ENEE2301

Chapter 7
Circuit Analysis with Capacitors and Inductors

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Reading Assignment: Chapter 7 in Electric Circuits, 10th Ed. by Nilsson

Chapter 7 – Circuit Analysis with Capacitors and Inductors

KVL and KCL involving circuits with capacitors and inductors result in differential equations (D.E.) rather than algebraic equations.

The order of a differential equation is equal to highest derivative.

1st-order DE: $\frac{dx}{dt} + a_0 x(t) = f(t)$

2nd-order DE: $\frac{d^2x}{dt^2} + a_1 \frac{dx}{dt} + a_0 x(t) = f(t)$

.

.

nth-order DE: $\frac{d^n x}{dt^n} + a_{n-1} \frac{d^{n-1}x}{dt^{n-1}} + \cdot \cdot \cdot + a_1 \frac{dx}{dt} + a_0 x(t) = f(t)$

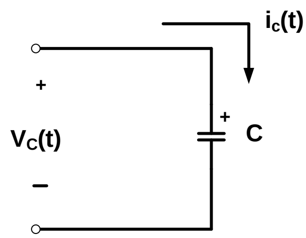
Capacitors and inductors:

- Resistors are passive elements which dissipate energy only.
- Two important passive linear circuit elements: capacitor, and inductor.
- Capacitors and inductors do not dissipate but store energy, which can be retrieved at a later time.
- Capacitors and inductors are called storage elements.

$$W_c(t) = \frac{1}{2} * C\, v_C^2(t)$$

$$W_L(t) = \frac{1}{2} * L\, i_L^2(t)$$

Some of the important characteristics of a capacitor:



$$i_C(t) = C\,\frac{dv_c\,(t)}{dt}$$

$$v_C(t) = v_C(0^-) + \frac{1}{C}\int_0^t i_C(t)dt \quad , \quad for\ t \geq 0$$

1. The current through a capacitor is zero if the voltage across it is not changing with time.

A capacitor is therefore an open circuit to DC.

2. A finite amount of energy can be stored in a capacitor even if the current through the capacitor is zero.

$$W_c(t) = \frac{1}{2} * C\, v_C^2(t)$$

3. The capacitor never dissipate energy, but only store it.

4. $v_C(t) = v_C(0^-) + \frac{1}{C}\int_{0^-}^{t} i_C(t)dt \quad ; \quad for\ t \geq 0.$

$at\ t = 0^+$

$$v_C(0^+) = v_C(0^-) + \frac{1}{C}\int_{0^-}^{0^+} i_C(t)dt$$

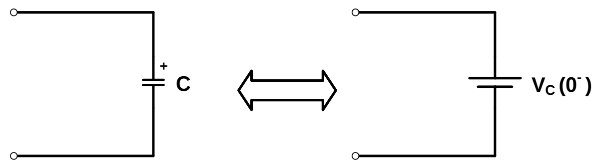
$$v_C(0^+) = v_C(0^-)$$

➔ The Capacitor is a continuous voltage device

➔ It is impossible to change the voltage across a capacitor by a finite amount in zero time, for this requires an infinite current through the capacitor.

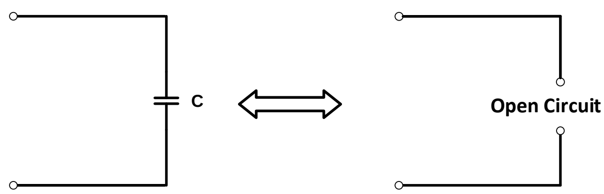
At $t = 0^+$

$i_C(t) = C\,\frac{dv_C(t)}{dt}$

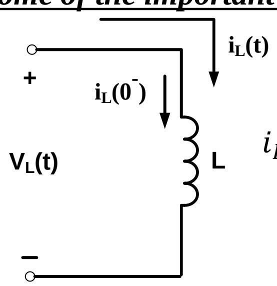


A capacitor is therefore an open circuit to DC.

At *t= 0-, and t= ∞* *(after the change and the circuit have reached steady state)*



Some of the important characteristics of an Inductor:



$$v_L(t) = L\,\frac{di_L(t)}{dt}$$

$$i_L(t) = i_L(0^-) + \frac{1}{L}\int_{0^-}^{t} v_L(t)dt \qquad ;\ for\ t \geq 0.$$

1. There is no voltage across an inductor if the current through it is not changing with time.

   An inductor is therefore a short circuit to DC.

2. A finite amount of energy can be stored in an inductor even if the voltage across inductor is zero.

3. The inductor never dissipate energy, but only store it.

$$4.\ i_L(t) = i_L(0^-) + \frac{1}{L}\int_{0^-}^{t} v_L(t)dt, \qquad for\ t \geq 0$$

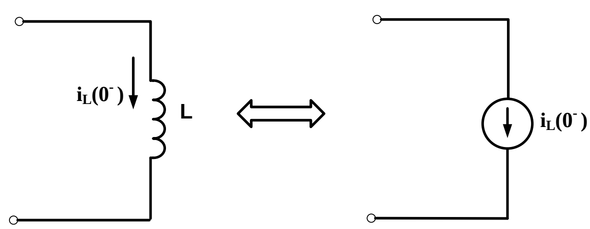
$at\ t = 0^+$

$$i_L(0^+) = i_L(0^-) + \frac{1}{L}\int_{0^-}^{0^+} v_L(t)\,dt$$

$$\therefore\ i_L\ (0^+) = i_L(0^-)$$

It is impossible to change the current through an inductor by a finite amount in zero time, for this requires an infinite voltage across the inductor.

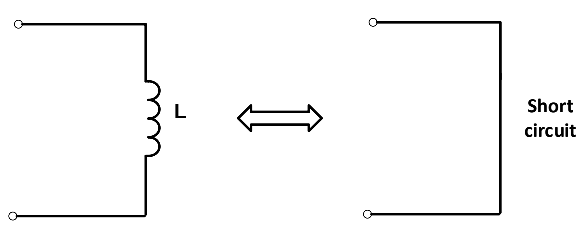
$at\ t = 0^+$



$$v_L(t) = L\,\frac{di_L\ (t)}{dt}$$

An inductor is therefore a short circuit to DC

At $t = 0^-$, and $t = \infty$ *(after the change)*



First order circuits:

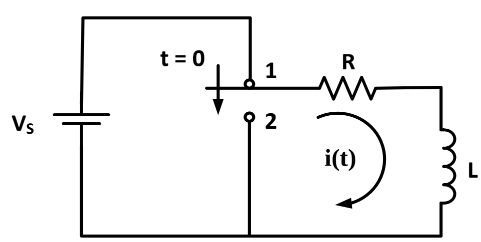
- A first-order circuit can only contain one energy storage element (a capacitor or an inductor) or a combination of capacitors or inductors that can be reduced to one capacitor or inductor.
- The circuit will also contain one or more resistances.
- A first-order circuit is characterized by a first-order differential equation.

Order of a circuit = **Order of the differential equation (DE) required to describe the circuit** = **The number of independent* energy storage elements (C's and L's)**

C's and L's are independent if they cannot be combined with other C's and L's (in series or parallel, for example)

Natural response of first-order circuits:

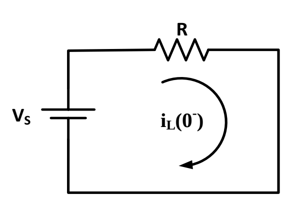
$Find\ i(t)\ \ for\ t > 0$



Solution Steps:

1) $For\ t < 0\ \ ;\quad t = 0^-$

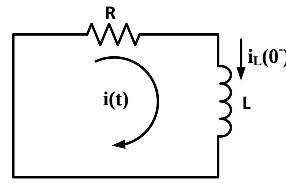
Switch in position 1
and L ==> short



$$i_L(0^-) = \frac{V_S}{R}$$

This is initial condition for current through the inductor

2) $For\ t > 0$



Switch have just switched to position 2
Current through inductor is equal to initial condition since the inductor is a continuous current device

KVL

$$R\,i(t) + L\,\frac{di\,(t)}{dt} = 0$$ Homogeneous first order differential equation

$$i(t) = A\,e^{st} \qquad for\ t > 0$$

$$R\,A\,e^{st} + L\,A\,S\,e^{st} = 0$$

$$A\,e^{st}\,(R + L\,S) = 0 \qquad \therefore\ S = -\frac{R}{L}$$

To find A:==>

==> To find A:

$$i(t) = A\,e^{st} \qquad for\ t > 0 \quad \Rightarrow \quad i(0^+) = A$$

$$i(0^+) = i_L(0^+) = i_L(0^-)$$

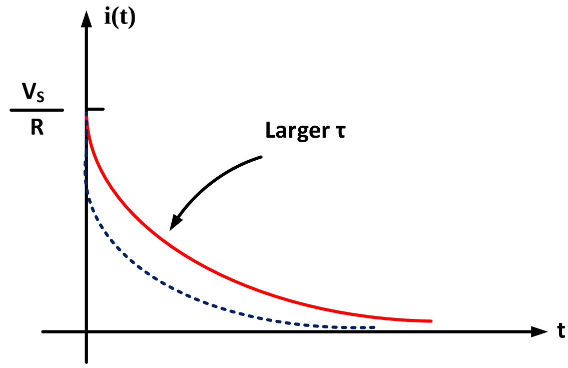
$$\therefore A = i_L(0^-) = \frac{V_S}{R}$$

$$\therefore\ i(t) = \frac{V_S}{R}\,e^{-\frac{R}{L}\,t} \qquad t > 0$$

$$let\ \tau = \frac{L}{R} \qquad \tau = Time\ constant$$

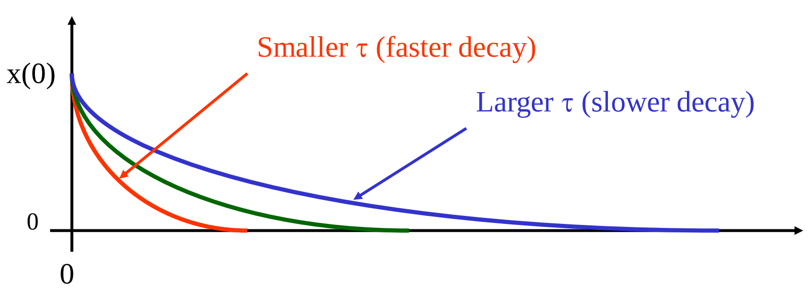
$$\therefore\ i(t) = \frac{V_S}{R}\,e^{-\frac{t}{\tau}} \qquad t > 0$$

t	$i(t)$
0	$\frac{V_S}{R}$
τ	$0.37\ \frac{V_S}{R}$
2τ	$0.14\ \frac{V_S}{R}$
3τ	$0.05\ \frac{V_S}{R}$
4τ	$0.018\ \frac{V_S}{R}$
5τ	$0.0067\ \frac{V_S}{R}$



- τ is related to the rate of exponential decay in a circuit as shown below

$$x(t) = x(0)e^{\frac{-t}{\tau}}$$



- It is typically easier to sketch a response in terms of multiples of τ than to be concerning with scaling of the graph (otherwise choosing an appropriate scale can be difficult).

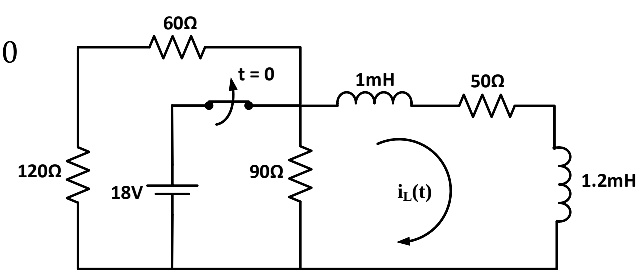
Time for a circuit to completely decay

From the last page note that **$x(5\tau) = x(0)e^{-5\tau/\tau} = x(0)e^{-5} = 0.007x(0)$**

This means that magnitude of x(t) is only 0.7% of its initial value by time 5τ (or the function has lost 99.3% of its original value). Technically a decaying exponential function never reaches zero, but we see that by time t = 5τ it is very close. So we generally use the approximation that:

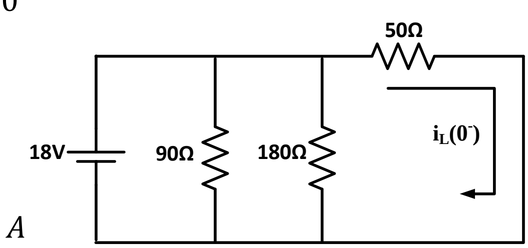
> 5τ = time for a circuit to decay or to reach steady state

$Find\ i_L(t)\ for\ t > 0$



Solution:

1) $For\ t < 0\ ;\quad t = 0^-$



Switch is closed
and L ==> short

$$i_L(0^-) = \frac{18\ V}{50\ \Omega} = 0.36\ A$$

This is initial condition for current through the inductor

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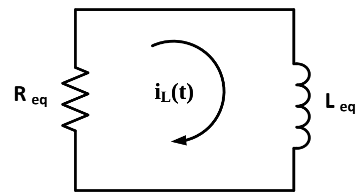
2) $For\ t > 0$

Switch is open



$$R_{eq} = (\,90\Omega\ //\ 180\Omega\,) + 50\Omega\ = 110\Omega$$

$$L_{eq} = 1\,mH + 1.2\,mH = 2.2\,mH$$

$$\tau = \frac{L_{eq}}{R_{eq}} = 20\ \mu s$$

$$i_L(t) = A\,e^{-\frac{t}{\tau}} \qquad for\ t > 0$$

$$i_L(t) = 0.36\,e^{-50{,}000\,t} \quad for\ t > 0$$

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Equivalent Resistance seen by an Inductor

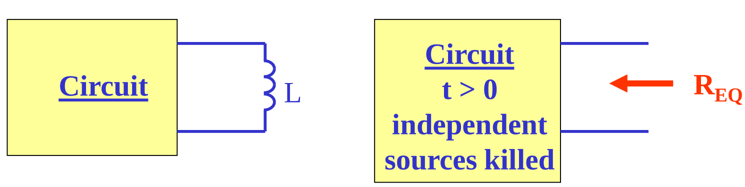
For the RL circuit in the previous example, it was determined that τ = L/R. As with the RC circuit, the value of R should actually be the equivalent (or Thevenin) resistance seen by the inductor.

In general, a first-order RL circuit has the following time constant:

$$\tau = \frac{L}{R_{EQ}}$$

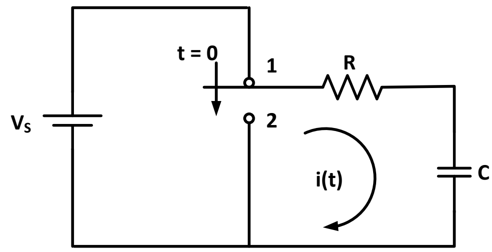
where

$$R_{EQ} = R\Big|_{\text{seen from the terminals of the inductor for } t>0 \text{ with independent sources killed}}$$



RC Circuits

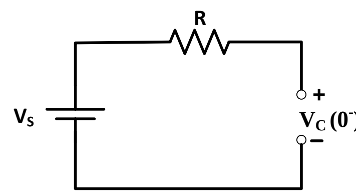
Find $i(t) \quad for\ t > 0$



1) For $t < 0 \quad ;\ t = 0^{-}$

Switch in position 1 and capacitor is open (DC or steady state)

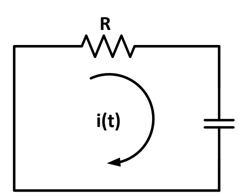
$$v_C(0^{-}) = V_S$$



Initial condition for voltage across the capacitor
(note cap is continuous voltage device, i.e voltage cannot change instantaneously)

2) For $t > 0$

Switch in position 2, cap acts as voltage source



$$R\ i(t) + v_C(0^{-}) + \frac{1}{C}\int_{0^{-}}^{t} i(t)dt = 0 \quad , \qquad t > 0$$

$$R\ \frac{di(t)}{dt} + \frac{1}{C}\ i(t) = 0$$

Homogeneous first order differential equation

$$\therefore\ i(t) = Ae^{st} \qquad t > 0 \quad \Rightarrow \quad R\ A\ S\ e^{st} + \frac{1}{C}\ A\ e^{st} = 0$$

$$A\ e^{st}\left(RS + \frac{1}{C}\right) = 0 \quad \Rightarrow \quad \therefore\ S = -\frac{1}{RC}$$

$$\therefore\ i(t) = A\, e^{-\frac{1}{RC}t} \qquad t > 0$$

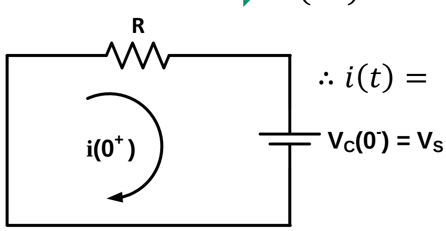
$$let\ \tau = RC = time\ constant$$

$$\therefore\ i(t) = A\, e^{-\frac{t}{\tau}} \qquad t > 0$$

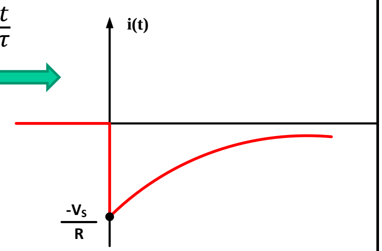
To find A: $i(t) = A\, e^{-\frac{t}{\tau}} \qquad t > 0 \;\Rightarrow\; i(0^+) = A$

To find $i(0^+)$

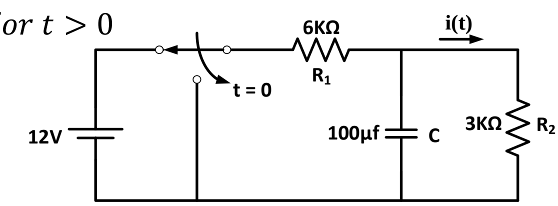
at $t = 0^+ \;\Rightarrow\; i(0^+) = -\dfrac{v_C(0^-)}{R} = -\dfrac{V_S}{R}$



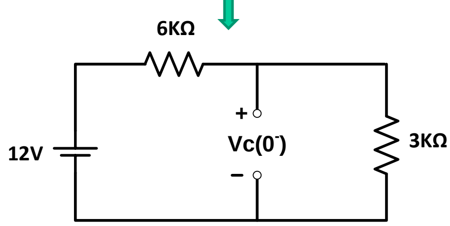
$$\therefore\ i(t) = -\frac{V}{R}\, e^{-\frac{t}{\tau}}$$



Calculate $v_C(t)$ and $i(t)$ for $t > 0$

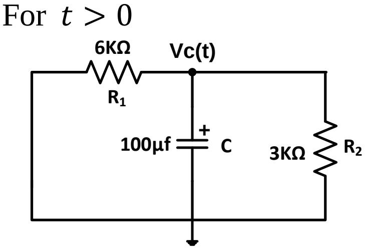


1) For $t < 0 \quad ; \; t = 0^-$



$$v_C(0^-) = \frac{3K}{3K + 6K}\, 12\,V = 4\,V$$

2) For $t > 0$



KCL: $\dfrac{v_C(t)}{6K} + \dfrac{v_C(t)}{3K} + C\,\dfrac{dv_C(t)}{dt} = 0 \;\Rightarrow\; \dfrac{dv_C(t)}{dt} + 5\,v_C(t) = 0$

$$\therefore\ v_C\,(t) = A\, e^{-\frac{t}{\tau}} \qquad t > 0$$

$$\tau = R_{eq}\, C$$

$$R_{eq} = 6K\Omega \;//\; 3K\Omega \;= 2K\Omega$$

$$C = 100\ \mu\text{F}$$

$$\tau = R_{eq}\, C = 0.2\ s$$

$$v_C(t) = A\, e^{-\frac{t}{0.2}} \quad t > 0$$

To find A:

$$v_C(0^+) = A = v_C(0^-) = 4\, V$$

$$\therefore v_C\,(t) = 4\, e^{-5t} \qquad t > 0$$

$$i(t) = \frac{v_C(t)}{R_2}$$

$$i(t) = \frac{4}{3}\, e^{-5t}\ mA \qquad t > 0$$

Equivalent Resistance seen by a Capacitor

For the RC circuit in the previous example, it was determined that $\tau = RC$. But what value of R should be used in circuits with multiple resistors?

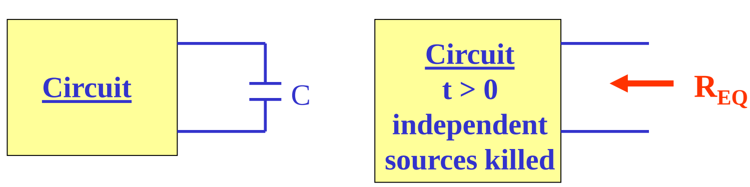
In general, a first-order RC circuit has the following time constant:

$$\tau = R_{EQ}\, C$$

where R_{eq} is the Thevenin resistance seen by the capacitor.

More specifically,

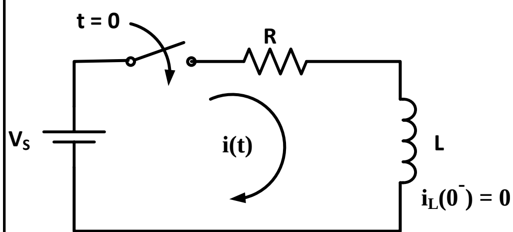
$$R_{EQ} = R\Big|_{\text{seen from the terminals of the capacitor for } t > 0 \text{ with independent sources killed}}$$



The step response of RC and RL circuits:

The response of a circuit to the sudden application of a constant voltage or current source is referred to as the step response of the circuit.

The step response of an RL circuit



Find it for t>0

KVL:

VS=R i(t)+L di(t)dt t>0

Nonhomogeneous first order differential equation

$$\therefore \quad i(t) = i_n(t) + i_f(t)$$

$$i_n(t) \equiv \quad natural\ response$$

$$i_f(t) \equiv \quad forced\ response$$

To find $i_f(t)$

Let $i_f(t) = k$

$$V_S = R\, i(t) + L\, \frac{di(t)}{dt}$$

$$V_S = R\, K + L\ (0)$$

$$V_S = R\, K$$

$$\therefore\ K = \frac{V_S}{R} = i_f(t)$$

Now

$$i(t) = i_n(t) + i_f(t) \qquad t > 0$$

$$i(t) = A\, e^{-\frac{t}{\tau}} + K \qquad t > 0$$

$$\tau = \frac{L}{R}$$

$$i(t) = A\, e^{-\frac{t}{\tau}} + \frac{V_S}{R} \qquad t > 0$$

To find A

$$i(t) = \frac{V_S}{R} + A\, e^{-\frac{t}{\tau}} \qquad for\ t > 0$$

$$i(0^+) = \frac{V_S}{R} + A$$

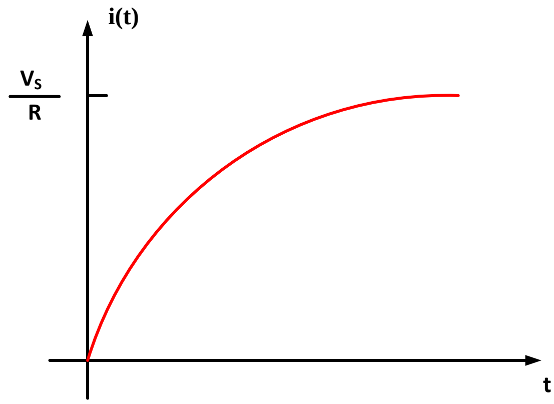
$$But \quad i(0^+) = i_L(0^+) = i_L(0^-) = 0$$

$$\therefore A = -\frac{V_S}{R}$$

$$\therefore\ i(t) = \frac{V_S}{R} - \frac{V_S}{R}\, e^{-\frac{t}{\tau}} \qquad t \ge 0$$

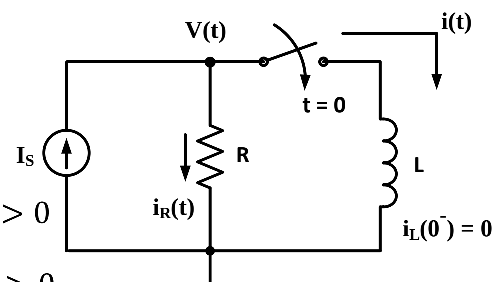
$$i(t) = \frac{V_S}{R}\left(1 - e^{-\frac{t}{\tau}}\right) \qquad t \geq 0$$

t	$i(t)$
0	0
τ	$0.63\ \frac{V_S}{R}$
2τ	$0.86\ \frac{V_S}{R}$
3τ	$0.95\ \frac{V_S}{R}$
4τ	$0.98\ \frac{V_S}{R}$
5τ	$0.99326\ \frac{V_S}{R}$



$$\tau = \frac{L}{R}$$

Find $i(t)$ for $t > 0$



$for\ t > 0$

KCL:

$$I_S = i_R(t) + i(t) \qquad t > 0$$

$$I_S = \frac{v(t)}{R} + i(t) \qquad t > 0$$

$$v(t) = v_R(t) = v_L(t) = L\,\frac{di(t)}{dt}$$

$$I_S = \frac{L}{R}\frac{di(t)}{dt} + i(t) \qquad t > 0$$

$$\therefore\ i(t) = i_n(t) + i_f(t)$$

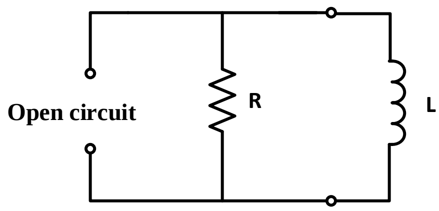
$$let\ i_f(t) = K$$

$$I_s = \frac{L}{R} * \ 0 + K$$

$$\therefore K = I_S = i_f(t)$$

$$\tau = \frac{L}{R_{eq}}$$

R_{eq} = *the thevenin resistor seen by the inductor*



$\therefore R_{eq} = R$

$i(t) = K + A\,e^{-\frac{t}{\tau}} \qquad ; t > 0$

$i(t) = I_S + A\,e^{-\frac{t}{\tau}} \qquad ; t > 0$

To find A

$i(0^+) = i_L(0^+) = i_L(0^-) = 0$

$i(0^+) = I_S + A = i_L(0^-) = 0$

$\therefore A = -I_S$

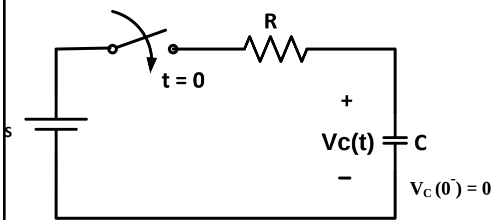
$\therefore i(t) = i_n(t) + i_f(t) \qquad ; t > 0$

$i(t) = A\,e^{-\frac{t}{\tau}} + I_S \qquad ; t > 0$

$i(t) = -I_S\,e^{-\frac{t}{\tau}} + I_S \qquad ; t > 0$

$\therefore i(t) = I_S\left(1 - e^{-\frac{t}{\tau}}\right) \qquad ; t \geq 0$

<u>The step response of an RC circuit:</u>

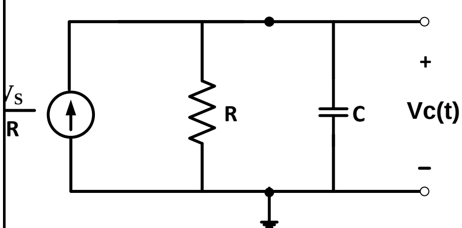


$Find\ v_C(t) for\ t > 0$

KCL:

$$\frac{V_S}{R} = i_R(t)\ + i_C(t) \qquad t > 0$$

<u>$for\ t > 0$</u>



$$\frac{V_S}{R} = \frac{v_C(t)}{R} + C\ \frac{dv_C(t)}{dt} \qquad t > 0$$

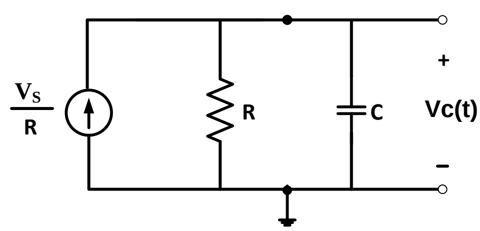
$$\frac{V_S}{R} = \frac{v_C(t)}{R} + C\ \frac{dv_C(t)}{dt}$$

First order nonhomogeneous differential equation

$$\therefore v_C(t) = v_{cn}(t) + v_{cf}(t) \qquad t > 0$$

$$v_C(t) = A\ e^{-\frac{t}{\tau}}\ +\ K$$

<u>To find K</u>



$$\frac{V_S}{R} = \frac{K}{R}\ +\ 0$$

$$\therefore K\ = V_S$$

$$\therefore v_C(t) = A\ e^{-\frac{t}{\tau}}\ +\ V_S \qquad t > 0$$

$$\tau\ = R_{eq}\ C$$

$$R_{eq}\ = R$$

To find A

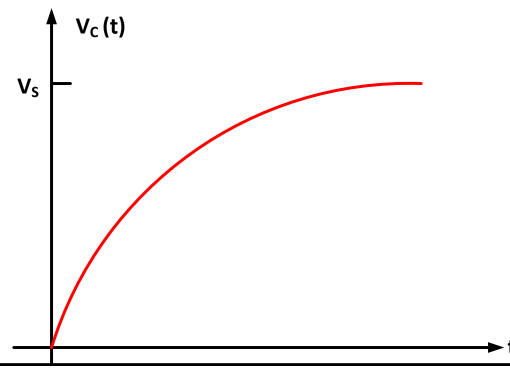
$$v_C(0^+) = V_C(0^-) = 0$$

$$v_C(0^+) = K + A = V_C(0^-) = 0$$

$$\therefore A = -K = -V_S$$

$$\therefore v_C(t) = V_S - V_S\, e^{-\frac{t}{\tau}} \qquad t \geq 0$$

$$v_C(t) = V_S\left(1 - e^{-\frac{t}{\tau}}\right) \qquad t \geq 0$$



General Solution

When all the independent sources are constant, the response of the first order circuit has the form

$$r(t) = r_n(t) + K \qquad t > 0$$

$$r(t) = A\, e^{-\frac{t}{\tau}} + K \qquad t > 0$$

$$r(\infty) = K$$

$$\therefore r(t) = A\, e^{-\frac{t}{\tau}} + r(\infty) \qquad t > 0$$

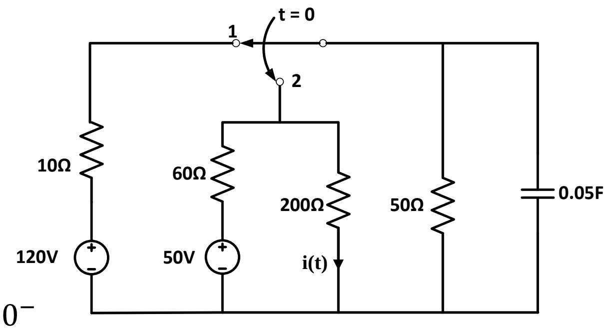
$$r(0^+) = A + r(\infty)$$

$$\therefore A = r(0^+) - r(\infty)$$

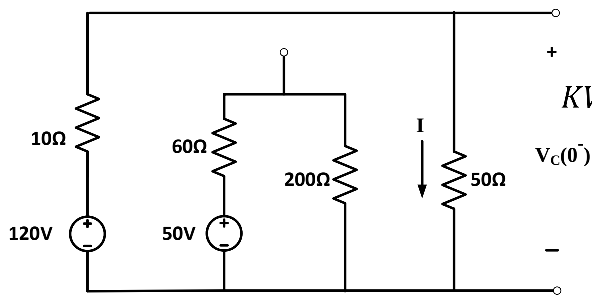
$$\therefore r(t) = r(\infty) + [r(0^+) - r(\infty)]\, e^{-\frac{t}{\tau}} \qquad t > 0$$

$$\tau = \frac{L}{R_{eq}} \quad or \quad \tau = R_{eq}C$$

Find $i(t) for\ t > 0$



$1\ ) For\ t < 0 \quad ;\ t\ = 0^-$

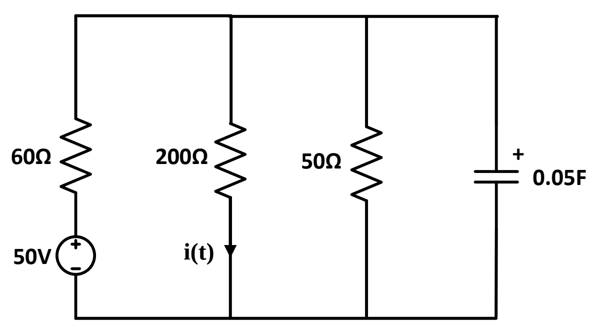


$$KVL: -120 + 10I + 50I = 0$$

$$\therefore\ I = 2A$$

$$v_C(0^-) = 50\ I = 100\ V$$

For $t > 0$

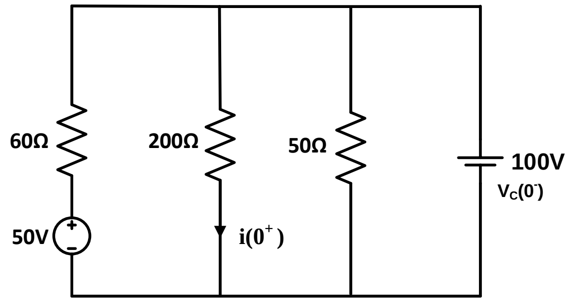


$$r(t) = r(\infty) + [r(0^+) - r(\infty)]\, e^{-\frac{t}{\tau}} \qquad t > 0$$

$$i(t) = i(\infty) + [i(0^+) - i(\infty)]\, e^{-\frac{t}{\tau}} \qquad t > 0$$

$\underline{To\ find\ i(0^+)}$

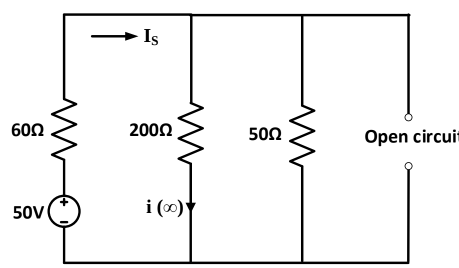
$at\ t = 0^+$



$$i(0^+) = \frac{v_C(0^-)}{200} = \frac{100}{200} = 0.5\ A$$

$\underline{To\ find\ i(\infty)}$

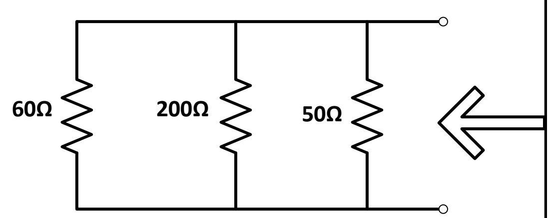
$at\ t = \infty$



$$i(\infty) = \frac{50}{50 + 200} I_S$$

$$I_S = \frac{50}{50\Omega // 200\Omega + 60\Omega} = 0.5\ A$$

$\therefore i(\infty) = 0.1\ A$



To find τ

$for\ t > 0$

$\tau = R_{eq}\ C$

$R_{eq} = R_{Th}\ \ seen\ by\ the\ capacitor$

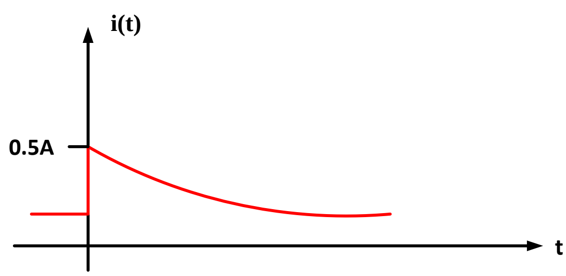
$$R_{Th} = 60\Omega \,//\, 200\Omega \,//\, 50\Omega$$

$$R_{Th} = 24\ \Omega$$

$$\tau = (0.5)(24) = 1.2\ s$$

$$\therefore\ \ i(t) = \left(0.1 + 0.4\, e^{-\frac{t}{1.2}}\right) A \qquad ;\ t > 0$$

$$i(t) = \left(0.1 + 0.4\, e^{-\frac{t}{1.2}}\right) A \qquad ;\ t > 0$$

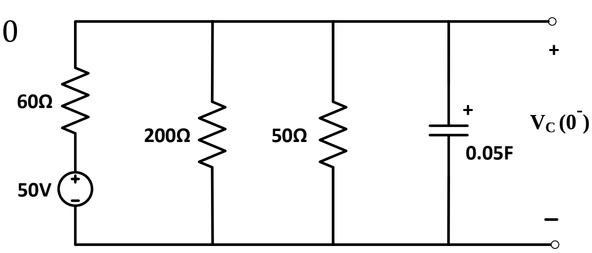


$$i(0^-) = 0.192\ A$$

$$i(0^+) = 0.5\ A$$

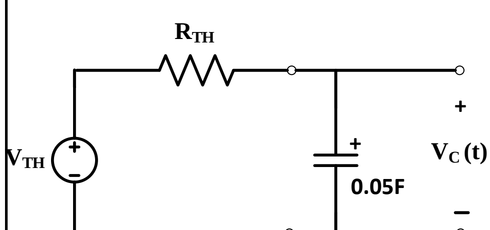
$$i(\infty) = 0.1\ A$$

To find $v_C(t)\ for\ t > 0$



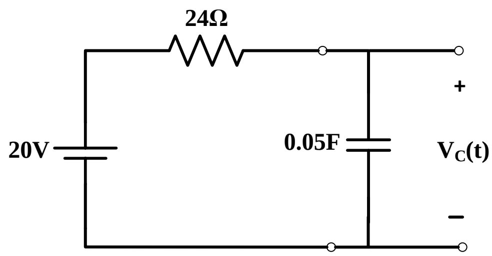
$for\ t > 0$

The circuit can be simplified to



$$R_{Th} = 60\ \Omega \,//\, 200\ \Omega \,//\, 50\ \Omega$$
$$= 24\ \Omega$$

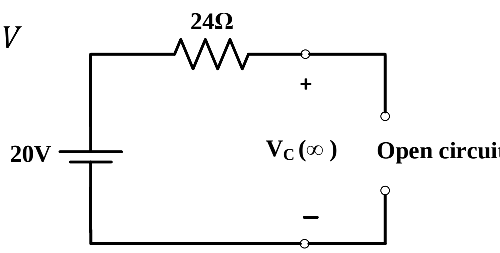
$$V_{Th} = \frac{50\ \Omega \,//\, 200\ \Omega}{50\ \Omega \,//\, 200\ \Omega\ + 60\ \Omega} \,.\, 50$$
$$=\ 20\ V$$



$$v_C(t) = v_C(\infty) + [v_C(0^+) - v_C(\infty)]\, e^{-\frac{t}{\tau}} \qquad t > 0$$

1) $v_C(0^+) = v_C(0^-) = 100\ V$

2) $To\ find\ v_C(\infty)$



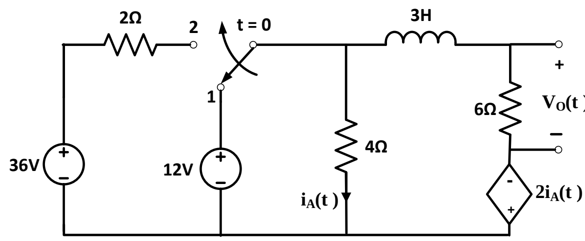
$$\therefore\ v_C(\infty) = 20\ V$$

3) $\tau = R_{Th}\, C = 1.2\ s$

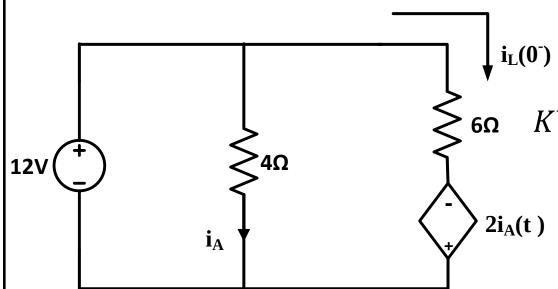
$$v_C(t) = 20 + (100 - 20)\, e^{-\frac{t}{1.2}}\ V \qquad t \geq 0$$

$$v_C(t) = \left(20 + 80\, e^{-\frac{t}{1.2}}\right)\ V \qquad t \geq 0$$

$Find\ v_O(t)\ for\ t > 0$



$For\ t < 0 \quad ;\ t = 0^-$



$$i_A = \frac{12}{4} = 3A$$

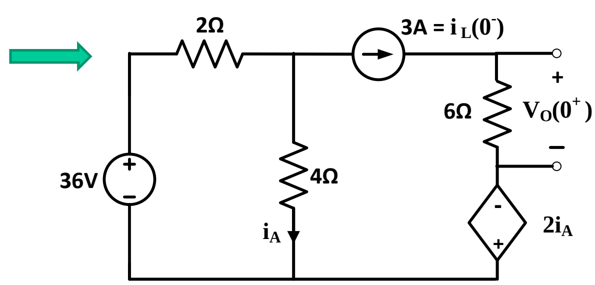
$KVL:$

$$-12 + 6\,i_L(0^-) - 2i_A = 0$$

$$\therefore\ i_L(0^-) = 3A$$

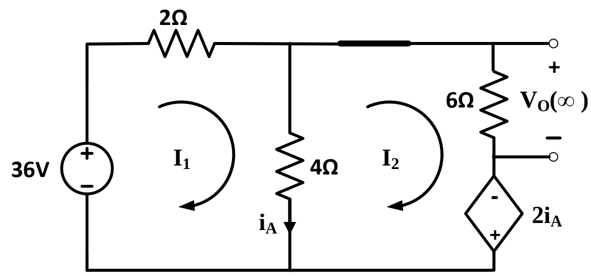
$$v_O(t) = v_O(\infty) + [v_O(0^+) - v_O(\infty)\,]\,e^{-\frac{t}{\tau}} \qquad t > 0$$

$To\ find\ v_O(0^+)$



$$v_O(0^+) = (3A)(6\Omega) = 18\,V$$

$To\ find\ v_O(\infty)$



$KVL\ for\ mesh\ 1:$

$$36 = 6I_1 - 4I_2$$

$KVL\ for\ mesh\ 2:$

$$2i_A = -4I_1 + 10I_2$$

$$i_A = I_1 - I_2$$

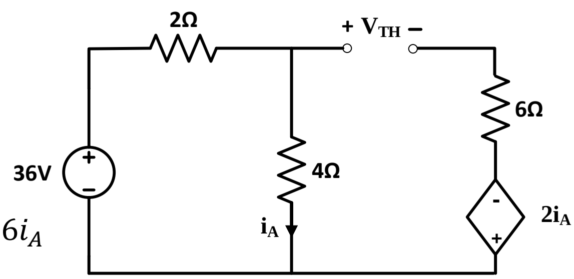
$solving\ for\ I_2\ ;\ we\ get\ I_2 = \frac{36}{8}\ A$

$$v_O(\infty) = (6\ \Omega)\left(\frac{36}{8}\ A\right) = 27\ V$$

$To\ find\ \tau = \frac{L}{R_{Th}}$

$To\ find\ R_{Th}$

$$R_{Th} = \frac{V_{Th}}{I_{SC}}$$

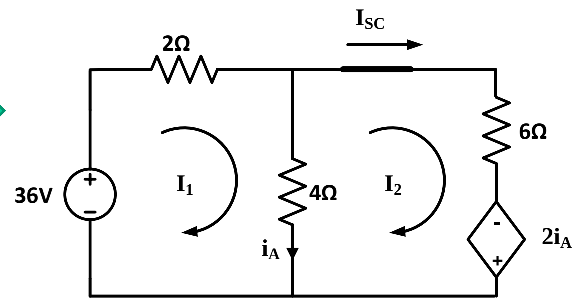


$$V_{Th} = (4\ \Omega)\ i_A + 2i_A = 6i_A$$

$$i_A = \frac{36}{6} = 6\ A$$

$\therefore\ V_{Th} = 36\ V$

$To\ find\ I_{SC}$



$KVL\ for\ mesh\ 1:$

$$36 = 6I_1 - 4I_2$$

$KVL\ for\ mesh\ 2:$

$$2i_A = -4I_1 + 10I_2$$

$i_A = I_2 - I_1$

$Solving\ for\ I_2 = I_{SC} = \frac{36}{8}\ A$

$\therefore\ R_{Th} = \frac{V_{Th}}{I_{SC}} = 8\ \Omega$

$\therefore\ \tau = \frac{L}{R_{Th}} = \frac{3}{8}\ s$

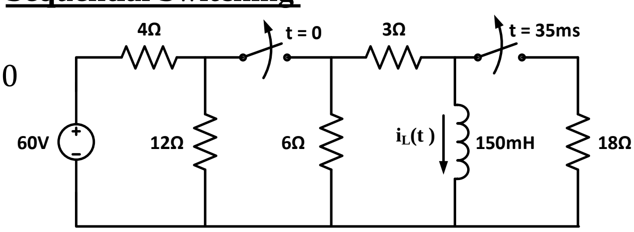
$\therefore v_O(t) = v_O(\infty) + [v_O(0^+) - v_O(\infty)]\, e^{-\frac{t}{\tau}} \qquad t > 0$

$\therefore v_O(t) = 27 + [18 - 27]\, e^{-\frac{t}{\tau}} \qquad t > 0$

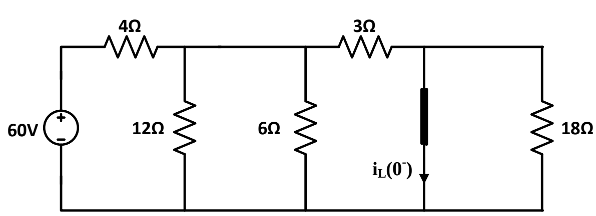
$\therefore v_O(t) = \left(27 - 9\, e^{-\frac{8}{3}t}\right) V \qquad for\ \ t > 0$

Sequential Switching

$Find\ i_L(t)\ \ for\ t > 0$



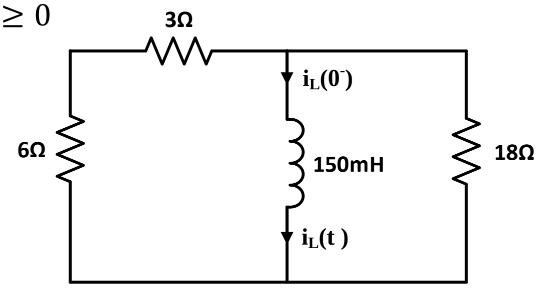
$For\ t < 0 \qquad ;\ t = 0^-$



iL(0-)=2/(2+3)*15

$i_L(0^-) = 6\ A$

2) *For* $35\,ms \geq t \geq 0$



Source-free circuit

$\therefore\ i_L(t) = A\,e^{-\frac{t}{\tau}} \quad ,t \geq 0$

$\tau_1 = \dfrac{L}{R_{Th_1}}$

$R_{Th_1} = 18\,\Omega\,//\,(6\,\Omega + 3\Omega) = 6\Omega$

$\therefore\ \tau_1 = \dfrac{150\,mH}{6\,\Omega} = 25\,ms$

<u>To find A</u>

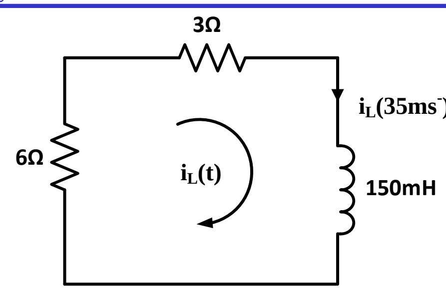
$i(0^+) = A = i_L(0^-) = 6\,A$

$\therefore\ i_L(t) = 6\,e^{-40t}\,A\,, \quad 0 \leq t \leq 35\,ms$

at $t = 35\,ms$

$i_L(35\,ms) = 1.48\,A$

3) *For* $t > 35\,ms$



Source-free circuit

$\therefore\ i_L(t) = A\,e^{-\left(\frac{t-35ms}{\tau_2}\right)}$

$\tau_2 = \dfrac{L}{R_{Th_2}}$

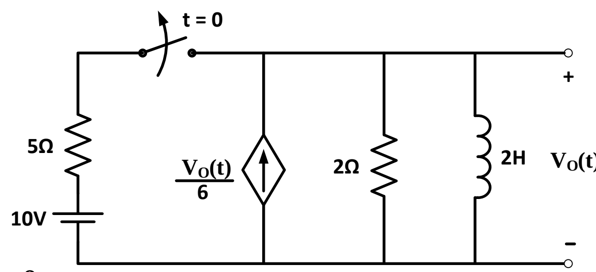
$R_{Th_2} = 6\,\Omega + 3\Omega = 9\,\Omega$

$\therefore\ \tau_2 = 16.67\,ms$

$\therefore\ i_L(t) = 1.48\,e^{-\left(\frac{t-35ms}{16.67ms}\right)}$

$i_L(t) = \left(1.48\,e^{-60(t-0.035)}\right)A \quad ;\ t > 35ms$

Circuits with dependent sources

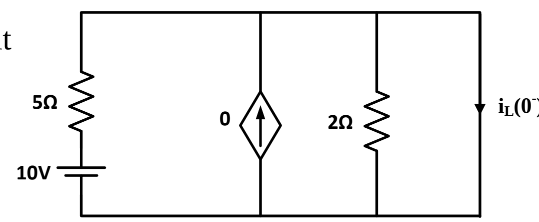


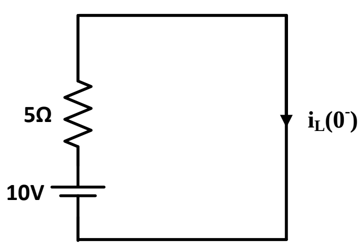
1) $For\ t < 0 \quad ; t = 0^-$

$$\frac{v_O}{6} = 0$$ Open circuit

$2\Omega\ //\ 0\ = 0$

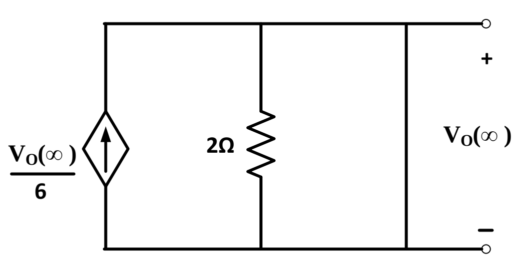
2Ω Open circuit





$\therefore\ i_L(0^-) = 2\ A$

2) At $t = \infty$

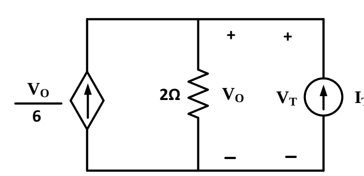


$\therefore\ v_O(\infty) = 0$

3) To find τ

$$\tau = \frac{L}{R_{Th}}$$

$$R_{Th} = \frac{V_T}{I_T}$$



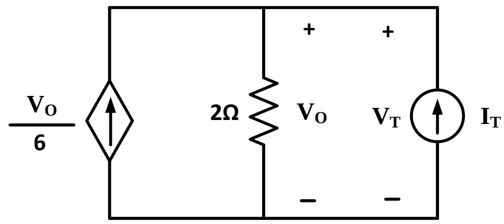
KCL:

$$\frac{v_O}{6} + I_T = \frac{v_O}{2}$$

$$v_O = v_T$$

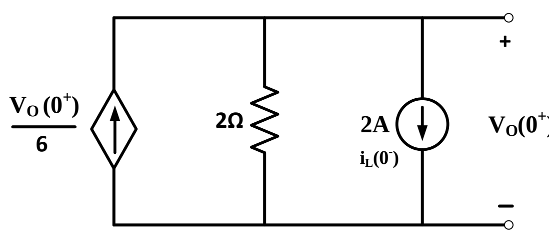
$$\therefore \frac{V_T}{I_T} = 3\Omega = R_{Th}$$

$$\therefore \tau = \frac{L}{R_{Th}} = \frac{2}{3}\, s$$



$$v_O(t) = v_O(\infty) + [v_O(0^+) - v_O(\infty)]\, e^{-\frac{t}{\tau}} \qquad t > 0$$

4) At $t = 0^+$



KCL:

$$\frac{v_O(0^+)}{6} = 2 + \frac{v_O(0^+)}{2}$$

$$\therefore v_O(0^+) = -6\, V$$

Now

$$v_O(t) = v_O(\infty) + [v_O(0^+) - v_O(\infty)]\, e^{-\frac{t}{\tau}} \qquad ;\ t > 0$$

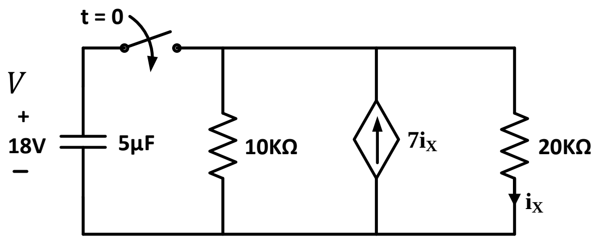
$$\therefore v_O(t) = -6\, e^{-\frac{t}{\tau}}\ \ V \quad ; t > 0$$

$$\therefore v_O(t) = -6\, e^{-1.5t}\ \ V \quad ; t > 0$$

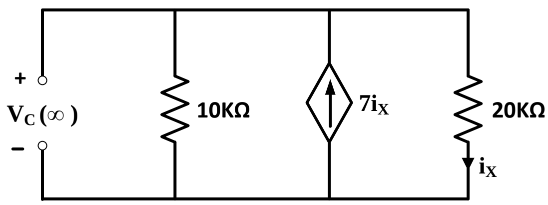
Unbounded response

Find $v_C(t)\ for\ t > 0$

$v_C(0^+) = v_C(0^-) = 18\ V$



1) To find $v_C(\infty)$



Since the circuit is dead

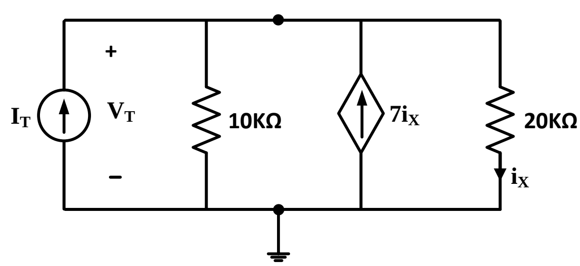
(no independent sources)

$\therefore v_C(\infty) = 0$

3) To find τ

$\tau = R_{Th}\ C$

$R_{Th} = \frac{V_T}{I_T}$



$KCL:$

$I_T + 7i_x = \frac{V_T}{10K} + \frac{V_T}{20K}$

$i_x = \frac{V_T}{20K}$

$\therefore \frac{V_T}{I_T} = -5K = R_{Th}$ **

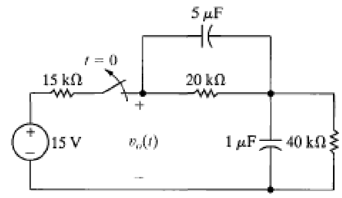
$\therefore \tau = R_{Th}\ C = -25\ ms$

$\therefore v_C(t) = 18\ e^{+40t}\ V \qquad for\ t \geq 0$

Rth and is negative which leads to unbounded response i.e. the output keeps increasing

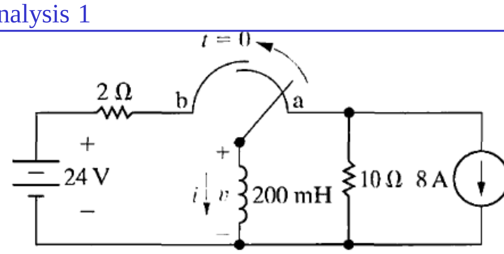
Assessment 7.4

The switch in the circuit shown has been closed for a long time before being opened at $t = 0$.
Find $v_o(t)$ for $t > 0$.



Answer $8e^{-25t} + 4e^{-10t}$ V

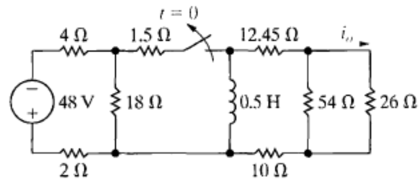
Assessment 7.5



Assume that the switch in the circuit shown has been in position b for a long time, and at $t = 0$ it moves to position
Find : $i(t)$, $t > 0$; and $v(t)$, $t > 0_+$.

Answer $-8 + 20e^{-50t}$ A, $t \geq 0$;
$-200e^{-50t}$ V, $t \geq 0^+$.

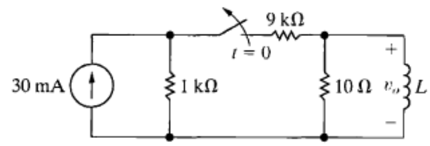
Problem 7.6
The switch in the circuit has been closed a long time. At $t = 0$ it is opened.
Find $i_o(t)$ for $t > 0$.



Answer

$$4.05e^{-80t}\,\text{V}, \qquad t \geq 0^+$$

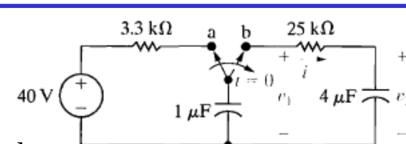
Problem 7.10
In the circuit ,the switch has been closed for a long time before opening at $t = 0$.
Find the value of L so that $v_o(t)$ equals to 0.5 $v_o(0+)$ when $t = 1$ ms



Answer

$$\therefore \quad L = \frac{10 \times 10^{-3}}{\ln 2} = 14.43\,\text{mH}$$

Problem 7.23
The switch in the circuit has been in position a for a long time and $v_2 = 0$ V.
At $t = 0$, the switch is thrown to position b.
Calculate
i, v_1 and v_2 for $t > 0+$.



Answer

i(t)= $1.6e^{-50t}\,\text{mA}, \qquad t \geq 0^+$

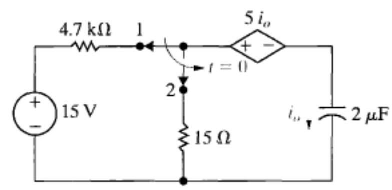
V1(t)= $32e^{-50t} + 8\,\text{V}, \qquad t \geq 0$

V2(t)= $-8e^{-50t} + 8\,\text{V}, \qquad t \geq 0$

Problem 7.30

The switch in the circuit has been in position 1 for a long time before moving to position 2 at $t = 0$.

Find $i_0(t)$ for $t > 0+$.



Answer

$$v_o = 15e^{-25{,}000t}\ \text{V}, \qquad t \geq 0$$

$$i_o = -\frac{v_o}{20} = -0.75e^{-25{,}000t}\ \text{A}, \qquad t \geq 0^+$$

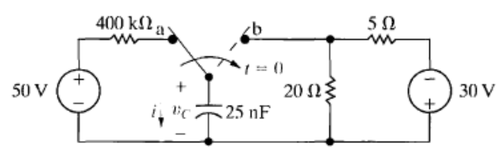
Problem 7.55

Assume that the switch in the circuit has been in position a for a long time and that at $t = 0$ it is moved to position b.

Find

(a) v_C, $t > 0$;

(b) i, $t > 0+$.

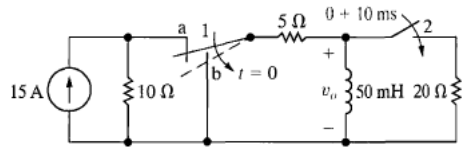


Answer

(a) $-24 + 74e^{-10^7 t}\ \text{V}, \qquad t \geq 0$

(b) $-18.5e^{-10^7 t}\ \text{A}, \qquad t \geq 0^+$

Problem 7.71



The action of the two switches in the circuit seen in is as follows.
For $t < 0$, switch 1 is in position a and switch 2 is open.
This state has existed for a long time.
At $t = 0$, switch 1 moves instantaneously from position a to position b, while switch 2 remains open. Ten milliseconds after switch 1 operates, switch 2 closes, remains closed for 10 ms and
then opens.
Find $v_o(t)$ for t > 20 ms.

Answer
$$v_o = (50 \times 10^{-3})(-165)e^{-100(t-0.02)}$$
$$= -8.26e^{-100(t-0.02)}\text{ V}, \qquad t > 20^{+}\text{ ms}$$