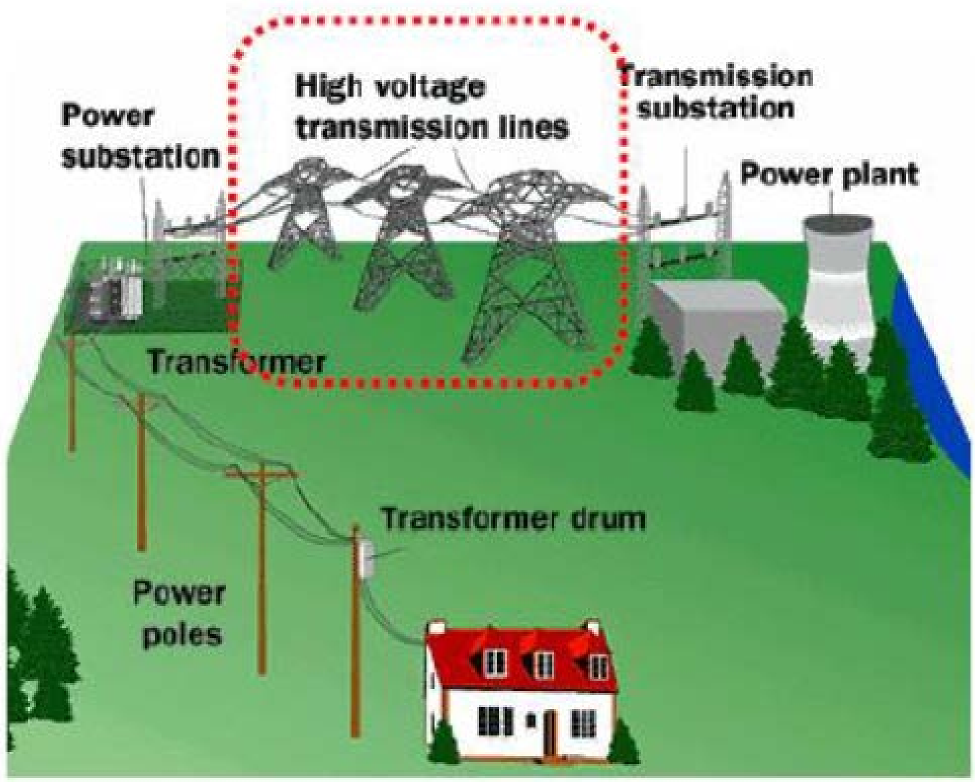
Chapter 11
Balanced Three Phase Circuits

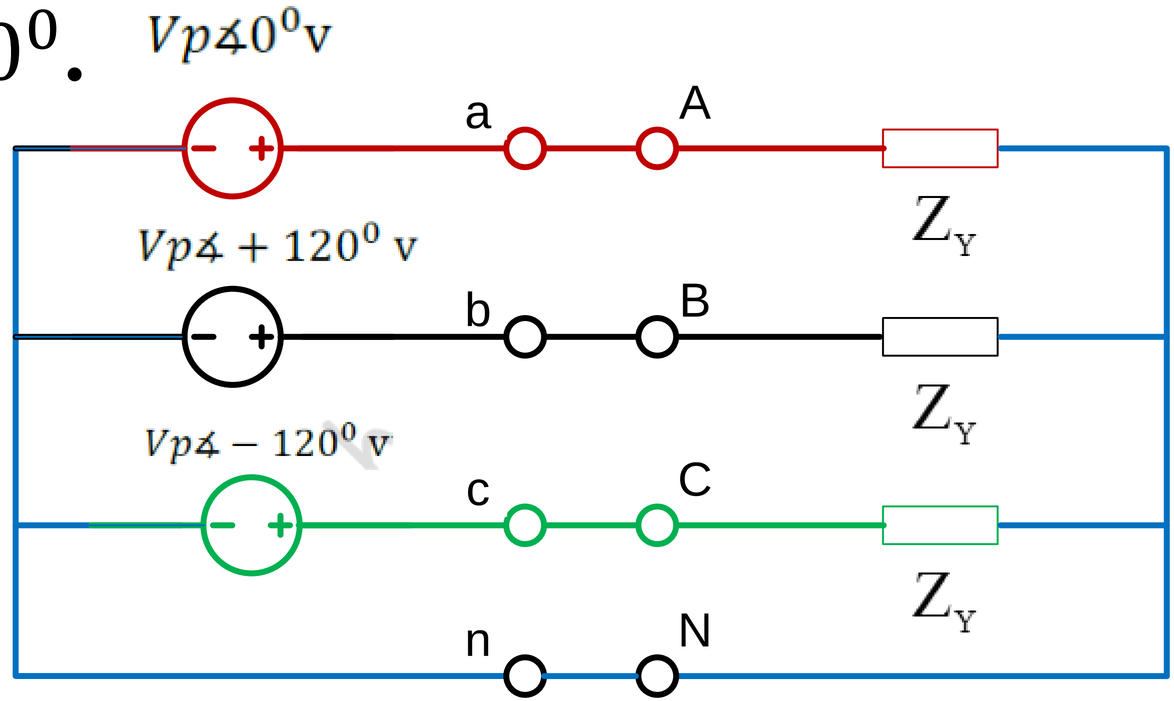


ENEE2301 - Birzeit University

Balanced Three-Phase Circuits

What is a Three-Phase Circuit?

It is a system produced by a generator consisting of three sources having the same amplitude and frequency but out of phase with each other by 120^0.



Advantages

1- Almost all the electric power is generated and distributed in three-phase.

2- The instantaneous power in a three-phase system is constant

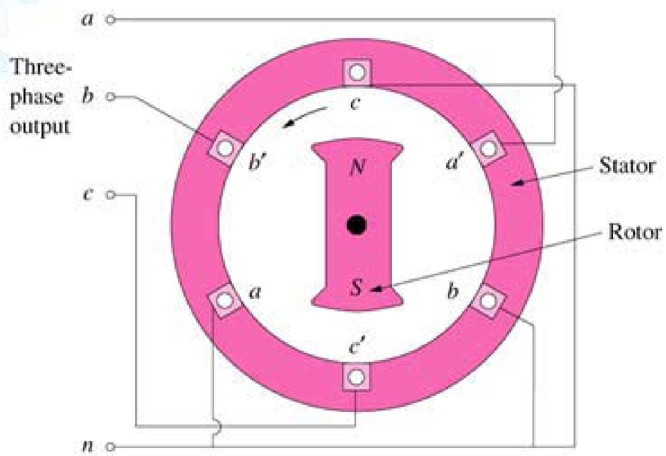
==> There is less vibration in the rotating machinery which in turn performs more efficiently.

3- The amount of power loss in the three-phase system is only half the power loss in the cables for the single phase system.

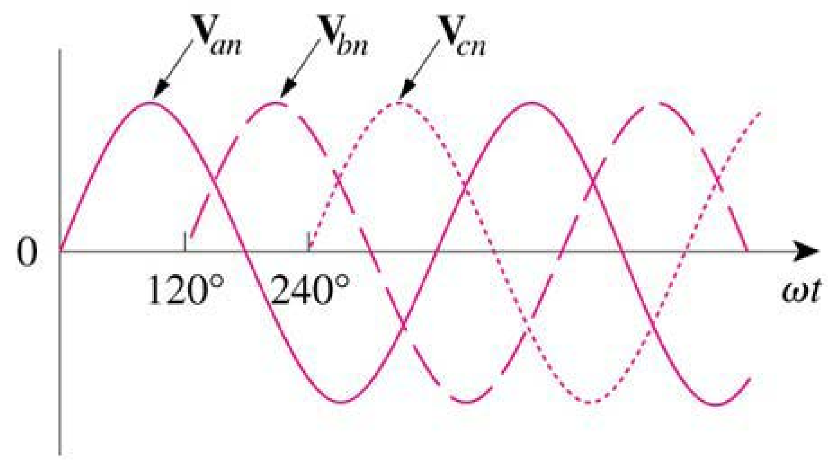
4- Thinner conductors can be used to transmit the same KVA at the same voltage.

Balanced Three Phase Generator

A three-phase generator consists of a rotating magnet (rotor) surrounded by a stationary winding (stator).



A three-phase generator



The generated voltages

The Three – Phase Generator:

a) Has three induction coils.

b) Placed 120^0 a part on the rotor.

c) The three coils have an equal number of turns.

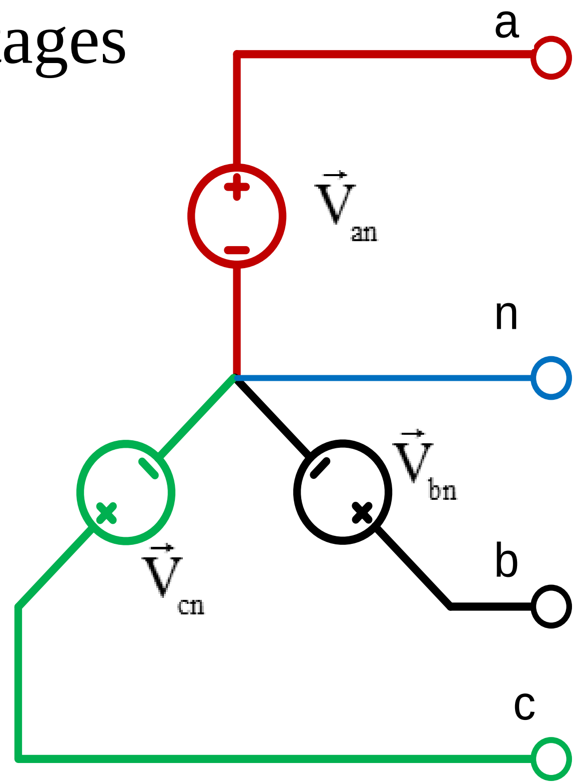
d) The voltage induced across each coil will have the same peak value, shape and frequency.

Balanced Three-Phase Sources:

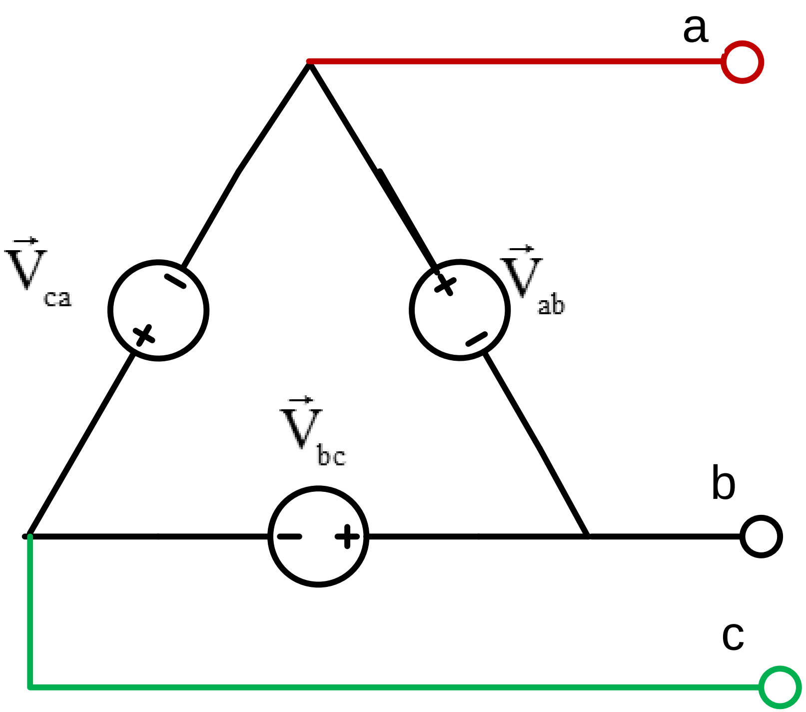
- Two possible configurations

1- The Y-connected source

$\vec{V}_{an}$, $\vec{V}_{bn}$ and $\vec{V}_{cn}$ are called phase voltages



2) The Δ- connected source



$\vec{V}_{ab}, \vec{V}_{bc}$ and $\vec{V}_{ca}$ are called line to line voltages

The Phase Sequence

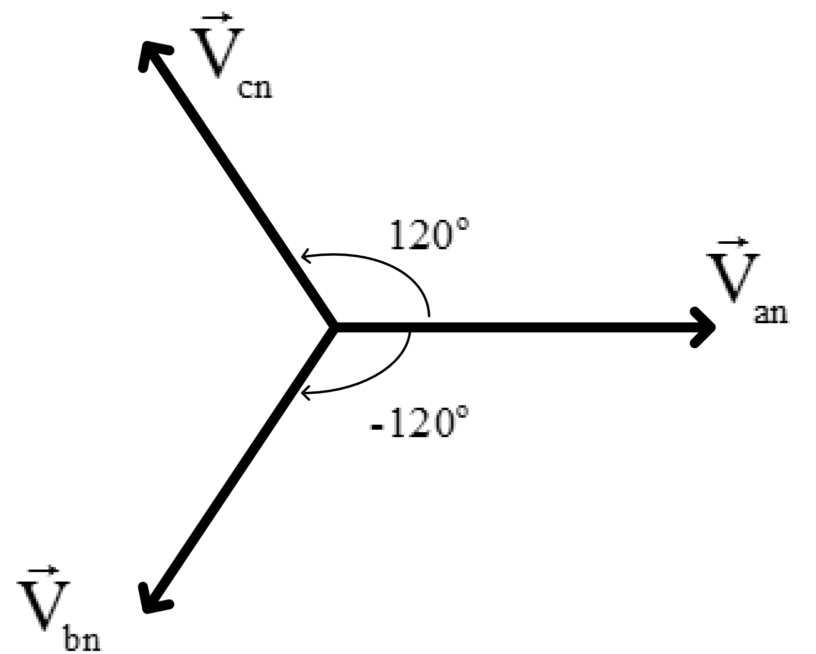
- The phase sequence is the time order in which the voltages pass through their respective maximum values

1- **abc sequence (positive sequence)**

$$\overrightarrow{V_{an}} = \overrightarrow{V_a} = V_P \measuredangle 0^0$$

$$\overrightarrow{V_{bn}} = \overrightarrow{V_b} = V_P \measuredangle -120^0$$

$$\overrightarrow{V_{cn}} = \overrightarrow{V_c} = V_P \measuredangle +120$$

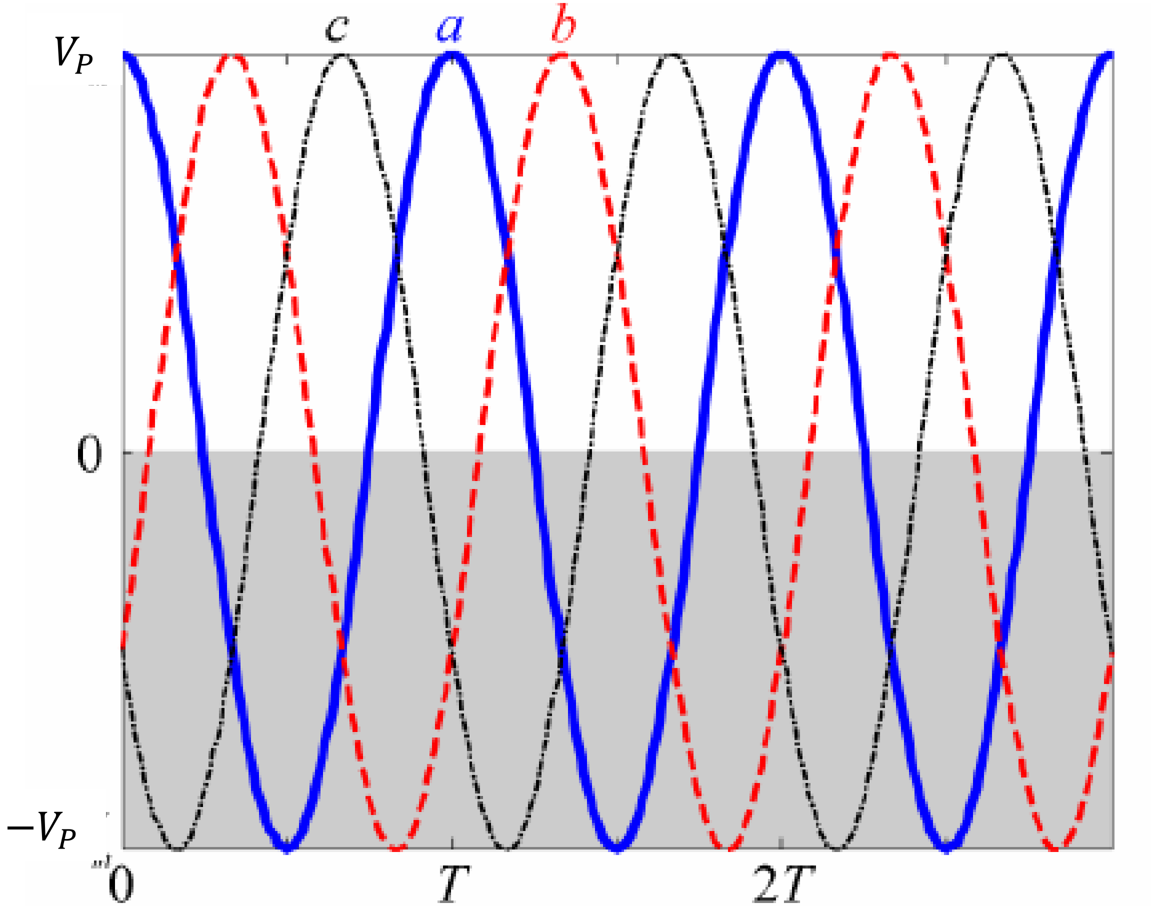


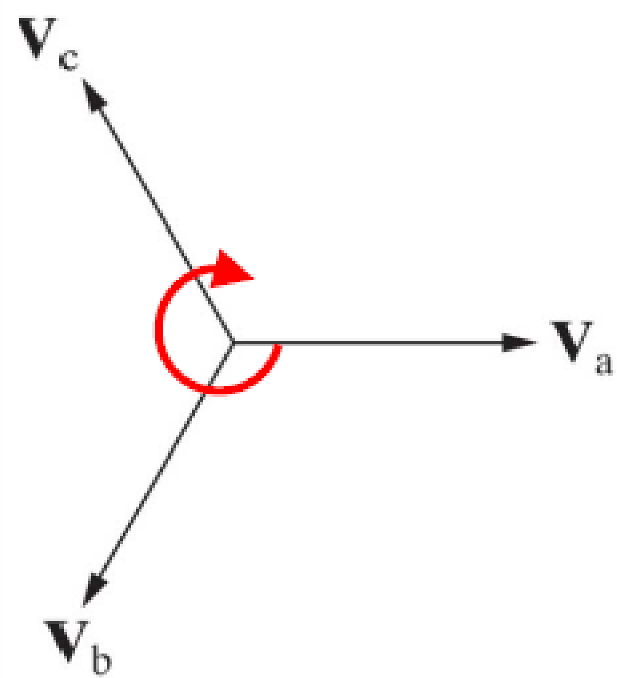
abc sequence

$$\overrightarrow{V_{an}} = \overrightarrow{V_a} = V_P\measuredangle 0^0$$

$$\overrightarrow{V_{bn}} = \overrightarrow{V_b} = V_P\measuredangle -120^0$$

$$\overrightarrow{V_{cn}} = \overrightarrow{V_c} = V_P\measuredangle +120$$



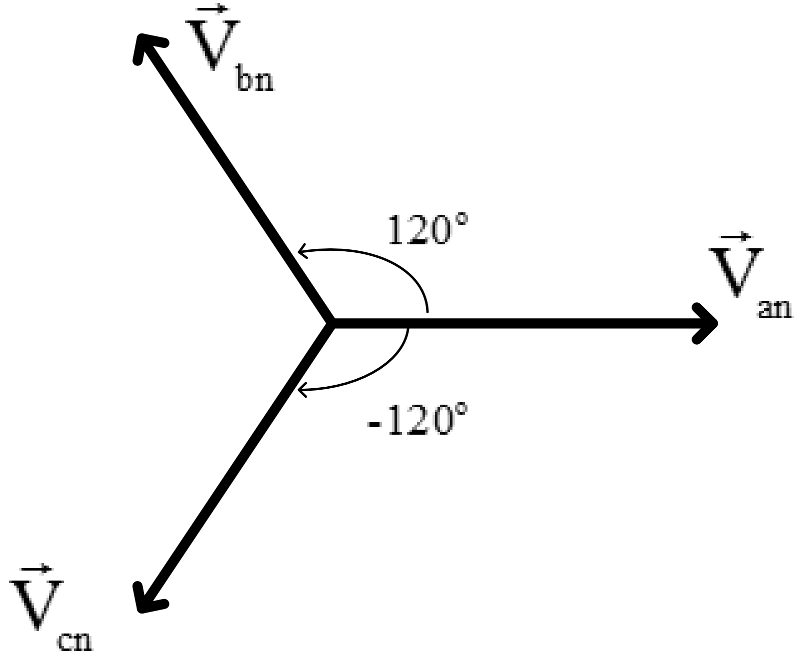


2- acb sequence (Negative sequence)

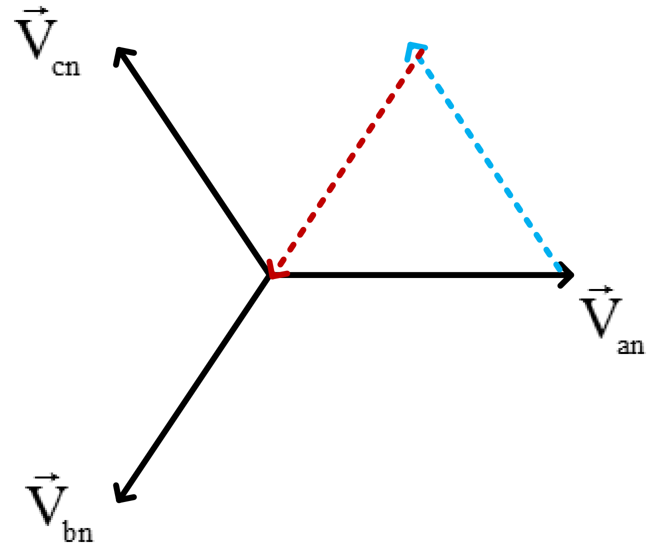
$$\overrightarrow{V_{an}} = \overrightarrow{V_a} = V_P \measuredangle 0^0$$

$$\overrightarrow{V_{bn}} = \overrightarrow{V_b} = V_P \measuredangle +120^0$$

$$\overrightarrow{V_{cn}} = \overrightarrow{V_c} = V_P \measuredangle -120$$



$$\overrightarrow{V_{an}} + \overrightarrow{V_{bn}} + \overrightarrow{V_{cn}} = 0$$



$$\overrightarrow{V_{an}} = \overrightarrow{V_a} = V_P \measuredangle 0^0$$

$$\overrightarrow{V_{bn}} = \overrightarrow{V_{\boldsymbol{b}}} = V_P \measuredangle - \mathbf{120}^0$$

$$\overrightarrow{V_{cn}} = \overrightarrow{V_{\boldsymbol{c}}} = V_P \measuredangle + \mathbf{120}^0$$

$$\overrightarrow{V_{an}} = V_P$$

$$\overrightarrow{V_{bn}} = V_P Cos(-120^o) + jV_P Sin(-120^o)$$

$$\overrightarrow{V_{bn}} = V_P \left(-\frac{1}{2} - j\frac{\sqrt{3}}{2}\right)$$

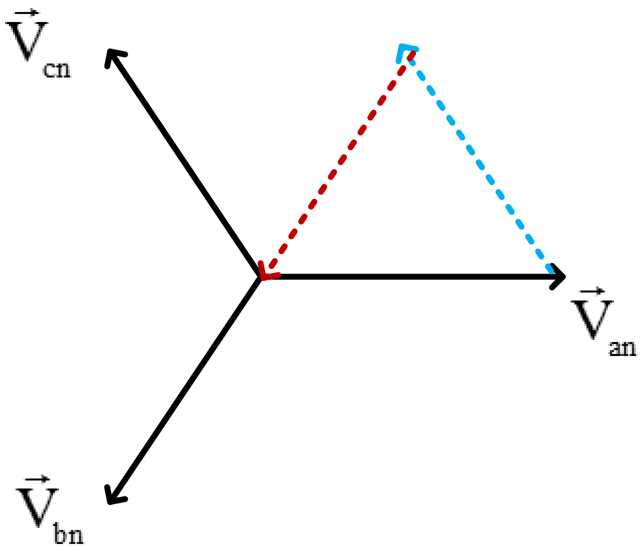
$$\overrightarrow{V_{cn}} = V_P Cos(+120^o) + jV_P Sin(+120^o)$$

$$\overrightarrow{V_{cn}} = V_P \left(-\frac{1}{2} + j\frac{\sqrt{3}}{2}\right)$$

$$\therefore \quad \overrightarrow{V_{an}} + \overrightarrow{V_{bn}} + \overrightarrow{V_{cn}} = 0$$

$$\therefore \quad \overrightarrow{V_{an(t)}} + \overrightarrow{V_{bn(t)}} + \overrightarrow{V_{cn(t)}} = 0$$

Balanced set



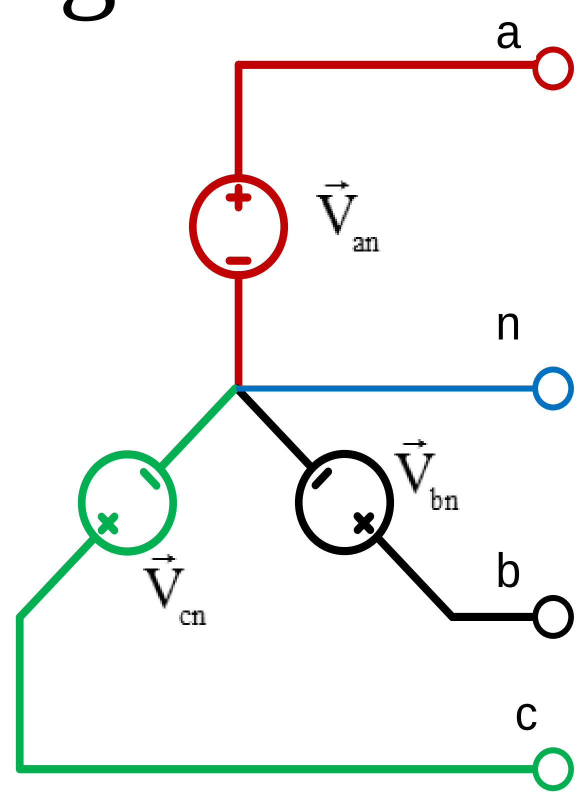
Line to Line Voltages

let

$$\overrightarrow{V_{an}} = V_P\measuredangle 0^0$$

$$\overrightarrow{V_{bn}} = V_P\measuredangle -\mathbf{120}^0$$

$$\overrightarrow{V_{cn}} = V_P\measuredangle +\mathbf{120}^0$$



⇩

$$\overrightarrow{V_{ab}} = \overrightarrow{V_{an}} + \overrightarrow{V_{nb}}$$

$$\overrightarrow{V_{ab}} = \overrightarrow{V_{an}} - \overrightarrow{V_{bn}}$$

$$\overrightarrow{V_{ab}} = V_P\measuredangle 0^0 - V_P\measuredangle -\mathbf{120}^0$$

$$\overrightarrow{V_{ab}} = V_P - V_P\left(Cos(-120^o) + jSin(-120^o)\right)$$

$\vec{V}_{ab}, \vec{V}_{bc}, \vec{V}_{ca}$ are line to line voltages

$$\overrightarrow{V_{ab}} = Vp - Vp\left(-\frac{1}{2} - j\frac{\sqrt{3}}{2}\right)$$

$$\overrightarrow{V_{ab}} = Vp\left(1 + \frac{1}{2} + j\frac{\sqrt{3}}{2}\right)$$

$$\overrightarrow{V_{ab}} = Vp\left(\frac{3}{2} + j\frac{\sqrt{3}}{2}\right)$$

$$\overrightarrow{V_{ab}} = vp\sqrt{\left(\frac{3}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2} \measuredangle \tan^{-1}\left(\frac{\frac{\sqrt{3}}{2}}{\frac{3}{2}}\right)$$

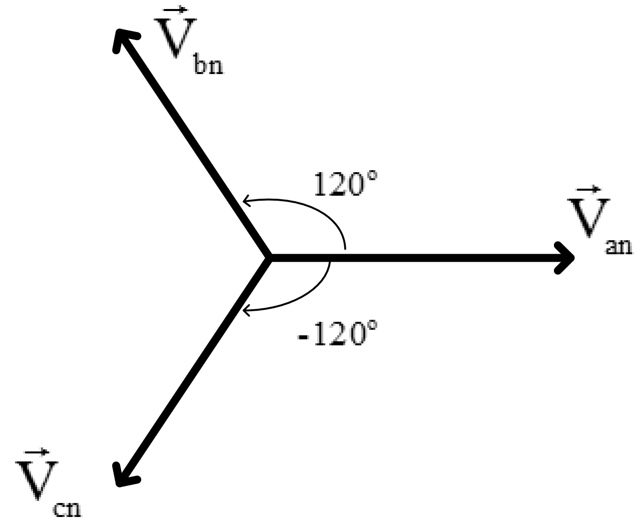
$$\overrightarrow{V_{ab}} = Vp\sqrt{3} \measuredangle +30^0 \text{ v}$$

$$\overrightarrow{V_{ab}} = \sqrt{3}\ \overrightarrow{V_{an}} \measuredangle +30^0 \text{ v}$$

$$\overrightarrow{V_{bc}} = \sqrt{3}\ \overrightarrow{V_{bn}} \measuredangle +30^0 \text{ v}$$

$$\overrightarrow{V_{ca}} = \overrightarrow{V_{cn}}\sqrt{3} \measuredangle +30^0 \text{ v}$$

For negative sequence:



$$\overrightarrow{V_{an}} = Vp\measuredangle 0^0 \text{v}$$

$$\overrightarrow{V_{bn}} = Vp\measuredangle + 120^0 \text{ v}$$

$$\overrightarrow{V_{cn}} = Vp\measuredangle - 120^0 \text{ v}$$

$$\overrightarrow{V_{ab}} = Vp\sqrt{3} \;\; \measuredangle - 30^0 \text{ v}$$

$$\therefore \overrightarrow{V_{ab}} = \sqrt{3} \;\; \overrightarrow{V_{an}} \measuredangle - 30^0 \text{ v}$$

$$\therefore \overrightarrow{V_{bc}} = \sqrt{3} \;\; \overrightarrow{V_{bn}} \measuredangle - 30^0 \text{ v}$$

$$\therefore \overrightarrow{V_{bc}} = Vp\sqrt{3} \;\; \measuredangle + 90^0 \text{ v}$$

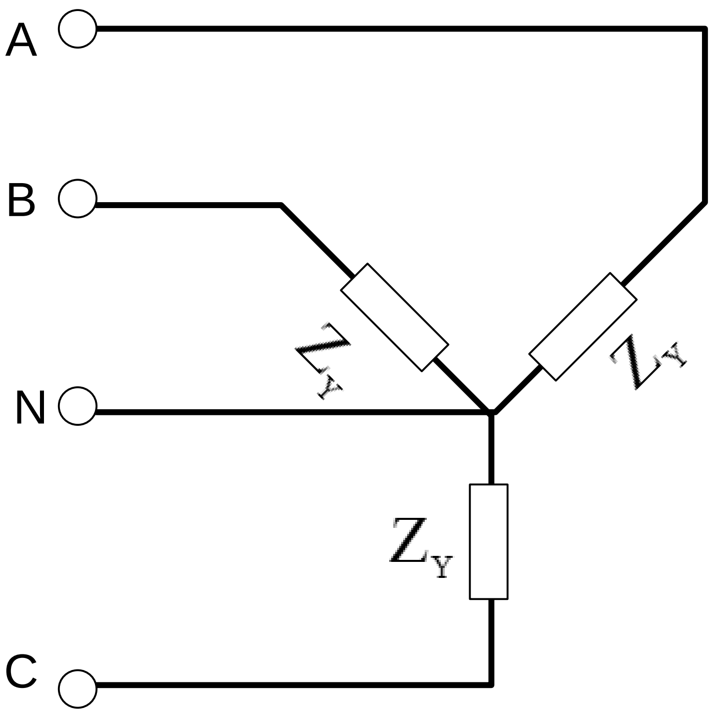
$\therefore \quad \overrightarrow{V_{ca}} = V_p\sqrt{3} \measuredangle -150^0 \text{ v}$

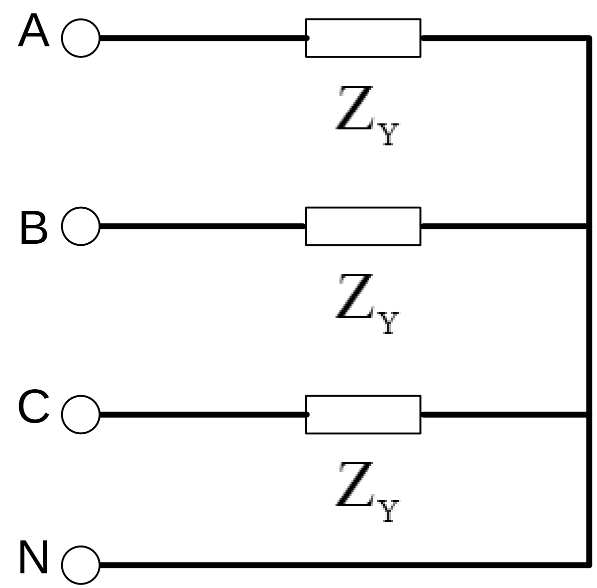
$\therefore \quad \overrightarrow{V_{ca}} = \sqrt{3}\ \overrightarrow{V_{cn}} \measuredangle -30^0 \text{ v}$

Balanced Three Phase Loads

- A balanced load has equal impedances on all the phases. Two connection schemes are used:

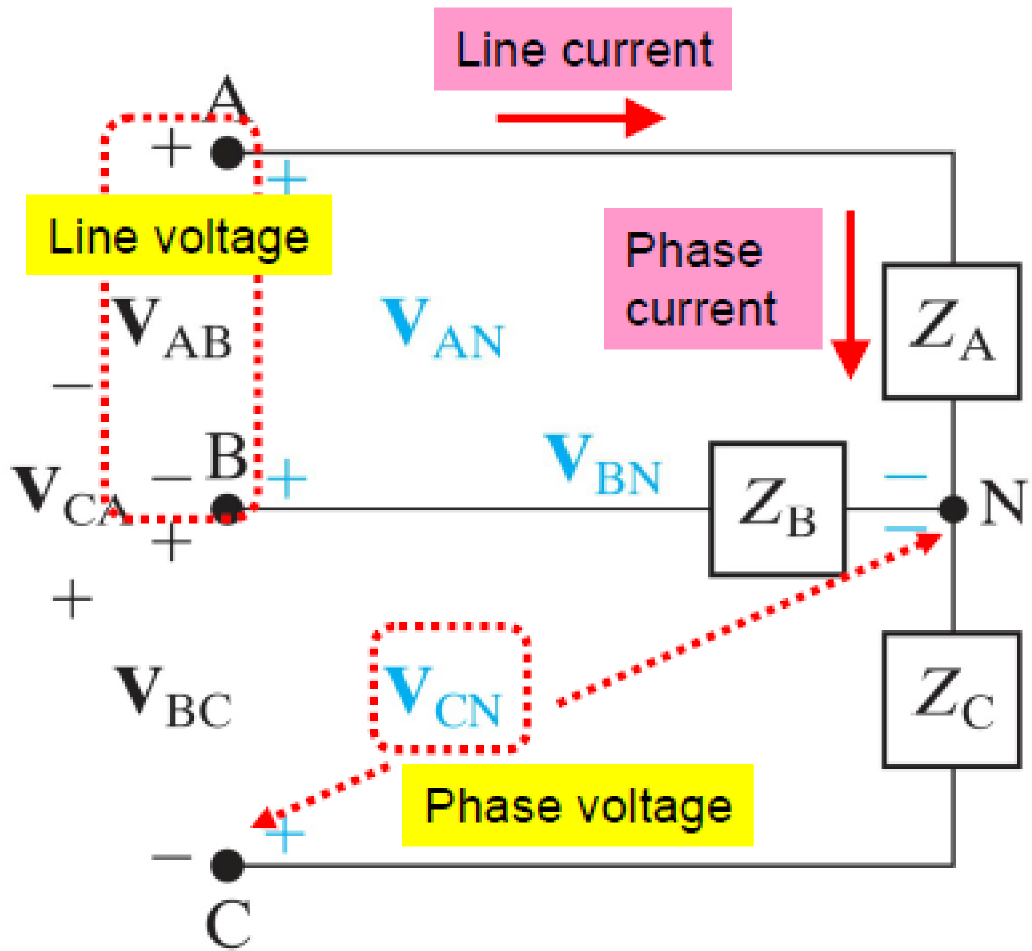
1- Y-connected load





Unknowns to be solved

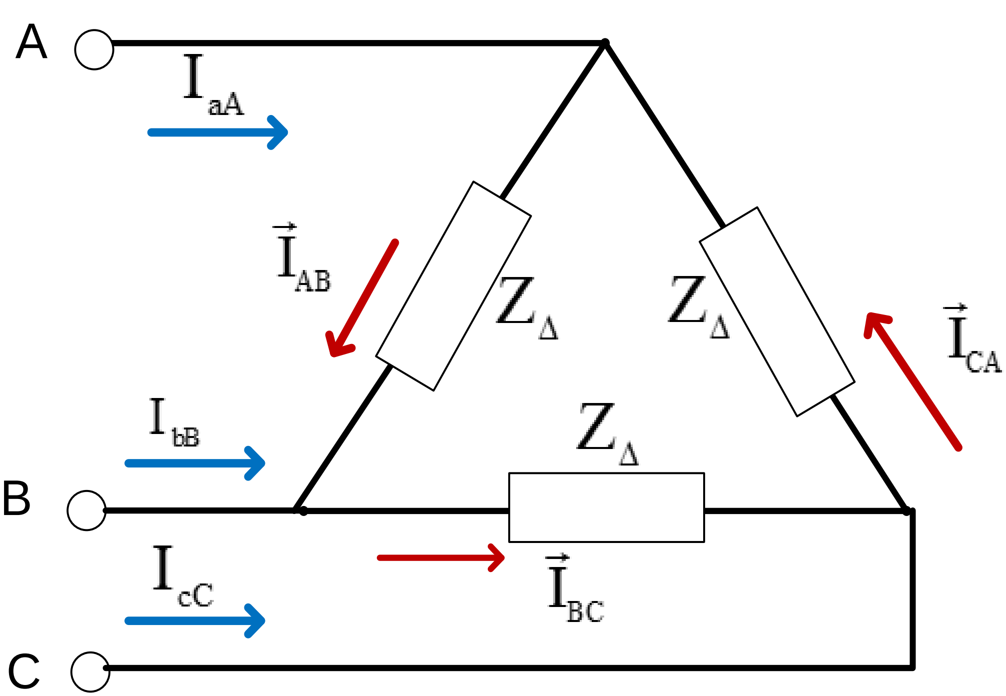
- Line (line-to-line) voltage: voltage across any pair of lines.
- Phase (line-to-neutral) voltage: voltage across a single phase.

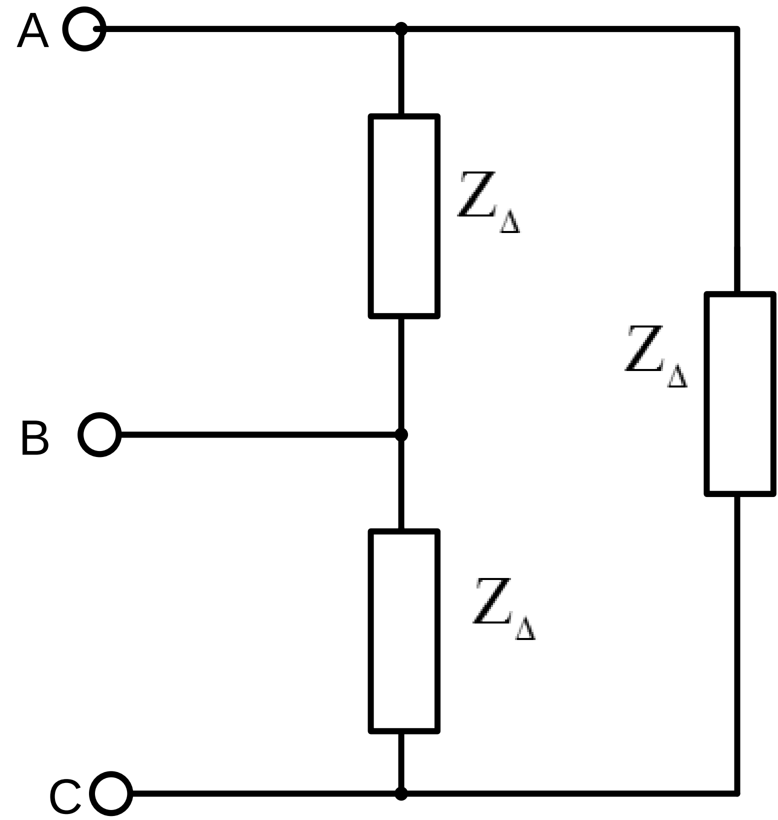


- For Y-connected load, line current equals phase current.

Balanced Three Phase Loads

2- Δ-connected load





$$Z_Y = \frac{Z_\Delta}{3}$$

$$Z_\Delta = 3Z_Y$$

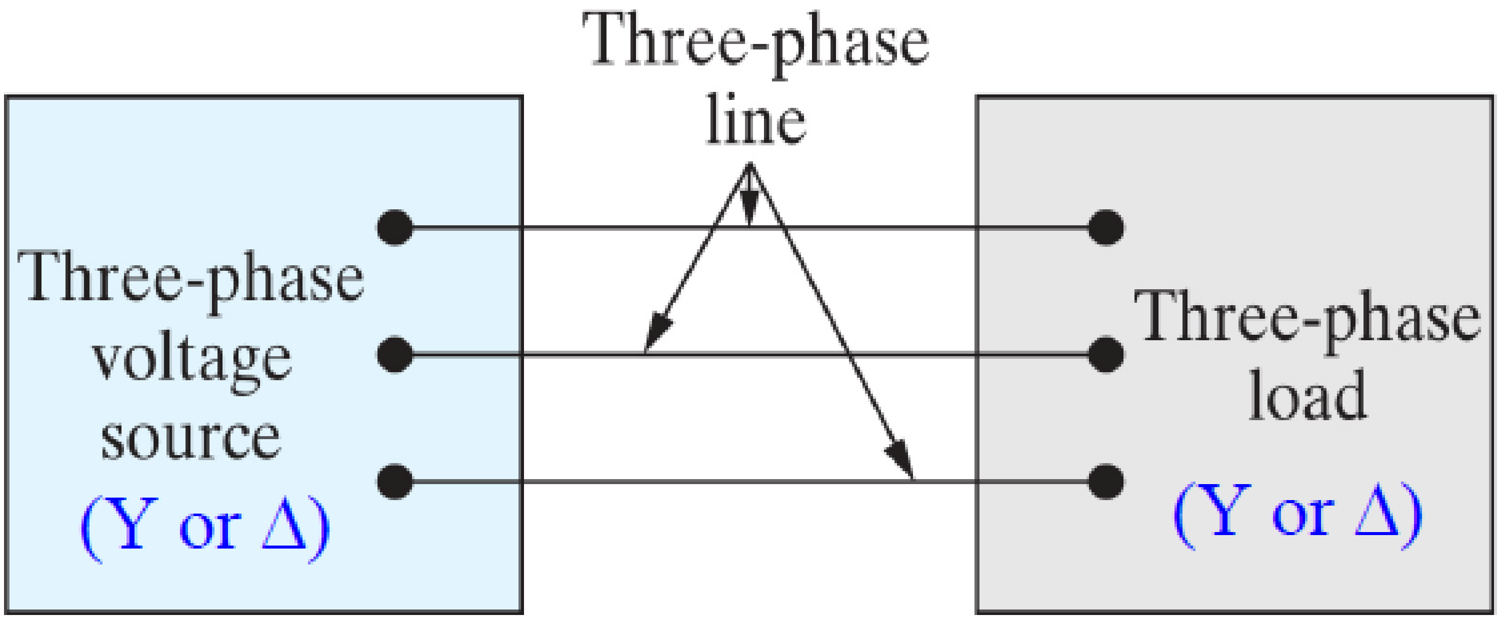
Three Phase Connections

- Both the three phase source and the three phase load can be connected either Wye or Delta

⇨ We have 4 possible connection types.

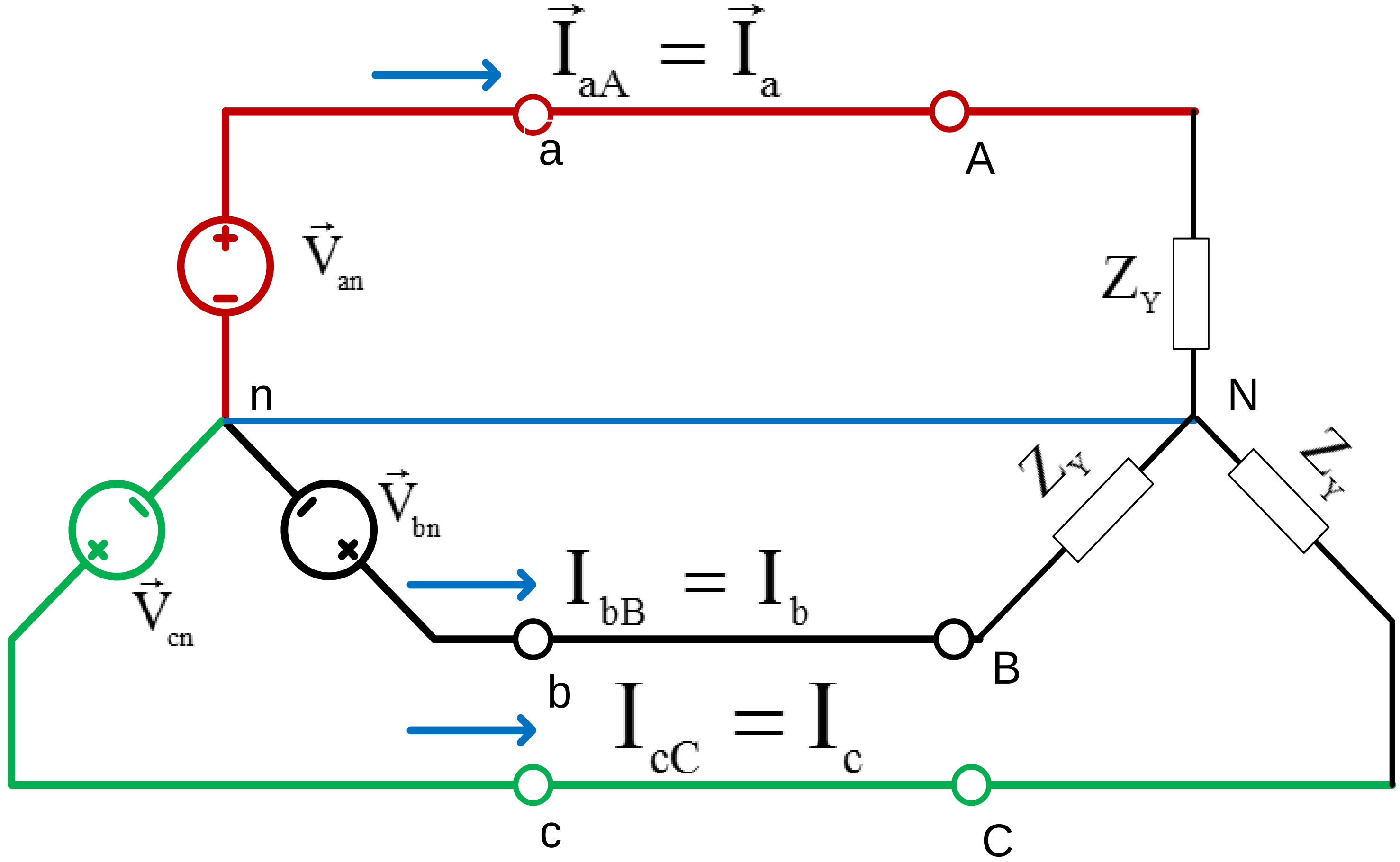
- Y-Y Connection
- Y- Δ Connection
- Δ – Δ Connection
- Δ - Y Connection

Three-phase systems



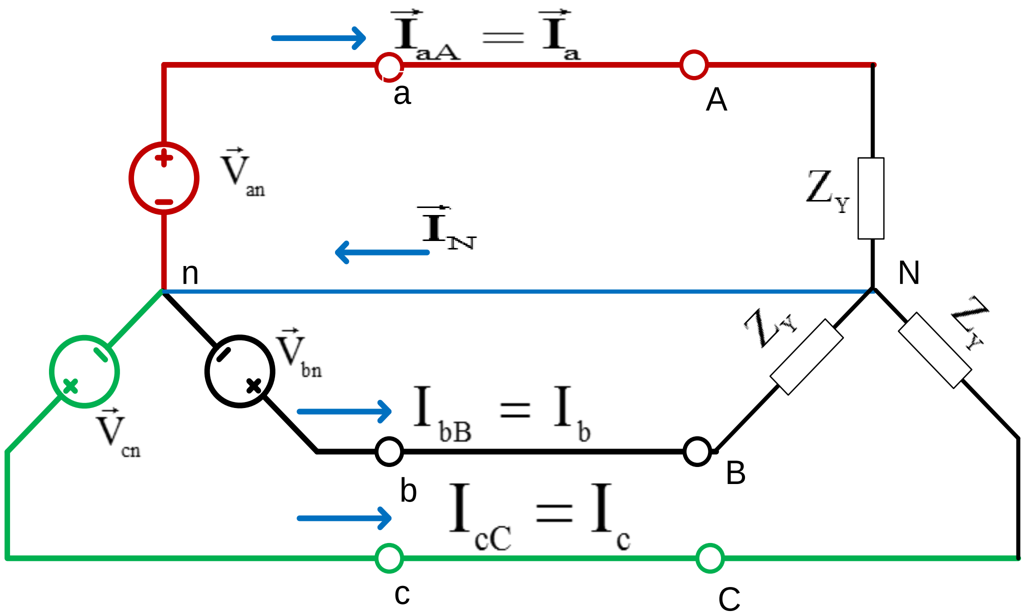
- Source-load can be connected in four configurations: Y-Y, Y-Δ, Δ-Y, Δ-Δ.
- It's sufficient to analyze Y-Y, while the others can be treated by Δ-Y and Y-Δ transformations.

Balanced Y-Y System



$$\vec{I}_{aA} = \vec{I}_a = \frac{\vec{V}_{an}}{Z_Y} \qquad \vec{I}_{bB} = \vec{I}_b = \frac{\vec{V}_{bn}}{Z_Y} \qquad \vec{I}_{cC} = \vec{I}_c = \frac{\vec{V}_{cn}}{Z_Y}$$

Balanced Y-Y System



$$\text{KCL}: \vec{I}_N = \vec{I}_a + \vec{I}_b + \vec{I}_c$$

$$= \frac{\vec{V}_{an}}{Z_Y} + \frac{\vec{V}_{bn}}{Z_Y} + \frac{\vec{V}_{cn}}{Z_Y}$$

$$= \frac{1}{Z_Y}\left(\vec{V}_{an} + \vec{V}_{bn} + \vec{V}_{cn}\right)$$

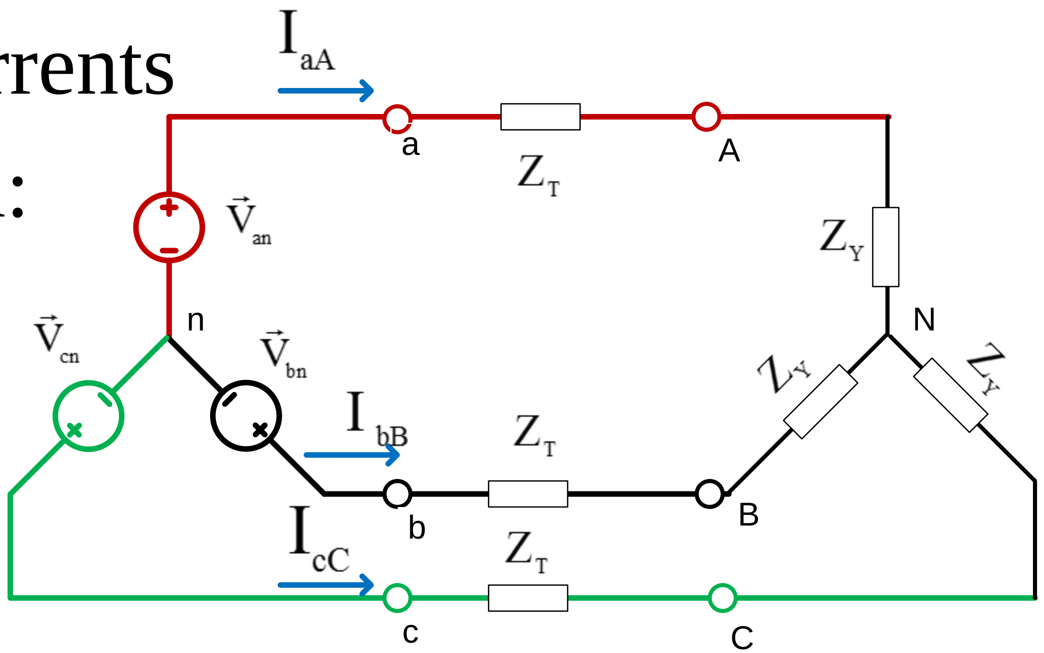
$$\vec{I}_N = \frac{1}{Z_Y}(Zero) = 0$$

$\Rightarrow$ it could be replaced by open circuit

Example

- Calculate The line currents

If the following is given:



$$\vec{V}_{an} = 120 \angle 0^0 \ \text{V rms}$$

$$\vec{V}_{bn} = 120 \angle -120^0 \ \text{V rms}$$

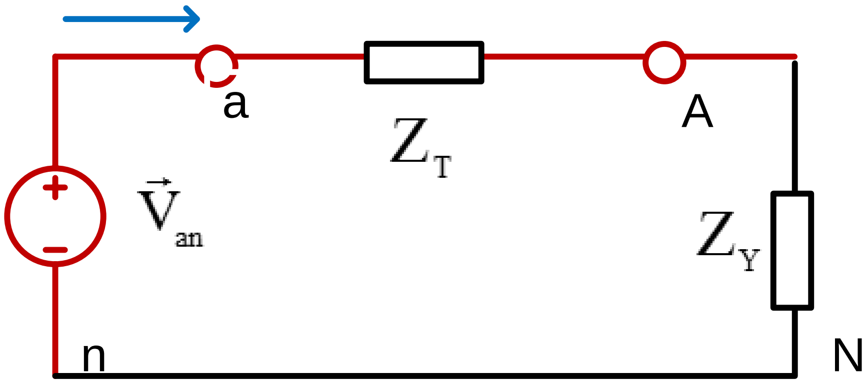
$$\vec{V}_{cn} = 120 \angle +120^0 \ \text{V rms}$$

$$Z_Y = (1 + j1)\Omega$$

$$Z_T = (20 + j10)\Omega$$

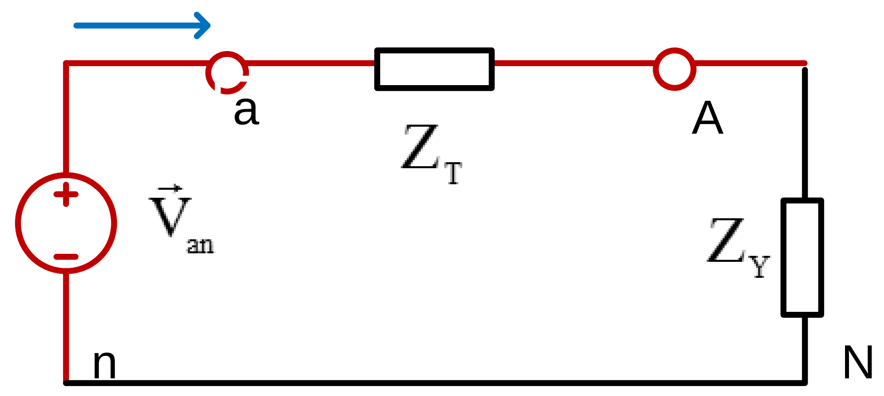
Solution

We can use single phase Representation (since we have balanced three phase system)



Solution

$$\vec{I}_{aA} = \frac{\vec{V}_{an}}{Z_T + Z_Y} = \frac{120\ \angle 0^0}{(1+j1)+(20+j10)}$$



$$\therefore \vec{I}_{aA} = \frac{120\ \angle 0^0}{(21+j11)} = 5.06\angle -27.65^o\ \text{A rms}$$

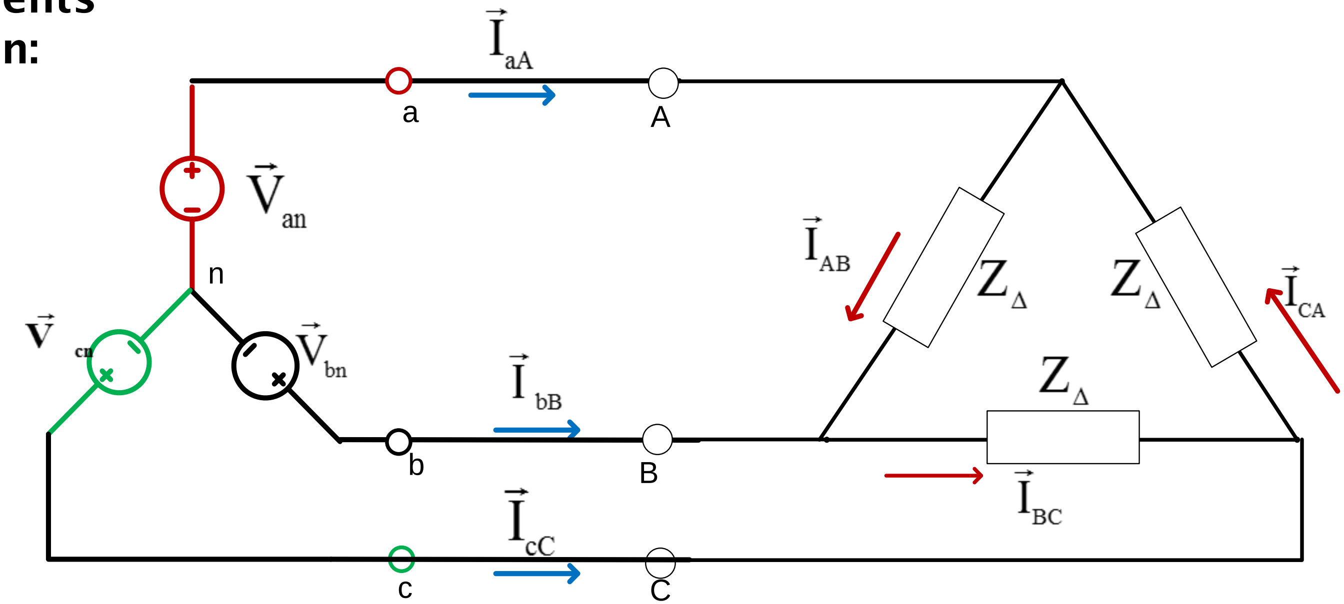
$$\therefore \vec{I}_{bB} = 5.06\angle -27.65^o - 120^o = 5.67\angle -147.65^o\ \text{A rms}$$

$$\therefore \vec{I}_{cC} = 5.06\angle -27.65^o + 120^o = 5.67\angle 92.35^o\ \text{A rms}$$

Note: It is enough to find one of the line currents and the other two are shifted by an angle equal to the phase voltage shift

Balanced Y-Δ system

Example
Calculate The line currents
If the following is given:



$$\vec{V}_{an} = 120 \angle 30^0$$

$$Z_\Delta = (6 + j6)\Omega$$

$$Z_T = 0$$

Positive Sequence

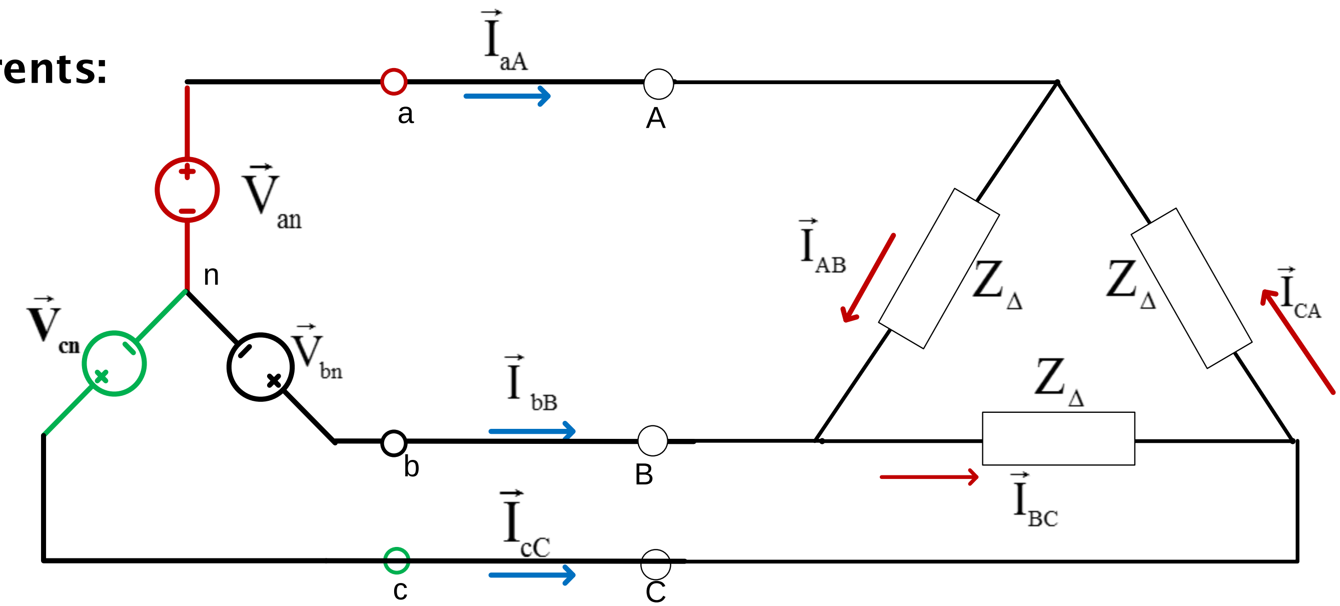
$\therefore$

$$\vec{V}_{bn} = 120 \angle -90^0$$

$$\vec{V}_{cn} = 120 \angle 150$$

Balanced Y-Δ system

Calculate The line currents:



$$\vec{V}_{ab} = \vec{V}_{AB} = 120\sqrt{3}\ \langle 60^0 \ \ \text{V rms}$$

$$\vec{I}_{AB} = \frac{\vec{V}_{AB}}{Z_\Delta} = 24.5\langle 15^0 \quad \text{A rms}$$

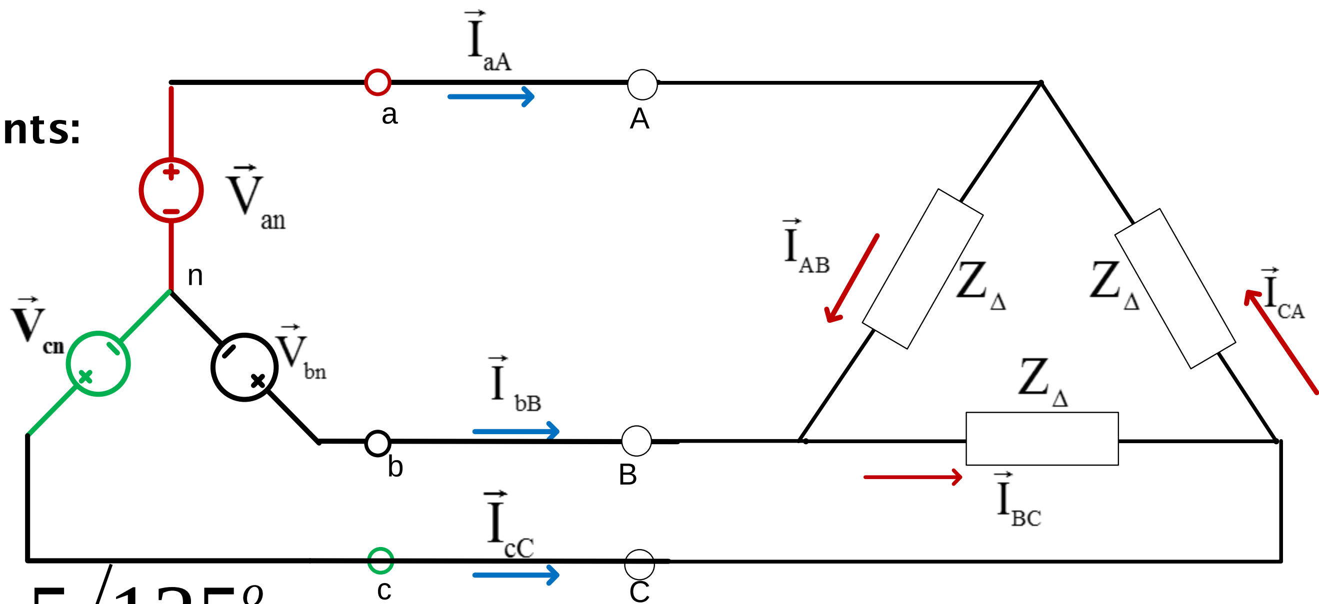
$$\therefore \vec{I}_{BC} = 24.5\langle -105^0 \quad \text{A rms}$$

$$\therefore \vec{I}_{CA} = 24.5\langle 135^0 \quad \text{A rms}$$

$\vec{I}_{AB}, \vec{I}_{BC}, \vec{I}_{CA}$ are the phase currents of the load

Balanced Y-Δ system

Calculate The line currents:



KCL @ A

$$\vec{I}_{aA} = \vec{I}_{AB} - \vec{I}_{CA}$$

$$= 24.5\angle 15^o - 24.5\angle 135^o$$

$$= 42.44\angle -15^o \text{ A rms}$$

$$\vec{I}_{aA} = \sqrt{3}\,\vec{I}_{AB}\angle -30^o$$

Line current lags the phase current by only -30^o for abc sequence

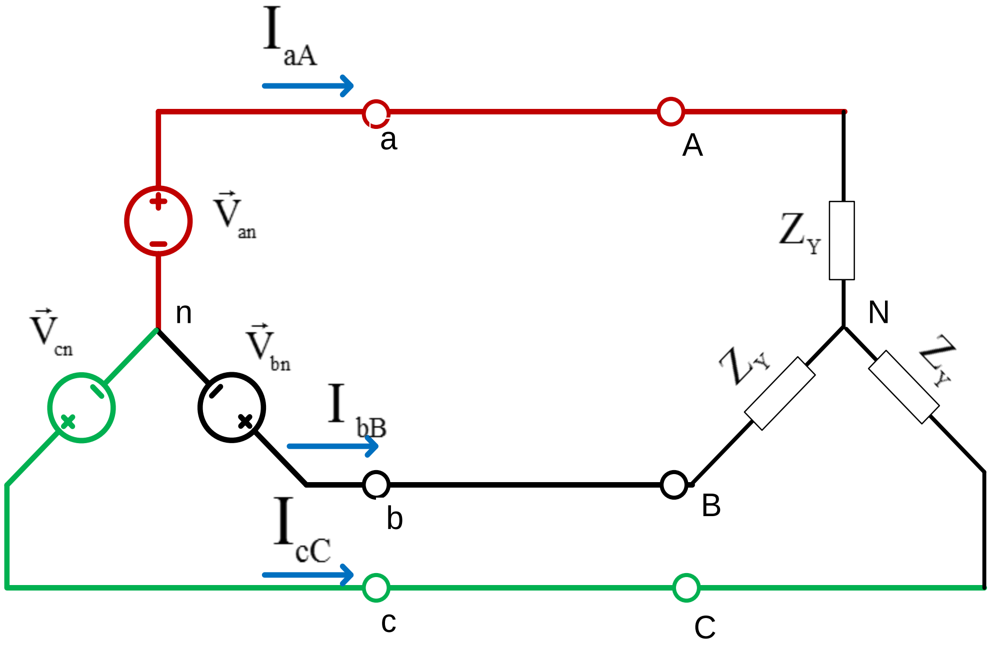
$$\therefore \vec{I}_{bB} = 42.44\angle -135^o \text{ A rms}$$

$$\therefore \vec{I}_{cC} = 42.44\angle 105^o \text{ A rms}$$

Second Method

Using Δ-Y transformation for the load

$$Z_Y = \frac{Z_\Delta}{3}$$

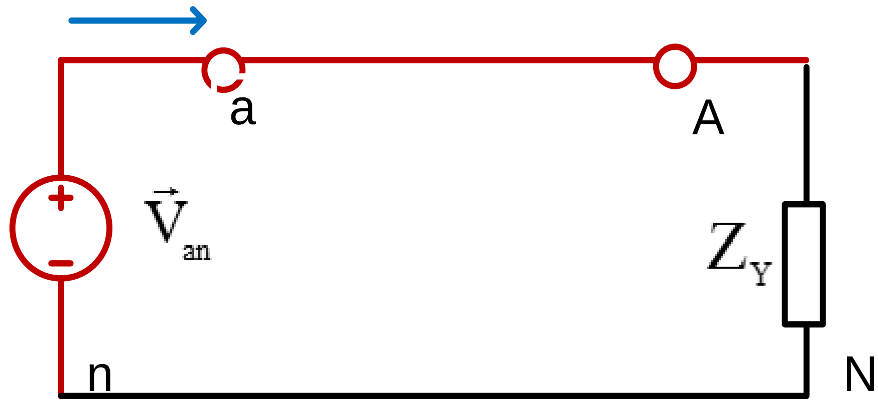


Second Method

$$Z_Y = \frac{6 + j6}{3} = 2 + j2$$

$$\vec{I}_{aA} = \frac{\vec{V}_{an}}{Z_Y} = \frac{120 \angle 30^0}{(2 + j2)}$$

$$\therefore \vec{I}_{aA} = 42.44 \angle -15^o \text{ A rms}$$



$$\therefore \vec{I}_{bB} = 42.44 \angle -15^o - 120^o = 42.44 \angle -135^o \text{ A rms}$$

$$\therefore \vec{I}_{cC} = 42.44 \angle -15^o + 120^o = 42.44 \angle 105^o \text{ A rms}$$

Example 2

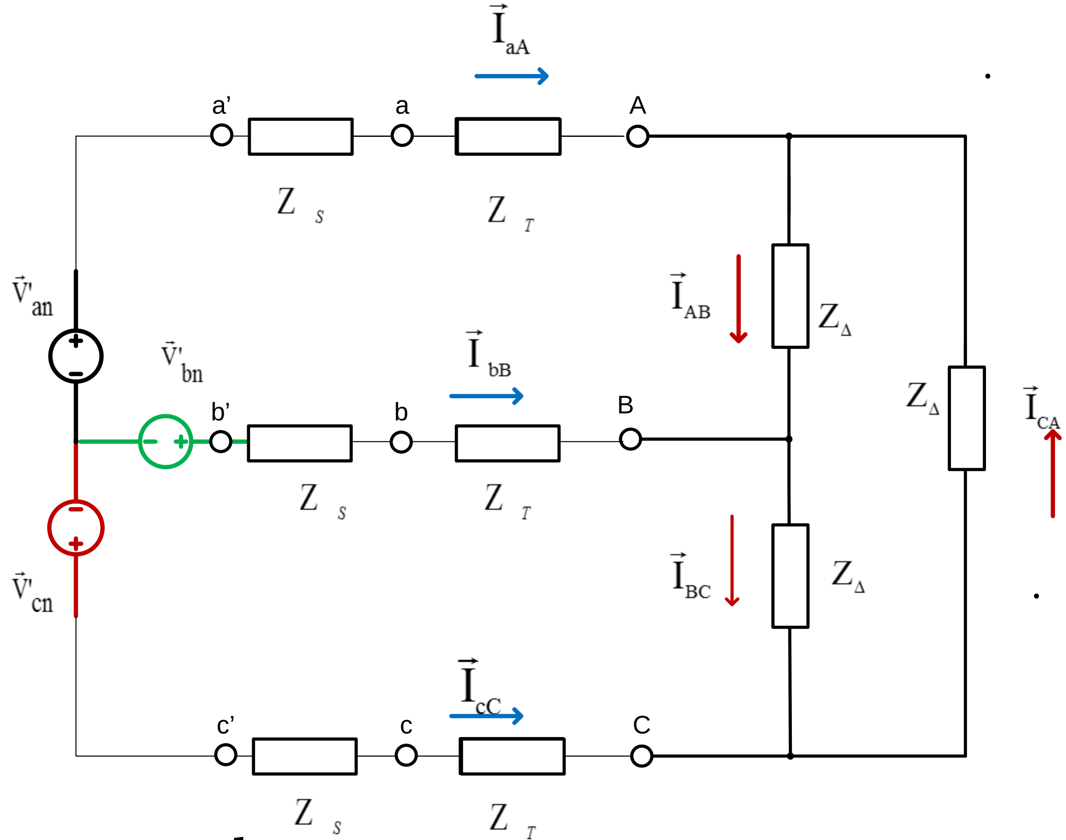
$\vec{V}_{an} = 120 \angle 0^0 \text{ V rms}$

$Z_\Delta = (118.5 + j85.8)\Omega$

$Z_T = (0.3 + j0.9)\Omega$

$Z_S = (0.2 + j0.5)\Omega$

abc sequence



1) Calculate The line currents:

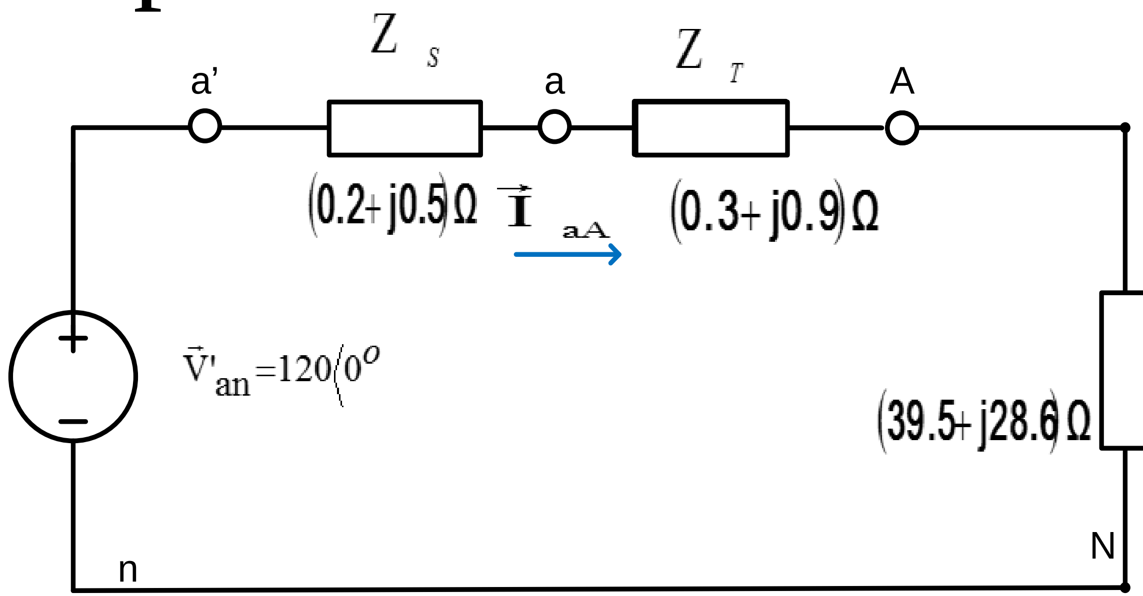
2) Calculate The phase currents of the load

3) Calculate The phase voltage at the load terminal *VAB, VBC, VCA*

Single Phase Representation

$$Z_Y = \frac{Z_\Delta}{3} = \frac{118.5 + j85.8}{3}$$

$$= 39.5 + j28.6$$



$$\vec{I}_{aA} = \frac{120}{Z_S + Z_T + Z_Y} = \frac{120}{0.2 + j0.5 + 0.3 + j0.9 + 39.5 + j28.6 +}$$

$$\vec{I}_{aA} = 2.4 \langle -36.87^o \text{ A rms}$$

$$\vec{I}_{bB} = 2.4 \langle -156.87^o \text{ A rms}$$

$$\vec{I}_{cC} = 2.4 \langle 83.13^o \text{ A rms}$$

2) Calculate The phase currents of the load

$$\vec{I}_{AB} = \frac{1}{\sqrt{3}} \vec{I}_{aA} \langle 30^{o} \text{ A rms}$$

$$\therefore \vec{I}_{AB} = \left(\frac{1}{\sqrt{3}} \langle 30^{o} \right) \left(2.4 \langle -36.87^{o} \right) \text{ A rms}$$

$$\therefore \vec{I}_{AB} = \left(1.39 \langle -6.87^{o} \right) \text{ A rms}$$

$$\therefore \vec{I}_{BC} = \left(1.39 \langle -126.87^{o} \right) \text{ A rms}$$

$$\therefore \vec{I}_{CA} = \left(1.39 \langle 113.13^{o} \right) \text{ A rms}$$

3) Calculate The phase voltage at the load terminal V_{AB}, V_{BC}, V_{CA}

First Method

$$\vec{V}_{AB} = Z_{\Delta} \vec{I}_{AB}$$

$$\therefore \vec{V}_{AB} = (118.5 + j85.8)\left(1.39 \angle -6.87^{o}\right) \text{ A rms}$$

$$\therefore \vec{V}_{AB} = 202.72 \angle 29.04^{o} \text{ V rms}$$

$$\therefore \vec{V}_{BC} = 202.72 \angle -90.96^{o} \text{ V rms}$$

$$\therefore \vec{V}_{CA} = 202.72 \angle 149.04^{o} \text{ V rms}$$

3) Calculate The phase voltage at the load terminal V_{AB}, V_{BC}, V_{CA}

Second Method: From single phase representation

$$\vec{V}_{AN} = Z_Y \vec{I}_{aA}$$

$$\therefore \vec{V}_{AN} = (39.5 + j28.6)\left(2.4\langle -36.87^{o}\right) \text{ V rms}$$

$$\therefore \vec{V}_{AN} = 117.04\langle -0.96^{o} \text{ V rms}$$

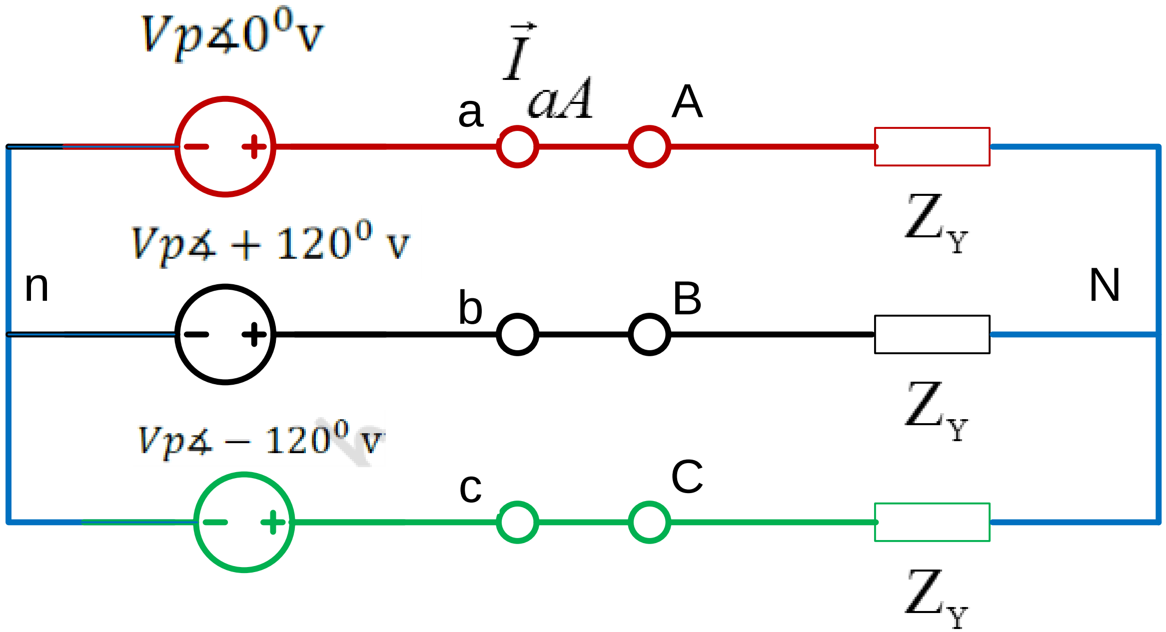
$$\therefore \vec{V}_{AB} = \sqrt{3}\langle +30^{o} \ \vec{V}_{AN}$$

$$\therefore \vec{V}_{AB} = \sqrt{3}\langle +30^{o} \left(117.04\langle -0.96^{o}\right) = 202\langle 29.04^{o} \text{ V rms}$$

$$\therefore \vec{V}_{BC} = 202.72\langle -90.96^{o} \text{ V rms}$$

$$\therefore \vec{V}_{CA} = 202.72\langle 149.04^{o} \text{ V rms}$$

Power in Balanced Three Phase System



The total instantaneous power in a balanced three phase system is constant

$$\vec{V}_{AN}(t) = \sqrt{2}V_P \cos(\omega t)$$

$$\therefore \vec{V}_{BN}(t) = \sqrt{2}V_P \cos\left(\omega t - 120^O\right)$$

$$\therefore \vec{V}_{CN}(t) = \sqrt{2}V_P \cos\left(\omega t + 120^O\right)$$

$$\vec{i}_{aA}(t) = \sqrt{2}\ \mathrm{I_P}\ \cos(\omega \mathrm{t} - \theta)$$

$$\vec{i}_{bB}(t) = \sqrt{2}\ \mathrm{I_P}\ \cos\left(\omega \mathrm{t} - \theta - 120^{o}\right)$$

$$\vec{i}_{cC}(t) = \sqrt{2}\ \mathrm{I_P}\ \cos\left(\omega \mathrm{t} - \theta + 120^{o}\right)$$

$$P(t) = P_{\mathrm{a}}(t) + P_{\mathrm{b}}(t) + P_{\mathrm{c}}(t)$$

$$P_a(t) = 2\,\mathrm{V_P I_P}\ \cos(\omega \mathrm{t})\cos(\omega \mathrm{t} - \theta)$$

$$P_b(t) = 2\,\mathrm{V_P I_P}\ \cos\left(\omega \mathrm{t} - 120^{\mathrm{o}}\right)\cos\left(\omega \mathrm{t} - \theta - 120^{\mathrm{o}}\right)$$

$$P_c(t) = 2\,\mathrm{V_P I_P}\ \cos\left(\omega \mathrm{t} + 120^{\mathrm{o}}\right)\cos\left(\omega \mathrm{t} - \theta + 120^{\mathrm{o}}\right)$$

Using

$$\cos(\alpha)\cos(\beta) = \frac{1}{2}\left[\cos(\alpha+\beta) + \cos(\alpha-\beta)\right]$$

$$P(t) = \mathrm{V_P I_P}\ \left[3\cos(\theta)\right]$$

$$= 3\mathrm{V_P I_P}\ \cos(\theta)$$

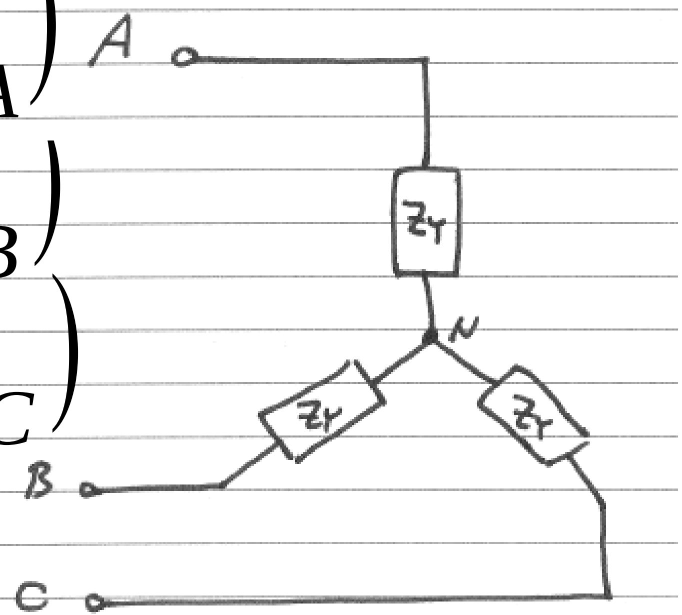
Power Calculations in Balanced 3Φ Systems

1) Average Power in Balanced Y load

$$P_A = V_{AN} I_{AN} \, Cos(\theta_{vA} - \Phi_{iA})$$

$$P_B = V_{BN} I_{BN} \, Cos(\theta_{vB} - \Phi_{iB})$$

$$P_C = V_{CN} I_{CN} \, Cos(\theta_{vC} - \Phi_{iC})$$



$$V_{AN} = V_{BN} = V_{CN} = V_{\Phi}$$

$$I_{AN} = I_{BN} = I_{CN} = I_{\Phi}$$

$$(\theta_{vA} - \Phi_{iA}) = (\theta_{vB} - \Phi_{iB}) = (\theta_{vC} - \Phi_{iC}) = \theta_{\Phi}$$

$$P_A = P_B = P_C = V_\Phi I_\Phi \ \mathrm{Cos}(\theta_\Phi)$$

$$P_T = 3V_\Phi I_\Phi \ \mathrm{Cos}(\theta_\Phi)$$

but

$$V_\Phi = \frac{V_L}{\sqrt{3}} \quad \& \quad I_\Phi = I_L$$

$$\therefore P_T = 3\frac{V_L}{\sqrt{3}} I_L \ \mathrm{Cos}(\theta_\Phi)$$

$$= \sqrt{3} V_L I_L \ \mathrm{Cos}(\theta_\Phi)$$

2) Reactive Power in balanced Y load

$$Q_A = Q_B = Q_C = \mathrm{V}_\Phi \mathrm{I}_\Phi \ \mathrm{Sin}\left(\theta_\Phi\right)$$

$$Q_T = 3\mathrm{V}_\Phi \mathrm{I}_\Phi \ \mathrm{Sin}\left(\theta_\Phi\right)$$

$$\therefore Q_T = \sqrt{3}\mathrm{V}_L \mathrm{I}_L \ \mathrm{Sin}\left(\theta_\Phi\right)$$

2) Complex Power in balanced Y load

$$\vec{S}_A = \vec{S}_B = \vec{S}_C = \mathrm{V}_\Phi \mathrm{I}_\Phi{}^* = \mathrm{P}_\Phi + \mathrm{jQ}_\Phi$$

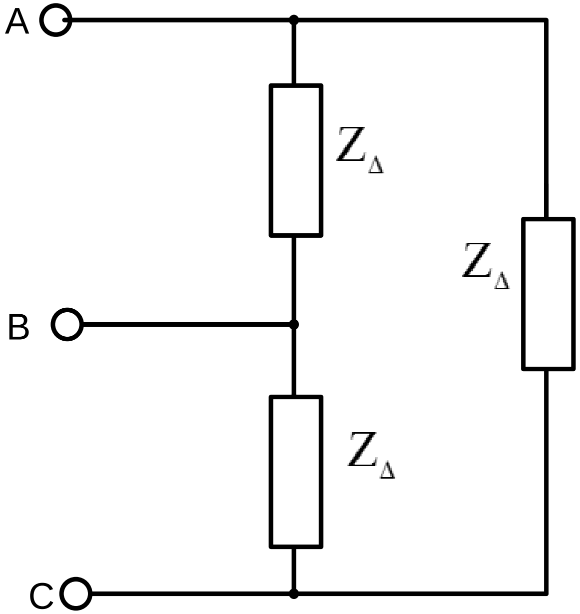
$$\therefore \vec{S}_T = 3\vec{S}_\Phi = 3\vec{\mathrm{V}}_\Phi \vec{\mathrm{I}}_\Phi{}^* = \sqrt{3}\mathrm{V}_L \mathrm{I}_L \langle \theta_\Phi$$

1) Average Power in Balanced Δ load

$$P_A = V_{AB} I_{AB} \, Cos\left(\theta_{v_{AB}} - \Phi_{i_{AB}}\right)$$

$$P_B = V_{BC} I_{BC} \, Cos\left(\theta_{v_{BC}} - \Phi_{i_{BC}}\right)$$

$$P_C = V_{CA} I_{CA} \, Cos\left(\theta_{v_{CA}} - \Phi_{i_{CA}}\right)$$



$$V_{AB} = V_{BC} = V_{CA} = V_{\Phi}$$

$$I_{AB} = I_{BC} = I_{CA} = I_{\Phi}$$

$$\left(\theta_{v_{AB}} - \Phi_{i_{AB}}\right) = \left(\theta_{v_{BC}} - \Phi_{i_{BC}}\right) = \left(\theta_{v_{CA}} - \Phi_{i_{CA}}\right) = \theta_{\phi}$$

$$P_A = P_B = P_C = V_\Phi I_\Phi \, \mathrm{Cos}(\Phi)$$

$$P_T = 3V_\Phi I_\Phi \, \mathrm{Cos}(\Phi)$$

but

$$V_\Phi = V_L \qquad \& \qquad I_\Phi = \frac{I_L}{\sqrt{3}}$$

$$\therefore P_T = 3V_L \frac{I_L}{\sqrt{3}} \mathrm{Cos}(\Phi)$$

$$= \sqrt{3} V_L I_L \, \mathrm{Cos}(\Phi)$$

This is the same formula derived earlier for the Y connected circuit

Reactive Power in Balanced Δ load

$$Q_A = Q_B = Q_C = Q_\Phi = \mathrm{V}_\Phi \mathrm{I}_\Phi \,\mathrm{Sin}\left(\theta_\Phi\right)$$

$$Q_T = 3\mathrm{V}_\Phi \mathrm{I}_\Phi \,\mathrm{Sin}\left(\theta_\Phi\right)$$

$$= \sqrt{3}\mathrm{V}_L \mathrm{I}_L \,\mathrm{Sin}\left(\theta_\Phi\right)$$

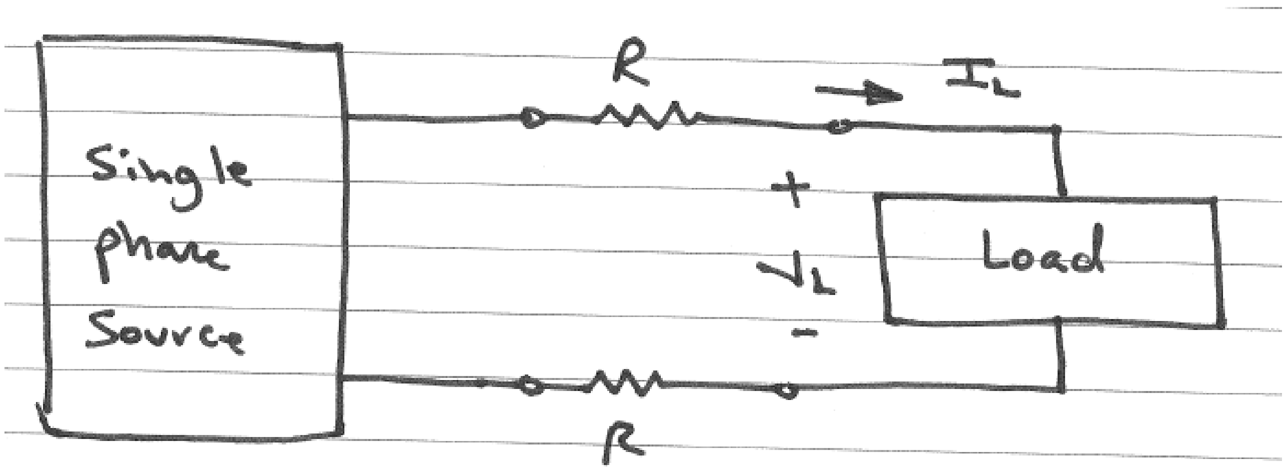
Complex Power in balanced Δ load

$$\vec{S}_A = \vec{S}_B = \vec{S}_C = \mathrm{V}_\Phi \mathrm{I}_\Phi{}^* = \vec{S}_\Phi = P_\Phi + jQ_\Phi$$

$$\therefore \vec{S}_T = 3\vec{S}_\Phi = 3\vec{\mathrm{V}}_\Phi \vec{\mathrm{I}}_\Phi{}^* = \sqrt{3}\mathrm{V}_L \mathrm{I}_L \langle \theta_\Phi$$

Comparing the Power Loss

a) A Single Phase System



$$P_{Loss} = 2\, I_L^2 . R$$

$$I_L = \frac{P_L}{VL\, .\, Pf}$$

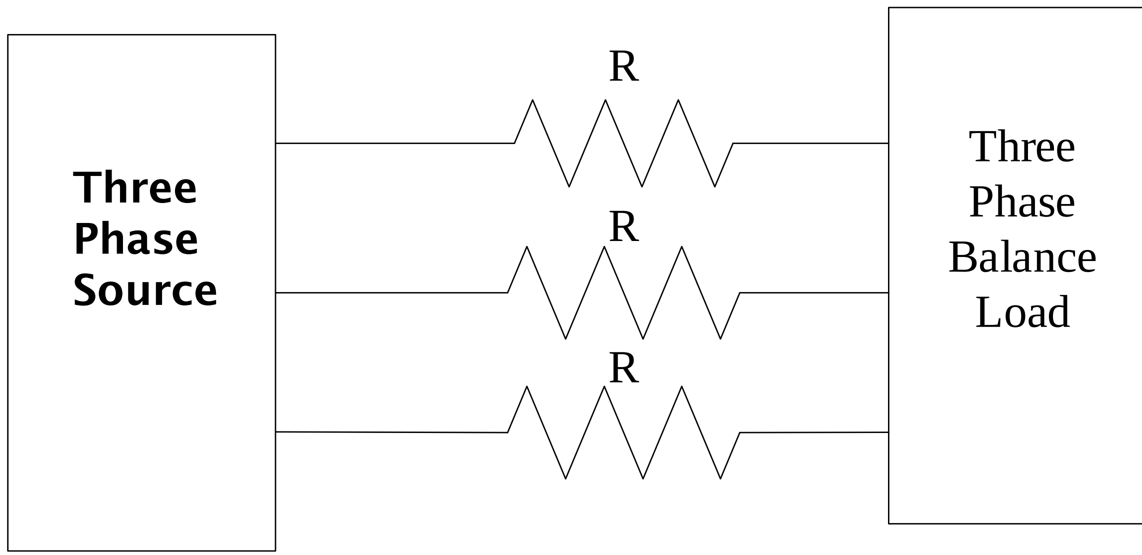
$$P_{Loss} = 2\frac{P_L^2}{V_L^2 . Pf^2}\, .\, R$$

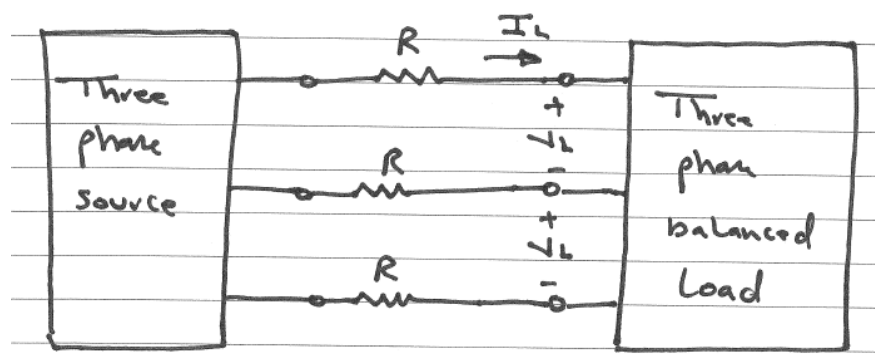
b) a three phase system

$$P_{Loss} = 3\, I_L^{\,2}.R$$

$$I_L = \frac{P_L}{\sqrt{3}\, VL\, .\, Pf}$$

$$P_{Loss} = \frac{P_L^2}{V_L^2 . Pf^2} \cdot R$$





This proves that power loss is half power loss in single phase circuit

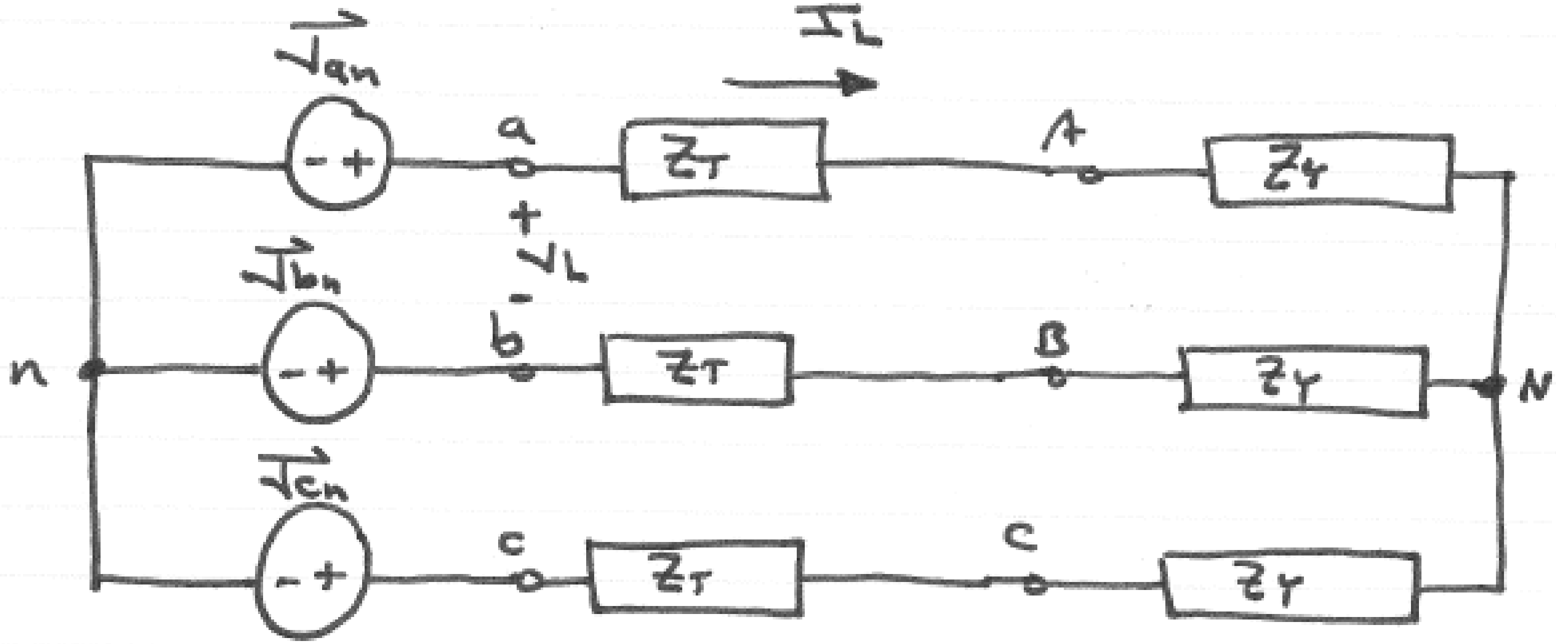
Example

- A balanced 3 Φ load requires 480 kW at a lagging power factor of 0.8. (Y-Y system)
- The load is fed from a line having an impedance of (0.005 + j0.025) Ω / Φ
- The line voltage at the terminal of the load is 600 V_{rms}

1) Calculate The Magnitude of the line current

2) Calculate the magnitude of the line voltage at the sending end of the line

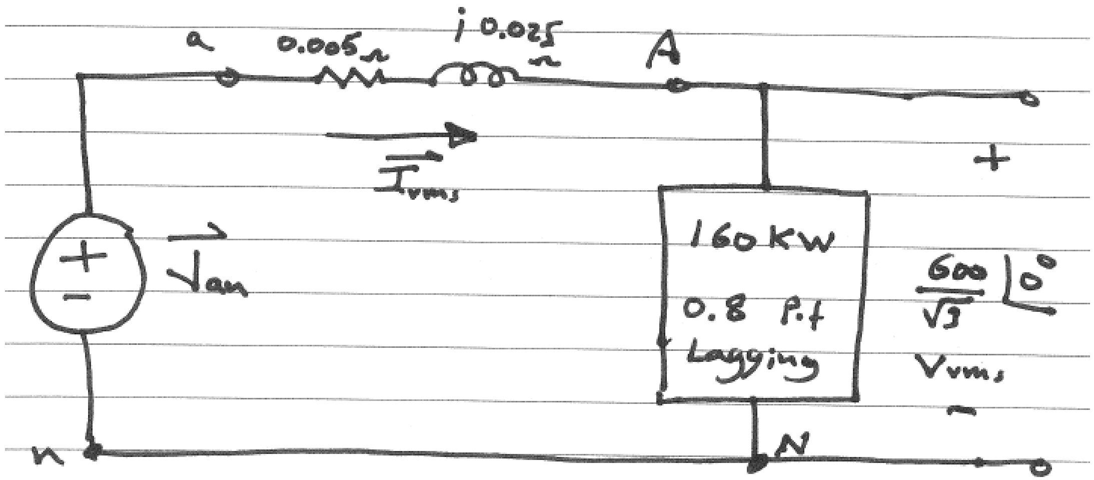
3) Calculate the power factor at the sending end of the line



$\vec{V}_{an}$
$\vec{V}_{bn}$
$\vec{V}_{cn}$
I_L
V_L
a
b
c
A
B
C
n
N
Z_T
Z_T
Z_T
Z_Y
Z_Y
Z_Y

Solution

- Single Phase Representation



P_{av} **=160 kW**

$\boldsymbol{Q} = P_{av}\ \tan[\cos^{-1}(\boldsymbol{PF})]$

$= \mathbf{120}\ \boldsymbol{k}VAR$

$\vec{S} = P_{av} + jQ$

$\vec{S} = 160 + j120\ \ kVA$

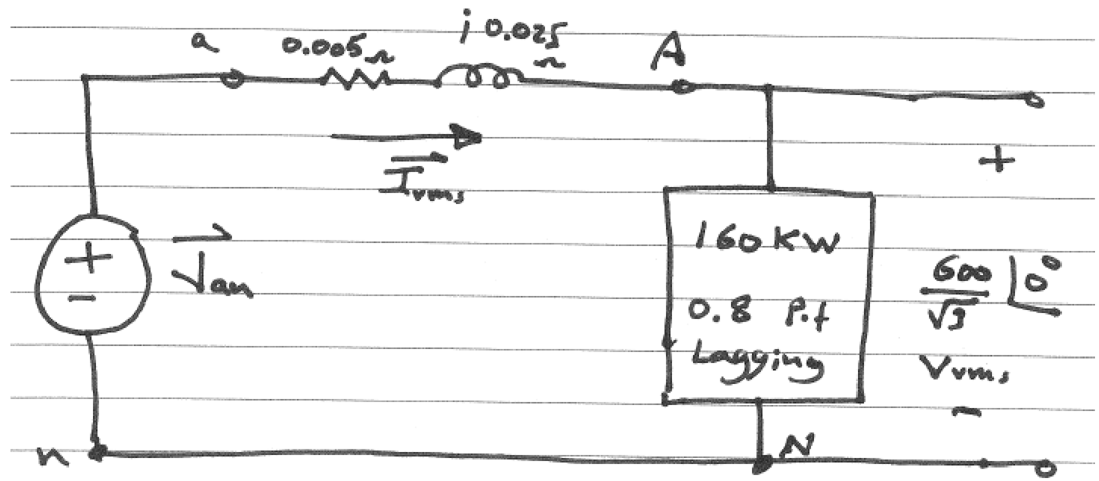
$$\vec{S} = 160 + j120 \text{ kVA}$$

$$\vec{S} = \vec{V}_{rms} . \vec{I}^{*}_{rms}$$

$$\therefore \vec{\mathrm{I}}_{\mathrm{rms}} = 577.35\angle -36.87^{o} \ \mathrm{A}_{\mathrm{rms}}$$

$$\therefore \mathrm{I}_{\mathrm{L}} = 577.35 \ \mathrm{Arms}$$

$$\vec{V}_{an} = (0.005 + j0.025)\vec{I}_{rms} + \frac{600}{\sqrt{3}}\langle 0^{o}$$



$$\vec{V}_{an} = 357.51\langle 1.57^{o}\ V_{rms}$$

$$\therefore V_{an} = 357.51\, V_{rms}$$

$$\therefore V_{L} = \sqrt{3}\, V_{an}$$

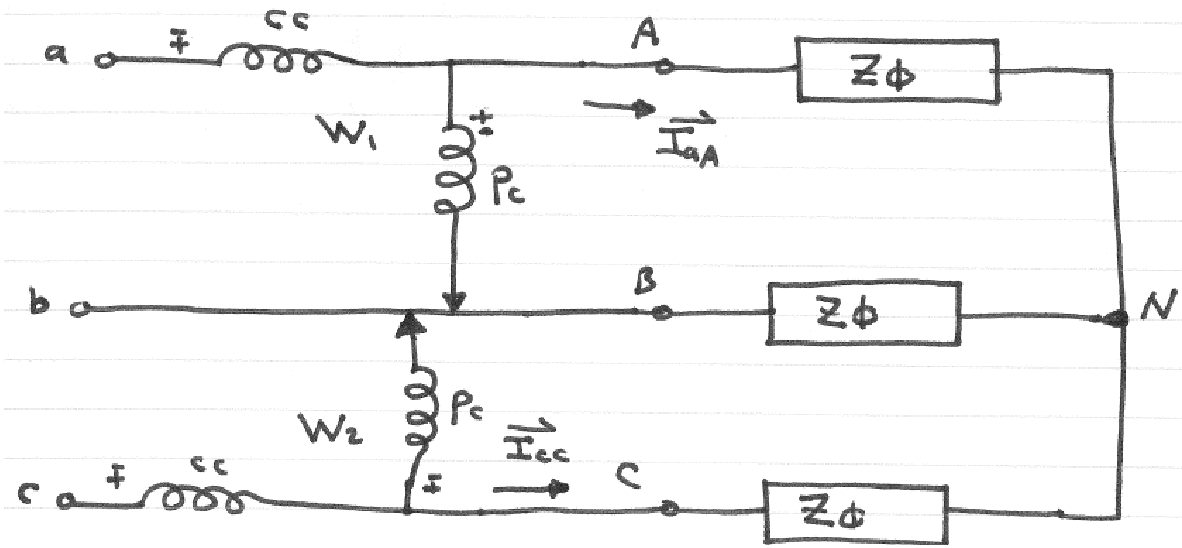
$$\therefore V_{L} = 619.23\, V_{rms}$$

$$Pf = \text{Cos}\left(\theta_v - \Phi_i\right)$$

$$Pf = \text{Cos}\left(1.57^{o} + 36.87^{o}\right)$$

$$\text{Pf} = 0.783 \text{ lagging}$$

Measuring Avg Power in 3-Φ System
The Two- Wattmeter Method



$$Z_\phi = |Z|\langle\theta_Z;\ \theta_Z \equiv \text{impedance angle}$$

$$W_1 = V_{AB} \cdot I_{aA} \operatorname{Cos}\theta_1$$

$$\theta_1 \equiv \text{The angle between } \overline{V}_{AB} \text{ and } \overline{I}_{aA}$$

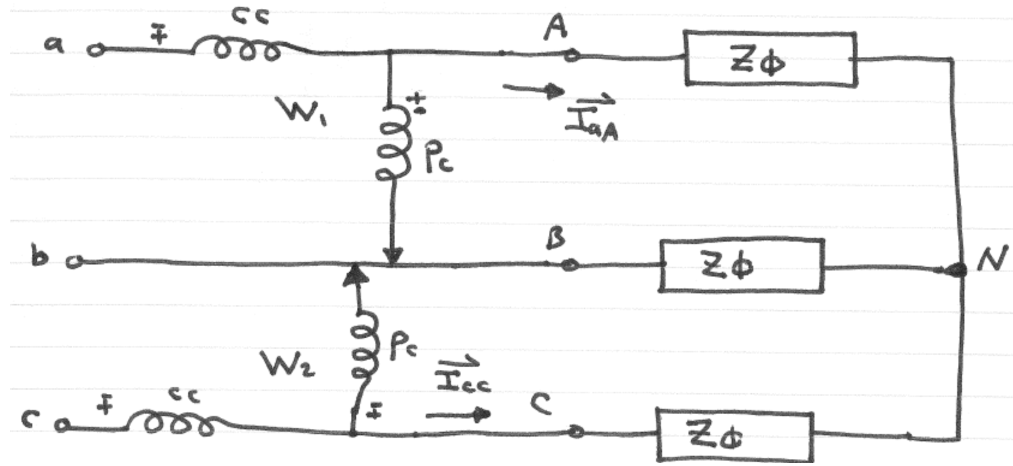
For a positive phase sequence:

$$\vec{V}_{AB} = \vec{V}_{AN}\sqrt{3}\langle 30^{o}$$

$$\bar{I}_{aA} = \bar{I}_{AN}$$

$$\therefore \theta_1 = \theta_z + 30^{o}$$

$$\therefore W_1 = V_L \cdot I_L \operatorname{Cos}\left(\theta_z + 30^{o}\right)$$



$$W_2 = V_{CB} \cdot I_{cC} \cos\theta_2$$

$\theta_2 \equiv$ The angle between $\overline{V}_{CB}$ and $\overline{I}_{cC}$

$$\therefore \theta_2 = \theta_Z - 30^o$$

Since we have V_{CB} not V_{BC}

$$\overline{V}_{CB} = -\overline{V}_{BC}$$

$$\overline{V}_{CB} = \overline{V}_{BC} \langle 180^o$$

$$\overline{V}_{CB} = \overline{V}_{CA} \langle -240^o \langle 180^o$$

$$\overline{V}_{CB} = \overline{V}_{CA} \langle -60^o$$

$$\overline{V}_{CB} = \overline{V}_{CA} \angle -60^o$$

$$\overline{V}_{CB} = \sqrt{3}\,\overline{V}_{CN} \angle +30^o \angle -60^o$$

$$\overline{V}_{CB} = \sqrt{3}\,\overline{V}_{CN} \angle -30^o$$

$$\overline{I}_{cC} = \overline{I}_{CN}$$

$$\therefore \theta_2 = \theta_z - 30^o$$

$$\therefore W_2 = V_L \cdot I_L \operatorname{Cos}\left(\theta_z - 30^o\right)$$

$$W_1 = V_L \cdot I_L \cos(\theta_Z + 30^{\circ})$$

$$W_2 = V_L \cdot I_L \cos(\theta_Z - 30^{\circ})$$

$$P_T = W_1 + W_2$$

$$\cos(\theta_Z + 30^{\circ}) = \cos\theta_Z . \cos 30^{\circ} - \sin\theta_Z . \sin 30^{\circ}$$

$$\cos(\theta_Z - 30^{\circ}) = \cos\theta_Z . \cos 30^{\circ} + \sin\theta_Z . \sin 30^{\circ}$$

$$\therefore W_1 + W_2 = V_L \cdot I_L (2 \cos\theta_Z \cos 30^{\circ})$$

$$W_1 + W_2 = \sqrt{3}\, V_L \cdot I_L (\cos\theta_Z)$$

→ **This is the same expression as that for total power derived earlier**

Example

Calculate the reading of each Wattmeter

if $\vec{V}_{AN} = 120\langle 0^{o}$ $\quad Z_{\Phi} = (8 + j6)\Omega$

- Solution

$$Z_{\Phi} = 10\langle 36.87^{o}\ \Omega$$

$$\bar{I}_{aA} = \frac{120\langle 0^{o}}{Z_{\Phi}} = 12\langle -36.87^{o}\ A_{RMS}$$

a) $Z_\Phi = (8 + j6)\Omega$ $\theta_Z = 36.87^o$

$I_L = 12\ A_{rms}$

$V_L = \sqrt{3}(120)\ V_{rms}$

$$W_1 = V_L \cdot I_L \cos(\theta_Z + 30^o)$$

$$W_1 = \sqrt{3}(120).12.\cos(36.87^o + 30^o)$$

$$= 979.7\ \text{Watt}$$

$$W_2 = V_L \cdot I_L \cos(\theta_Z - 30^o)$$

$$W_2 = \sqrt{3}(120).12.\cos(36.87^o - 30^o)$$

$$= 2476.25\ \text{Watt}$$

b) $Z_{\Phi} = \left(8 - j6\right)\Omega \qquad \theta_Z = -36.87^{o}$

$I_L = 12\ A_{rms}$

$V_L = \sqrt{3}(120)\ V_{rms}$

$$W_1 = \sqrt{3}(120).12.\ Cos(-36.87^{o} + 30^{o})$$

$$= 2476.25\ \text{Watt}$$

$$W_2 = \sqrt{3}(120).12.\ Cos(-36.87^{o} - 30^{o})$$

$$= 979.75\ \text{Watt}$$

c) $Z_\Phi = \left(5 + j5\sqrt{3}\right)\Omega = 10\ \angle 60^o$ $\theta_Z = 60^o$

$I_L = 12\ A_{rms}$

$V_L = \sqrt{3}(120)\ V_{rms}$

$$W_1 = \sqrt{3}(120).12.\ Cos(60^o + 30^o)$$
$$= 0\ \ Watt$$

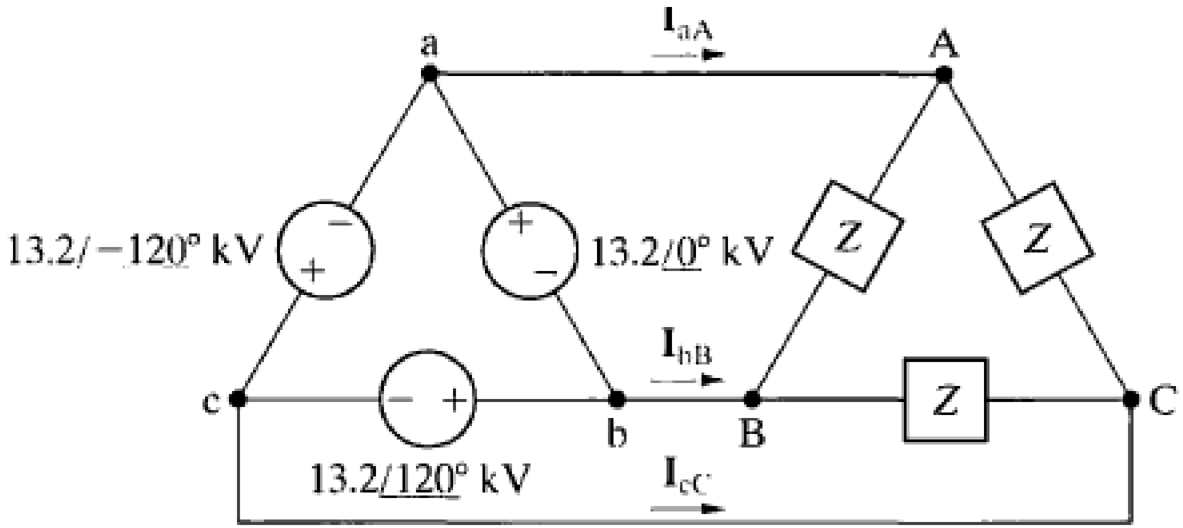
$$W_2 = \sqrt{3}(120).12.\ Cos(60^o - 30^o)$$
$$= 2160\ \ Watt$$

11.17 The impedance Z in the balanced three-phase circuit in Fig. P11.17 is $100 - j75\ \Omega$. Find

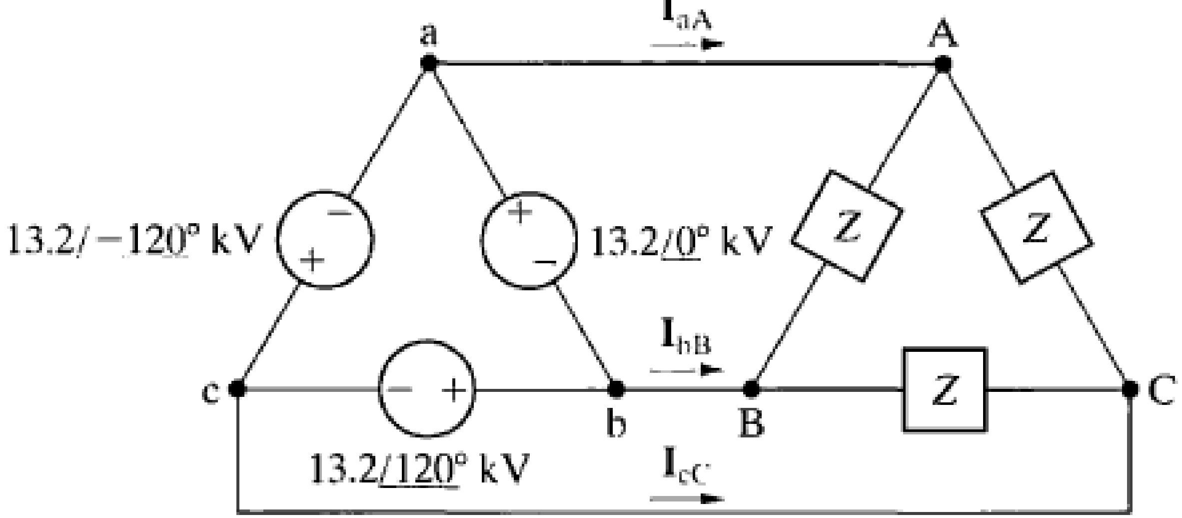
a) $\mathbf{I}_{AB}$, $\mathbf{I}_{BC}$, and $\mathbf{I}_{CA}$,

b) $\mathbf{I}_{aA}$, $\mathbf{I}_{bB}$, and $\mathbf{I}_{cC}$,

c) $\mathbf{I}_{ba}$, $\mathbf{I}_{cb}$, and $\mathbf{I}_{ac}$.



IAB??



$$\text{[a]}\quad \mathbf{I}_{AB} = \frac{13{,}200\angle 0^\circ}{100 - j75} = 105.6\angle 36.87^\circ \text{ A (rms)}$$

$$\mathbf{I}_{BC} = 105.6\angle 156.87^\circ \text{ A (rms)}$$

$$\mathbf{I}_{CA} = 105.6\angle -83.13^\circ \text{ A (rms)}$$

[b] $\mathbf{I}_{aA} = \sqrt{3}\angle -30^\circ \mathbf{I}_{AB} = 182.9\angle 66.87^\circ$ A (rms)

$\mathbf{I}_{bB} = 182.9\angle -173.13^\circ$ A (rms)

$\mathbf{I}_{cC} = 182.9\angle -53.13^\circ$ A (rms)

[c] $\mathbf{I}_{ba} = \mathbf{I}_{AB} = 105.6\angle 36.87^\circ$ A (rms)

$\mathbf{I}_{cb} = \mathbf{I}_{BC} = 105.6\angle 156.87^\circ$ A (rms)

$\mathbf{I}_{ac} = \mathbf{I}_{CA} = 105.6\angle -83.13^\circ$ A (rms)