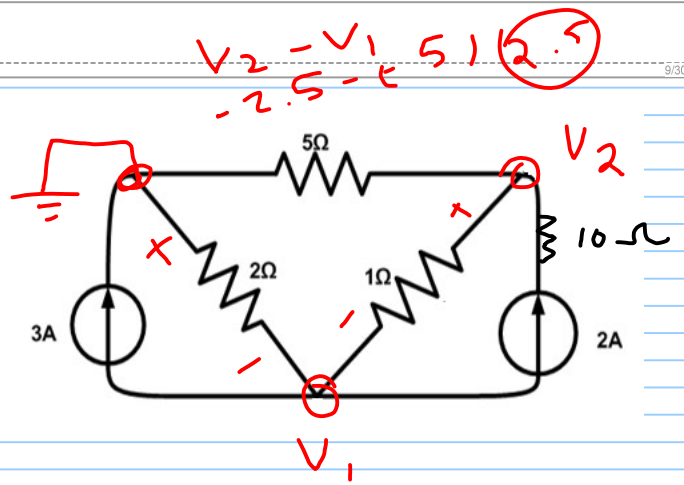


$$\underline{\underline{CN = 4}}$$

Nodal  
method

@ node  $V_1$

All currents are out }  
OR " ~ " IN }



$$\underline{I_{out}} \quad 3 + \frac{V_1}{2} + \frac{V_1 - V_2}{1} + 2 = 0$$

$$\sum I_{out} = 0 \quad \text{OR}$$

$$\sum I_{IN} = 0$$

$$(1.5) \quad V_1 - V_2 = -5 \quad \text{--- (1)}$$

@ Node  $V_2$

$$\underline{I_{out}} \quad -2 + \frac{V_2 - V_1}{1} + \frac{V_2}{5} = 0$$

$$-V_1 + 1.2V_2 = 2 \quad \text{--- (2)}$$

$$V_1 = -5 \text{ V}$$

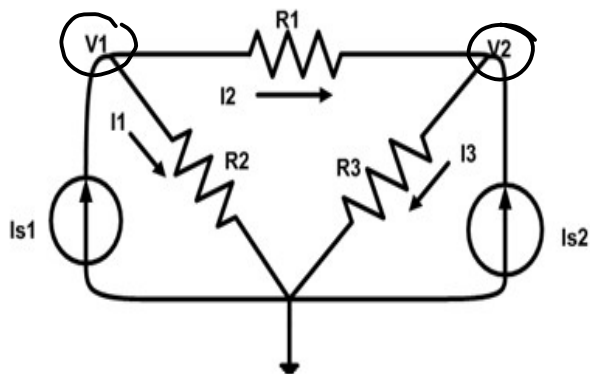
$$V_2 = -2.5 \text{ V}$$

@ node  $V_1$

$$\left(\frac{1}{R_1} + \frac{1}{R_2}\right)V_1 - \left(\frac{1}{R_1}\right)V_2 = I_{S1}$$

@ node  $V_2$

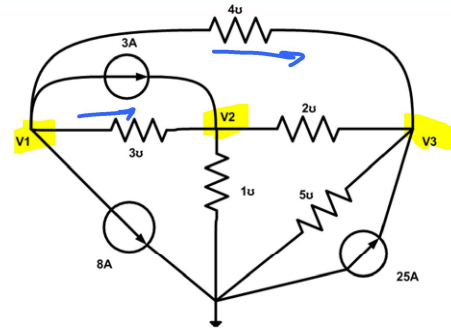
$$-\left(\frac{1}{R_1}\right)V_1 + \left(\frac{1}{R_1} + \frac{1}{R_3}\right)V_2 = I_{S2}$$



@ node 1

$$\sum I_{out} = \sum I_{in}$$

$$7V_1 - 3V_2 - 4V_3 = -11 \quad \text{--- (1)}$$



@ node  $V_2$

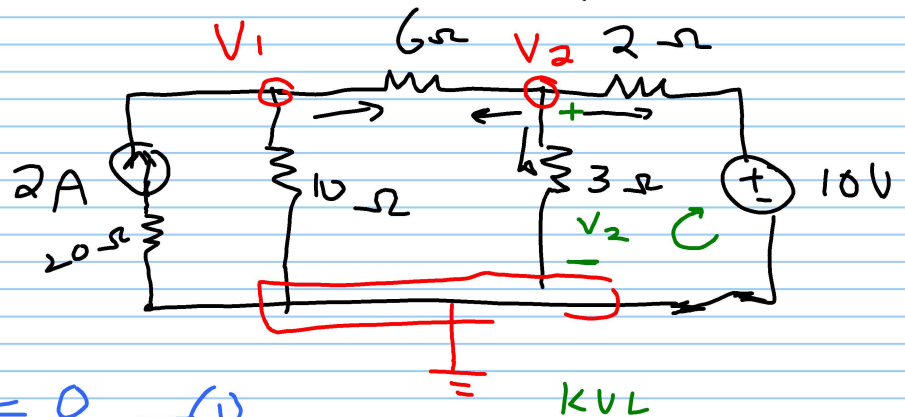
$$3(V_1 - V_2) + 4(V_1 - V_3) + 3 + 8 = 0$$

$$-(3)V_1 + (3+2+1)V_2 - (2)V_3 = 3 \quad \text{--- (2)}$$

@ node  $V_3$

$$-4V_1 - 2V_2 + (5+2+4)V_3 = 25 \quad \text{--- (3)}$$

EX



$$\sum I_{out} = \sum I_{in}$$

node 1

$$-2 + \frac{V_1}{10} + \frac{V_1 - V_2}{6} = 0 \quad \text{--- (1)}$$

node 2

$$\sum I_{out} = 0$$

$$\frac{V_2 - V_1}{6} + \frac{V_2}{3} + \frac{V_2 - 10}{2} = 0 \quad \text{--- (2)}$$

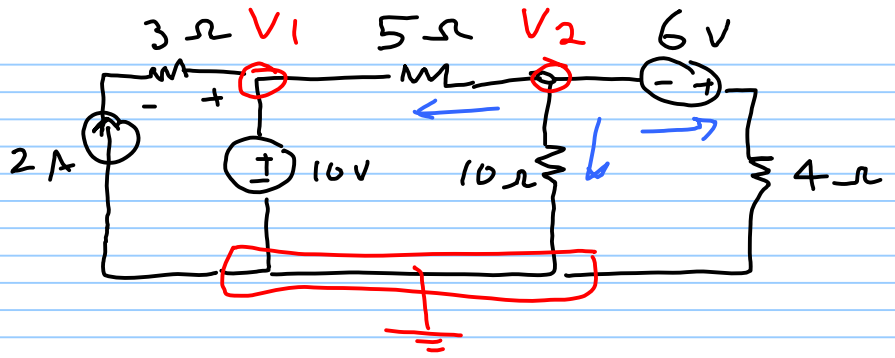
$$-V_2 + 2I_x + 10 = 0$$

$$I_x = \frac{V_2 - 10}{2}$$

$$V_1 = \text{---}$$

$$V_2 = \text{---} \quad \checkmark$$

EX1



$$(-2) + (V_1 - 10) = 0$$

$$V_1 = 10 \text{ V} \quad (\text{smiley face})$$

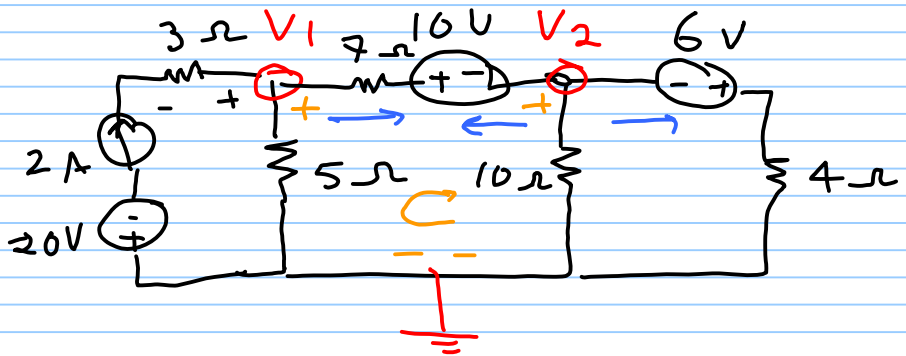
@ node V2

$$\frac{V_2 - 10}{5} + \frac{V_2}{10} + \frac{V_2 + 6}{4} = 0$$

$$0.55 V_2 = 2 - 1.5$$

$$V_2 = \frac{0.5}{0.55} = 0.909 \text{ Volt}$$

EX2



$$\sum I_{out} = 2 \text{ A}$$

@ node V1

$$-2 + \frac{V_1}{5} + \frac{V_1 - 10 - V_2}{7} = 0 \quad \text{--- (1)}$$

@ node V2

$$\frac{V_2 - V_1 + 10}{7} + \frac{V_2}{10} + \frac{V_2 + 6}{4} = 0 \quad \text{--- (2)}$$

## Special case

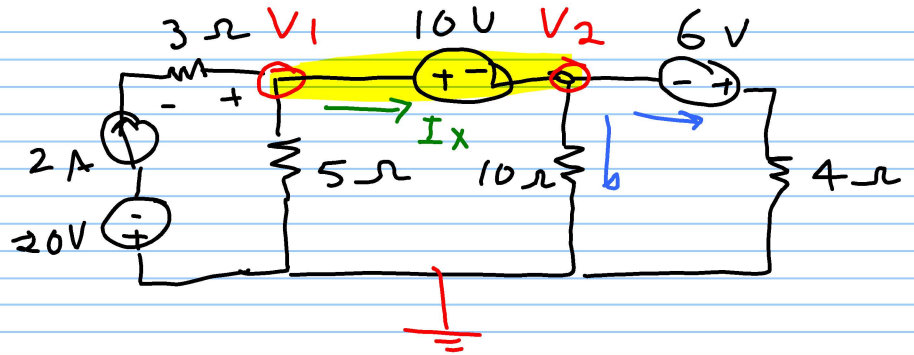
EX

@ node  $V_1$

$$-2 + \frac{V_1}{5} + \underline{I_x} = 0$$

$$-2 + \frac{V_1}{5} + \frac{V_2}{10} + \frac{V_2 + 6}{4} = 0 \quad \text{--- (1) KVL}$$

$$\underline{V_1 - V_2 = 10} \quad \text{--- (2) KVL @ supernode}$$



EX

$$I_o = \frac{V_2}{3k} \quad \text{--- (1)}$$

@ node  $V_1$

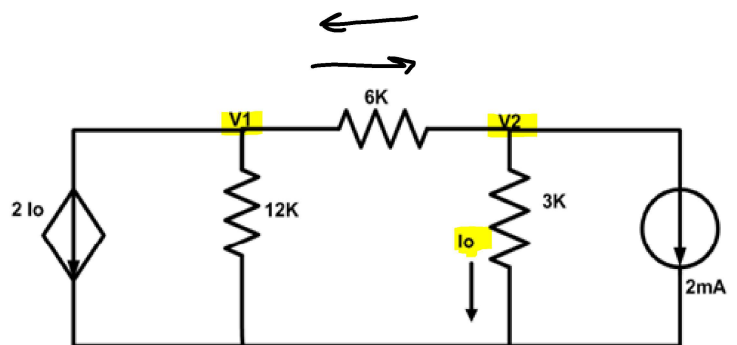
$$2I_o + \frac{V_1}{12k} + \frac{V_1 - V_2}{6k} = 0$$

$$2\left(\frac{V_2}{3k}\right) + \frac{V_1}{12k} + \frac{V_1 - V_2}{6k} = 0 \quad \text{--- (1) } \checkmark$$

@ node  $V_2$

$$\frac{V_2 - V_1}{6k} + \frac{V_2}{3k} + 2mA = 0 \quad \text{--- (2) } \checkmark$$

$$V_1 = -\frac{24}{5} V, \quad V_2 = \frac{12}{5} V$$



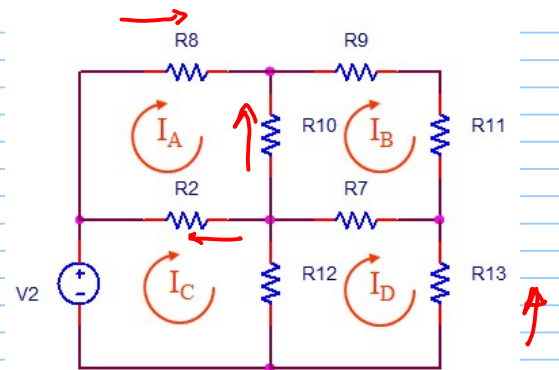
## Mesh method

$$I_{R_3} = I_A$$

$$I_{R_2} = I_A - I_C$$

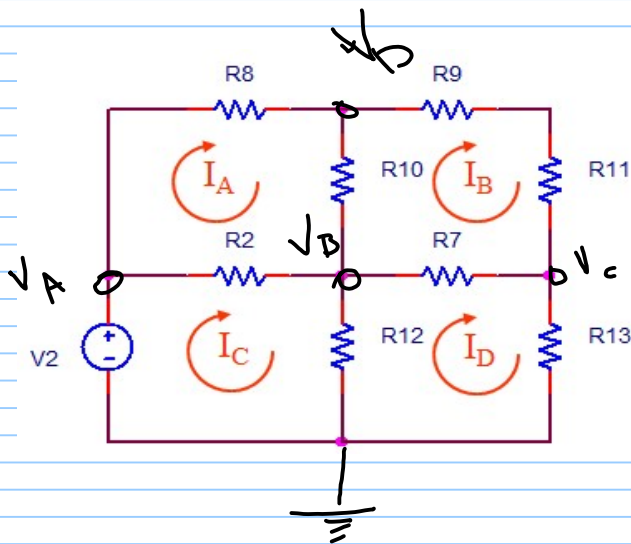
$$I_{R_{13}} = -I_D$$

$$I_{R_{10}} = I_B - I_A$$



4 equations

4 unknowns  $I_A, I_B, I_C$   
 ~~$I_D$~~



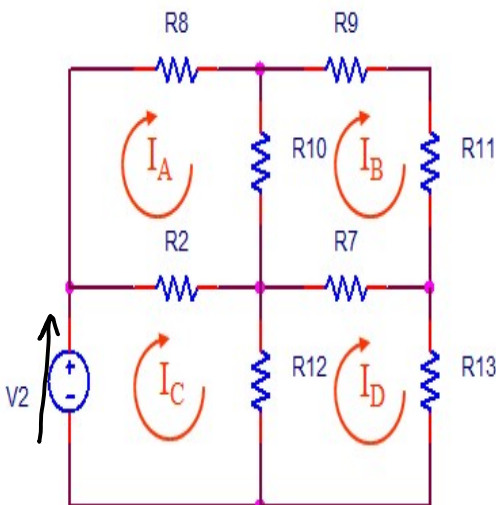
$$V_A = V_2$$

Nodal method

3 eqs

3 unknowns

$$V_B, V_C, V_D$$



## Mesh KVL

$$R_8 I_A + R_{10} (I_A - I_B) + R_2 (I_A - I_C) = 0$$

$$(R_8 + R_{10} + R_2) I_A - R_{10} I_B - R_2 I_C = 0 \quad (1)$$

$$- (R_{10}) I_A + (R_{10} + R_9 + R_{11} + R_7) I_B$$

$$- 0 I_C - R_7 I_D = 0 \quad (2)$$

$$-V_2 + R_2 (I_C - I_A) + R_{12} (I_C - I_D) = 0$$

$$(R_2 + R_{12}) I_C - R_2 I_A - R_{12} I_D = V_2 \quad (3)$$

$$(R_{12} + R_7 + R_{13})I_D - 0I_A - R_7I_B - R_{12}I_C = 0 \quad \text{--- (4)}$$

EX]

$$-42 + 6I_1 + 3(I_1 - I_2) = 0$$

$$9I_1 - 3I_2 = 42 \quad \text{--- (1)}$$

$$-3I_1 + 7I_2 = 10 \quad \text{--- (2)}$$

6A, 4A

special case 1 | current source exists  
only in one mesh

EX]

$$I_2 = -5A \quad \checkmark$$

$$-10 + 4I_1 + 6(I_1 - 5) = 0$$

$$I_1 = -2A \quad \checkmark$$

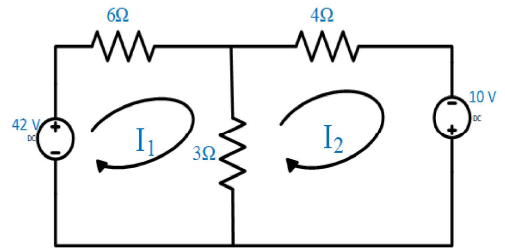


Figure 3: example 1 of mesh analysis

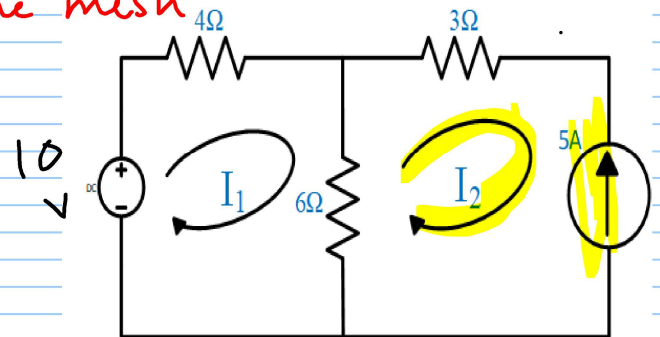


Figure 6: mesh with current source.

Case 2:

Current source exists between two meshes, a **Super mesh** is obtained

→ KVL for mesh (2)

$$(1+2+3)I_2 - (1)I_1 - (3)I_3 = 0 \text{ --- (1)}$$

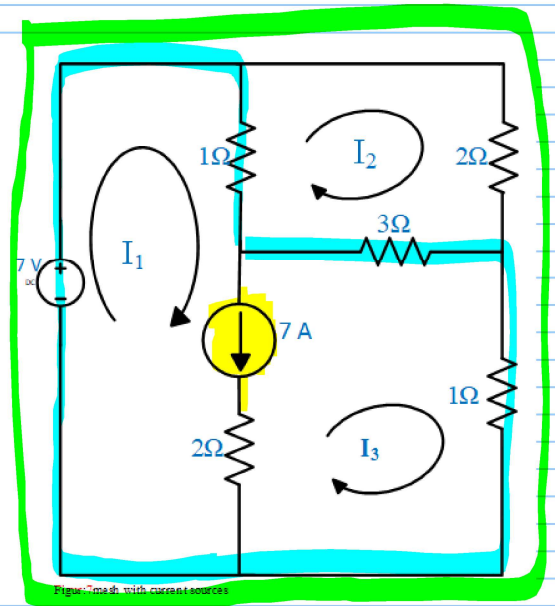
→ constraint equation

$$I_1 - I_3 = 7 \text{ --- (2)}$$

→ **super mesh equation**

$$-7 + (1)(I_1 - I_2) + (3)(I_3 - I_2) + (1)I_3 = 0$$

$$I_1 - 4I_2 + 4I_3 = 7 \text{ --- (3)}$$



$$I_3 = -7A \times$$

**OR**

$$-7 + 2I_2 + I_3 = 0 \text{ --- (3)*}$$

EX Mesh Analysis with dependent sources

$$V_x = (3)(I_3 - I_2)$$

$$I_1 = 15A \checkmark$$

→ constraint equation

$$\frac{V_x}{9} = I_3 - I_1$$

$$\frac{3}{9}(I_3 - I_2) = I_3 - 15$$

$$\frac{2}{9}I_2 + \frac{2}{3}I_3 = 15$$

$$I_2 + 2I_3 = 45 \text{ --- (1)}$$

$$(1)(I_2 - 15) + (2)I_2 + (3)(I_2 - I_3) = 0$$

$$6I_2 - 3I_3 = 15 \text{ --- (2)}$$

$$I_2 = 11A \checkmark$$

$$I_3 = 17A \checkmark$$

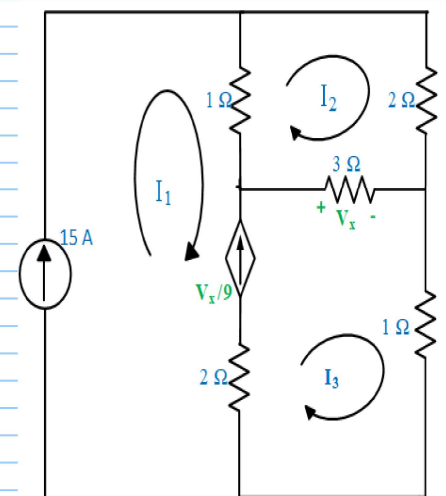


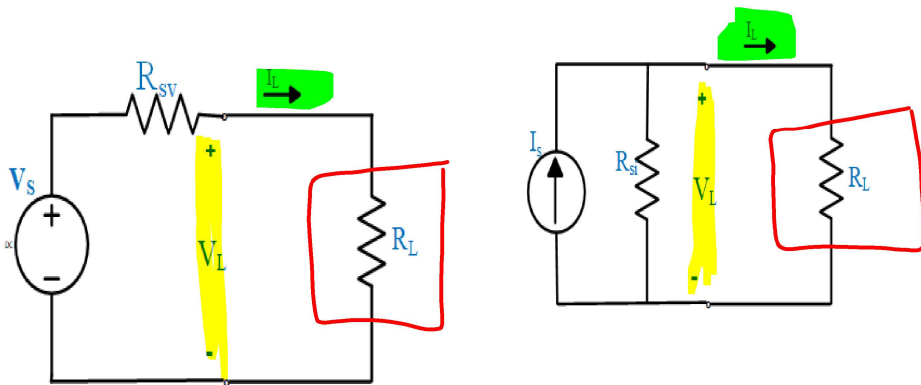
Figure Shows mesh with dependent current source.

Node or mesh: How to choose?

- Use the one with fewer equations.
- Use the method you like best.

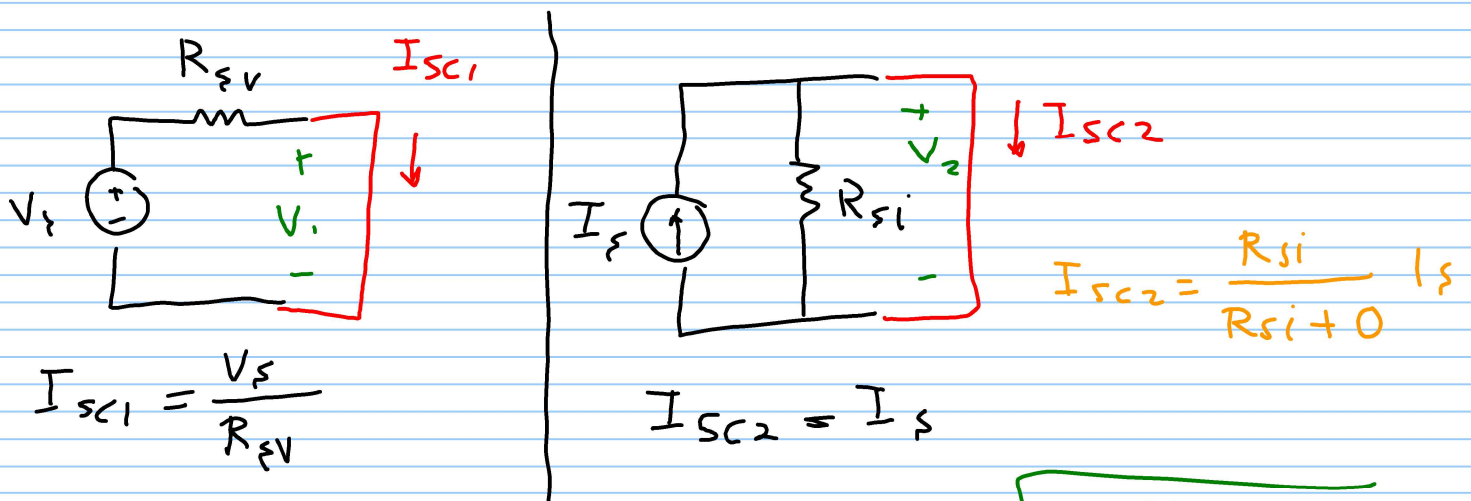
slides

### Source Transformation



Two sources are equivalent, if each produces identical current and identical voltage in any load which is placed across its terminal.

↳ let  $R_L = \text{Zero}$  (short circuit s.c.)



$$I_{sc1} = \frac{V_s}{R_{sv}}$$

$$I_{sc2} = I_s$$

$$I_{sc2} = \frac{R_{si}}{R_{si} + 0} I_s$$

for  $I_{sc1} = I_{sc2}$

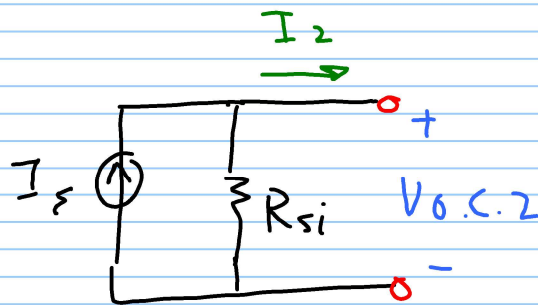
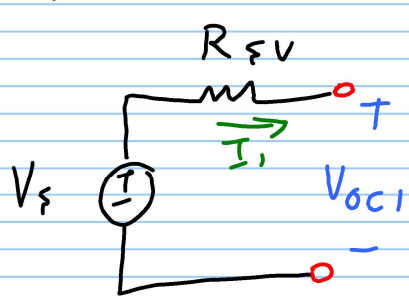
$$I_s = \frac{V_s}{R_{sv}}$$

$$V_1 = V_2 = \text{Zero}$$



let  $R_L = \infty$  (open circuit)

$I_1 = I_2 = \text{zero}$

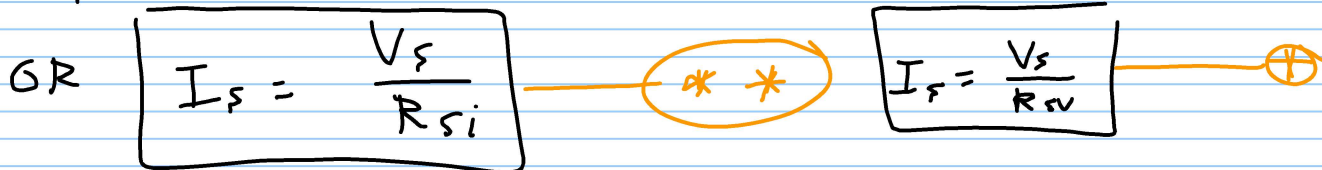


$V_{oc1} = V_s$

$V_{oc2} = I_s R_{si}$

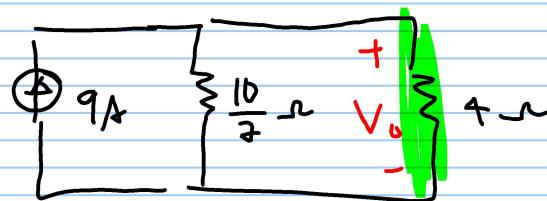
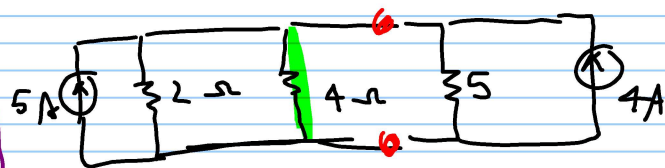
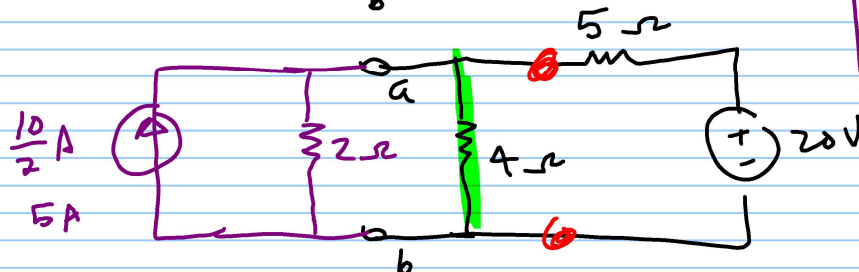
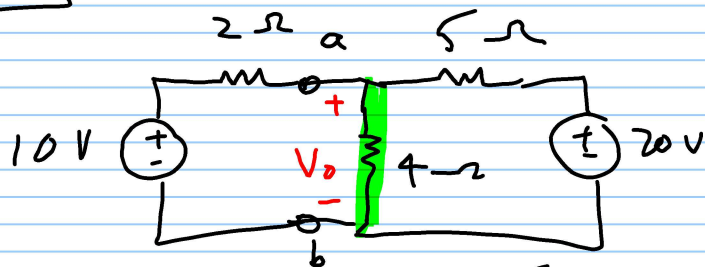
For  $V_{oc1} = V_{oc2}$

$V_s = I_s R_{si}$



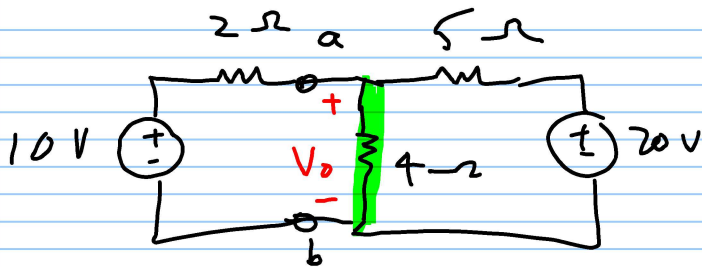
$R_{si} = R_{sV}$

Ex 1 find  $V_o$



$\left( \frac{10/7}{10/4 + 4} \cdot 9 \right) (4) = V_o$

$V_o = 9.47 \text{ Volt}$



OR  
Nodal

$$\frac{V_o - 10}{2} + \frac{V_o}{4} + \frac{V_o - 20}{5} = 0$$

$$V_o \left( \frac{1}{2} + \frac{1}{4} + \frac{1}{5} \right) = \frac{10}{2} + \frac{20}{5}$$

$$V_o (0.95) = 9$$

$$V_o = 9.473 \text{ Volt.}$$

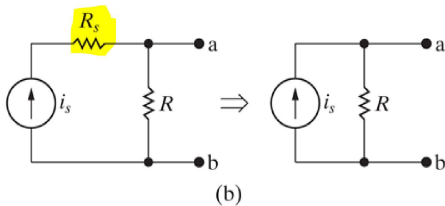
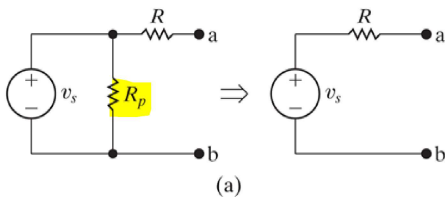


Figure: 04-39a,b

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↙  
Thevenin &  
Norton eq. circuit

- The two circuits depicted in Fig. 4.39(a) are equivalent with respect to terminals a,b because they produce the same voltage and current in any resistor  $RL$  inserted between nodes a,b.
- The same can be said for the circuits in Fig. 4.39(b).

Example: Find  $V_o$  using source transformation

*neglect*

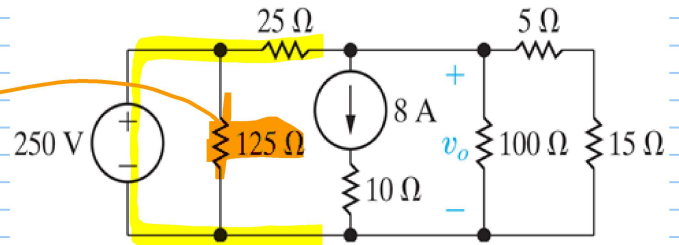
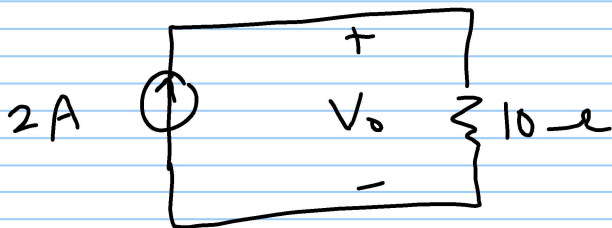
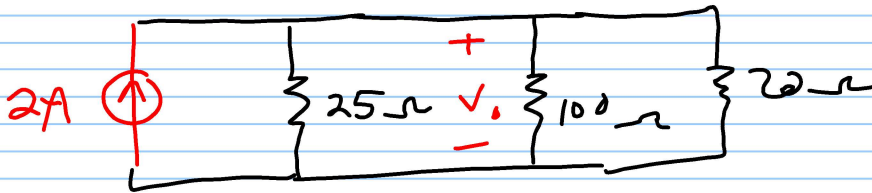
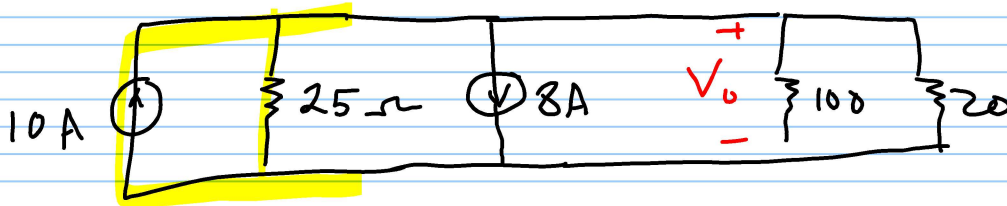
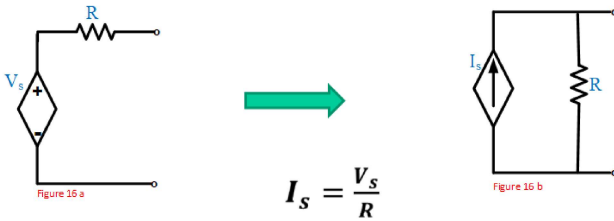


Figure: 04-40Ex4.9  
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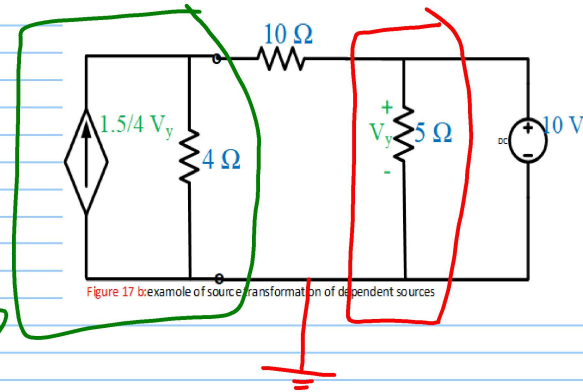
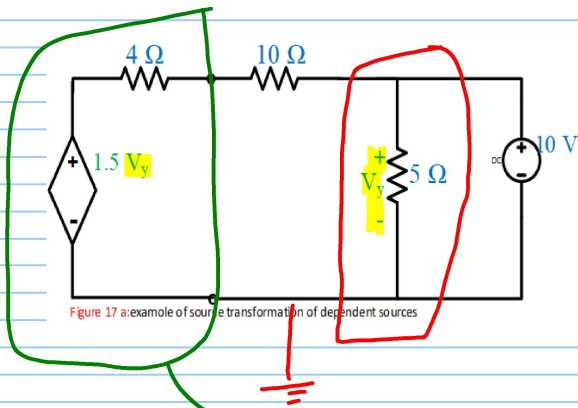


$$V_o = (2A)(10\Omega) = 20 \text{ Volt.}$$

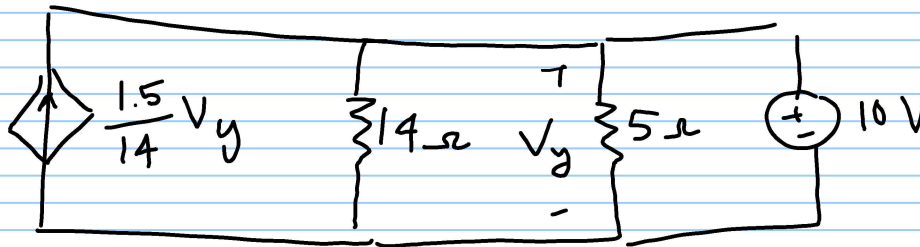
## Dependent sources



The control variable must be outside the transformation.



OR



## The Superposition Theorem

In a linear network, the voltage across or the current through any element may be calculated by adding algebraically all the individual voltages or currents caused by the **separate independent sources acting alone**, i.e. with:

- ② 1) All other **independent voltage** sources replaced by short circuits.
- ③ 2) All other **independent current** sources replaced by open circuits.
- ① ➤ **Dependent sources are left intact because they are controlled by circuit variables.**

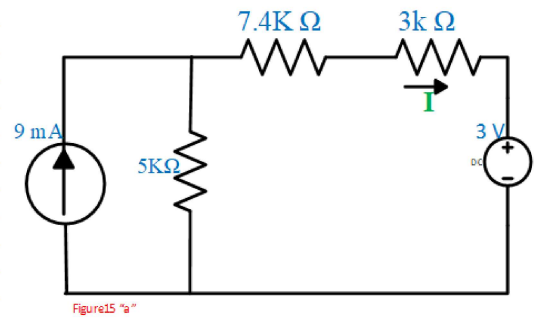


### Steps to apply superposition principle

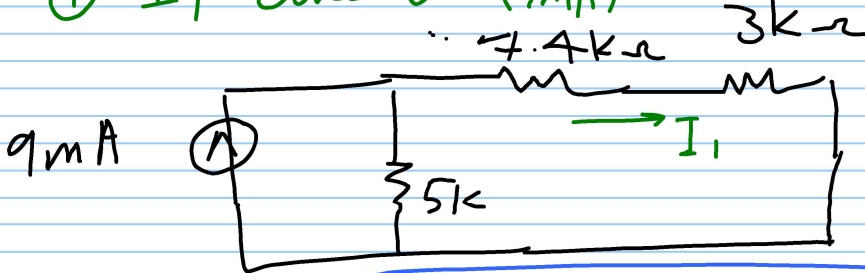
- Turn off all independent sources except one source. Find the output (voltage or current) due to that source using nodal, mesh, ...
- Repeat step 1 for each of the other independent sources.
- Find the total contribution by adding algebraically all contributions due to each independent sources.

EX)

find (I) using  
superposition

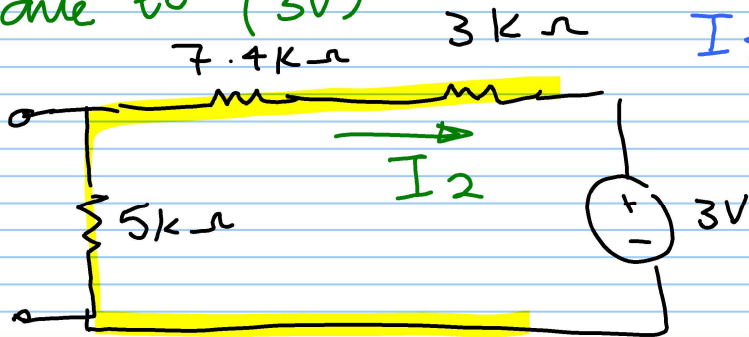


①  $I_1$  due to (9mA)



$$I_1 = \frac{5k}{5k + 7.4k + 3k} \times 9m$$
$$= 2.922 \text{ mA}$$

②  $I_2$  due to (3V)



$$I_2 = \frac{-3}{15.4k}$$
$$= -0.194 \text{ mA}$$

$$\therefore I = I_1 + I_2$$
$$= 2.992 - 0.194$$
$$= 2.728 \text{ mA}$$

Use superposition to solve for  $i_x$

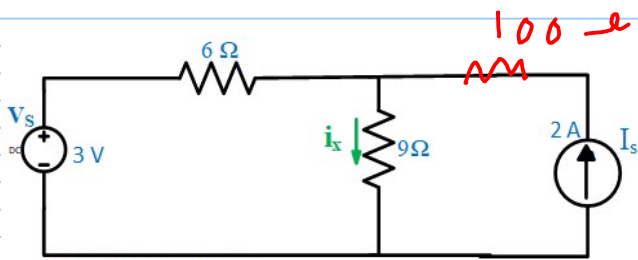
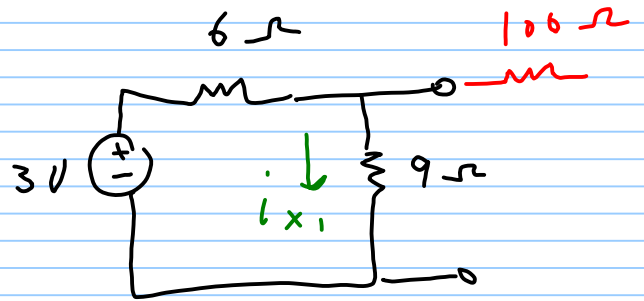


Figure 1. "a" example of superposition

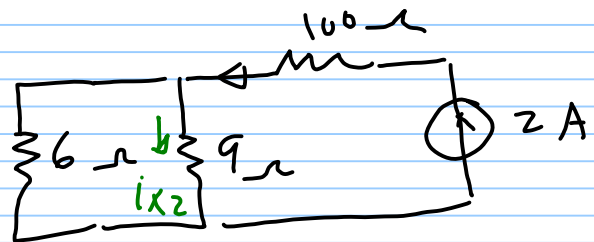
①  $i_{x1}$  due to (3V)

$$i_{x1} = \frac{3}{9+6} = 0.2 \text{ A}$$



②  $i_{x2}$  due to 2A

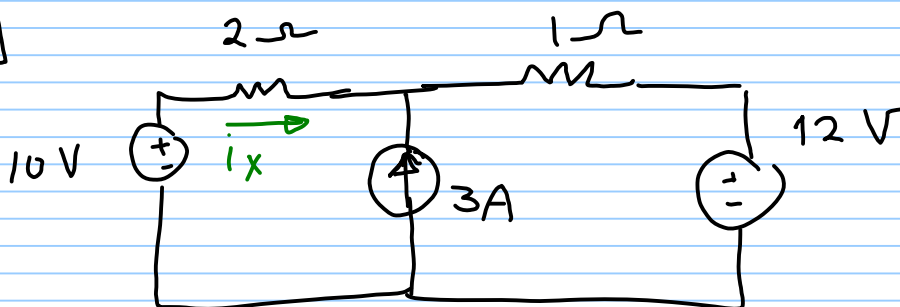
$$i_{x2} = \frac{6}{6+9} 2\text{A} = 0.8 \text{ A}$$



$$\text{So } i_x = i_{x1} + i_{x2} = 0.2 + 0.8 = 1 \text{ A}$$



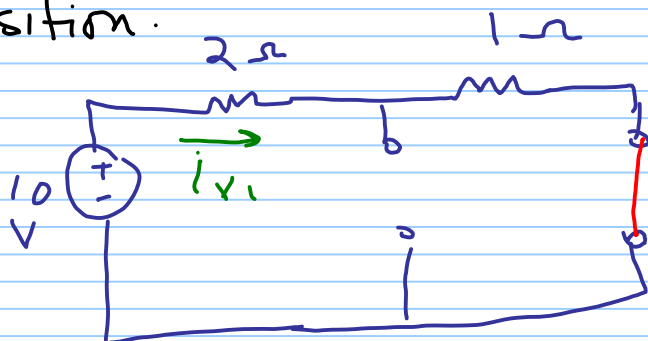
EX1



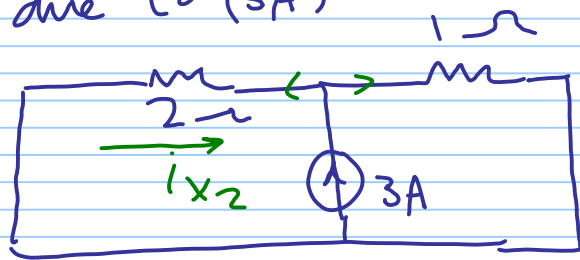
find  $I_x$  using superposition.

①  $i_{x1}$  due to (10V)

$$i_{x1} = \frac{10}{3} \text{ A}$$

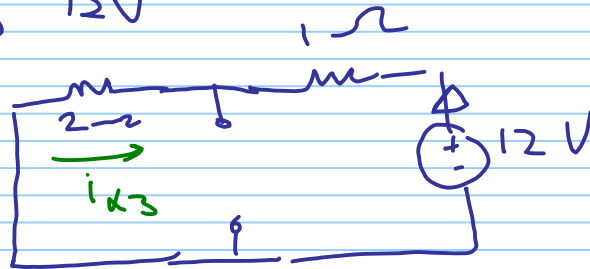


②  $i_{x2}$  due to (3A)



$$i_{x2} = \frac{1}{3} \cdot 3 = -1 \text{ A}$$

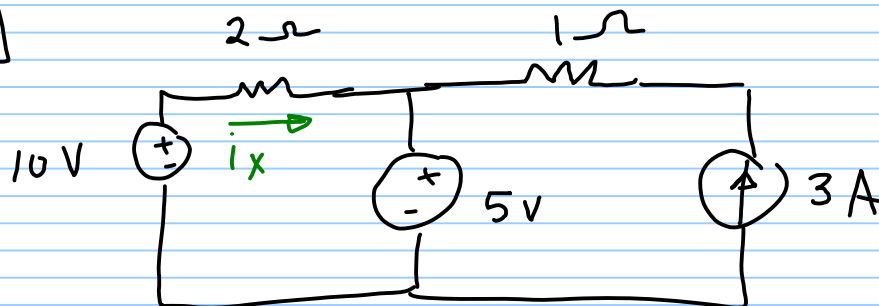
③  $i_{x3}$  due to 12V



$$i_{x3} = -\frac{12}{3} = -4 \text{ A}$$

$$\begin{aligned} \therefore I_x &= i_{x1} + i_{x2} + i_{x3} \\ &= 3.33 - 1 - 4 \\ &= -1.666 \text{ A} \end{aligned}$$

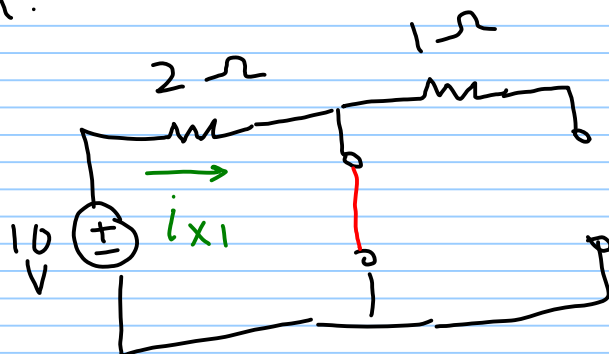
EX1



find  $I_x$  using superposition.

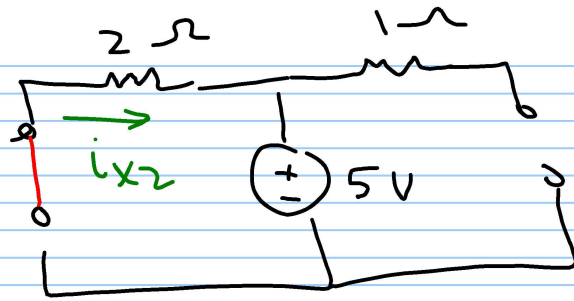
①  $i_{x1}$  due to (10V)

$$i_{x1} = \frac{10}{2} = 5 \text{ A}$$



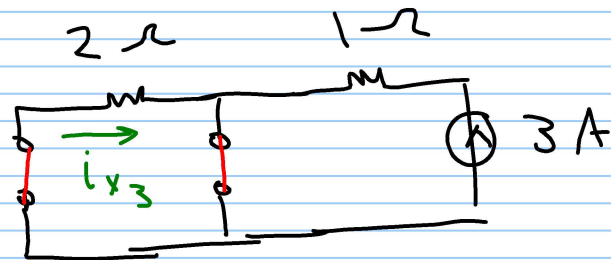
②  $i_{x_2}$  due to (5V)

$$i_{x_2} = -\frac{5}{2} = -2.5A$$



③  $i_{x_3}$  due to (3A)

$$i_{x_3} = \text{Zero}$$



$$\begin{aligned} \text{or } i_x &= i_{x_1} + i_{x_2} + i_{x_3} \\ &= 5 - 2.5 + 0 \\ &= 2.5A \end{aligned}$$

Find  $i_x$  using superposition

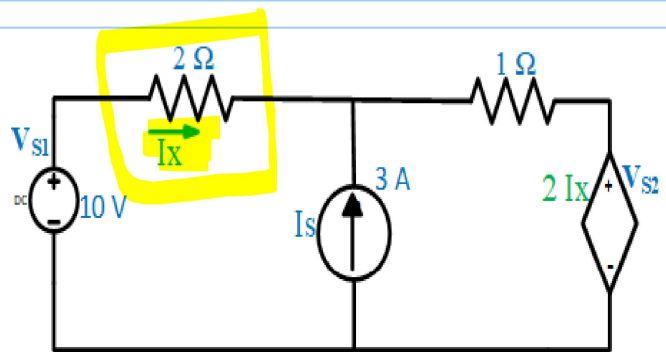


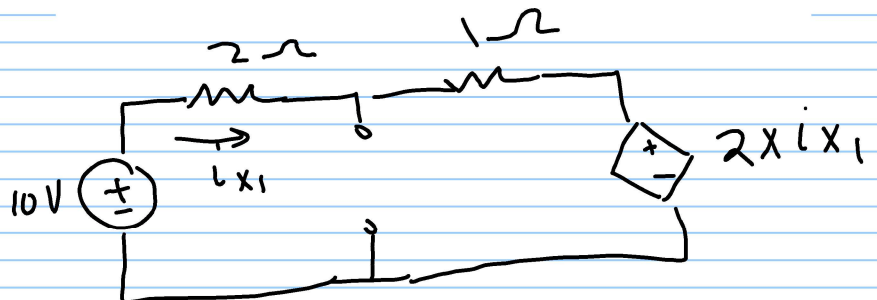
Figure 1.11: Superposition with a dependent source

①  $i_{x_1}$  due to 10V

KVL

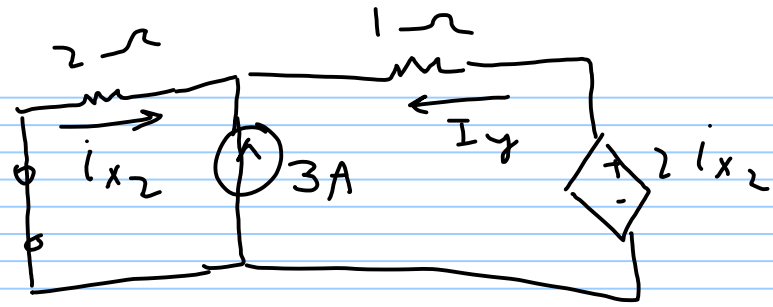
$$-10 + 3i_{x_1} + 2i_{x_1} = 0$$

$$i_{x_1} = 2A$$





②  $i_{x2}$  due to (3A)



KVL

$$2i_{x2} - I_y + 2i_{x2} = 0$$

$$4i_{x2} - I_y = 0$$

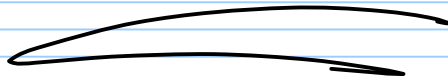
KCL  $I_{x2} + I_y = 3$

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$$5i_{x2} = 3$$

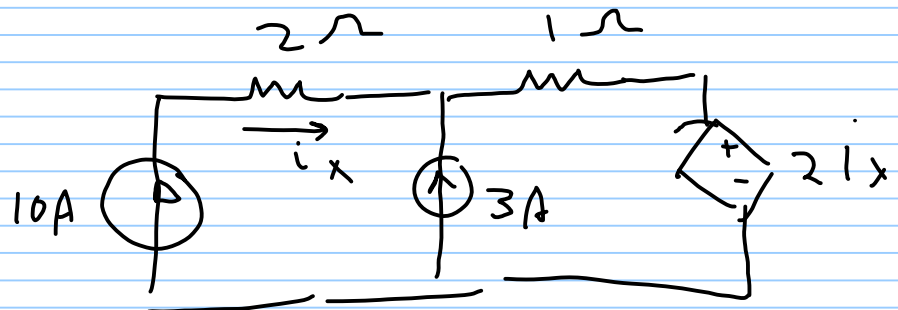
$$i_{x2} = 3/5 = -0.6 \text{ A}$$

$$\therefore I_x = i_{x1} + i_{x2} = 2 - 0.6 = 1.4 \text{ A}$$



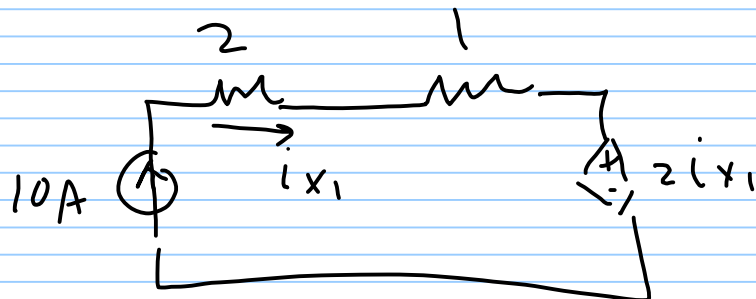
Ex Find  $i_x$

$i_x = 10 \text{ A}$



①  $i_{x1}$  due to (10A)

$$i_{x1} = 10 \text{ A}$$



(2)  $i_{x2}$  due to (3A)

$$i_{x2} = 0$$

