $\vec{V_s}$ on



$$\vec{I_s} = \frac{12\angle 0^\circ}{Z_{eq}}$$

$$Z_{eq} = 1 + j1 \,||\, (1 - j1) = (2 + j1)\,\Omega$$

$$\therefore \vec{I_s} = \frac{12\angle 0^\circ}{2 + j1} \text{ A}$$

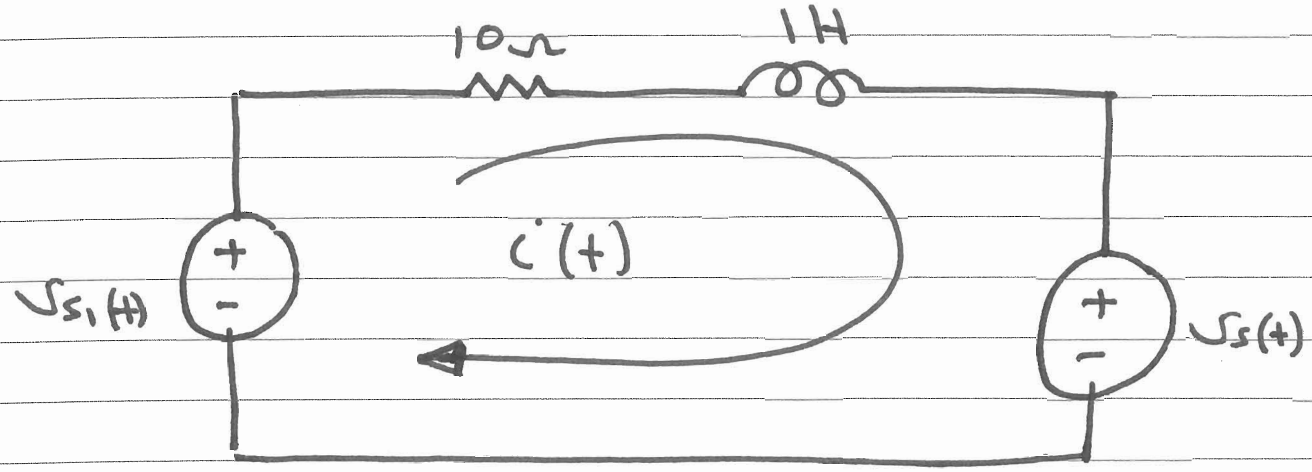
$$\vec{I_{o2}} = \vec{I_s}\,\frac{j1}{j1 + 1 - j1}$$

$$\vec{I_{o2}} = \vec{I_s} \cdot j1 = \frac{12}{1 - j2} \text{ A}$$

$$\therefore \vec{I_o} = \vec{I_{o1}} + \vec{I_{o2}}$$

$$\vec{I_o} = \left(\frac{8}{5} + j\,\frac{26}{5}\right) \text{A}$$

The Power of Superposition



$$v_{s_1}(t) = 100 \cos 10t \ \underline{V}$$

$$v_{s_2}(t) = 50 \cos (20t - 10^\circ) \ \underline{V}$$

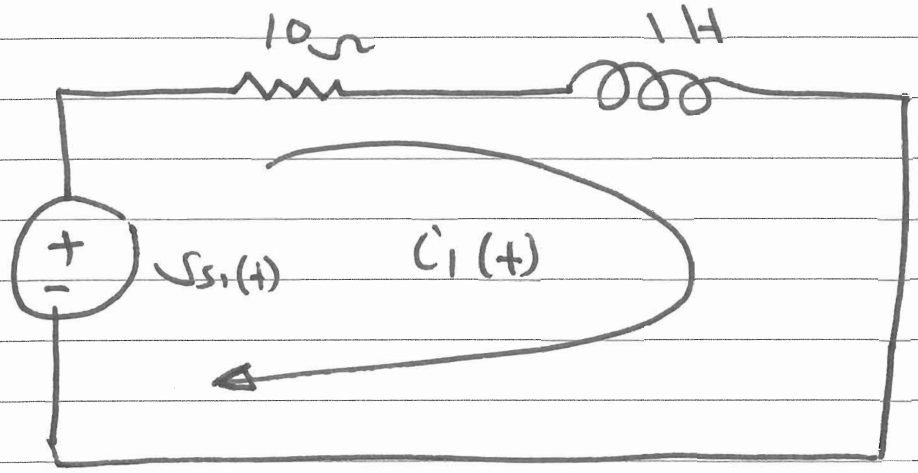
note that $\omega_1 = 10$ r/s and

$\omega_2 = 20$ r/s

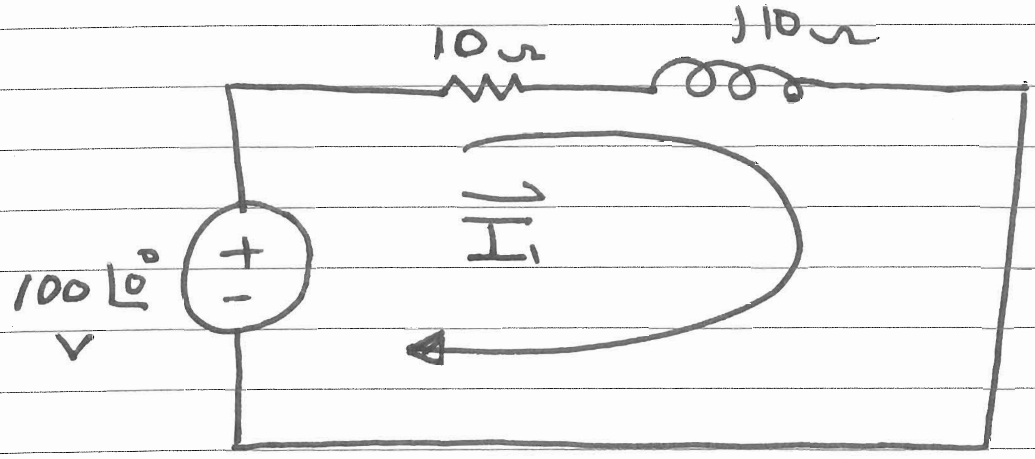
$\therefore$ Superposition is the Only method of analysis.

$$i(t) = i_1(t) + i_2(t)$$

1) Let $v_{s2}(t)$ OFF, and $v_{s1}(t)$ on



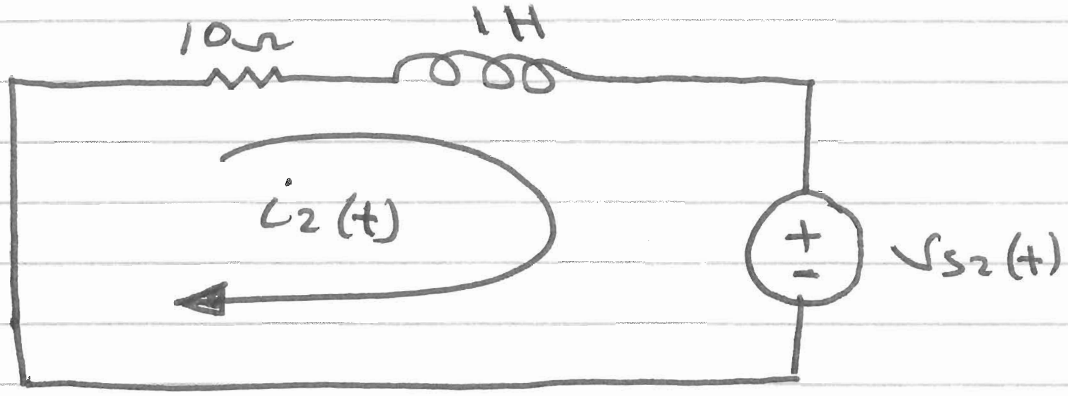
$v_{s1}(t) = 100 \cos 10t$ V



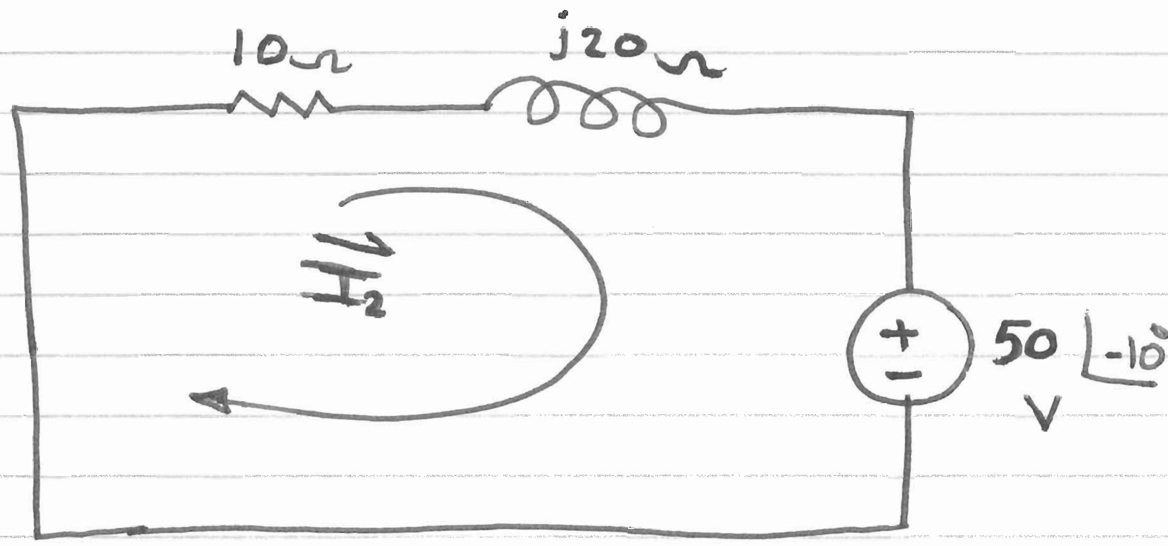
$$\vec{I}_1 = \frac{100\angle 0^\circ}{10 + j10} = 7.07\angle -45^\circ \text{ A}$$

$$\therefore i_1(t) = 7.07 \cos(10t - 45^\circ) \text{ A}$$

2) let $v_{s1}(t)$ OFF, and $v_{s2}(t)$ On



$$v_{s2}(t) = 50 \cos(20t - 10^\circ)\ \text{V}$$



$$\vec{I_2} = \frac{-50\angle -10^\circ}{10 + j20} = \frac{50\angle 170^\circ}{10 + j20}$$

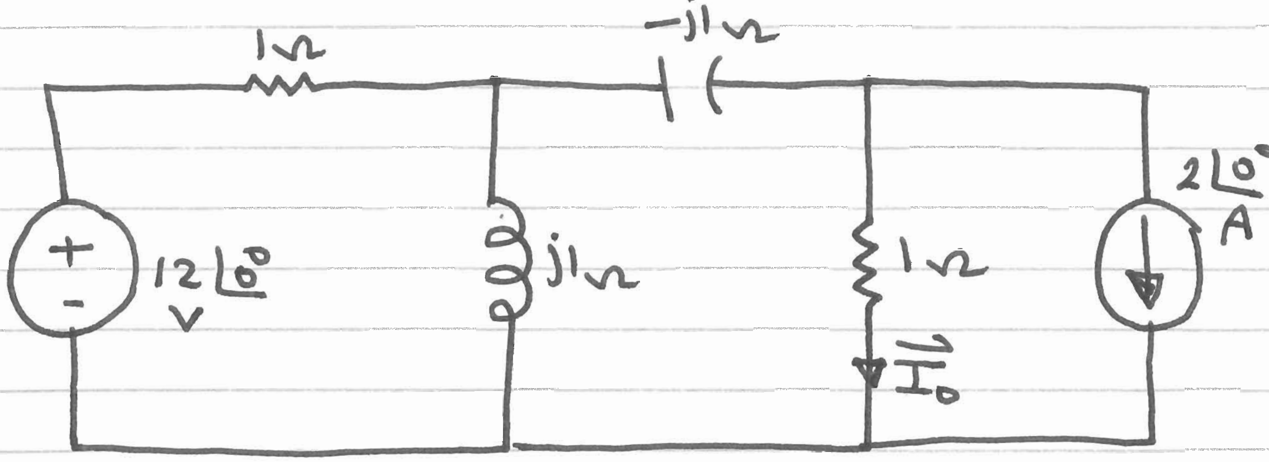
$$\vec{I_2} = 2.24\angle 106.57^\circ\ \text{A}$$

$$\therefore\ i_2(t) = 2.24 \cos(20t + 106.57^\circ)\ \text{A}$$

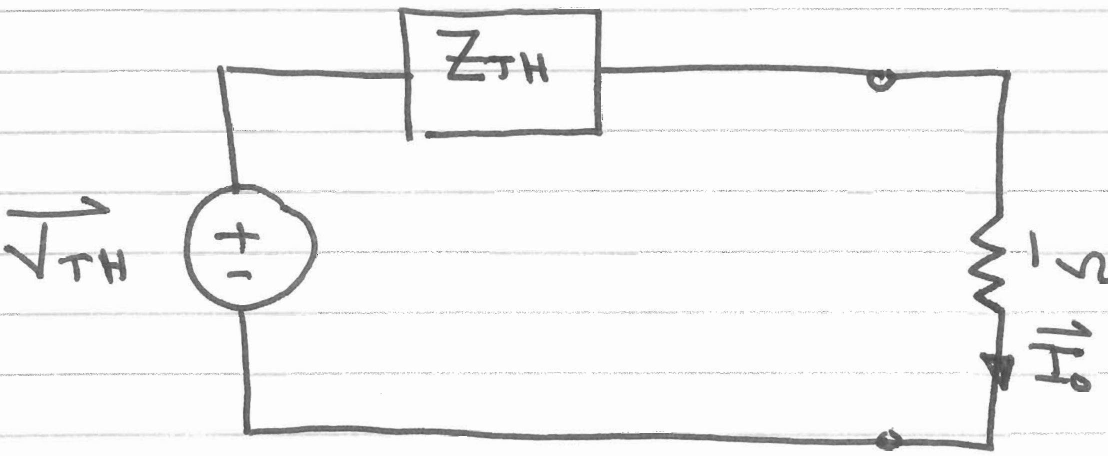
$$\therefore \; i(t) = i_1(t) + i_2(t)$$

$$i(t) = 7.07 \cos(10t - 45^\circ) \text{ A}$$
$$+ 2.24 \cos(20t + 106.57^\circ) \text{ A}$$

Thevenin's and Norton's Theorems

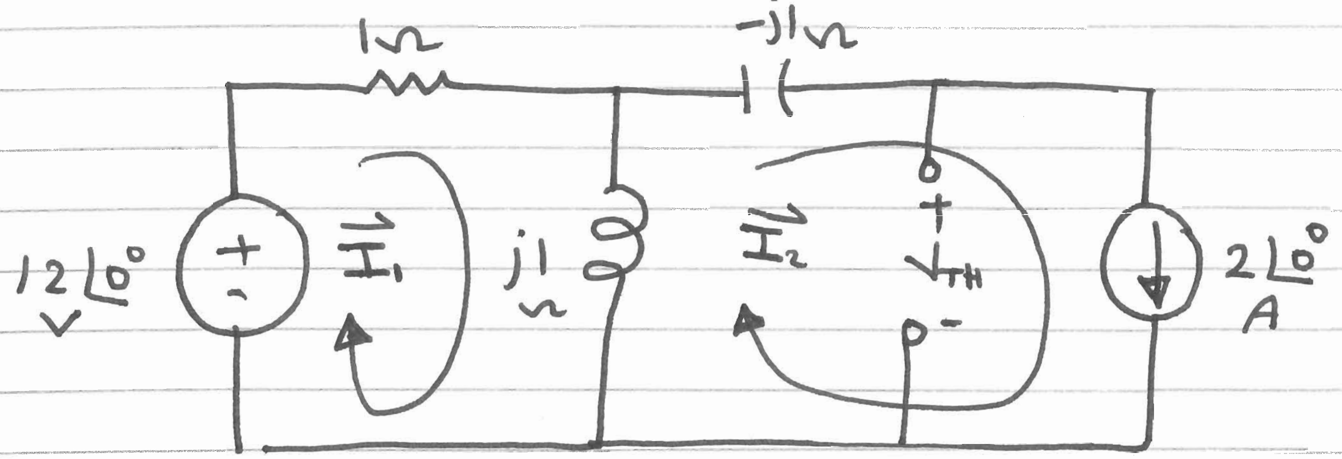


Find $\vec{I_o}$ using Thevenin's theorem



$$\vec{I_o} = \frac{\vec{V_{TH}}}{Z_{TH} + 1\Omega}$$

1) To find $\vec{V}_{TH}$



$$\vec{V}_{TH} = -(-j1\Omega)\,\vec{I}_2 + j1\Omega\,(\vec{I}_1 - \vec{I}_2)$$

$$\vec{I}_2 = 2\angle 0^\circ \quad \text{constrain equation}$$

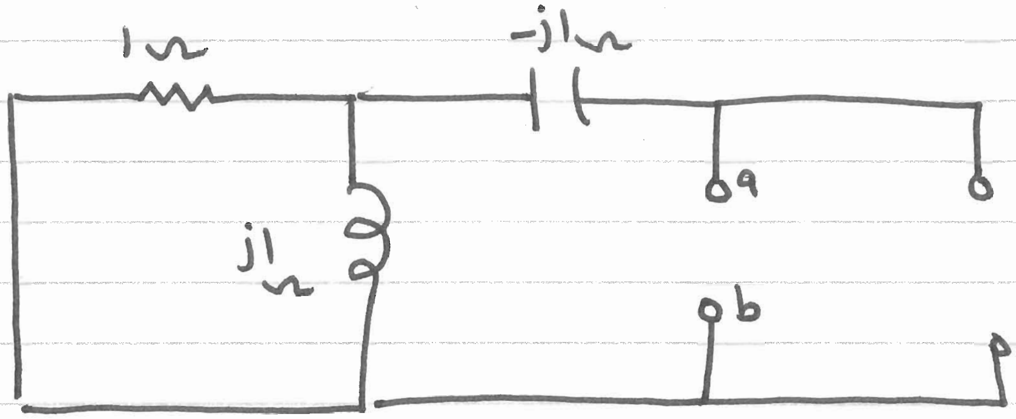
KVL for mesh 1 :

$$12\angle 0^\circ = (1+j1)\,\vec{I}_1 - j1\,\vec{I}_2$$

$$\therefore\ \vec{I}_1 = \left(\frac{12+j2}{1+j1}\right) A$$

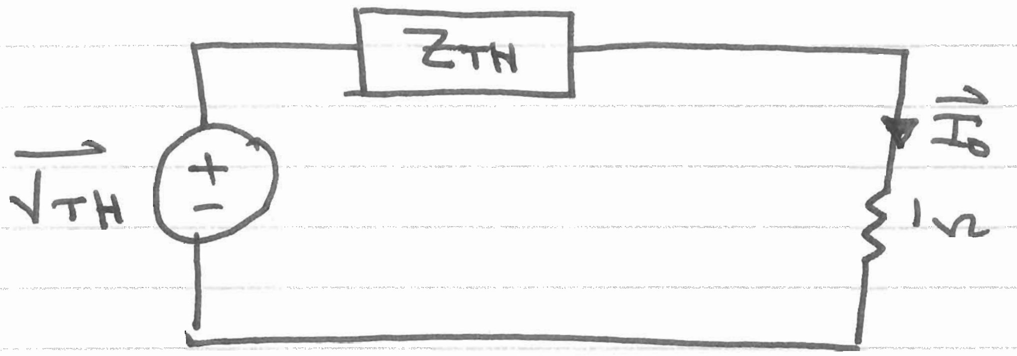
$$\therefore\ \vec{V}_{TH} = \left(\frac{-2+j12}{1+j1}\right) V$$

2) To find Z_{TH}, set all the independent sources to zero



$$Z_{TH} = -j1 + (1 \parallel j1)$$

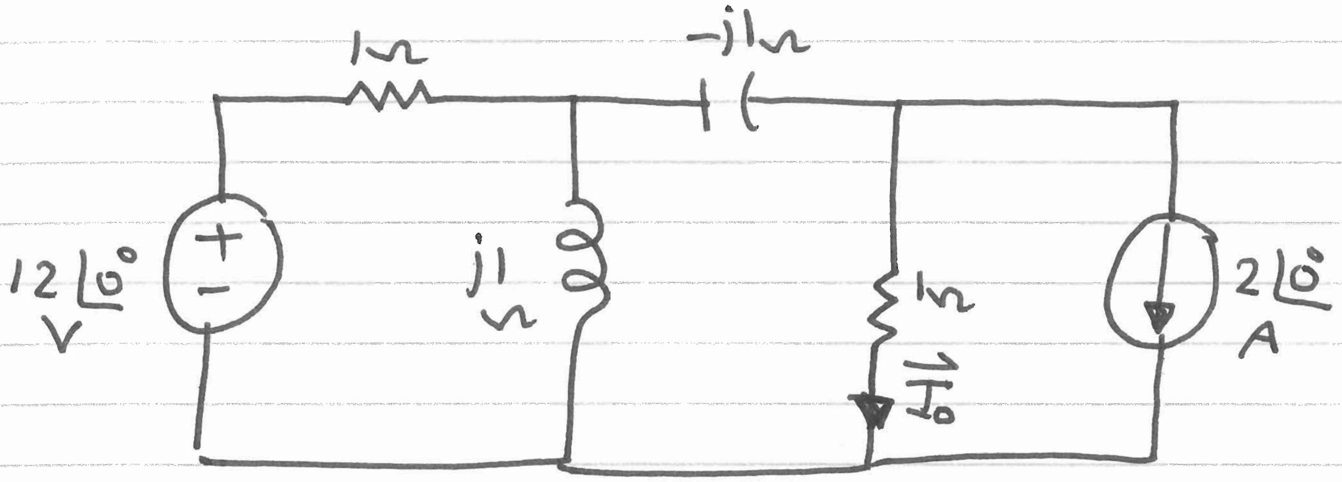
$$Z_{TH} = \left(\frac{1}{2} - j\frac{1}{2}\right)\,\Omega$$



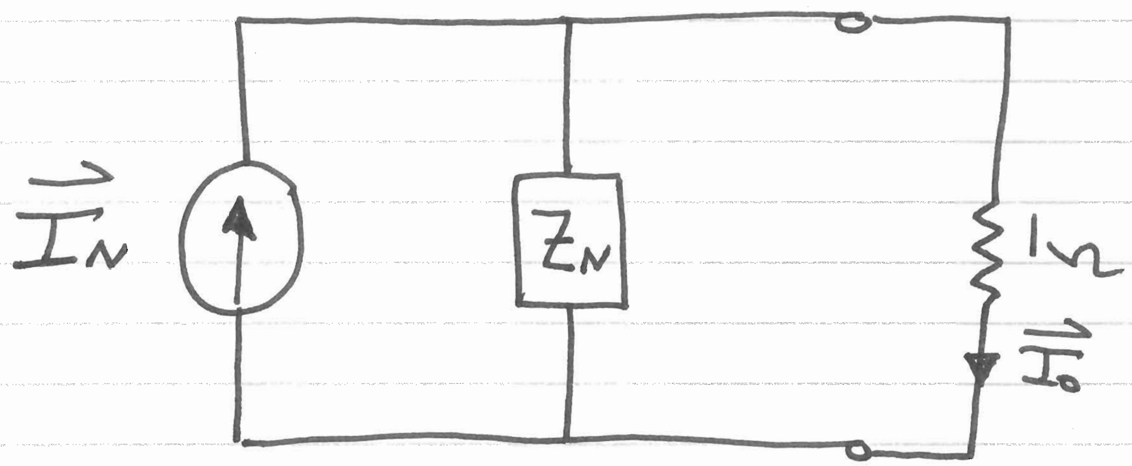
$$\vec{I_o} = \frac{\vec{V_{TH}}}{Z_{TH} + 1\Omega}$$

$$I_o = \left(\frac{8}{5} + j\,\frac{26}{5}\right) A$$

Norton's Theorem

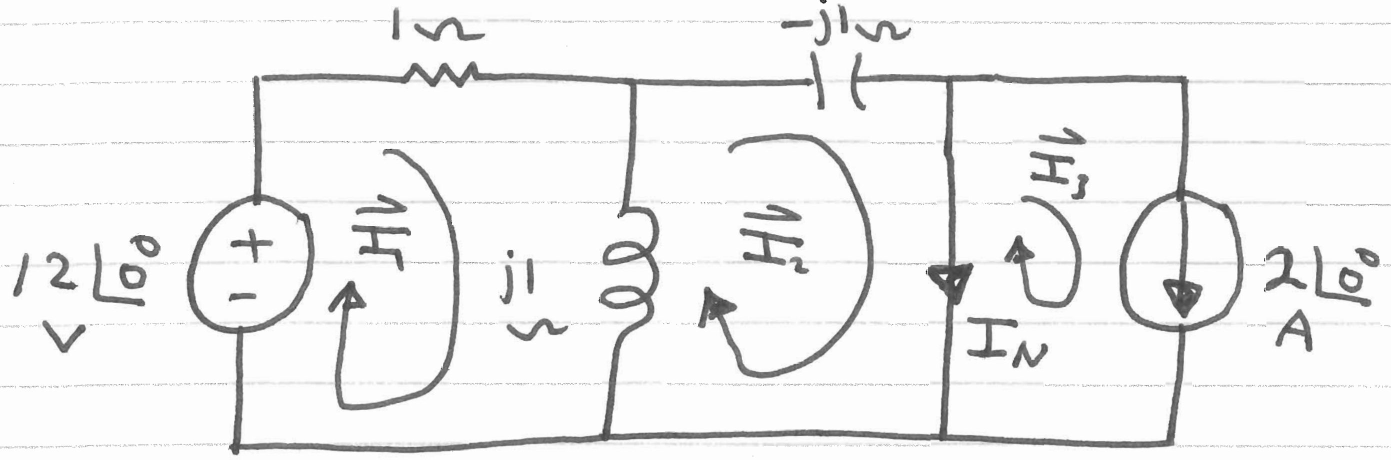


Find $\vec{I_o}$ using Norton's theorem



$$\vec{I_o} = \vec{I_N} \frac{Z_N}{Z_N + 1\Omega}$$

1) To find I_N



$$\vec{I}_N = \vec{I}_2 - \vec{I}_3$$

$$\vec{I}_3 = 2\angle 0^\circ \text{ A} \qquad \text{Constrain equation}$$

KVL for mesh 1 :

$$12\angle 0^\circ = (1+j1)\vec{I}_1 - j1\,\vec{I}_2$$

KVL for mesh 2 :

$$0 = -j1\,\vec{I}_1 + (j1-j1)\vec{I}_2$$

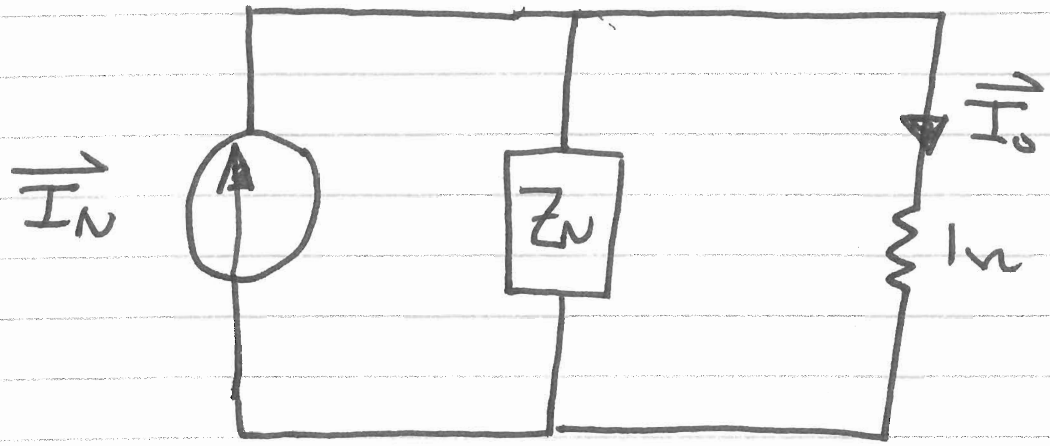
$$0 = -j1\,\vec{I}_1$$

$$\therefore \vec{I}_1 = 0$$

$$\therefore \vec{I}_2 = 12\angle 90^\circ \text{ A}$$

$$\therefore \vec{I}_N = \vec{I}_2 - \vec{I}_3 = -2 + j12$$

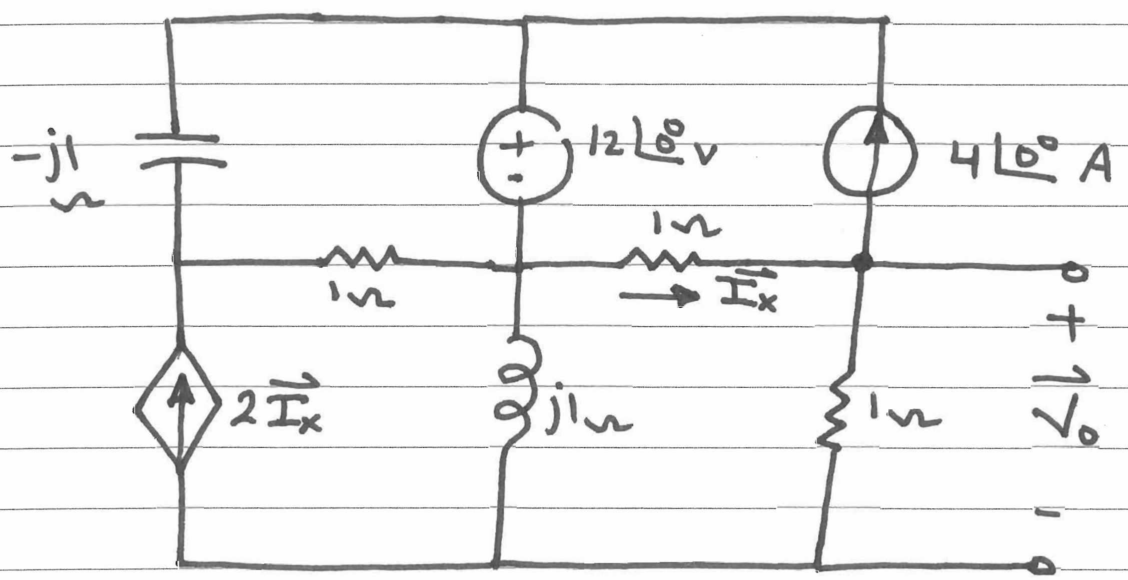
$$Z_N = Z_{TH} = \left(\frac{1}{2} - j\frac{1}{2}\right)\Omega$$



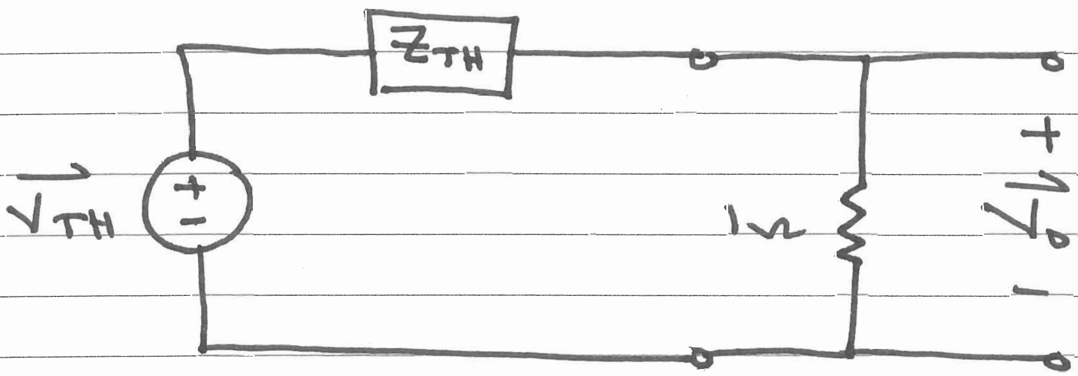
$$\vec{I}_o = \vec{I}_N \frac{Z_N}{Z_N + 1\Omega}$$

$$\vec{I}_o = \left(\frac{8}{5} + j\frac{26}{5}\right) A$$

Thevenin's Theorem

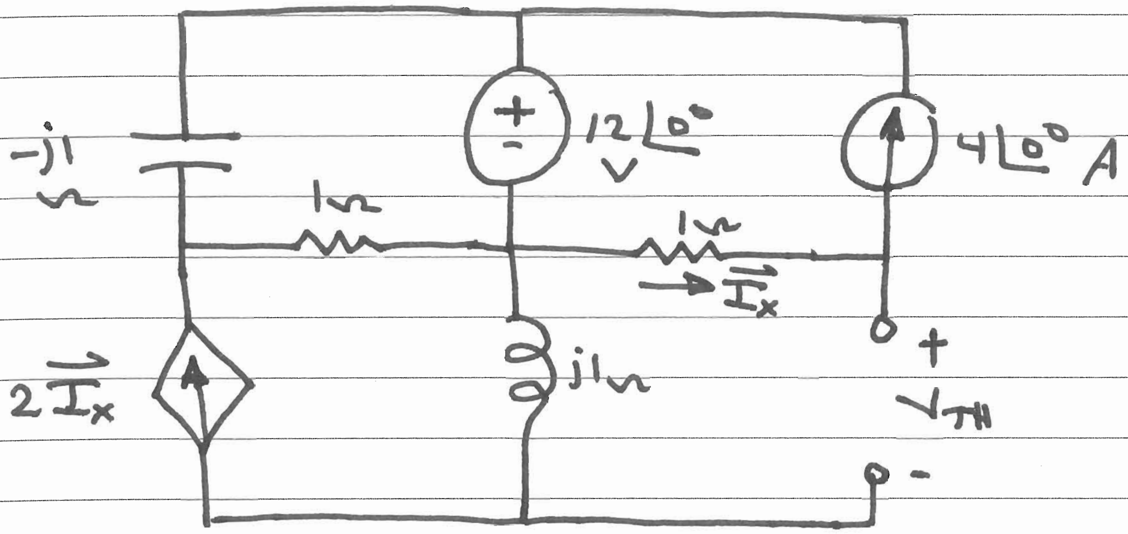


Find $\vec{V_o}$ using Thevenin's theorem



$$\vec{V_o} = \frac{1\Omega}{1\Omega + Z_{TH}} \vec{V_{TH}}$$

1) To find $\vec{V}_{TH}$



$$V_{TH} = -1\Omega\,\vec{I}_x + j1\Omega\,(2\vec{I}_x)$$

$$\vec{I}_x = 4\angle 0^\circ \text{ A}$$

$$\therefore\ \vec{V}_{TH} = (-4 + j8)\text{ V}$$

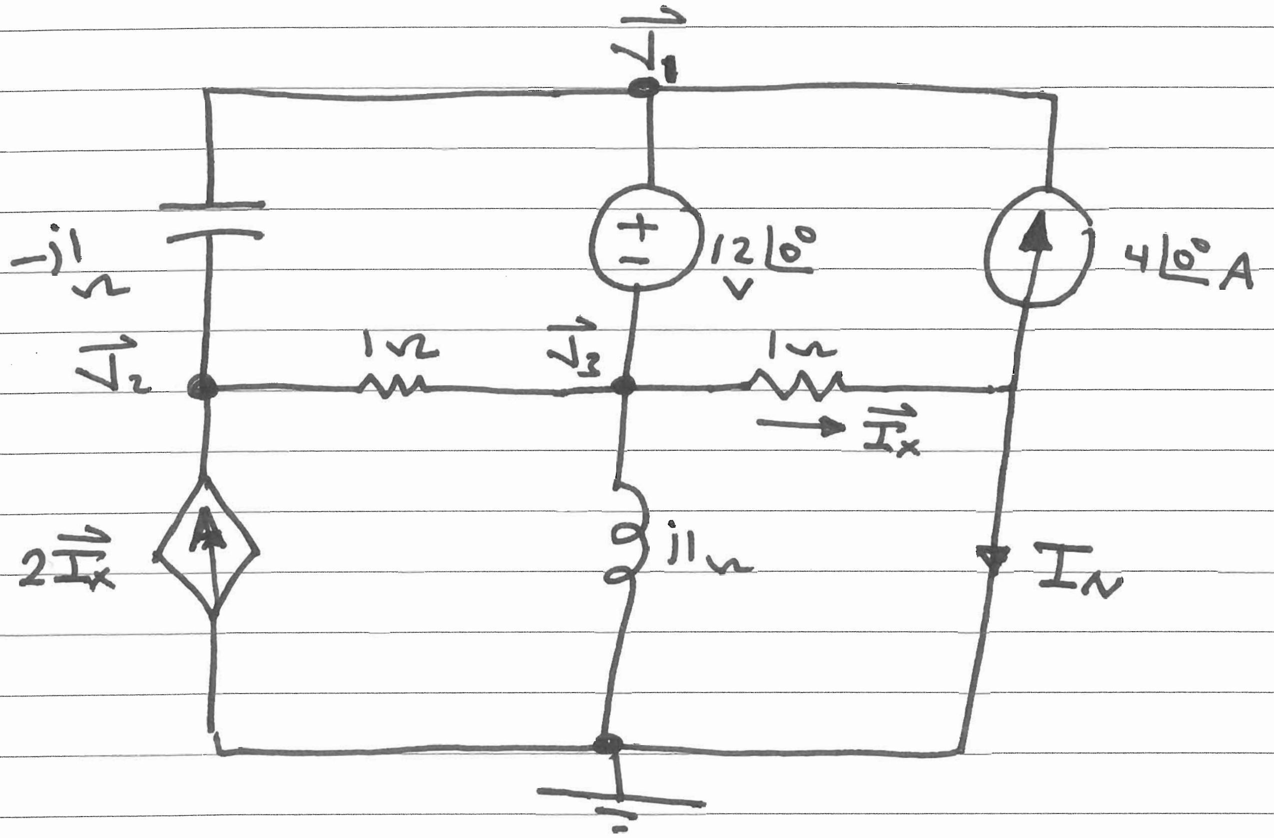
2) To find Z_{TH}

a) $Z_{TH} = \dfrac{\vec{V}_{TH}}{\vec{I}_N}$

b) $Z_{TH} = \dfrac{\vec{V}_T}{\vec{I}_T}\Bigg|$ all independent sources are set to zero

a) $\vec{Z}_{TH} = \dfrac{\vec{V}_{TH}}{\vec{I}_N}$

To find $\vec{I_N}$



$$\vec{I}_N = \vec{I}_x - 4\angle 0^\circ$$

$$\vec{I}_x = \frac{\vec{V}_3}{1\Omega} = \vec{V}_3$$

Nodal Analysis

$\vec{V}_2 - \vec{V}_3 = 12\angle 0^\circ$ Constrain equation

KCL at node 2 :

$$2\vec{I}_x = \left(1 + \frac{1}{-j1}\right)\vec{V}_2 + jV_1 - 1V_3$$

KCL for the Supernode (1,3)

$$4\angle 0^\circ = \left(\frac{1}{-j1}\right)V_1 + \left(1+1+\frac{1}{j1}\right)V_3 - \left(1+\frac{1}{-j1}\right)V_2$$

Solving for $\vec{V_3}$

$$\vec{V_3} = \frac{4j}{1-j1}$$

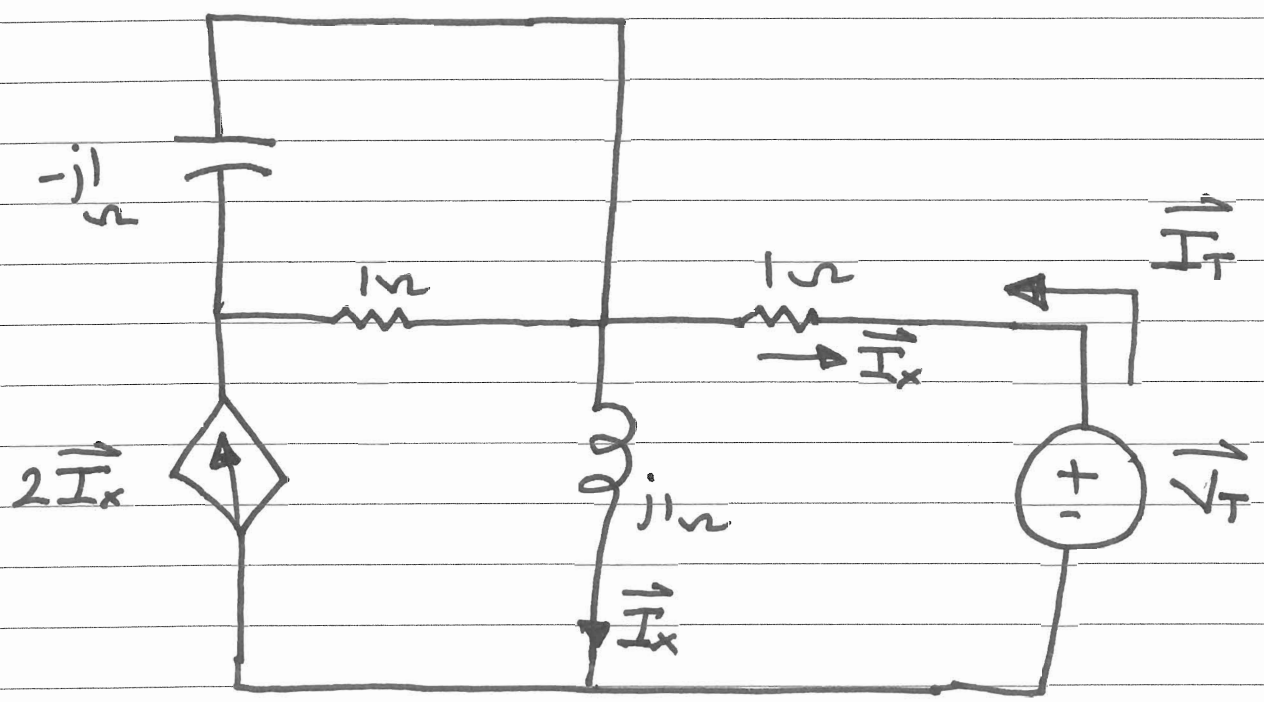
$$\therefore I_N = -\left(\frac{8+j4}{1+j1}\right)$$

$$\therefore Z_{TH} = \frac{\vec{V_{TH}}}{\vec{I_N}}$$

$$Z_{TH} = (1-j1)\ \Omega$$

$$\therefore \vec{V_o} = \frac{-4+j8}{1+1-j} = 4\angle 143.13^\circ\ V$$

b) $Z_{TH} = \dfrac{\vec{V_T}}{\vec{I_T}} \Bigg|$ independent source are zero



$$\vec{V_T} = -1(\vec{I_x}) + j1(\vec{I_x})$$

$$\vec{V_T} = (-1+j1)\vec{I_x}$$

$$\vec{I_x} = -\vec{I_T}$$

$$\vec{V_T} = (-1+j1)\vec{I_T}$$

$$\therefore Z_{TH} = \frac{\vec{V_T}}{\vec{I_T}} = (1-j1)\,\Omega$$