ENEE2301

Network Reduction Techniques and Network Theorems 2

Ch4

Reading Assignment: Sections 4.9 - 4.16, in *Electric Circuits, 10th Ed.* by Nilsson

Network Reduction Techniques and Network Theorems

Chapter 4, Sections 9-16, in the text by Nilsson covers several useful network reduction techniques and network theorems.

The purposes of these techniques and theorems are:

- To provide alternate analysis methods
- To provide methods for simplifying circuits
- To provide methods for representing circuits in the simplest possible form
- To gain insight into circuit behavior

Topics to be covered

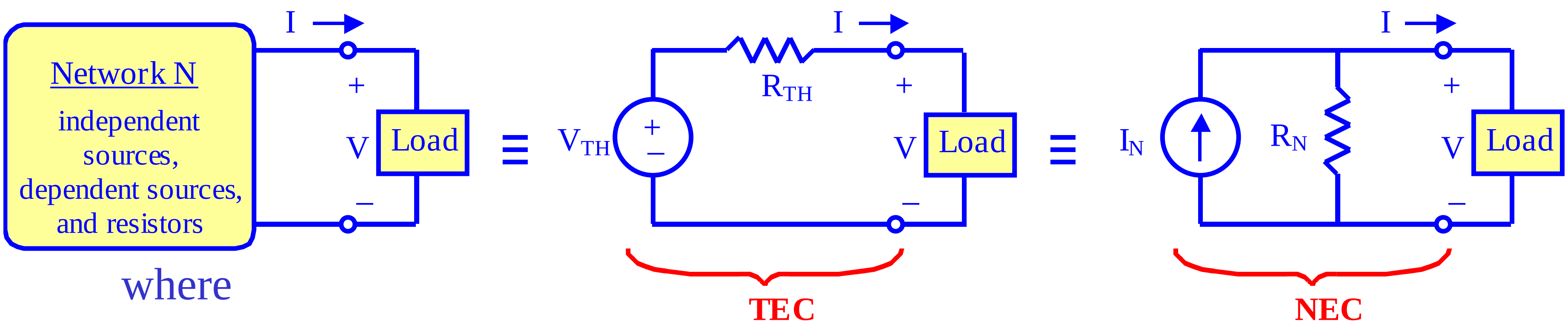
- Source Transformations
- Superposition
- Thevenin's and Norton's Theorems
- Maximum Power Transfer Theorem

Thevenin's & Norton's Theorems

Any one-port network N may be represented by either of the following types of equivalent circuits:

Thevenin Equivalent Circuit (TEC) – consisting of a voltage source and a series impedance

Norton Equivalent Circuit (NEC) – consisting of a current source and a parallel impedance

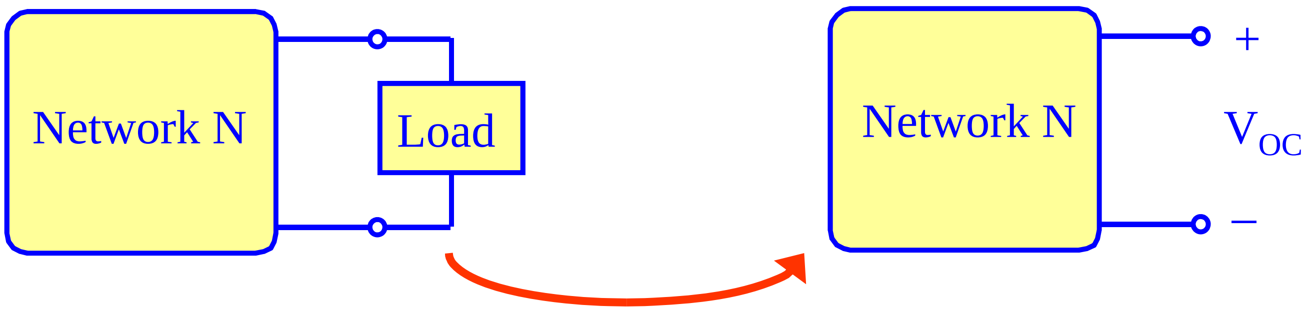


$V_{TH} = V_{OC}$ = Thevenin voltage or open-circuit voltage

$I_N = I_{SC}$ = Norton current or short-circuit current

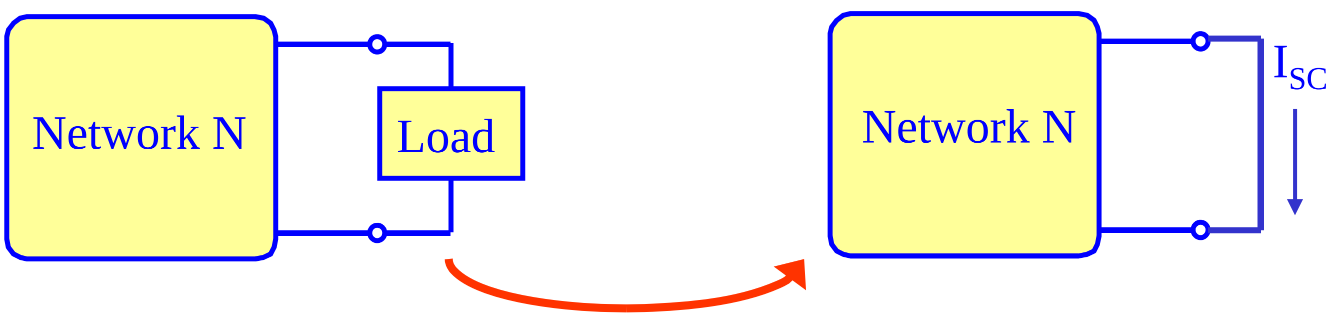
$R_{TH} = R_N$ = Thevenin or Norton resistance $= \dfrac{V_{OC}}{I_{SC}} = R_{EQ}\Big|$ Seen by the load with independent sources killed

Illustration of V_{OC} and I_{SC}:



Remove the load and the voltage across the open terminals is V_{OC}

$V_{TH} = V_{OC} =$ Thevenin voltage or open-circuit voltage



Replace the load by a short circuit (wire) and the current through the short is I_{SC}

$I_N = I_{SC} =$ Norton current or short-circuit current

There are 3 ways to find the TEC or NEC for a given circuit:

Examples using each of the three methods will be provided on the following pages.

1) Reduce the circuit into the form of a TEC or NEC using source transformations
   - Not possible with dependent sources, though a partial reduction may be useful
   - Recall that not all sources are transformable

2) Find V_{oc} or I_{sc}. Also find $R_{Th} = R_{eq}\Big|_{\text{Seen by the load with independent sources killed}}$
   - For a simple circuit, this can often be done by combining series & parallel R's.
   - If the circuit has dependent sources, R_{Th} can be found by adding an external voltage or current source (any value) to the output terminals and by finding :

$$R_{Th} = \frac{V_T}{I_T} = \frac{\text{Terminal voltage}}{\text{Terminal current}}$$

3) Find V_{oc} and I_{sc}. Also calculate $R_{Th} = \dfrac{V_{oc}}{I_{sc}}$. This is the most general method and is probably the best choice for circuits with dependent sources.

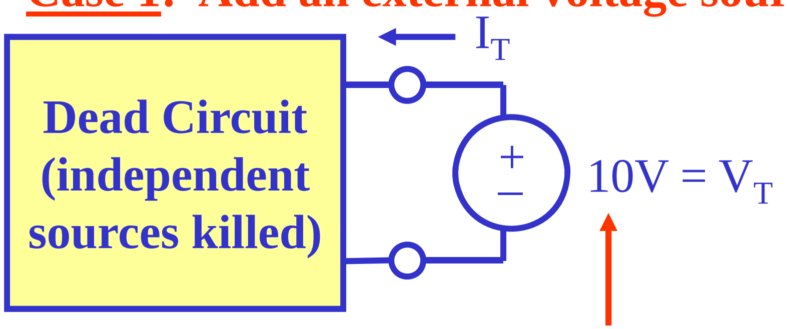
Finding Thevenin resistance by adding external sources

As indicated in Method 2 for finding a TEC or NEC, R_{TH} can also be found as follows:

$$R_{th} = \left.\frac{\text{terminal voltage}}{\text{terminal current}}\right| \text{With independent sources killed}$$

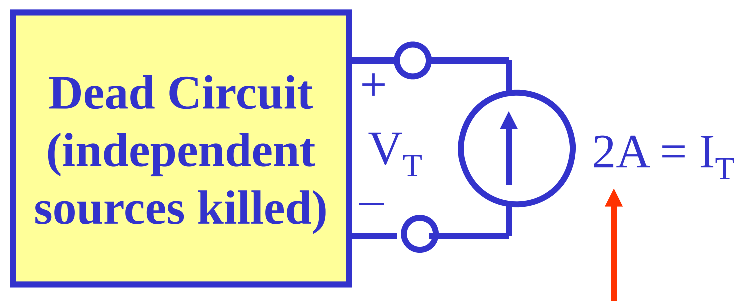
Discussion: If the dead circuit really acts like a resistor (R_{TH}), then we can add any voltage source across the circuit (resistor), solve for the current, and use Ohm's Law to find R_{TH}. Similarly, we can add any value current source and solve for the voltage. This technique is illustrated below.

Case 1: Add an external voltage source



Add any value of voltage source and solve for I_T

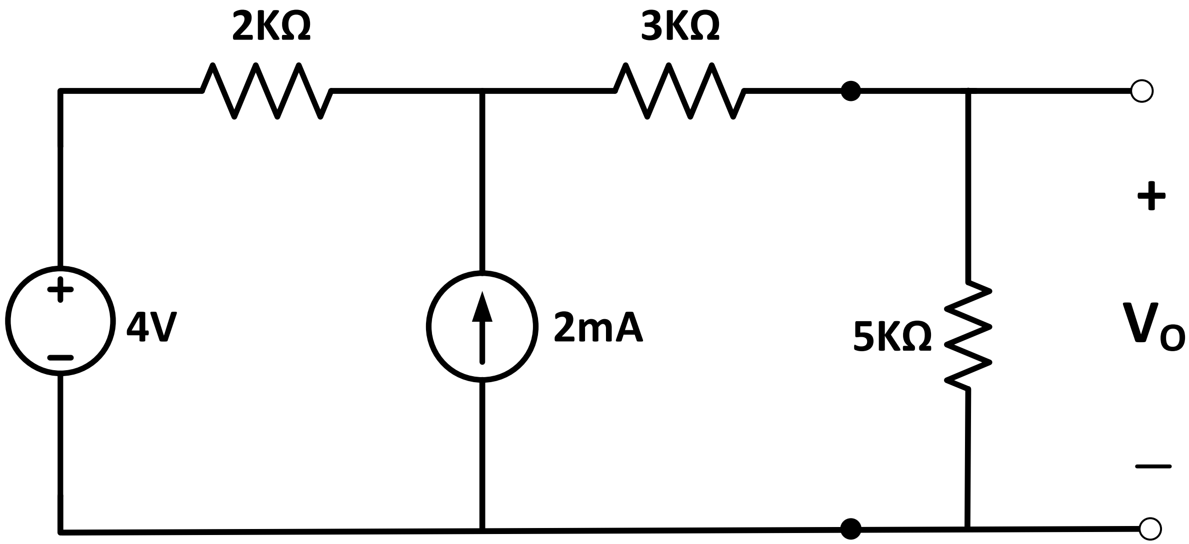
Case 2: Add an external current source

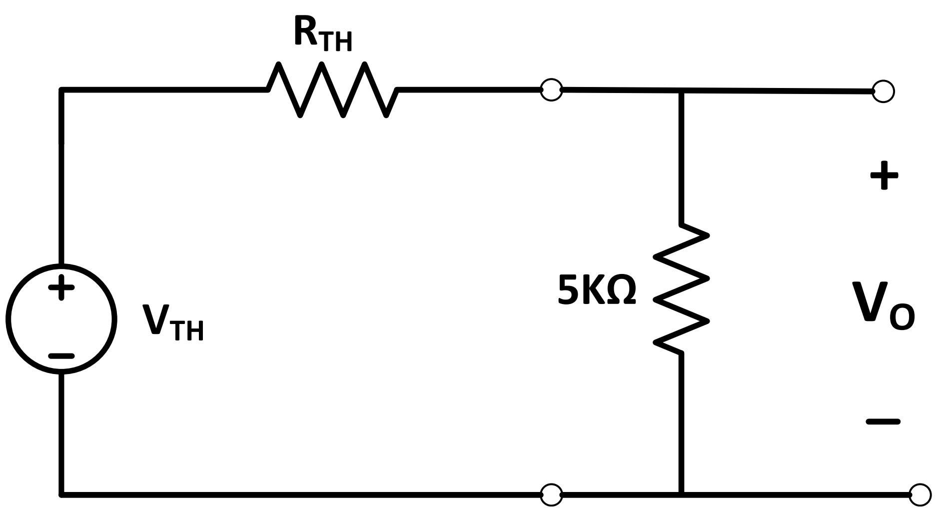


Add any value of current source and solve for V_T

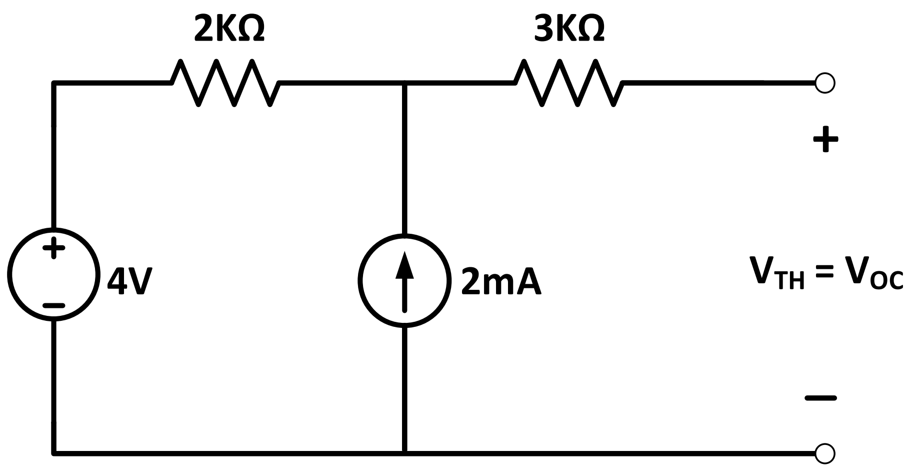
In either case above, calculate $R_{th} = \dfrac{V_T}{I_T} = \dfrac{\text{terminal voltage}}{\text{terminal current}}$

Example: Find Vo using Thevenin's Theorem



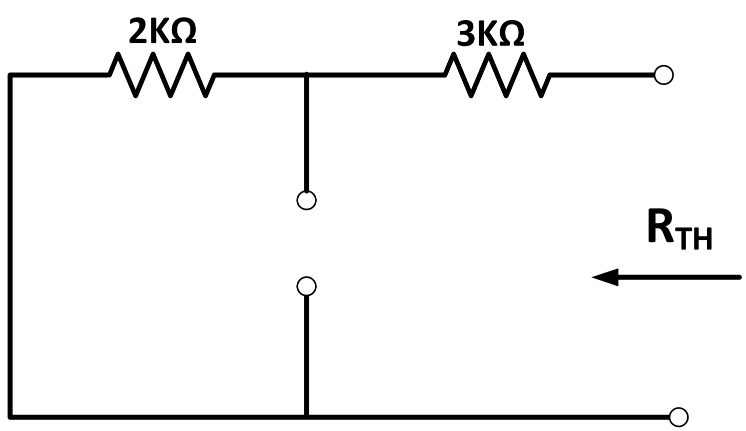


1) To find V_{Th}:



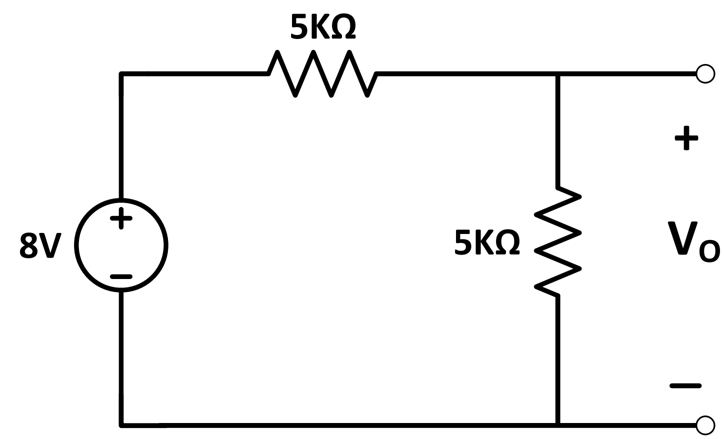
$$V_{Th} = [(2K)(2mA)] + 4 = 8\ V$$

2) To find R_{Th}:



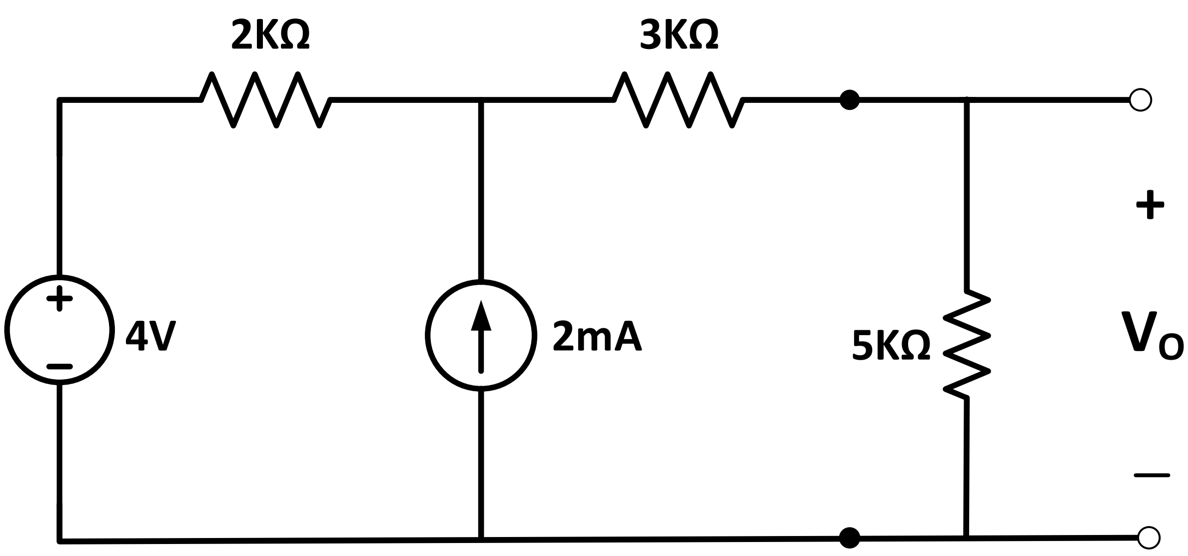
$$R_{Th} = 3K\Omega + 2K\Omega = 5\ K\Omega$$

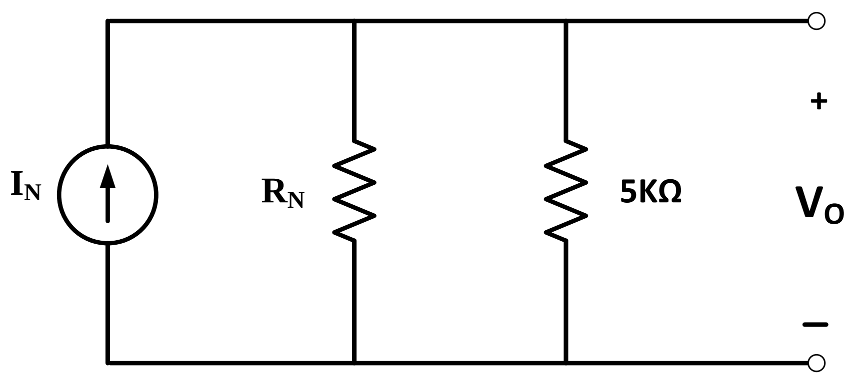
3) To find V_O:



$$V_O = \frac{5}{5+5} * 8 = 4\ V$$

Example: Find Vo using Norton's Theorem

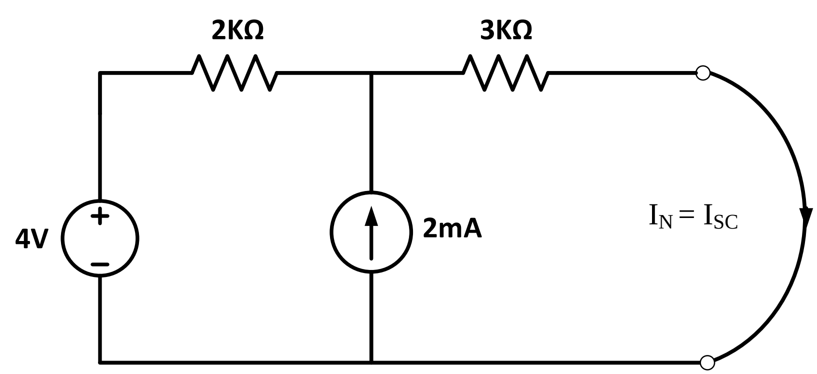


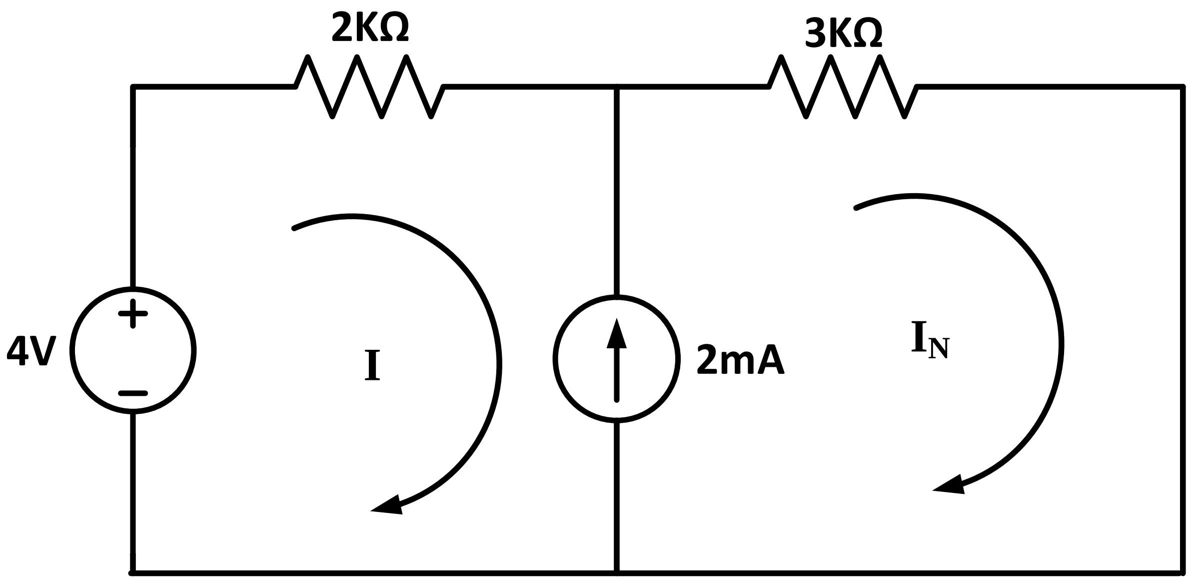


$$V_O = (\ R_N\ //\ 5K\Omega)\ I_N$$

1) To find I_N:

$$I_N = I_{sc}$$





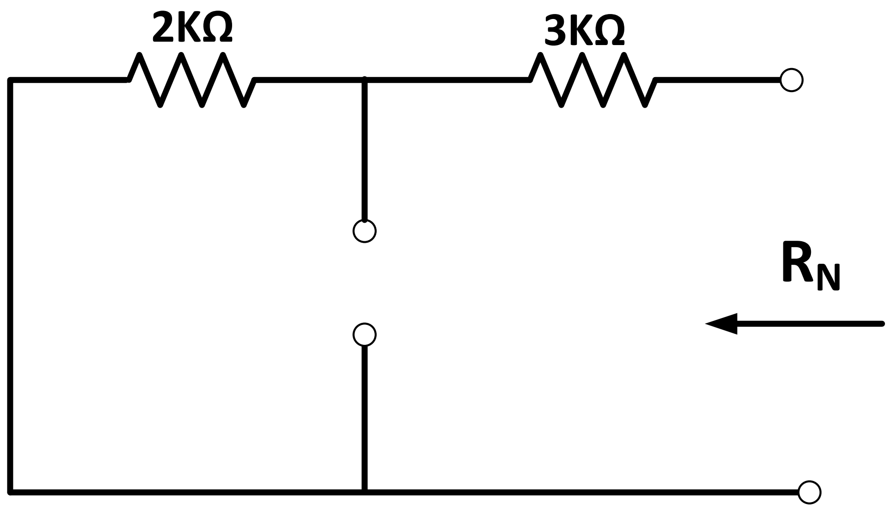
$2mA = I_N - I$ …………….. Constraint equation

$4 = (2K)\, I + (3K)\, I_N$ …………….. Supermesh equation

$\therefore I_N = 1.6\, mA$

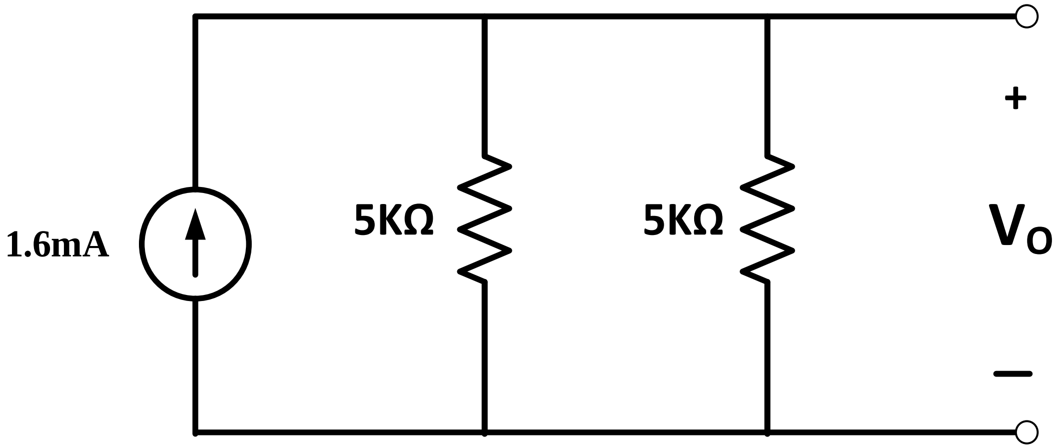
2) To find $R_N = R_{Th}$

Turn off all the independent sources



$$R_N = 3K\Omega + 2K\Omega = 5K\Omega$$

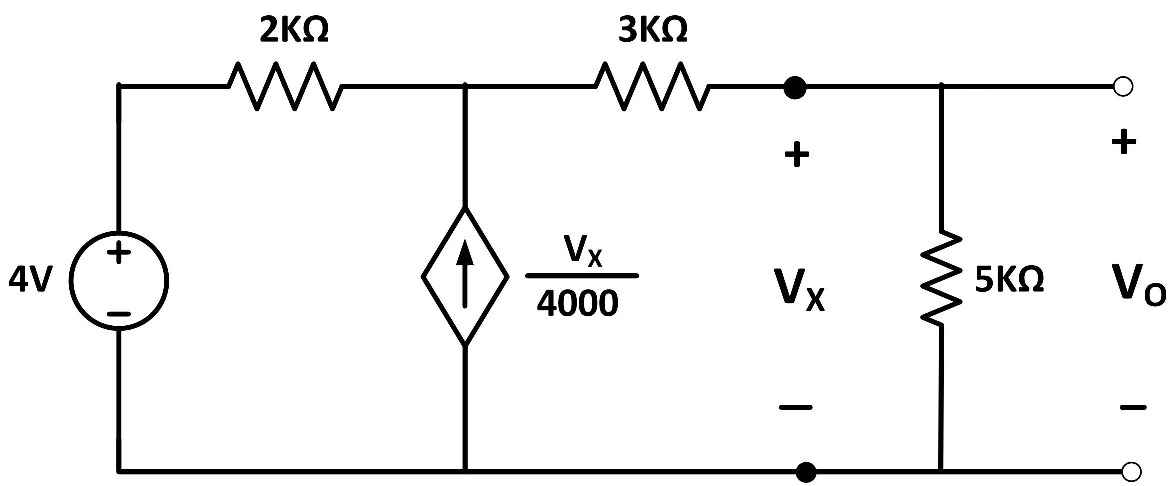
3) To find V_O



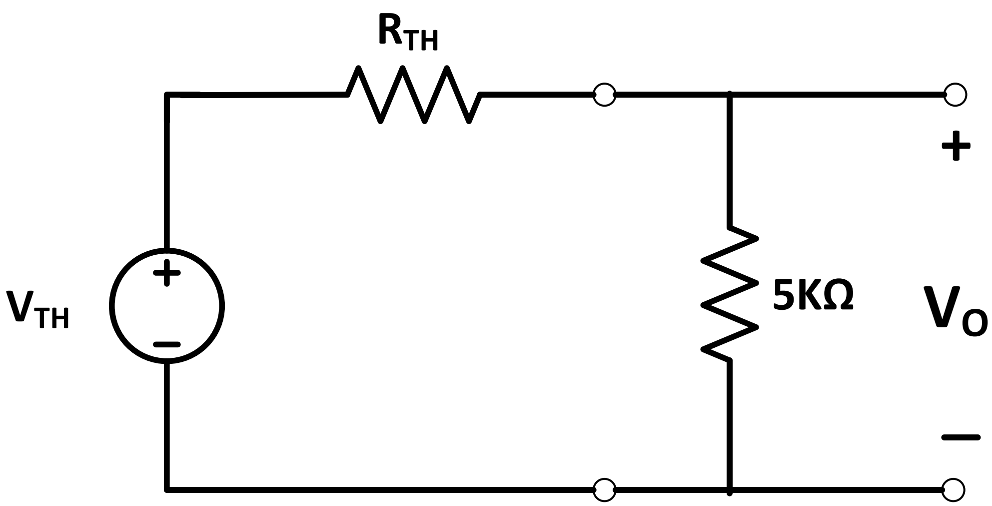
$$V_O = (5K\Omega \,//\, 5K\Omega)\,(1.6\,mA)$$

$$V_O = 4\,V$$

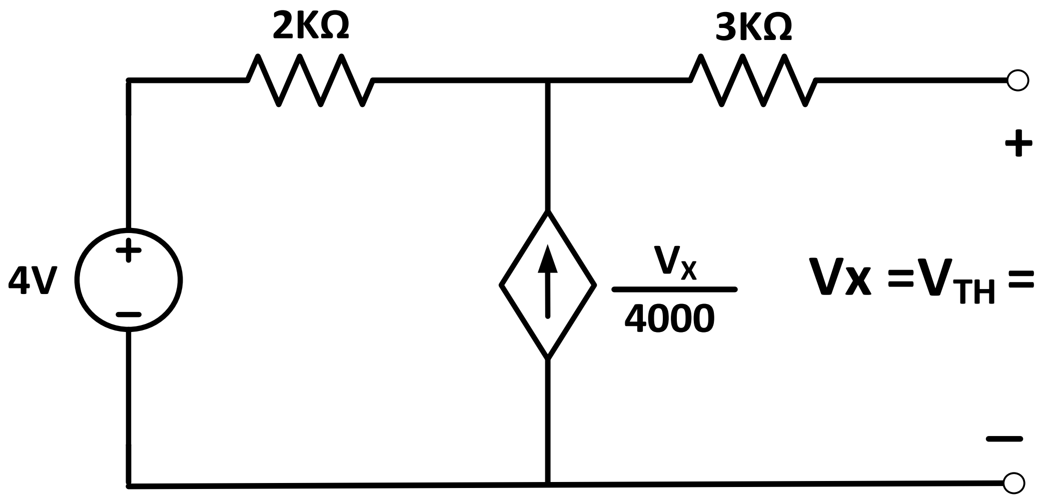
Find V_o using Thevenin's theorem



After finding TEC and putting back the 5kohm load , the circuit will be:



1) To find V_{Th}



Vx = V_{TH} = V_{OC}

$$V_{Th} = (2K)\left(\frac{V_x}{4000}\right) + 4$$

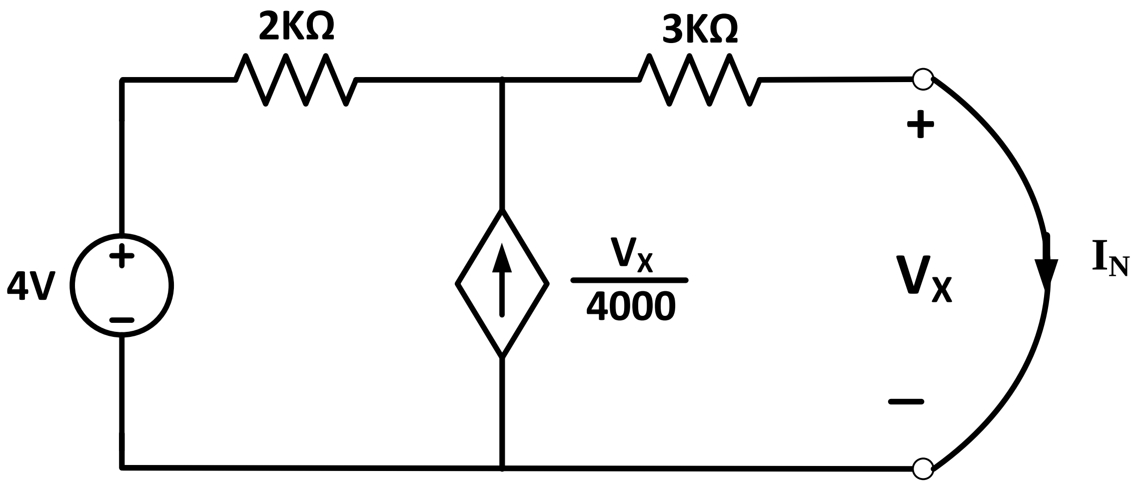
Note that current through the 3 kohm
Is equal to zero (open circuit)

$$V_x = V_{Th}$$

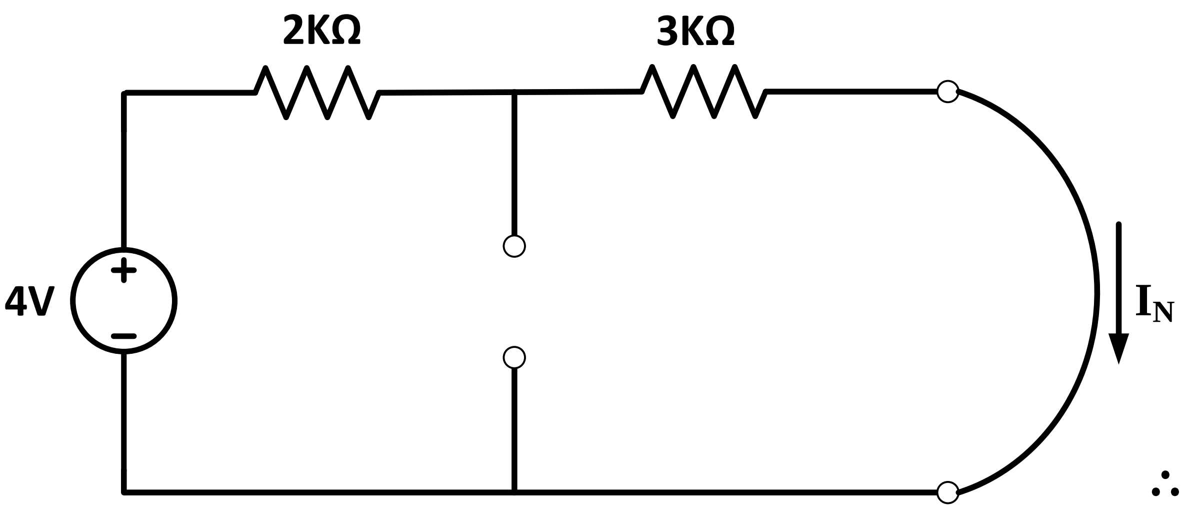
$$\therefore V_{Th} = 8\ V$$

1) To find R_{Th}

a) Method 1: $R_{Th} = \frac{V_{Th}}{I_N}$



$V_x = 0$ ⟹ $\frac{V_x}{4000} = 0$ ⟹ Open circuit.

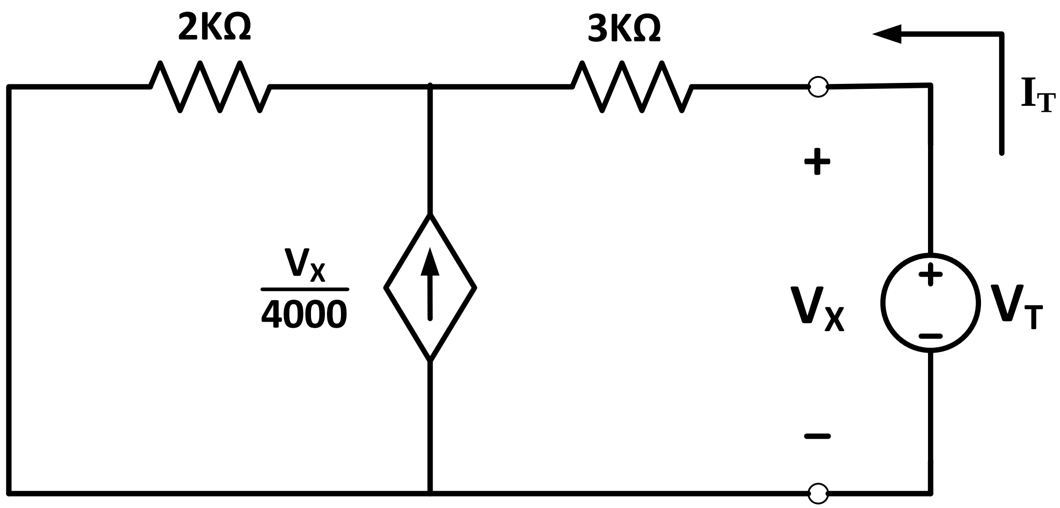


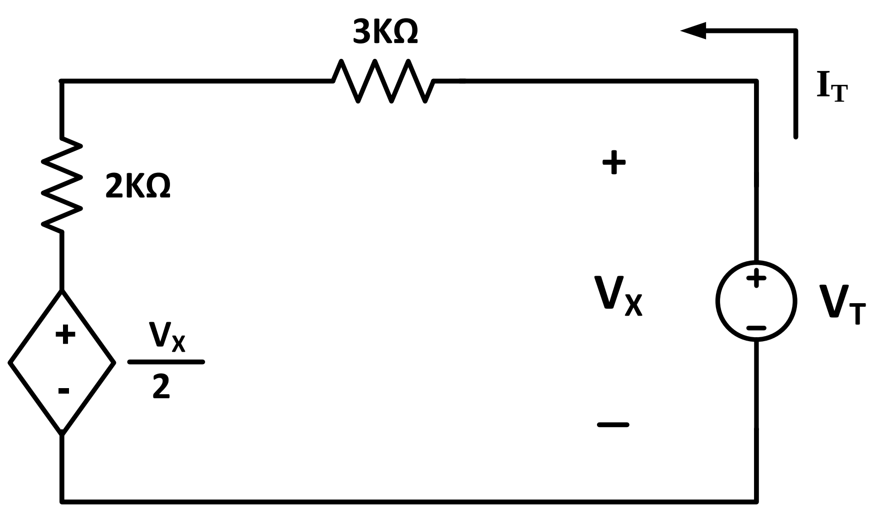
$$I_N = \frac{4\ V}{5\ K\Omega} = 0.8\ mA$$

$$\therefore R_{Th} = \frac{8\ V}{0.8\ mA} = 10\ K\Omega$$

Method 2: *RTh=VT/IT*

while All independent sources set to zero



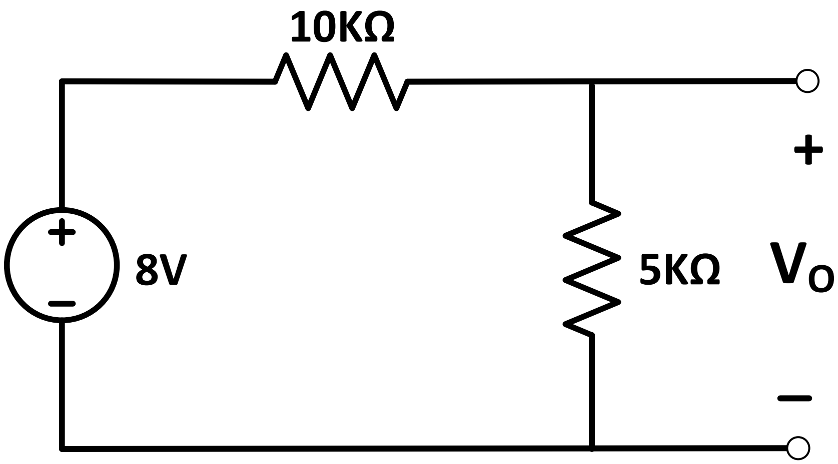


$$-V_T + 3K\ I_T + 2K\ I_T + \frac{V_x}{2} = 0$$

$$V_x = V_T$$

$$\therefore R_{Th} = \frac{V_T}{I_T} = 10\ K\Omega$$

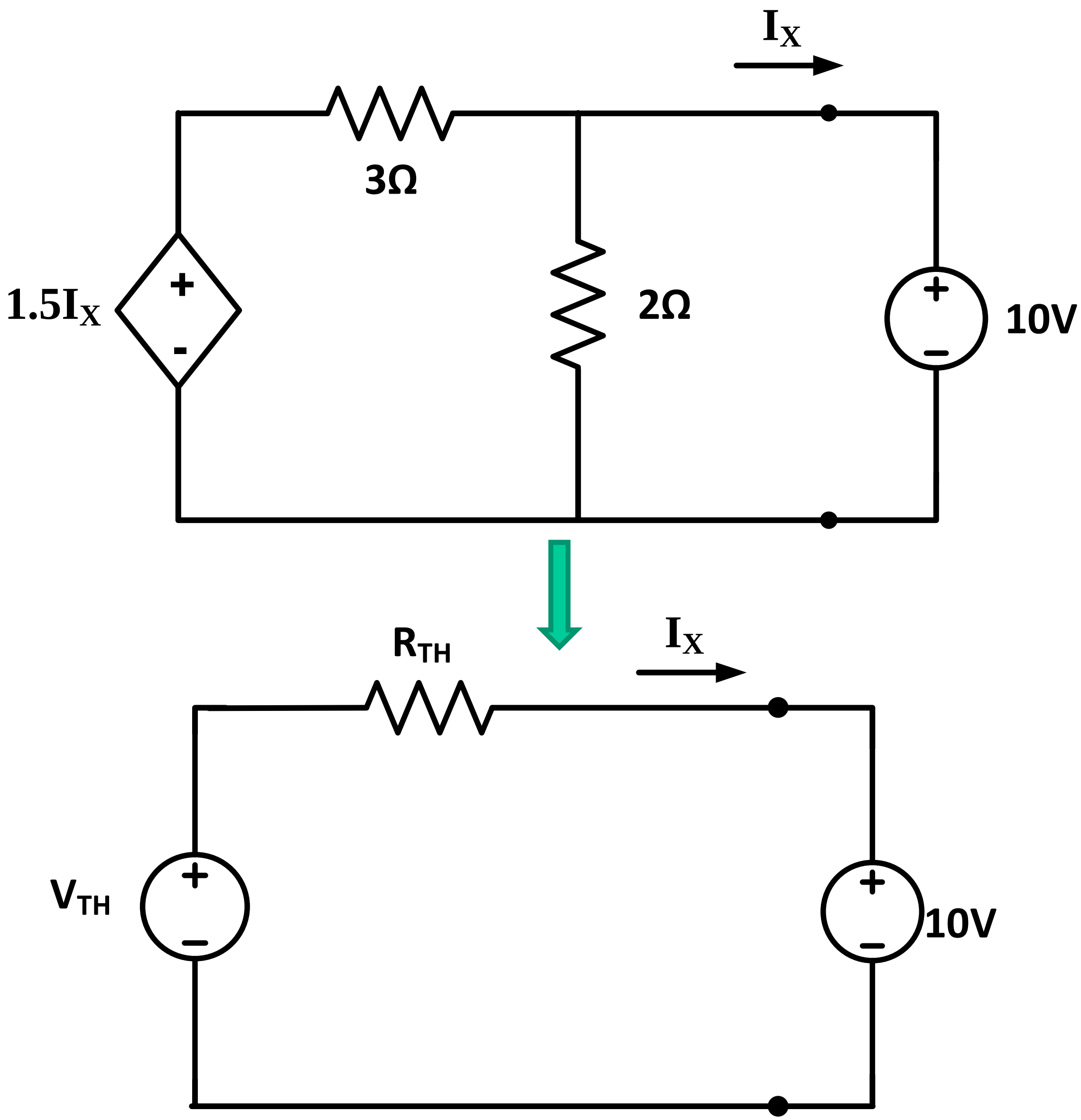
Equivalent Circuit



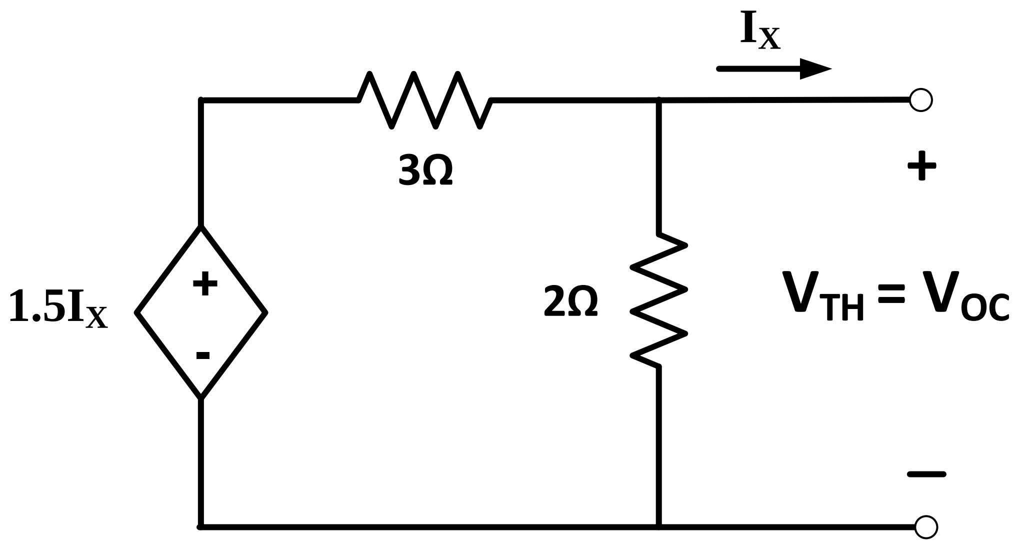
$$V_O = \frac{5K}{5K + 10K}\ (8\ V)$$

$$V_O = \frac{8}{3}\ V$$

Find I_x using Thevenin's theorem

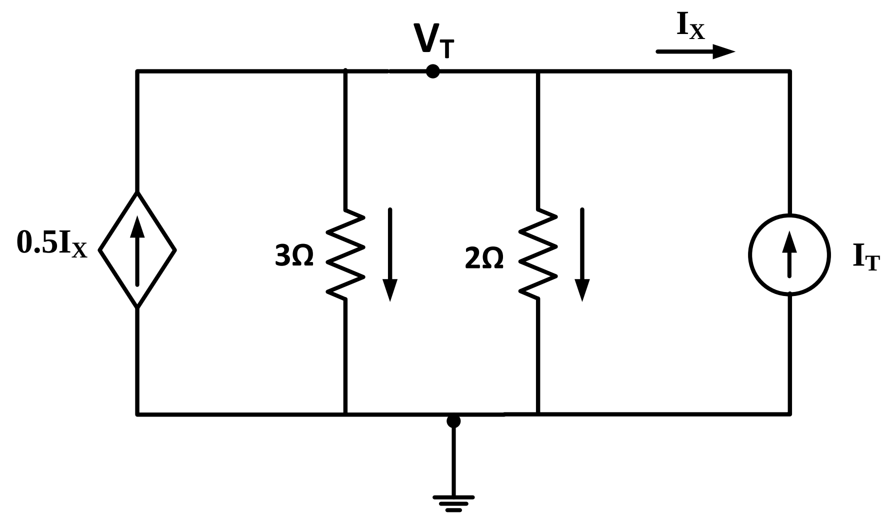


1) To find V_{Th}

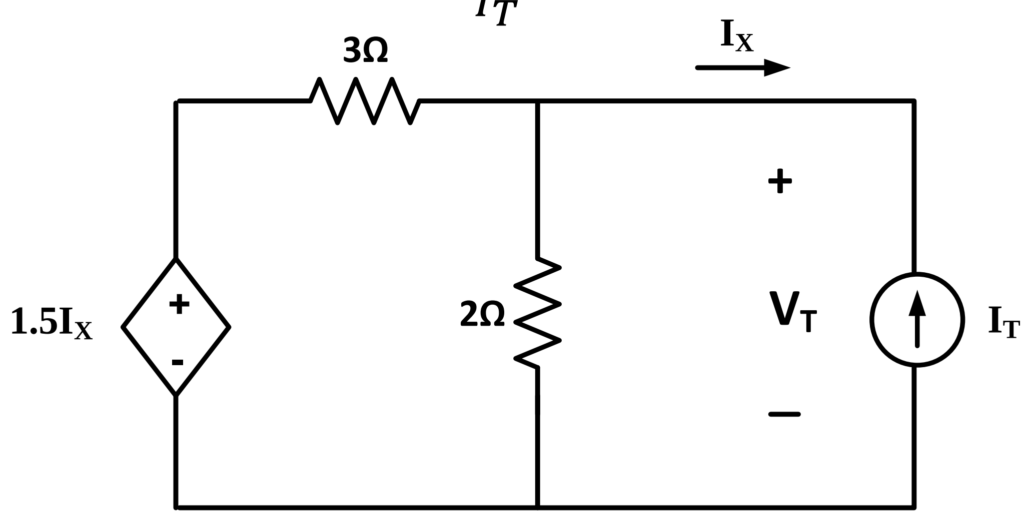


Since there is no independent sources

$$\therefore V_{Th} = 0$$

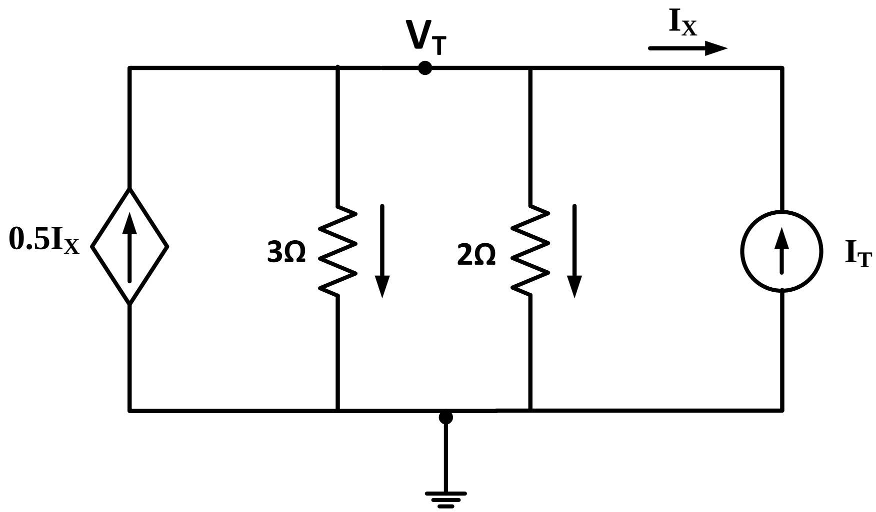


2) To find R_{Th}: $\frac{V_T}{I_T}$



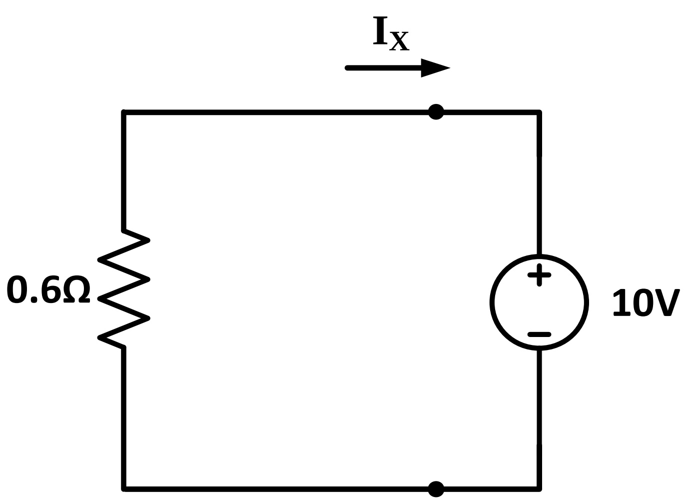
KCL

$$0.5\,I_X + I_T = \frac{V_T}{3} + \frac{V_T}{2}$$



$$I_X = -I_T$$

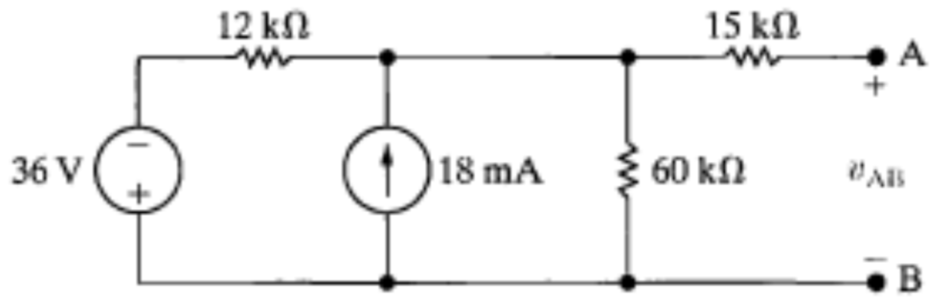
$$\therefore R_{Th} = \frac{V_T}{I_T} = 0.6\,\Omega$$



$$I_X = -\frac{10}{0.6} = -16.67\,A$$

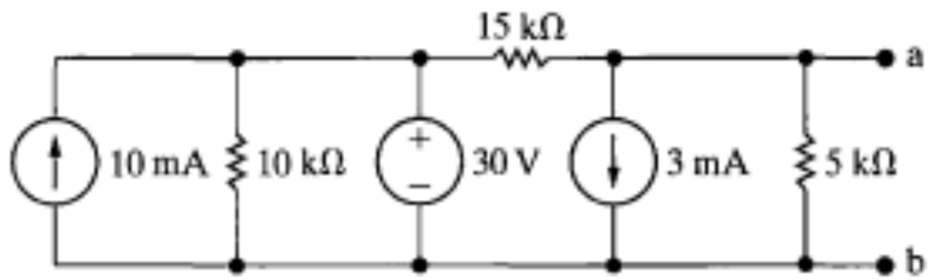
Assessment 4.18

A voltmeter with an internal resistance of 100 kΩ is used to measure the voltage v_{AB} in the circuit shown. What is the voltmeter reading?

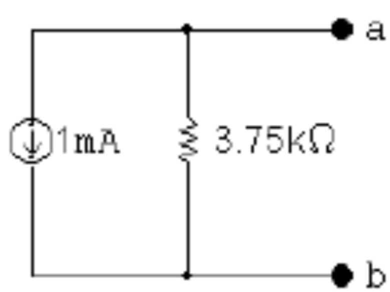


Answer: 120 V.

Problem 4.64
Find the Norton equivalent with respect to the terminals a,b

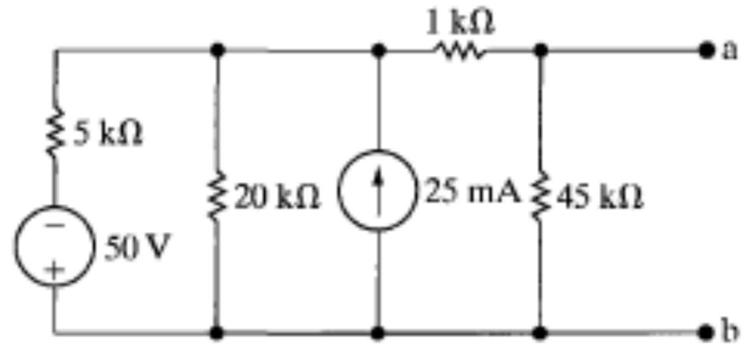


Answer



Problem 4.71

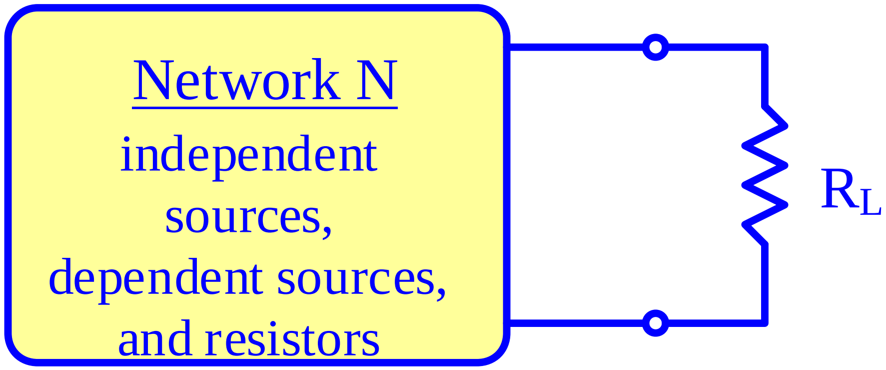
A voltmeter with a resistance of 85.5 *kW* is used to measure the voltage v_{ab} in the circuit. What is the voltmeter reading?



Answer $\therefore \quad v_{\text{Th}} = 54 \text{ V} \qquad R_{\text{Th}} = 4.5 \text{ k}\Omega$

Maximum Power Transfer Theorem

Suppose that a general network N has a resistive load as shown below.



Now we might consider two questions:

- For what value of R_L is maximum power delivered to R_L?
- What is the maximum power that can be delivered to R_L?

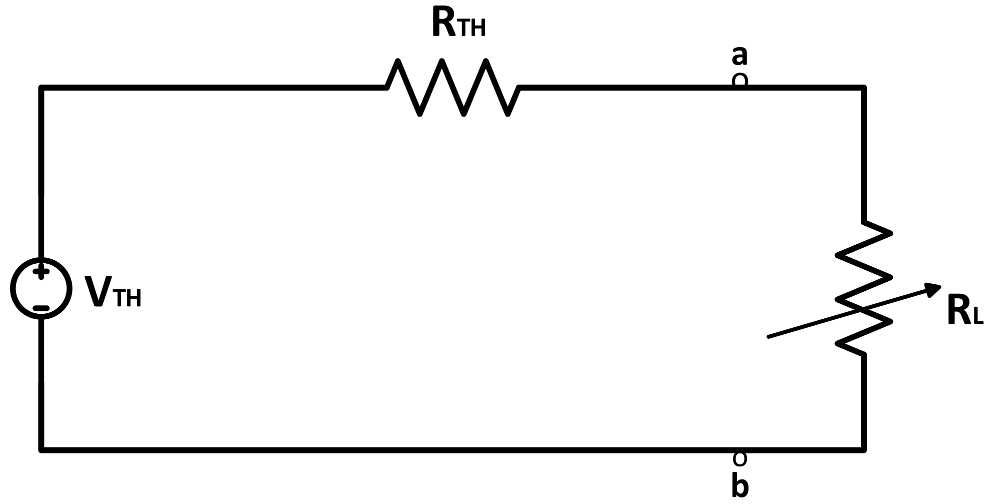
To answer these questions (see next page),

1) Replace N by a Thevenin Equivalent Circuit
2) Determine a general expression for power to R_L
3) Solve $\frac{dP_L}{dR_L} = 0$ to find where P_L is maximum

Maximum Power Transfer Theorem

Show that: $R_L = R_{th}$ for maximum power

$$P_{max} = \frac{V_{th}^2}{4R_{th}}$$



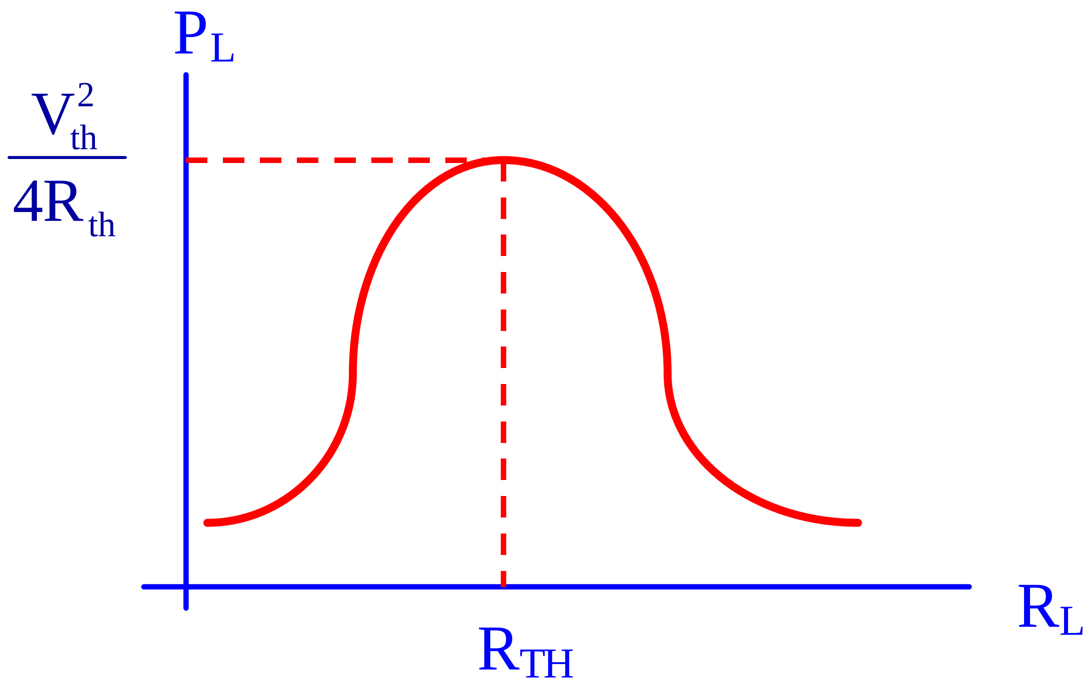
A load resistance will receive maximum power from a circuit when the resistance of the load is exactly the same at the Thevenin's resistance looking back at the circuit .

$$R_L = R_{TH}$$

$$P_L = \frac{V_L^2}{R_L}$$

$$V_L = \frac{R_L}{R_L + R_{TH}} . V_{TH}$$

The relationship between P_L and R_L can be illustrated by the graph shown below.



$$P_L = \frac{V_{TH}^2 \,.\, R_L}{(R_L + R_{TH})^2}$$

$$\frac{\partial P_L}{\partial R_L} = \frac{V_{TH}^2 \left[(R_L+R_{TH})^2 - 2R_L(R_L+R_{TH})\right]}{(R_{TH}+R_L)^4}$$

$$\frac{\partial P_L}{\partial R_L} = 0$$

$$(R_L + R_{TH})^2 - 2R_L(R_L + R_{TH}) = 0$$

$$(R_L + R_{TH})[(R_L + R_{TH}) - 2R_L] = 0$$

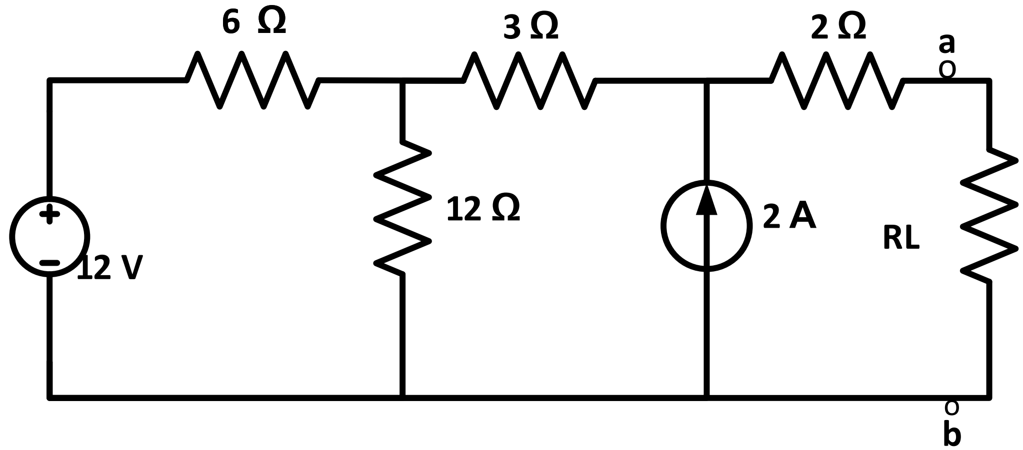
$$\therefore R_{TH} - R_L = 0$$

$$\therefore R_L = R_{TH}$$

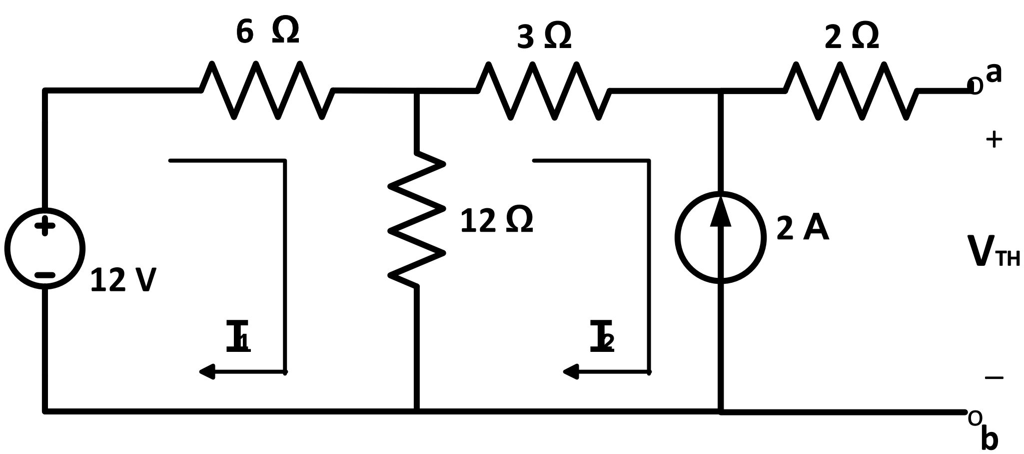
$$P_{L,max} = \frac{V_{TH}^2}{4\,R_L} = \frac{V_{TH}^2}{4\,R_{TH}}$$

Find the value of R_L for maximum power transfer in the circuit shown .

Find the maximum power .



To find V_{TH}



$I_2 = -2\ A$ constrain equation

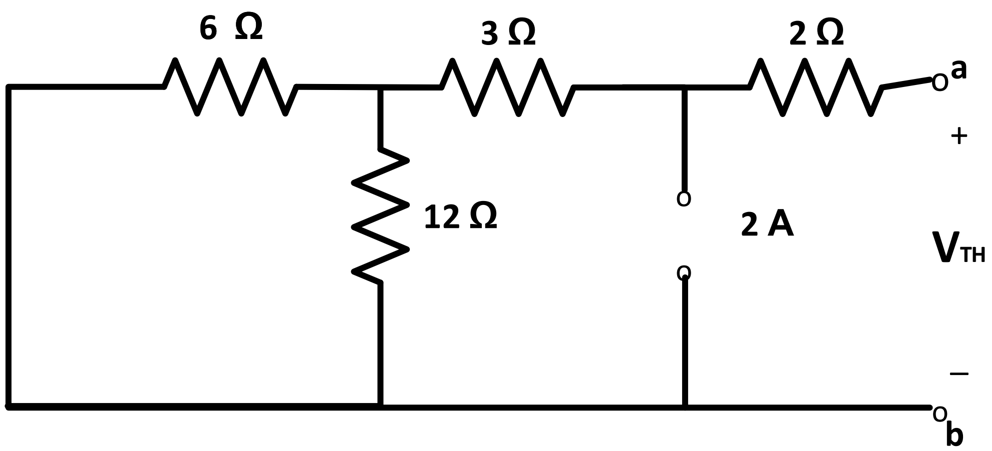
$$12 = 18\ I_1 - 12\ I_2$$

$$\therefore\ I_1 = -\frac{2}{3}\ A$$

$$V_{TH} = -3\ I_2 - 6\ I_1 + 12$$

$$\therefore\ V_{TH} = 22\ V$$

To find R_{TH}



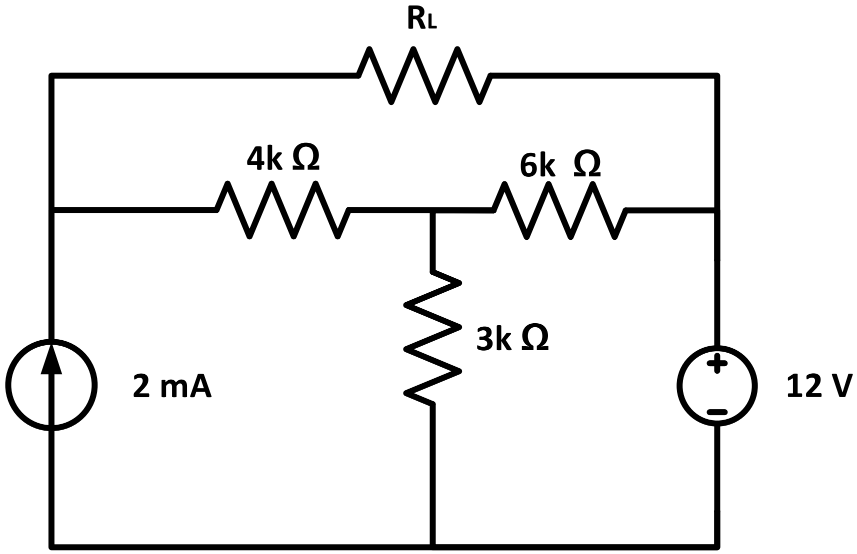
$$R_{TH} = 2 + 3 + 6||12$$
$$= 2 + 3 + 4$$
$$= 9\ \Omega$$

$$\therefore R_L = R_{TH} = 9\ \Omega$$

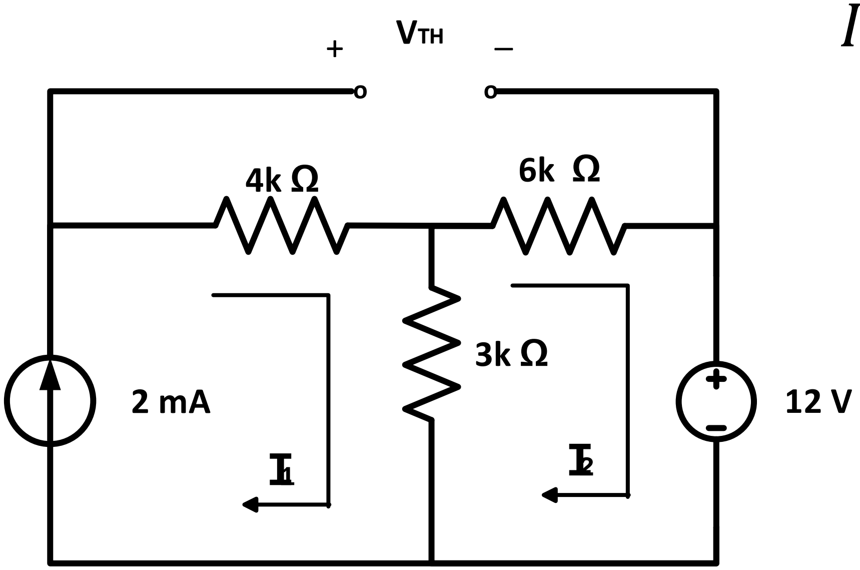
$$\therefore\ P_{L,max} = \frac{V_{TH}^2}{4\ R_{TH}} = 13.44\ W$$

Find the value of R_L for maximum power transfer in the circuit shown .

Find the maximum power .



To find V_{TH}



$I_1 = 2\ mA$ constrain equation

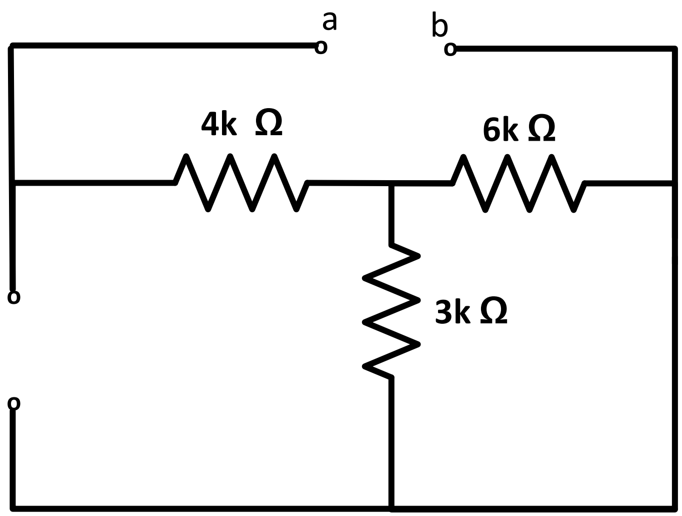
$$-12 = 9k\ I_2 - 3k\ I_1$$

$$\therefore\ I_2 = \frac{1}{3}\ mA$$

$$V_{TH} = 4k\ I_1 + 6k\ I_2$$

$$V_{TH} = 10\ V$$

To find R_{TH}



$$R_{TH} = 4k + 3k||6k$$

$$= 4k + 2k$$

$$= 6k\ \Omega$$

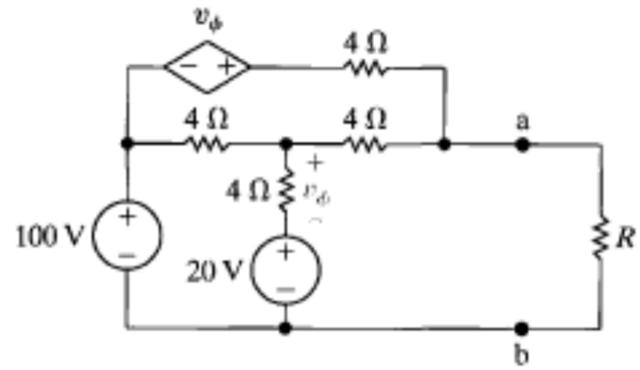
$$\therefore R_L = R_{TH} = 6k\ \Omega$$

$$P_{L,max} = \frac{V_{TH}^2}{4\ R_{TH}} = \frac{25}{6}\ mW$$

Assessment 4.21

a) Find the value of *R* that enables the circuit shown to deliver maximum power to the terminals a,b.

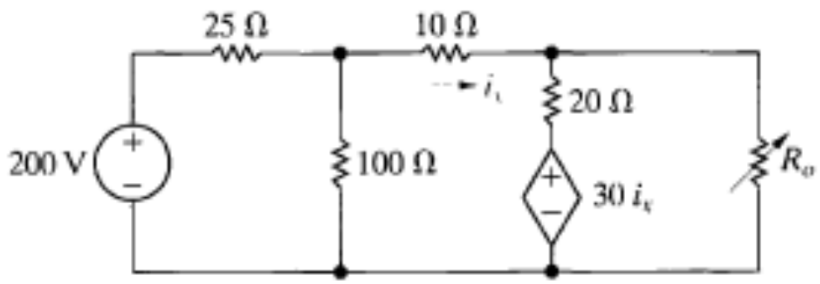
b) Find the maximum power delivered to *R*.



Answer: (a) 30 ;

(b) 1.2 kΩ.

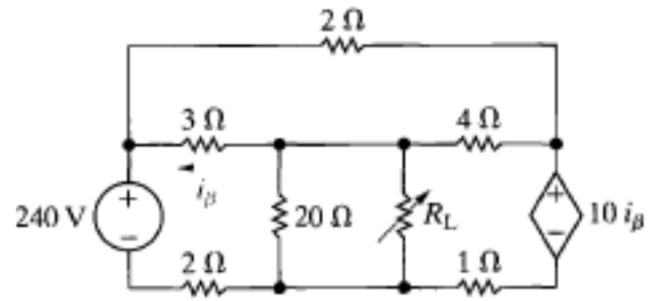
Problem 4.83



The variable resistor *(Ro)* in the circuit adjusted until the power dissipated in the resistor is 250 W. Find the values of R_0 that satisfy this condition.

Answer $R_o = 22.5\,\Omega$ or $R_o = 2.5\,\Omega$

Problem 4.87
The variable resistor *(RL)* in the circuit adjusted for maximum power transfer
a) Find the numerical value of *RL*.
b) Find the maximum power transferred to *RL*.



Answer

$$R_L = R_{Th} = 6\,\Omega$$

$$p_{max} = \frac{12^2}{6} = 24 \text{ W}$$