Reading Assignment: Sections 4.1-4.8 in *Electric Circuits, 9th Edition* by Nilsson

Chapter 4 – Methods of Analysis for Resistive Circuits

Mesh Equations (or Mesh Analysis) – result in a set of simultaneous, independent KVL equations.

Systematic procedure is introduced for writing these equations that will give a clear approaches that can be used even for very large circuits.

Mesh Equations

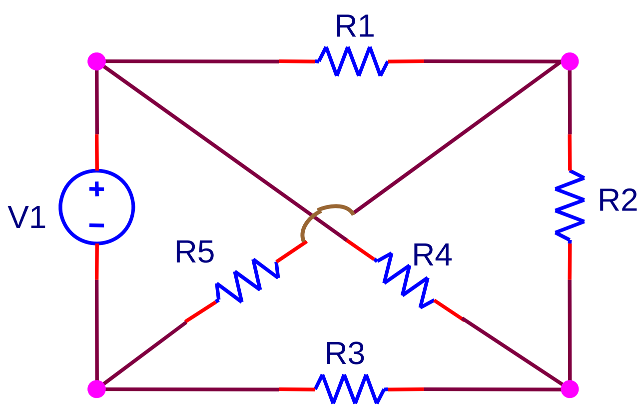
Mesh equations (or mesh analysis) are a set of simultaneous KVL equations.

Mesh equations have one restriction: Mesh analysis can only be used if a circuit is ***planar***.

A circuit is planar if it could be drawn on a 2D surface with no crossovers.

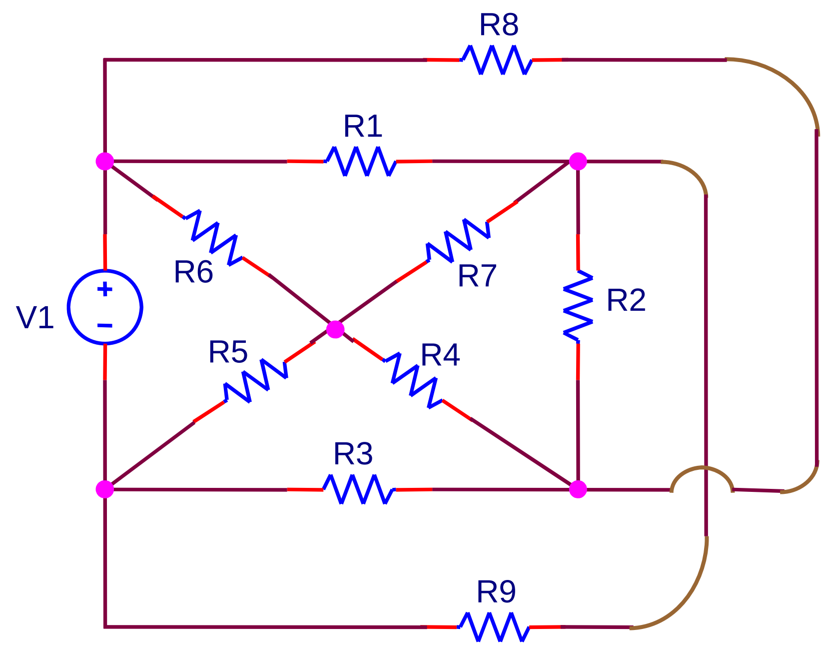
Example:

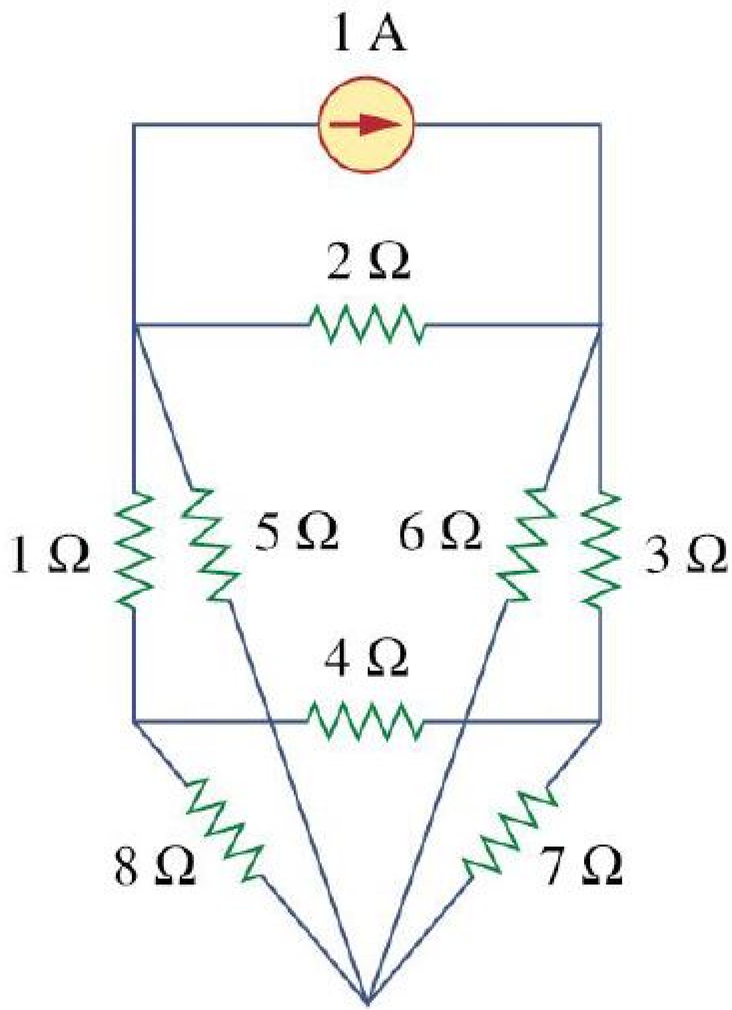
Is the following circuit planar?



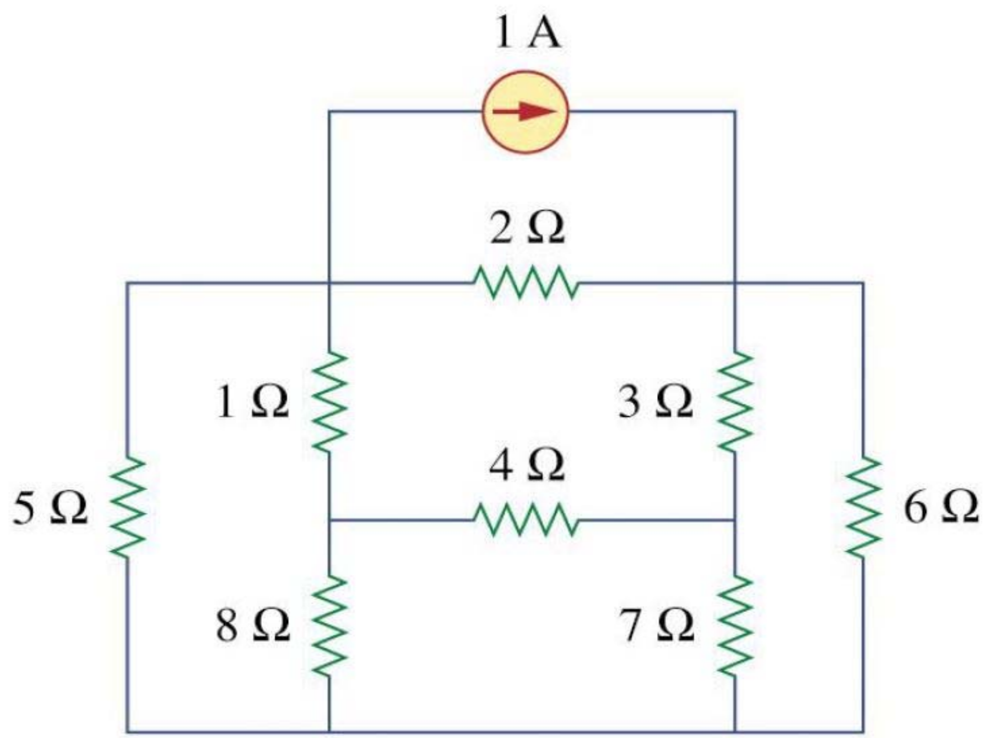
Example:

Is the following circuit planar?



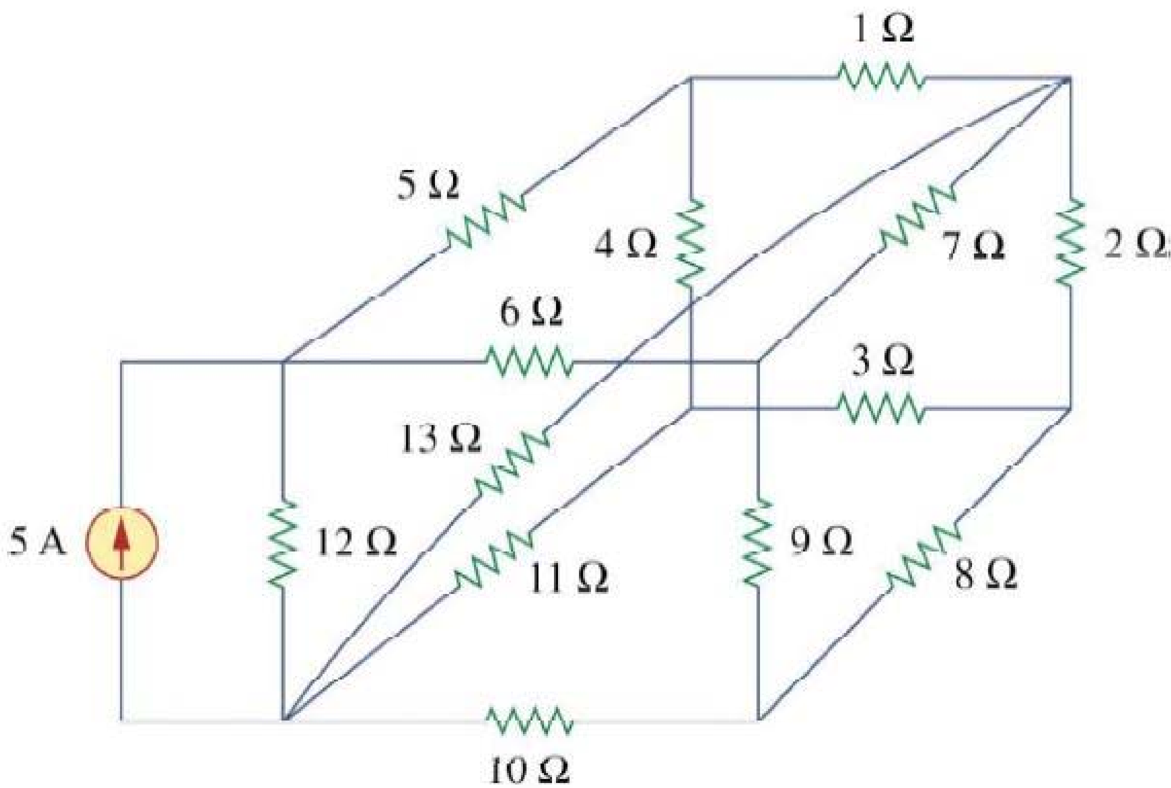


Redraw



Planner?

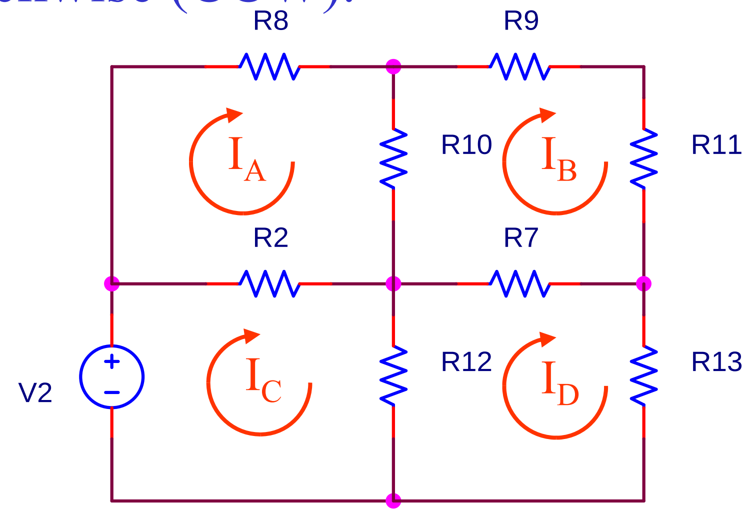
Yes , it is planner



Non-planner

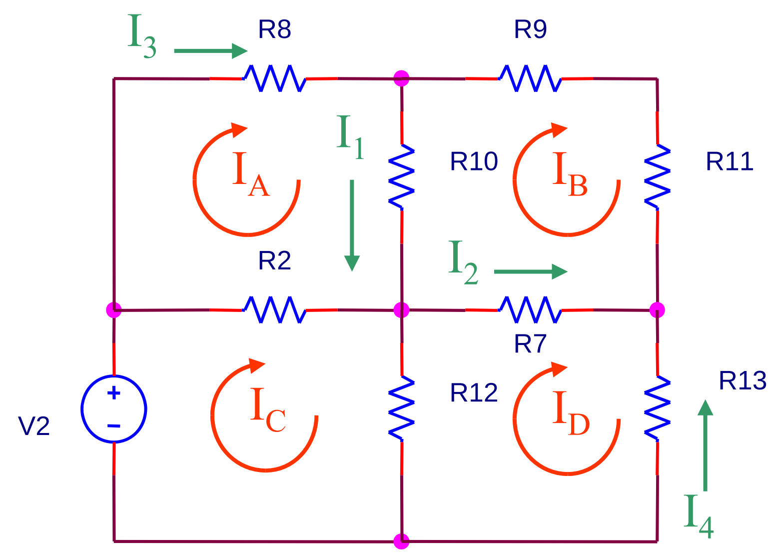
Mesh current – a current associated with a mesh. Mesh currents are generally drawn all clockwise (CW) or all counter clockwise (CCW).

Example: Mesh currents I_A, I_B, I_C, and I_D are shown in the circuit to the right.



Component currents – note that a component current is made up of either one or two mesh currents.

Example: Define the component currents I_1, I_2, I_3, and I_4 in terms of mesh currents.



$I_1 = I_A - I_B$

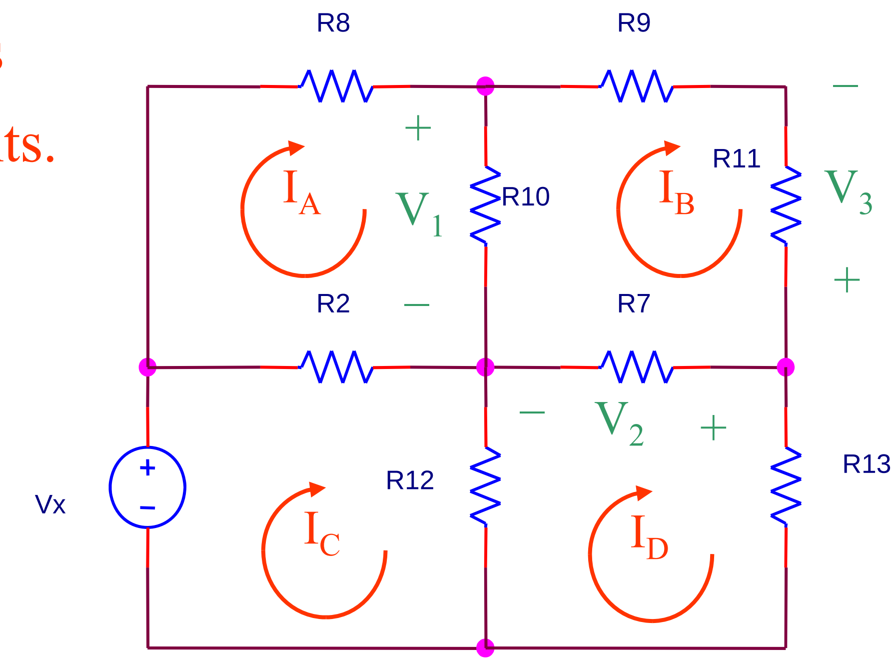
$I_2 = I_D - I_B$

$I_3 = I_A$

$I_4 = -I_D$

Expressing resistor voltages in terms of mesh currents:

Example: Define the resistor voltages V_1, V_2, and V_3 in terms of mesh currents.



$V_1 = (I_A - I_B) * R10$

$V_2 = (I_B - I_D) * R7$

$V_3 = -I_B * R11$

Mesh Equations – Procedure:

1) Be sure that the circuit is planar (redraw it if necessary).
2) Label the mesh currents (generally all CW or all CCW).
3) If the circuit contains any current sources on the outer edge, the corresponding mesh currents are defined. If the circuit contains any internal current sources, a ***supermesh*** is required (more information later).
4) Write a KVL equation in each mesh with no current sources and one KVL equation around each supermesh. Express resistor voltages in terms of mesh currents (see below).
5) Solve the equations simultaneously. In general, the number of mesh equations is:

 # Mesh Equations = # meshes - # current sources

Mesh Analysis
Example

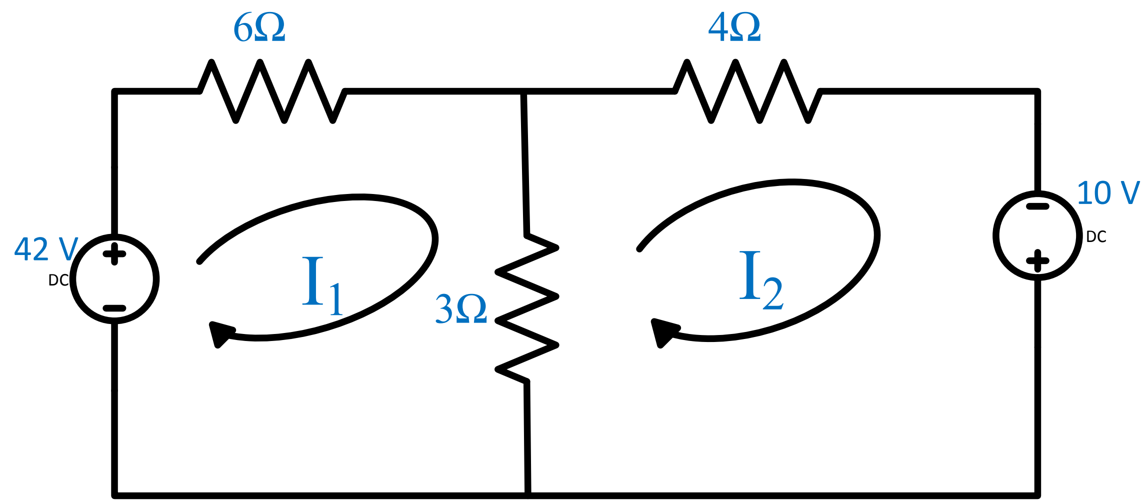


Figure3:example 1 of mesh analysis

KVL for mesh (1)

$$42 = 6I_1 + 3(I_1 - I_2)$$

$$\boldsymbol{42 = 9I_1 - 3I_2} \text{ ------------ } \textit{(1)}$$

KVL for mesh (2):

$$10 = 4I_2 + 3(I_2 - I_1)$$

$$\boldsymbol{10 = -3I_1 + 7I_2} \text{ ------------- } \textit{(2)}$$

Solving (1) & (2):

- $I_1 = 6\,A$ *and* $I_2 = 4\,A$

In general

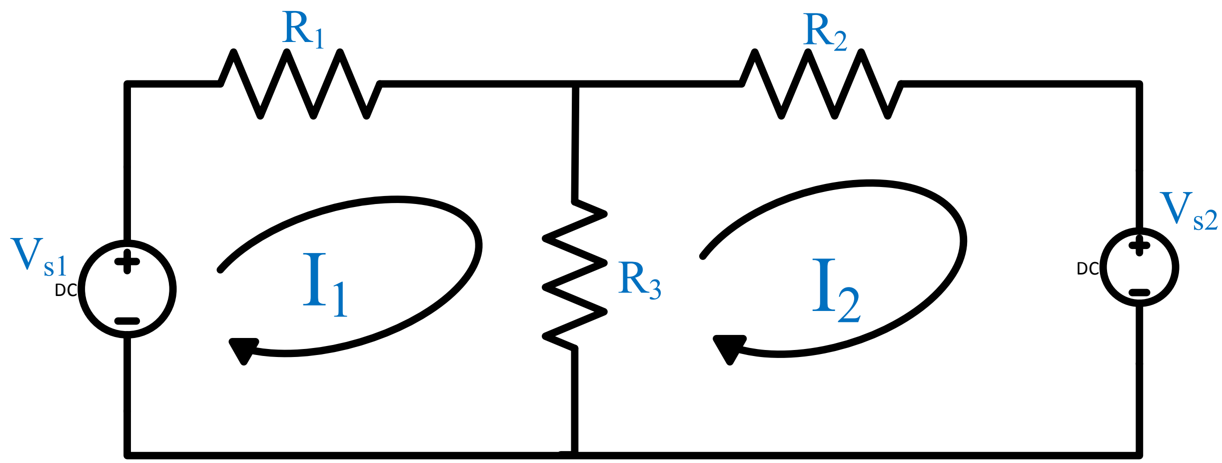


Figure5:Applying KVL for mesh analysis

KVL for mesh (1):

$$-V_{s1} + R_1 I_1 + R_3(I_1 - I_2) = 0$$

$V_{s1} = (R_1 + R_3)I_1 - R_3 I_2$, where $(R_1 + R_3)$ = self-resistance of mesh (1).

- R_3 = mutual resistance between meshes (1) & (

KVL for mesh (2):

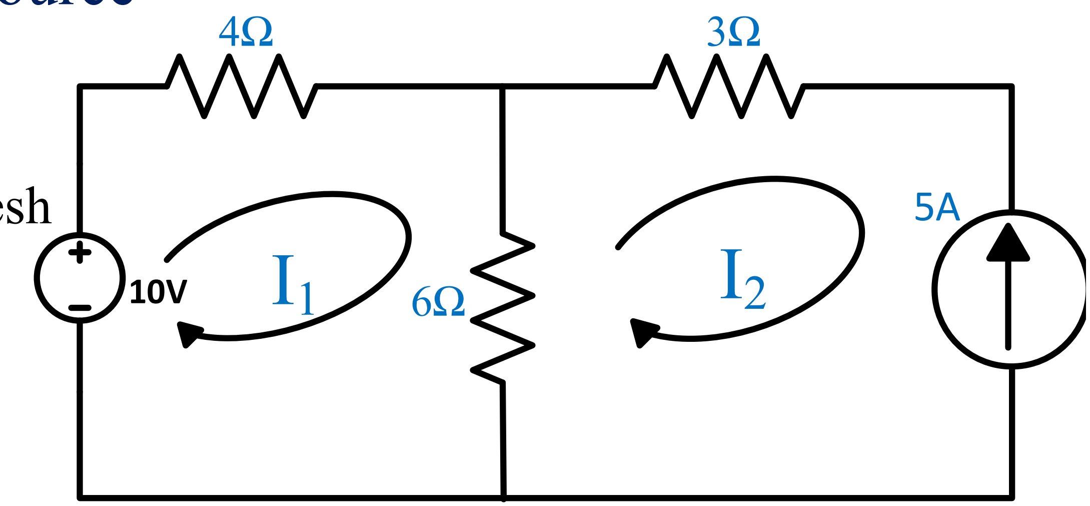
$$-V_{s2} = -R_3 I_1 + (R_2 + R_3)I_2$$

$(R_2 + R_3)$ = Self-resistance of mesh (2).

Mesh analysis: with current source

Case 1:

Current source exist only in one mesh



Figur6:mesh with current sourcee.

KVL for mesh (1):

$$10 = 10I_1 - 6I_2$$

Constrain equation:

$$I_2 = -5A$$

- $I_1 = -2A$

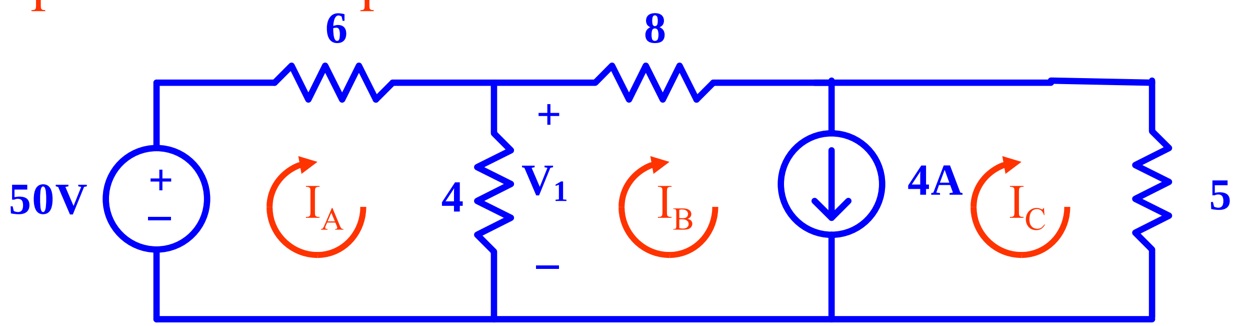
Note: Since I2 is known and there is no need to write mesh equation for mesh 2

Supermesh

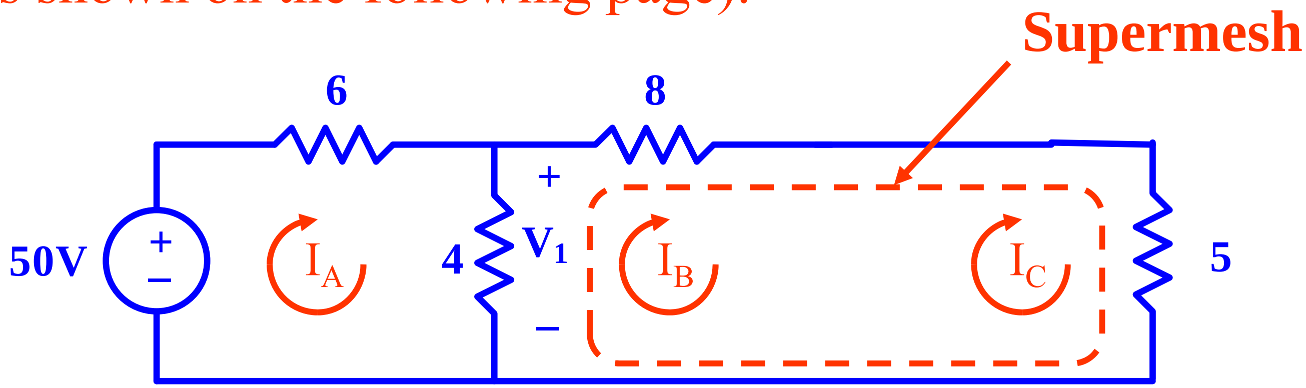
If a circuit contains an internal current source, a ***supermesh*** is required in order to perform mesh analysis. A ***supermesh*** is the new, larger mesh that is created by removing the internal current source. A new mesh current is not added. The supermesh simply shows the path for a KVL equation around the ***supermesh***.

Example:

1) Note that the following circuit has an internal current source, so a supermesh is required.

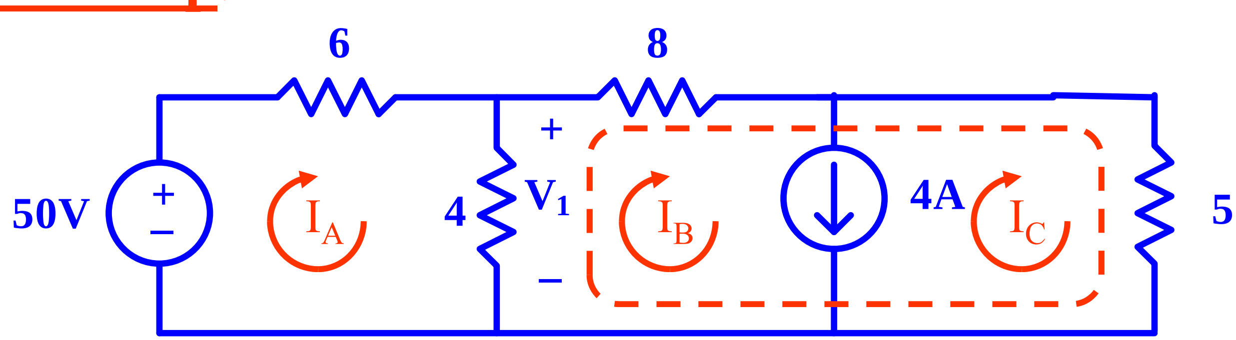


2) The supermesh is the new, larger mesh created by removing the current source (as shown on the following page).



3) Note that the supermesh defines a path for a KVL equation. No new mesh current is defined.

4) Also note that the internal current source can be used to form a relationship between currents I_B and I_C. In general, this is referred to as the **supermesh relationship**.



Supermesh relationship:
$I_B - I_C = 4$

Case 2:

Current source exists between two meshes, a **Super mesh** is obtained.

Mesh analysis: with current sources

KVL for mesh (2):

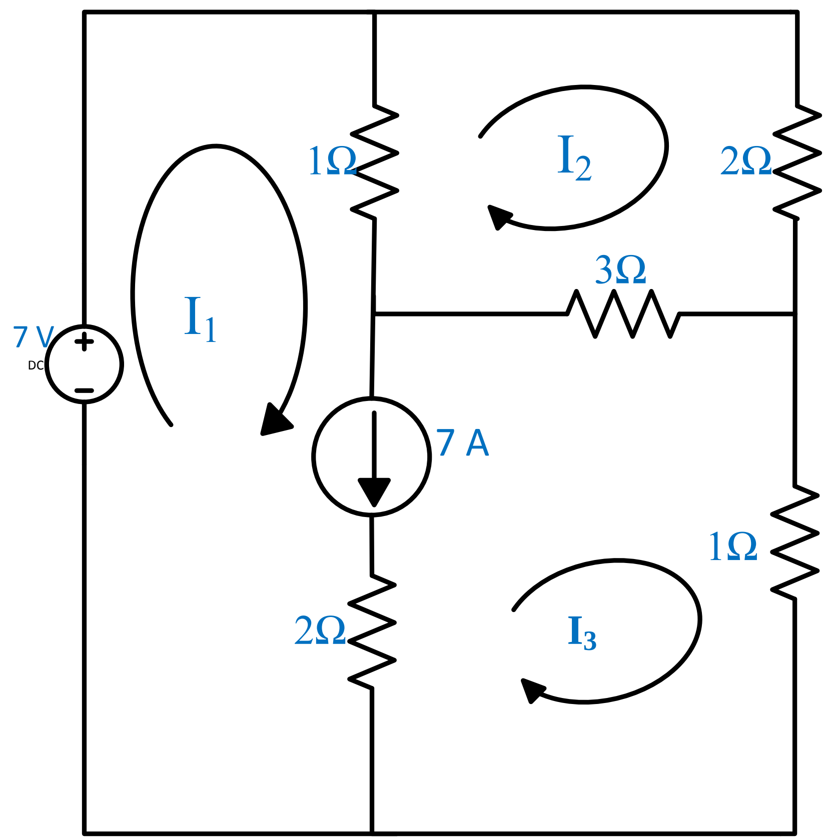
$$0 = 6I_2 - I_1 - 3I_3$$

Constrain equation:

$$I_1 - I_3 = 7$$

Super mesh equation:

$$7 = I_1 + 4I_3 - 4I_2$$



Figur:7mesh with current sources

KVL for mesh (1):

$$-7 + 1(I_1 - I_2) + V + 2(I_1 - I_3) = 0$$

$$\mathbf{7 = 1(I_1 - I_2) + V + 2(I_1 - I_3) \quad (1)}$$

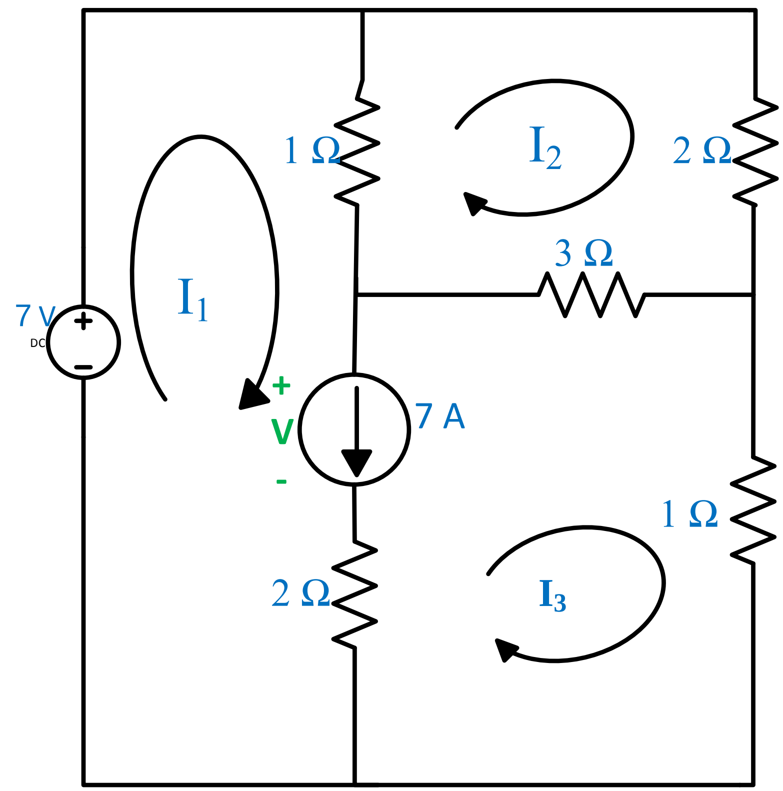
KVL for mesh (3):

$$3(I_3 - I_2) + I_3 + 2(I_3 - I_1) - V = 0$$

$$\mathbf{-2I_1 - 3I_2 + 6I_3 - V = 0 \quad (2)}$$

Adding (1) & (2):

$$\mathbf{I_1 - 4I_2 + 4I_3 = 7}$$



Figur8:mesh with current sourcee.

$$7 = 3I_1 - I_2 - 2I_3 + V \quad (1)$$

$$0 = -2I_1 - 3I_2 + 6I_3 - V \quad (2)$$

Mesh Analysis with dependent sources

KVL for mesh (2):

$$1(I_2 - I_1) + 2I_2 + 3(I_2 - I_3) = 0$$

$$-I_1 + 6I_2 - 3I_3 = 0$$

Constrain equation:

$$I_1 = 15\ A$$

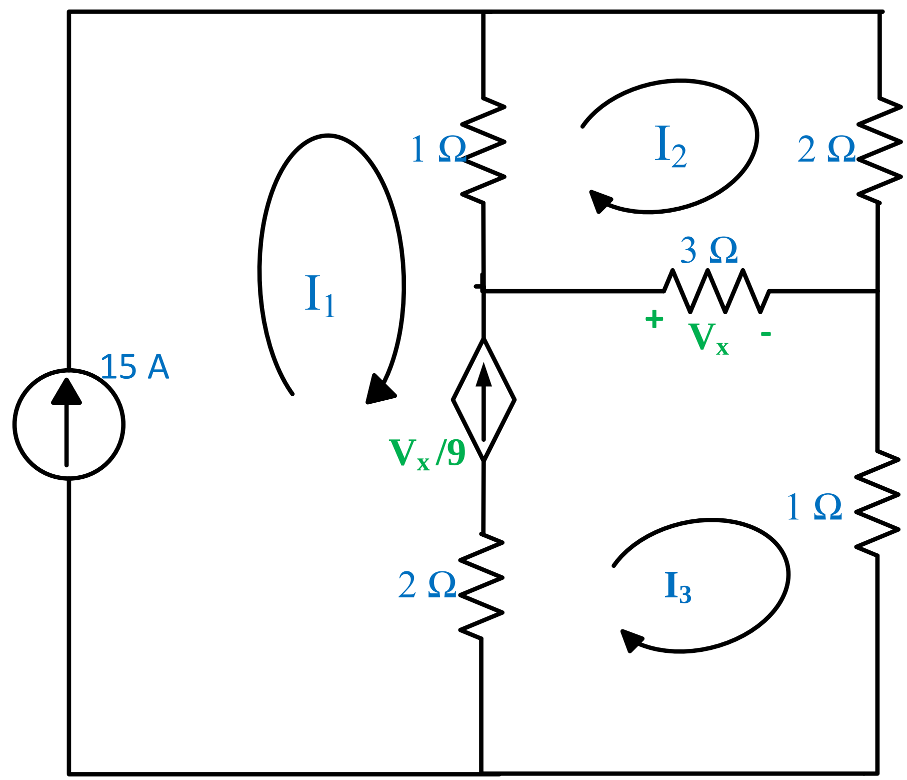
Constrain equation:

$$I_3 - I_1 = \frac{v_x}{9}$$

$$v_x = 3(I_3 - I_2)$$

- $I_1 = 15\ A\ ; I_2 = 11\ A$

$$I_3 = 17\ A$$

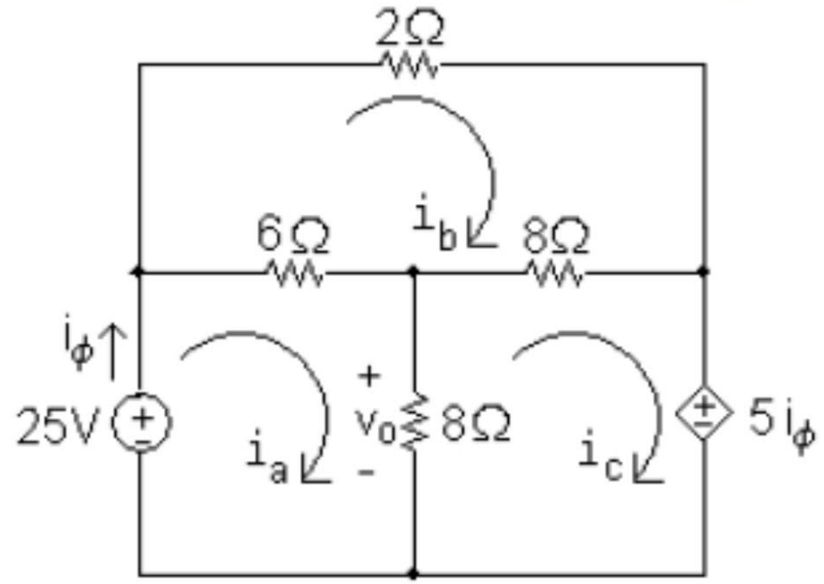


Figur:9mesh with dependant current source.

Node or mesh: How to choose?

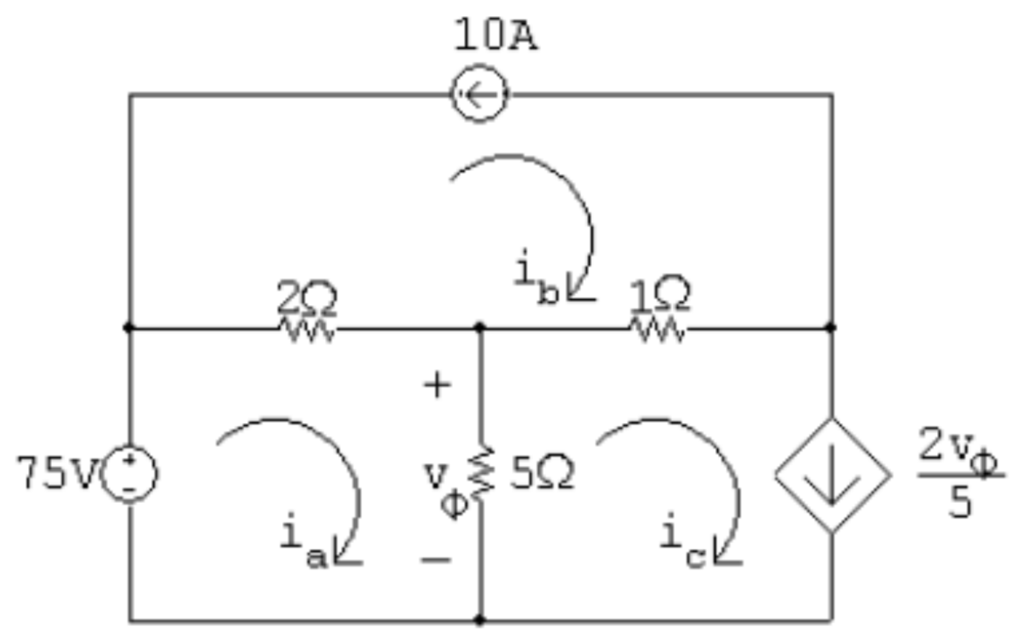
- Use the one with fewer equations.
- Use the method you like best.

Assessment 4.9: Use the mesh-current method to find v_o in the circuit shown.



Answer: 16V

Assessment 4.11: Use the mesh-current method to find *mesh current* i_a



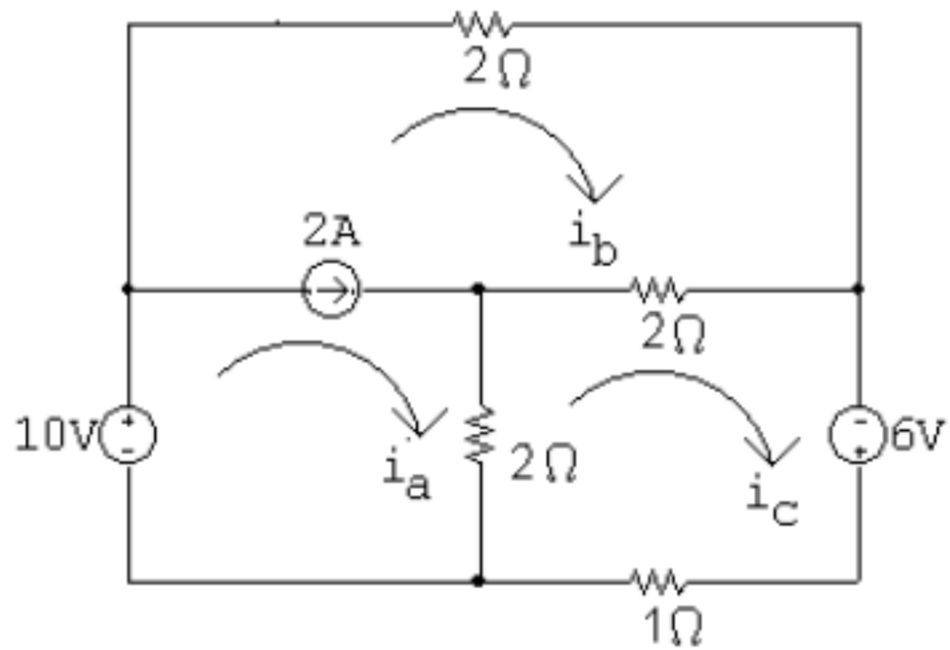
Answer: 15A

Assessment 4.12:

a) Use the mesh-currcnt method to find how much power the 4 A current source delivers to the circuit

b) Find the total power delivered to the circuit.

c) Check your calculations by showing that the total power developed in the circuit equals the total power dissipated



Answer: a) –162.92 W

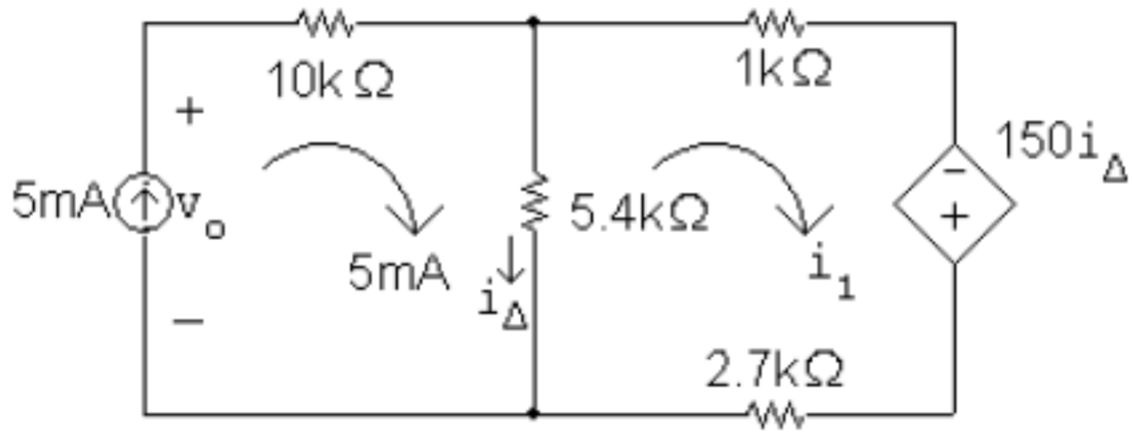
b) 518.52 W

Problem 4.44:

a) Use the mesh-current method to solve for i_Δ

b) Find the power delivered by the independent current source.

c) Find the power delivered by the dependent voltage source.

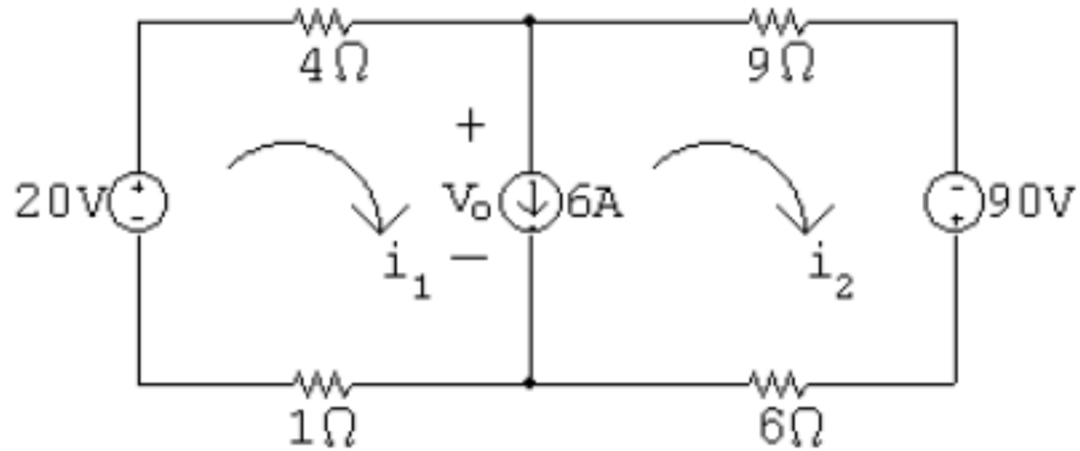


Answer: a) i_Δ=2mA

b) 304 mW

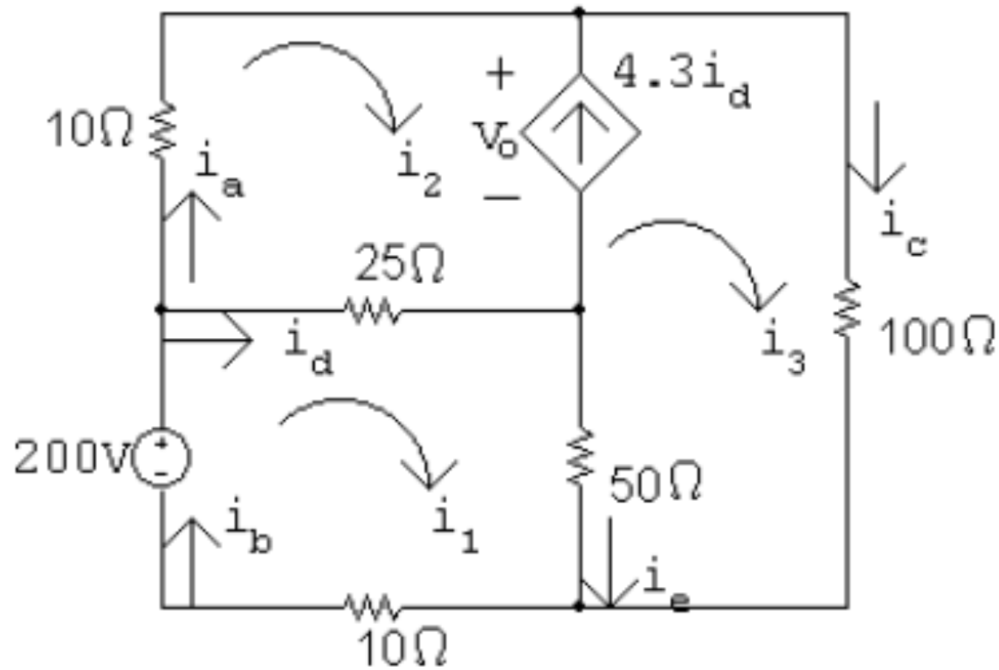
c) 0.9 mW

Problem 4.48: Use the mesh-current method to find the total power dissipated in the circuit



Answer: 740 W

Problem 4.51: Use the mesh-current method to find the branch currents in *ia* — *ie* in the circuit



Answer:

$$i_a = i_2 = 5.7 \text{ A}; \qquad i_b = i_1 = 4.6 \text{ A}$$

$$i_c = i_3 = 0.97 \text{ A}; \qquad i_d = i_1 - i_2 = -1.1 \text{ A}$$

$$i_e = i_1 - i_3 = 3.63 \text{ A}$$

Solution of simultaneous equations

- Solve the following

$$21i_1 - 9i_2 - 12i_3 = -33,$$

$$-3i_1 + 6i_2 - 2i_3 = 3,$$

$$-8i_1 - 4i_2 + 22i_3 = 50.$$

- Use Cramer's rule

$$x_k = \frac{N_k}{\Delta}.$$

$$\Delta = \begin{vmatrix} 21 & -9 & -12 \\ -3 & 6 & -2 \\ -8 & -4 & 22 \end{vmatrix}$$

$$\Delta = \begin{vmatrix} 21 & -9 & -12 \\ -3 & 6 & -2 \\ -8 & -4 & 22 \end{vmatrix}$$

$$\Delta = 21(1)\begin{vmatrix} 6 & -2 \\ -4 & 22 \end{vmatrix} - 3(-1)\begin{vmatrix} -9 & -12 \\ -4 & 22 \end{vmatrix} - 8(1)\begin{vmatrix} -9 & -12 \\ 6 & -2 \end{vmatrix}$$

$$\Delta = 21(132 - 8) + 3(-198 - 48) - 8(18 + 72)$$

$$= 2604 - 738 - 720 = 1146.$$

$$N_1 = \begin{vmatrix} -33 & -9 & -12 \\ 3 & 6 & -2 \\ 50 & -4 & 22 \end{vmatrix}, \qquad N_1 = 1146,$$

$$N_2 = \begin{vmatrix} 21 & -33 & -12 \\ -3 & 3 & -2 \\ -8 & 50 & 22 \end{vmatrix}, \qquad N_2 = 2292,$$

$$N_3 = \begin{vmatrix} 21 & -9 & -33 \\ -3 & 6 & 3 \\ -8 & -4 & 50 \end{vmatrix}. \qquad N_3 = 3438.$$

$$i_1 = \frac{N_1}{\Delta} = 1\text{ A},$$

$$i_2 = \frac{N_2}{\Delta} = 2\text{ A},$$

$$i_3 = \frac{N_3}{\Delta} = 3\text{ A}.$$