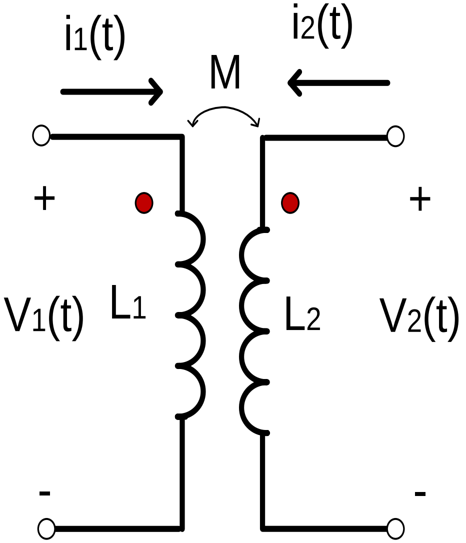
ENEE2301

Network Analysis 1



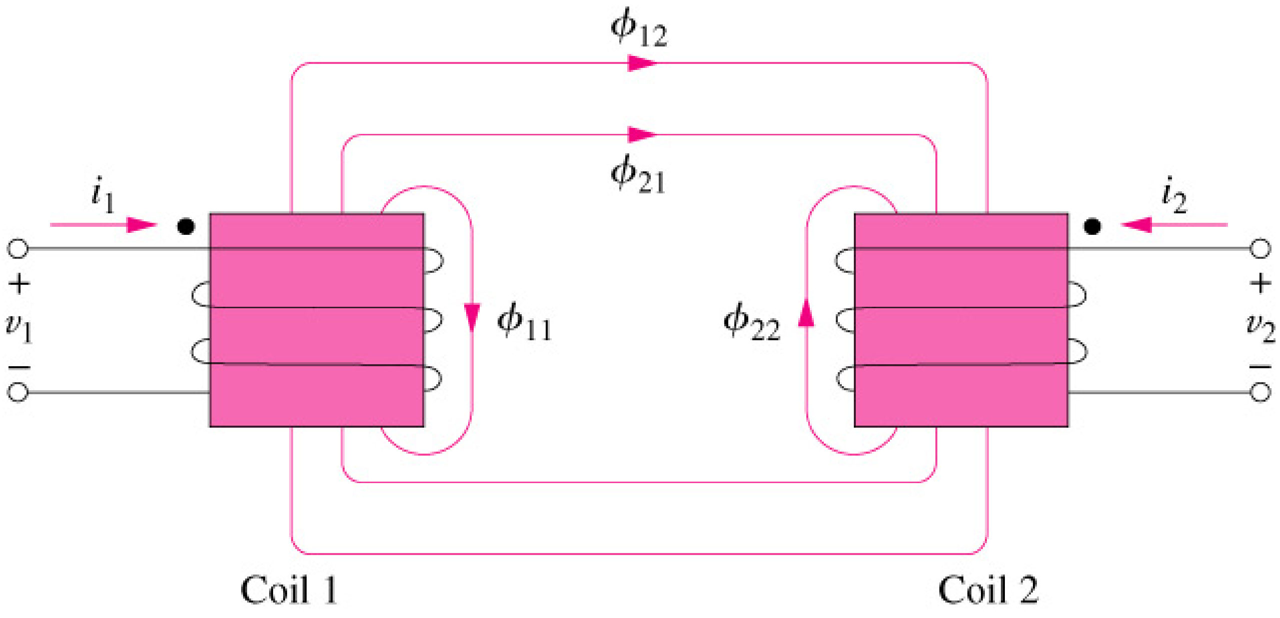
Transformers

Transformers

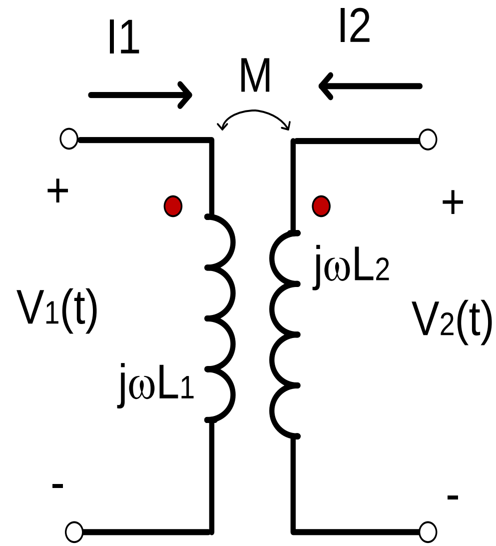
- Transformers are used in a wide variety of applications.
- In electric power transmission and distribution systems they step up the voltage at the sending end to reduce transmission losses
- and step down at the receiving end to make it safer and easier to utilize.
- Transformers change voltages and currents to any desired amplitude, large or small.
- They transform impedances and match load impedances to source impedances for maximum power transfer.

- **Transformers are constructed of two coils placed so that the Changing flux developed by one will link the other**
- **When two coils are placed close to each other, a changing flux in one coil will cause an induced voltage in the second coil.**
- **The coils are said to have mutual inductance M, which can either add or subtract from the total inductance depending on if the fields are aiding or opposing.**

- **The coil to which the source is applied is called the primary coil**
- **The coil to which the load is applied is called the secondary coil**



The Ideal Transformer



$\mathbf{V_1} = j\omega L_1 \ \mathrm{I}_1 + j\omega \mathbf{M} \ \mathrm{I}_2$ ------------------------ **(1)**

$\mathbf{V_2} = j\omega \mathbf{M} \ \mathrm{I}_1 + j\omega L_2 \ \mathrm{I}_2$ -------------------------- **(2)**

From (1)

$$\mathrm{I}_1 = \frac{V_{1} - j\omega \mathbf{M} \ \mathrm{I}_2}{j\omega L_1}$$ -------------------------- **(3)**

Substitute (3) in to (2)

$$\mathbf{V_2} = j\omega L_2 \ \mathrm{I}_2 + \frac{M\mathrm{V}_1}{L_1} - \frac{j\omega M^2 I_2}{L_1}$$

M = K$\sqrt{L_1 L_2}$

For ideal transformer k=1

M=$\sqrt{L_1 L_2}$

$$\mathbf{V_2} = j\boldsymbol{\omega} L_2 \ \mathrm{I}_2 + \frac{\boldsymbol{M}\mathrm{V}_1}{L_1} - \frac{j\boldsymbol{\omega}\boldsymbol{M}^2 I_2}{L_1}$$

$$\mathbf{V_2} = j\boldsymbol{\omega} L_2 \ \mathrm{I}_2 + \frac{\sqrt{L_1 L_2}}{L_1}\ \mathbf{V_1} - \frac{j\boldsymbol{\omega} L_1 L_2 I_2}{L_1}$$

$$\mathbf{V_2} = \sqrt{\frac{L_2}{L_1}}\ \mathbf{V_1} = \frac{N_2}{N_1}\mathrm{V}_1$$

Remember $L_1 = K\ N_1^{\ 2}$

$L_2 = K\ N_2^{\ 2}$

$$\mathbf{V_2} = \mathbf{n}\ \mathbf{V_1}$$

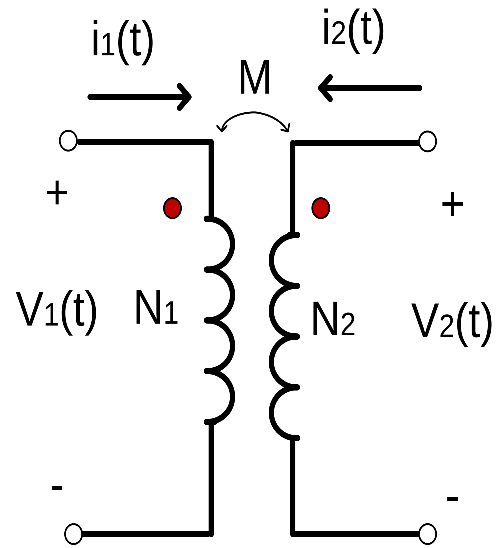
$$\mathbf{n} = \frac{N_2}{N_1} \equiv \textbf{Turns ratio}$$

The ideal transformer

$$V_1(t) = N_1 \frac{d\Phi}{dt}$$

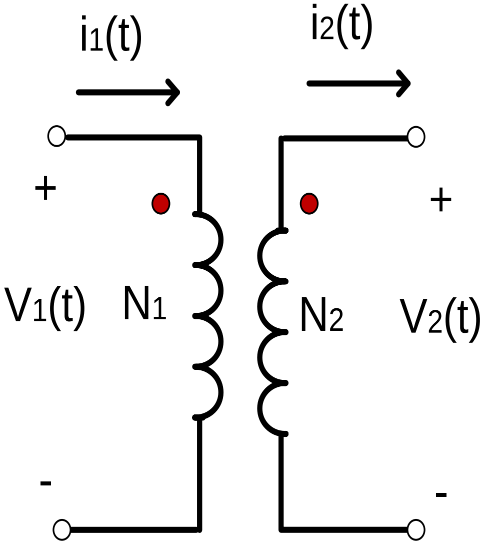
$$V_2(t) = N_2 \frac{d\Phi}{dt}$$

$$\frac{V_1(t)}{V_2(t)} = \frac{N_1}{N_2}$$



In an ideal transformer all flux generated by coil 1 is seen by coil 2 and there is no leakage flux
There is some techniques in transformer winding and in core geometry that allows the real transformers to approach the ideal characteristics

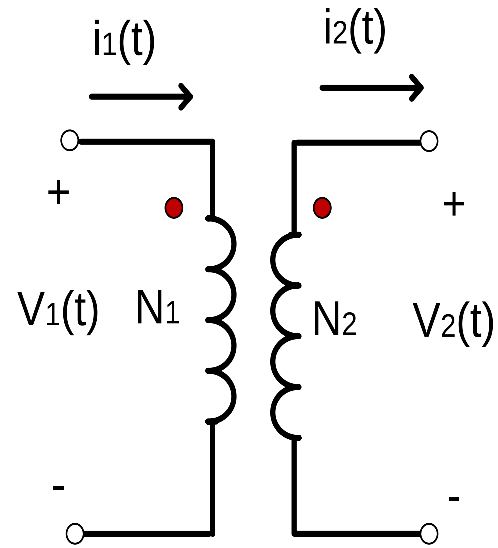
The ideal transformer



$$V_1(t) = N_1 \frac{d\Phi}{dt}$$

$$V_2(t) = N_2 \frac{d\Phi}{dt}$$

$$\frac{V_1(t)}{V_2(t)} = \frac{N_1}{N_2}$$



$$V_1(t) = N_1 \frac{d\Phi}{dt}$$

Assuming P1=P2 (100% efficiency- no losses)

$$V_2(t) = N_2 \frac{d\Phi}{dt}$$

$$V_1(t)i_1(t) = V_2(t)i_2(t)$$

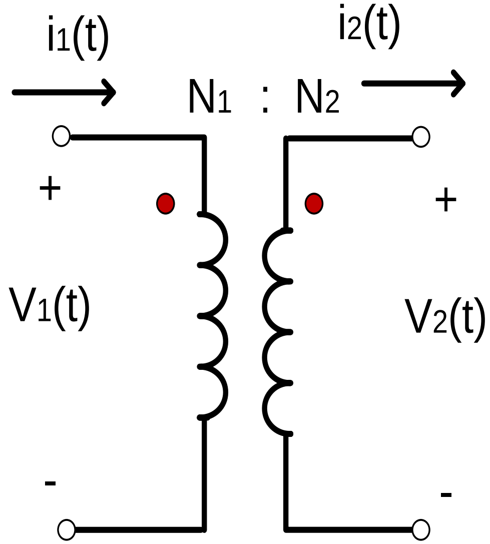
$$\frac{V_1(t)}{V_2(t)} = \frac{N_1}{N_2}$$

$$\therefore \frac{i_1(t)}{i_2(t)} = \frac{V_2(t)}{V_1(t)} = \frac{N_2}{N_1}$$

Ideal Transformer

$$\frac{i_1(t)}{i_2(t)} = \frac{N_2}{N_1}$$

$$\frac{V_2(t)}{V_1(t)} = \frac{N_2}{N_1}$$



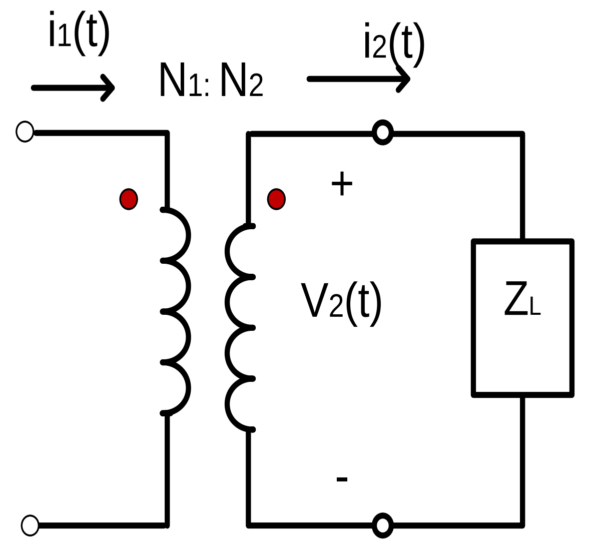
Since the equations are algebraic,

they are unchanged for phasors

$$\frac{\boldsymbol{I_1}}{\boldsymbol{I_2}} = \frac{\boldsymbol{V_2}}{\boldsymbol{V_1}} = \frac{N_2}{N_1}$$

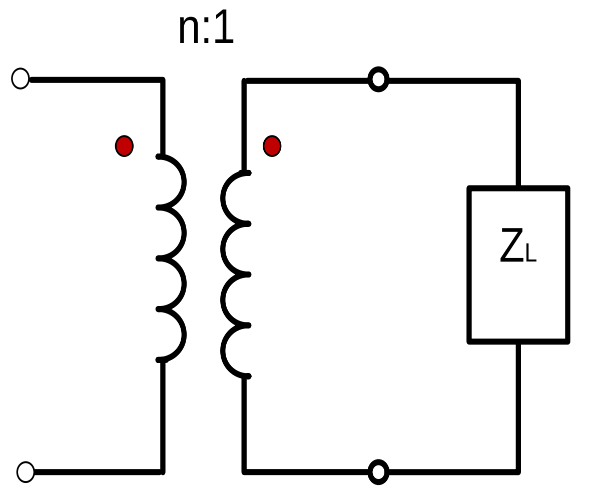
Impedance Reflection

$$\mathbf{Z_{TH}} = \frac{V_1}{I_1} = \frac{\frac{N_1}{N_2}\mathrm{V}_2}{\frac{N_2}{N_1}\mathrm{I}_2}$$



$$\mathrm{Z_{TH}} = \frac{N_1^2}{N_2^2}\frac{\mathrm{V}_2}{\mathrm{I}_2}$$

$$\mathbf{Z_{TH}} = \frac{N_1^2}{N_2^2}\,\mathbf{Z_L}$$

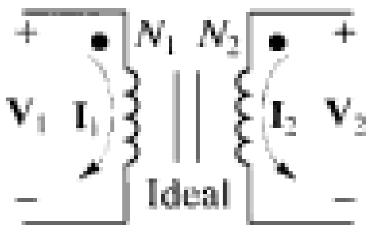


$$\textbf{Let } \frac{N_1}{N_2} = \boldsymbol{a}$$

$$\mathbf{Z_{TH}} = \boldsymbol{a}^2\ \mathbf{Z_L}$$

Important

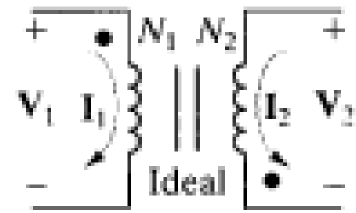
Dot Convention for Ideal Transformer



$$\frac{\mathbf{V}_1}{N_1} = \frac{\mathbf{V}_2}{N_2},$$

$$N_1\mathbf{I}_1 = -N_2\mathbf{I}_2$$

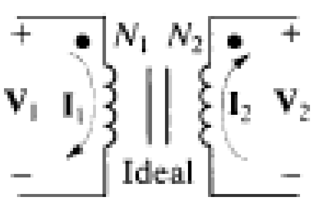
(a)



$$\frac{\mathbf{V}_1}{N_1} = -\frac{\mathbf{V}_2}{N_2},$$

$$N_1\mathbf{I}_1 = N_2\mathbf{I}_2$$

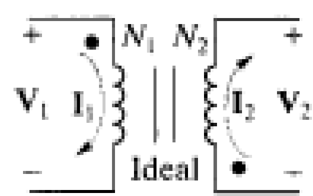
(b)



$$\frac{\mathbf{V}_1}{N_1} = \frac{\mathbf{V}_2}{N_2},$$

$$N_1\mathbf{I}_1 = N_2\mathbf{I}_2$$

(c)

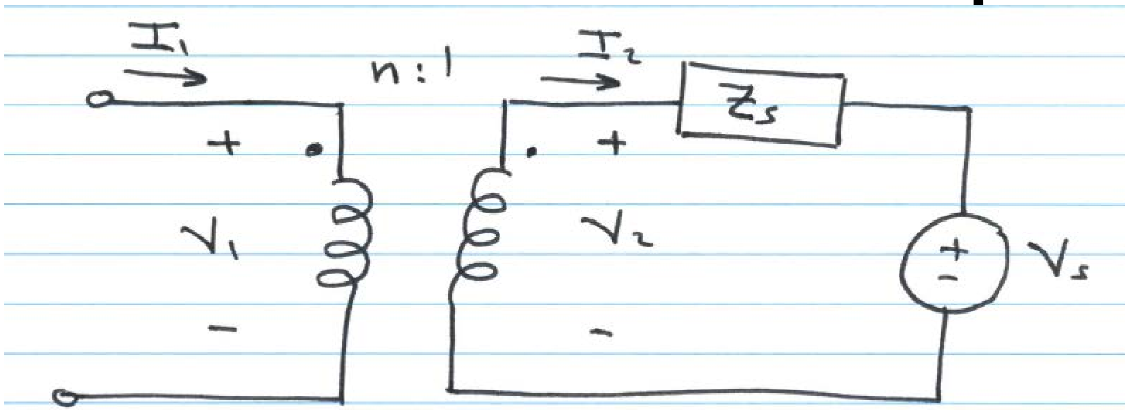


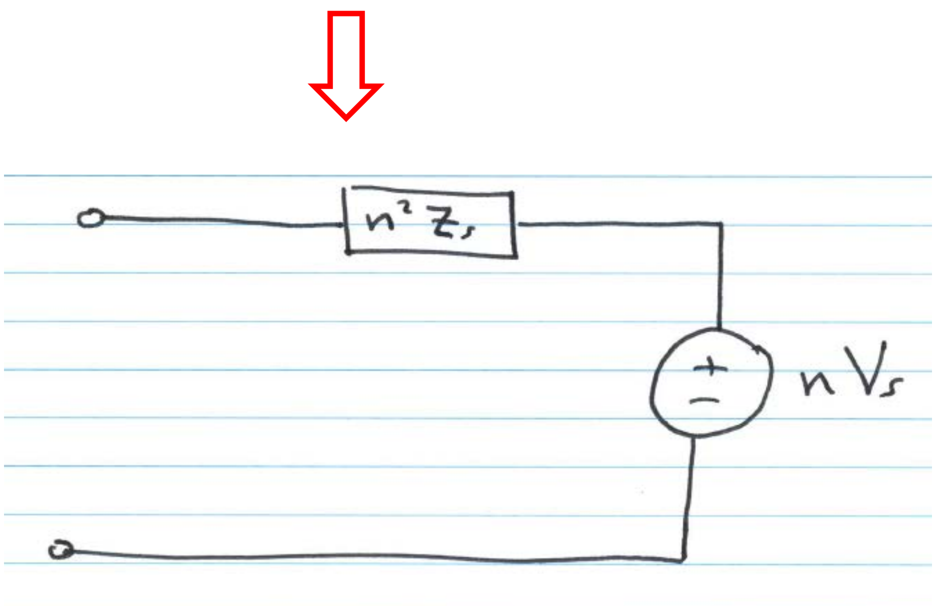
$$\frac{\mathbf{V}_1}{N_1} = -\frac{\mathbf{V}_2}{N_2},$$

$$N_1\mathbf{I}_1 = -N_2\mathbf{I}_2$$

(d)

Example

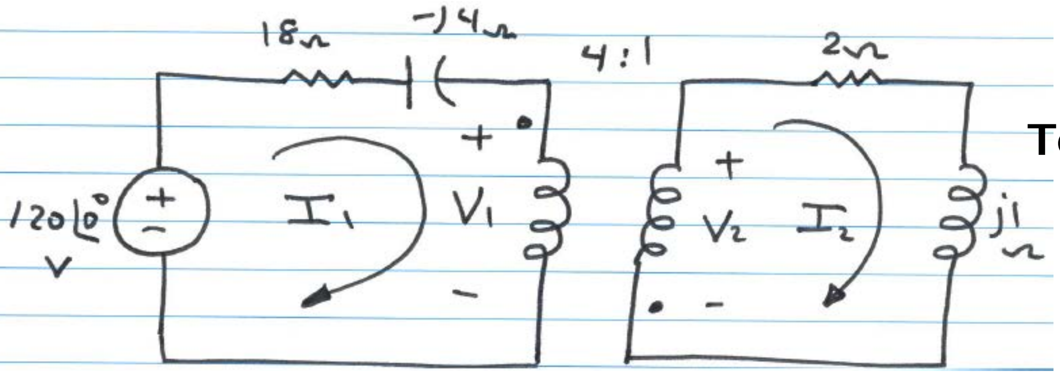




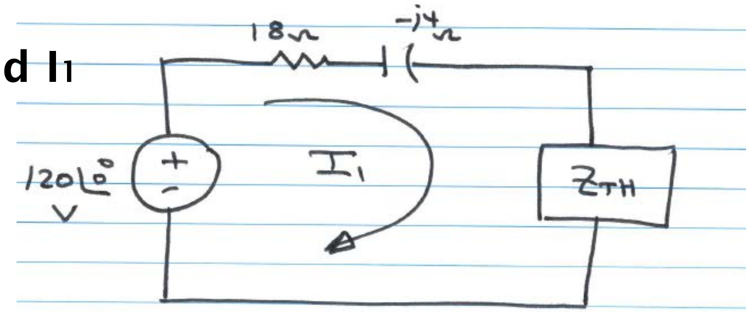
$$V_{TH} = V_1 = nV_2 = nV_S$$

$$Z_{TH} = n^2 Z_S$$

Find I_1 and I_2



To Find I_1

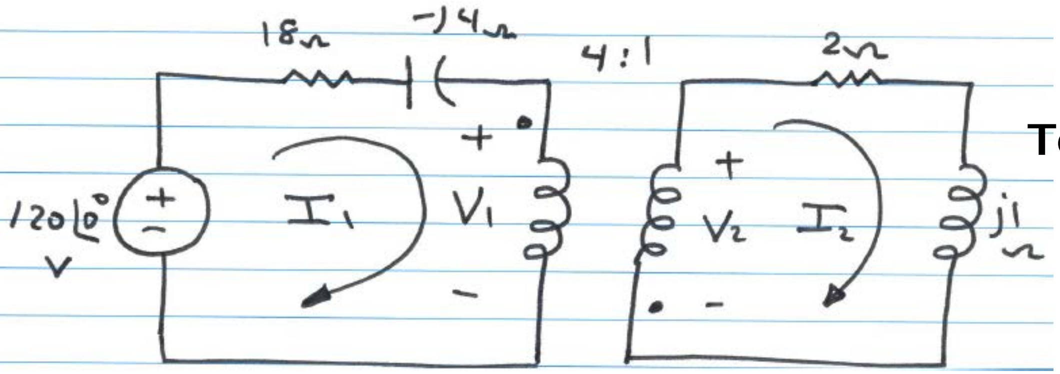


$$Z_{TH} = \frac{N_1^2}{N_2^2} Z_L = 4^2 (2 + j1) = 32 + j16$$

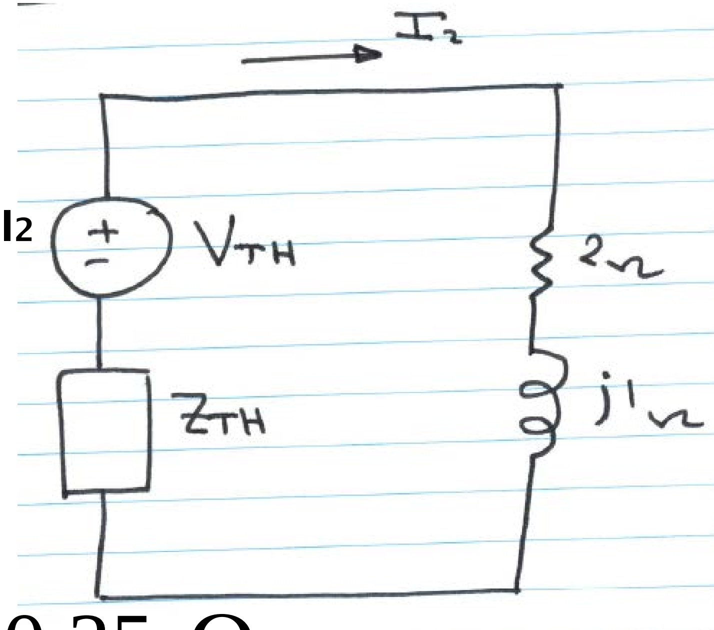
$$I_1 = \frac{120\angle 0}{18 - j4 + 32 + j16} = \frac{120\angle 0}{50 + j12}$$

$$= 2.33\angle -13.5^o \ \ \text{A}$$

Find I_1 and I_2



To Find I_2 ⇨

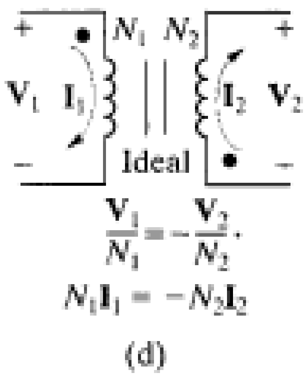


$$Z_{TH}=\frac{N_2^2}{N_1^2}Z_s=\frac{(18-j4)}{4^2}=1.125-j0.25\ \Omega$$

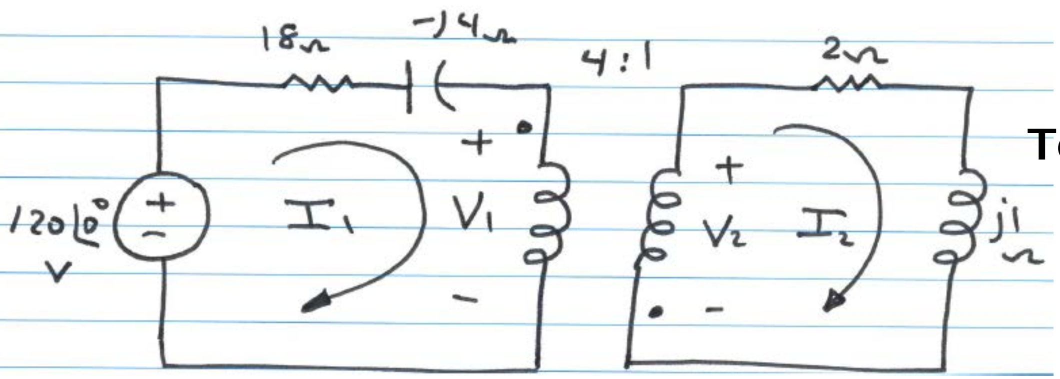
$$V_{TH}=-\frac{120\angle 0}{4}=-30\angle 0\ \text{V}$$

$$I_2=\frac{V_{TH}}{Z_{TH}+2+j1}$$

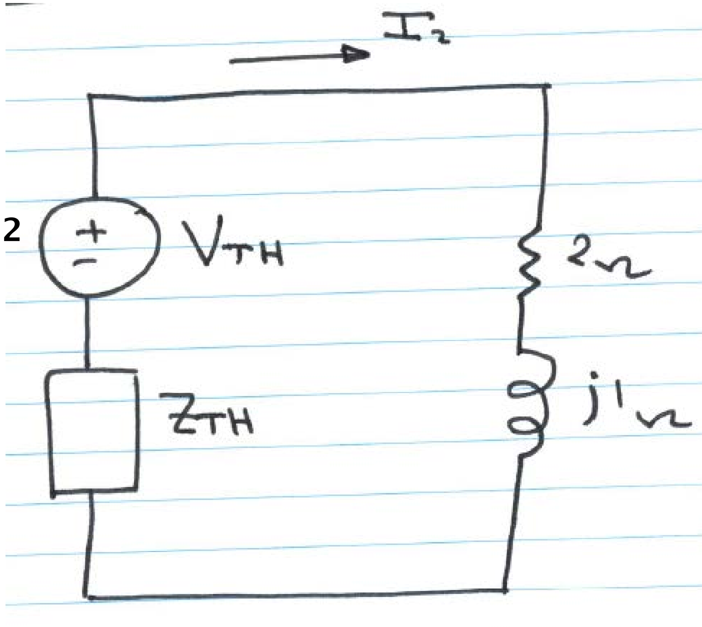
$$=9.335\angle 166.5^o\ \text{A}$$



Find V1 and V2



To Find I2



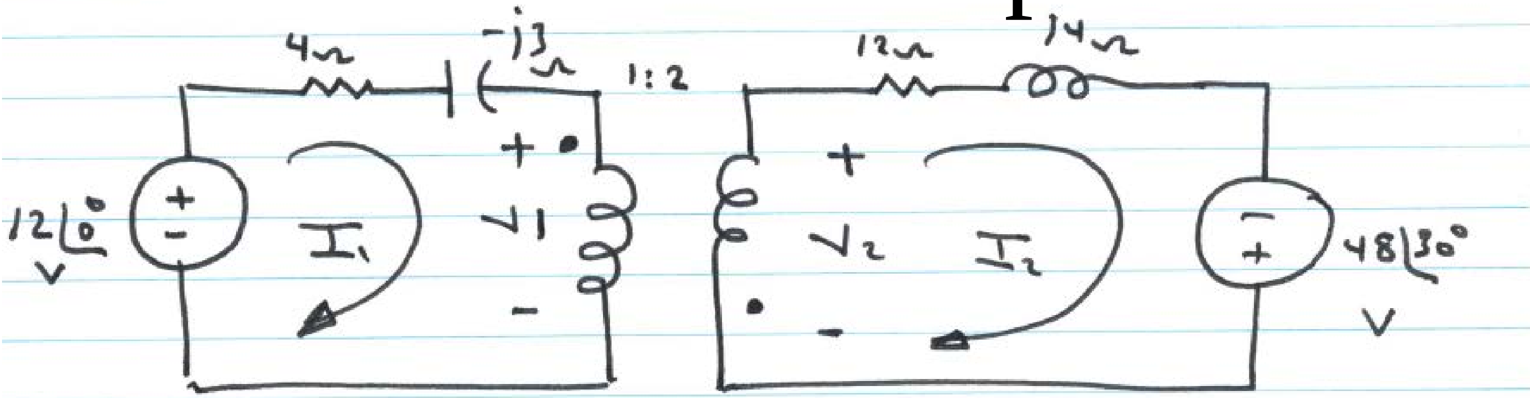
$$V_2 = I_2(2 + j1)$$

$$V_2 = 20.87\langle 193.07^o \text{ V}$$

$$V_1 = -4V_2$$

$$= 83.49\langle 3.07^o \text{ V}$$

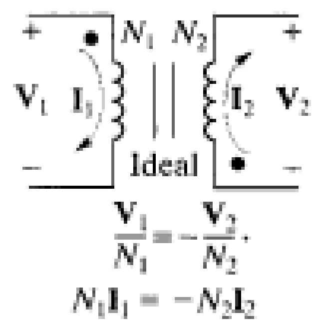
Example



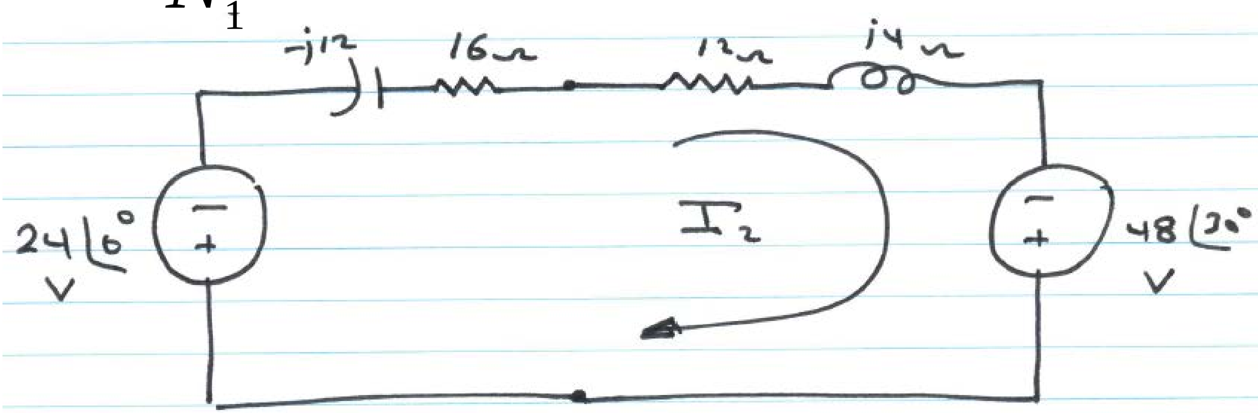
To Find I2

$$V_{TH1} = -\frac{N_2}{N_1}V_1 = -2(12\ \angle 0^{O}) = -24\ \angle 0^{O}$$

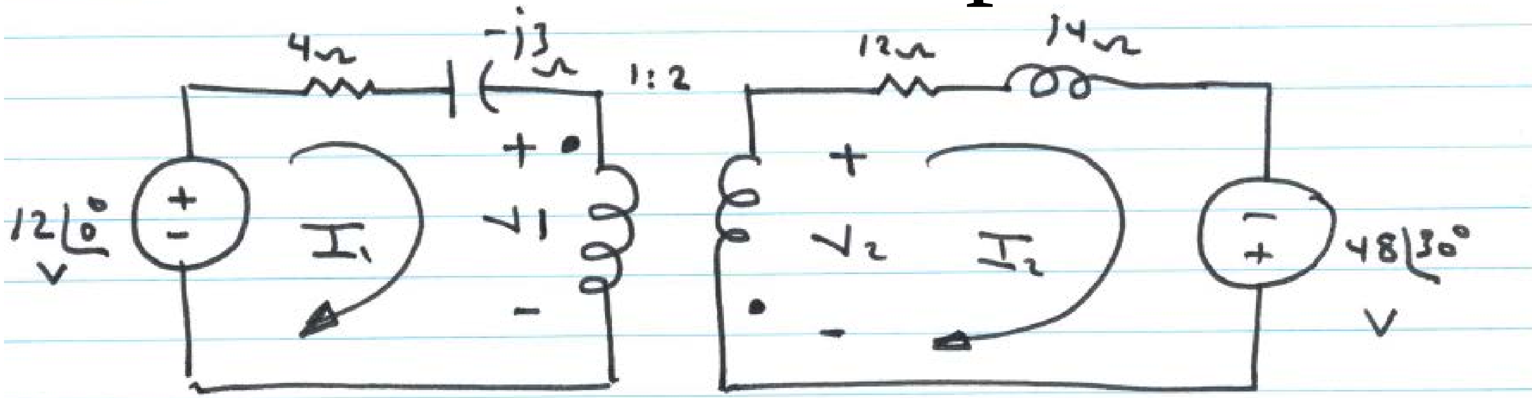
$$Z_{TH1} = \frac{N_2^2}{N_1^2}Z_s = 2^2(4 - j3) = 16 - j12\ \Omega$$



(d)



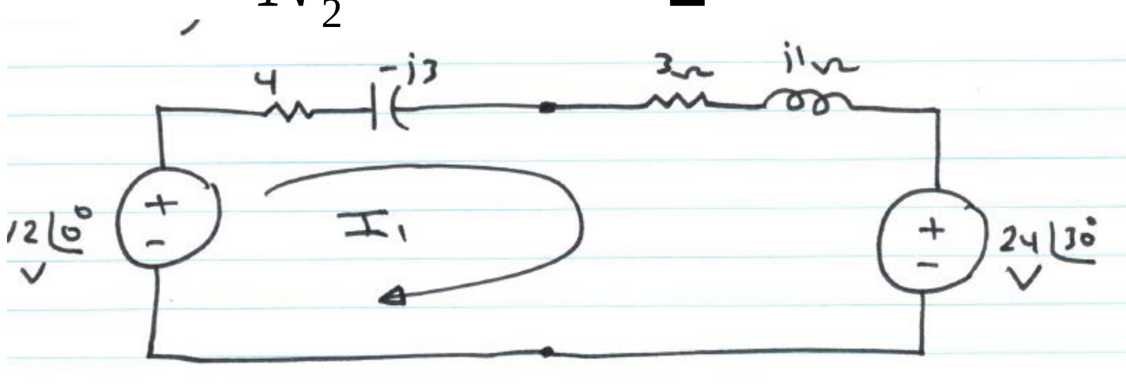
Example

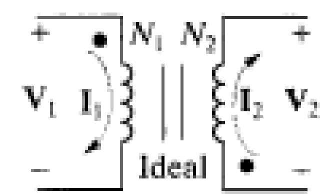


To Find I1

$$V_{TH2} = -\frac{N_1}{N_2}V_2 = -\frac{1}{2}(48\angle 30^O) = -24\angle 30^O$$

$$Z_{TH2} = \frac{N_1^2}{N_2^2}Z_L = \frac{(12 + j4)}{2^2} = 3 + j1\ \Omega$$



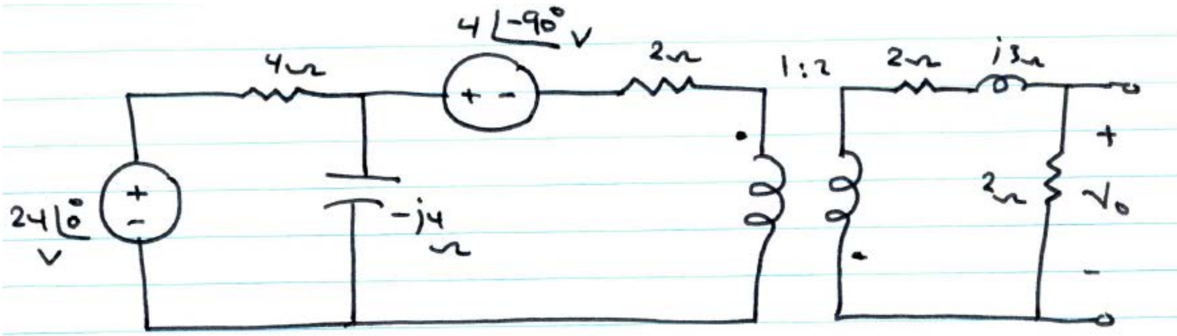


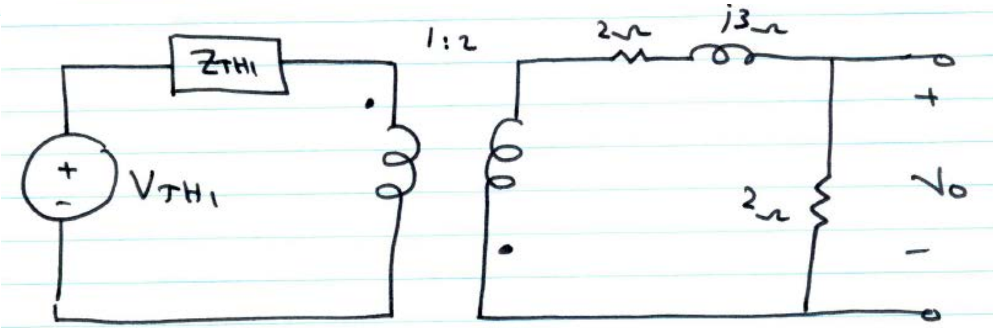
$$\frac{V_1}{N_1} = -\frac{V_2}{N_2},$$

$$N_1 I_1 = -N_2 I_2$$

(d)

Example: Find Vo





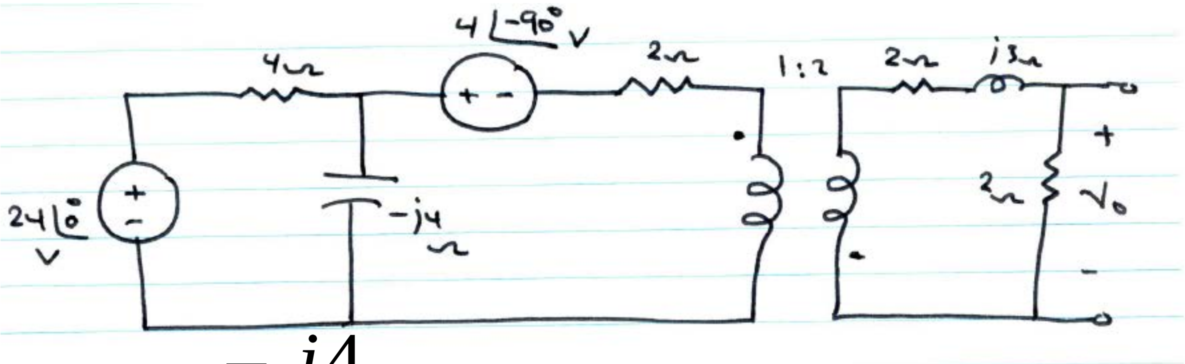
$$Z_{TH1} = 2 + (4 // -j4)$$

$$= 2 + \left(\frac{-j16}{4 - j4}\right)\left(\frac{4 + j4}{4 + j4}\right) = 2 + \left(\frac{-j64 + 64}{32}\right)$$

$$= 2 + (2 - j2) = 4 - j2\ \Omega$$

Important

Example: Find Vo

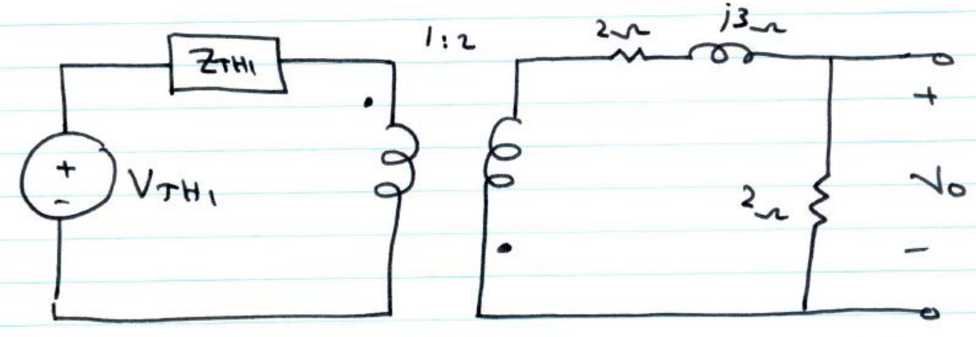


$$V_{TH} = \frac{-j4}{4-j4} 24\langle 0^o - 4\langle -90^o$$

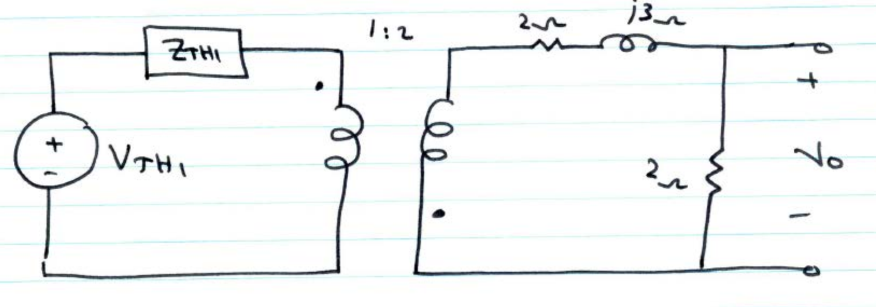
$$\frac{-j4}{4-j4} = \frac{-j4}{4-j4} x \frac{4+j4}{4+j4} = \frac{-j16+16}{32} = \frac{1}{2} - j\frac{1}{2}$$

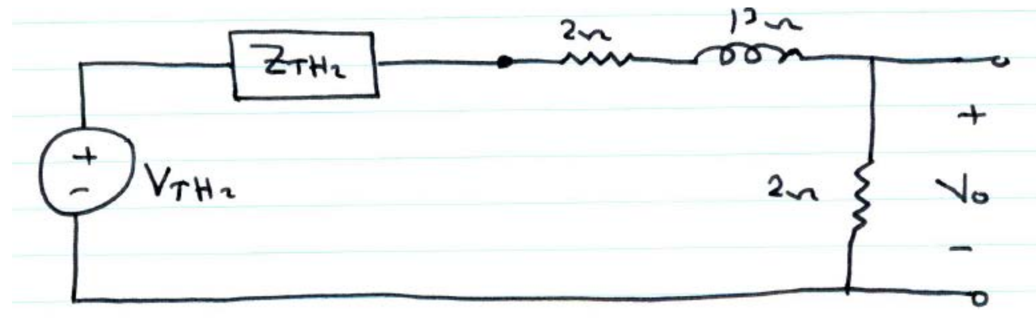
$$V_{TH} = \left(\frac{1}{2} - j\frac{1}{2}\right) 24\langle 0^o - (-j4)$$

$$= 12 - j12 + j4 = 12 - j8$$



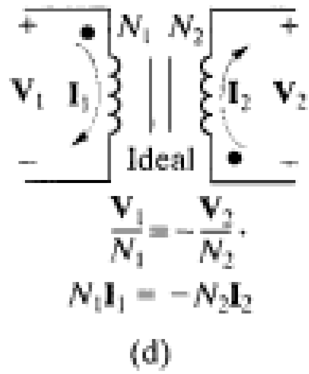
$$V_{TH1} = 14.42\langle -33.69^o \ \ V$$

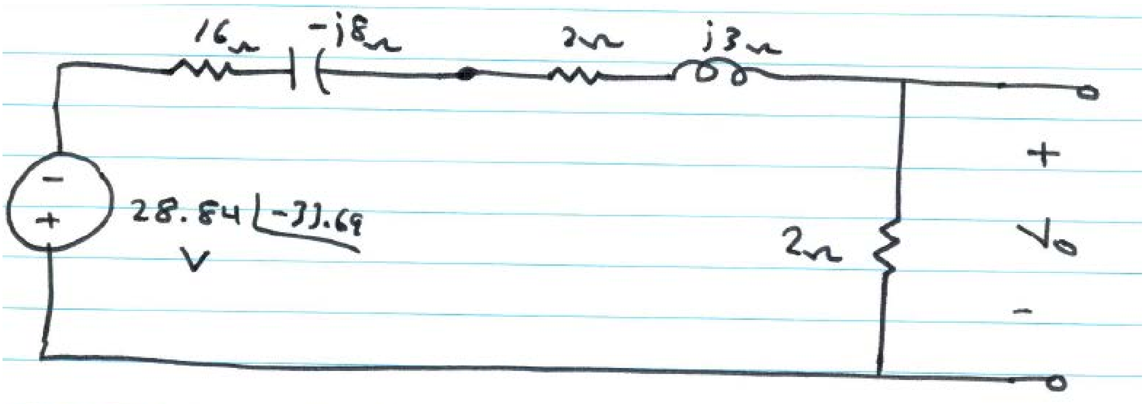




$$Z_{TH2} = \left(2^2\right) Z_{TH1} = 4(4 - j2) = 16 - j8\,\Omega$$

$$V_{TH2} = -2V_{TH1} = -28.84\,\langle -33.69^o \;\; V$$





$$V_o = \frac{2}{(2 + 2 + 16 + j3 - j8)}\left(-28.84\langle -33.69^o\right)$$

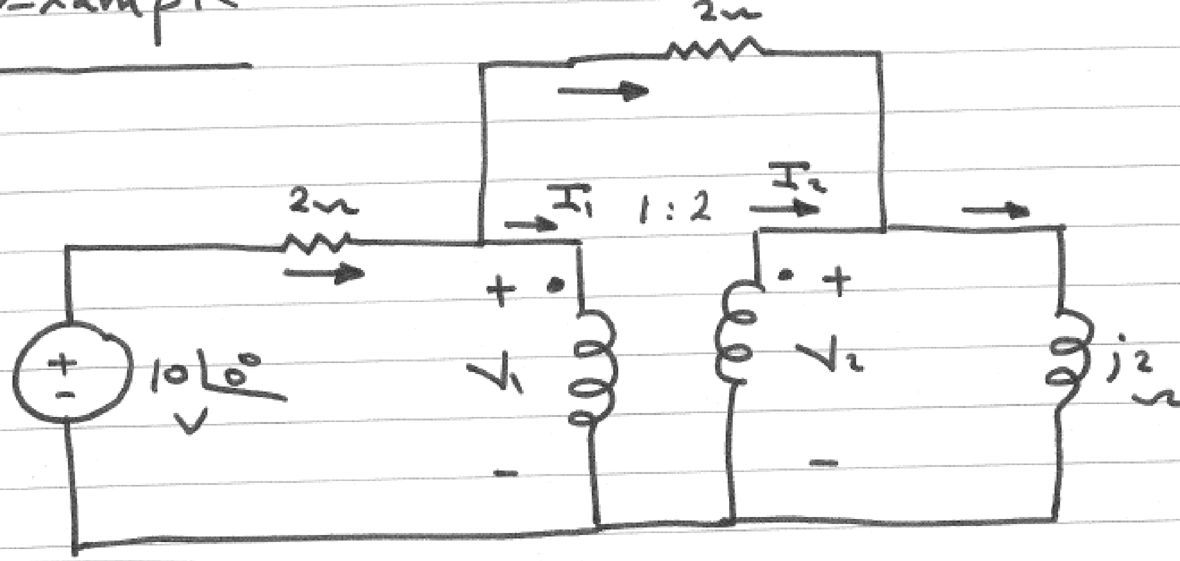
$$V_o = 2.8\,\langle 160.35^o \;\; V$$

$$\frac{2}{(2+2+16+j3-j8)}=$$

$$=\frac{2}{(20-j5)}\frac{(20+j5)}{(20+j5)}=\frac{40+j10}{425}=0.09701\langle 14.036^{O}$$

$$\mathrm{V_o}=(0.09701\langle 14.036)(-28.84\langle -33.69^{o})=2.8\langle 160.35^{o}$$

Example



KCL :

$$\frac{10 - V_1}{2} = \frac{V_1 - V_2}{2} + I_1$$

KCL :

$$I_2 + \frac{V_1 - V_2}{2} = \frac{V_2}{j2}$$

$$V_2 = 2V_1$$

$$I_1 = 2I_2$$

$$\therefore \quad I_1 = 5\angle 0^\circ \ A$$

$$I_2 = 2.5\angle 0^\circ \ A$$

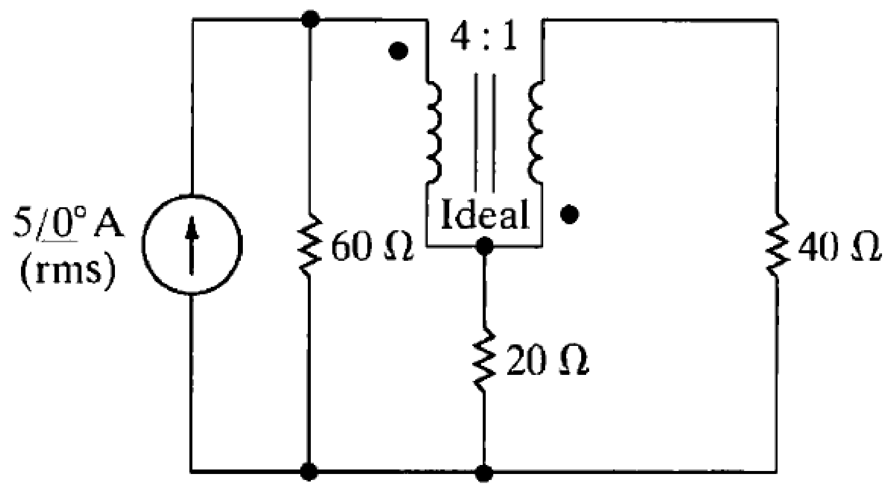
$$V_1 = \sqrt{5}\angle 63^\circ \ V$$

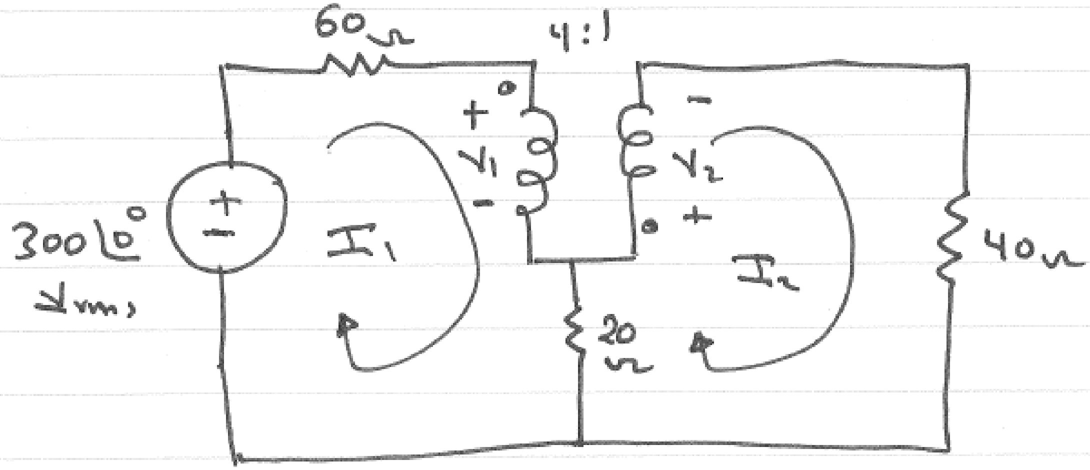
$$V_2 = 2\sqrt{5}\angle 63^\circ \ V$$

10.39 a) Find the average power delivered by the sinusoidal current source in the circuit of Fig. P10.39.

b) Find the average power delivered to the 20 Ω resistor.

Figure P10.39





$$300\angle 0^\circ = 60\vec{I}_1 + \vec{V}_1 + 20(\vec{I}_1 - \vec{I}_2)$$

$$0 = 40\vec{I}_2 + 20(\vec{I}_2 - \vec{I}_1) + \vec{V}_2$$

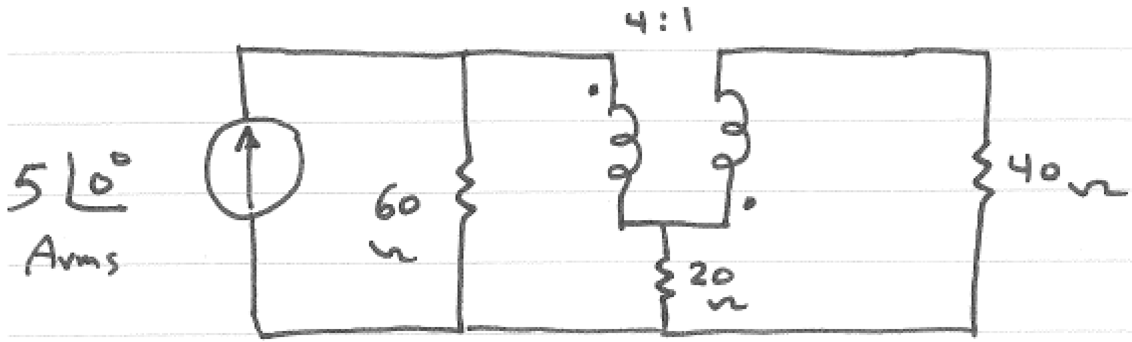
$$\vec{V}_2 = \frac{\vec{V}_1}{4} \quad ; \quad \vec{I}_2 = -4\vec{I}_1$$

Solving:

$$\vec{V}_1 = 260\angle 0^\circ \text{ V rms}$$

$$\vec{V}_2 = 65\angle 0^\circ \text{ V rms}$$

$$\vec{I}_1 = 0.25\angle 0^\circ \text{ A rms} \quad ; \quad I_2 = 1\angle 180^\circ \text{ A rms}$$



$$\vec{V}_{5\angle 0^\circ A} = \vec{V}_1 + 20(\vec{I}_1 - \vec{I}_2) = 285\angle 0^\circ \text{ V rms}$$

$$P_{5\angle 0^\circ A} = (5)(285) = 1425 \text{ W}$$

$$\vec{I}_{20\Omega} = \vec{I}_1 - \vec{I}_2 = 1.25\angle 0^\circ \text{ A rms}$$

$$P_{20\Omega} = (1.25)^2 \cdot 20 = 31.25 \text{ W}$$

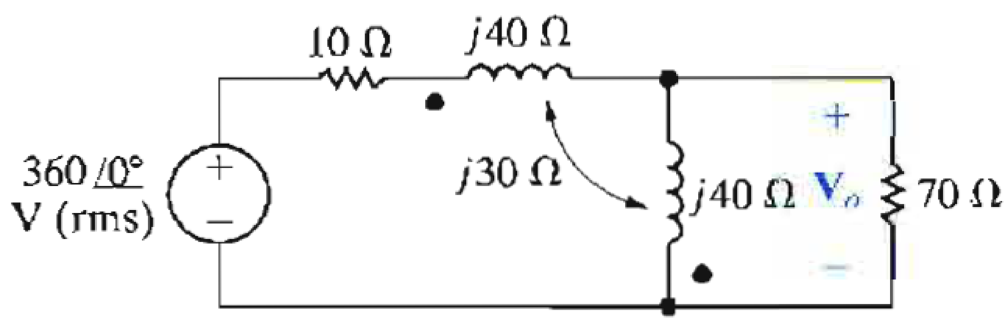
10.44 For the frequency-domain circuit in Fig. P10.44, calculate:

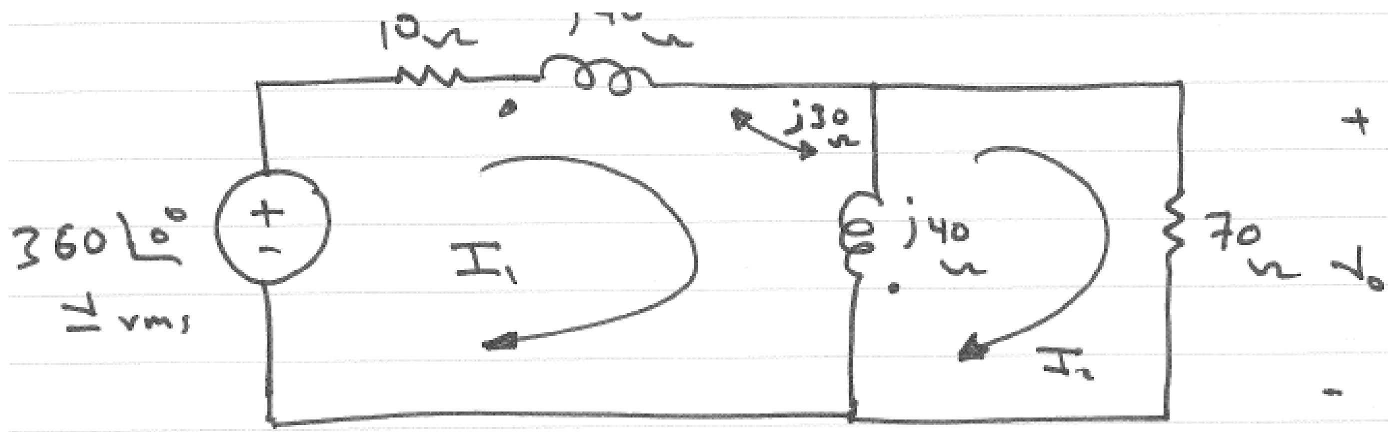
a) the rms magnitude of $\mathbf{V}_o$.

b) the average power dissipated in the 70 Ω resistor.

c) the percentage of the average power generated by the ideal voltage source that is delivered to the 70 Ω load resistor.

Figure P10.44





$$360\angle 0^\circ = 10\vec{I}_1 + j40\vec{I}_1 - j30(\vec{I}_1 - \vec{I}_2) + j40(\vec{I}_1 - I_2) - j30\vec{I}_1$$

$$360\angle 0^\circ = (10 + j20)\vec{I}_1 - j10\vec{I}_2$$

$$0 = 70\vec{I}_2 + j40(\vec{I}_2 - \vec{I}_1) + j30\vec{I}_1$$

$$0 = -j10\vec{I}_1 + (70 + j40)\vec{I}_2$$

solving:

$$\vec{I}_1 = 16.1245\angle{-60.25^\circ} \text{ A rms}$$

$$\vec{I}_2 = 2\angle{0^\circ} \text{ A rms}$$

$$\vec{V}_0 = 140\angle{0^\circ} \text{ V rms}$$

b) $P_{70\Omega} = |I_2|^2 \cdot 70 = 280 \text{ W}$

c) $P_{360\angle 0^\circ} = V I \cos\theta = (360)(16.1245)\cos(0+60.25^\circ)$

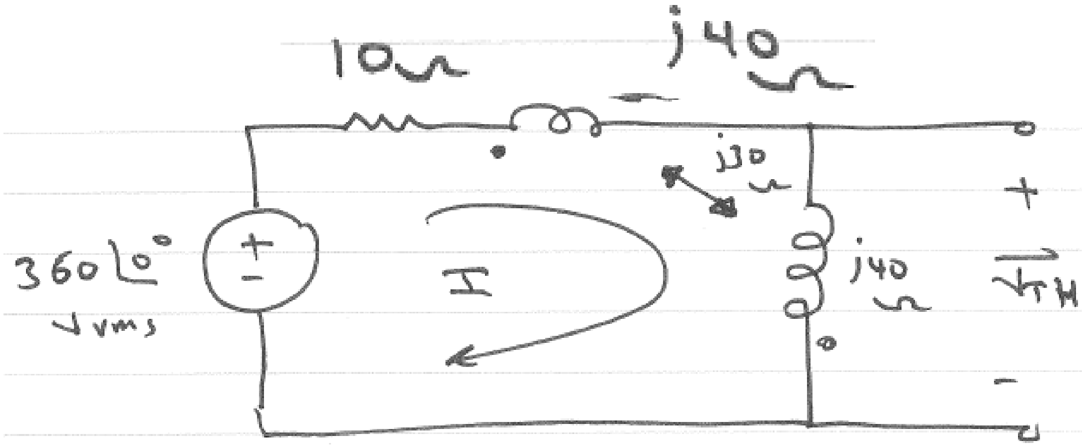
$P_{360\angle 0^\circ} = 2880 \text{ W}$

$\%\text{ delivered} = \frac{280}{2880} \times 100\% = 9.72\%$

10.45 The 70 Ω resistor in the circuit in Fig. P10.44 is replaced with a variable impedance Z_o. Assume Z_o is adjusted for maximum average power transfer to Z_o.

a) What is the maximum average power that can be delivered to Z_o?

b) What is the average power developed by the ideal voltage source when maximum average power is delivered to Z_o?



$$360\angle 0^\circ = 10\vec{I} + j40\vec{I} - j30\vec{I} + j40\vec{I} - j30\vec{I}$$

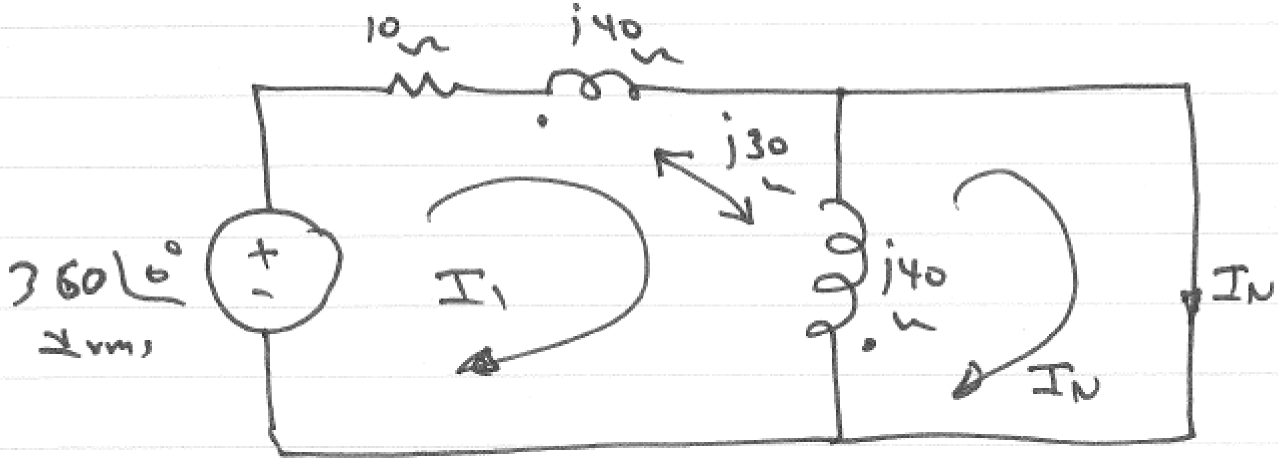
$$360\angle 0^\circ = (10 + j20)\vec{I}$$

$$\vec{I} = 7.2 - j14.4 \text{ Arms}$$

$$\vec{V}_{TH} = j40\vec{I} - j30\vec{I} = j10\vec{I}$$

$$\vec{V}_{TH} = 144 + j72 = 161\angle 26.56 \text{ Vrms}$$

To find I_N



$$360\angle 0^\circ = (10+j20)\vec{I_1} - j10\,\vec{I_N}$$

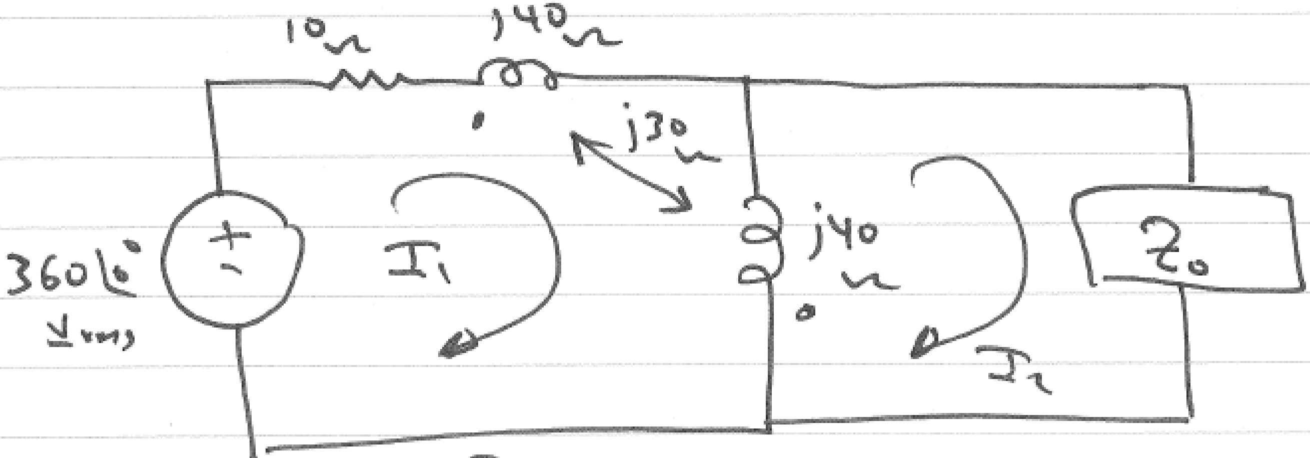
$$0 = -j10\,\vec{I_1} + j40\,\vec{I_N}$$

$$\therefore \quad \vec{I_N} = 2.215 - j3.877 \text{ Arms}$$

$$Z_{TH} = \frac{\vec{V}_{TH}}{\vec{I}_N} = 2 + j36\ \Omega$$

$$\therefore\ Z_0 = Z_{TH}^* = 2 - j36\ \Omega$$

$$P_{L,max} = \frac{|V_{TH}|^2}{4R_{TH}} = 3240\ W$$



$$\vec{I}_2 = \frac{\vec{V}_{TH}}{4} = 36 + j18\ A_{rms}$$

$$360\angle 0° = (10 + j20)\vec{I}_1 - j10\vec{I}_2$$

$$\therefore\ \vec{I}_1 = 18\angle 0°\ A_{rms}$$