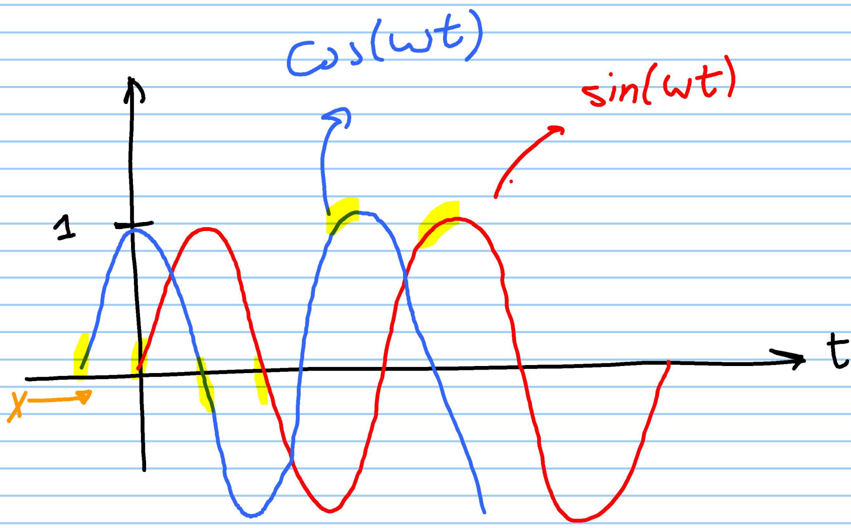
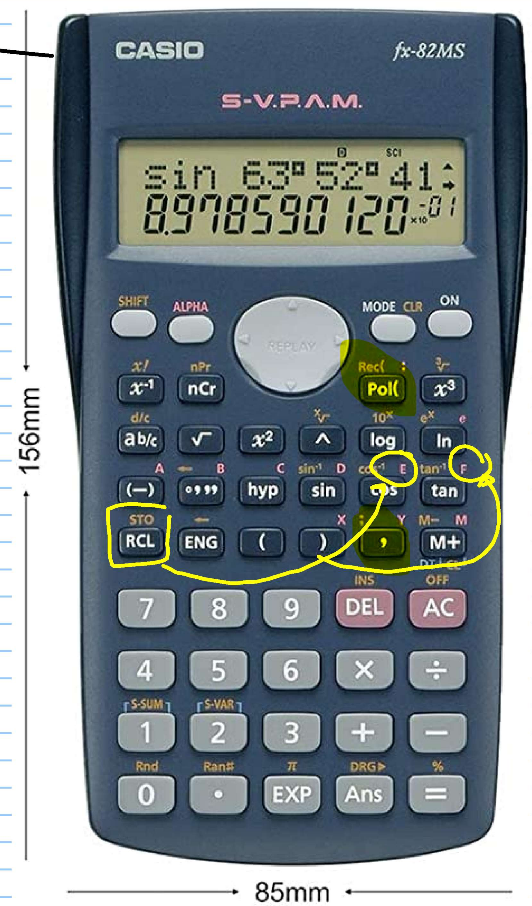
Note Title 12/16/2020

chapter 9:



$$\sin(\omega t) = \cos(\omega t - 90^\circ)$$

$$\cos(\omega t) = \sin(\omega t + 90^\circ)$$



* $Z = 10 + j10$ (Rectangular)

$= \boxed{14.14} \angle \boxed{45^\circ}$ (Polar)

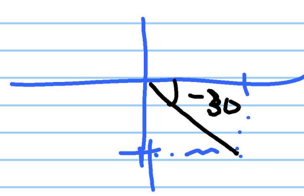
[Pol(] 10 [,] 10 [=]

14.14 → magnitude (stored in E)

45° → angle (stored in F)

* $Z = 25 \angle -30^\circ$

$= 21.6 - j12.5$



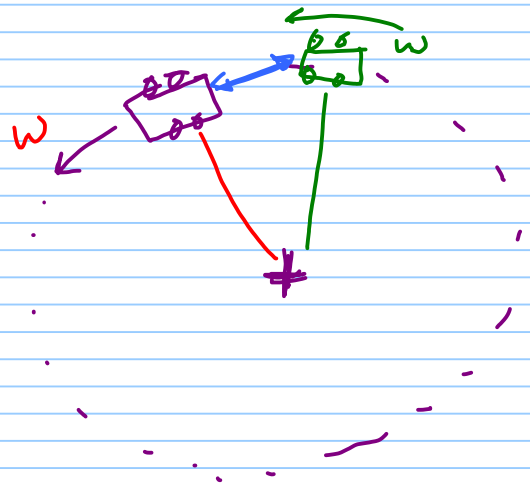
[Rec(] 25 [,] [(-)] 30 [=]

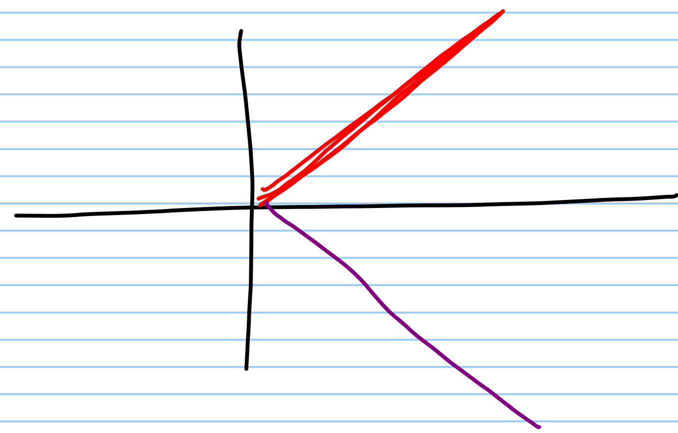
$$(10\angle 20^\circ) + (15\angle -35^\circ) = X$$

Rec.

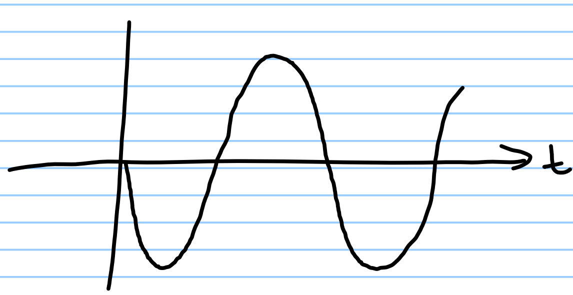
$$X_1 + jY_1 + X_2 + jY_2$$

$$(X_1 + X_2) + j(Y_1 + Y_2)$$





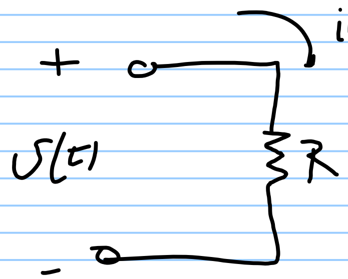
$\sin wt$



$-\sin wt$

phasor Relationships for circuit elements

Resistors



$\Rightarrow v(t) = R\, i(t)$

if $i(t) = I_m \cos(\omega t + \theta_i)$

$\therefore v(t) = R I_m \cos(\omega t + \theta_i)$

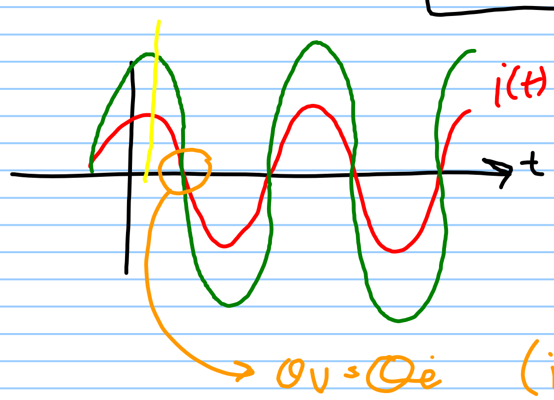
→ Phasor transformation:-

$$\vec{I} = I_m \angle \theta_i \, , \quad \vec{V} = R I_m \angle \theta_i$$

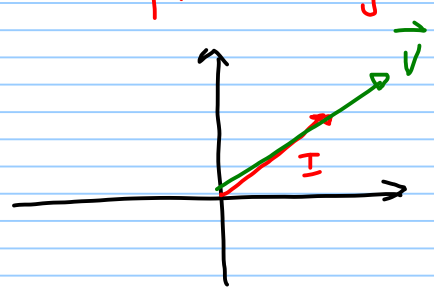
$$\boxed{\vec{V} = R \vec{I}}$$

Note for (R), there is no phase shift between the current and voltage.

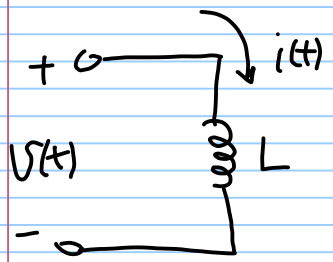
$\vec{V}$ & $\vec{I}$ are in phase



phasor diagram



Inductors



$\rightarrow v(t) = L \frac{d}{dt} i(t)$

$\rightarrow$ let $i(t) = I_m \cos(\omega t + \theta_i)$

$\therefore v(t) = L \frac{d}{dt} i(t) = -I_m \, \omega L \, \underline{\sin}(\omega t + \theta_i)$

$\hookrightarrow$ cosine {reference}

$\rightarrow v(t) = -\omega L \, I_m \cos(\omega t + \theta_i - 90^\circ)$

In phasor representation

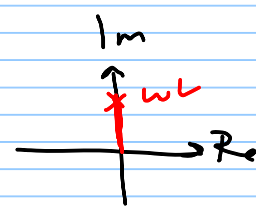
$$\vec{V} = -\omega L \, I_m \, e^{j(\theta_i - 90^\circ)}$$

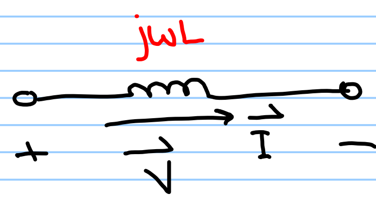
$$= -\omega L \, I_m \, e^{j\theta_i} \underbrace{e^{-j90^\circ}}_{-j}$$

$$e^{-j90^\circ} = \cos(-90^\circ) + j\sin(-90^\circ) = -j$$

$$= j\omega L \, I_m \, e^{j\theta_i}$$

$$\boxed{\vec{V} = j\omega L \, \vec{I}}$$

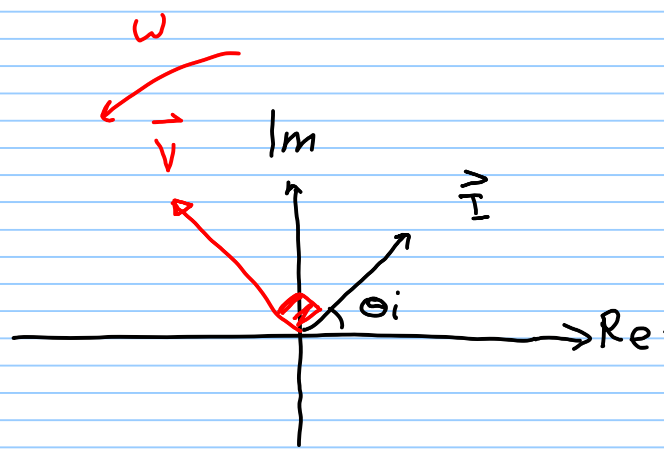




$$\vec{V} = j\omega L \, \vec{I}$$

$$= (\omega L \angle 90^\circ)(I_m \angle \theta_i)$$

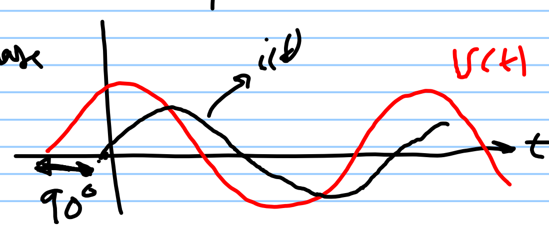
$$= \omega L I_m \angle \theta_i + 90^\circ$$



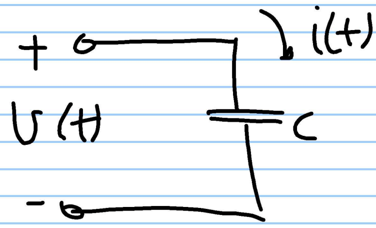
NOTE: The voltage & current are out of phase

v leads i by 90°

i lags v by 90°



Capacitors



$$\rightarrow \quad i(t) = C \frac{d}{dt} v(t)$$

let $v(t) = V_m \cos(\omega t + \theta_v)$

$$\therefore i(t) = -C V_m \omega \sin(\omega t + \theta_v)$$

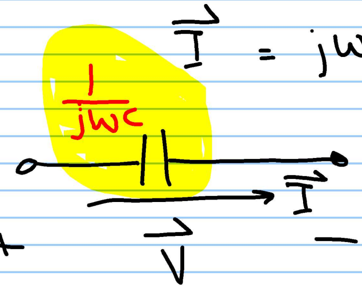
$$= -\omega C V_m \cos(\omega t + \theta_v - 90^\circ)$$

$$\longrightarrow \vec{I} = -\omega C V_m e^{j(\theta_v - 90^\circ)}$$

$$= -\omega C V_m e^{j\theta_v} e^{-j90^\circ}$$

$$\vec{I} = j\omega C \underbrace{V_m e^{j\theta_v}}$$

$$\vec{I} = j\omega C \vec{V}$$

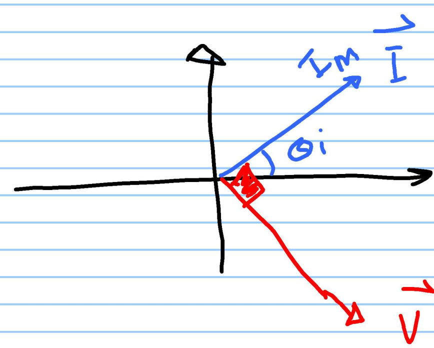


$$\vec{V} = \frac{1}{j\omega C} \vec{I}$$

$$= -j \frac{1}{\omega C} \vec{I}$$

$$= \left(\frac{1}{\omega C} \angle -90^\circ\right) \left(I_m \angle \theta_i\right)$$

$$= \frac{I_m}{\omega C} \angle \theta_i - 90^\circ$$



i leads v by 90°

v lags i by 90°

Impedance & Reactance

$$\vec{V} = R\,\vec{I}$$
$$\vec{V} = j\omega L\,\vec{I}$$
$$\vec{V} = \frac{1}{j\omega c}\,\vec{I}$$
$$\vec{V} = Z\,\vec{I}$$

↳ The Impedance Z is the ratio of a circuit element's voltage phasors to it's current phasor.

Z is NOT a phasor ⚠

complex number ✓

1 Z → Impedance (Ω)

2 $Y = \frac{1}{Z}$ → admittance ($\mho$)

3 Reactance ?

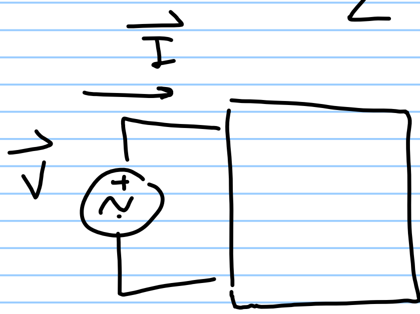
	Impedance Ω	Reactance Ω
resistor	R	—
inductor	$Z_L = j\omega L$	$X_L = \omega L$
capacitor	$Z_c = \frac{1}{j\omega c} = -j\frac{1}{\omega c}$	$X_c = \frac{-1}{\omega c}$

↳ Reactance is the imaginary part of the impedance

+ve → Inductor
−ve → capacitor
} ⚠

In General, if Z is a combination of R, L, C in frequency domain, then it will be in a form of

$$Z = R + jX$$



$$Z = \frac{\vec{V}}{\vec{I}} = \frac{V_m \angle \theta_v}{I_m \angle \theta_i}$$

$$= \frac{V_m}{I_m} \angle \theta_v - \theta_i$$

$$Z = |Z| \angle \theta_z$$

$$= R + jX$$

$R \equiv$ Resistive part

$X \equiv$ Reactive part

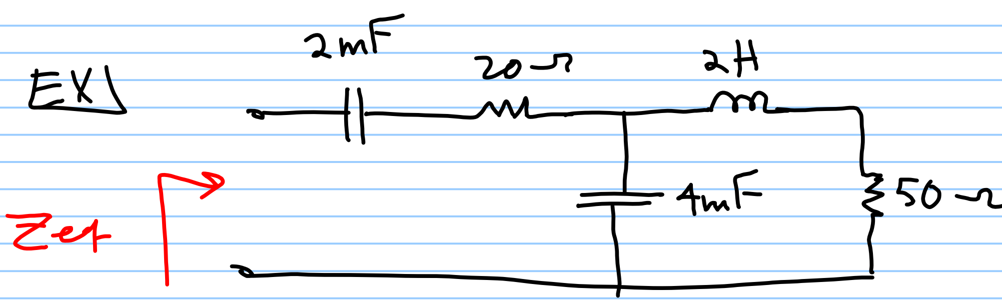
$$|Z| = \sqrt{R^2 + X^2}$$

$$\theta_z = \tan^{-1} \frac{X}{R}$$

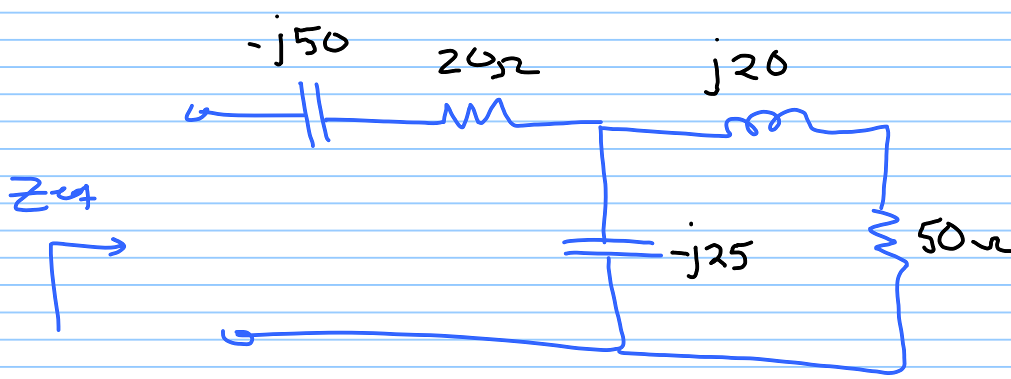
$$R = |Z| \cos \theta_z$$

$$X = |Z| \sin \theta_z$$

EX1

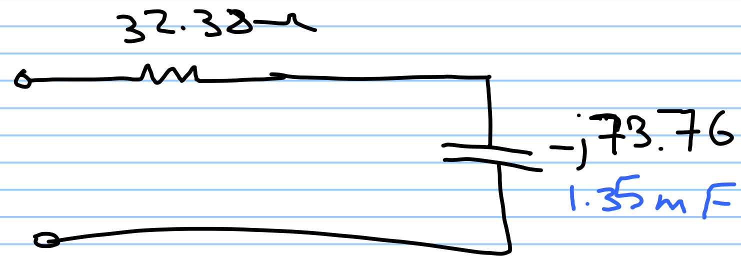


find Z_{eq} if $\omega = 10$ rad/sec.



$$Z_{eq} = \{[50+j20] // [-j25]\} + [20-j50]$$

$$= 32.38 - j73.76 \ \Omega$$



$$-j\frac{1}{\omega C}$$

$$\frac{1}{\omega C} = 73.76$$

$$\frac{1}{10kC} = 73.76$$

$$C = \frac{1}{737.6} = 1.35\,mF$$