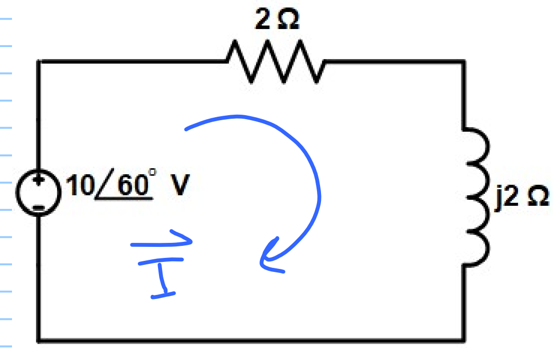
Example :

find the average power absorbed by each element.



$$P_{av} = \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i)$$

$$P_{av.j2} = \text{Zero}.$$

$$\vec{I} = \frac{10\angle 60^\circ}{2+j2} = 3.53\angle 15^\circ \text{ A}$$

$$\vec{V}_R = \frac{2}{2+j2} * 10\angle 60^\circ = 7.07\angle 15^\circ \text{ Volt}.$$

OR $\vec{V}_R = \vec{I} R = 7.06\angle 15^\circ$ Volt.

$$P_{av_R} = \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i)$$

$$= \frac{1}{2} * 7.07 * 3.53 \cos(15^\circ - 15^\circ)$$

$$= 12.47 \text{ W}$$

$$P_{av_R} = \frac{I_m^2 R}{2} = \frac{1}{2} \times (3.53)^2 \times (2) = 12.46 \text{ W}$$

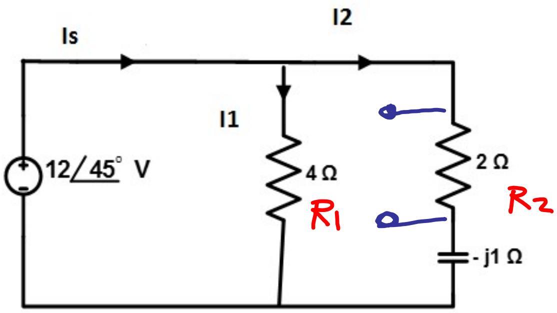
→ The average power supplied by the source

$$P_{av_{Vs}} = \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i)$$

$$= \frac{1}{2}(10)(3.53)\cos(60^\circ - 15^\circ)$$

$$= 12.48 \text{ W}$$

Example :



Determine the average power absorbed by each resistor . Determine the total average power supplied by the source .

$$\rightarrow P_{av_{R_1}} = \frac{V_m^2}{2R_1} = \frac{12^2}{2\times 4} = 18\,W.$$

$$\boxed{V_{R_2} \rightarrow \frac{2}{2-j} * 12\angle 45}$$

$$\vec{I_2} = \frac{12\angle 45^\circ}{2-j} = 5.366\angle 71.56^\circ\ A$$

$$\rightarrow P_{av_{R_2}} = \frac{1}{2} I_{m_2}^2 R_2 = \frac{1}{2}\times 5.366^2 \times 2 = 28.79\,W$$

$$\rightarrow P_{av_{Vs}} = 18 + 28.79 = 46.79\,W$$

del.

OR $\vec{I_s} = \frac{\vec{V_s}}{Z_{eq}} = \vec{I_1} + \vec{I_2}$

$$= \frac{12\angle 45}{4} + 5.366\angle 71.56$$

$$= 8.16\angle 62.1^\circ\ A$$

$$P_{av_{Vs}} = \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i)$$

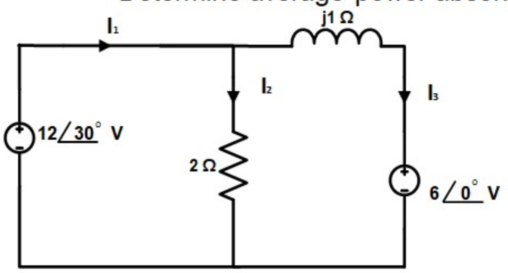
$$= \frac{1}{2}\times 12\times 8.16 \times \cos(45 - 62.1)$$

$$= 46.785\,W$$

$$P_{abs.} = P_{del.}$$

Example :

Determine average power absorbed or supplied by each element .



$$\vec{I_2} = \frac{12}{2}\angle 30^\circ = 6\angle 30^\circ \text{ A}$$

$$\vec{I_3} = \frac{(12\angle 30^\circ)-(6\angle 0^\circ)}{\underset{1\angle 90^\circ}{j1}} = 7.43\angle -36.2 \text{ A}$$

$$\therefore \vec{I_1} = \vec{I_2} + \vec{I_3} = 6\angle 30^\circ + 7.43\angle -36.2 = 11.28\angle -7.08 \text{ A}$$

$$P_{av_{2\Omega}} = \frac{1}{2} I_{2m}^2 \times 2 = 36\text{W (absorbed)}$$

$$\frac{1}{2} I_m^2 R$$

$$P_{av_{j1\Omega}} = \text{Zero}$$

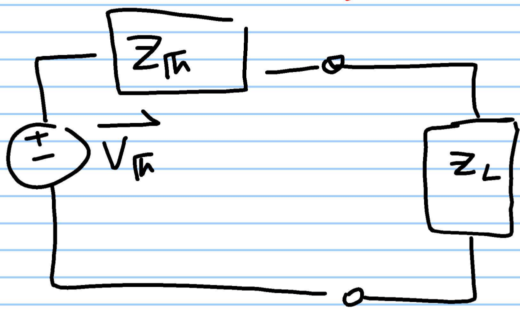
$$P_{av_{12\angle 30^\circ}} = \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i) = \frac{1}{2} 12 \times 11.28 \cos(30-(-7.08))$$

$$= 54\text{W} \quad \text{(supplied)}$$

$$P_{av_{6\angle 0^\circ}} = \frac{1}{2} \times 6 \times 7.43 \cos(0 - (-36.2))$$

$$= 18\text{W (absorbed)}$$

Maximum Power Transfer



$$Z_{Th} = R_{Th} + j\,X_{Th}$$

$$Z_L = R_L + j\,X_L$$

$$\to P_L = \frac{1}{2} I_{Lm}^2 R_L$$

$$\vec{I} = \frac{\vec{V}_{Th}}{Z_{Th} + Z_L} = \frac{\vec{V}_{Th}}{(R_{Th}+R_L) + j(X_{Th}+X_L)}$$

$$\therefore P_L = \frac{1}{2} \frac{V_{Th}^2 \cdot R_L}{(R_{Th}+R_L)^2 + (X_{Th}+X_L)^2}$$

(1) $\frac{\partial P_L}{\partial X_L} = 0$ & (2) $\frac{\partial P_L}{\partial R_L} = 0$

$$* \Rightarrow \frac{\partial P_L}{\partial X_L} = \frac{1}{2} \frac{-2V_{Th}^2 R_L (X_L + X_{Th})}{\left[(R_{Th}+R_L)^2 + (X_{Th}+X_L)^2\right]^2}$$

for $\frac{\partial P_L}{\partial X_L} = 0 \longrightarrow \boxed{X_L = -X_{Th}}$

$$* \Rightarrow \frac{\partial P_L}{\partial R_L} = \frac{V_{Th}^2 \left[(R_L + R_{Th})^2 + (X_L + X_{Th})^2 - 2R_L(R_L + R_{Th})\right]}{2\left[(R_L+R_{Th})^2 + (X_L+X_{Th})^2\right]^2}$$

for $\frac{\partial P_L}{\partial R_L} = 0 \longrightarrow \boxed{R_L = R_{Th}}$

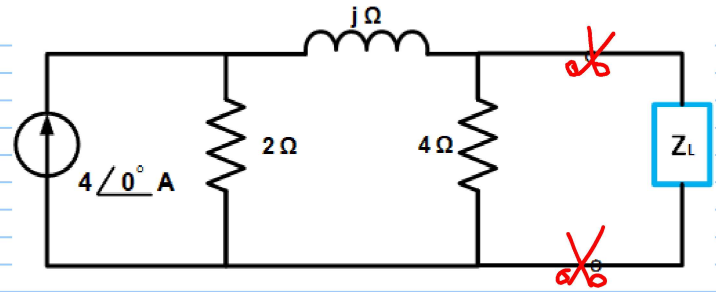
$$\boxed{Z_L = Z_{Th}^*}$$

$$P_{L,max} = \frac{1}{8} \frac{V_{Th}^2}{R_{Th}}$$

$R_{Th} = R_L$

Example: Find Z_L for maximum average power transfer.

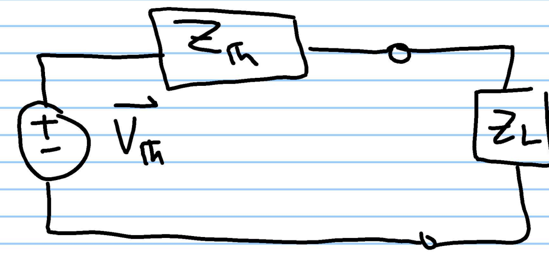
Compute the maximum average power supplied to the load.



$$Z_{Th} = 4 /\!/ (2+j)$$

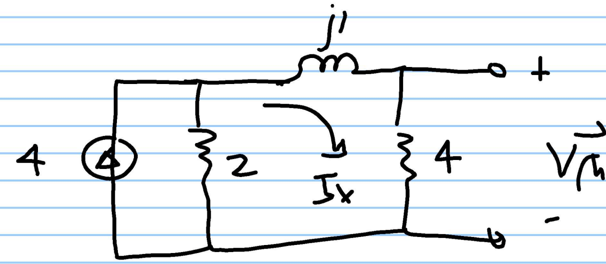
$$= 1.4 + j0.43 \ \Omega$$

$$Z_L = Z_{Th}^{*} = 1.4 - j0.43 \ \Omega$$



$$P_{max} = \frac{1}{8} \frac{V_{Th}^2}{R_{Th}}$$

$$\vec{V}_{Th} = \underbrace{\left(\frac{2 \times 4}{6+j}\right)}_{I_x} \times 4$$



$$= 5.28 \angle -9.46^{\circ}$$

$$\therefore P_{max} = \frac{1}{8} \frac{(5.28)^2}{1.4} = 2.489 \text{ W}$$

$$V_{TH} = 2I - 4\angle 0^{\circ}$$

$$I = \frac{V_X + 4\angle 0^{\circ}}{2 + j4} \longrightarrow I = \frac{-2I + 4}{2 + j4}$$

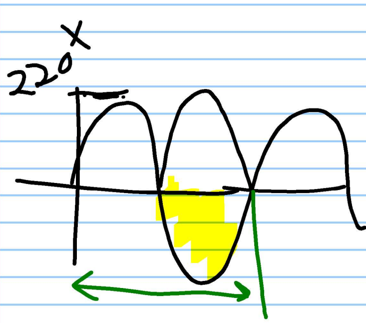
$$V_X = -2I$$

$$2I + j4I + 2I = 4$$

$$(4 + j4) I = 4$$

$$I = 0.707 \angle -45^{\circ} \ A$$

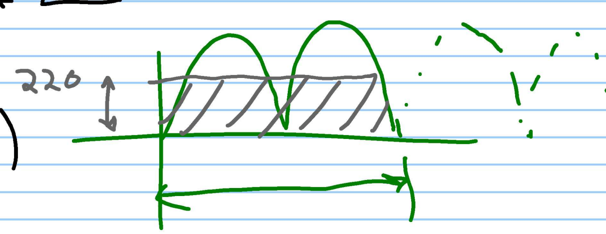
$$I = \frac{4}{\sqrt{4^2+4^2} \angle 45^{\circ}}$$



X

$220 \cos(2\pi \times 50t + 0^{\circ})$

rms



$$V_{rms_{eff}} = \frac{V_m}{\sqrt{2}} \qquad I_{rms} = \frac{I_m}{\sqrt{2}}$$

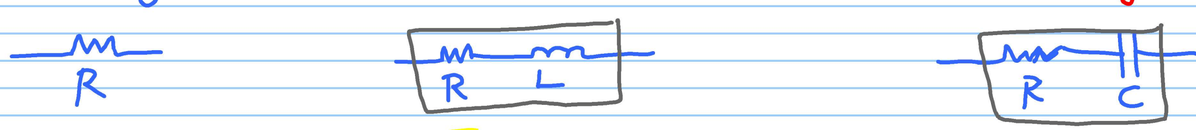
$$\begin{cases} P_{av} = \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i) \\ \quad = V_{rms} I_{rms} \cos(\theta_v - \theta_i) \end{cases}$$

$$P_{av_R} = \frac{V_{rms}^2}{R} = I_{rms}^2 R = V_{rms} I_{rms}$$

$$P_{av} = \underbrace{V_{rms} I_{rms}}_{P_a \ (VA)} \ \underbrace{\cos(\theta_v - \theta_i)}_{PF} \quad (W)$$

apparent power   Power Factor

unity   lagging PF   leading PF



$\theta_v = \theta_i$

$PF = 1$

i lags v by θ

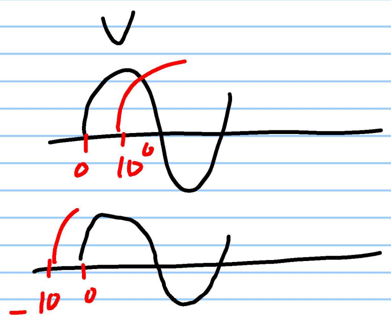
$90^\circ > \theta_v - \theta_i > 0$

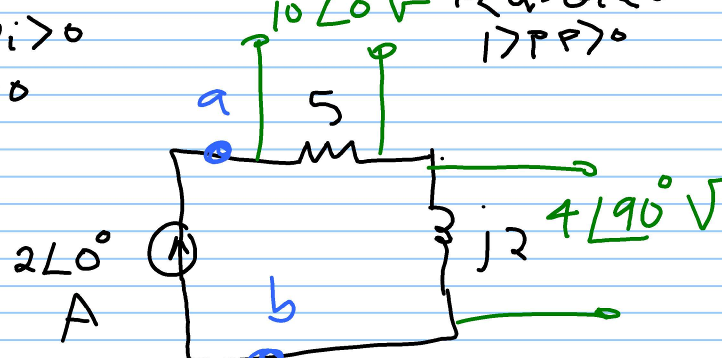
$1 > PF > 0$

i leads v by θ

$-90 < \theta_v - \theta_i < 0$

$1 > PF > 0$





$\cos(\theta) = \cos(-\theta)$

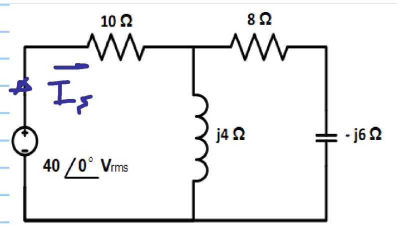
P.F. $= \cos(21.8) = 0.928$ lagging

$$\begin{cases} V_{ab} = (2\angle 0)(5 + j2) \\ \quad = 10 + j4 = 10.77\angle 21.8^\circ \end{cases}$$

→ Power factor is either leading or lagging, reffering to the phase of the current with respect to the voltage.

Ex]

Calculate the power factor seen by the source and the average power supplied by the source.



$$Z = \left[(8-j6)//j4\right]+10$$

$$= 12.69\angle 20.62\ \Omega$$

$$\vec{I_s} = \frac{40\angle 0}{Z} = 3.152\angle -20.62\ A$$

$$PF = \cos(\theta_v - \theta_i) = \cos(20.62)$$

$$= 0.936 \text{ lagging}$$

$$P_{av} = P_a \cdot PF = V_{rms}\, I_{rms} \times 0.936$$

$$= 40 \times 3.152 \times 0.936$$

$$= 118\ W.$$

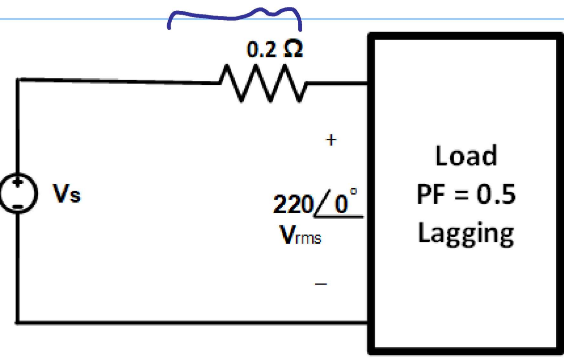
Example :

An industrial load consumes 11 kW at 0.5 PF lagging from a 220 V_{rms} line . The transmission line resistance from the power company to the plant is 0.2Ω .

1) Determine the average power that must be supplied by the power company .

2) Repeat (1) if the power factor is changed to unity

Transmission line



(1)
$$P_{av_{Load}} = V_{rms} I_{rms} \cdot PF$$

$$I_{rms} = \frac{P_{av}}{V_{rms}\ PF} = \frac{11K}{220 \times 0.5} = \boxed{100\,A}$$

$$P_{Loss} = (100)^2 \times (0.2) = 2KW$$

$$\therefore P_{av_{supplied}} = P_{av_{Load}} + P_{loss} = 13\,KW.$$

(2)
$$P_{av_{Load}} = V_{rms} \cdot I_{rms} \cdot PF$$

$$11 \times 10^3 = 220 \cdot I_{rms} \cdot 1 \qquad \boxed{I_{rms} = 50A}$$

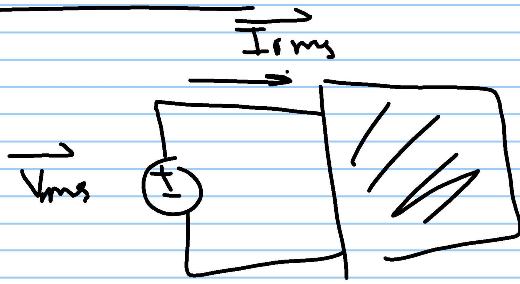
$$P_{Losses} = I_{rms}^2 R = (50)^2 \times 0.2 = 500W$$

$$P_{av_{supplied}} = 0.5k + 11k = 11.5KW$$

Complex Power

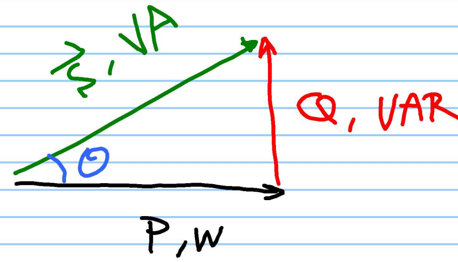
$\vec{V}_{rms} = V_{rms} \angle \theta_v$

$\vec{I}_{rms} = I_{rms} \angle \theta_i$



$\vec{S} \equiv$ complex power

$\vec{S} = \vec{V}_{rms} \cdot \vec{I}_{rms}^{\,*}$



$= V_{rms} I_{rms} \angle \theta_v - \theta_i$

$\theta = \theta_v - \theta_i$

$= \underbrace{V_{rms} I_{rms} \cos(\theta_v - \theta_i)} + j \underbrace{V_{rms} I_{rms} \sin(\theta_v - \theta_i)}$

$\vec{S} = P_{av} + jQ$

average power

Reactive Power

R

$\theta_v - \theta_i = 0$

$Q = VI \sin\theta$
$=$ Zero

L

$\theta_v - \theta_i = 90°$

$Q_L = V_{rms} I_{rms}$

$= (\omega L I_{rms}) I_{rms}$

$= \omega L I_{rms}^2$

$= \dfrac{V_{rms}^2}{\omega L}$

$\boxed{X_L = \omega L}$

C

$\theta_v - \theta_i = -90°$

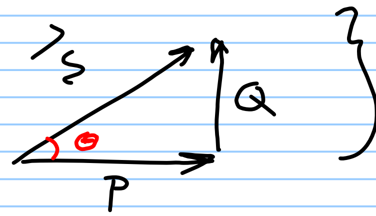
$Q_C = -V_{rms} I_{rms}$

$= -V_{rms} (\omega C V_{rms})$

$= -\omega C V_{rms}^2$

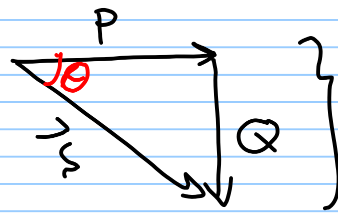
$= -\dfrac{I_{rms}^2}{\omega C}$

$\boxed{X_C = \dfrac{-1}{\omega C}}$



Inductive

$\theta = \theta_v - \theta_i$



Capacitive

$$\vec{S} = P + jQ$$

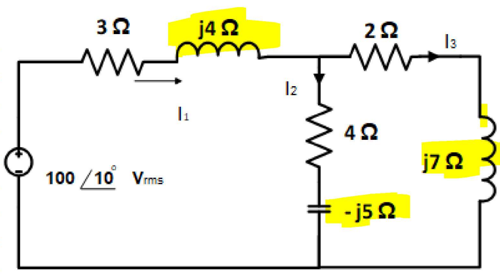
$$= \sqrt{P^2 + Q^2} \angle \tan^{-1} \frac{Q}{P}$$

$$PF = \cos\theta = \frac{P_{av}}{|\vec{S}|} \qquad \sin\theta = \frac{Q_L}{|\vec{S}|}$$

$$\tan\theta = \frac{Q}{P} \longrightarrow \theta = \tan^{-1}\frac{Q}{P}$$

$$P_a = |\vec{S}| = \sqrt{P_{av}^2 + Q^2} = V_{rms} I_{rms}$$

What are the VARs consumed by the circuit



$$Q = V_{rms} I_{rms} \sin(\theta_v - \theta_i)$$

$$\vec{I}_1 = \frac{100\angle 10^\circ}{Z_{tot}}$$

$$\rightarrow Z_{tot} = (3+j4) + (2+j7) \,//\, (4-j5)$$

$$= 10.35 + j4.55 = 11.3\angle 23.7^\circ \ \Omega$$