Homework Solutions ch. 8 EE 231

8.8

$$v(t) = -8e^{-250t} + 32e^{-1000t} \quad V \quad \text{for } t > 0$$

$$s_1 = -250, \quad s_2 = -1000$$

$$s_1 = -\alpha + \sqrt{\alpha^2 - \omega_0^2} = -250$$
$$s_2 = -\alpha - \sqrt{\alpha^2 - \omega_0^2} = -1000$$

$$\therefore \quad s_1 + s_2 = -1250 = -2\alpha$$

$$\therefore \quad \alpha = 625 \text{ r/s}$$

$$\text{But} \quad \alpha = \frac{1}{2RC} = 625$$

$$C = 0.1 \mu F$$

$$\therefore \quad R = 8 k\Omega$$

$$s_1 - s_2 = 1000 - 250 = 750 = 2\sqrt{\alpha^2 - \omega_0^2}$$

$$\therefore \quad \omega_0 = 500 \text{ r/s}$$

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

$$\therefore \quad L = 40 H$$

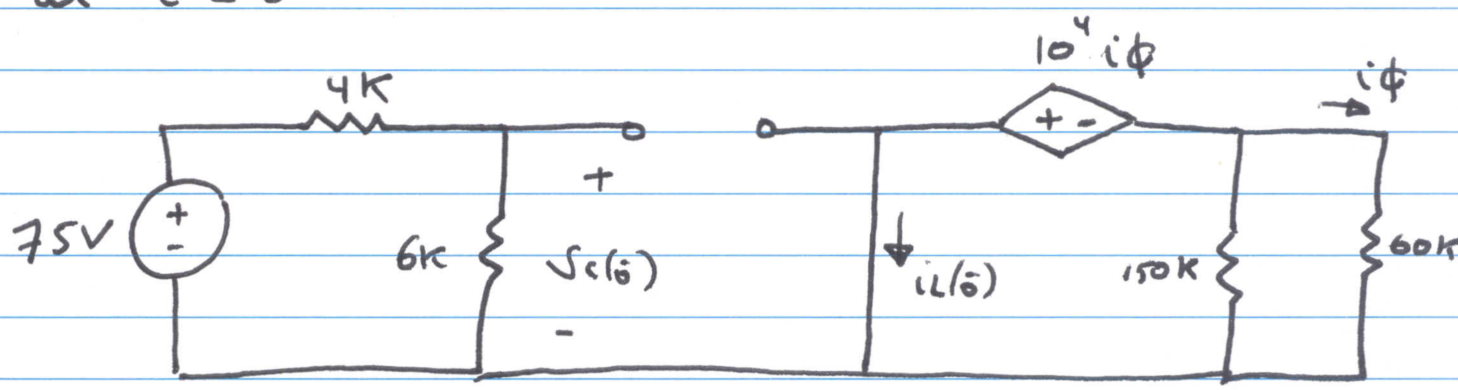
b) $$i_R(t) = \frac{v(t)}{R} = -e^{-250t} + 4e^{-1000t} \quad mA \quad \text{for } t > 0$$

$$i_C(t) = C\frac{dv(t)}{dt} = 0.2e^{-250t} - 3.2e^{-1000t} \quad mA \quad \text{for } t > 0$$

$$i_L(t) = -(i_R(t) + i_C(t)) = 0.8e^{-250t} - 0.8e^{-1000t} \quad mA \quad \text{for } t > 0$$

P 8.21

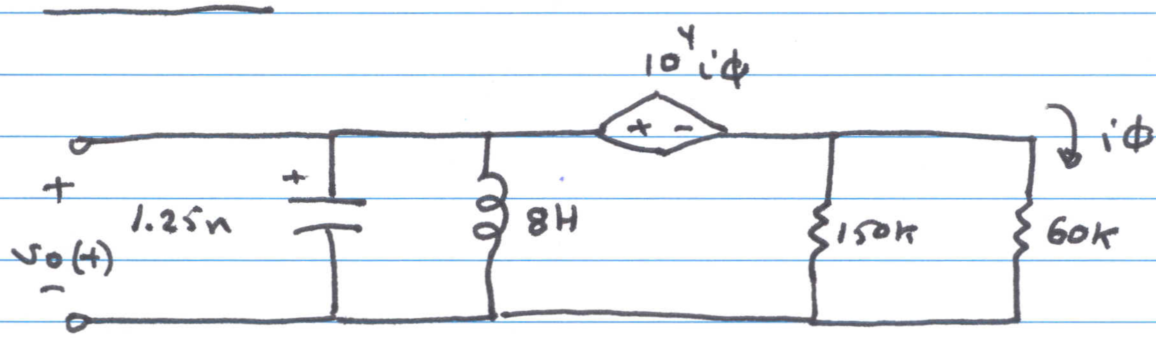
at $t = 0^-$



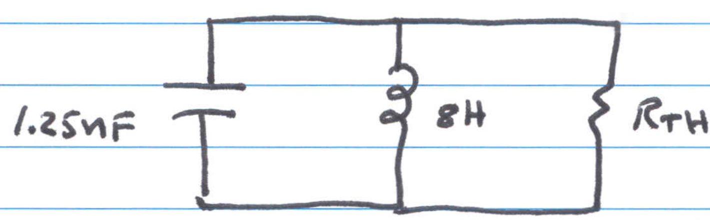
$$v_c(0^-) = \frac{6K}{6K+4K} 75 = 45V$$

$$i_L(0^-) = 0$$

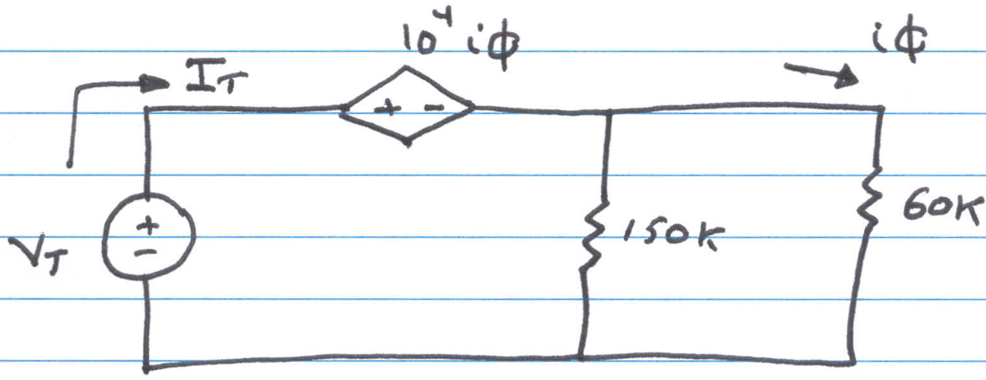
For $t > 0$



The circuit can be simplied to the following



$$R_{TH} = \frac{V_T}{I_T}$$



$$V_T = 10^4 i\phi + 60K\, i\phi$$

$$i\phi = \frac{150K}{150K + 60K} I_T$$

$$\therefore R_{TH} = \frac{V_T}{I_T} = 50K$$

$$\omega_o = \frac{1}{\sqrt{LC}} = 10{,}000 \ r/s$$

$$\alpha = \frac{1}{2RC} = 8000 \ r/s$$

$\alpha < \omega_o$ $\therefore$ underdamped case

$$\omega_d = \sqrt{\omega_o^2 - \alpha^2} = 6000$$

$$V_o(t) = e^{-8000t}\left[B_1 \cos 6000t + B_2 \sin 6000t\right]$$

To find B_1 and B_2 we need

$v(0^+)$ and $\dfrac{dv(0^+)}{dt}$

$$v(0^+) = v_c(0^+) = v_c(0^-) = 45\ \underline{v}$$

KCL :

$$i_c(t) + i_R(t) + i_L(t) = 0$$

$$C\frac{dv_c(t)}{dt} + \frac{v_c(t)}{R} + i_L(0^-) + \frac{1}{L}\int_{0^-}^{t} v_c(t) = 0$$

$$C\frac{dv_c(0^+)}{dt} + \frac{v_c(0^+)}{R} + i_L(0^-) + 0 = 0$$

$$\therefore \frac{dv_c(0^+)}{dt} = -7.2 \times 10^6$$

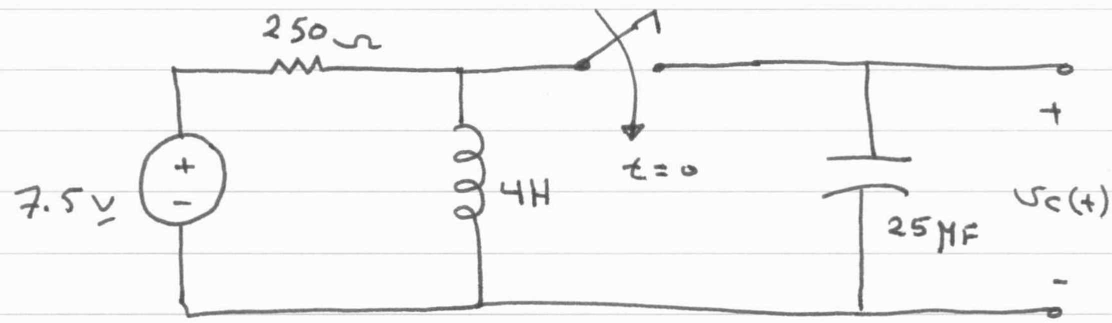
$$v_o(0^+) = B_1 = 45$$

$$\frac{dv_o(0^+)}{dt} = 6000 B_2 - 8000 B_1 = -7.2 \times 10^6$$

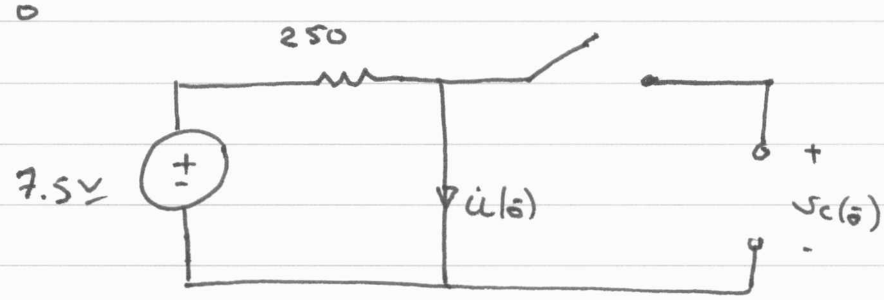
$$\therefore B_1 = 45 \quad \text{and} \quad B_2 = -60$$

$$\therefore v_o(t) = \left[45 \cos 6000t - 60 \sin 6000t\right] e^{-8000t} \ \underline{\underline{v}}$$

8.36

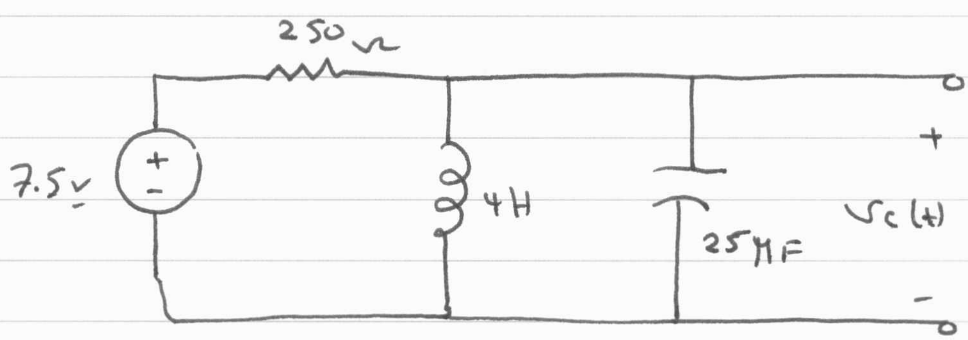


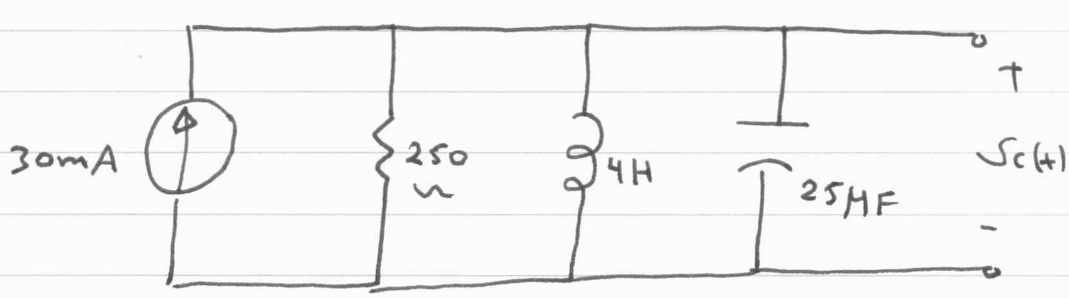
at $t = 0^-$



$$i_L(0^-) = \frac{7.5}{250} = 30\,mA \quad ; \quad v_c(0^-) = 0$$

for $t > 0$





$$\alpha = \frac{1}{2RC} = 80\ r/s \quad ; \quad \omega_0 = \frac{1}{\sqrt{LC}} = 100\ r/s$$

since $\omega_o > \alpha$

$\therefore$ underdamped Case

$$\omega_d = \sqrt{\omega_o^2 - \alpha^2} = 60 \text{ r/s}$$

$$v_c(t) = v_n(t) + v_f(t)$$

$$v_f(t) = v_c(\infty) = 0$$

$$\therefore \quad v_c(t) = e^{-\alpha t}\left(B_1 \cos \omega_d t + B_2 \sin \omega_d t\right)$$

$$v_c(t) = e^{-80t}\left(B_1 \cos 60t + B_2 \sin 60t\right) \; \underline{v} \quad \text{for } t > 0$$

$$v_c(0^+) = v_c(0^-) = 0$$

To find $\dfrac{d v_c(0^+)}{dt}$:

for $t > 0$

$$30mA = \frac{v_c(t)}{R} + i_L(0^-) + \frac{1}{L}\int_{0^-}^{t} v_c(t)\,dt + C\frac{dv_c(t)}{dt}$$

at $t = 0$

$$30mA = \frac{v_c(0^+)}{R} + i_L(0^-) + 0 + C\frac{dv_c(0^+)}{dt}$$

$$\therefore \quad \frac{dv_c(0^+)}{dt} = 0$$

$$v_c(t) = e^{-80t}\left(B_1 \cos 60t + B_2 \sin 60t\right) \; \underline{v} \quad \text{for } t > 0$$

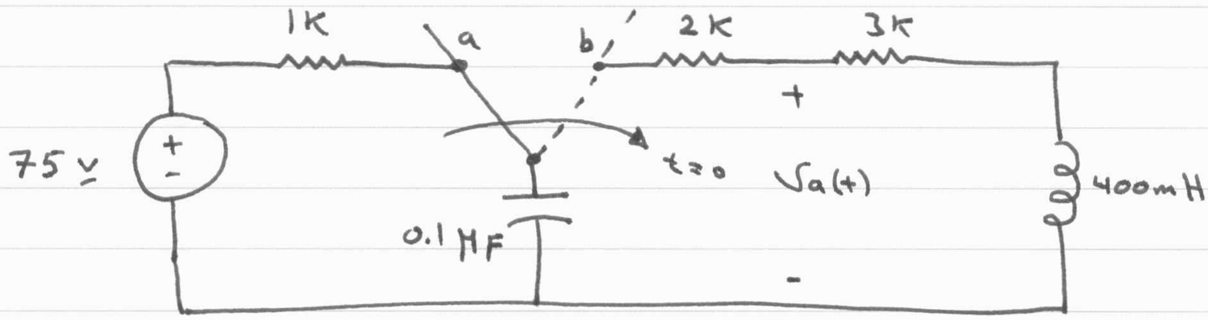
$$v_c(0^+) = B_1 = 0 \quad ; \quad \therefore B_1 = 0$$

$$\frac{dv_c(0^+)}{dt} = -\alpha B_1 + \omega_d B_2 = 0$$

$\therefore \quad B_2 = 0$

$\therefore \quad v_c(t) = 0 \quad$ for $t \geqslant 0$

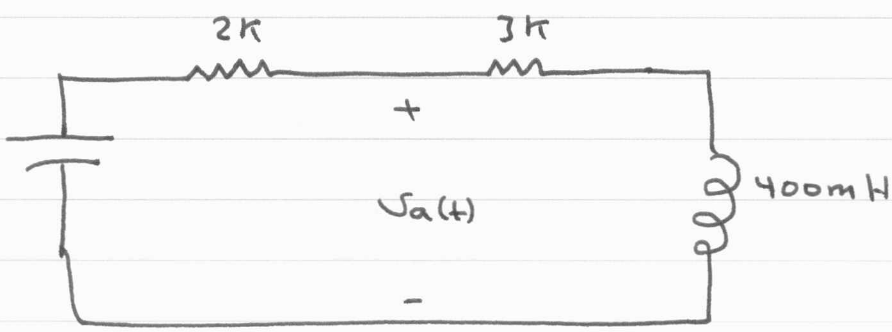
8.46



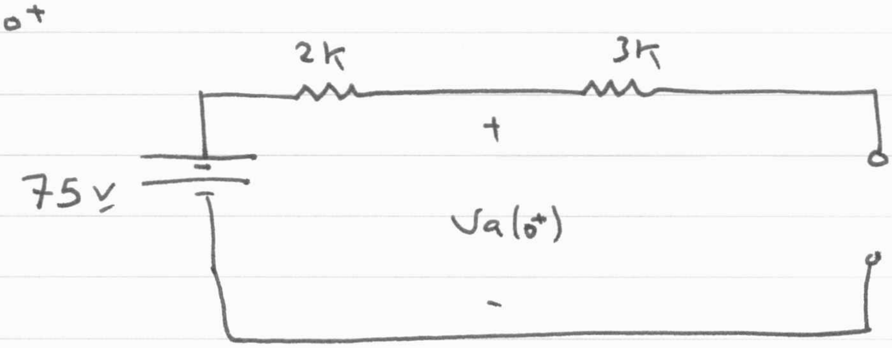
at $= 0^-$

$v_c(0^-) = 75\ \underline{V}$, $i_L(0^-) = 0$

for $t > 0$



at $= 0^+$



$\therefore \quad v_a(0^+) = 75\ \underline{V}$

for $t > 0$

b) $\quad v_a(t) = 3K\, i_L(t) + L \dfrac{di_L(t)}{dt}$

$$v_a(0^+) = 3k\, i_L(0^+) + 0.4 \frac{d i_L(0^+)}{dt}$$

$$\therefore \frac{d i_L(0^+)}{dt} = 187.5$$

for $t > 0$

KVL : $$5k\, i_L(t) + 0.4 \frac{d i_L(t)}{dt} + 10\times 10^6 \int_{0^-}^{t} i_L(t)\, dt = 75$$

Diff : $$5k \frac{d i_L(t)}{dt} + 0.4 \frac{d^2 i_L(t)}{dt^2} + 10\times 10^6\, i_L(t) = 0$$

at $t = 0^+$

$$5k \frac{d i_L(0^+)}{dt} + 0.4 \frac{d^2 i_L(0^+)}{dt^2} + 10\times 10^6\, i_L(0^+) = 0$$

$$\therefore \frac{d^2 i_L(0^+)}{dt^2} = -2343750$$

$$\frac{d v_a(0^+)}{dt} = 3k \frac{d i_L(0^+)}{dt} + 0.4 \frac{d^2 i_L(0^+)}{dt^2}$$

$$\frac{d v_a(0^+)}{dt} = -375000$$

c) $$\alpha = \frac{R}{2L} = \frac{5k}{0.8} = 6250 \text{ r/s}$$

$$\omega_0 = \frac{1}{\sqrt{LC}} = 5000 \text{ r/s}$$

$\therefore$ overdamped case

$$s_1 = -\alpha + \sqrt{\alpha^2 - \omega_0^2} = -2500 \text{ r/s}$$

$$s_2 = -\alpha - \sqrt{\alpha^2 - \omega_0^2} = -10000 \text{ r/s}$$

$$v_a(t) = A_1 e^{-2500t} + A_2 e^{-10000t} \quad \text{for } t > 0$$

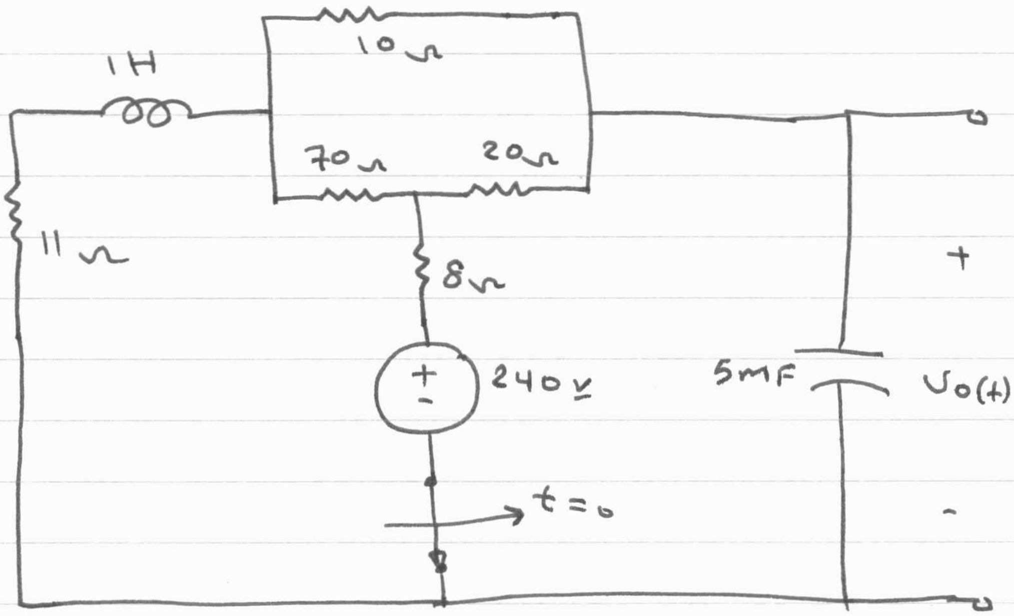
$$v_a(0^+) = A_1 + A_2 = 75$$

$$\frac{dv_a(0^+)}{dt} = -2500 A_1 - 10000 A_2 = -375000$$

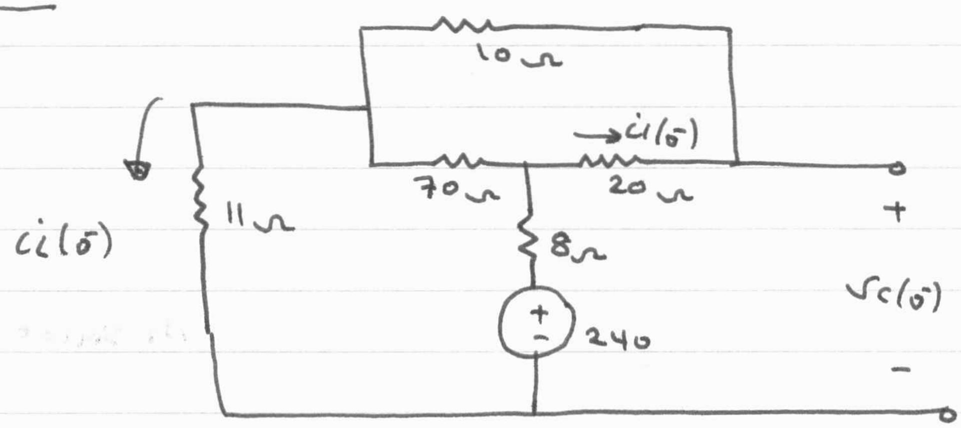
$$\therefore A_1 = 50 \quad , \quad A_2 = 25$$

$$\therefore \quad v_a(t) = 50 e^{-2500t} + 25 e^{-10000t} \text{ V} \quad \text{for } t > 0$$

8. 49



at = 0^-

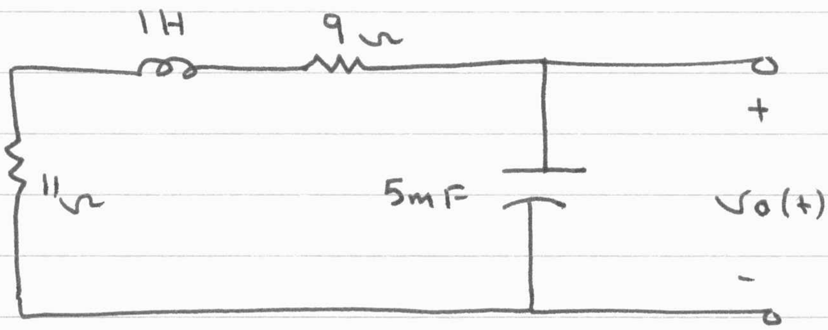


$$i_L(0^-) = \frac{240}{8 + (20+10) \| 70 + 11} = 6A$$

$$i_1(0^-) = \frac{70}{70+20+10} \cdot 6 = 4.2\,A$$

$$v_c(0^-) = -20\, i_1(0^-) - 8\, i_L(0^-) + 240 = 108\ V$$

For $t > 0$



$$\alpha = \frac{R}{2L} = \frac{20}{2} = 10 \ \text{r/s}$$

$$w_0 = \frac{1}{\sqrt{LC}} = 14.142 \ \text{r/s}$$

Since $w_0 > \alpha$

underdamped ; $w_d = \sqrt{w_0^2 - \alpha^2} = 10$

$s_1 = -10 + j10$ r/s
$s_2 = -10 - j10$ r/s

$$v_o(t) = e^{-10t}\left(B_1 \cos 10t + B_2 \sin 10t \right)$$

$v_o(0^+) = v_c(0^-) = 108$ V

$$\frac{dv_o(0^+)}{dt} = -\frac{i_L(0^+)}{C} = -1200$$

$v_o(0^+) = B_1 = 108$

$$\frac{dv_o(0^+)}{dt} = -\alpha B_1 + w_d B_2$$

$\therefore \ B_2 = -12$

$\therefore \ v_o(t) = 108 e^{-10t} \cos 10t - 12 e^{-10t} \sin 10t$ V for $t > 0$