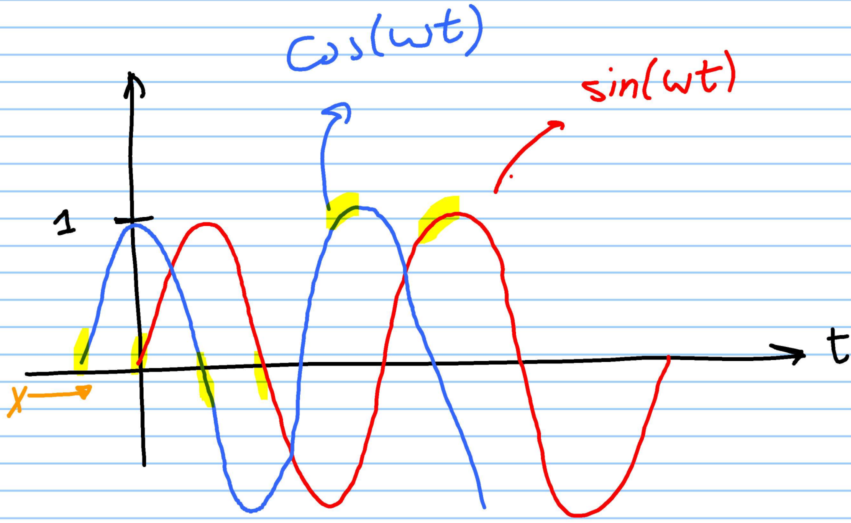
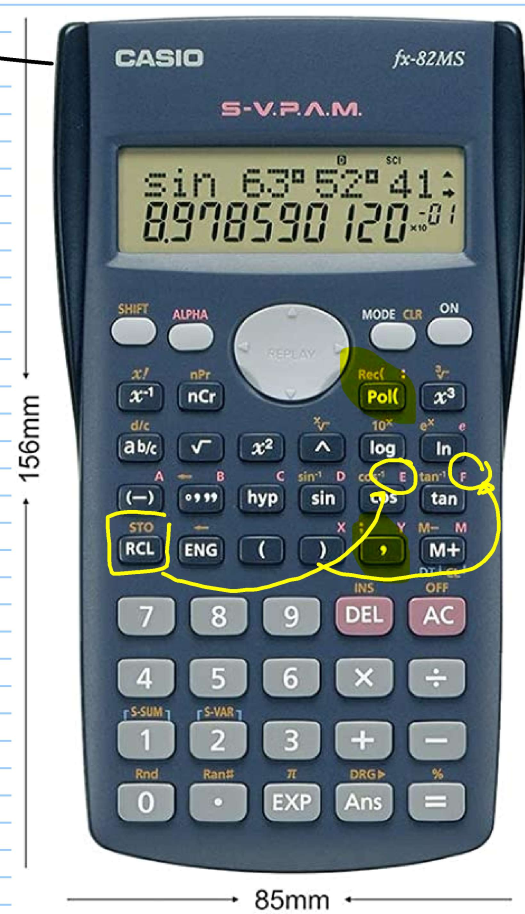
chapter 9:



$$\sin(wt) = \cos(wt - 90^\circ)$$
$$\cos(wt) = \sin(wt + 90^\circ)$$



* $Z = 10 + j10$ (Rectangular)

$= \boxed{14.14} \angle \boxed{45^\circ}$ (Polar)

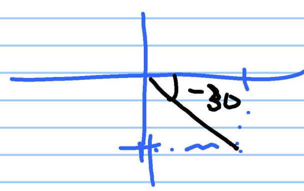
$\boxed{\text{Pol(}}$ 10 $\boxed{,}$ 10 $\boxed{=}$

14.14 → magnitude (stored in E)

45° → angle (stored in F)

* $Z = 25 \angle -30^\circ$

$= 21.6 - j12.5$



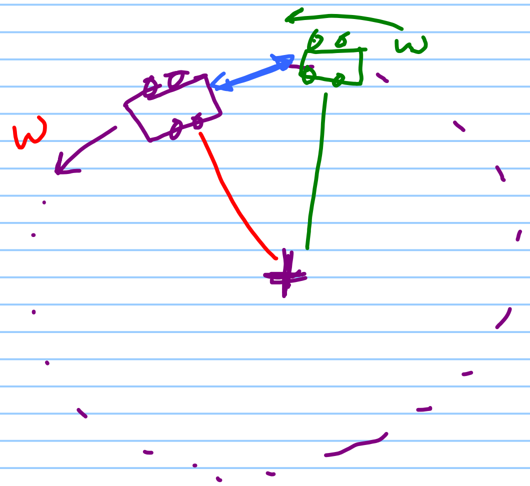
$\boxed{\text{Rec}}$ 25 $\boxed{,}$ $\boxed{-}$ 30 $\boxed{=}$

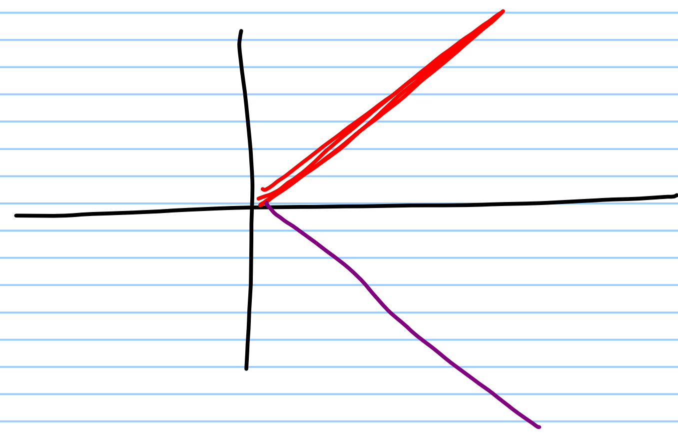
$$(10\angle 20^\circ) + (15\angle -35^\circ) = X$$

Rec.

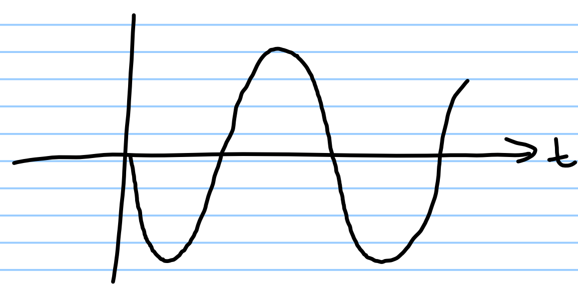
$$x_1 + jy_1 + x_2 + jy_2$$

$$(x_1 + x_2) + j(y_1 + y_2)$$





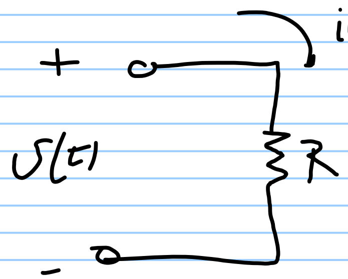
$\sin \omega t$



$-\sin \omega t$

phasor Relationships for circuit elements

Resistors



$\Rightarrow$ $v(t) = R\, i(t)$

if $i(t) = I_m \cos(\omega t + \theta_i)$

$\therefore v(t) = R I_m \cos(\omega t + \theta_i)$

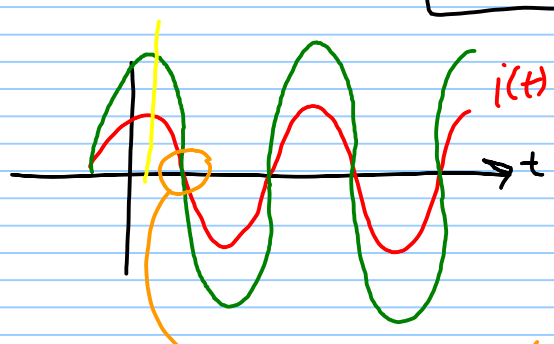
$\rightarrow$ Phasor transformation:-

$$\vec{I} = I_m \angle \theta_i \quad , \quad \vec{V} = R I_m \angle \theta_i$$

$$\boxed{\vec{V} = R\,\vec{I}}$$

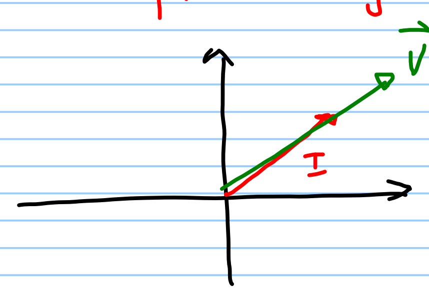
Note for (R), there is no phase shift between the current and voltage.

$$\boxed{\vec{V} \;\&\; \vec{I} \text{ are in phase}}$$

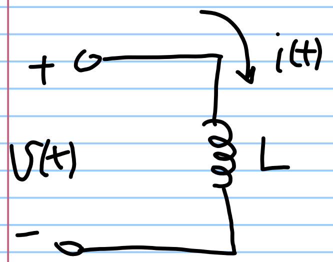


$\theta_v = \theta_i$ (in phase)

phasor diagram



Inductors



→ $v(t) = L \frac{d}{dt} i(t)$

→ let $i(t) = I_m \cos(\omega t + \theta_i)$

$\therefore\ v(t) = L \frac{d}{dt} i(t) = -I_m \omega L \sin(\omega t + \theta_i)$

↳ cosine {reference}

→ $v(t) = -\omega L I_m \cos(\omega t + \theta_i - 90°)$

In phasor representation

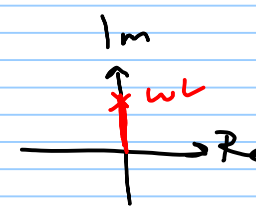
$$\vec{V} = -\omega L I_m e^{j(\theta_i - 90°)}$$

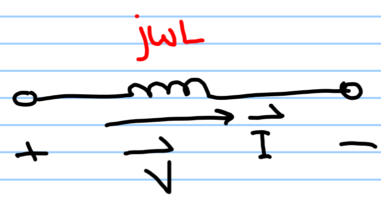
$$= -\omega L I_m e^{j\theta_i} \underbrace{e^{-j90°}}_{-j}$$

$$= j\omega L I_m e^{j\theta_i}$$

$$\boxed{\vec{V} = j\omega L \vec{I}}$$

$$e^{-j90°} = \cos(-90°) + j\sin(-90°)$$
$$= -j$$

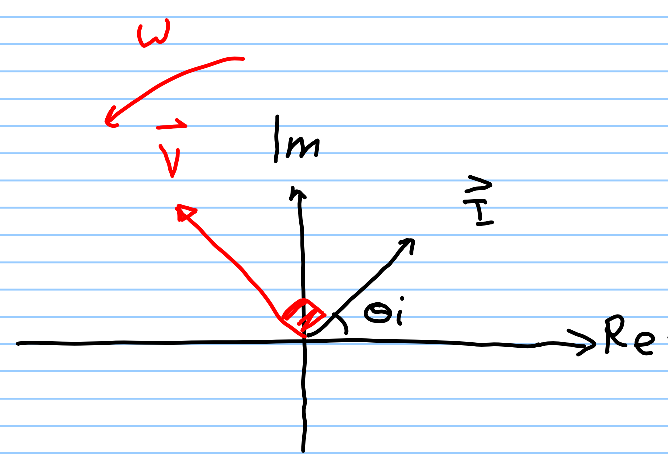




$$\vec{V} = j\omega L \vec{I}$$

$$= (\omega L \angle 90°)(I_m \angle \theta_i)$$

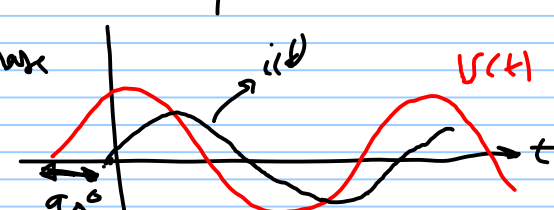
$$= \omega L I_m \angle \theta_i + 90°$$



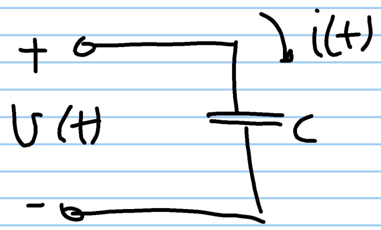
NOTE The voltage & current are out of phase

v leads i by 90°

i lags v by 90°



Capacitors



$\rightarrow \quad i(t) = C \dfrac{d}{dt} v(t)$

let $\quad v(t) = V_m \cos(\omega t + \theta_v)$

$\therefore \quad i(t) = -C V_m \omega \sin(\omega t + \theta_v)$

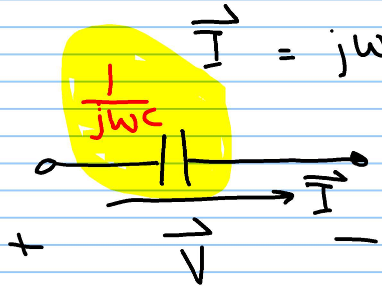
$= -\omega C V_m \cos(\omega t + \theta_v - 90^\circ)$

$\longrightarrow \quad \vec{I} = -\omega C V_m e^{j(\theta_v - 90)}$

$= -\omega C V_m e^{j\theta_v} e^{-j90^\circ}$

$\vec{I} = j\omega C \underbrace{V_m e^{j\theta_v}}$

$\vec{I} = j\omega C \quad \vec{V}$

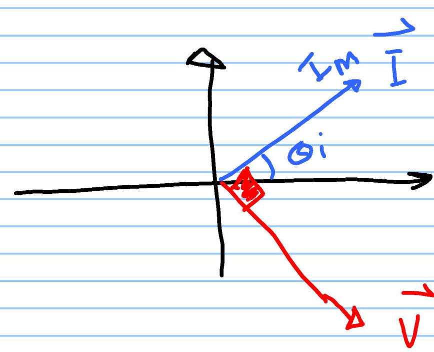


$\vec{V} = \dfrac{1}{j\omega C} \vec{I}$

$= -j \dfrac{1}{\omega C} \vec{I}$

$= \left(\frac{1}{\omega C} \angle -90^\circ\right) \left(I_m \angle \theta_i\right)$

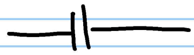
$= \dfrac{I_m}{\omega C} \angle \theta_i - 90^\circ$



i leads v by 90°

v lags i by 90°

Impedance & Reactance

	$\vec{V} = R\,\vec{I}$
	$\vec{V} = j\omega L\,\vec{I}$
	$\vec{V} = \frac{1}{j\omega C}\,\vec{I}$

$$\vec{V} = Z\,\vec{I}$$

↳ The Impedance Z is the ratio of a circuit element's voltage phasors to it's current phasor.

$\boxed{Z \text{ is NOT a phasor}}$ ⚠

complex number ✓

1 $Z \rightarrow$ Impedance (Ω)

2 $Y = \frac{1}{Z} \rightarrow$ admittance ($℧$)

3 Reactance ?

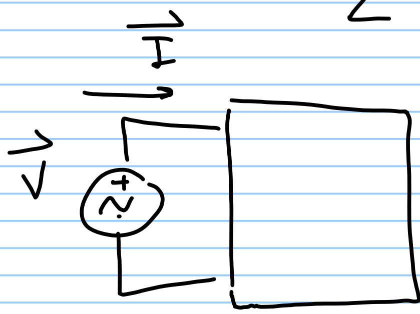
	Impedance Ω	Reactance Ω
	R	—
	$Z_L = j\omega L$	$X_L = \omega L$
	$Z_C = \frac{1}{j\omega C} = -j\frac{1}{\omega C}$	$X_C = \frac{-1}{\omega C}$

↳ Reactance is the imaginary part of the impedance

$+ve \longrightarrow$ Inductor
$-ve \longrightarrow$ capacitor
⚠

In General, if Z is a combination of R, L, C in frequency domain, then it will be in a form of

$$Z = R + jX$$



$$Z = \frac{\vec{V}}{\vec{I}} = \frac{V_m \angle \theta_v}{I_m \angle \theta_i}$$

$$= \frac{V_m}{I_m} \angle \theta_v - \theta_i$$

$$Z = |Z| \angle \theta_Z$$

$$= R + jX$$

$R \equiv$ Resistive part

$X \equiv$ Reactive part

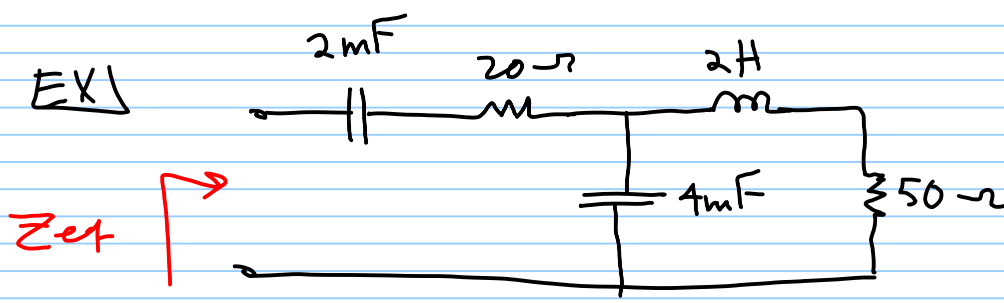
$$|Z| = \sqrt{R^2 + X^2}$$

$$\theta_Z = \tan^{-1} \frac{X}{R}$$

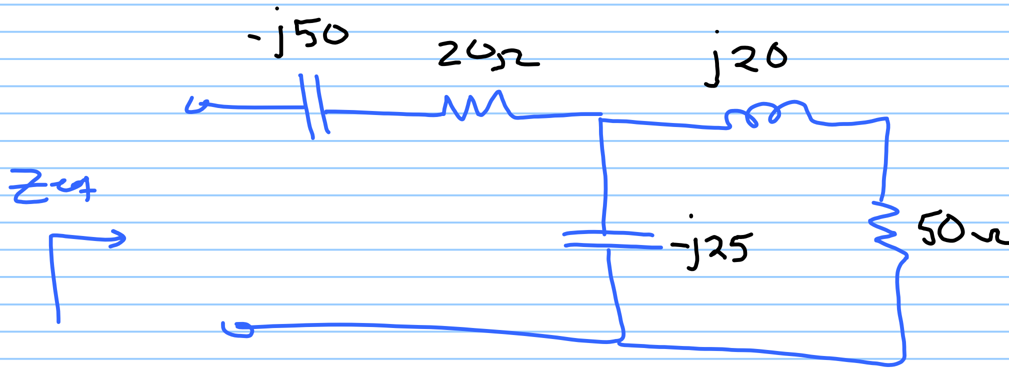
$$R = |Z| \cos \theta_Z$$

$$X = |Z| \sin \theta_Z$$

EX1

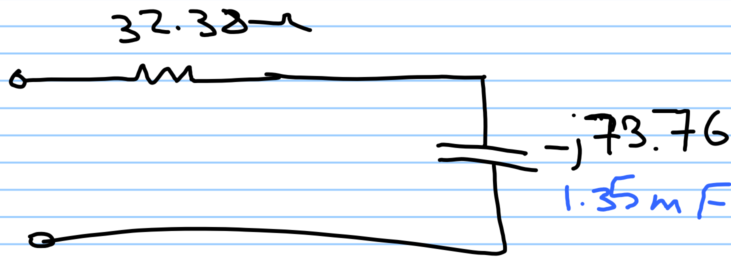


find Z_{eq} if $\omega = 10$ rad/sec.



$$Z_{eq} = \{[50+j20] // [-j25]\} + [20-j50]$$

$$= 32.38 - j73.76 \ \Omega$$



$-j\frac{1}{\omega C}$

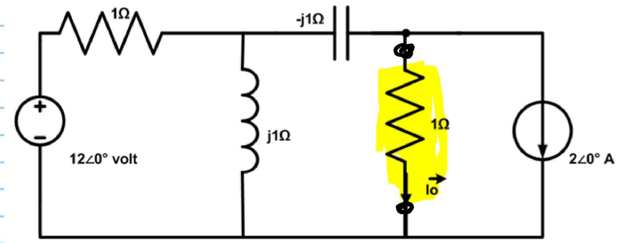
$$\frac{1}{\omega C} = 73.76$$

$$\frac{1}{10k C} = 73.76$$

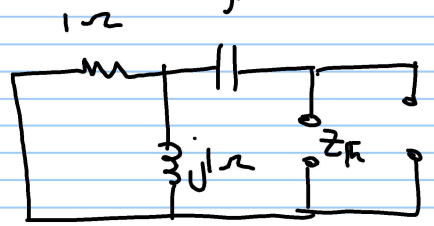
$$C = \frac{1}{737.6} = 1.35\,mF$$

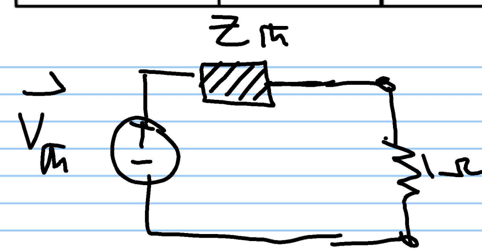
Thevenin's and Norton's theorems

find $\overrightarrow{Io}$ using thevenin's theorem



$Z_{th} \rightarrow$



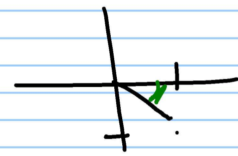


$$Z_{th} = (1 // j1) + (-j1)$$

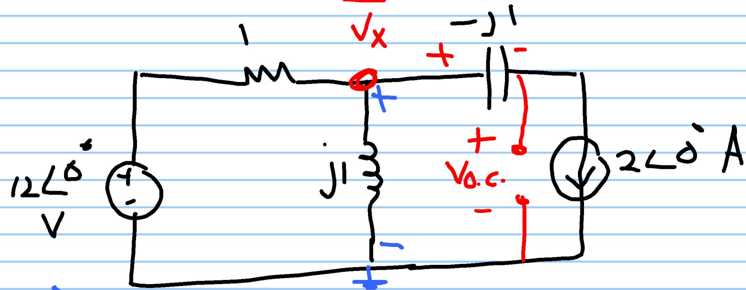
$$= \frac{j}{1+j} - j = \frac{1\angle 90}{\sqrt{2}\angle 45} - j$$

$$= \frac{1}{\sqrt{2}}\angle 45 - j$$

$$= \frac{1}{2} + \frac{1}{2}j - j = \frac{1}{2} - j\frac{1}{2} = \frac{\sqrt{2}}{2}\angle -45 \ \Omega$$



$\vec{V}_{th} = \vec{V}_{o.c.}$



$$\frac{\vec{V}_x - 12\angle 0^\circ}{1} + \frac{\vec{V}_x}{j1} + 2\angle 0^\circ = 0$$

$$\vec{V}_x \left(1 + \frac{1}{j1}\right) - 12 + 2 = 0$$

$$\vec{V}_x = \frac{10}{1 + \frac{1}{j}} = \frac{10}{1 - j} = 5 + j5 \ V$$

$$= 5\sqrt{2}\angle 45^\circ \ V$$

$\therefore$ KVL

$$-\vec{V}_{o.c.} + \vec{V}_c + \vec{V}_x = 0$$

$$\vec{V}_{o.c} = \vec{V}_c + \vec{V}_x = -(-j)(2) + 5 + j5$$

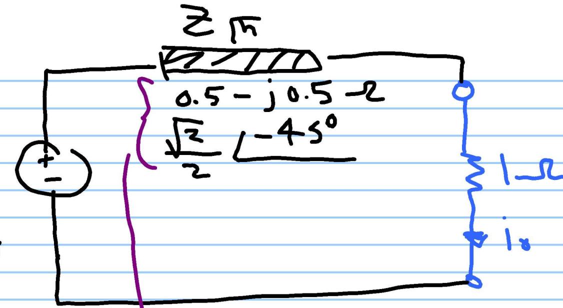
$$= j2 + 5 + j5 = 5 + j7 \ V$$

$$= \sqrt{74}\angle 54.46^\circ \ V$$

$\vec{V}_{th}$

$5+j7$

$\sqrt{74}\angle 54.46^\circ$



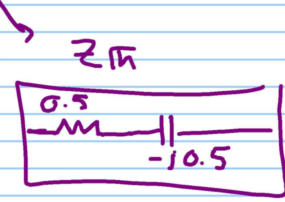
$$\therefore \vec{I}_o = \frac{\vec{V}_{th}}{\vec{Z}_{eq}}$$

$$= \frac{\sqrt{74}\angle 54.46^\circ}{0.5 - j0.5 + 1}$$

$$= \frac{\sqrt{74}\angle 54.46^\circ}{\frac{\sqrt{10}}{2}\angle -18.43^\circ}$$

$$= 5.44\angle 72.89^\circ \text{ A} \qquad 5.44\cos(\omega t + 72.89^\circ)\text{ A}$$

$$\simeq 1.6 + j5.2 \text{ A} \checkmark$$



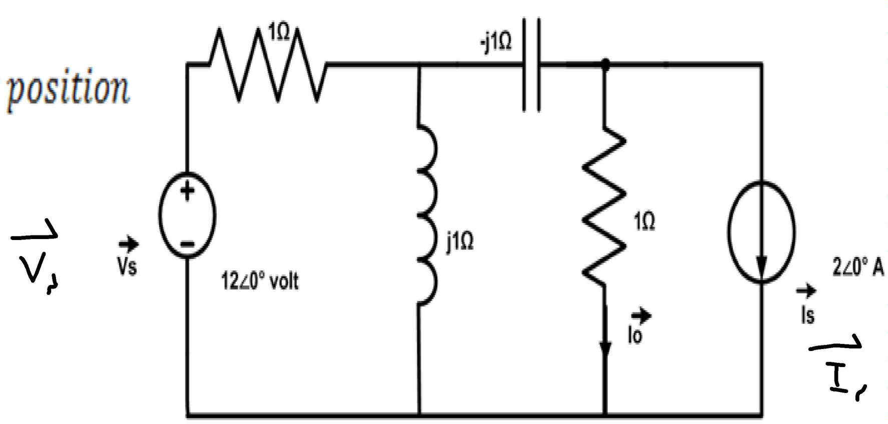
series, parallel, KVL, KCL, VDR, CDR

source transformation, thevenin, Norton

Nodal, Mesh, superposition

Superposition

find $\vec{Io}$ using super position

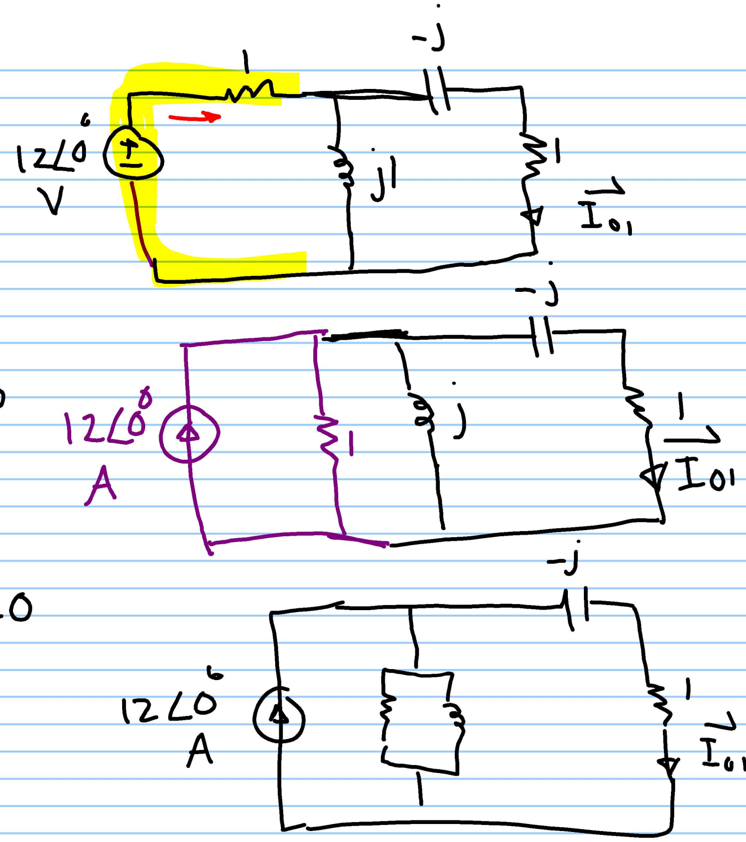


$$\vec{I}_o = \vec{I}_{o1} + \vec{I}_{o2}$$

$\vec{I}_{o1}$: due to $\vec{V}_s$

$\vec{I}_{o2}$: due to $\vec{I}_s$

① $\vec{I}_{o1}$ due to $\vec{V}_f$



$$\vec{I}_{01} = \frac{(1//j)}{(1//j)+(1-j)} * (12\angle 0)$$

$$= \frac{j/_{1+j}}{j/_{1+j} + (1-j)} * 12\angle 0$$

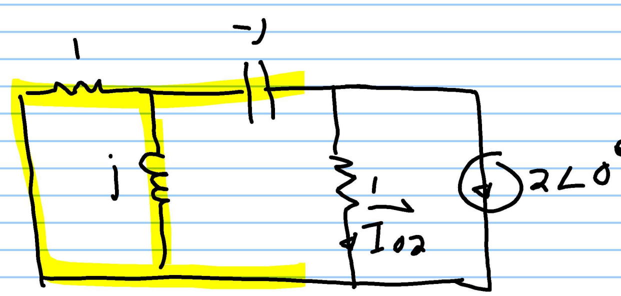
$$= \frac{j}{j+(1-j)(1+j)} * 12\angle 0$$

$$= (0.2+0.4j)(12\angle 0)$$

$$= 2.4 + j4.8 \text{ A}$$

$$= 5.366\angle 63.43° \text{ A}$$

$\vec{I}_{o2}$ due to $\vec{I}_f$



$$\vec{I}_{o2} = -\frac{[(1//j)-j]}{[(1//j)-j]+1} * 2\angle 0°$$

$$= -\frac{j/_{1+j} - j}{j/_{1+j} - j + 1} * 2\angle 0$$

$$= -\frac{0.5-0.5j}{1.5-0.5j} * 2\angle 0 = \frac{-0.5+0.5j}{1.5-0.5j} * 2\angle 0$$

$$= -0.8+0.4j \text{ A}$$

$\therefore \vec{I_o} = \vec{I_{o1}} + \vec{I_{o2}}$

$= (2.4 + 4.8j) + (-0.8 + 0.4j)$

$= 1.6 + 5.2j \text{ A}$ ✓

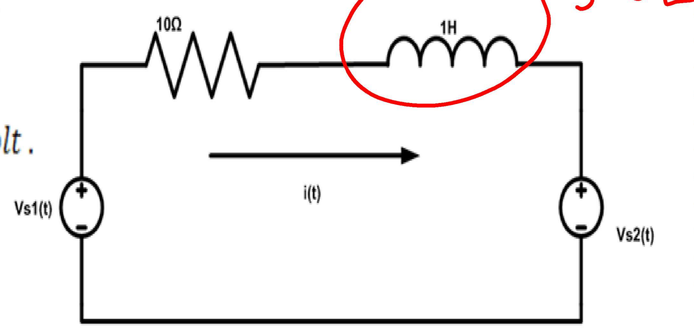
Power of super position :

$Vs1(t) = 100\cos 10t \text{ volt}$

$Vs2(t) = 50\cos(20t - 10°) \text{ volt}.$

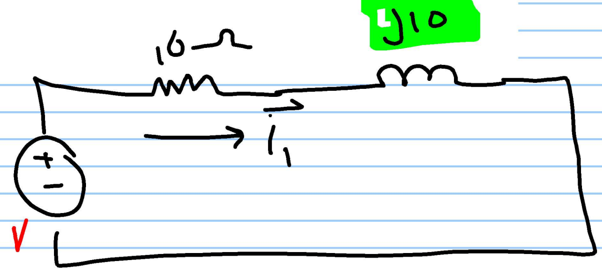
note that $w1 = 10 \ {}^{rad}/_{sec}$

and $w2 = 20 \ {}^{rad}/_{sec}$



$\therefore$ *super position is the only method of analysis.*

$i(t) = i1(t) + i2(t).$

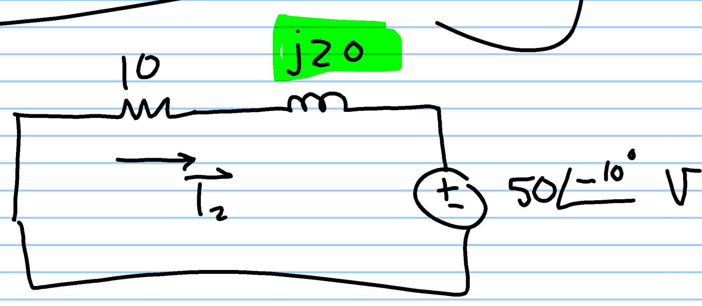


$\vec{I_1}$

$\therefore \vec{I_1} = \dfrac{100\angle 0}{10 + j10}$

$= 5 - j5 \text{ A}$

$= 7.07\angle -45° \text{ A}$



$\vec{I_2} = -\dfrac{50\angle -10°}{10 + j20}$

$= -0.637 + j2.143 \text{ A}$

$= 2.235\angle 106.5° \text{ A}$

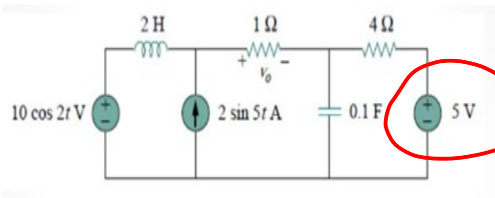
$\therefore \vec{I} = \vec{I_1} + \vec{I_2}$

= You can't add them!!

$$\therefore\ i(t) = i_1(t) + i_2(t)$$

$$= 7.07\cos(16t - 45^\circ) + 2.235\cos(20t + 106.5^\circ)$$

Superposition (dc+ac) sources



DC !!

Find Vo(t) using the superposition theorem