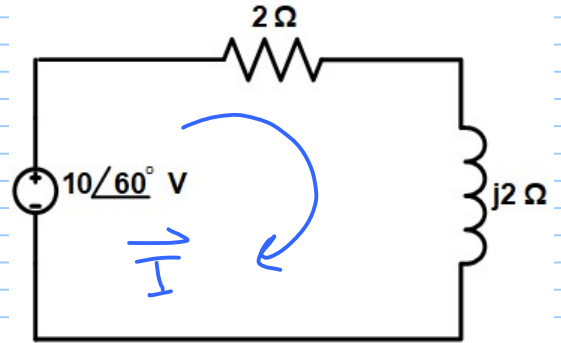


Example :

find the average power absorbed by each element.



$$P_{av} = \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i)$$

$$P_{av, j2} = Z_{avg}$$

$$\vec{I} = \frac{10\angle 60^\circ}{2 + j2} = 3.53\angle 15^\circ \text{ A}$$

$$\vec{V}_R = \frac{2}{2 + j2} * 10\angle 60^\circ = 7.07\angle 15^\circ \text{ Volt.}$$

OR $\vec{V}_R = \vec{I} R = 7.06\angle 15^\circ \text{ Volt.}$

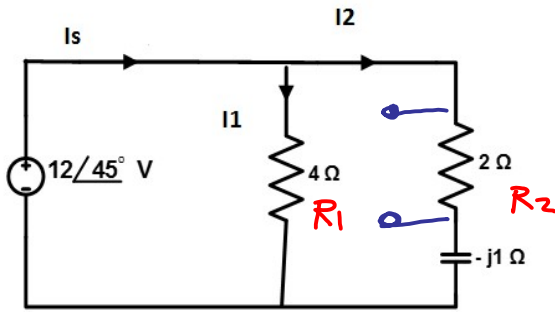
$$\begin{aligned} P_{av, R} &= \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i) \\ &= \frac{1}{2} * 7.07 * 3.53 \cos(15^\circ - 15^\circ) \\ &= 12.47 \text{ W} \end{aligned}$$

$$P_{av, R} = \frac{I_m^2 R}{2} = \frac{1}{2} * (3.53)^2 * (2) = 12.46 \text{ W}$$

→ The average power supplied by the source

$$\begin{aligned} P_{av} &= \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i) \\ &= \frac{1}{2} (10) (3.53) \cos(60^\circ - 15^\circ) \\ &= 12.48 \text{ W} \end{aligned}$$

Example :



Determine the average power absorbed by each resistor .
Determine the total average power supplied by the source .

$$\rightarrow P_{av R_1} = \frac{V_m^2}{2R_1} = \frac{12^2}{2 \times 4} = \underline{\underline{18 W}}$$

$$\rightarrow \vec{I}_2 = \frac{12 \angle 45^\circ}{2 - j} = 5.366 \angle 71.56^\circ \text{ A}$$

$$\frac{V_{R_2}}{2 - j} * 12 \angle 45^\circ$$

$$\rightarrow P_{av R_2} = \frac{1}{2} I_{m_2}^2 R_2 = \frac{1}{2} \times 5.366^2 \times 2 = \underline{\underline{28.79 W}}$$

$$\rightarrow P_{av V_S} = 18 + 28.79 = \underline{\underline{46.79 W}}$$

del.

$$\underline{\underline{OR}} \quad \vec{I}_f = \frac{\vec{V}_f}{Z_{eq}} = \vec{I}_1 + \vec{I}_2$$

$$= \frac{12 \angle 45^\circ}{4} + 5.366 \angle 71.56^\circ$$

$$= \underline{\underline{8.16 \angle 62.1^\circ \text{ A}}}$$

$$P_{av V_S} = \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i)$$

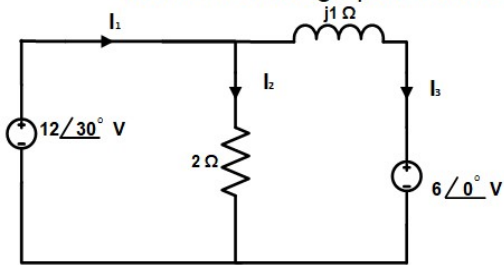
$$= \frac{1}{2} \times 12 \times 8.16 \times \cos(45 - 62.1)$$

$$= \underline{\underline{46.785 W}}$$

$$P_{abs.} = P_{del.}$$

Example :

Determine average power absorbed or supplied by each element .



$$\vec{I}_2 = \frac{12 \angle 30^\circ}{2} = 6 \angle 30^\circ \text{ A}$$

$$\vec{I}_3 = \frac{(12 \angle 30^\circ) - (6 \angle 0^\circ)}{j1 \angle 90^\circ} = 7.43 \angle -36.2^\circ \text{ A}$$

$$\vec{I}_1 = \vec{I}_2 + \vec{I}_3 = 6 \angle 30^\circ + 7.43 \angle -36.2^\circ = 11.28 \angle -7.08^\circ \text{ A}$$

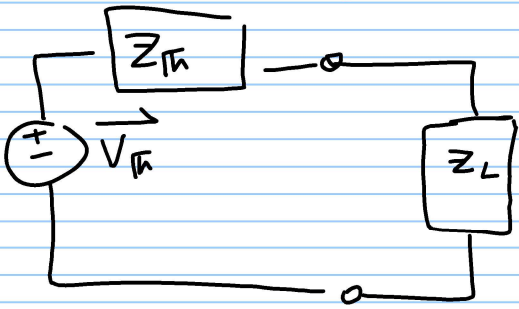
$$\rightarrow P_{av_{2\Omega}} = \frac{1}{2} I_{2m}^2 \times 2 = 36 \text{ W (absorbed)}$$
$$\frac{1}{2} I_m^2 R$$

$$\rightarrow P_{av_{j1\Omega}} = \text{Zero}$$

$$\rightarrow P_{av_{12\angle 30^\circ}} = \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i) = \frac{1}{2} 12 \times 11.28 \cos(30 - (-7.08))$$
$$= 54 \text{ W (supplied)}$$

$$\rightarrow P_{av_{6\angle 0^\circ}} = \frac{1}{2} \times 6 \times 7.43 \cos(0 - (-36.2))$$
$$= 18 \text{ W (absorbed)}$$

Maximum Power Transfer



$$Z_{th} = R_{th} + jX_{th}$$

$$Z_L = R_L + jX_L$$

$$\rightarrow P_L = \frac{1}{2} I_{Lm}^2 R_L$$

$$\begin{aligned} \vec{I} &= \frac{\vec{V}_{th}}{Z_{th} + Z_L} \\ &= \frac{V_{th}}{(R_{th} + R_L) + j(X_{th} + X_L)} \end{aligned}$$

$$\therefore P_L = \frac{1}{2} \frac{V_{th}^2 \cdot R_L}{(R_{th} + R_L)^2 + (X_{th} + X_L)^2}$$

$$\textcircled{1} \frac{\partial P_L}{\partial X_L} = 0 \quad \& \quad \textcircled{2} \frac{\partial P_L}{\partial R_L} = 0$$

$$* \Rightarrow \frac{\partial P_L}{\partial X_L} = \frac{1}{2} \frac{-2 V_{th}^2 R_L (X_L + X_{th})}{[(R_{th} + R_L)^2 + (X_{th} + X_L)^2]^2}$$

$$\text{for } \frac{\partial P_L}{\partial X_L} = 0 \rightarrow \boxed{X_L = -X_{th}}$$

$$* \Rightarrow \frac{\partial P_L}{\partial R_L} = \frac{V_{th}^2 [(R_L + R_{th})^2 + (X_L + X_{th})^2 - 2R_L(R_L + R_{th})]}{2 [(R_L + R_{th})^2 + (X_L + X_{th})^2]^2}$$

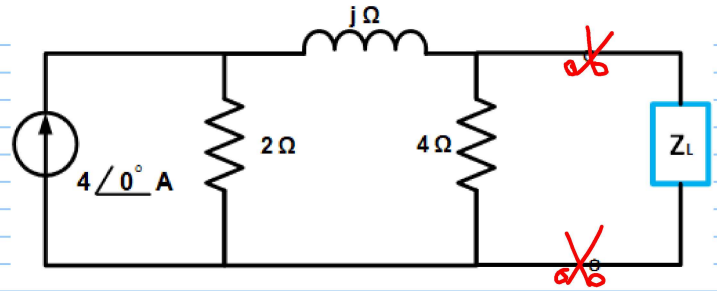
$$\text{for } \frac{\partial P_L}{\partial R_L} = 0 \rightarrow \boxed{R_L = R_{th}}$$

$$\boxed{Z_L = Z_{th}^*}$$

$$P_{L, \max} = \frac{1}{8} \frac{V_{th}^2}{R_{th}}$$

$$R_{th} = R_L$$

Example : Find Z_L for maximum average power transfer .
 Compute the maximum average power supplied to the load .



$$Z_{Th} = 4 \parallel (2 + j)$$

$$= 1.4 + j0.43 \Omega$$

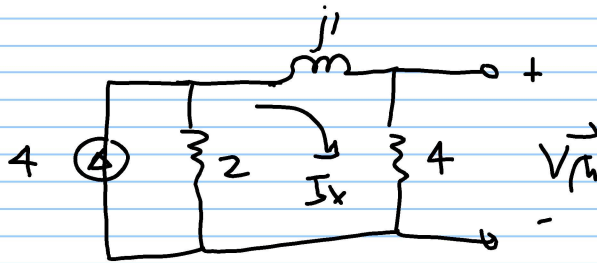
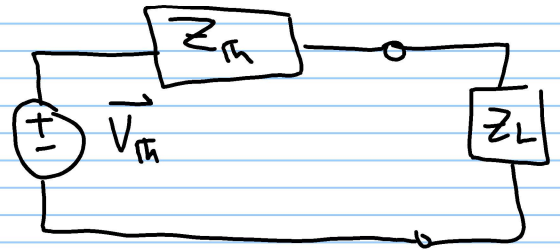
$$Z_L = Z_{Th}^* = 1.4 - j0.43 \Omega$$

$$P_{max} = \frac{1}{8} \frac{V_{Th}^2}{R_{Th}}$$

$$V_{Th} = \left(\frac{2 \times 4}{6 + j} \right) \times 4$$

$$= 5.28 \angle -9.46^\circ$$

$$\therefore P_{max} = \frac{1}{8} \frac{(5.28)^2}{1.4} = 2.489 \text{ W}$$



$$V_{Th} = 2I - 4 \angle 0^\circ$$

$$I = \frac{V_x + 4 \angle 0^\circ}{2 + j4}$$

$$V_x = -2I$$

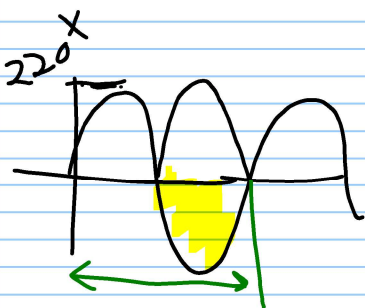
$$I = 0.707 \angle -45^\circ \text{ A}$$

$$I = \frac{-2I + 4}{2 + j4}$$

$$2I + j4I + 2I = 4$$

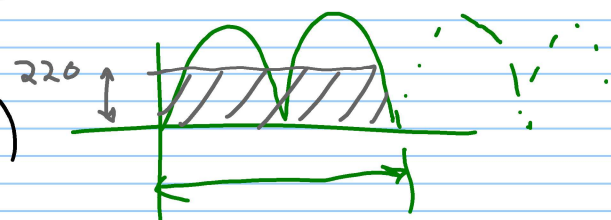
$$(4 + j4)I = 4$$

$$I = \frac{4}{\sqrt{4^2 + 4^2}} \angle 45^\circ$$



$$220 \cos(2\pi \times 50t + 0^\circ)$$

rms



$$\underline{V_{rms\ eff} = \frac{V_m}{\sqrt{2}}}$$

$$\underline{I_{rms} = \frac{I_m}{\sqrt{2}}}$$

$$P_{av} = \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i) \quad (\text{frowny face})$$

$$= V_{rms} I_{rms} \cos(\theta_v - \theta_i) \quad (\text{happy face})$$

$$P_{av\ R} = \frac{V_{rms}^2}{R} = I_{rms}^2 R \quad (\text{happy face}) = V_{rms} I_{rms}$$

$$P_{av} = \underbrace{V_{rms} I_{rms}}_{P_a \text{ (VA)}} \underbrace{\cos(\theta_v - \theta_i)}_{PF} \quad (W)$$

P_a (VA)

PF

apparent Power

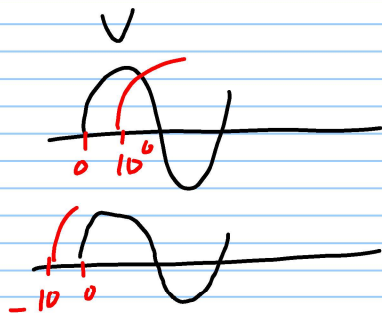
Power Factor

unity



$$\theta_v = \theta_i$$

$$PF = 1$$



$$\cos(\theta) = \cos(-\theta)$$

lagging PF



i lags v by θ

$$90^\circ > \theta_v - \theta_i > 0$$

$$1 > PF > 0$$

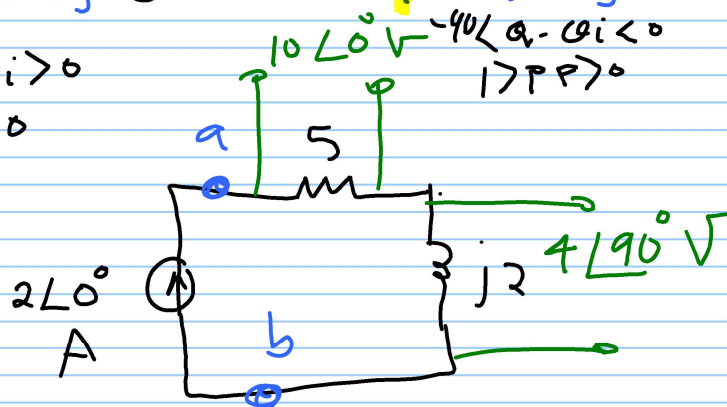
leading PF



i leads v by θ

$$-90^\circ < \theta_v - \theta_i < 0$$

$$1 > PF > 0$$



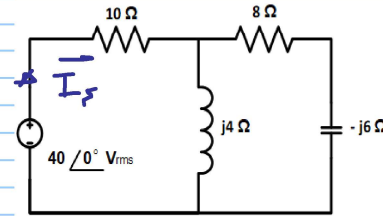
P.F. = $\cos(21.8^\circ) = 0.928$ lagging.

$$\left\{ \begin{aligned} V_{ab} &= (2L0^\circ)(5 + j2) \\ &= 10 + j4 = 10.77 \angle 21.8^\circ \end{aligned} \right.$$

→ Power factor is either leading or lagging, referring to the phase of the current with respect to the voltage.

EX 1

Calculate the power factor seen by the source and the average power supplied by the source.



$$Z = [(8 - j6) \parallel j4] + 10$$
$$= 12.69 \angle 20.62 \text{ } \Omega$$

$$\vec{I}_s = \frac{40 \angle 0^\circ}{Z} = 3.152 \angle -20.62 \text{ A}$$

$$PF = \cos(0^\circ - (-20.62)) = \cos(20.62)$$
$$= 0.936 \text{ lagging}$$

$$P_{av} = P_a \cdot PF = V_{rms} I_{rms} \times 0.936$$
$$= 40 \times 3.152 \times 0.936$$
$$= 118 \text{ W}$$

Example :

An industrial load consumes **11 kW** at 0.5 PF lagging from a $220 V_{rms}$ line . The transmission line resistance from the power company to the plant is 0.2Ω .

- 1) Determine the average power that must be supplied by the power company .
- 2) Repeat (1) if the power factor is changed to unity

① $P_{av, Load} = V_{rms} I_{rms} \cdot PF$

$$I_{rms} = \frac{P_{av}}{V_{rms} PF} = \frac{11K}{220 \times 0.5}$$
$$= \boxed{100 A}$$
$$P_{Loss} = (100)^2 \times 0.2 = 2 KW$$

$$\therefore P_{av, supplied} = P_{av, Load} + P_{Loss} = \underline{\underline{13 KW}}$$

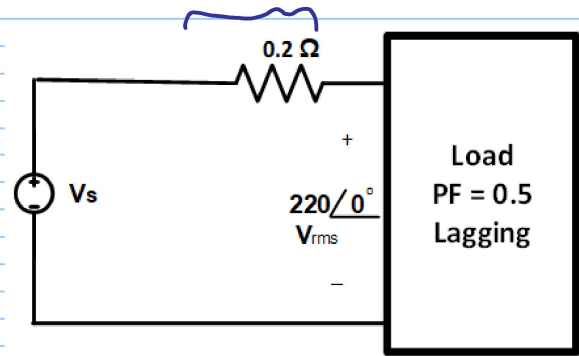
② $P_{av, Load} = V_{rms} \cdot I_{rms} \cdot PF$

$$11 \times 10^3 = 220 \cdot I_{rms} \cdot 1 \quad \boxed{I_{rms} = 50 A}$$

$$P_{Losses} = I_{rms}^2 R = (50)^2 \times 0.2 = 500W$$

$$P_{av, supplied} = 0.5k + 11k = \underline{\underline{11.5 KW}}$$

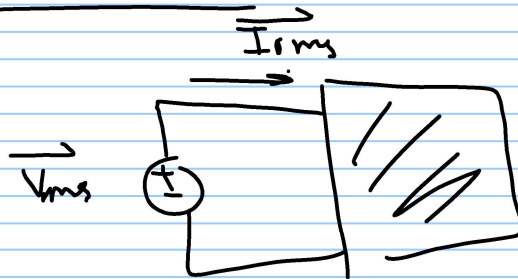
Transmission line



Complex Power

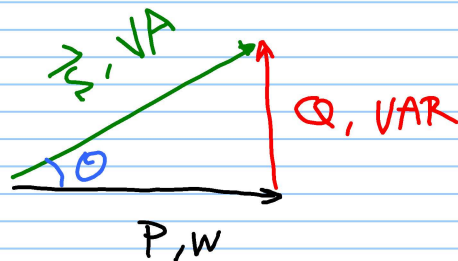
$$\vec{V}_{rms} = V_{rms} \angle \theta_v$$

$$\vec{I}_{rms} = I_{rms} \angle \theta_i$$



\vec{S} \equiv complex power

$$\vec{S} = \vec{V}_{rms} \cdot \vec{I}_{rms}^*$$



$$= V_{rms} I_{rms} \angle \theta_v - \theta_i$$

$$\theta = \theta_v - \theta_i$$

$$\vec{S} = \underbrace{V_{rms} I_{rms} \cos(\theta_v - \theta_i)}_{P_{av} \leftarrow \text{average power}} + j \underbrace{V_{rms} I_{rms} \sin(\theta_v - \theta_i)}_{Q \leftarrow \text{Reactive Power}}$$

R
 $\theta_v - \theta_i = 0$

$$Q = VI \sin 0 = 0$$

L
 $\theta_v - \theta_i = 90^\circ$

$$Q_L = V_{rms} I_{rms}$$

$$= (\omega L I_{rms}) I_{rms}$$

$$= \omega L I_{rms}^2$$

$$= \frac{V_{rms}^2}{\omega L}$$

$$X_L = \omega L$$

C
 $\theta_v - \theta_i = -90^\circ$

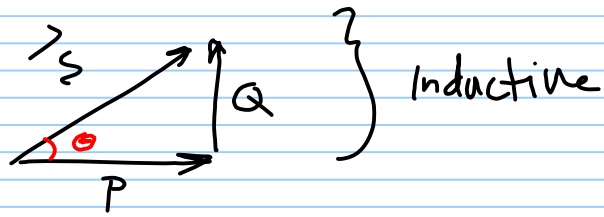
$$Q_C = -V_{rms} I_{rms}$$

$$= -V_{rms} (\omega C V_{rms})$$

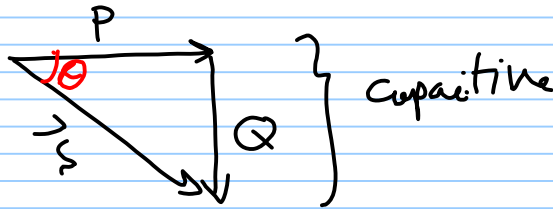
$$= -\omega C V_{rms}^2$$

$$= -\frac{I_{rms}^2}{\omega C}$$

$$X_C = \frac{-1}{\omega C}$$



$$\theta = \theta_v - \theta_i$$



$$\vec{S} = P + jQ$$

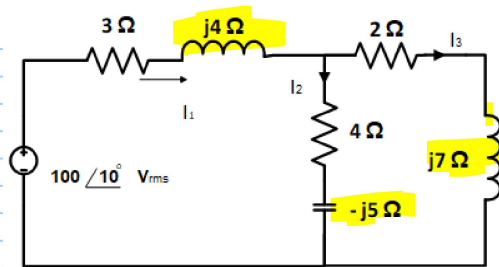
$$= \sqrt{P^2 + Q^2} \angle \tan^{-1} \frac{Q}{P}$$

$$PF = \cos \theta = \frac{P_{av}}{|\vec{S}|} \quad \left. \vphantom{PF} \right\} \quad \sin \theta = \frac{Q_L}{|\vec{S}|}$$

$$\tan \theta = \frac{Q}{P} \rightarrow \theta = \tan^{-1} \frac{Q}{P}$$

$$P_a = |\vec{S}| = \sqrt{P_{av}^2 + Q^2} = V_{rms} I_{rms}$$

What are the VARs consumed by the circuit



$$Q = V_{rms} I_{rms} \sin(\theta_v - \theta_i)$$

$$\vec{I}_1 = \frac{100 \angle 10^\circ}{Z_{tot}}$$

$$\begin{aligned} \rightarrow Z_{tot} &= (3 + j4) + (2 + j7) \parallel (4 - j5) \\ &= 10.35 + j4.55 = 11.3 \angle 23.7^\circ \Omega \end{aligned}$$

$$\therefore \vec{I}_1 = 8.84 \angle -13.7^\circ \text{ A}$$

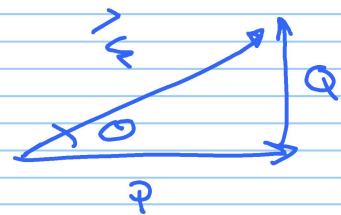
$$\begin{aligned} \rightarrow Q &= (100)(8.84) \sin(10 - (-13.7^\circ)) \\ &= 355 \text{ VAR} \end{aligned}$$

$$Q_L = \omega L I_{rms}^2 = X_L I_{rms}^2$$

$$Q_C = -\frac{I_{rms}^2}{\omega C} = X_C I_{rms}^2$$

Example : A certain industrial plant consumes 1 MW at 0.7 lagging power factor and a 2300 V_{rms} .

What is the minimum capacitor required to improve the power factor to 0.9 lagging. ($\omega = 377 \text{ rad/s}$)



\rightarrow 0.7 lagging

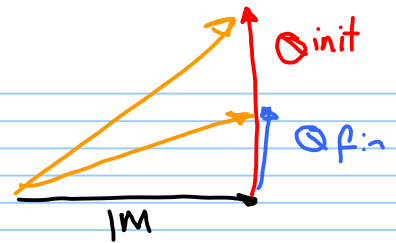
$$\begin{aligned} Q_{init} &= P \tan \theta \\ &= P \tan [\cos^{-1} 0.7] \\ &= 1 \text{ M} \tan(\cos^{-1} 0.7) \\ &= 1.02 \text{ M VAR} \end{aligned}$$

$$\begin{aligned} \cos \theta &= \frac{P}{|S|} \\ \tan \theta &= \frac{Q}{P} \end{aligned}$$

\rightarrow 0.9 lagging

$$\begin{aligned} Q_{fin} &= P \tan \cos^{-1}(0.9) = 1 \text{ M} \times \tan(\cos^{-1} 0.9) = \\ &= 0.484 \text{ M VAR} \end{aligned}$$

$$\begin{aligned} \therefore Q_c &= Q_{\text{final}} - Q_{\text{init}} \\ &= -0.536 \text{ MVAR} \end{aligned}$$



$$Q_c = -\frac{V_{\text{rms}}^2}{X_c} = -\omega C V_{\text{rms}}^2$$

$$\therefore C = \frac{Q_c}{\omega V_{\text{rms}}^2} = \frac{0.536 \text{ M}}{377 \times 2300} = 269 \mu\text{F}$$

$$\rightarrow I_{\text{ini}} = \frac{P_{\text{av}}}{V_{\text{rms}} \cos \theta} = \frac{1 \text{ M}}{2300 \times 0.7} = 621 \text{ A}$$

$$\rightarrow I_{\text{final}} = \frac{P_{\text{av}}}{V_{\text{rms}} \text{ PF}} = \frac{1 \text{ M}}{2300 \times 0.9} = 483 \text{ A}$$