ENEE2301
Nodal Analysis
Ch4-L1

Instructor

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Reading Assignment: Sections 4.1-4.8 in *Electric Circuits, 9th Edition* by Nilsson

Chapter 4 – Methods of Analysis for Resistive Circuits

- In Chapters 2 and 3 KVL and KCL were applied in a somewhat arbitrary manner. No systematic procedure was introduced as to where to write the equations and how many equations to write. This arbitrary approach would be difficult to use for larger, more complex circuits.
- In this chapter we will develop methods for writing and solving simultaneous independent circuit equations.

Two methods are introduced in this chapter and are commonly used throughout electrical engineering:

1. **Node Equations** (or Nodal Analysis) – result in a set of simultaneous, independent KCL equations
2. **Mesh Equations** (or Mesh Analysis) – result in a set of simultaneous, independent KVL equations.

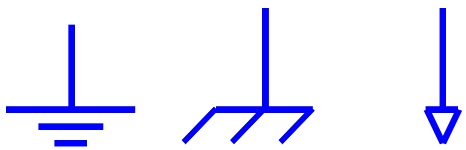
Systematic procedures will be introduced for writing these equations that will give a clear approaches that can be used even for very large circuits.

Node Equations

First, a couple of definitions:

Ground (or reference) – a node in the circuit used as a reference point for measuring node voltages. As far as node voltages are concerned, the ground node is the 0V point in the circuit.

Ground symbols:

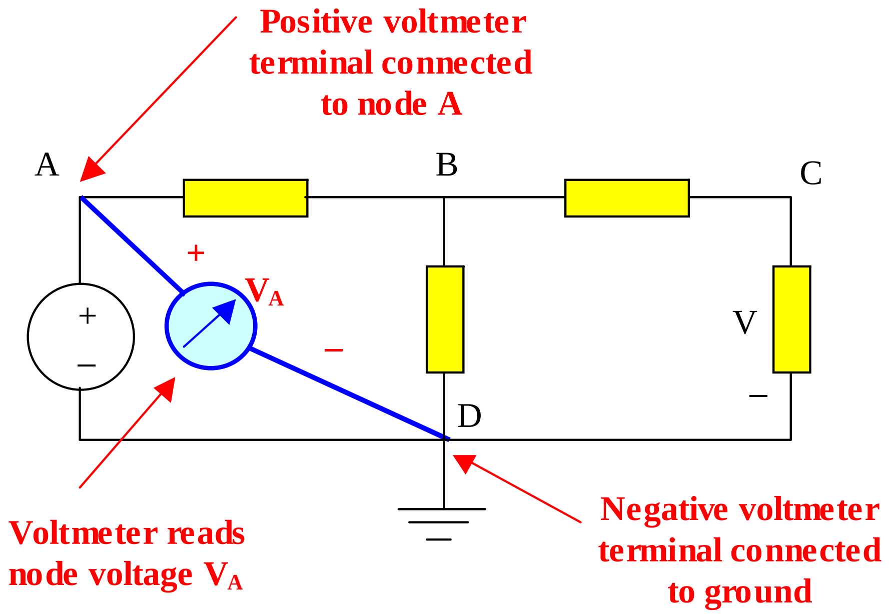


Node voltage – the voltage at a node with respect to ground. For example, V_A is the voltage at node A, meaning that the positive terminal is at A and the negative terminal is at the ground node.

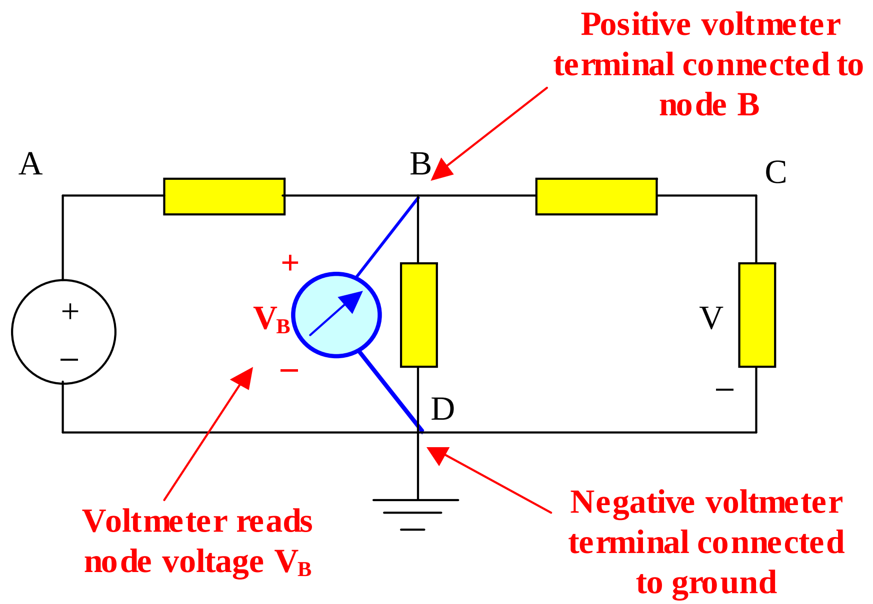
Note: Node voltages are ***relative measurements***. The value of a node voltage changes if a different ground node is used.

Illustration – The use of a ground essentially means that a common node will be used as the negative terminal for all voltages (node voltages) in the circuit. This would be like using a voltmeter to measure various voltages where the negative lead of the meter always stayed on the ground node and the positive lead then moved to various other nodes to measure "node voltages."

Case 1 – Using a voltmeter to measure node voltage V_A



Case 2 – Using a voltmeter to measure node voltage V_B

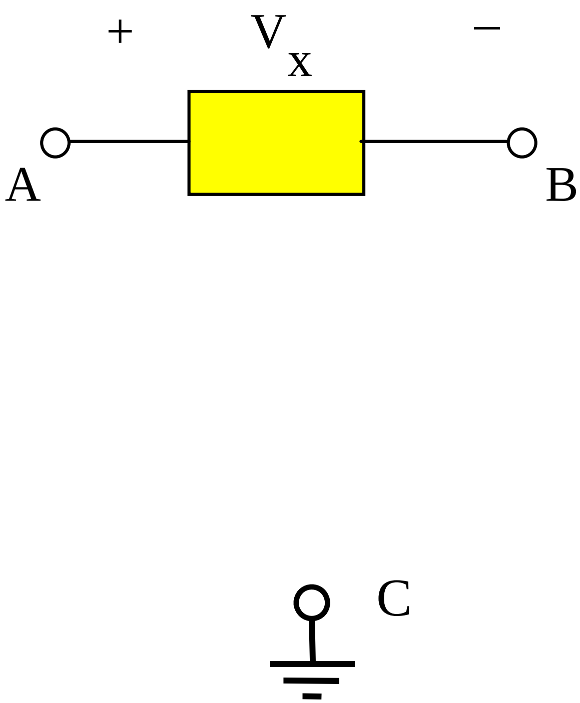


Note – In all cases the negative side of the meter is connected to the ground node in order to measure node voltages.

Component Voltages

Recall that node voltages are relative quantities. If a different reference is used, the node voltages change.

Component voltages are not relative quantities. Component voltages can be determine from node voltages as illustrated below.

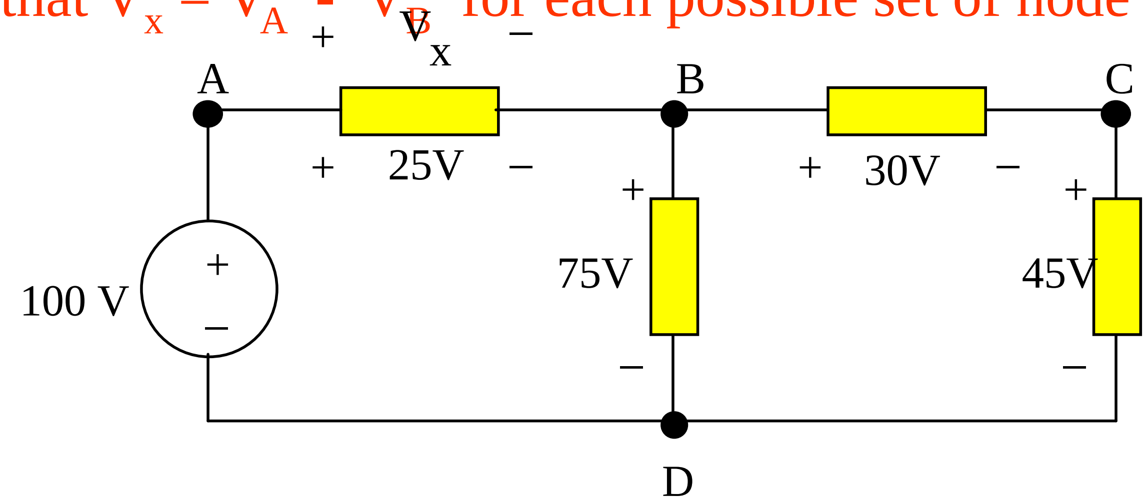


$$V_x = V_A - V_B$$

(prove that this is true for any ground C)

Example (HW): The circuit below includes the value of the component voltages.

- Determine the corresponding node voltages (fill out the table).
- Show that $V_x = V_A - V_B$ for each possible set of node voltages.



Voltage	Node A used as ground	Node B used as ground	Node C used as ground	Node D used as ground
V_A	**0**	**25**	25+30	100
V_B	-25	0	30	75
V_C				
V_D				
$V_x = V_A - V_B$	**25**	**25**	**25**	**25**

Node equations – Procedure

1) Label each node.
2) Select a node as the reference (or ground) node.

   Note: It is generally easier to pick the ground adjacent to a voltage source.
3) If the circuit has no voltage sources, skip to step 4. Otherwise:
   A) For any voltage source or group of voltage sources adjacent to the ground, all node voltages adjacent to the sources can be determined so no KCL equation will be required.
   B) For any voltage source or group of voltage sources not adjacent to the ground, a ***supernode*** is required (to be discussed later).
4) Write a KCL equation at each node not adjacent to a voltage source and not at the ground node. (Also write a KCL equation for each supernode.) Express resistor currents in terms of node voltages.
5) Solve the simultaneous KCL equations. In general, the number of equations required is:

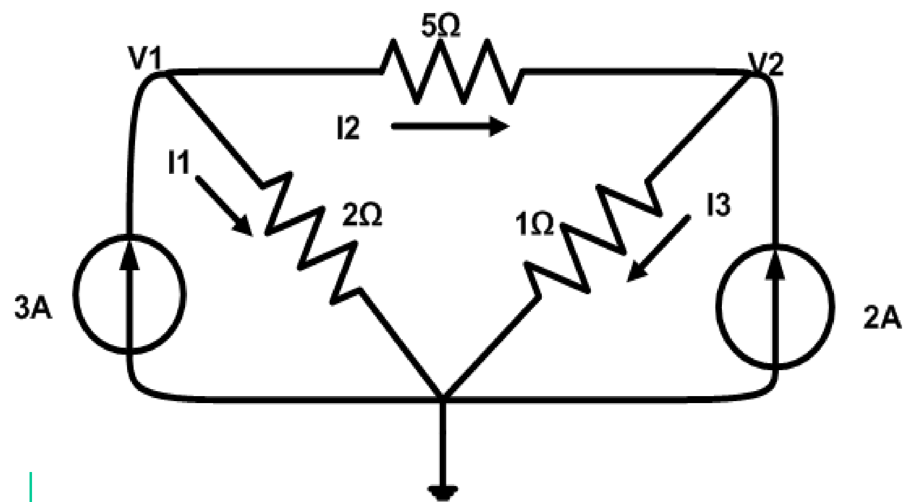
 # Node Equations = #nodes - #voltage sources - 1

Nodal and mesh analysis

- As circuit get more complicated , we need an organized method of applying KVL,KCL and Ohm’s
- Nodal analysis assigns voltages to each node then we apply KCL
- Mesh analysis assigns current to each mesh ,and then we apply KVL

The nodal analysis method

- Assign voltages to every node relative to a reference node.
- Apply KCL to node 1



$$3 = I1 + I2$$

$$3 = \frac{V1}{2} + \frac{V1 - V2}{5}$$

$$3 = 0.7\,V1 - 0.2\,V2 \qquad (1)$$

Apply KCL to node 2

$$2 + I2 = I3$$

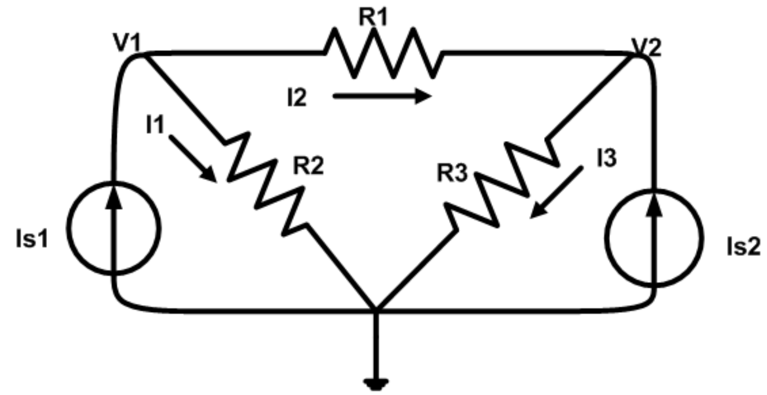
$$2 = I3 - I2$$

$$2 = \frac{V2}{1} - \frac{V1 - V2}{5}$$

$$2 = -0.2\,V1 + 1.2\,V2 \qquad (2)$$

SOLVING (1) & (2) YIELDS

- $V1 = 5\ V$
- $V2 = 2.5\ V$



- Applying KCL to node 1

$$Is1 = I1 + I2$$

$$Is1 = \frac{V1}{R2} + \frac{V1 - V2}{R1}$$

$$Is1 = \left(\frac{1}{R2} + \frac{1}{R1}\right)V1 - \frac{1}{R1}\,V2$$

$$Is1 = (G2 + G1)V1 - G1\,V2$$

Self conductance of node 1 = G2 +G1

Mutual conductance = - G1

- Applying KCL to node 2

$$Is2 = I3 - I2$$

$$Is2 = \frac{V2}{R3} - \frac{V1 - V2}{R1}$$

$$Is2 = -\frac{1}{R1}\,V1 + \left(\frac{1}{R3} + \frac{1}{R1}\right)V2$$

$$Is2 = -GV1 + (G3 + G1)V2$$

Self conductance of node 2 = (G3+G1)

Mutual conductance between nodes 1 and 2 = - G_1

Writing Nodal Equations by inspection

- KCL at node 1:

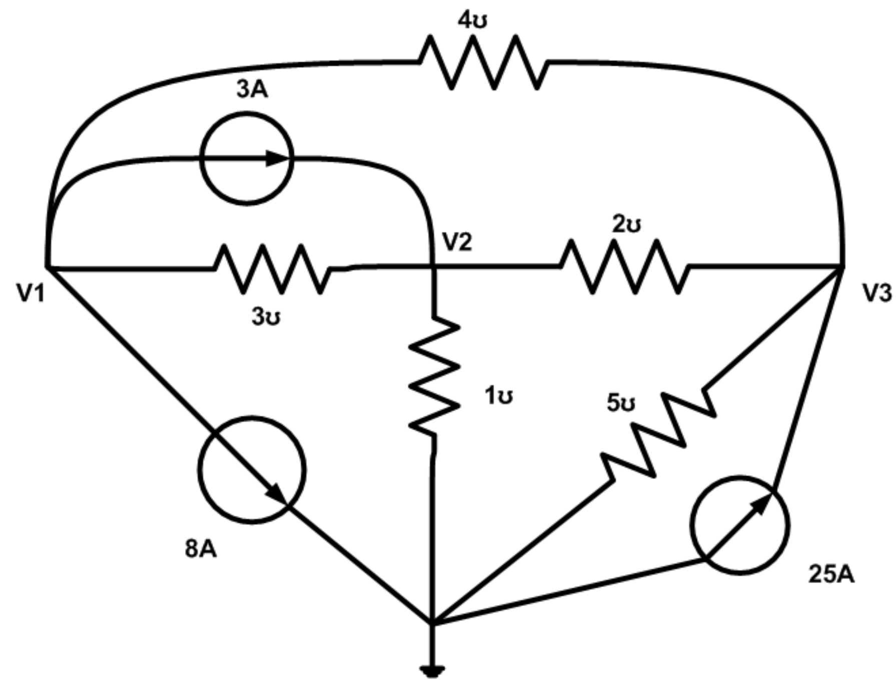
  7V1-3V2-4V3=-11

- KCL at node 2

  -3V1+6V2-2V3=3

- KCL at node 3:

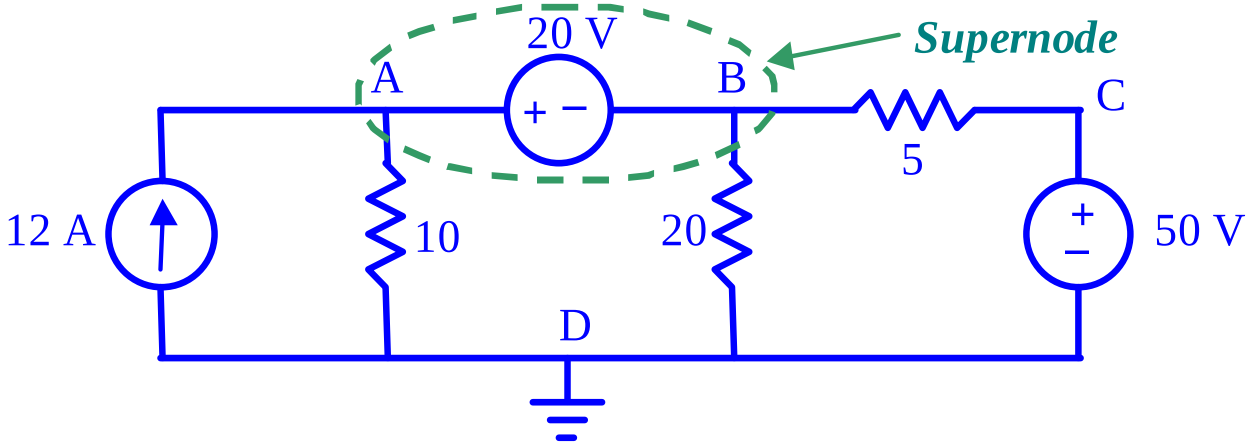
  -4V1-2V2+11V3=25



- Solving:

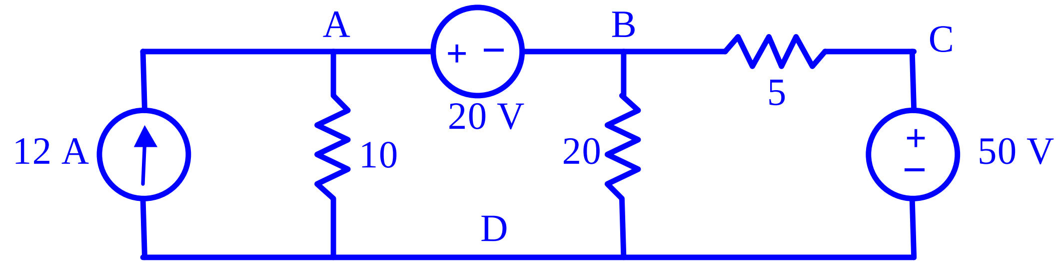
V1=1V; V2=2 V; V3=3V

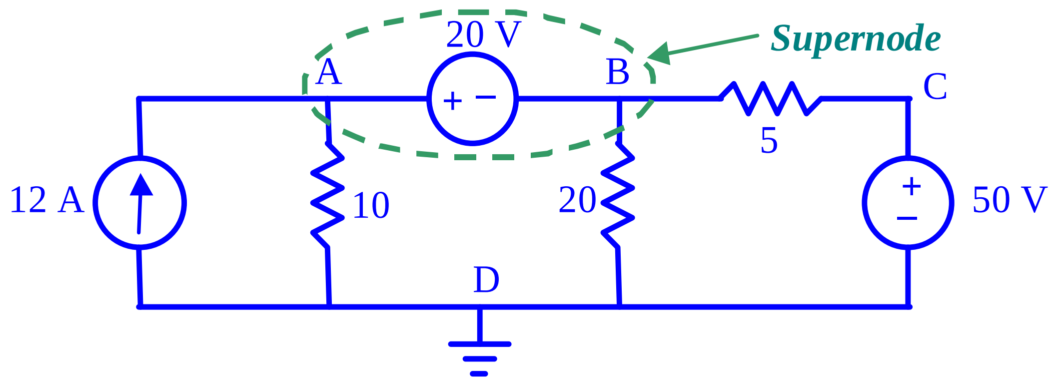
Nodal Analysis with voltage sources



- Note that the voltage difference between nodes A and B is 20V, so:

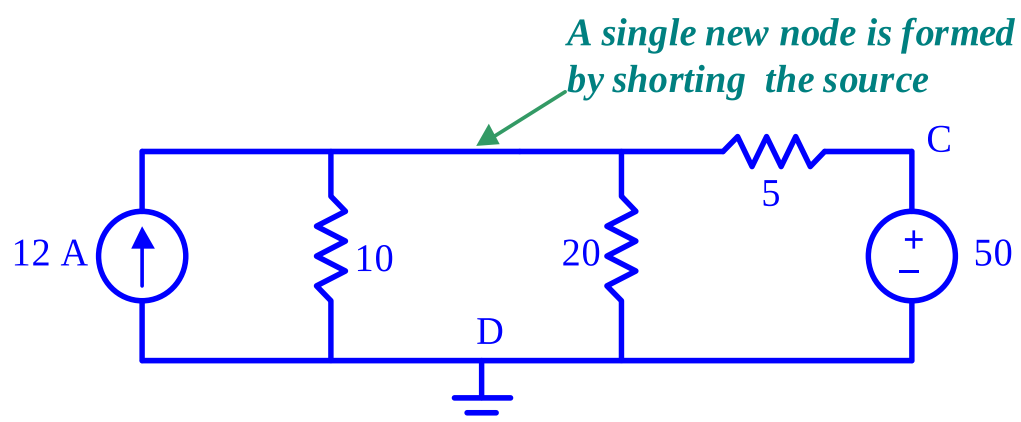
$V_A - V_B = 20$





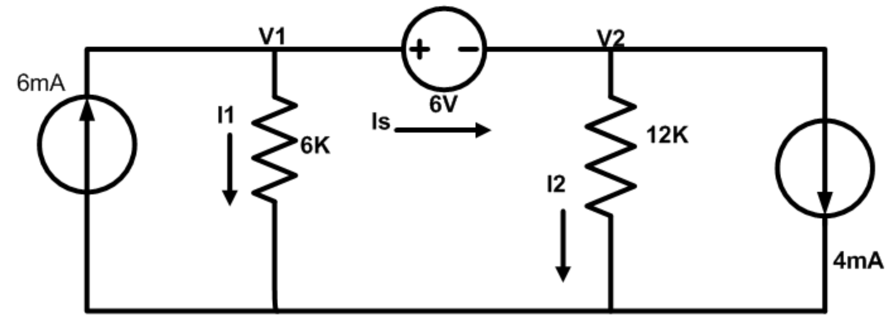
- Note that the voltage of node C is 50V, so :

$V_C = 50$



- Think of the supernode as the node that would exist if the voltage source were replaced by a short (a wire).

Voltage sources and the super-node



- Constraint equation :

$$V1 - V2 = 6 \qquad (1)$$

- KCL at node 1:

$$6mA = I1 + Is$$

$$6mA = \frac{V1}{6k} + Is \qquad (2)$$

- KCL at node 2

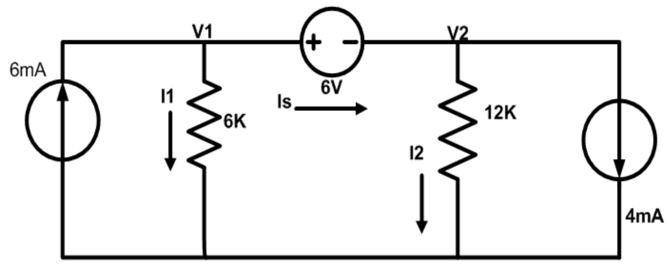
$$Is = I2 + 4\,mA$$

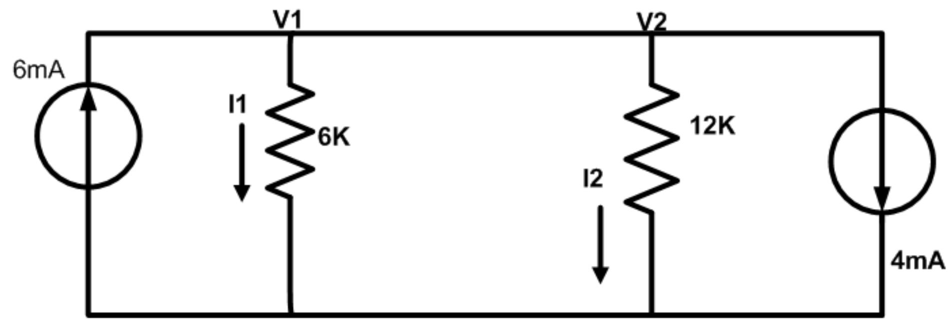
$$4\,mA = Is - I2$$

$$4mA = Is - \frac{V2}{12k} \qquad (3)$$

- Subtracting 3 from 2:
- $2mA = \frac{V1}{6k} + \frac{V2}{12\,k} \qquad (4)$
- This is the super-node equation
- solving (1) and (4) we get
- $V1 = 10\ v$
- $V2 = 4\ v$

Supernode equation by inspection



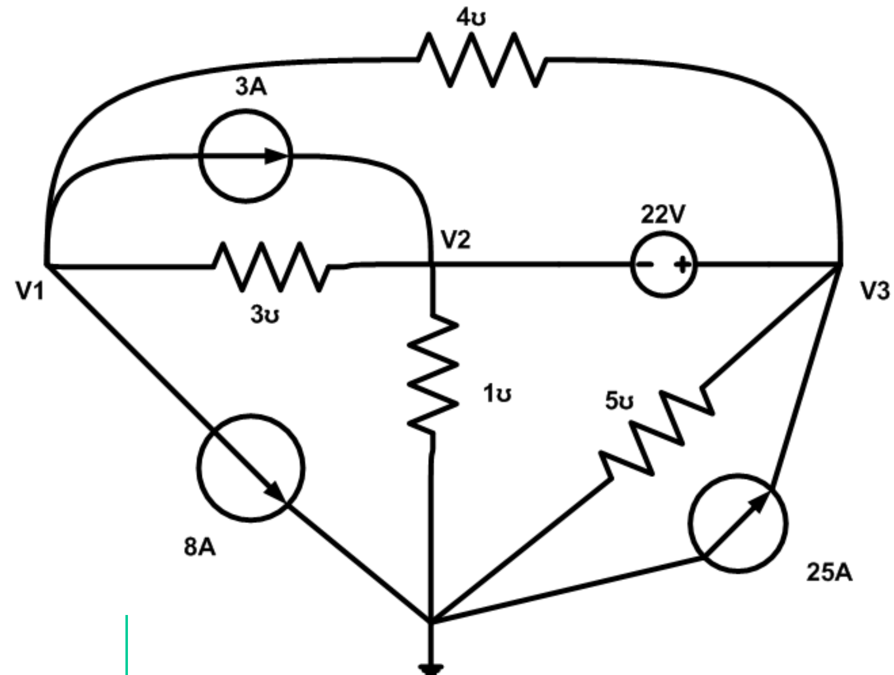


$$6mA = I1 + I2 + 4\ mA$$

$$2\ mA = I1 + I2$$

$$2mA = \frac{V1}{6k} + \frac{V2}{12\ k}$$

This is equation 4 from previous slide



Constrain equation

$$V3 - V2 = 22$$

KCL at node 1:

$$7\,V1 - 3\,V2 - 4\,V3 = -11$$

Super node equation :

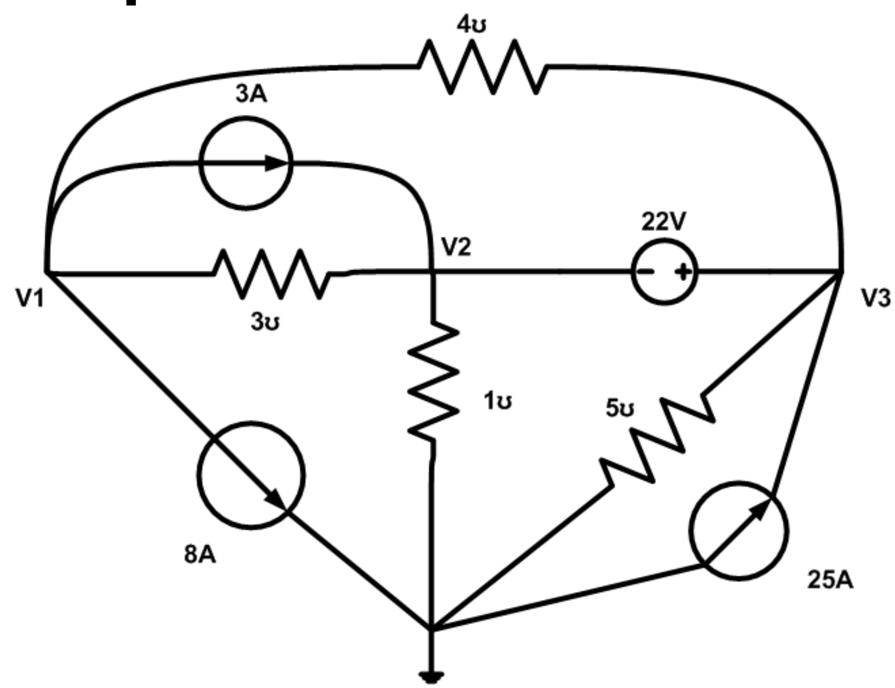
$$-7\,V1 + 4\,V2 + 9V3 = 28$$

Solving for $V1$

$$\xrightarrow{yields} V1 = -4.5\ v$$

Super-node equation by inspection

- 2&3 forms a super-node



$$25 + 3 = (1 + 3)V2 + (5 + 4)V3 - (3 + 4)V1$$

$$28 = 4\,V2 + 9\,V3 - 7V1$$

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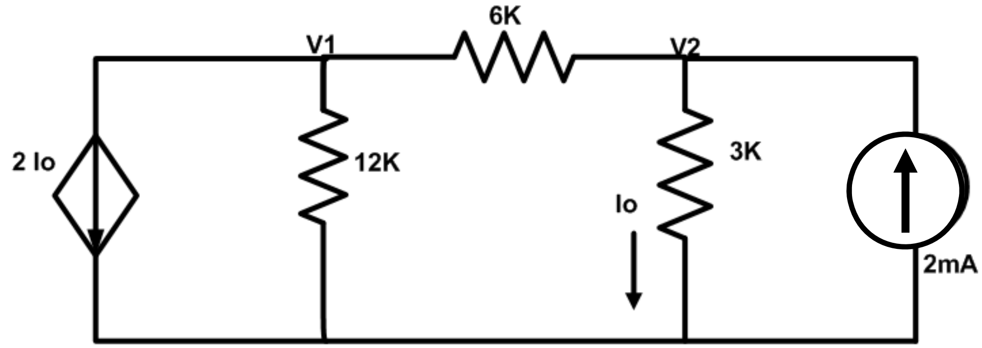
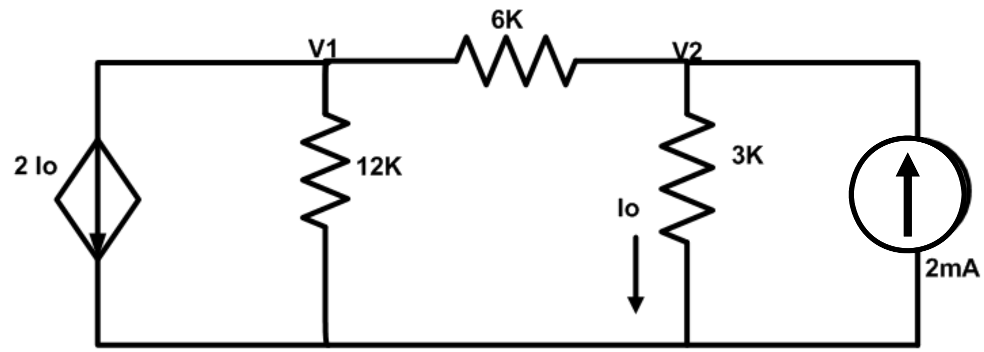


KCL at node 1:

$$-2Io = \left(\frac{1}{12k} + \frac{1}{6k}\right)V1 - \frac{1}{6k}\,V2$$

$$Io = \frac{V2}{3\,k}$$

$$0 = \left(\frac{1}{12k} + \frac{1}{6k}\right)V1 + \left(\frac{2}{3k} - \frac{1}{6k}\right)V2$$



- KCL an node 2

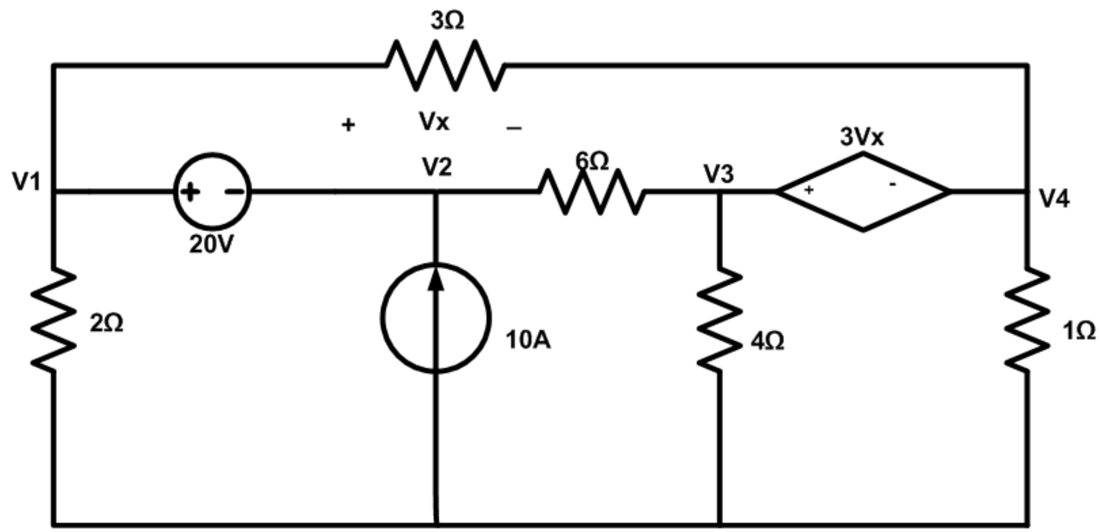
$$2\ mA = -\frac{1}{6k}\ V1 + \left(\frac{1}{3k} + \frac{1}{6k}\right) V2$$

Solving for V1 and V2 , we get

$$V1 = -\frac{24}{5}\ v \quad ; \quad V2 = \frac{12}{5}\ v$$

Nodal Analysis : super-node

KCL at supernode 1,2 :



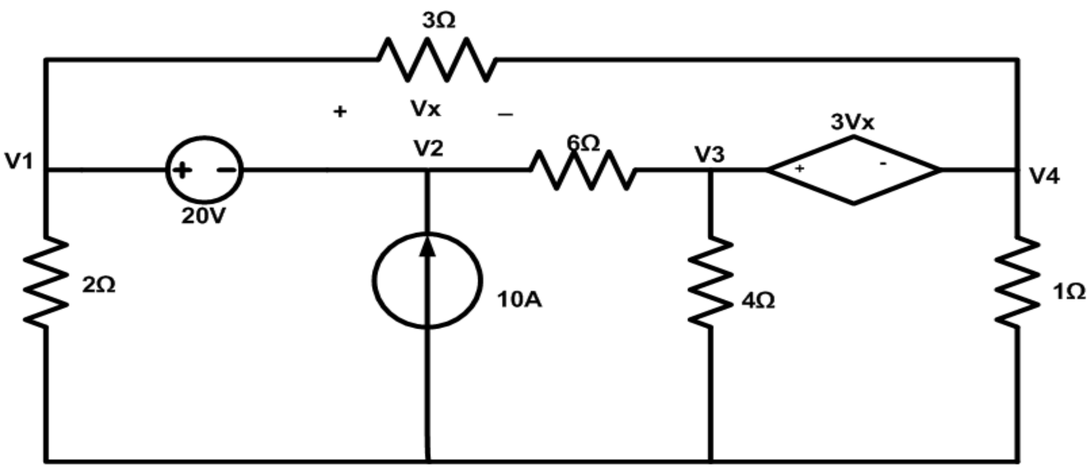
$$10 = \left(\frac{1}{2} + \frac{1}{3}\right) V1 + \frac{1}{6} V2 - \frac{1}{6} V3 - \frac{1}{3} V4$$

KCL at supernode 3,4 :

$$0 = \left(\frac{1}{4} + \frac{1}{6}\right) V3 + \left(\frac{1}{1} + \frac{1}{3}\right) V4 - \frac{1}{6} V2 - \frac{1}{3} V1$$

Constraint equation :

$$V1 - V2 = 20$$



Constraint Equations

$$V3 - V4 = 3\,Vx$$

$$Vx = V1 - V4$$

Simplifying

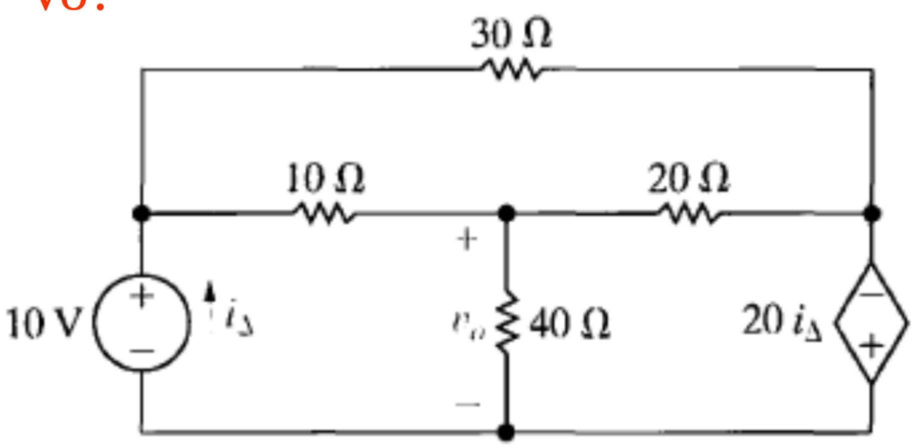
$$0 = 3\,V1 - V3 - 2\,V4$$

What is the current through the independent voltage source ?

- Solve the Following Problems

Assessment 4.4: Analyze the following circuit using node equations and find Vo?

Assign reference & label nodes



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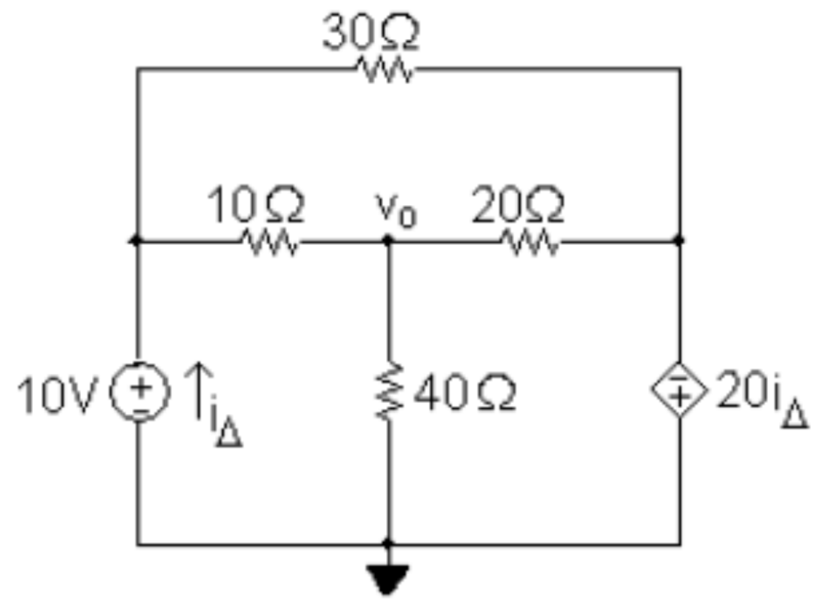
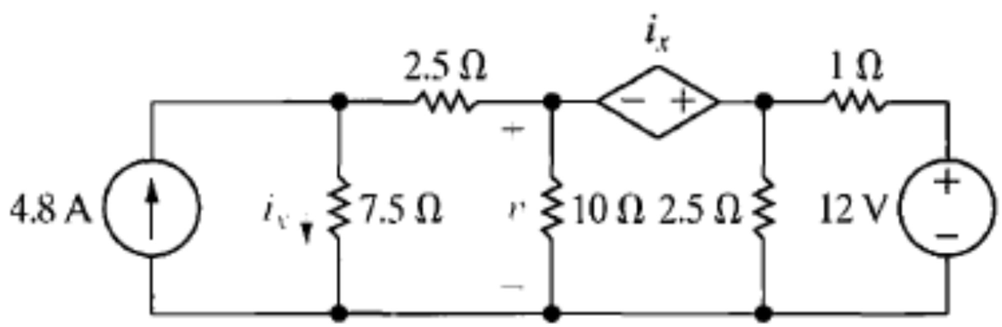
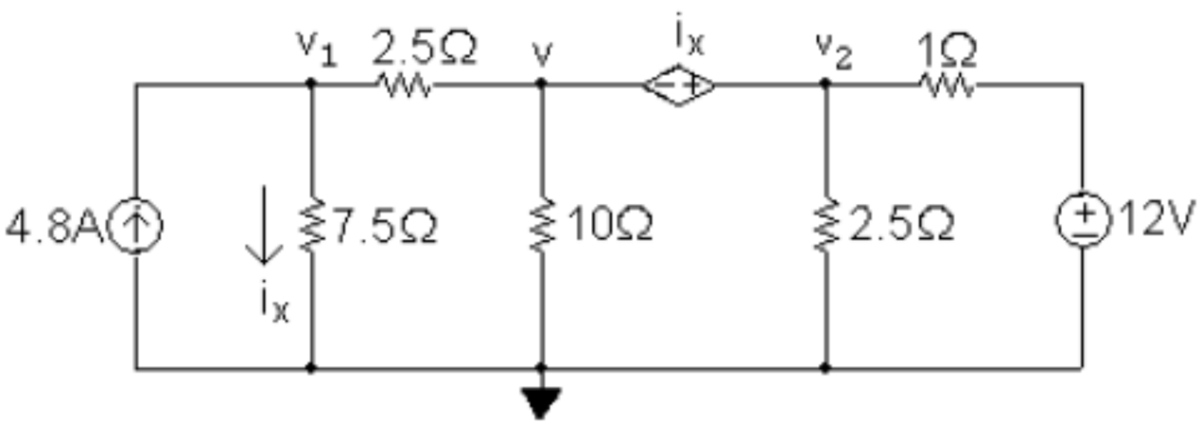


Answer: 24V

Assessment 4.5 : Analyze the following circuit using node equations and find v (across the 10 ohm resistor)?

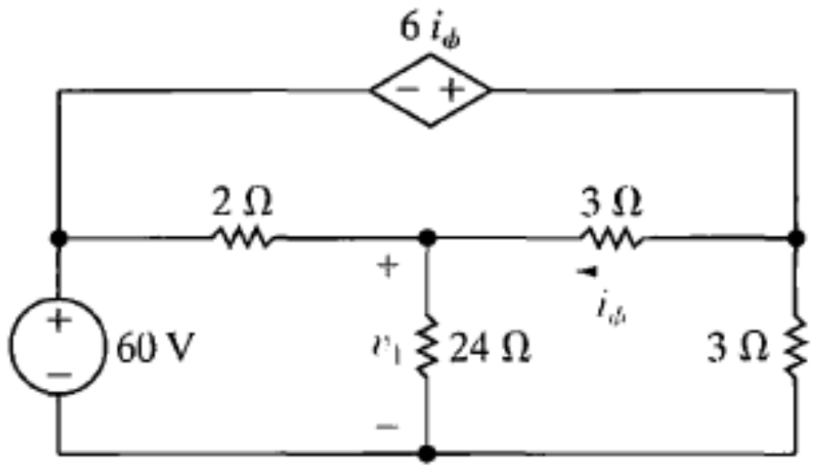


Assign reference & label nodes

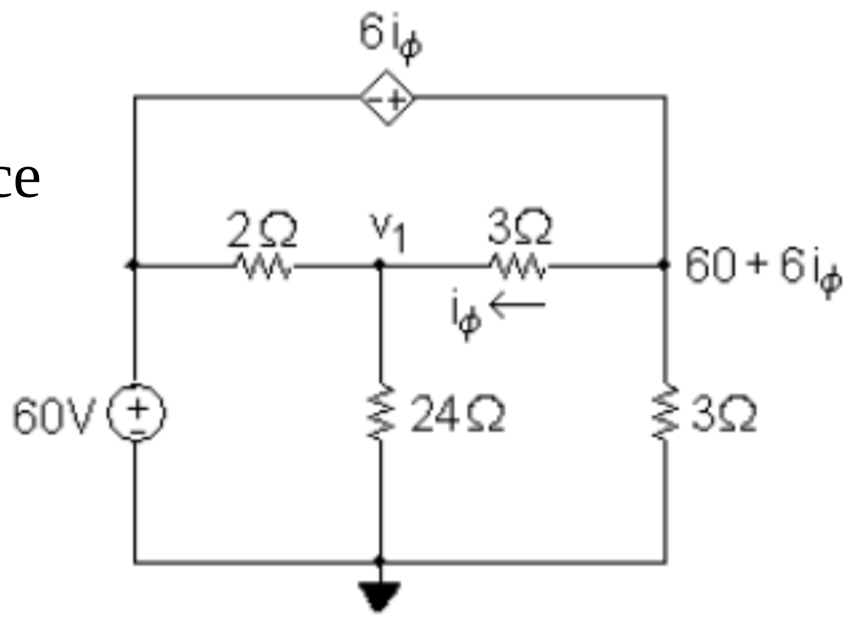


Answer: 8V

Assessment 4.6 : Analyze the following circuit using node equations and find V1 (across the 24 ohm resistor)?

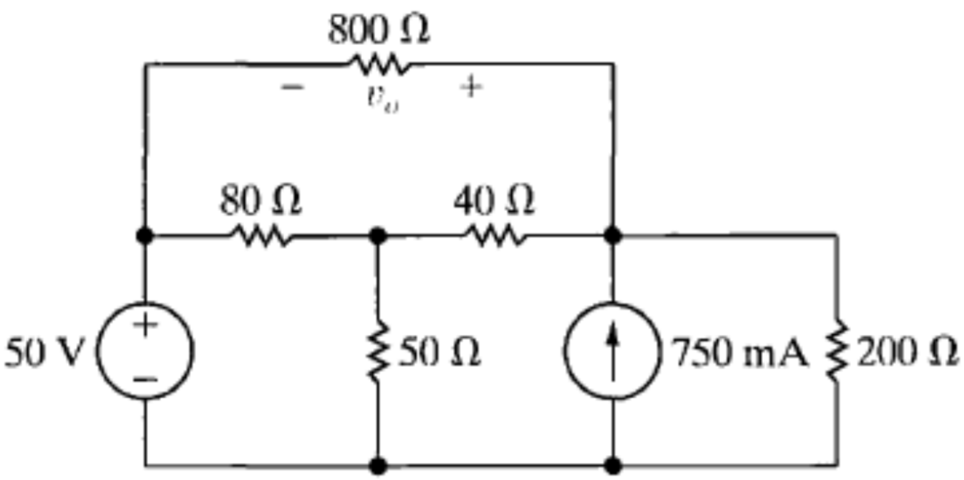


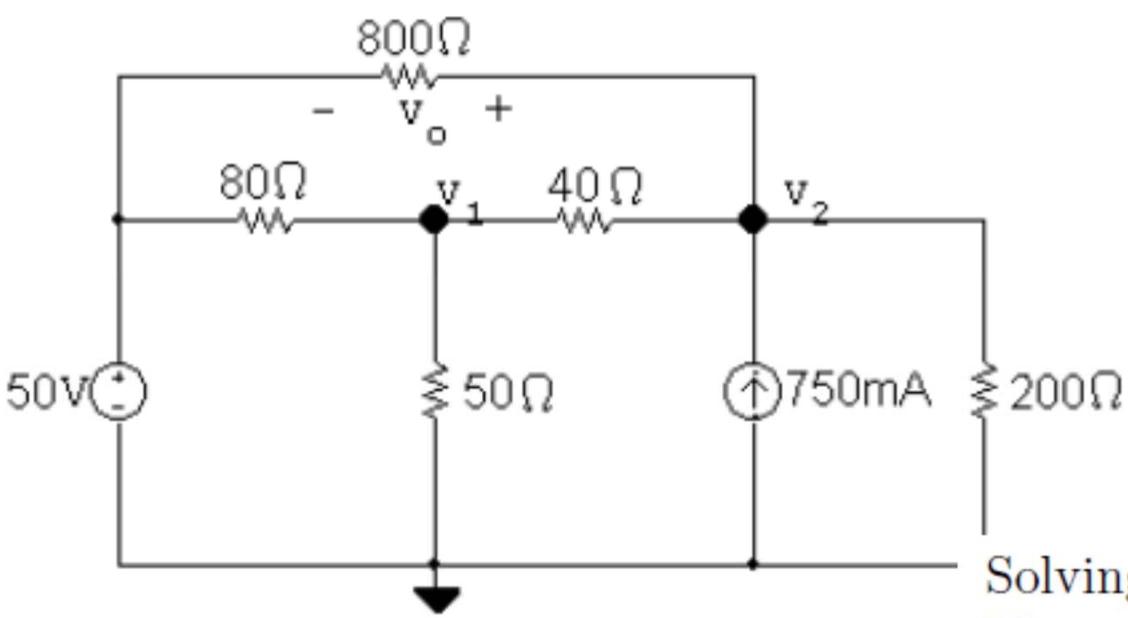
Assign reference & label nodes



Answer: 48V

Problem 4.24 Analyze the following circuit using node equations and find Vo (across the 800 ohm resistor)?

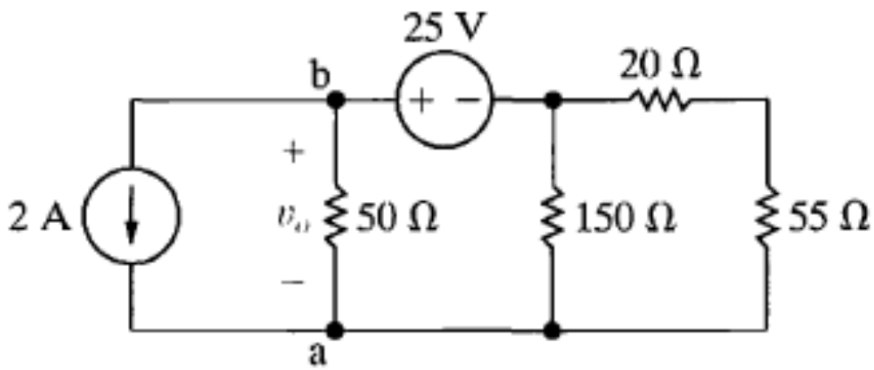




Solving, $v_1 = 34$ V; $v_2 = 53.2$ V.

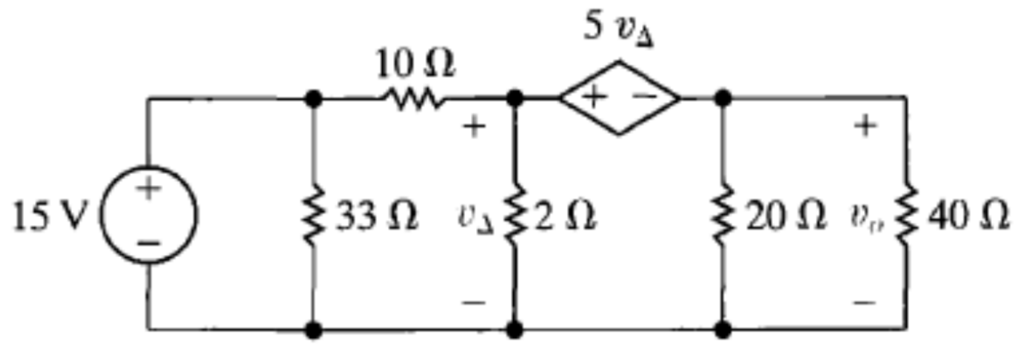
Thus, $v_o = v_2 - 50 = 53.2 - 50 = 3.2$ V.

Problem 4.26 Analyze the following circuit using node equations and find Vo (across the 50 ohm resistor)?



Answer: -37.5 V..

Problem 4.27 Analyze the following circuit using node equations and find Vo (across the 40 ohm resistor)?



Answer: -20 V