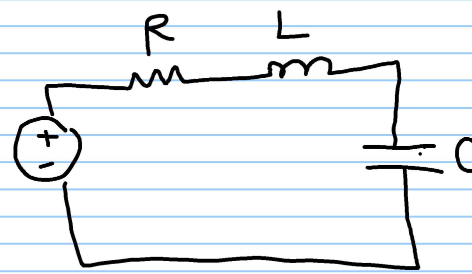
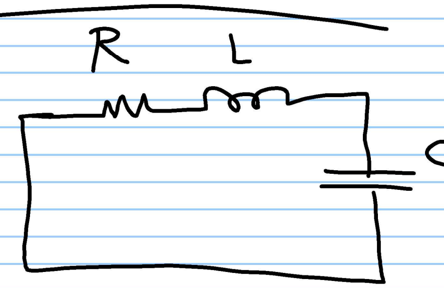
chapter 8

Natural & step Response of RLC circuits
2^{nd} - order

series

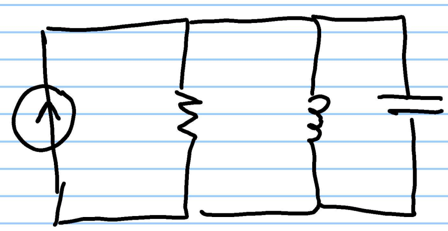


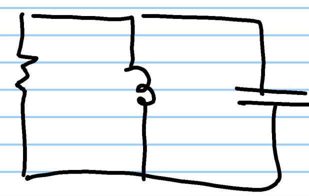
step



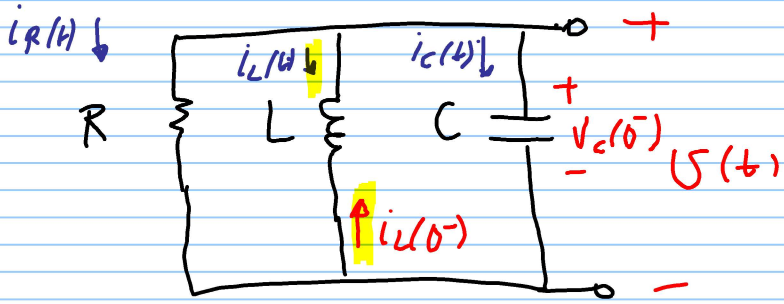
Naural

parallel





* Natural Response of parallel RLC circuits



$v_L = L\,\frac{di_L}{dt}$

$\frac{1}{L} v_L = \frac{d}{dt} i_L$

$\frac{1}{L}\int v + IC = i_L$

let $\underline{v_c(0^-) = 0}$; $\underline{i_L(0^-) = 10\text{ A}}$

KCL $i_R(t) + i_L(t) + i_C(t)$

$$\frac{d}{dt}\left(\underbrace{\frac{v(t)}{R}} + \underbrace{\frac{1}{L}\int_0^t v(t)\,dt - i_L(0^-)} + \underbrace{C\frac{d}{dt}v(t)} = 0\right)$$

$$\frac{1}{R}\frac{d}{dt}v(t) + \frac{1}{L}v(t) + C\frac{d^2}{dt^2}v(t) = 0$$

$$\boxed{\frac{d^2}{dt^2}v(t) + \frac{1}{RC}\frac{d}{dt}v(t) + \frac{1}{LC}v(t) = 0}$$

2^{nd}-order homogeneous differential equation

$\therefore$ $\underline{v(t) = Ae^{st}}$ for $t>0$ $\quad A_1e^{s_1t} + A_2e^{s_2t}$

$$As^2e^{st} + \frac{1}{RC}Ase^{st} + \frac{1}{LC}Ae^{st} = 0$$

$$Ae^{st}\left[s^2 + \frac{1}{RC}s + \frac{1}{LC}\right] = 0$$

$$\therefore\ s^2 + \frac{1}{RC}s + \frac{1}{LC} = 0$$ characteristic equation

$$s^2 + \underbrace{2\xi\omega_n}_{2\xi\omega_0}s + \underbrace{\omega_n^2}_{\omega_0^2}$$

$$s_{1,2} = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$\to s_1 = -\frac{1}{2RC} + \sqrt{\left(\frac{1}{2RC}\right)^2 - \frac{1}{LC}}$$

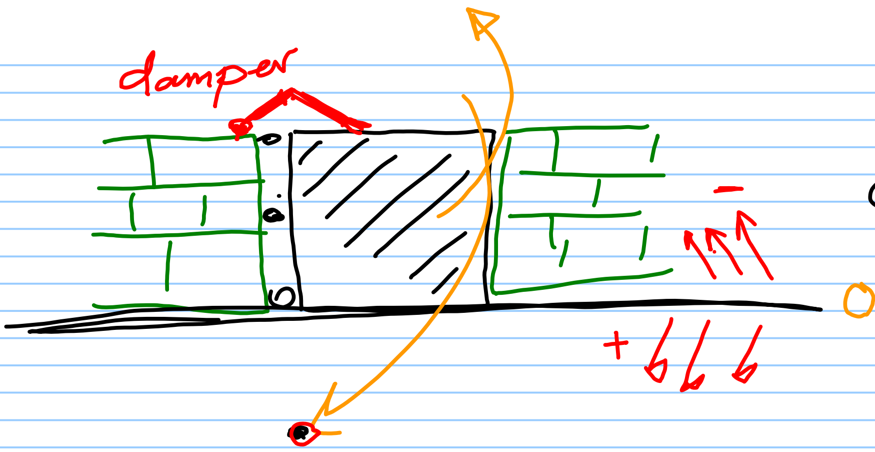
$$s_2 = -\frac{1}{2RC} - \sqrt{\left(\frac{1}{2RC}\right)^2 - \frac{1}{LC}}$$

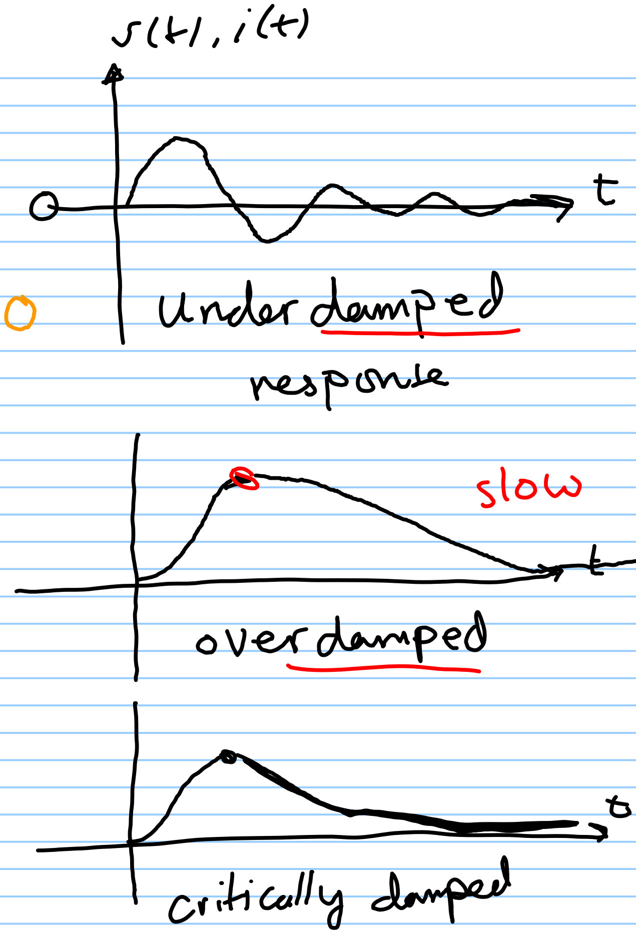
let $\omega_0 = \frac{1}{\sqrt{LC}}$ resonant frequency

& $\alpha = \frac{1}{2RC}$ damping coefficient

$$s_1 = -\alpha + \sqrt{\alpha^2 - \omega_0^2}$$

$$s_2 = -\alpha - \sqrt{\alpha^2 - \omega_0^2}$$





$$s_1 = -\alpha + \sqrt{\alpha^2 - \omega_0^2}$$

$$s_2 = -\alpha - \sqrt{\alpha^2 - \omega_0^2}$$

1) if $\alpha > \omega_0$, the solutions are real, unequal & the response is overdamped

2) if $\alpha < \omega_0$, the solutions are complex conjugates & the response is termed underdamped

3) if $\alpha = \omega_0$, the solutions are real and equal, & the response is termed critically damped.

$\boxed{1}$ overdamped case

$$\boxed{\alpha > \omega_0} \qquad \alpha = \frac{1}{2RC}$$

$$s_1 = -\alpha + \sqrt{\alpha^2 - \omega_0^2} \quad \text{-ve}$$

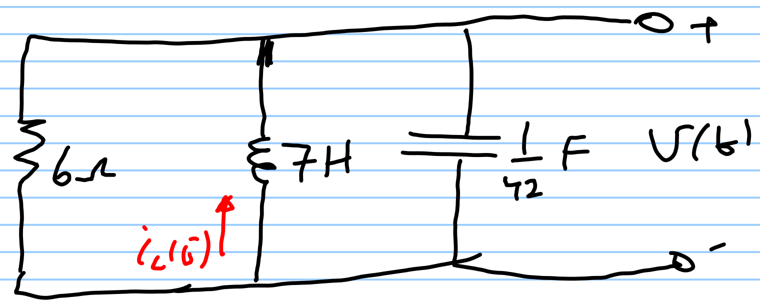
$$s_2 = -\alpha - \sqrt{\alpha^2 - \omega_0^2} \quad \text{-ve}$$

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

real & unequal (real & distinct)

$$\therefore \boxed{v(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t}} \qquad t > 0$$

EX overdamped parallel RLC



if $i_L(0^-) = 10A$ & $V_c(0^-) = 0$. find $v(t)$ for $t > 0$

① $\alpha = \frac{1}{2RC} = 3.5$, $\omega_0 = \frac{1}{\sqrt{LC}} = 2.45$

$\alpha > \omega_0 \longrightarrow$ overdamped

② $s_1 = -\alpha + \sqrt{\alpha^2 - \omega_0^2} = -1$

$s_2 = -\alpha - \sqrt{\alpha^2 - \omega_0^2} = -6$

$$\therefore v(t) = A_1 e^{-t} + A_2 e^{-6t}, \quad t > 0$$

③ to find $A_1, A_2 \longrightarrow$ from the initial conditions

$v(0^+)$ & $\frac{d}{dt} v(0^+)$

$$v(t) = A_1 e^{-t} + A_2 e^{-6t}$$

math

$$\left.\begin{aligned} &\rightarrow v(0^+) = A_1 + A_2 \\ &\rightarrow \frac{d}{dt} v(0^+) = -A_1 - 6A_2 \end{aligned}\right\}$$

$$s_1 A_1 + s_2 A_2$$

circuit

$v(0^+) \rightarrow$ The initial voltage on the capacitor

$d/dt\, v(0^+) \rightarrow$??

KCL $\quad i_c(t) + i_R(t) + i_L(t) = 0$

$$\underbrace{C \frac{d}{dt} v(t)}_{i_c(0^+)} + \frac{v(t)}{R} + \frac{1}{L}\int_0^t v(t)\,dt \pm I_0 = 0$$

$t \rightarrow 0^+$

$$\underbrace{C \frac{d}{dt} v(0^+)}_{i_c(0^+)} + \frac{v(0^+)}{R} \mp I_0 = 0 \qquad v(0^+) = V_0$$

$$\therefore \frac{d}{dt} v(0^+) = \left(\mp I_0 - \frac{V_0}{R}\right)\frac{1}{C}$$

$$\boxed{\frac{d}{dt} v(0^+) = \frac{i_c(0^+)}{C} = \frac{\mp I_0 - \frac{V_0}{R}}{C}}$$

Summary