ENEE2301

SINUSOIDAL STEADY-STATE POWER CALCULATIONS

CH 10

Overview

- Nearly all electric energy is supplied in the form of sinusoidal voltages and currents (i.e. AC, alternating currents), because

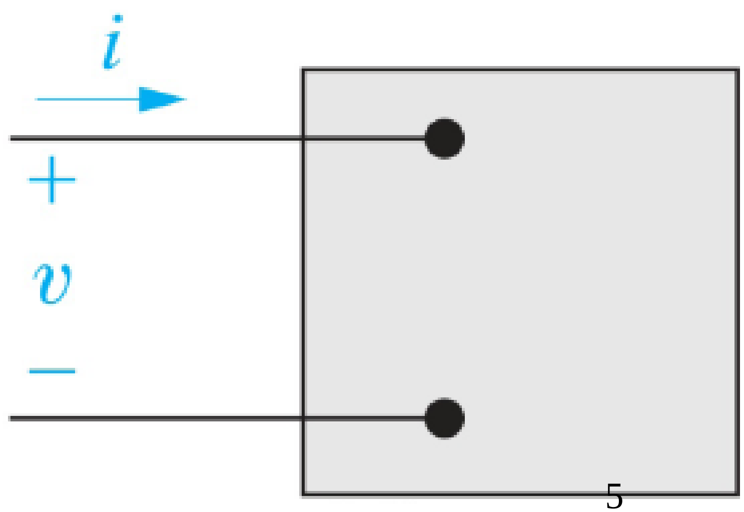
1. Generators generate AC naturally.
2. Transformers must operate with AC.
3. Transmission relies on AC.
4. It is expensive to transform from DC to AC.

Instantaneous Power

- "Instantaneous" power is the product of the instantaneous terminal voltage and current, or

$$p(t) = \pm v(t) \cdot i(t).$$

- Positive sign is used if the passive sign convention is satisfied (current is in the direction of voltage drop).



Sinusoidal power formula

$$\begin{cases} v(t) = V_m \cos(\omega t + \theta_v), \\ i(t) = I_m \cos(\omega t + \theta_i), \end{cases} \Rightarrow \begin{cases} v(t) = V_m \cos(\omega t + \phi), \\ i(t) = I_m \cos(\omega t), \\ \phi = \theta_v - \theta_i; \end{cases}$$

By $\cos\alpha\cos\beta = \dfrac{\cos(\alpha-\beta)}{2} + \dfrac{\cos(\alpha+\beta)}{2}$,

$$\Rightarrow p(t) = V_m I_m \cos(\omega t + \phi)\cos(\omega t)$$

$$= \frac{V_m I_m}{2}\cos\phi + \frac{V_m I_m}{2}\cos(2\omega t + \phi).$$

Constant, P_{avg} Oscillating at frequency 2ω

Example :

$$v(t) = 4\ \cos(\omega t + 60°) \quad V$$

$$Z(j\omega) = 2\ \angle 30° \quad \Omega$$

Find $p(t)$

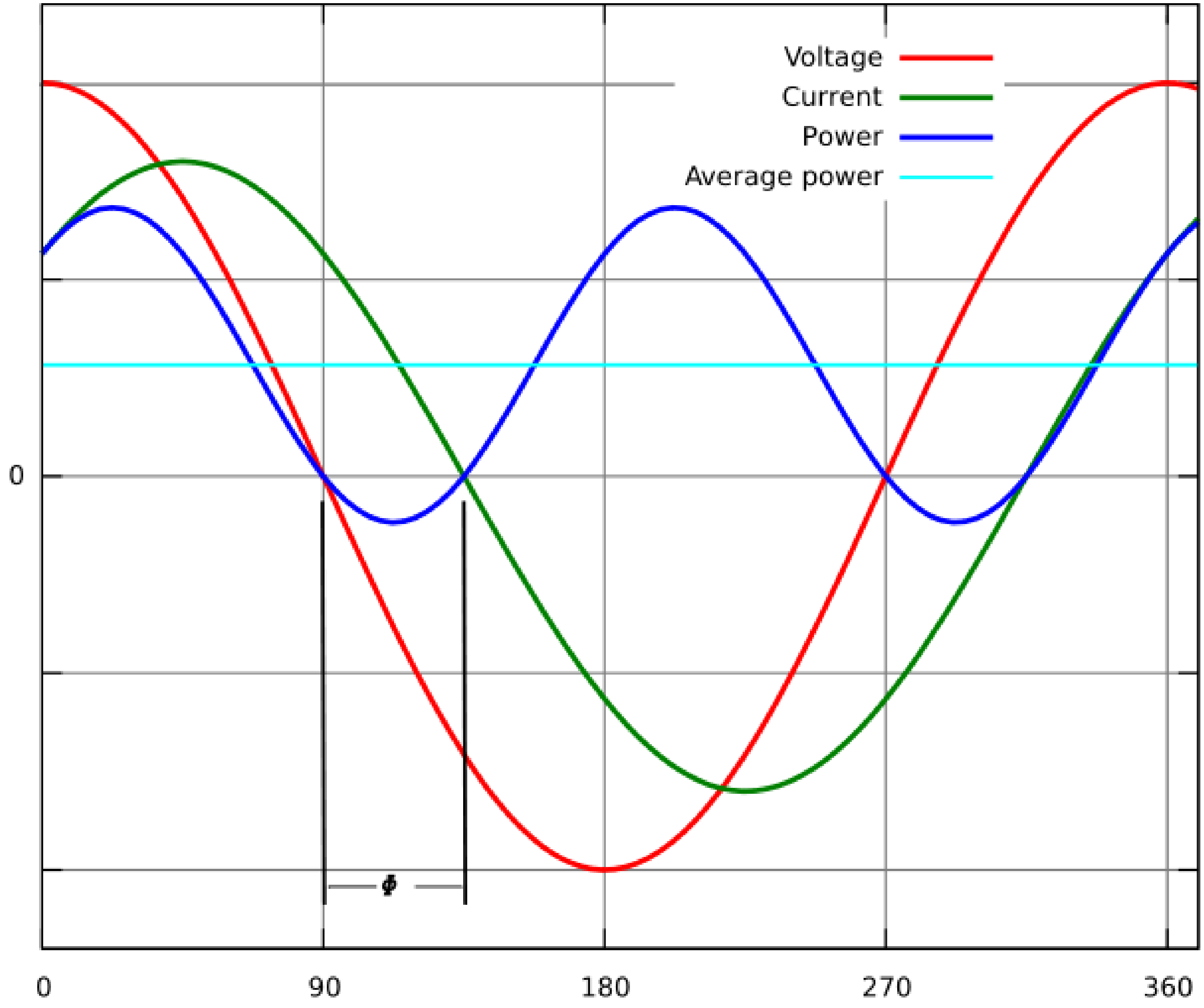
$$\boldsymbol{I} = \frac{\boldsymbol{V}}{Z} = \frac{4\ \angle 60°}{2\ \angle 30°} = 2\ \angle 30° \quad A$$

$$\therefore\ i(t) = 4\ \cos(\omega t + 30°) \quad A$$

$$p(t) = v(t)\, i(t)$$

$$= 4\ \cos(30°) + 4\cos(2\omega t + 90°)$$

$$= 3.46 + 4\cos(2\omega t + 90°)$$


Voltage
Current
Power
Average power
0
ϕ
0
90
180
270
360

<u>Average Power</u> : Real Power

$$P_{av} = \frac{1}{T}\int_0^T p(t)\,dt = \frac{1}{2} V_m I_m \cos(\theta_v - \emptyset_i)$$

$$\theta_z = \theta_v - \emptyset_i$$

$$P_{av} = \frac{1}{2} V_m I_m \cos(\theta_z)$$

1) For Resistor :

$$\theta_v - \emptyset_i = 0 \rightarrow \theta_z = 0$$

$$\therefore P_{av} = \frac{1}{2} V_m I_m = \frac{Vm^2}{2R} = \frac{Im^2 R}{2}$$

Always positive for a resistor since they dissipate energy

2) For Inductor : current and voltage are out of phase by 90 degrees (current lags voltage)

$$\theta_v - \emptyset_i = 90°$$

$$\therefore P_{av} = 0$$

3) For Capacitor :

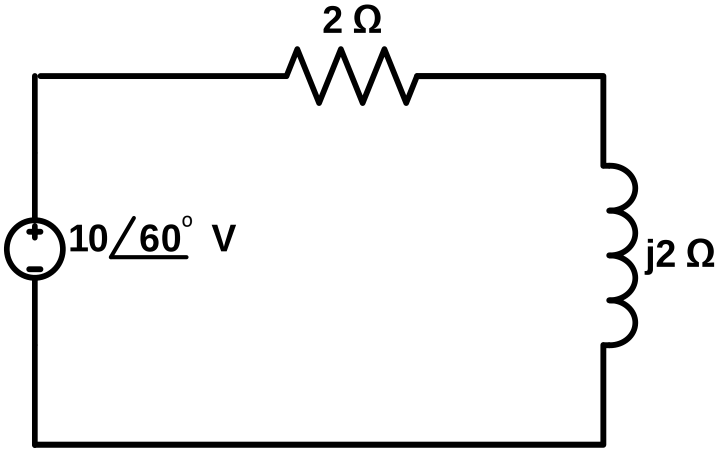
$$\theta_v - \emptyset_i = -90°$$

$$\therefore P_{av} = 0$$

$\therefore$ *Reactive impedances* (L and C) *absorb* ***NO*** *average power*

Example :

Find the average power absorbed by each element .



$$\boldsymbol{I} = \frac{10\ \angle 60°}{2+j2} = 3.53\ \angle 15°\ A$$

$$P_{av} = \frac{1}{2}\ V_m\ I_m\ \cos(\theta_v - \emptyset_i)$$

$$P_{av,j2} = 0$$

$$P_{av,2} = \frac{I_m^2 R}{2} = \frac{3.53^2 * 2}{2} = 12.5\ W$$

To calculate the average power supplied by the source.

$$P_{av,vs} = \frac{V_m\, I_m}{2} \cos(\theta_v - \emptyset_i)$$

$$I_m = 3.53$$

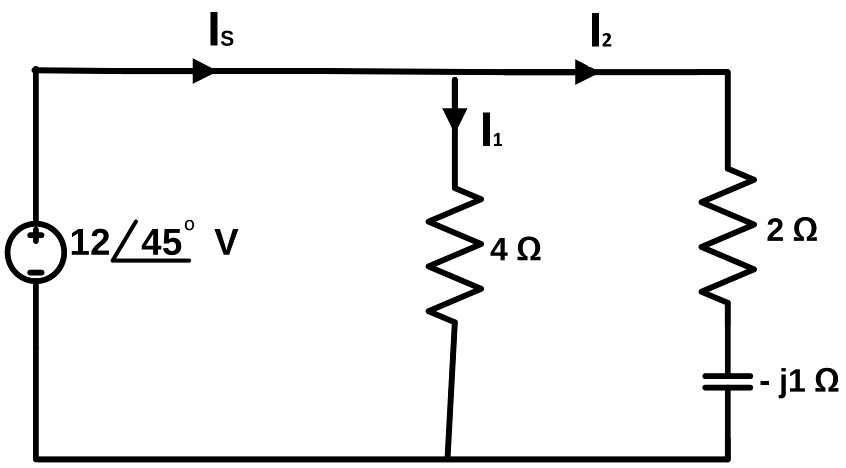
$$V_m = 10\ V$$

$$\theta_v = 60° \quad ; \ \emptyset_i = 15°$$

$$\therefore P_{av,vs} = \frac{10 * 3.53}{2} \cos(60° - 15°)$$

$$= 12.5\ W$$

Example :



Determine the average power absorbed by each resistor . Determine the total average power absorbed and the average power supplied by the source .

$$I_1 = \frac{12\ \angle 45°}{4} = 3\ \angle 45°\ A$$

$$I_2 = \frac{12\ \angle 45°}{2 - j} = 5.36\ \angle 71.57°\ A$$

$$I_s = I_1 + I_2 = 8.15\ \angle 62.1°\ A$$

$$1)\ P_{4\Omega} = \frac{I_{1m}^2 * 4}{2} = 18\ W$$

$$2)\ P_{2\Omega} = \frac{I_{2m}^2 * 2}{2} = 24.7\ W$$

∴ Total Average power absorbed = 46.7 W

$$P_{Vs} = \frac{V_m \, I_m}{2} \cos(\theta_v - \emptyset_i)$$

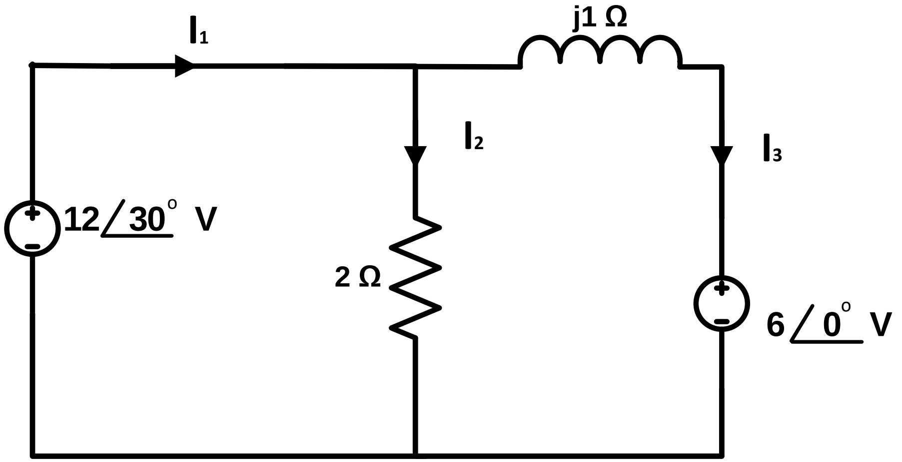
$$P_{Vs} = \frac{12 * 8.16}{2} \cos(45 - 62.1)$$

$$P_{Vs} = 46.7 \; W$$

$$P_{Vs} = P_{4\Omega} + P_{2\Omega} + P_{-j}$$

Example :

Determine average power absorbed or supplied by each element .



$$P_{12\angle 30^\circ} = \frac{1}{2} V_m \, I_m \, \cos(\theta_v - \phi_i)$$

$$= \frac{12 * 11.29}{2} \cos(30^\circ - (-7.07))$$

$$= 54 \, W$$

supply

$$\boldsymbol{I}_2 = \frac{12 \angle 30^\circ}{2} = 6 \angle 30^\circ$$

$$\boldsymbol{I}_3 = \frac{12 \angle 30^\circ - 6 \angle 0^\circ}{j} = 7.43 \angle -36.19^\circ$$

$$\boldsymbol{I}_1 = \boldsymbol{I}_2 + \boldsymbol{I}_3 = 11.29 \angle -7.07^\circ$$

$$P_{2\Omega} = \frac{{I_{2m}}^2 .2}{2} = 36 \, W$$

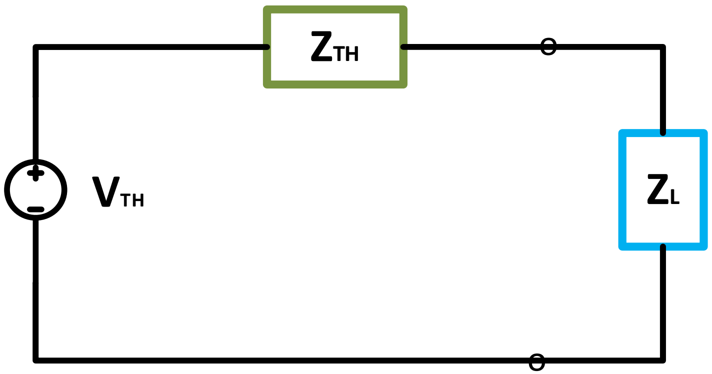
$$P_{6\angle 0^\circ} = \frac{1}{2}\, V_m\, I_m\, \cos(\theta_v - \emptyset_i)$$

$$= \frac{6 * 7.43}{2} \cos(0^\circ - (-36.19^\circ))$$

$$= 18\, W$$

absorbed

Maximum Average Power Transfer



$$Z_{TH} = R_{TH} + jX_{TH}$$

$$Z_L = R_L + jX_L$$

$$P_L = \frac{I_{Lm}^2 . R_L}{2}$$

$$\boldsymbol{I} = \frac{\boldsymbol{V}_{TH}}{Z_{TH}+Z_L}$$

$$\boldsymbol{I} = \frac{\boldsymbol{V}_{TH}}{(R_{TH}+R_L)+j(X_{TH}+X_L)}$$

$$P_L = \frac{I_{Lm}^2 . R_L}{2}$$

$$P_L = \frac{1}{2}\frac{V_{TH}^2 \, . \, R_L}{(R_{TH}+R_L)^2 + (X_{TH}+X_L)^2}$$

$$\frac{\partial P_L}{\partial R_L} = 0 \qquad ; \qquad \frac{\partial P_L}{\partial X_L} = 0$$

$$\frac{\partial P_L}{\partial X_L} = \frac{-2\, V_{TH}^2 \, . \, R_L \, (X_{TH}+X_L)}{2[(R_{TH}+R_L)^2+(X_{TH}+X_L)^2]^2}$$

For $\frac{\partial P_L}{\partial X_L} = 0 \quad \longrightarrow \quad X_L = -X_{TH}$

$$\frac{\partial P_L}{\partial R_L} = \frac{V_{TH}^2 \, \left[(R_L+R_{TH})^2+(X_{TH}+X_L)^2-2R_L(R_L+R_{TH})\right]}{2[(R_{TH}+R_L)^2+(X_{TH}+X_L)^2]^2}$$

For $\frac{\partial P_L}{\partial R_L} = 0 \quad \longrightarrow \quad R_L = \sqrt{R_{TH}^2 + (X_{TH}+X_L)^2}$

$$X_L = -X_{TH}$$

$$\therefore \; R_L = R_{TH}$$

$$\therefore\ Z_L = Z_{TH}^*$$

$$P_{L;max} = \frac{1}{8}\frac{V_{TH}^2}{R_L}$$

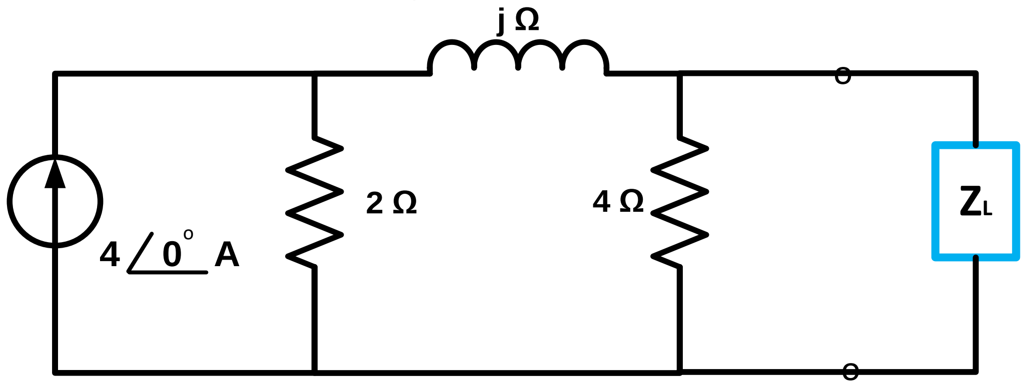
∴ For maximum average power transfer

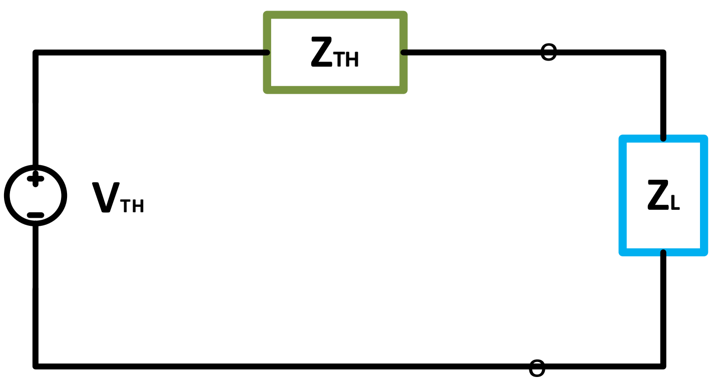
$$\therefore\ Z_L = Z_{TH}^*$$

$$P_{L;max} = \frac{1}{8}\frac{V_{TH}^2}{R_L}$$

Example :Find Z_L for maximum average power transfer .

Compute the maximum average power supplied to the load .





$$\boldsymbol{V}_{TH} = 4\angle 0° \frac{2}{2+j+4} \,.\, 4 = 5.28\angle -9.46° \; V$$

$$Z_{TH} = 4\Omega \,||\, (2+j)\Omega$$

$$Z_{TH} = (1.4 + j0.43)\Omega$$

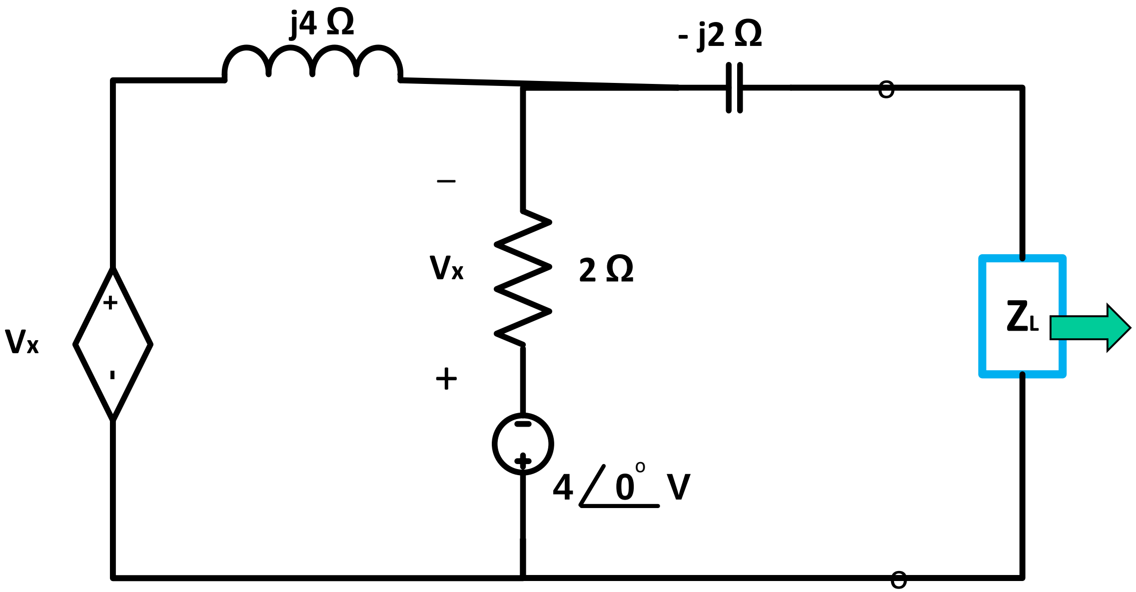
$$\therefore \; Z_L = (1.4 - j0.43)\Omega$$

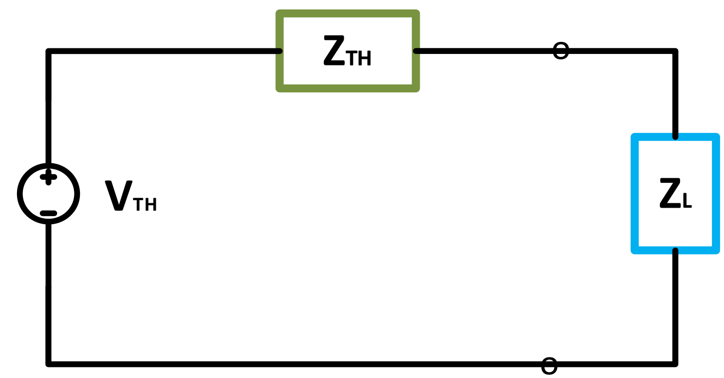
$$P_{;max} = \frac{1}{8}\frac{V_{TH}^2}{R_L}$$

$$P_{L;max} = 2.489 \; W$$

Example : Find Z_L for maximum average power transfer .

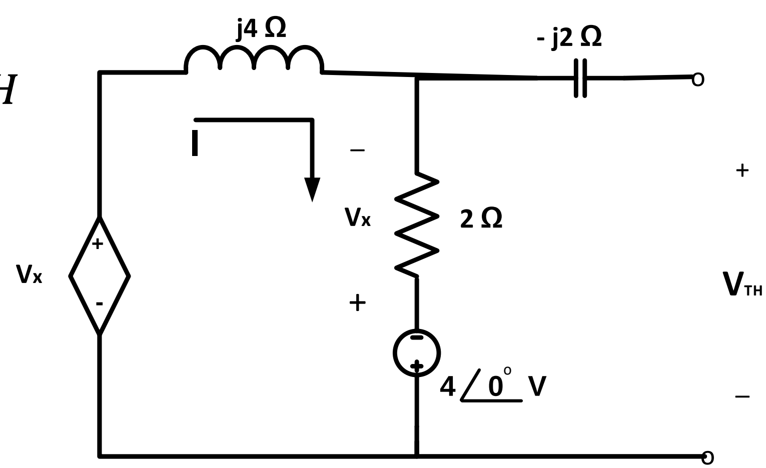
Compute the maximum average power supplied to the Z_L





$$\boldsymbol{V}_{TH} = 2\boldsymbol{I} - 4\angle 0°$$

$$Z_L = Z_{TH}^*$$



$$\boldsymbol{I} = \frac{V_X + 4\angle 0°}{2 + j4}$$

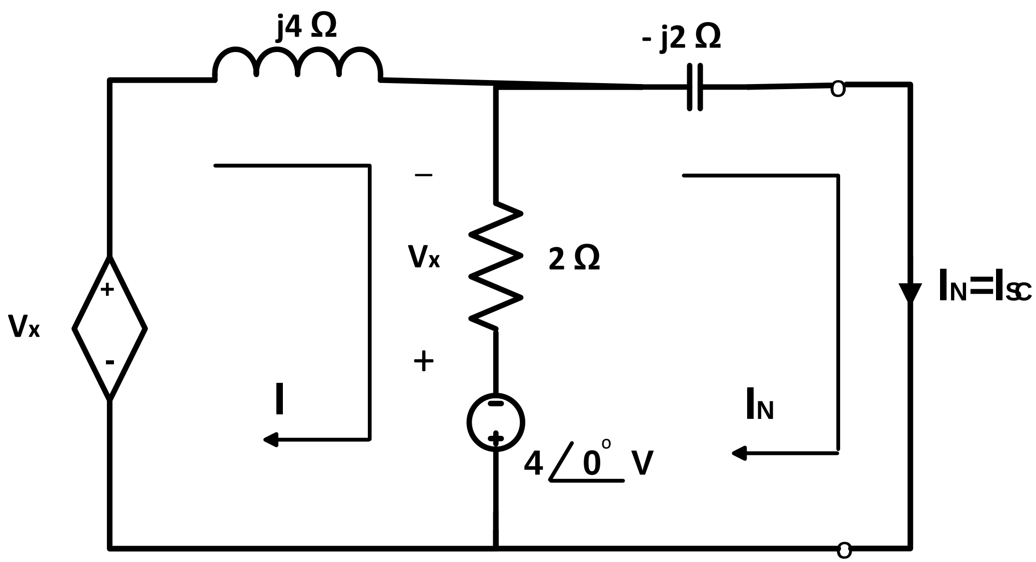
$$\boldsymbol{V}_X = -2\boldsymbol{I}$$

$$\boldsymbol{I} = 0.707\angle -45° \ A$$

$$\therefore \ \boldsymbol{V}_{TH} = (-3 - j) \ V$$

$$= 3.16\angle 198.43°$$

$$\boldsymbol{Z}_{TH} = \frac{\boldsymbol{V}_{TH}}{\boldsymbol{I}_N}$$



Solving for $\boldsymbol{I}_N$

$$\boldsymbol{I}_N = (-1 - j2)\ A$$

$$= 2.24\ \angle 243.43°\ A$$

$$\therefore\ \boldsymbol{Z}_{TH} = \frac{\boldsymbol{V}_{TH}}{\boldsymbol{I}_N} = 1.41\ \angle -45°\ \Omega$$

$$= 1 - j\ \Omega$$

$$\therefore\ \boldsymbol{Z}_L = \boldsymbol{Z}_{TH}^* = 1.41\ \angle 45°\ \Omega\ = 1 + j\ \Omega$$

KVL for mesh 1 :

$$\boldsymbol{V}_X + 4\ \angle 0° = (2 + j4)\ \boldsymbol{I} - 2\boldsymbol{I}_N$$

$$\boldsymbol{V}_X = 2(\boldsymbol{I}_N - \boldsymbol{I})$$

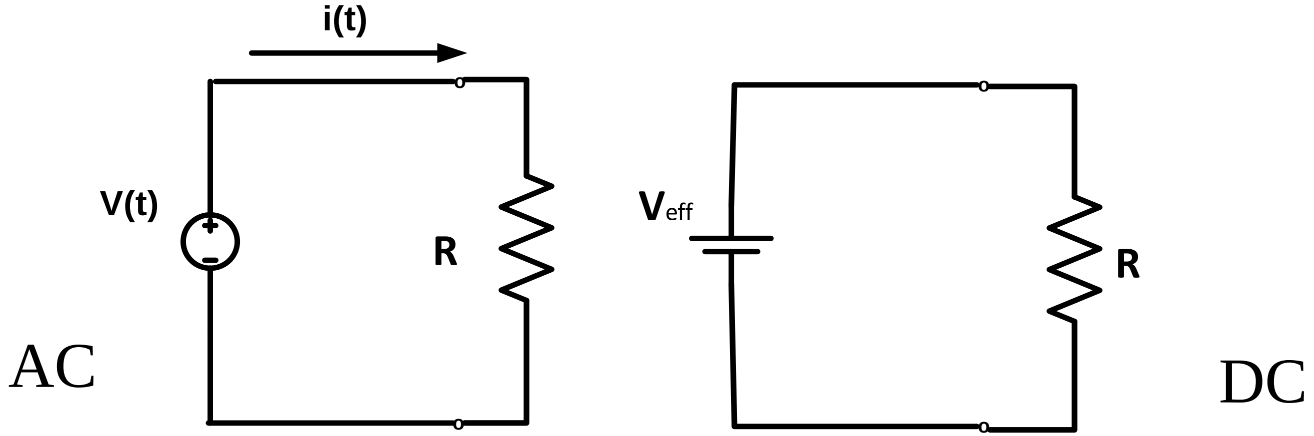
$$\therefore\ P_{L;max} = \frac{V_{TH}^2}{8\ R_{TH}} = 1.25\ W$$

KVL for mesh 2 :

$$-4\ \angle 0° = -2\boldsymbol{I} + (2 - j2)\boldsymbol{I}_N$$

Effective or RMS Value

The effective value of a periodic voltage (current) is the dc voltage (current) that delivers the same average power to a resistor as the periodic voltage (current) .



Let $v(t) = V_m \cos(\omega t + \theta_v)$ $\qquad P_1 = P_2$

$$\therefore P_1 = \frac{V_m^2}{2R}$$

$$P_2 = \frac{V_{eff}^2}{R}$$

$$\therefore \frac{V_m^2}{2R} = \frac{V_{eff}^2}{R}$$

$$\therefore V_{eff} = \frac{V_m}{\sqrt{2}}$$

RMS : Root Mean Square

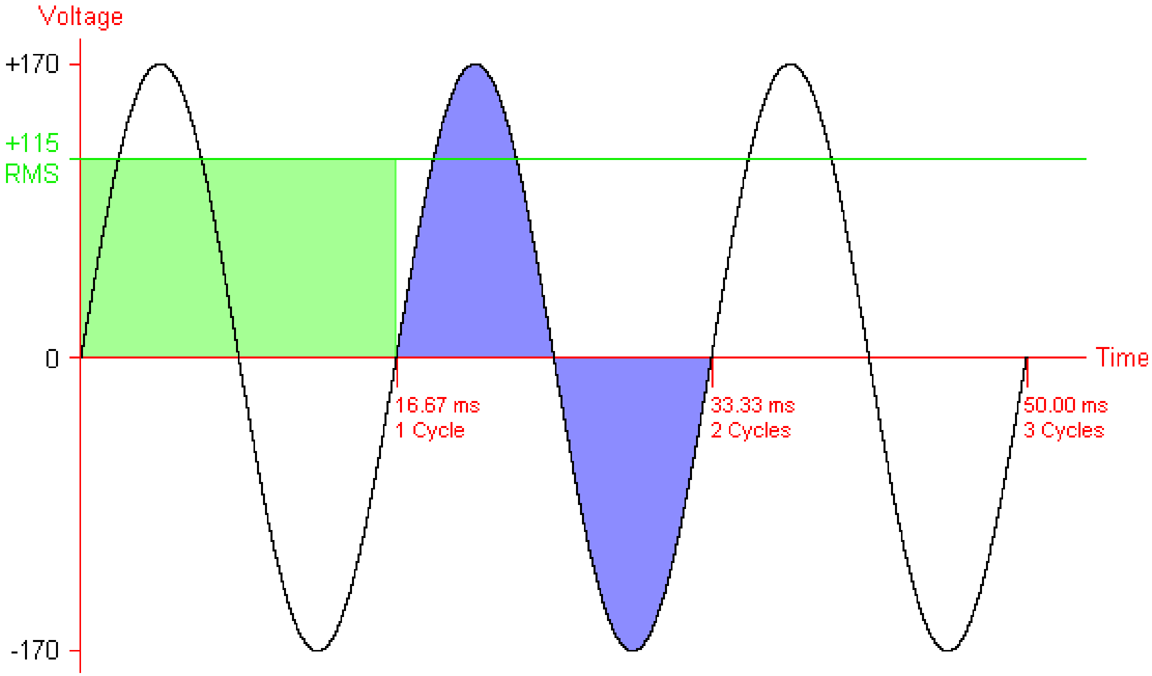
Let $v(t) = V_m \cos(\omega t + \theta_v)$

$$V_{RMS} = \sqrt{\frac{1}{T}\int_0^T V_m^2 \cos^2(\omega t + \theta_v)\,dt}$$

$$V_{RMS} = V_m\sqrt{\frac{1}{T}\int_0^T \cos^2(\omega t + \theta_v)\,dt}$$

$$V_{RMS} = V_m\sqrt{\frac{1}{T}\int_0^T \frac{1}{2}(1 + \cos 2(\omega t + \theta_v))dt}$$

$$V_{RMS} = \frac{V_m}{\sqrt{2}}$$


Voltage
+170
+115
RMS
0
-170
Time
16.67 ms
1 Cycle
33.33 ms
2 Cycles
50.00 ms
3 Cycles

$$P_{av} = \frac{1}{2} V_m \, I_m \, \cos(\theta_v - \emptyset_i)$$

$$P_{av} = V_{rms} \, I_{rms} \, \cos(\theta_v - \emptyset_i)$$

For a Resistor

$$P_{av} = V_{rms} \, I_{rms} \, \cos(\theta_v - \emptyset_i)$$

$$V_{rms} = R \, I_{rms} \qquad ; \theta_v - \emptyset_i = 0$$

$$\therefore \; P_{av} = \frac{V_{rms}^2}{R}$$

$$\therefore \; P_{av} = I_{rms}^2 \, R$$

Apparent Power and Power factor

$$P_{av} = V_{rms}\, I_{rms}\, \cos(\theta_v - \emptyset_i)$$

define $P_{apparent} = V_{rms}\, I_{rms} = P_a$

$P_{apparent}$ measured in VA

$PF \equiv$ Power Factor

$$PF = \cos(\theta_v - \emptyset_i)$$

$$\therefore\ P_{av} = P_a\, .\, PF$$

1) For Resistor

$$\theta_v - \emptyset_i = 0°$$

$$\therefore\ PF = 1$$

2) For Inductor

$$\theta_v - \emptyset_i = +90°$$

$$\therefore\ PF = 0$$

3) For Capacitor

$$\theta_v - \emptyset_i = -90°$$

$$\therefore\ PF = 0$$

4) For Inductive load

$$90° > \theta_v - \emptyset_i > 0°$$

$$\therefore\ 1 > PF > 0$$ **lagging power factor**

5) For Capacitive load

$$0° > \theta_v - \emptyset_i \succ -90°$$

$$\therefore\ 1 > PF > 0$$

leading power factor

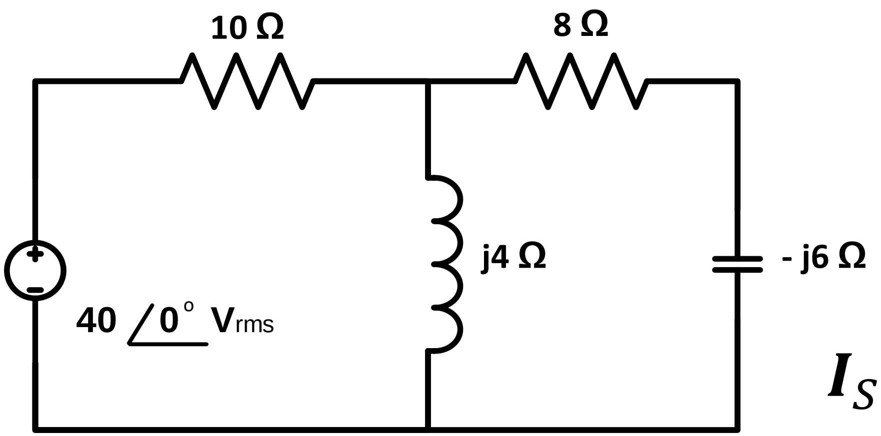
$$PF = \cos(\theta_v - \emptyset_i)$$

$$\cos(\alpha) = \cos(-\alpha)$$

Power factor is either leading or lagging referring to the phase of current with respect to the voltage .

Example :

Calculate the power factor seen by the source and the average power supplied by the source .



$$\boldsymbol{Z} = 10 + j4||(8 - j6)$$

$$= 12.69\angle 20.62°$$

$$\boldsymbol{I}_S = \frac{40\angle 0°}{\boldsymbol{Z}} = 3.152\angle -20.62° \text{ A rms}$$

$$\theta_v = 0 \quad ;\theta_i = \angle -20.62°$$

$$PF = \cos(\theta_v - \emptyset_i)$$

$$= \cos(20.62)$$

$$= 0.936 \qquad \text{lagging}$$

The average power supplied by the source is equal to the average power absorbed by the circuit .

$$P_{av} = V_{rms}\, I_{rms}\, \cos(\theta_v - \emptyset_i)$$

$$V_{rms} = 40\ V_{rms}$$

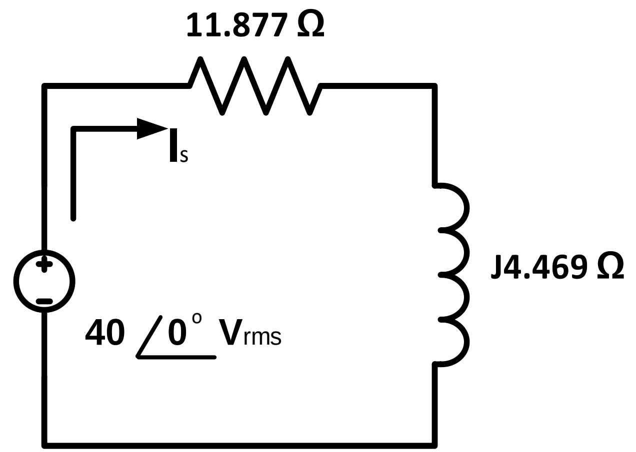
$$I_{rms} = 3.15\ A_{rms}$$

$$\theta_v = 0°$$

$$\theta_i = -20.62°$$

$$\therefore P_{av} = 40 * 3.152\ \cos\left(0 - (-20.62°)\right)$$

$$= 118\ W$$



$$\boldsymbol{Z} = 12.69\angle 20.62° \quad \Omega$$

$$= 11.877 + j4.469 \quad \Omega$$

$$P_{av} = I_{rms}^2 \, R$$

$$= 3.152^2 * 11.877$$

$$= 118 \; W$$

Also $P_{av} = P_{av;10\Omega} + P_{av;8\Omega} + P_{av;-j6} + P_{av;j4}$

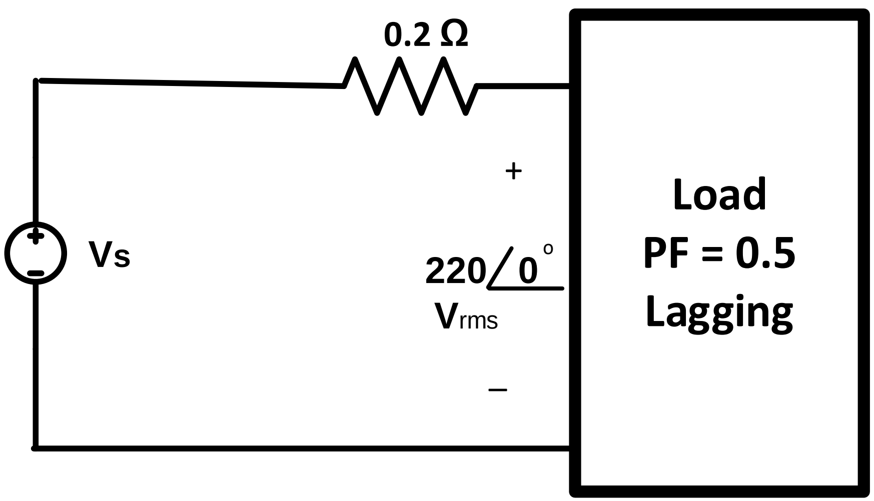
$$= P_{av;10\Omega} + P_{av;8\Omega}$$

Example :

An industrial load consumes 11 kW at 0.5 PF lagging from a 220 V_{rms} line . The transmission line resistive from the power company to the plant is 0.2Ω .

1) Determine the average power that must be supplied by the power company .

2) Repeat (1) if the power factor is changed to unity .



$$P_{av;Load} = V_{rms} . I_{rms} . PF$$

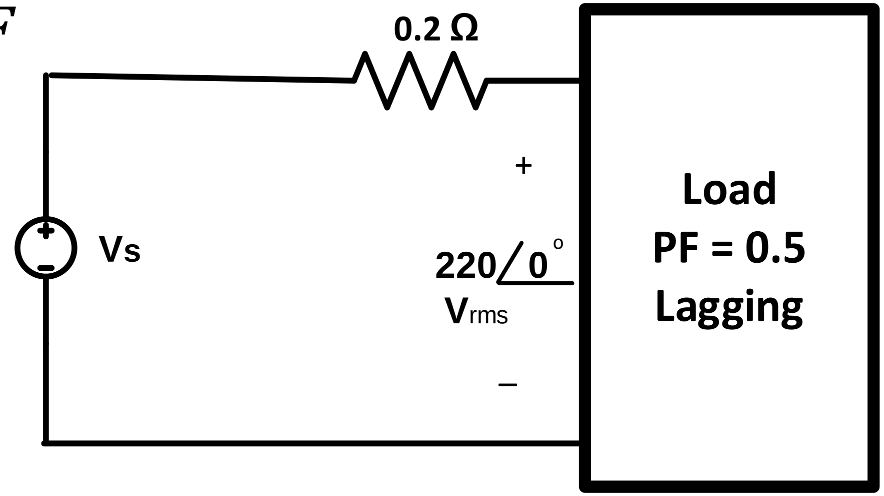
$$\therefore \; I_{rms} = \frac{P_{av;Load}}{V_{rms} . PF}$$

$$= \frac{11kW}{220 * 0.5} = 100 \; A_{rms}$$

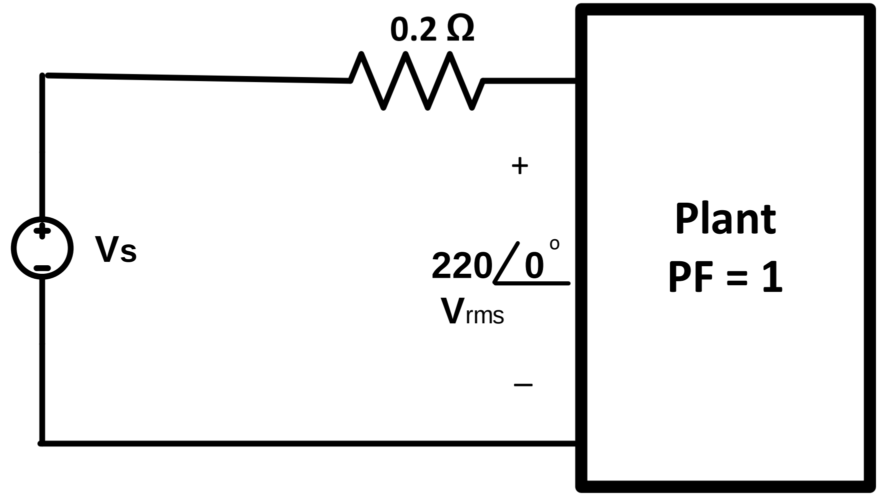
$$P_{loss} = I_{rms}^2 * 0.2 = 2 \; kW$$

$$\therefore P_{av;sup} = P_{av;Load} + P_{av;\,loss}$$

$$= 13 \; kW$$



$$P_{av;Load} = V_{rms}.I_{rms}.PF$$



$$\therefore\ I_{rms} = \frac{P_{av;Load}}{V_{rms}.PF} = 50\ A_{rms}$$

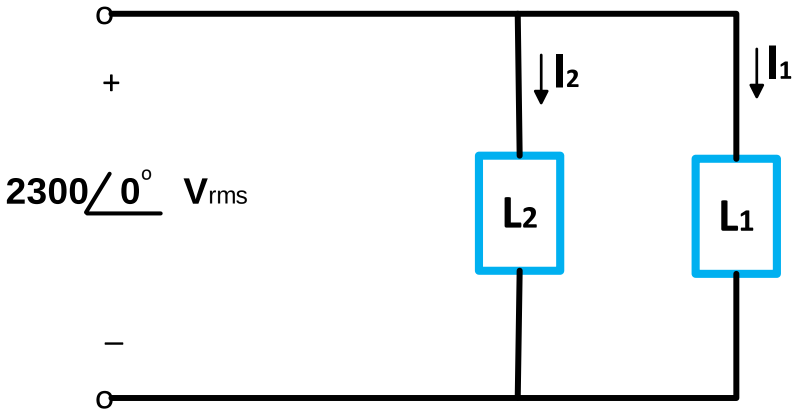
$$P_{loss} = I_{rms}^2.R = 50 * 0.2 = 0.5\ kW$$

$$\therefore P_{av;sup} = 0.5\ kW + 11\ kW$$

$$= 11.5\ kW$$

Example :

Find the power factor of the two loads .



Load 1 : 10 kW ; 0.9 lagging PF

Load 2 : 5kW ; 0.95 leading PF

$$\boldsymbol{I}_1 = \frac{10{,}000}{2300*0.9}\angle-\cos^{-1}0.9$$

$$= 4.83\angle-25.84° \quad A_{rms}$$

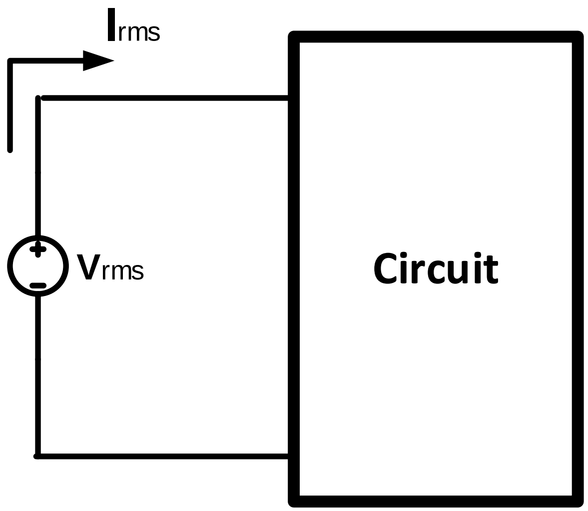
$$\boldsymbol{I}_2 = \frac{5000}{2300*0.95}\angle+\cos^{-1}0.95$$

$$= 2.288\angle18.195° \quad A_{rms}$$

$$\boldsymbol{I}_s = \boldsymbol{I}_1 + \boldsymbol{I}_2 = 6.78\angle-12° \quad A_{rms}$$

$$PF = \cos(\theta_v - \phi_i) = \cos\big(0° - (-12°)\big) = 0.978 \quad \text{lagging}$$

Complex Power



$$\boldsymbol{V}_{rms} = V_{rms}\angle\theta_v$$

$$\boldsymbol{I}_{rms} = I_{rms}\angle\emptyset_i$$

$$\boldsymbol{S} \equiv Complex\ Power$$

$$\boldsymbol{S} = \boldsymbol{V}_{rms} \,.\, \boldsymbol{I}^{*}_{rms}$$

$$= V_{rms} \,.\, I_{rms}\ \angle(\theta_v - \emptyset_i)$$

$$\boldsymbol{S} = V_{rms} \,.\, I_{rms}\ \cos(\theta_v - \emptyset_i) + j\, V_{rms} \,.\, I_{rms}\ \sin(\theta_v - \emptyset_i)$$

$$\boldsymbol{S} = P_{av} + j\,Q$$

$$P_{av} \equiv Average\ Power\ in\ Watt$$

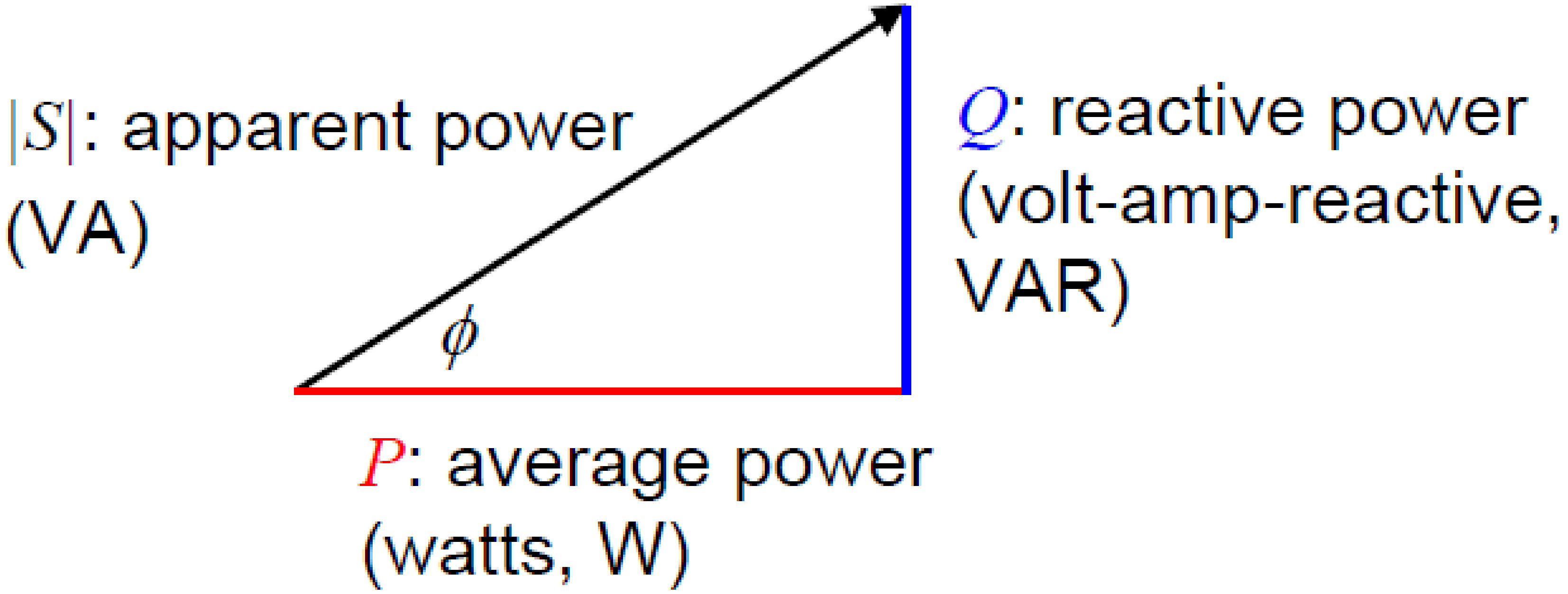
$$Q \equiv Reactive\ Power\ in\ VAR$$

$$\therefore\quad P_{av} = \Re\{\boldsymbol{S}\}$$

$$Q = \Im\{\boldsymbol{S}\}$$

- The complex power S (volt-amps, VA) is:

$$S = P + jQ$$



1) For pure resistance :

$$\theta_v - \emptyset_i = 0$$

$$\therefore \; Q_R = 0$$

2)For pure inductance :

$$\theta_v - \emptyset_i = +90°$$

$$\therefore \; Q_L = V_{rms} \; I_{rms}$$

$$V_{rms} = \omega L \, I_{rms}$$

$$\therefore \; Q_L = \omega L \, I_{rms}^2 = \frac{V_{rms}^2}{\omega L}$$

3) For pure Capacitance :

$$\theta_v - \emptyset_i = -90°$$

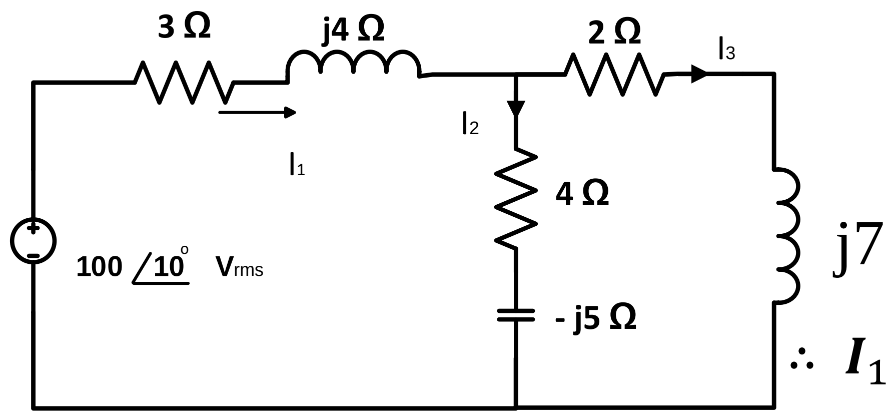
$$\therefore\ Q_c = -V_{rms}\ I_{rms}$$

$$I_{rms} = \omega c\, V_{rms}$$

$$\therefore\ Q_c = -\frac{I_{rms}^2}{\omega c}$$

$$= -\omega c\ V_{rms}^2$$

What are the VARs consumed by the circuit



$$\therefore \boldsymbol{I}_1 = \frac{100\,\angle 10°}{11.3\,\angle 23.7°} = 8.83\ \angle -13.7°\ \ A_{rms}$$

$$Q = 100 * 8.84\ \sin(10° - (-13.7°))$$

$$= 355\ \ VARs$$

$$Q = V_{rms}\,.\,I_{rms}\ \sin(\theta_v - \emptyset_i)$$

$$I_2 = \ 10.2\ \ A_{rms}$$

$$I_3 = \ 8.95\ \ A_{rms}$$

$$\boldsymbol{I}_1 = \frac{\boldsymbol{V}_s}{Z}$$

$$Z = (2 + j7)||(4 - j5)\ +\ 3 + j4$$

$$= 10.35 + j4.55 = 11.3\,\angle 23.7°\ \ \Omega$$

$$P_{av} = \frac{1}{2} V_m \, I_m \, \cos(\theta_v - \emptyset_i)$$

$$Q = V_{rms} \, . I_{rms} \, \sin(\theta_v - \emptyset_i)$$

$$\frac{Q}{P_{av}} = \tan(\theta_v - \emptyset_i)$$

$$Q = P_{av} \, \tan(\theta_v - \emptyset_i)$$

$$Q = P_{av} \, \tan[\cos^{-1}(PF)]$$

$$\boldsymbol{S} = P_{av} + j\, Q$$

$$= \sqrt{P_{av}^2 + Q^2} \angle \tan^{-1} \frac{Q}{P_{av}}$$

$$\therefore \; P_a = |\boldsymbol{S}| = \sqrt{P_{av}^2 + Q^2}$$ apparent power

$$\theta_v - \theta_i = \tan^{-1} \frac{Q}{P_{av}}$$

$$\theta_v - \emptyset_i = \tan^{-1}\frac{Q}{P_{av}}$$

To increase PF , we need to decrease Q .

∴ For inductive circuit we add a capacitor in parallel to increase the power factor .

Total Power (average, Reactive and complex)

$$P_{av\,T} = P_{av\,1} + P_{av\,2} + P_{av\,3} + \cdots + P_{av\,n}$$

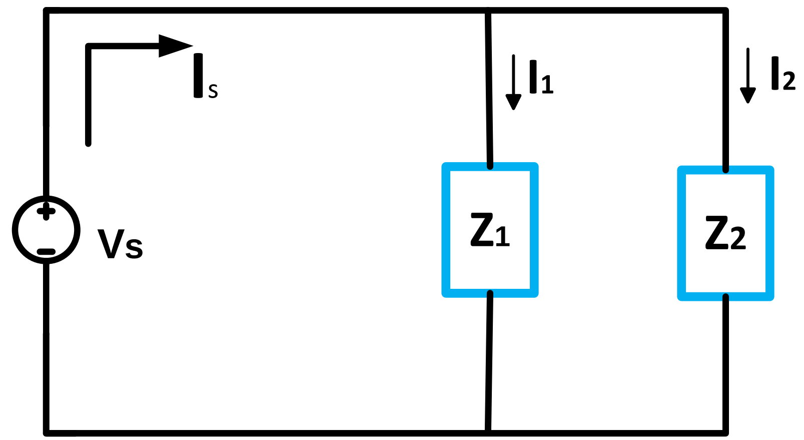
$$Q_T = Q_1 + Q_2 + Q_3 + \cdots + Q_n$$

$$\boldsymbol{S}_T = P_{av\,T} + j\,Q_T$$

$$= \boldsymbol{S}_1 + \boldsymbol{S}_2 + \boldsymbol{S}_3 + \cdots + \boldsymbol{S}_n$$

Conservation of AC Power

The complex, real and reactive power of the source equal the respective sum of the complex, real and reactive power of the individual loads .



$$\boldsymbol{S}_{source} = V_s \,.\, {I_s}^*$$

$$= V_s \,.\, (I_1 + I_2)$$

$$= V_s \,.\, {I_1}^* + V_s \,.\, {I_2}^*$$

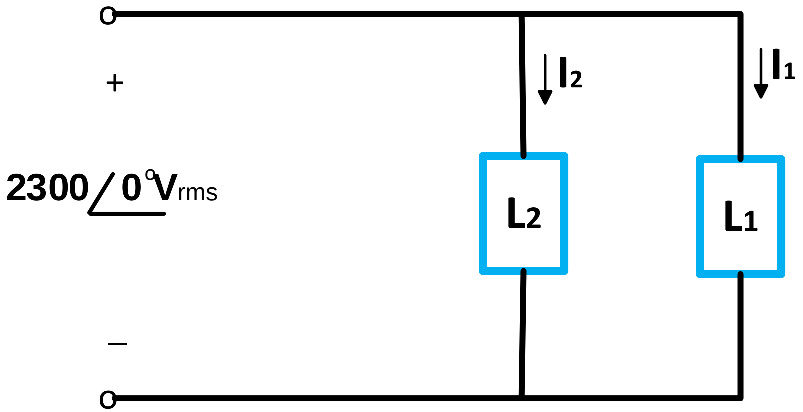
$$= \boldsymbol{S}_1 + \boldsymbol{S}_2$$

The same results can be obtained for a series connection .

Find the power factor of the two loads

Load 1 : 10 kW; 0.9 $lagging\ PF$

Load 2 : 5 kW ; 0.95 $leading\ PF$



$$\boldsymbol{S_1} = P_{av\,1} + j\,Q_1$$

$$Q_1 = P_{av\,1}\ \tan[\cos^{-1}(PF_1)]$$

$$= 4843\ VARs$$

$$\therefore\ \boldsymbol{S_1} = 10000 + j\,4843\ \ VA$$

$$\boldsymbol{S_2} = P_{av\,2} + j\,Q_2$$

$$Q_2 = -P_{av\,2}\ \tan[\cos^{-1}(PF_2)]$$

$$= -1643\ VARs \qquad \therefore\ \boldsymbol{S}_2 = 5000 - j\,1643\ \ VA$$

$$\boldsymbol{S}_T = \boldsymbol{S}_1 + \boldsymbol{S}_2$$

$$= 15000 + j\,3200$$

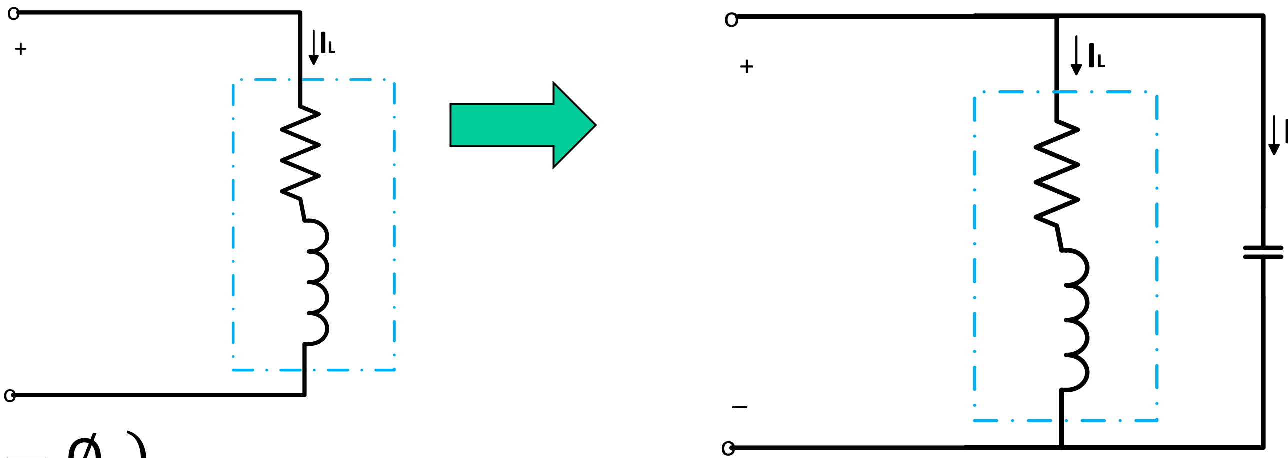
$$= 15337.5\ \angle 12.02° \quad VA$$

$$PF = \cos 12.02°$$

$$= 0.978 \quad \text{lagging}$$

Power Factor Correction

Power Factor correction is the process of increasing the power factor without altering the voltage or current to the original load .



$$PF = \cos(\theta_v - \phi_i)$$

For R :

PF = 1 ; $Q_R = 0$

∴ To improve the power factor we must decrease the Reactive Power .

∴ For inductive circuit, we add a capacitor in parallel to the load .

$$Q_c = Q_{Final} - Q_{init}$$

$$c = -\frac{Q_c}{\omega V_{rms}^2}$$

Example : A certain industrial plant consumes 1 MW at 0.7 lagging power factor and a $2300 \quad V_{rms}$.

What is the minimum capacitor required to improve the power factor to 0.9 lagging. $(\omega = 377 \quad rad/s)$

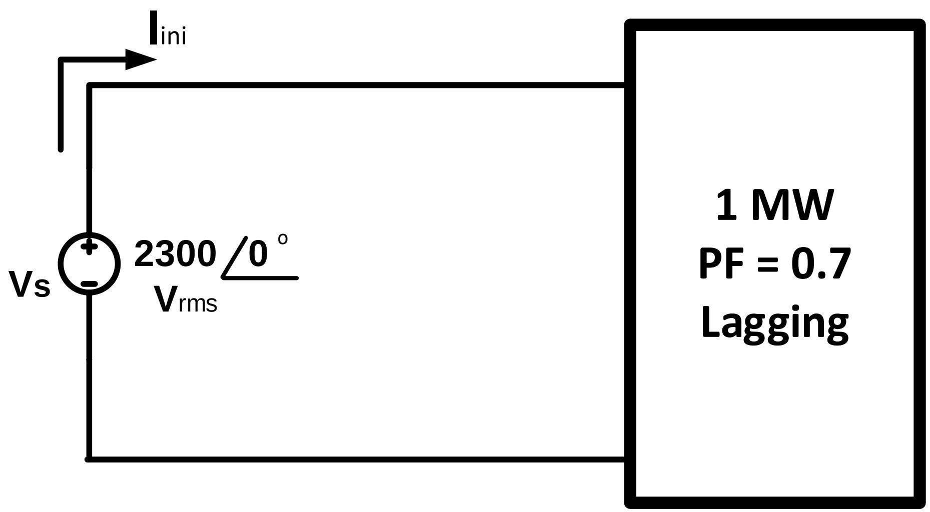
$$Q_{ini} = P_{av} \,.\tan[\cos^{-1}(PF_1)]$$
$$= 1MW.\tan[\cos^{-1}(0.7)]$$
$$= 1.02 \quad MVARs$$

$$Q_{Fin} = P_{av} \tan[\cos^{-1}(PF_2)]$$
$$= P_{av} \tan[\cos^{-1}(0.9)]$$
$$= 0.484 \quad MVARs$$

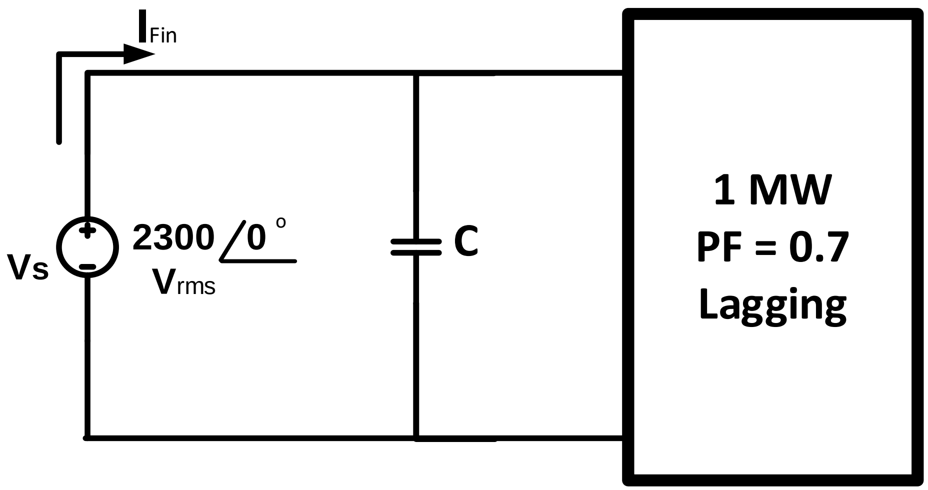
$$Q_c = Q_{Final} - Q_{init}$$
$$= -0.536 \quad MVARs$$

$$Q_c = -\frac{V_{rms}^2}{X_c}$$
$$= -\omega c \, V_{rms}^2$$

$$\therefore C = \frac{Q_c}{V_{rms}^2} = 269 \quad \mu F$$

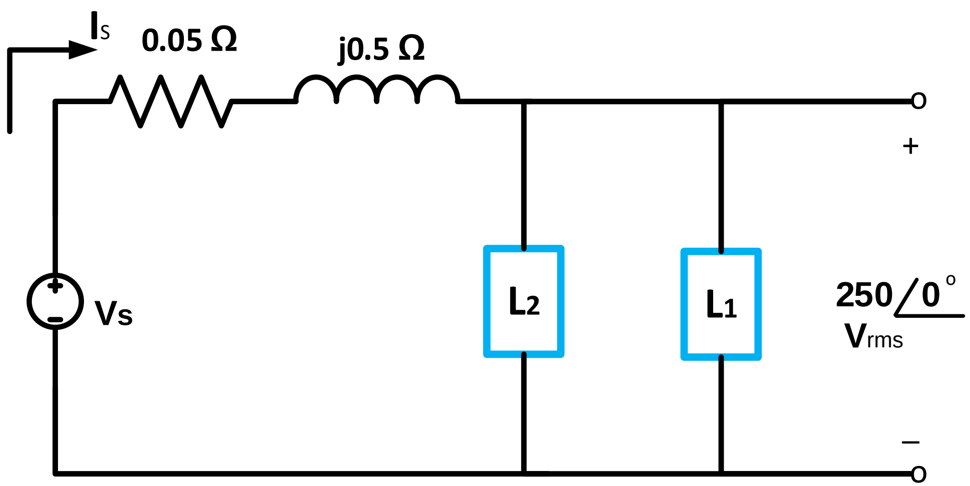


$$I_{ini} = \frac{P_{av}}{V_{rms}\, PF_1} = 621 \quad A_{rms}$$



$$I_{Fin} = \frac{P_{av}}{V_{rms}\, PF_2} = 483 \quad A_{rms}$$

Example :



Load 1 : 8 kW ; $0.8\ \ leading\ PF$

Load 2 : 20 kVA ; $0.6\ \ lagging\ PF$

1) Determine the power factor of two loads in parallel

2) Determine the apparent power required to supply the loads ; the magnitude of the current I_s ; the average power loss in the transmission line .

3) Compute the value of the capacitor that would correct the power factor to 1 if Placed in parallel with the two loads . ($\omega = 377\ \ r/s$)

4) Repeat step (2)

Load 1 : 8 kW ; 0.8 $leading\ PF$

Load 2 : 20 kVA ; 0.6 $lagging\ PF$

$$P_{av\,1} = 8000\,W$$

$$\therefore\ Q_1 = -P_{av\,1}\ \tan[\cos^{-1}(PF_1)] = -6000\ VAR$$

$$\therefore\ \boldsymbol{S}_1 = P_{av\,1} + j\,Q_1$$

$$= 8000 - j\,6000\ \ VA$$

$$P_{a\,2} = 20{,}000\,VA \qquad\qquad PF_2 = 0.6\ \ lagging$$

$$P_{av\,2} = P_{a\,2} * \ PF_2 = 12000\,W$$

$$\therefore\ Q_2 = \ P_{av\,2}\ \tan[\cos^{-1}(PF_2)] = +16000\ VAR$$

$$\therefore\ \boldsymbol{S}_2 = P_{av\,2} + j\,Q_2$$

$$= 12000 + j\,16000\ \ VA$$

$$\boldsymbol{S}_{L\,T} = \boldsymbol{S}_1 + \boldsymbol{S}_2$$

$$= 20{,}000 + j\,10{,}000$$

$$= 22360\,\angle 26.565° \quad VA$$

$$\therefore\; PF = \cos(26.565°) = .8544 \quad \text{lagging}$$

$$\boldsymbol{S}_{L\,T} = \boldsymbol{V}_{rms}\;\boldsymbol{I}_s^*$$

$$\therefore\; \boldsymbol{I}_s^* = \frac{\boldsymbol{S}_{L\,T}}{\boldsymbol{V}_{rms}} = \frac{22360}{250}\;\angle 26.565°$$

$$= 89.44\,\angle 26.565° \quad A_{rms}$$

Since $\boldsymbol{S}_{L\,T} = 22360\,\angle 26.565° \quad VA$

$$\therefore\; P_a = |\boldsymbol{S}_{L\,T}| = 22360 \quad VA$$

$$P_{av\;loss} = |\boldsymbol{I}_s|^2\,.\,(0.05) = 400 \quad W$$

3) since $\boldsymbol{S}_{L\,T} = 20\,000 + j\,10\,000$

$$\therefore\ Q_{ini} = 10\,000 \quad VAR$$

$$Q_{Fin} = 0$$

$$\therefore\ Q_c = Q_{Fin} - Q_{ini} = -10\,000 \quad VAR$$

$$\therefore C = \frac{Q_c}{\omega\, V_{rms}^2}$$

4) since $Q_{Fin} = 0$

$\therefore\ \boldsymbol{S}_F = P_a = 20\,000 \quad VA$

$\therefore\ P_a = P_{av} = 20\,000 \quad VA$

$\therefore\ \boldsymbol{S}_F = 20\,000 \angle 0° \quad VA$

$= \boldsymbol{V}_{rms}\ \boldsymbol{I}_s^*$

$$\boldsymbol{I}_s^* = \frac{20\,000 \angle 0°}{250 \angle 0°} = 80 \angle 0° \quad A$$

$\therefore\ \boldsymbol{I}_s = 80 \angle 0° \quad A$

$P_{av\ loss} = |\boldsymbol{I}_s|^2 \,.\, (0.05)$

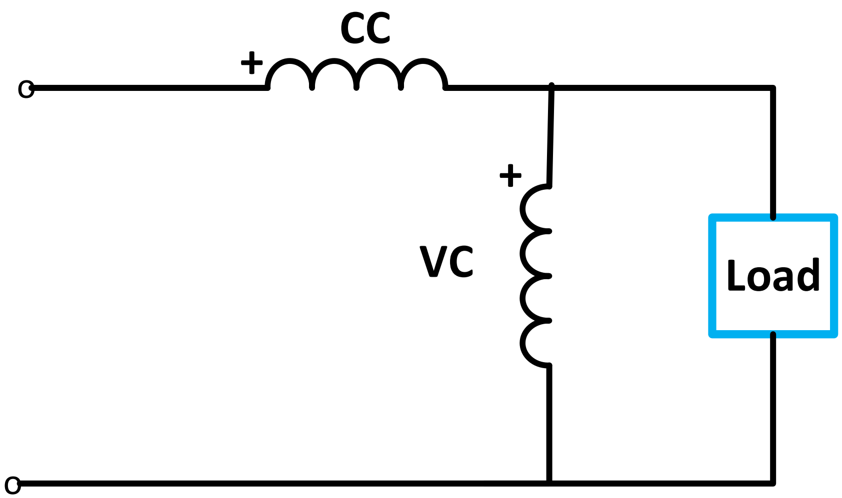
$= 320 \quad W$

Power Measurement

Wattmeter is the instrument for measuring the average power

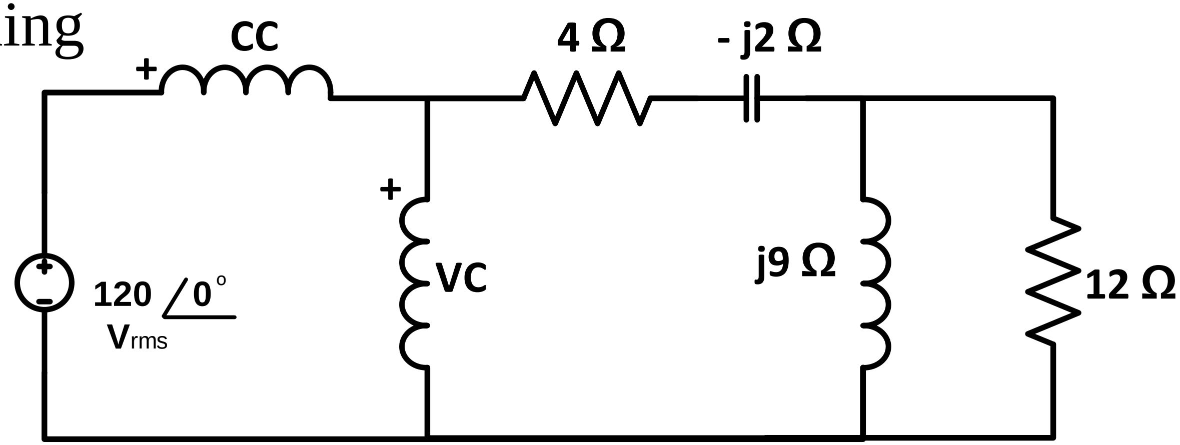
Two coils are used , the high impedance voltage coil and the low impedance current coil .

$$P = V_{rms} I_{rms} \cos(\theta_v - \theta_i)$$



Example :

Find the Wattmeter reading



$$Z = 4 - j2 \; + (\; j9||12 \;)$$

$$= 9.13 \angle 24.32° \qquad \Omega$$

$$\boldsymbol{I} = \frac{120 \angle 0°}{9.13 \angle 24.32°} \; 13.14 \angle -24.32° \qquad A_{rms}$$

$$P = (120)(13.14) \; \cos(0 + 24.32)$$

$$= 1436.9 \quad W$$

10.6 Find the average power dissipated in the 30 Ω resistor in the circuit seen in Fig. P10.6 if $i_g = 6 \cos 20{,}000t$ A.

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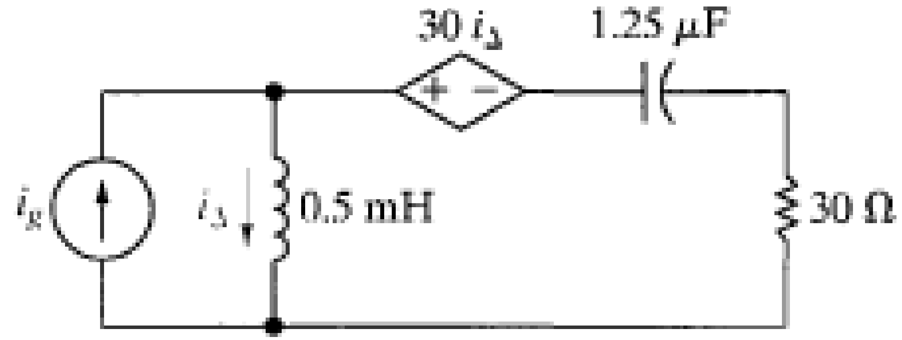
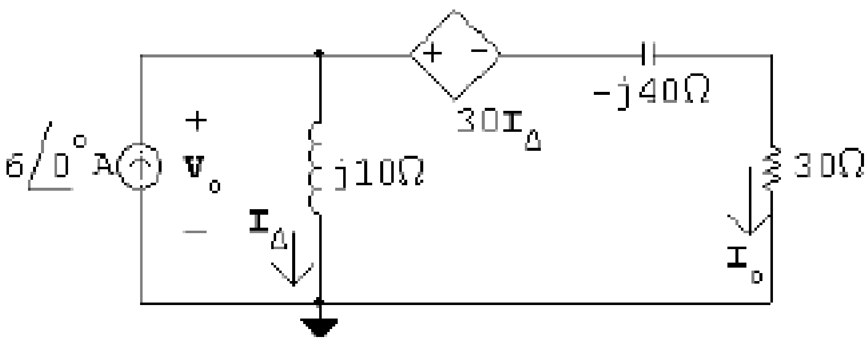
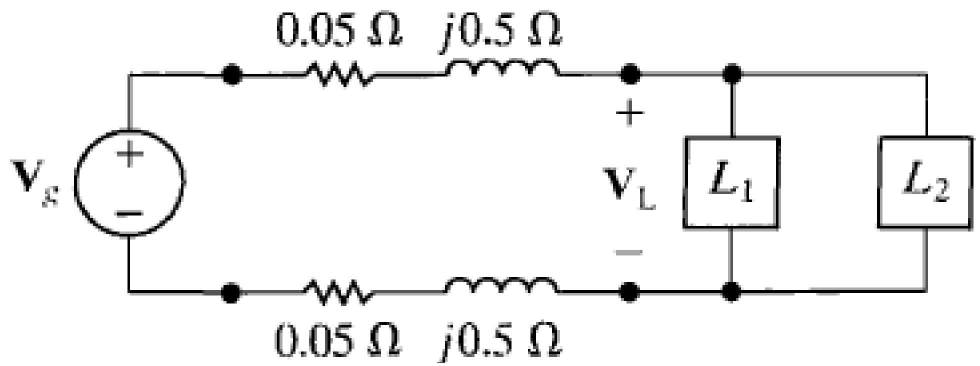


Answer



$$P_{30\Omega} = \frac{1}{2}|\mathbf{I}_o|^2 30 = 600\,\mathrm{W}$$

10.21 The two loads shown in Fig. P10.21 can be described as follows: Load 1 absorbs an average power of 60 kW and delivers 70 kVAR magnetizing reactive power; load 2 has an impedance of $24 + j7$.



The voltage at the terminals of the loads is $2500\sqrt{2}\cos 120\pi t$ V.

a) Find the rms value of the source voltage.

b) By how many microseconds is the load voltage out of phase with the source voltage?

c) Does the load voltage lead or lag the source voltage?

Answer

a) $V_g = 2514.86\underline{/2.735^\circ}$ Vrms

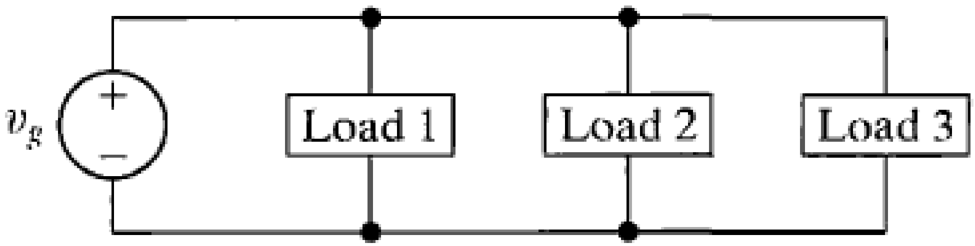
b) $t = 126.62\,\mu s$

c) $\mathbf{V}_L$ lags $\mathbf{V}_g$ by 2.735° or $126.62\,\mu s$

10.26 The three loads in the circuit in Fig. P10.26 can be described as follows: Load 1 is a 240 Ω resistor in series with an inductive reactance of 70 Ω; load 2 is a capacitive reactance of 120 Ω in series with a 160 Ω resistor; and load 3 is a 30 Ω resistor in series with a capacitive reactance of 40 Ω. The frequency of the voltage source is 60 Hz.

a) Give the power factor of each load.

b) Give the power factor of the composite load seen by the voltage source.



Answer

a)

$$Z_1 = 240 + j70 = 250\underline{/16.26^\circ}\ \Omega$$

$$\text{pf} = \cos(16.26^\circ) = 0.96 \text{ lagging}$$

$$Z_2 = 160 - j120 = 200\underline{/-36.87^\circ}\ \Omega$$

$$\text{pf} = \cos(-36.87^\circ) = 0.8 \text{ leading}$$

$$Z_3 = 30 - j40 = 50\underline{/-53.13^\circ}\ \Omega$$

$$\text{pf} = \cos(-53.13^\circ) = 0.6 \text{ leading}$$

b)

$$Z = \frac{1}{Y} = 37.44\underline{/-42.03^\circ}\ \Omega$$

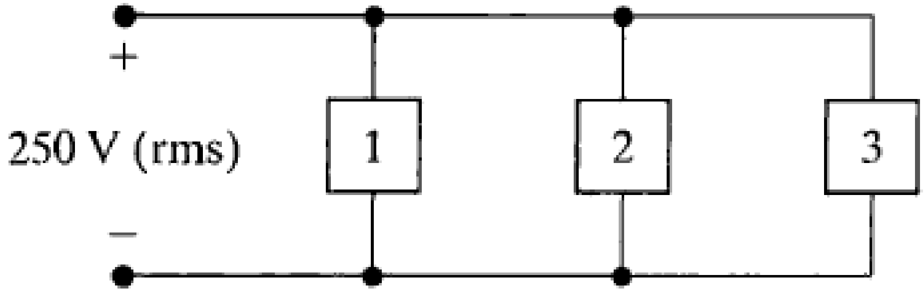
$$\text{pf} = \cos(-42.03^\circ) = 0.74 \text{ leading}$$

10.27 Three loads are connected in parallel across a 250 V (rms) line, as shown in Fig. P10.27. Load 1 absorbs 16 kW and 18 kVAR. Load 2 absorbs 10 kVA at 0.6 pf lead. Load 3 absorbs 8 kW at unity power factor.

a) Find the impedance that is equivalent to the three parallel loads.

b) Find the power factor of the equivalent load as seen from the line's input terminals.

Figure P10.27



Answer

a) $$Z = \frac{250}{120 - j40} = 1.875 + j0.625\,\Omega = 1.98\underline{/18.43^\circ}\,\Omega$$

b) $$\text{pf} = \cos(18.43^\circ) = 0.9487 \text{ lagging}$$

10.28 The three loads in Problem 10.27 are fed from a line having a series impedance $0.01 + j0.08\ \Omega$, as shown in Fig. P10.28.

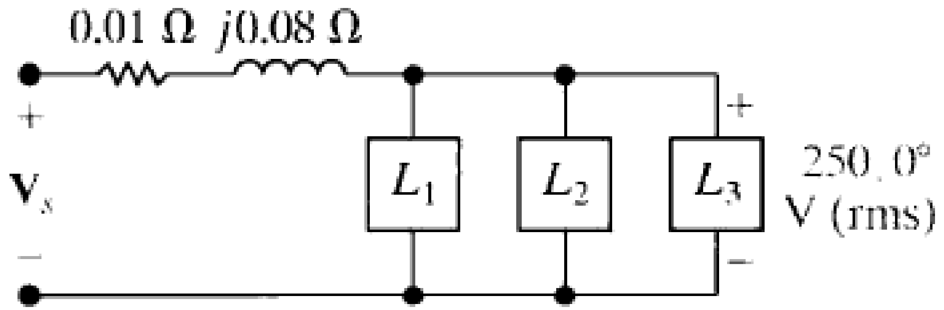
a) Calculate the rms value of the voltage ($\mathbf{V}_s$) at the sending end of the line.

b) Calculate the average and reactive powers associated with the line impedance.

c) Calculate the average and reactive powers at the sending end of the line.

d) Calculate the efficiency (η) of the line if the efficiency is defined as

$$\eta = (P_{\text{load}}/P_{\text{sending end}}) \times 100.$$



[a] From the solution to Problem 10.26 we have

$$\mathbf{I}_{\mathrm{L}} = 120 - j40 \text{ A (rms)}$$

$$\therefore \quad \mathbf{V}_s = 250\underline{/0^\circ} + (120 - j40)(0.01 + j0.08) = 254.4 + j9.2$$

$$= 254.57\underline{/2.07^\circ} \text{ V (rms)}$$

[b] $|\mathbf{I}_{\mathrm{L}}| = \sqrt{16{,}000}$

$$P_\ell = (16{,}000)(0.01) = 160 \text{ W} \qquad Q_\ell = (16{,}000)(0.08) = 1280 \text{ VAR}$$

[c] $P_s = 30{,}000 + 160 = 30.16 \text{ kW} \qquad Q_s = 10{,}000 + 1280 = 11.28 \text{ kVAR}$

[d] $\eta = \dfrac{30}{30.16}(100) = 99.47\%$

10.34 A group of small appliances on a 60 Hz system requires 20 kVA at 0.85 pf lagging when operated at 125 V (rms). The impedance of the feeder supplying the appliances is $0.01 + j0.08\ \Omega$. The voltage at the load end of the feeder is 125 V (rms).

a) What is the rms magnitude of the voltage at the source end of the feeder?

b) What is the average power loss in the feeder?

c) What size capacitor (in microfarads) across the load end of the feeder is needed to improve the load power factor to unity?

d) After the capacitor is installed, what is the rms magnitude of the voltage at the source end of the feeder if the load voltage is maintained at 125 V (rms)?

e) What is the average power loss in the feeder for (d)?

[a] $S_{\mathrm{L}} = 20{,}000(0.85 + j0.53) = 17{,}000 + j10{,}535.65\,\mathrm{VA}$

$$125\mathbf{I}_{\mathrm{L}}^* = (17{,}000 + j10{,}535.65); \qquad \mathbf{I}_{\mathrm{L}}^* = 136 + j84.29\,\mathrm{A\,(rms)}$$

$$\therefore \quad \mathbf{I}_{\mathrm{L}} = 136 - j84.29\,\mathrm{A\,(rms)}$$

$$\mathbf{V}_s = 125 + (136 - j84.29)(0.01 + j0.08) = 133.10 + j10.04$$

$$= 133.48\underline{/4.31^\circ}\,\mathrm{V\,(rms)}$$

$$|\mathbf{V}_s| = 133.48\,\mathrm{V\,(rms)}$$

[b] $P_\ell = |\mathbf{I}_\ell|^2(0.01) = (160)^2(0.01) = 256\,\mathrm{W}$

[c] $\dfrac{(125)^2}{X_{\mathrm{C}}} = -10{,}535.65; \qquad X_{\mathrm{C}} = -1.48306\,\Omega$

$$-\frac{1}{\omega C} = -1.48306; \qquad C = \frac{1}{(1.48306)(120\pi)} = 1788.59\,\mu\mathrm{F}$$

[d] $\mathbf{I}_\ell = 136 + j0\,\mathrm{A\,(rms)}$

$$\mathbf{V}_s = 125 + 136(0.01 + j0.08) = 126.36 + j10.88$$

$$= 126.83\underline{/4.92^\circ}\,\mathrm{V\,(rms)}$$

$$|\mathbf{V}_s| = 126.83\,\mathrm{V\,(rms)}$$

[e] $P_\ell = (136)^2(0.01) = 184.96\,\mathrm{W}$