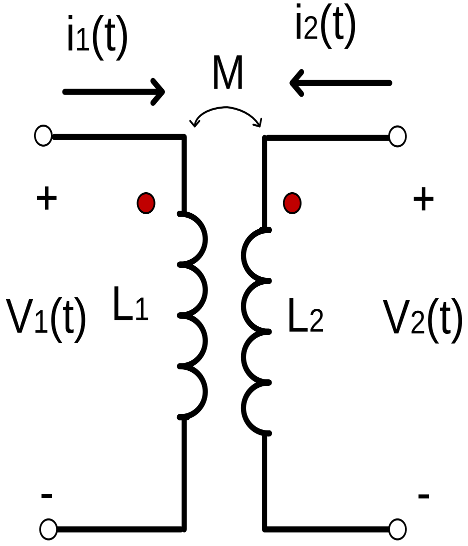
ENEE2301

Network Analysis 1

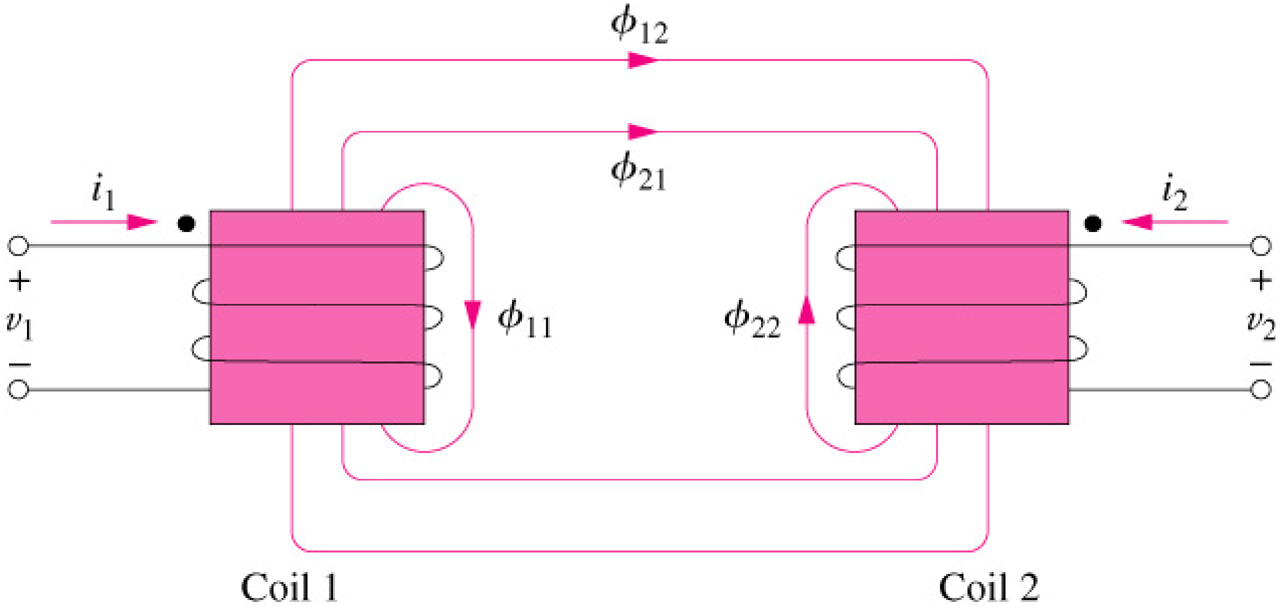


Transformers

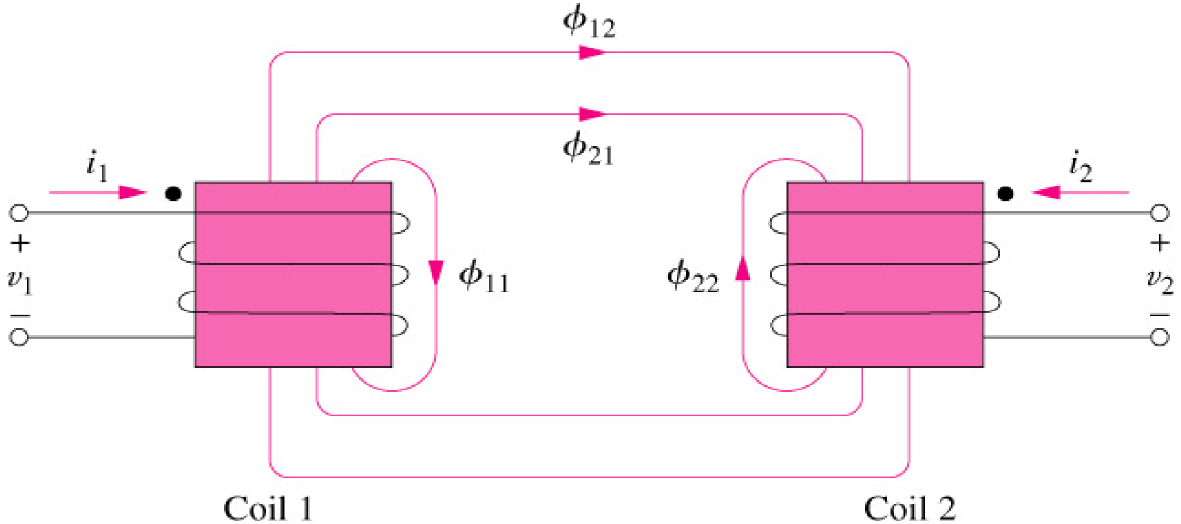
Transformers

- Transformers are used in a wide variety of applications.
- In electric power transmission and distribution systems they step up the voltage at the sending end to reduce transmission losses and step down at the receiving end to make it safer and easier to utilize.
- Transformers change voltages and currents to any desired amplitude, large or small.
- They transform impedances and match load impedances to source impedances for maximum power transfer.

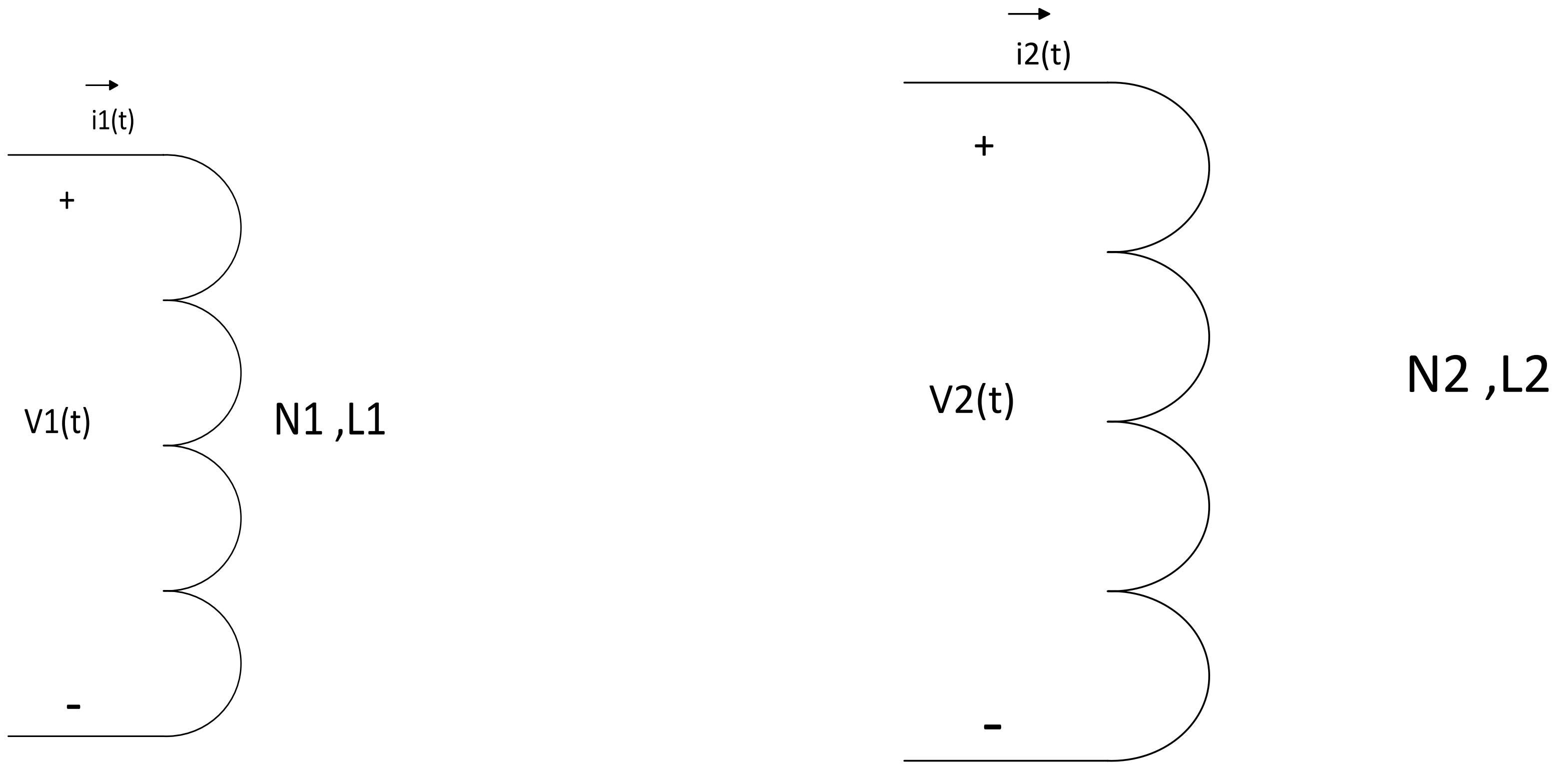
- **Transformers are constructed of two coils placed so that the Changing flux developed by one will link the other**
- **When two coils are placed close to each other, a changing flux in one coil will cause an induced voltage in the second coil.**



- **The coils are said to have mutual inductance M, which can either add or subtract from the total inductance depending on if the fields are aiding or opposing.**
- **The coil to which the source is applied is called the primary coil**
- **The coil to which the load is applied is called the secondary coil**



Mutually coupled circuit



$$V_1(t) = N_1 \frac{d\Phi_1}{dt} \qquad V_2(t) = N_2 \frac{d\Phi_2}{dt}$$

$$V_1(t) = N_1 \frac{d\Phi_1}{dt} \qquad\qquad V_2(t) = N_2 \frac{d\Phi_2}{dt}$$

$\Phi_1 =$ The magnetic fluxes threading through the coil 1

$\Phi_2 =$ The magnetic fluxes threading through the coil 2

$$\mathbf{\Phi_1 = K_1\ N_1 i_1(t)}$$

$$\mathbf{\Phi_2 = K_2\ N_2 i_2(t)}$$

K_1, K_2 are constants related to the geometry of construction and the permeability of the material used for the coils.

$$V_1(t) = N_1 \frac{d\Phi_1}{dt} \qquad V_2(t) = N_2 \frac{d\Phi_2}{dt}$$

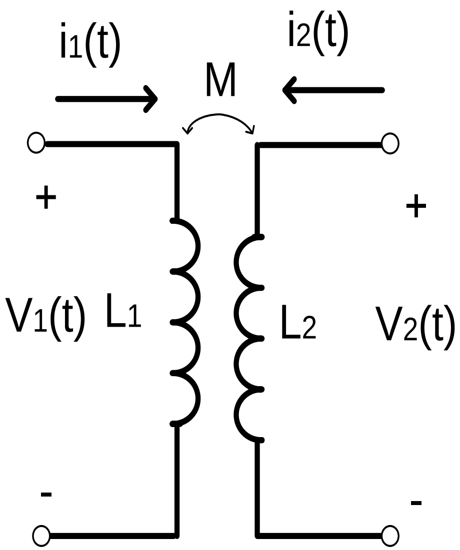
$$V_1(t) = N_1 \frac{d}{dt}\left[K_1 N_1 i_1(t)\right] \qquad V_2(t) = N_2 \frac{d}{dt}\left[K_2 N_2 i_2(t)\right]$$

$$V_1(t) = K_1 N_1^{\,2} \frac{d\left[i_1(t)\right]}{dt} \qquad V_2(t) = K_2 N_2^{\,2} \frac{d\left[i_2(t)\right]}{dt}$$

$$\text{Define } \; L_1 = K_1 N_1^{\,2} \qquad \text{Define } \; L_2 = K_2 N_2^{\,2}$$

$$\therefore V_1(t) = L_1 \frac{d\left[i_1(t)\right]}{dt} \qquad \therefore V_2(t) = L_2 \frac{d\left[i_2(t)\right]}{dt}$$

Now if we bring the two coils close to each other

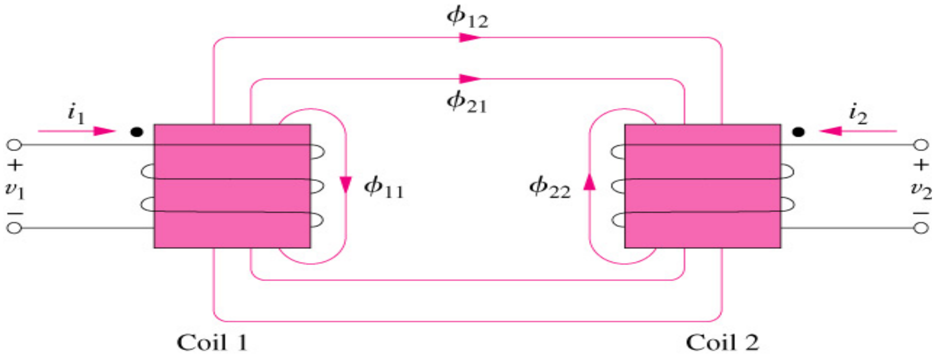


$$V_1(t) = N_1 \frac{d\Phi_1}{dt}$$

$$\phi_1 = \phi_{11} \pm \phi_{12}$$

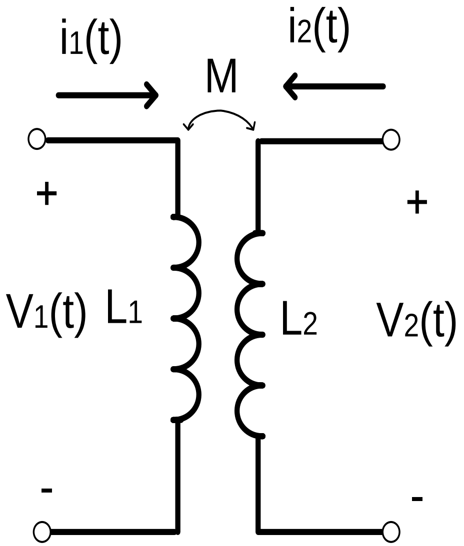
$\phi_{11} \equiv$ **the flux caused in coil 1 by current in coil 1**

$\phi_{12} \equiv$ **the flux caused in coil 1 by current in coil 2**



$$\phi_{11} = K_{11} N_1 i_1(t)$$

$$\phi_{12} = K_{12} N_2 i_2(t)$$

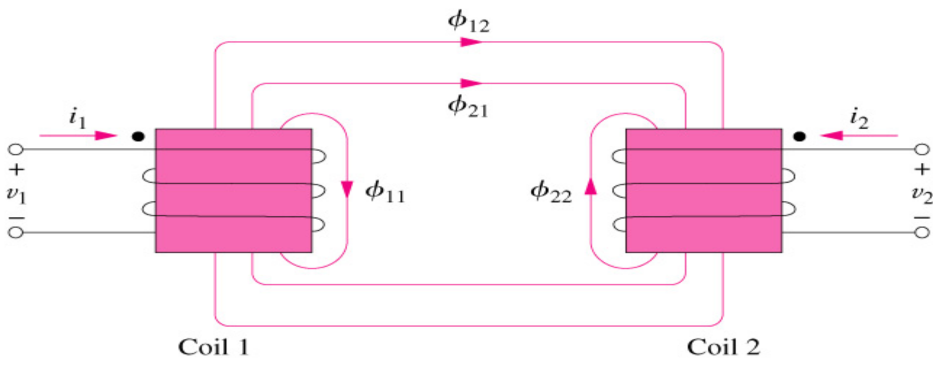


$$V_2(t) = N_2 \frac{d\Phi_2}{dt}$$

$$\phi_2 = \phi_{22} \pm \phi_{21}$$

$\phi_{22} \equiv$ **the flux caused in coil 2 by current in coil 2**

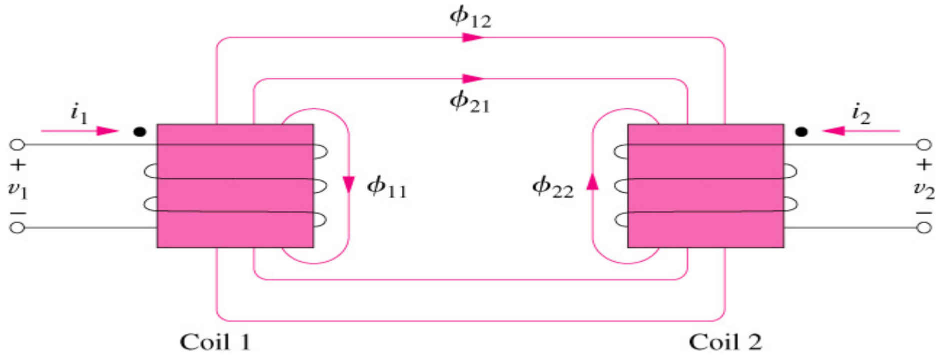
$\phi_{21} \equiv$ **the flux caused in coil 2 by current in coil 1**



$$\phi_{22} = \mathbf{K_{22}\, N_2 i_2(t)}$$

$$\phi_{21} = \mathbf{K_{21}\, N_1 i_1(t)}$$

$$\mathbf{K_{12} = K_{21} = K_m}$$



$$V_1(t) = N_1 \frac{d\Phi_1}{dt}$$

$$\phi_1 = \phi_{11} \pm \phi_{12}$$

$$K_{12} = K_{21} = K_m$$

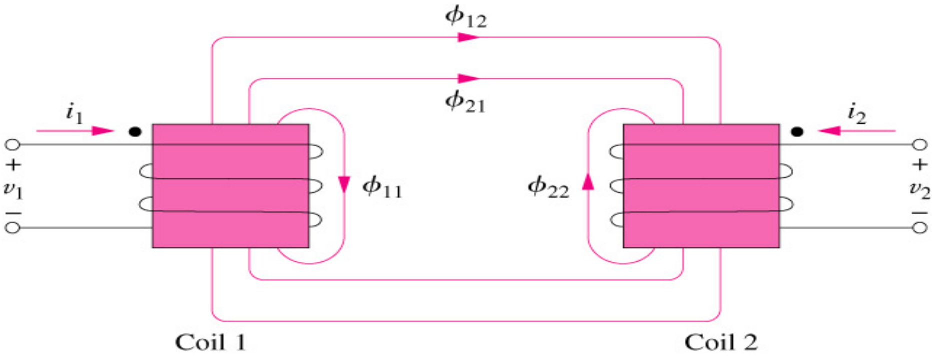
$$V_1(t) = N_1 \frac{d}{dt}\left(\left[K_{11} N_1 i_1(t) \pm K_{12} N_2 i_2(t)\right]\right)$$

$$V_1(t) = K_{11} N_1^{\,2} \frac{d}{dt}\left(i_1(t)\right) \pm K_{12} N_2 N_1 \frac{d}{dt}\left(i_2(t)\right)$$

Let $L_1 = K_{11} N_1^2$ **; self-inductance of coil 1**

$M = K_{12} N_1 N_2$ **; mutual inductance of coil1 and 2**

$$V_1(t) = L_1 \frac{d}{dt}\left(i_1(t)\right) \pm M \frac{d}{dt}\left(i_2(t)\right)$$



$$V_2(t) = N_2 \frac{d\Phi_2}{dt}$$

$$\phi_2 = \phi_{22} \pm \phi_{21}$$

$$K_{12} = K_{21} = K_m$$

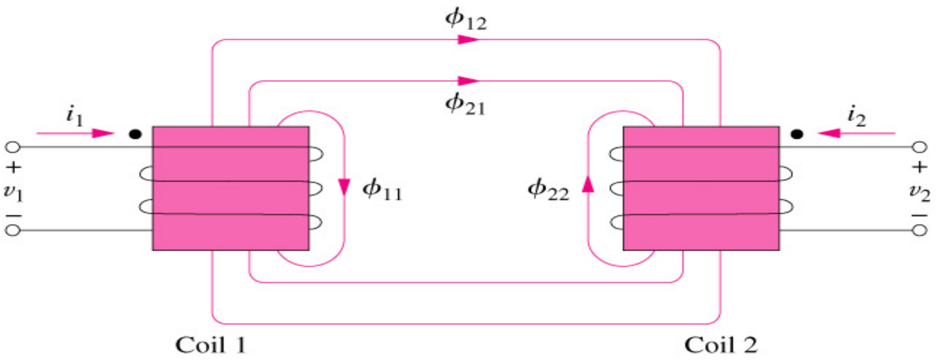
$$V_2(t) = N_2 \frac{d}{dt}\left(\left[K_{22}N_2 i_2(t) \pm K_{21}N_1 i_1(t)\right]\right)$$

$$V_2(t) = K_{22}N_2{}^2 \frac{d}{dt}\left(i_2(t)\right) \pm K_{21}N_2N_1 \frac{d}{dt}\left(i_1(t)\right)$$

Let $L_2 = K_{22}N_2{}^2$ **; self-inductance of coil 1**

$M = K_{21}N_1N_2$ **; mutual inductance of coil 1and 2**

$$V_2(t) = L_2 \frac{d}{dt}\left(i_2(t)\right) \pm M \frac{d}{dt}\left(i_1(t)\right)$$



$$M = K_{12} N_1 N_2$$

$$L_2 = K_{22} N_2^2$$

$$L_1 = K_{11} N_1^2$$

$$M = K_m \sqrt{\frac{L_1}{K_{11}}} \sqrt{\frac{L_2}{K_{22}}}$$

$$M = K_m \sqrt{\frac{L_1 L_2}{K_{11} K_{22}}} \qquad M = K\sqrt{L_1 L_2}$$

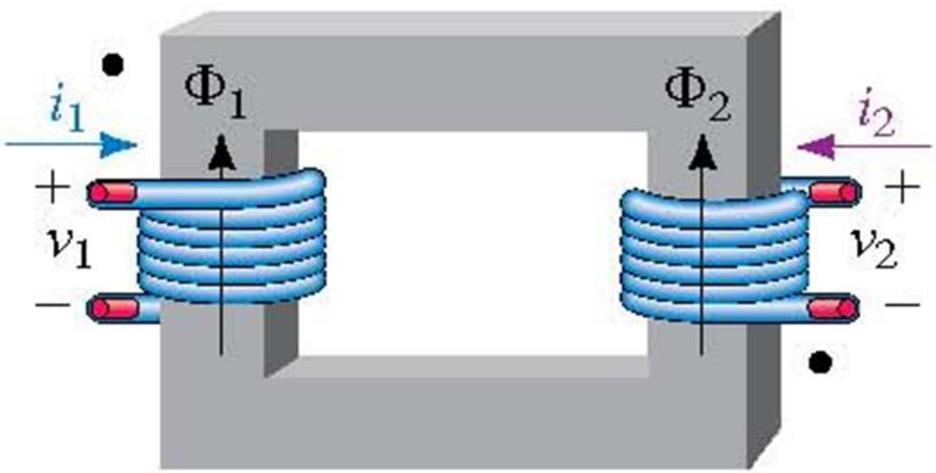
$$K = \frac{K_m}{\sqrt{K_{11} K_{22}}}$$

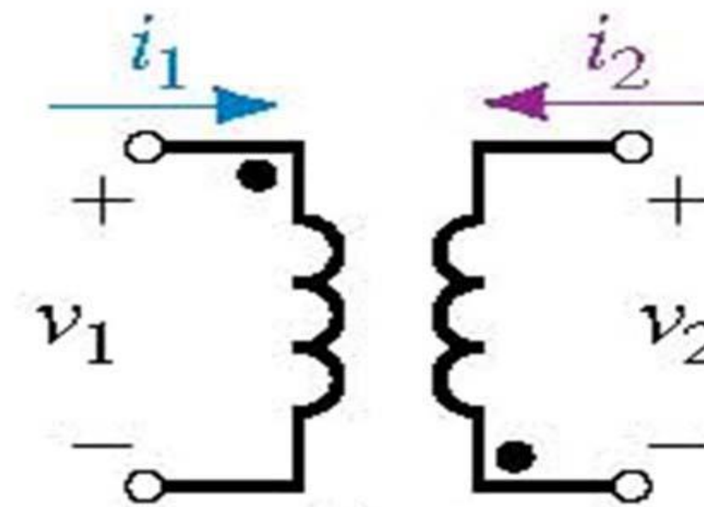
coeffecient of coupling

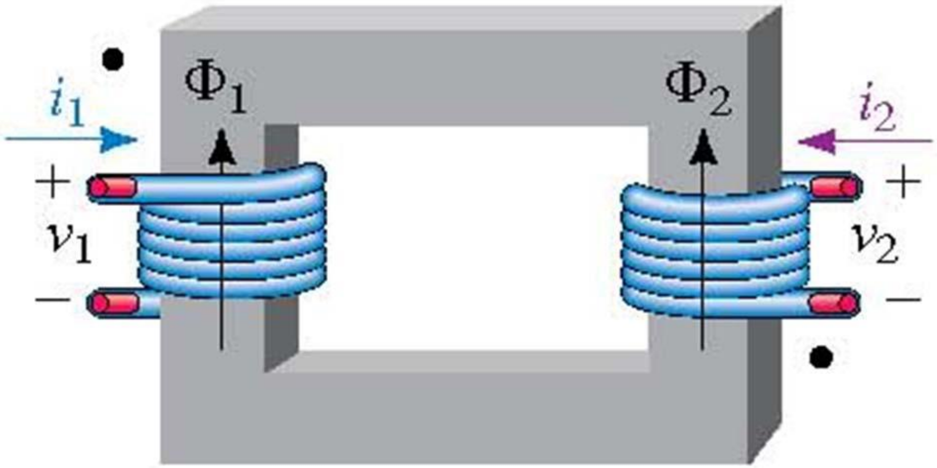
$$1 \geq k \geq 0$$

$$V_1(t) = L_1 \frac{d}{dt}\left(i_1(t)\right) \pm M \frac{d}{dt}\left(i_2(t)\right)$$

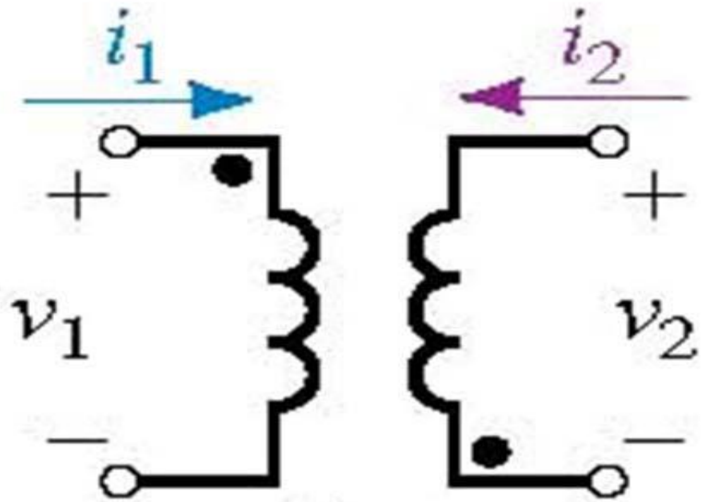
- Depending on the direction of the coil winding and the relative position of the coils, the voltage due to the mutual inductance either aids or opposes the voltage due to the self-inductance.



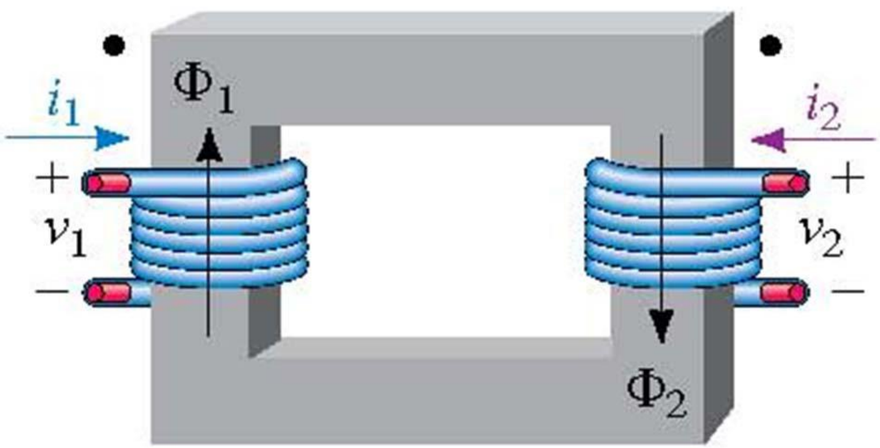




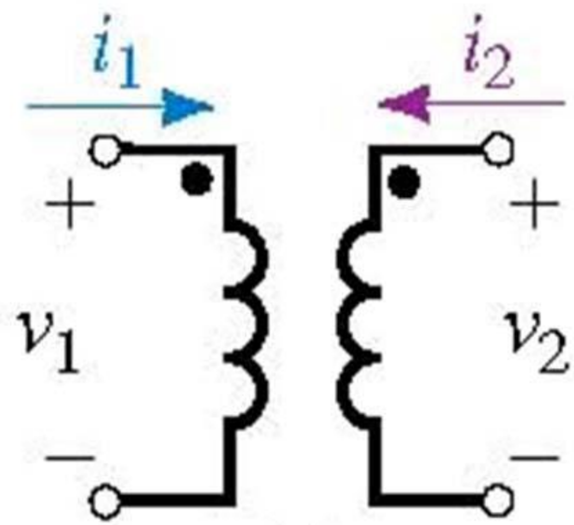
i_1
Φ_1
Φ_2
i_2
+
v_1
−
+
v_2
−



i_1
i_2
+
v_1
−
+
v_2
−

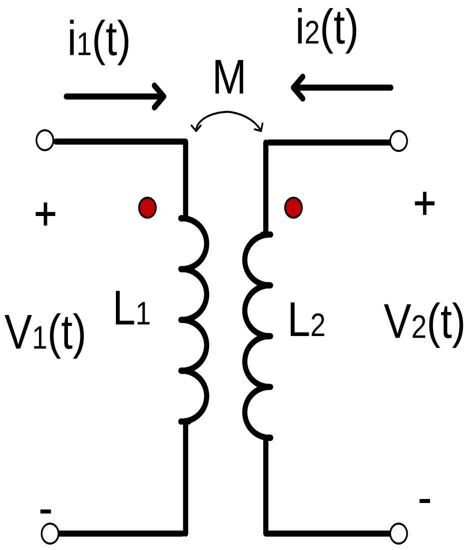


i_1
Φ_1
i_2
+
v_1
−
+
v_2
−
Φ_2



i_1
i_2
+
v_1
−
+
v_2
−

Dot Convention



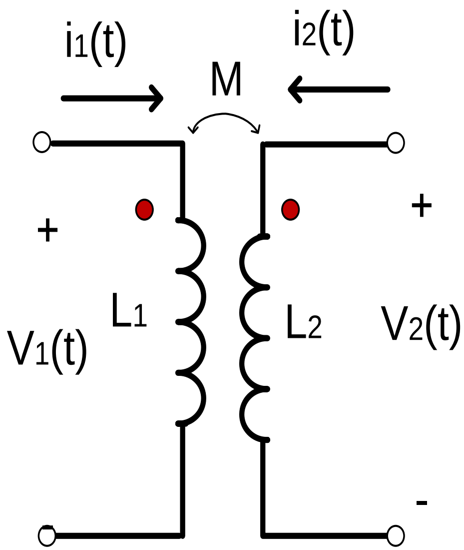
1-If both currents are directed into or away from corresponding terminals, the voltage due to the mutual inductance is of the same sign as the voltage due to self inductance

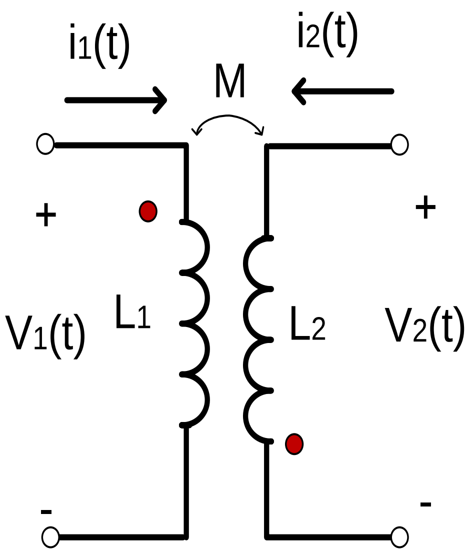
2-If one current enter a dotted terminal and the other enters an un-dotted terminal ,

Then voltage due to mutual and self inductance have opposite signs.

Important

Flux Direction



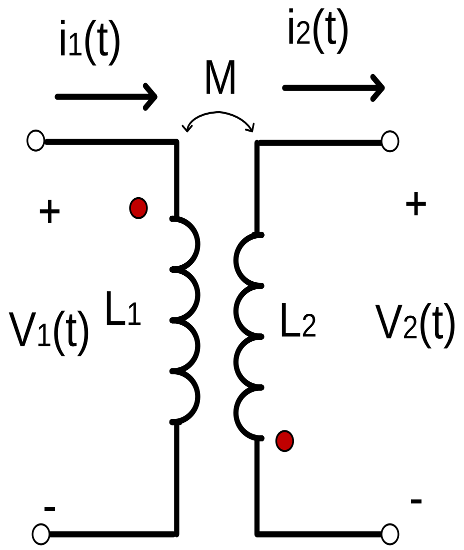


$$V_1(t) = L_1 \frac{d}{dt}\left(i_1(t)\right) + M \ \frac{d}{dt}\left(i_2(t)\right)$$

$$V_2(t) = L_2 \frac{d}{dt}\left(i_2(t)\right) + M \ \frac{d}{dt}\left(i_1(t)\right)$$

$$V_1(t) = L_1 \frac{d}{dt}\left(i_1(t)\right) - M \ \frac{d}{dt}\left(i_2(t)\right)$$

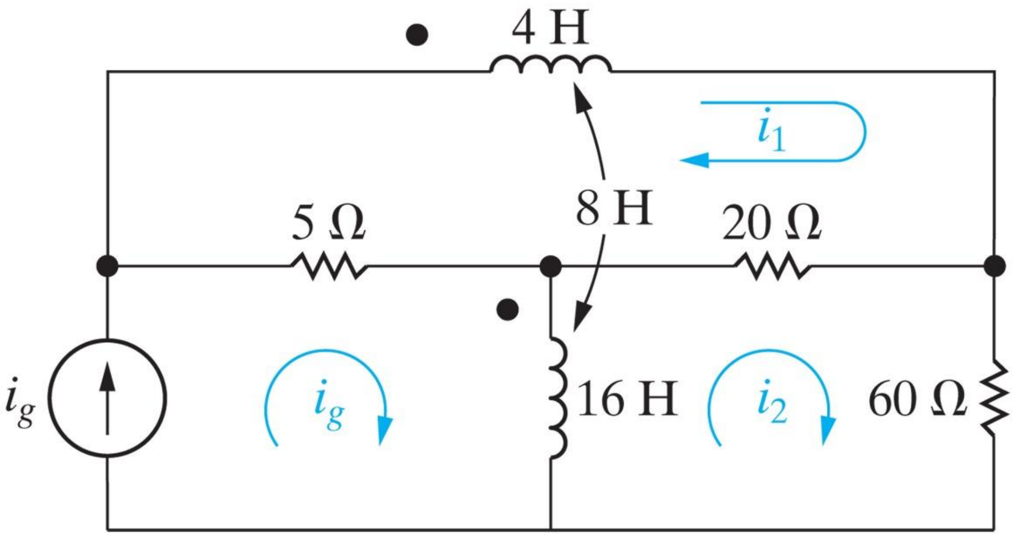
$$V_2(t) = L_2 \frac{d}{dt}\left(i_2(t)\right) - M \ \frac{d}{dt}\left(i_1(t)\right)$$



$$V_1(t) = L_1 \frac{d}{dt}\left(i_1(t)\right) + M \ \frac{d}{dt}\left(i_2(t)\right)$$

$$V_2(t) = -L_2 \frac{d}{dt}\left(i_2(t)\right) - M \ \frac{d}{dt}\left(i_1(t)\right)$$

Example



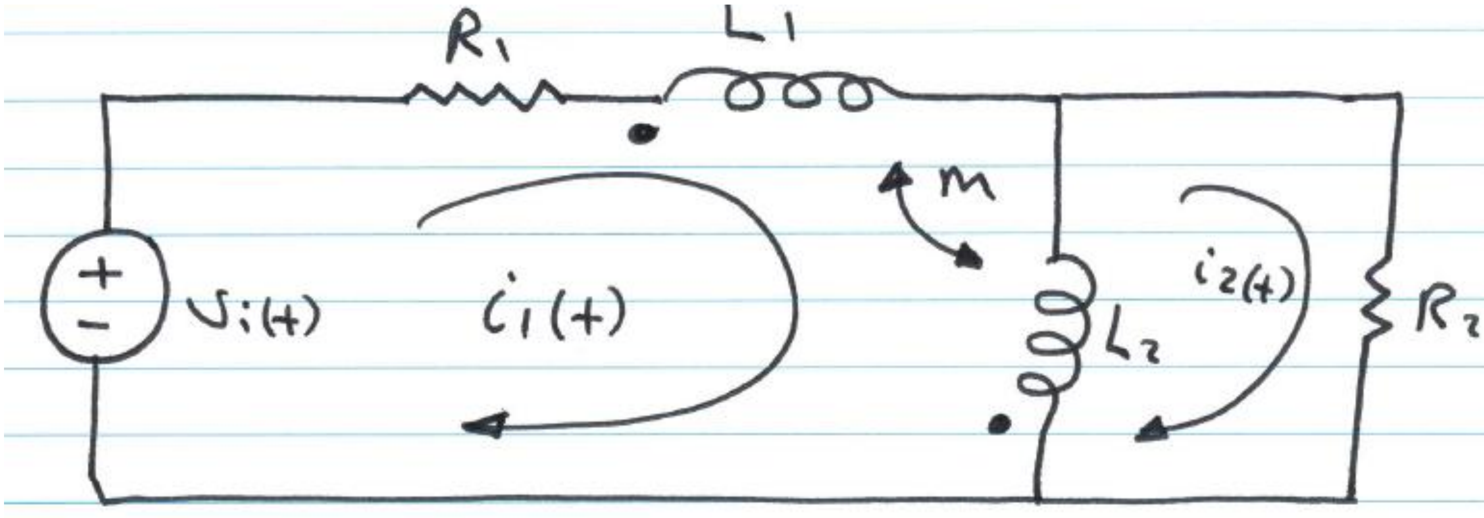
Mesh 1

$$0 = 4\frac{d}{dt}(i_1) + 8\ \frac{d}{dt}(i_g - i_2) + 20(i_1 - i_2) + 5(i_1 - i_g)$$

Mesh 2

$$0 = 20(i_2 - i_1) + 60(i_2) + 16\frac{d}{dt}(i_2 - i_g) - 8\frac{d}{dt}(i_1)$$

Example



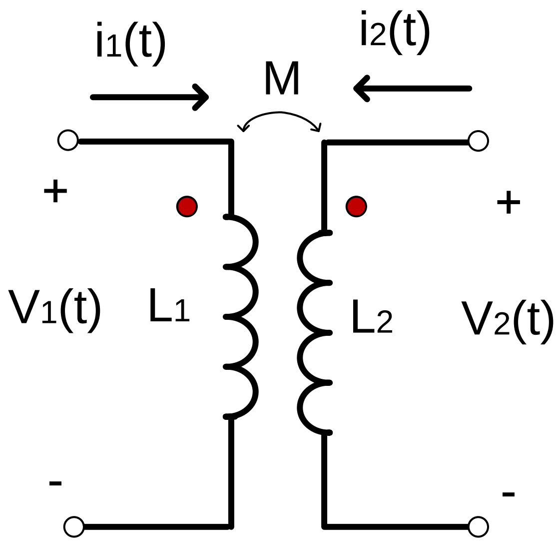
Mesh 1

$$V_i(t) = R_1 i_1(t) + L_1 \frac{d}{dt}\left(i_1(t)\right) - M \ \frac{d}{dt}\left(i_1 - i_2\right) + L_2 \frac{d}{dt}\left(i_1 - i_2\right) - M \ \frac{d}{dt}\left(i_1(t)\right)$$

Mesh 2

$$0 = R_2 i_2(t) + L_2 \frac{d}{dt}\left(i_2 - i_1\right) + M \ \frac{d}{dt}\left(i_1(t)\right)$$

Phasors and mutual inductance



$$V_1(t) = L_1 \frac{d}{dt}\left(i_1(t)\right) + M \ \frac{d}{dt}\left(i_2(t)\right)$$

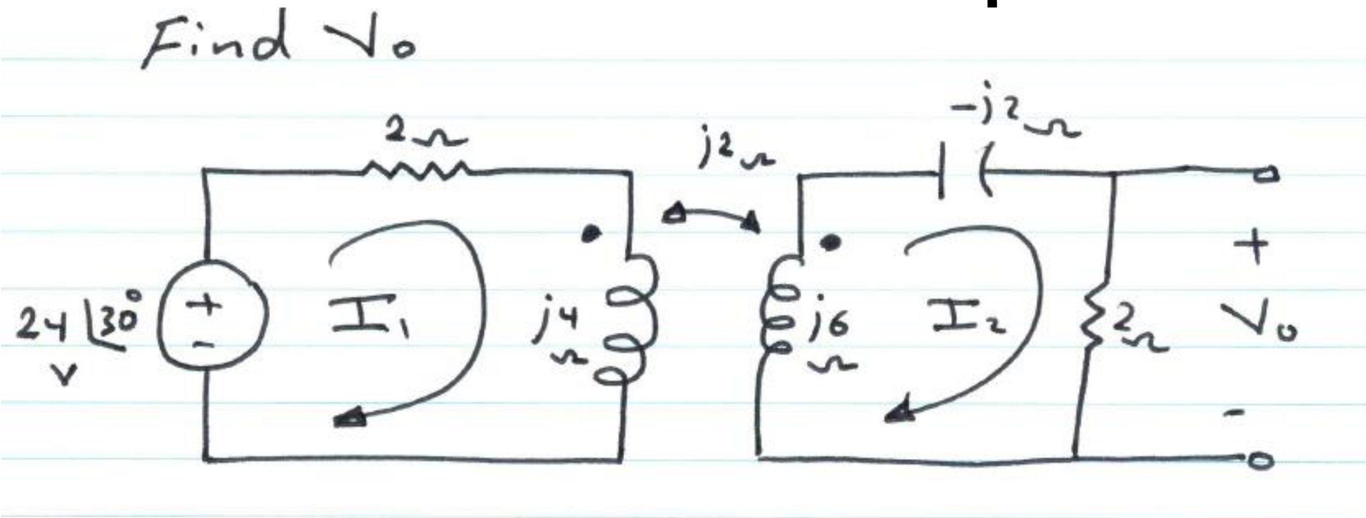
$$V_2(t) = L_2 \frac{d}{dt}\left(i_2(t)\right) + M \ \frac{d}{dt}\left(i_1(t)\right)$$

$$\Downarrow$$

$$\boldsymbol{V_1} = \boldsymbol{j\omega} \mathrm{L}_1 \boldsymbol{I}_1 + \boldsymbol{j\omega} \mathrm{M} \boldsymbol{I}_2$$

$$\boldsymbol{V_2} = \boldsymbol{j\omega} \mathrm{M} \boldsymbol{I}_1 + \boldsymbol{j\omega} \mathrm{L}_2 \boldsymbol{I}_2$$

Example



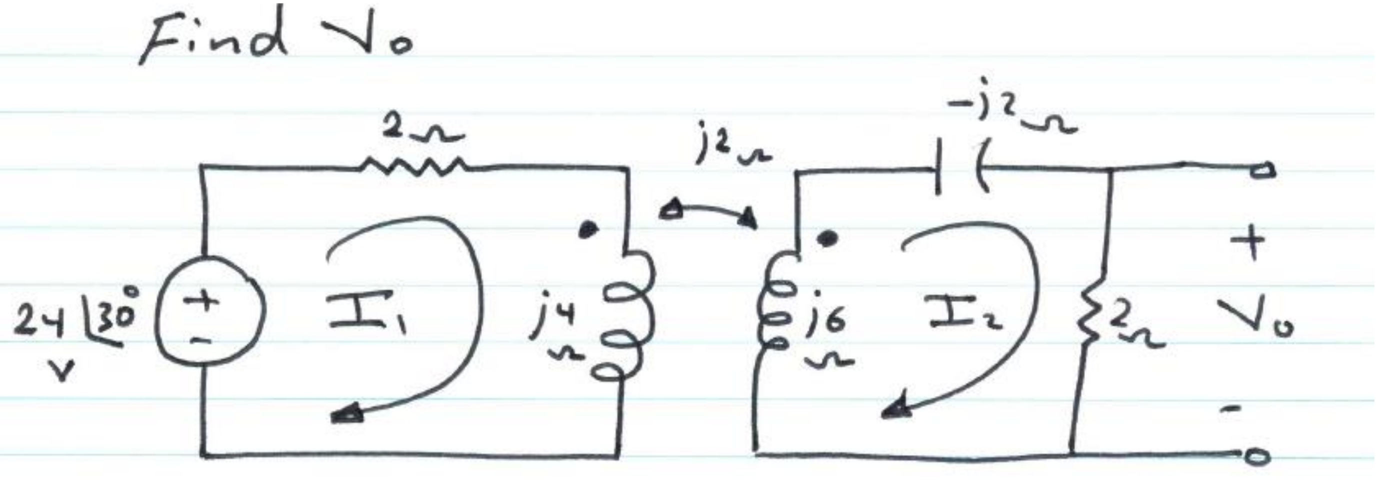
$$\boldsymbol{2}4\langle 30 = 2\boldsymbol{I}_1 + \boldsymbol{j}4\boldsymbol{I}_1 - \mathrm{j}2\boldsymbol{I}_2$$

$$\boldsymbol{2}4\langle 30 = \left(2 + \boldsymbol{j}4\right)\boldsymbol{I}_1 - \mathrm{j}2\boldsymbol{I}_2 \ldots\ldots\ldots(1)$$

$$0 = -\boldsymbol{j}2\boldsymbol{I}_2 + 2\boldsymbol{I}_2 + \boldsymbol{j}6\boldsymbol{I}_2 - \mathrm{j}2\boldsymbol{I}_1$$

$$0 = -\boldsymbol{j}2\boldsymbol{I}_1 + \left(2 + \boldsymbol{j}4\right)\boldsymbol{I}_2 \ldots\ldots\ldots\ldots(2)$$

Example



$$24\angle 30 = (2 + j4)I_1 - j2I_2 ....(1)$$

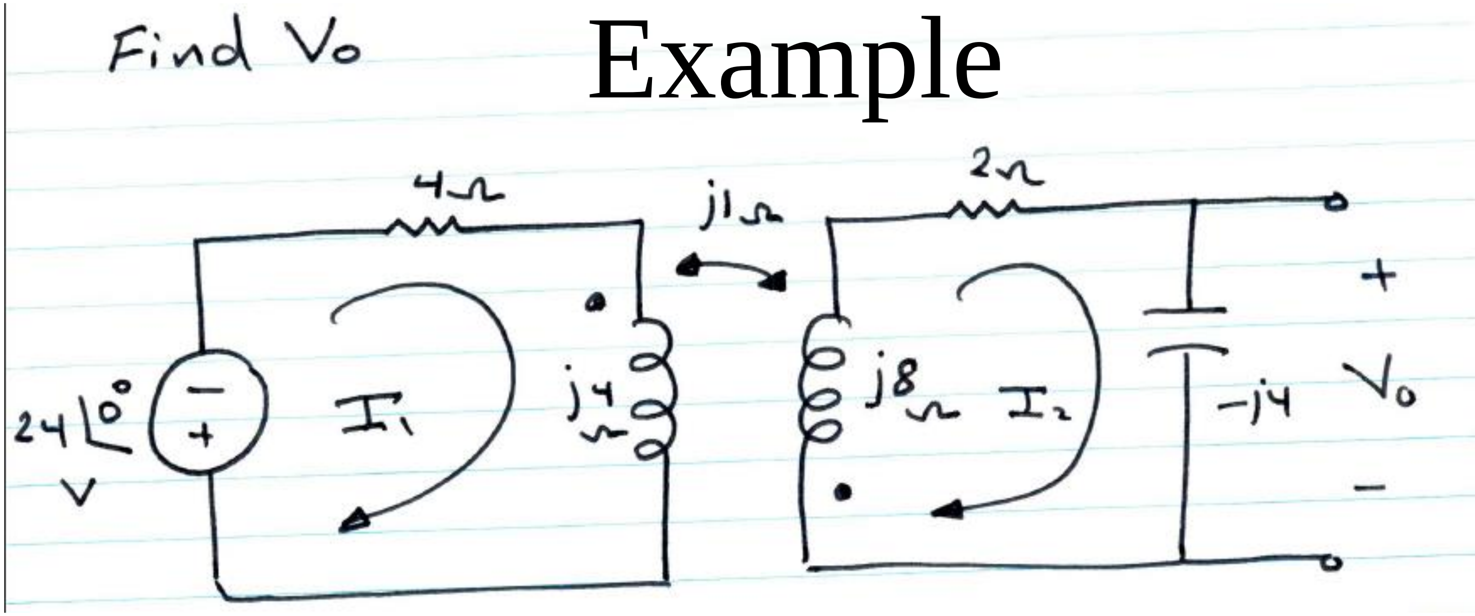
$$0 = -j2I_1 + (2 + j4)I_2 ....(2)$$

Solving (1) & (2) yields

$$I_2 = 2.685 \angle 3.24^o \text{ A}$$

$$V_o = 2I_2 = 5.37 \angle 3.24^o \text{ v}$$

Example



$$-\mathbf{2}4\langle 0^{o} = \left(4 + j4\right)I_1 + \mathrm{j}1I_2 \ldots\ldots\ldots(1)$$

$$0 = 2I_2 - j4I_2 + j8I_2 + j1I_1$$

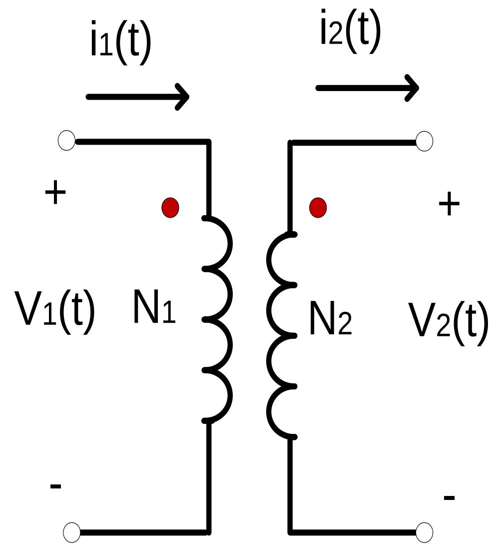
$$0 = j1I_1 + \left(2 + j4\right)I_2 \ldots\ldots\ldots\ldots(2)$$

$$\text{Solving (1) \& (2) yields}$$

$$I_2 = 0.96 \angle -16.26^o \text{ A}$$

$$V_o = -j4I_2 = 3.84 \angle -106.26^o \text{ V}$$

The ideal transformer



$$V_1(t) = N_1 \frac{d\Phi}{dt}$$

$$V_2(t) = N_2 \frac{d\Phi}{dt}$$

$$\frac{V_1(t)}{V_2(t)} = \frac{N_1}{N_2}$$

Assuming P1=P2 (100% efficiency- no losses)

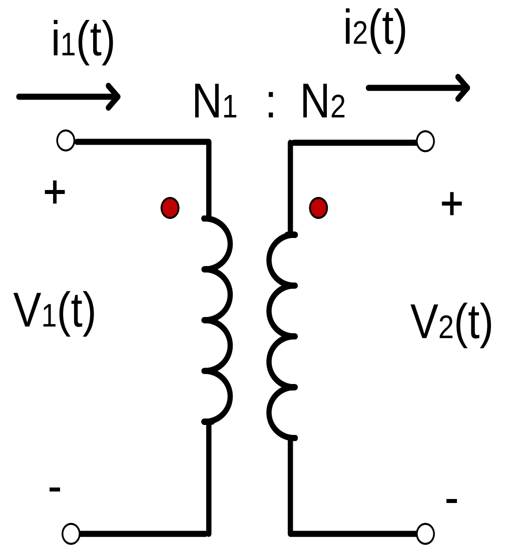
$$V_1(t)i_1(t) = V_2(t)i_2(t)$$

$$\therefore \frac{i_1(t)}{i_2(t)} = \frac{V_2(t)}{V_1(t)} = \frac{N_2}{N_1}$$

Ideal Transformer

$$\frac{i_1(t)}{i_2(t)} = \frac{N_2}{N_1}$$

$$\frac{V_2(t)}{V_1(t)} = \frac{N_2}{N_1}$$



Since the equations are algebraic, they are unchanged for phasors

$$\frac{\boldsymbol{I_1}}{\boldsymbol{I_2}} = \frac{\boldsymbol{V_2}}{\boldsymbol{V_1}} = \frac{N_2}{N_1}$$

Impedance Reflection

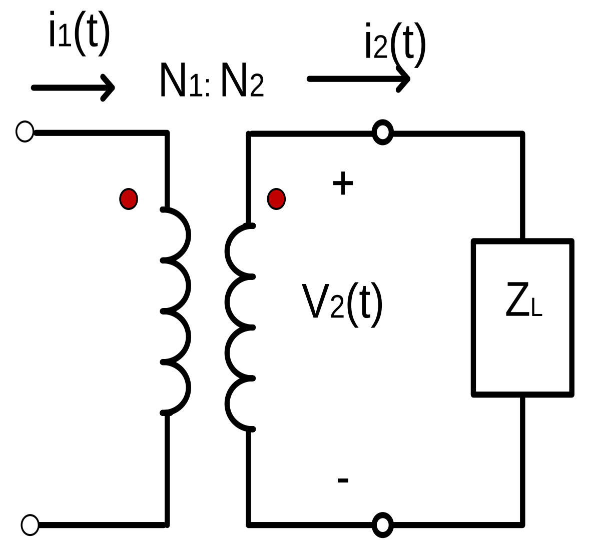
$$\mathbf{Z_{TH}} = \frac{V_1}{I_1} = \frac{\frac{N_1}{N_2}\,\mathrm{V_2}}{\frac{N_2}{N_1}\,\mathrm{I_2}}$$

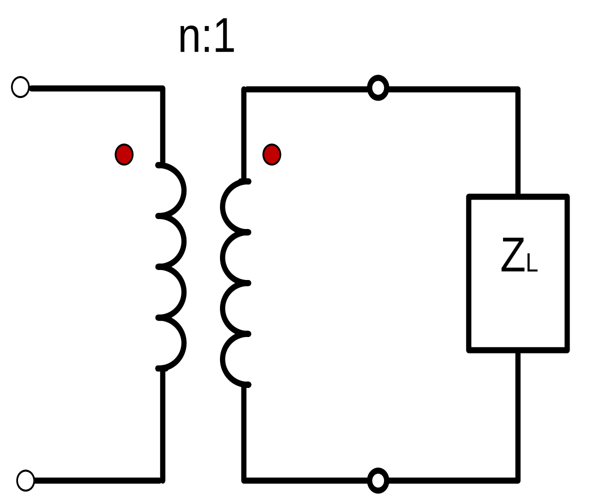
$$\mathrm{Z_{TH}} = \frac{N_1^2}{N_2^2}\frac{\mathrm{V_2}}{\mathrm{I_2}}$$

$$\mathbf{Z_{TH}} = \frac{N_1^2}{N_2^2}\,\mathbf{Z_L}$$

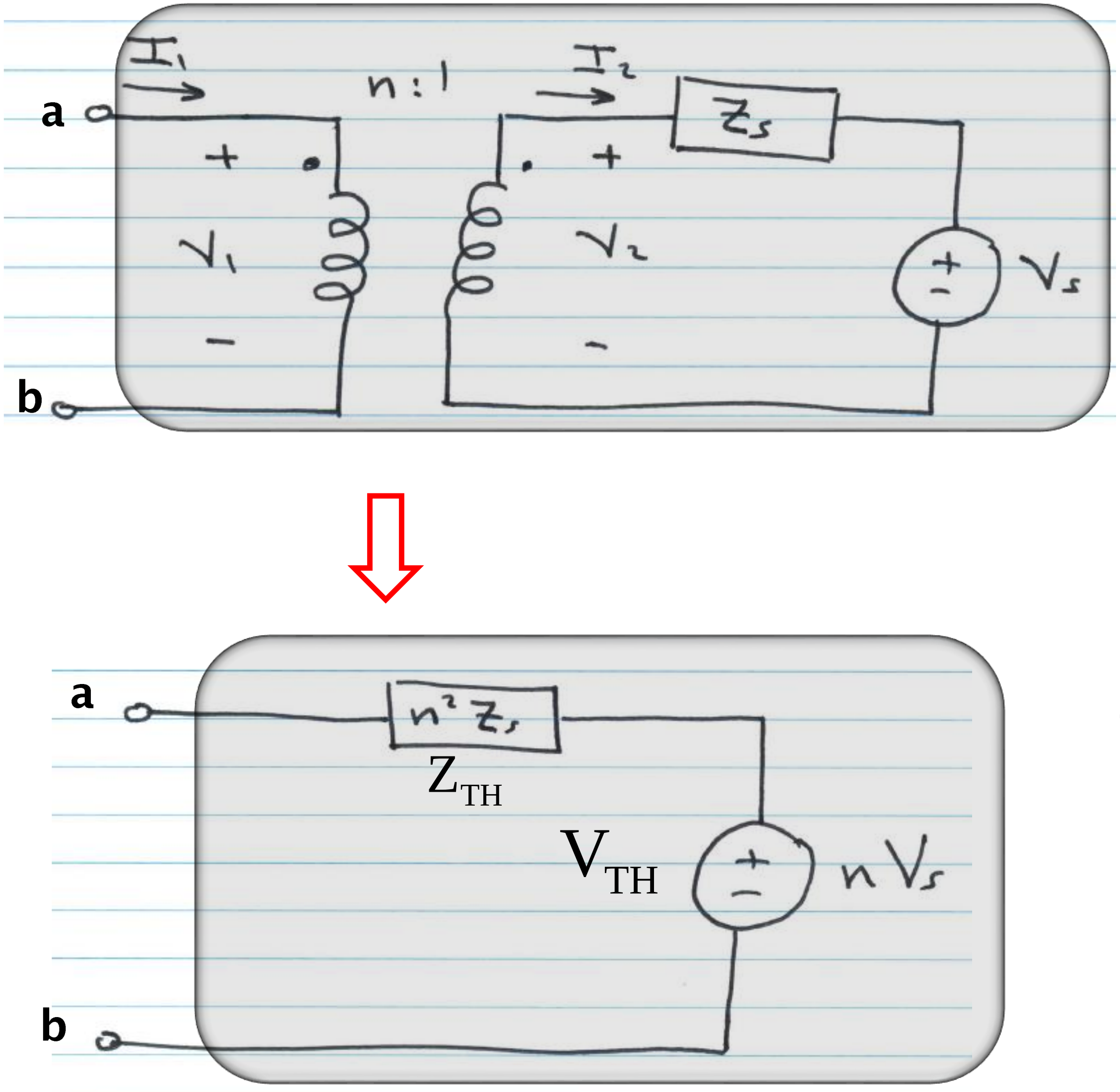
$$\textbf{Let}\ \frac{N_1}{N_2} = \boldsymbol{n}$$

$$\mathbf{Z_{TH}} = \boldsymbol{n}^2\ \mathbf{Z_L}$$





Example

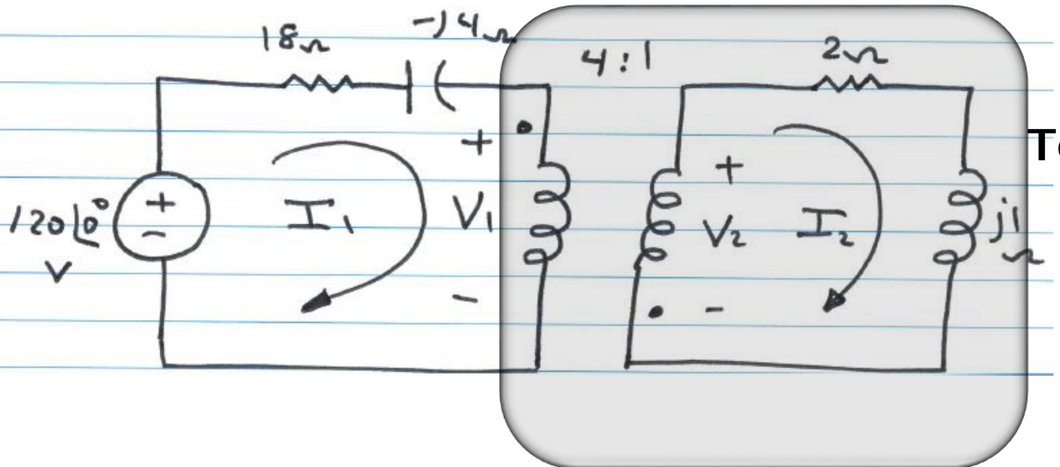


$$I_1 = I_2 = 0$$

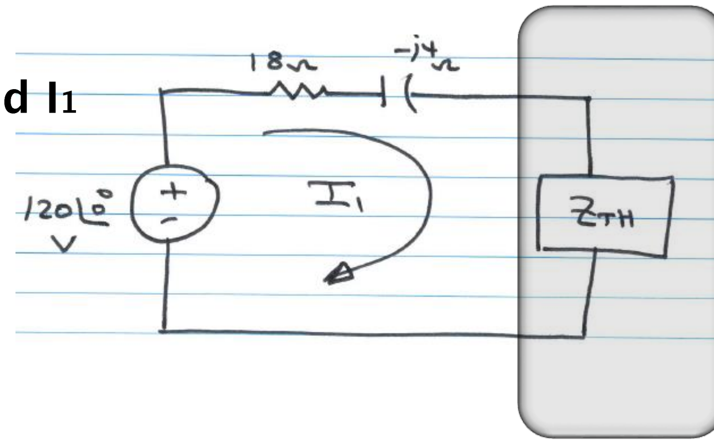
$$V_{TH} = V_1 = nV_2 = nV_S$$

$$Z_{TH} = n^2 Z_S$$

Find I_1 and I_2



To Find I_1

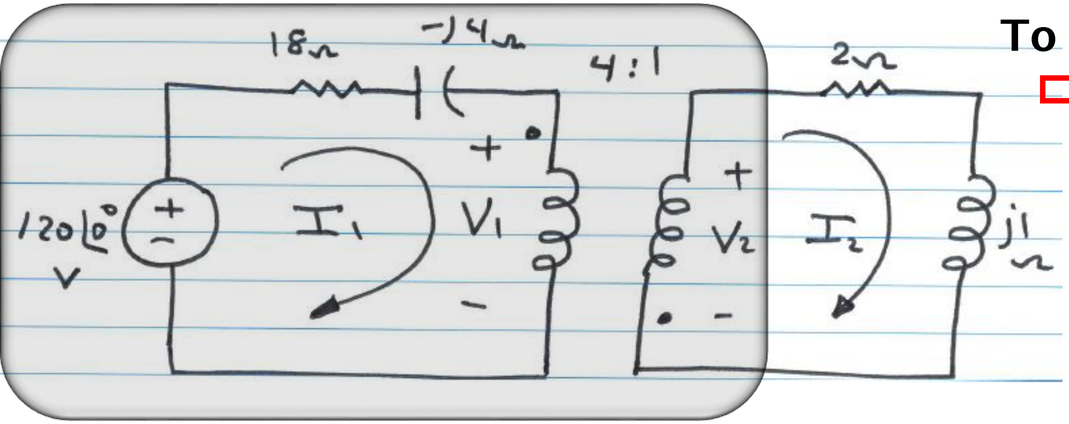


$$Z_{TH} = n^2 Z_L = 4^2(2 + j1) = 32 + j16$$

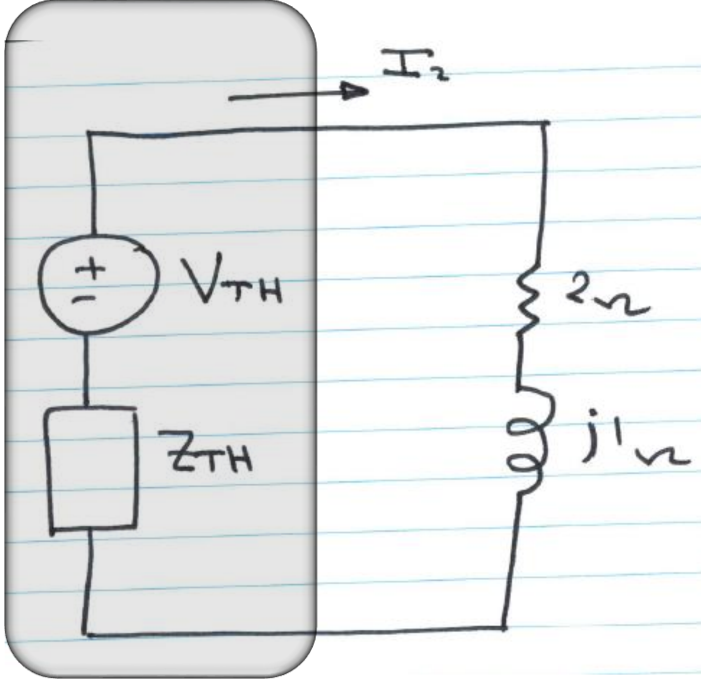
$$I_1 = \frac{120\angle 0}{18 - j4 + 32 + j16} = \frac{120\angle 0}{50 + j12}$$

$$= 2.33\angle -13.5^o \quad A$$

Find I_1 and I_2



To Find I_2



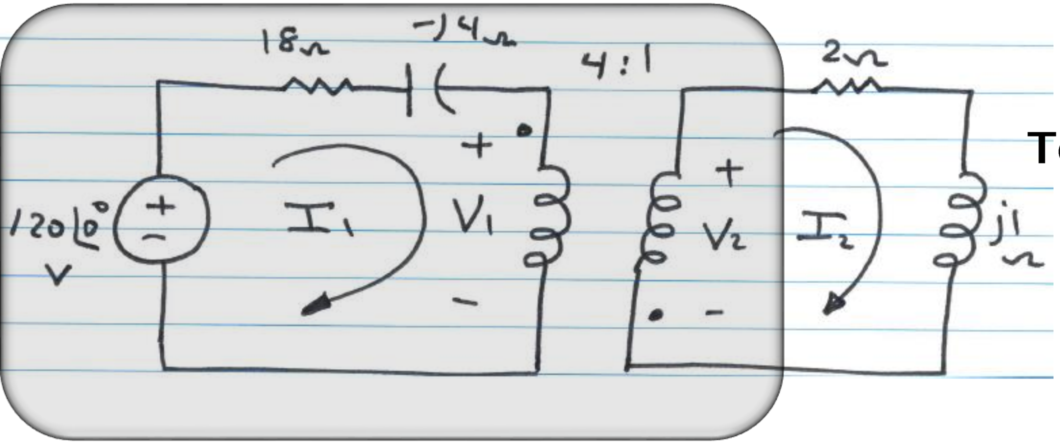
$$Z_{TH} = \frac{1}{n^2} Z_s = \frac{(18 - j4)}{4^2} = 1.125 - j0.25\ \Omega$$

$$V_{TH} = -\frac{120\langle 0}{4} = -30\langle 0\ \text{V}$$

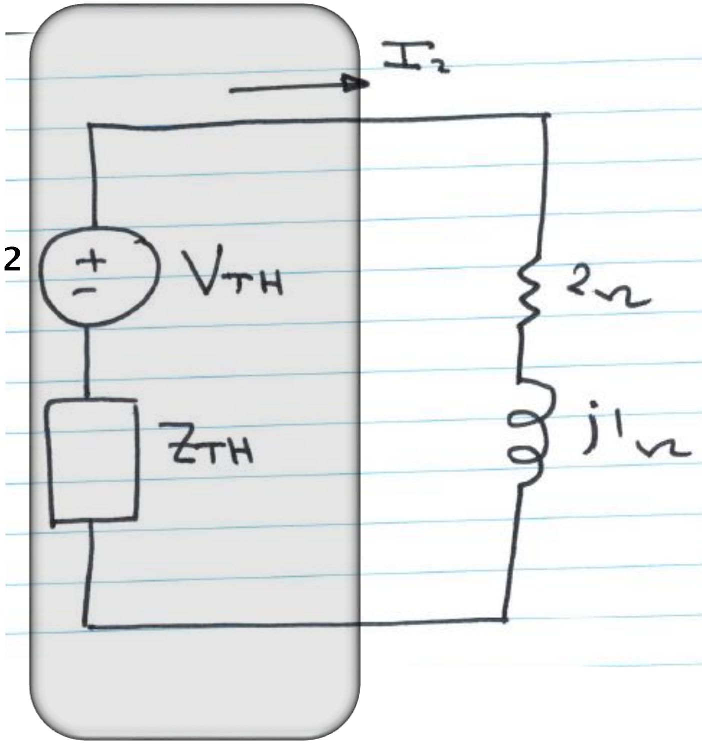
$$I_2 = \frac{V_{TH}}{Z_{TH} + 2 + j1}$$

$$= 9.335\langle 166.5^o\ \text{A}$$

Find V1 and V2



To Find I2



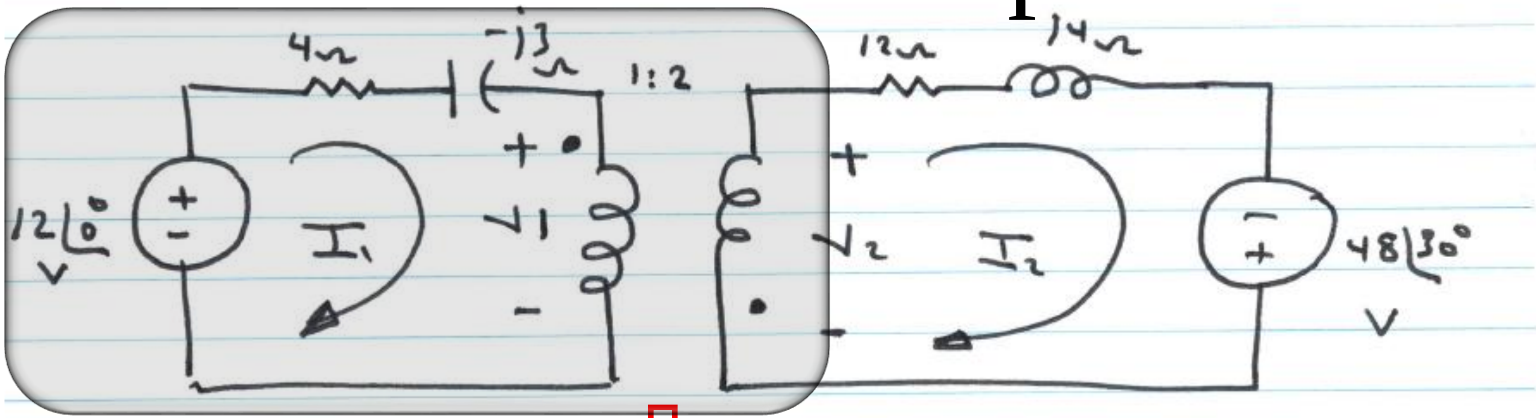
$$V_2 = I_2(2 + j1)$$

$$V_2 = 20.87\angle 193.07^o \text{ V}$$

$$V_1 = -4V_2$$

$$= 83.49\angle 3.07^o \text{ V}$$

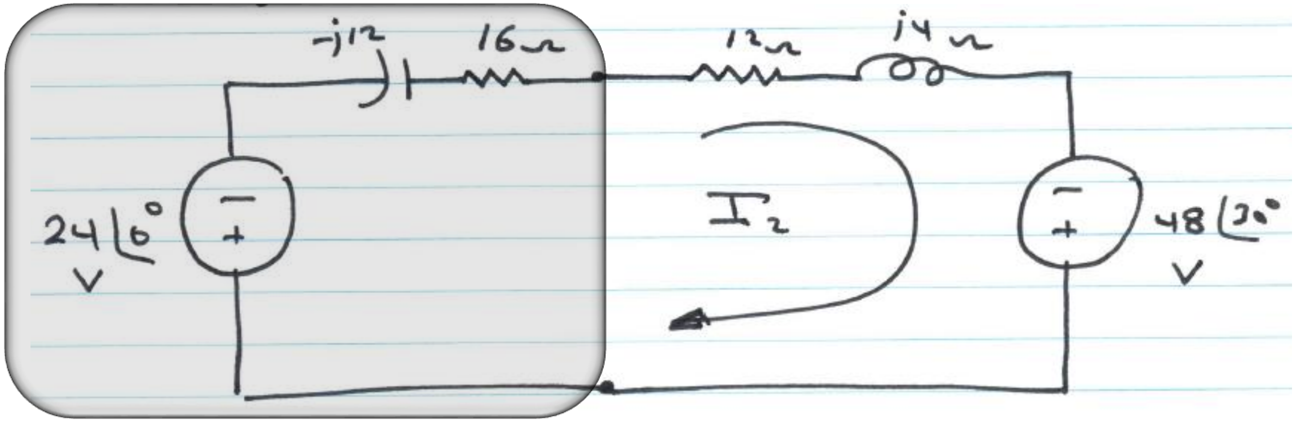
Example



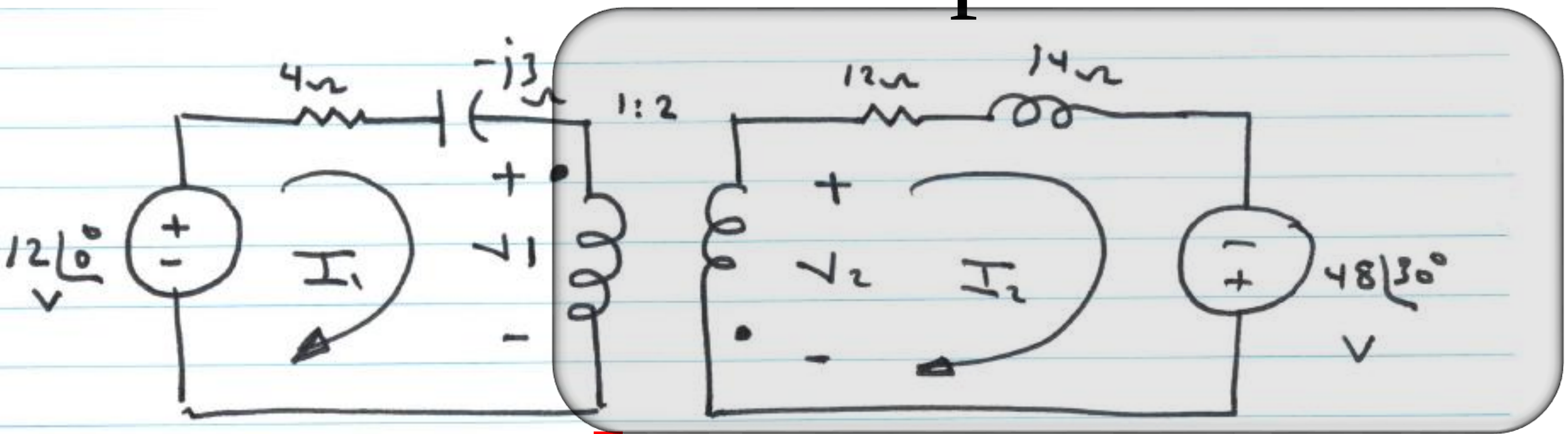
To Find I2

$$V_{TH1} = -nV_1 = -2(12\ \angle 0^O) = -24\ \angle 0^O$$

$$Z_{TH1} = n^2 Z_s = 2^2(4 - j3) = 16 - j12\ \Omega$$



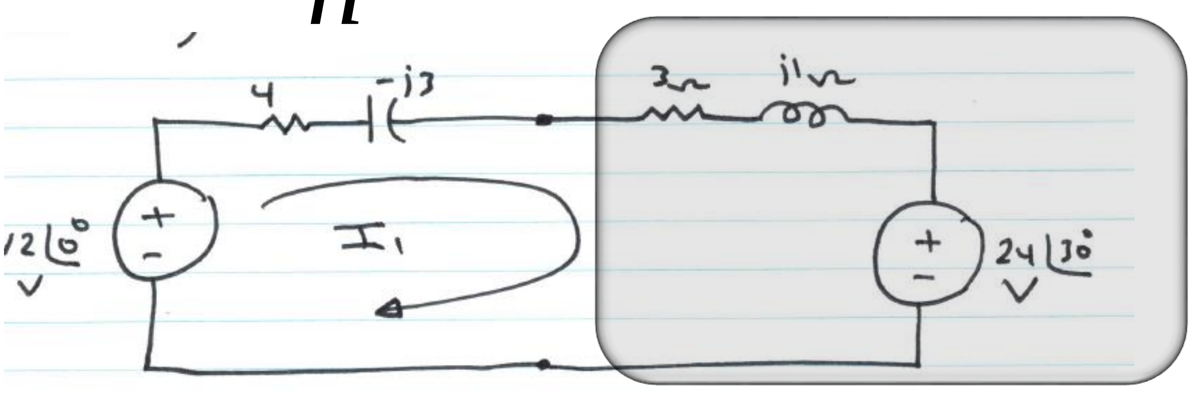
Example



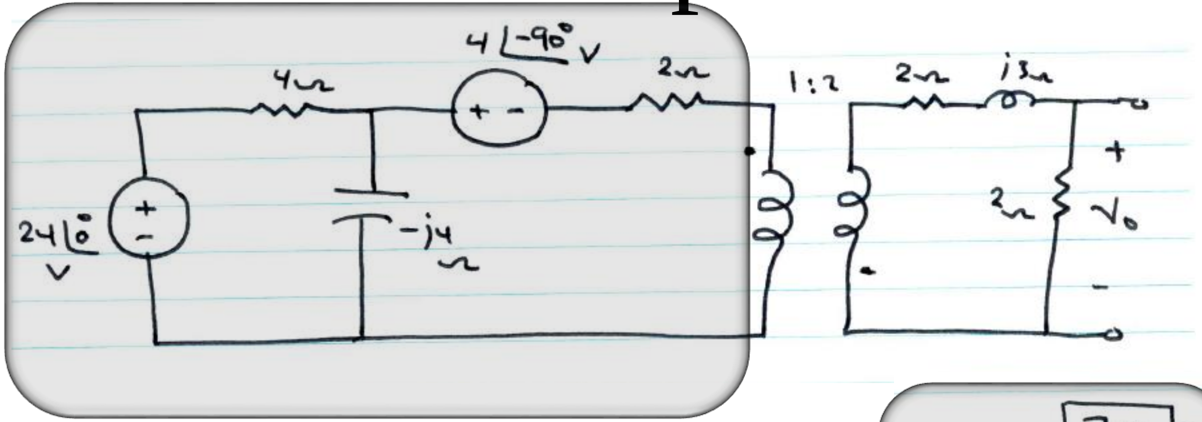
To Find I_1

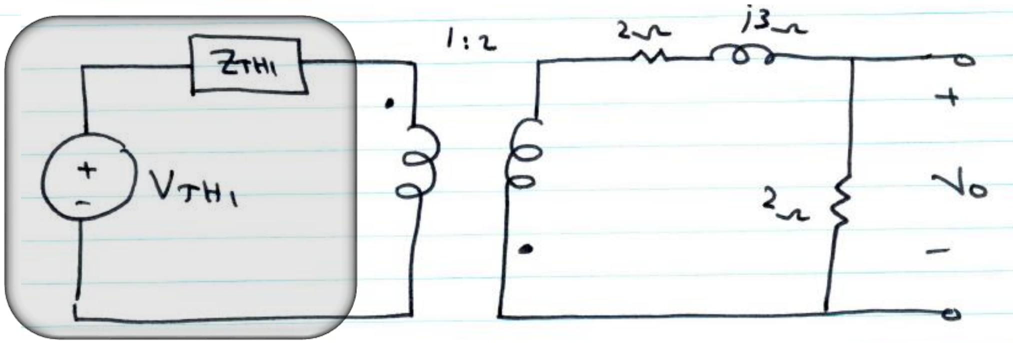
$$V_{TH2} = -\frac{1}{n}V_2 = -\frac{1}{2}(-48\angle 30^{\circ}) = +24\angle 30^{\circ}$$

$$Z_{TH2} = \frac{1}{n^2}Z_L = \frac{(12+j4)}{2^2} = 3 + j1\ \Omega$$



Example: Find Vo



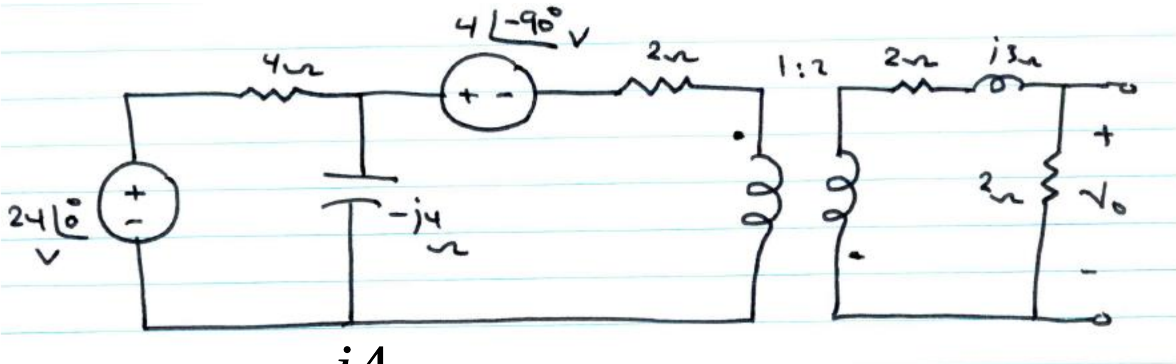


$$Z_{TH1} = 2 + \left(4 // - j4\right)$$

$$= 2 + \left(\frac{-j16}{4 - j4}\right)\left(\frac{4 + j4}{4 + j4}\right) = 2 + \left(\frac{-j64 + 64}{32}\right)$$

$$= 2 + \left(2 - j2\right) = 4 - j2 \ \Omega$$

Example: Find Vo

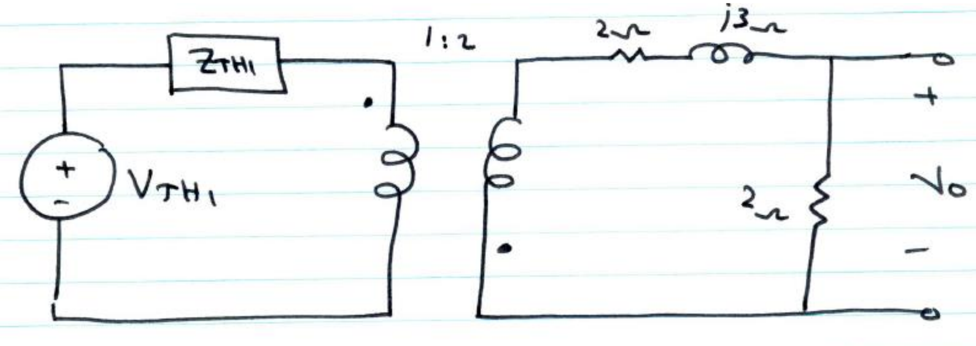


$$V_{TH1} = \frac{-j4}{4-j4} 24\langle 0^o - 4\langle -90^o$$

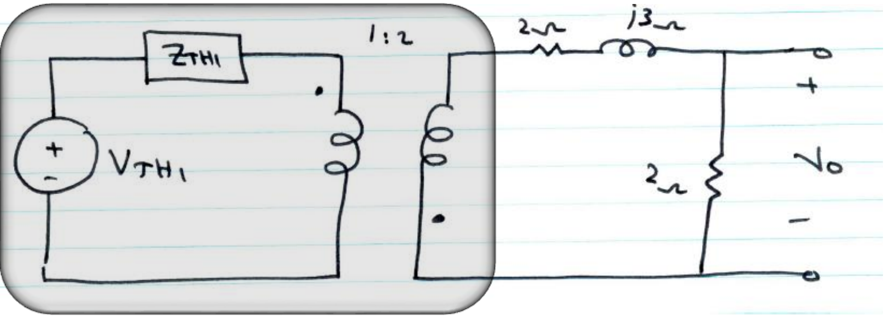
$$\frac{-j4}{4-j4} = \frac{-j4}{4-j4} x \frac{4+j4}{4+j4} = \frac{-j16+16}{32} = \frac{1}{2} - j\frac{1}{2}$$

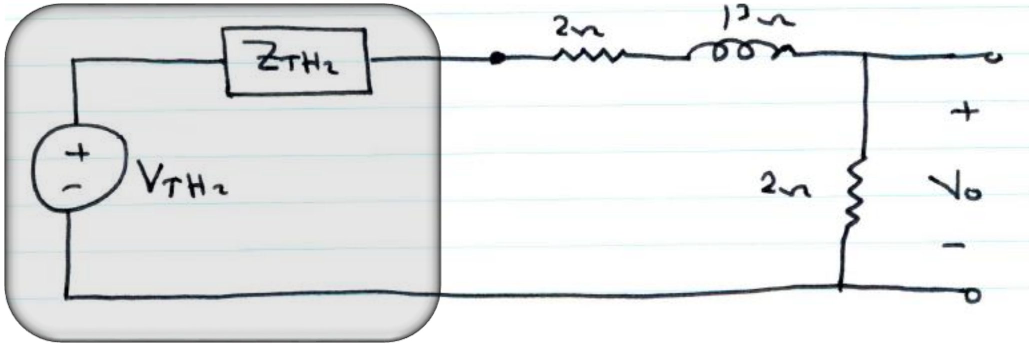
$$V_{TH1} = \left(\frac{1}{2} - j\frac{1}{2}\right) 24\langle 0^o - (-\,j4)$$

$$= 12 - j12 + j4 = 12 - j8$$



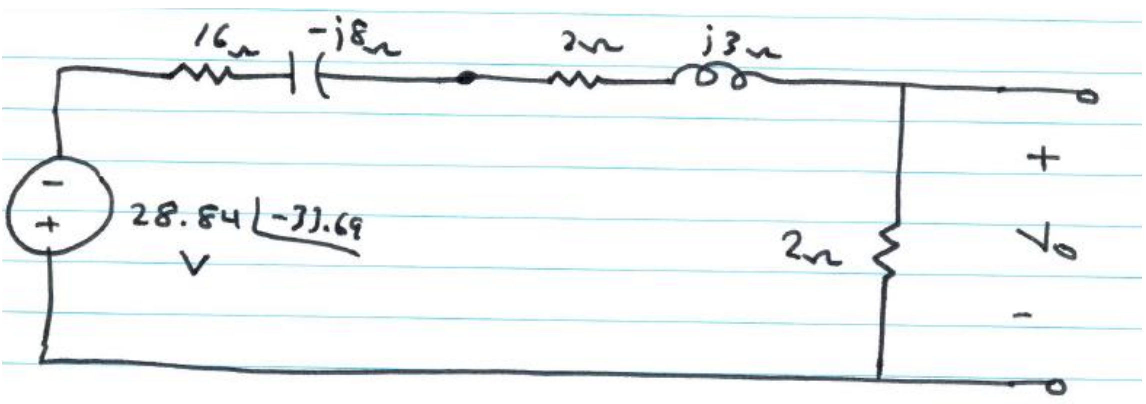
$$V_{TH1} = 14.42\langle -33.69^o \ \ V$$





$$Z_{TH2} = \left(2^2\right) Z_{TH1} = 4(4 - j2) = 16 - j8\,\Omega$$

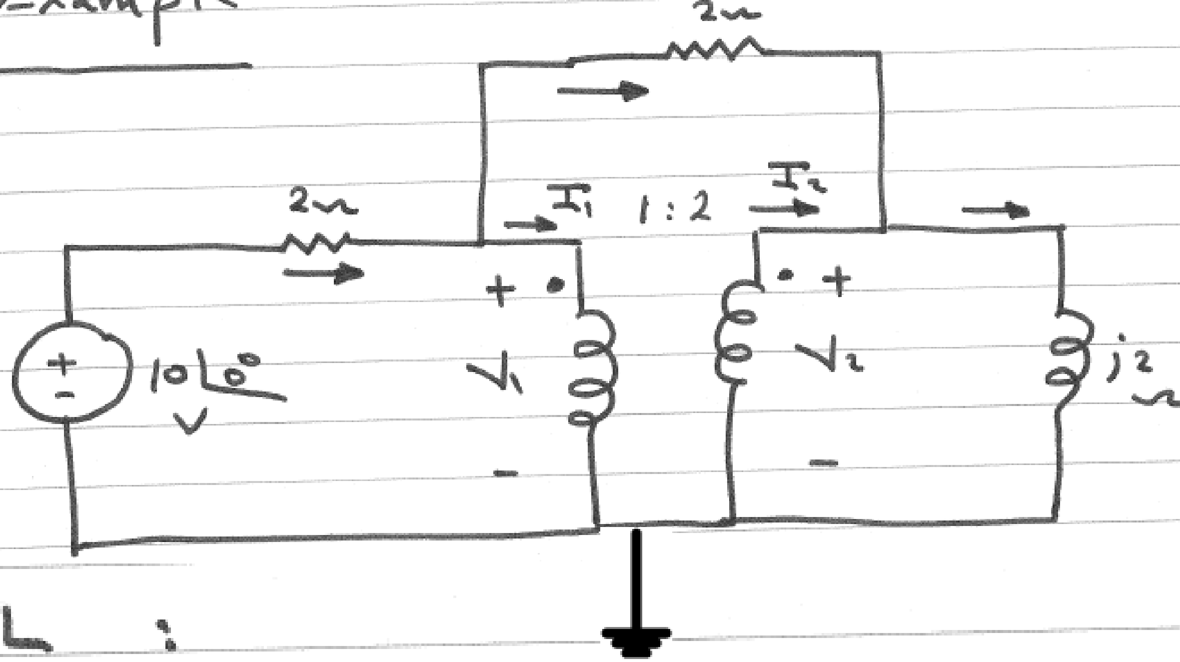
$$V_{TH2} = \text{-}2V_{TH1} = -28.84\langle -33.69^o \;\; V$$



$$V_o = \frac{2}{(2 + 2 + 16 + j3 - j8)}\left(-28.84\langle -33.69^o\right)$$

$$V_o = 2.8\langle 160.35^o \;\; V$$

Example



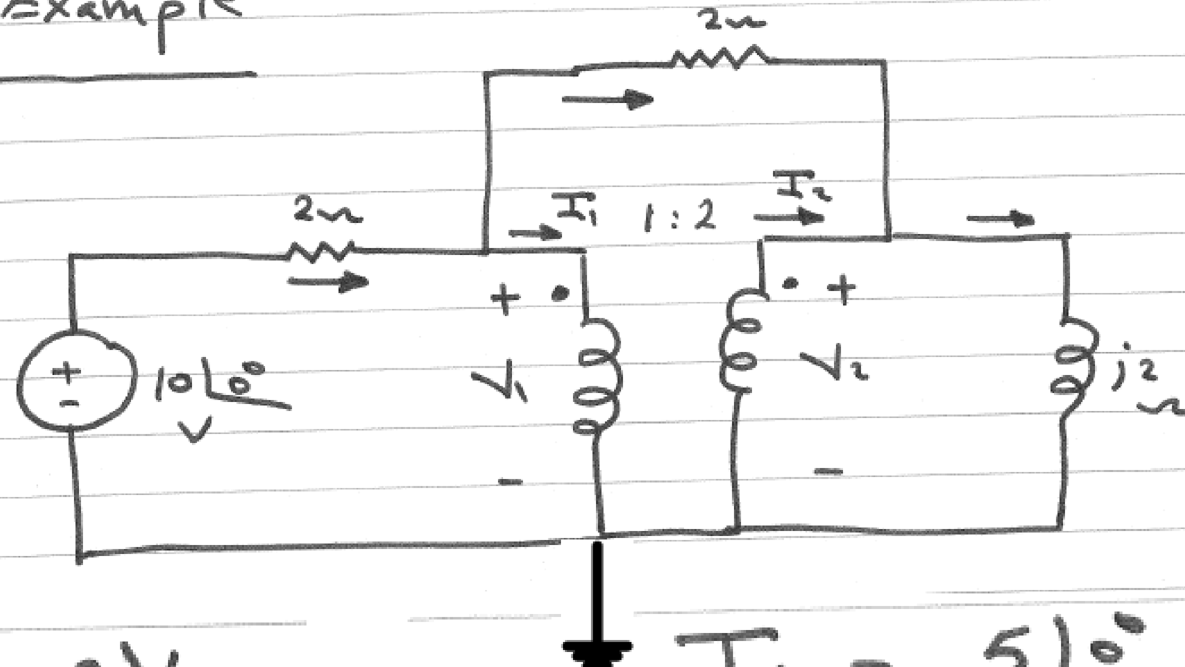
KCL :

$$\frac{10 - V_1}{2} = \frac{V_1 - V_2}{2} + I_1$$

KCL :

$$I_2 + \frac{V_1 - V_2}{2} = \frac{V_2}{j2}$$

Example



$V_2 = 2V_1$

$I_1 = 2I_2$

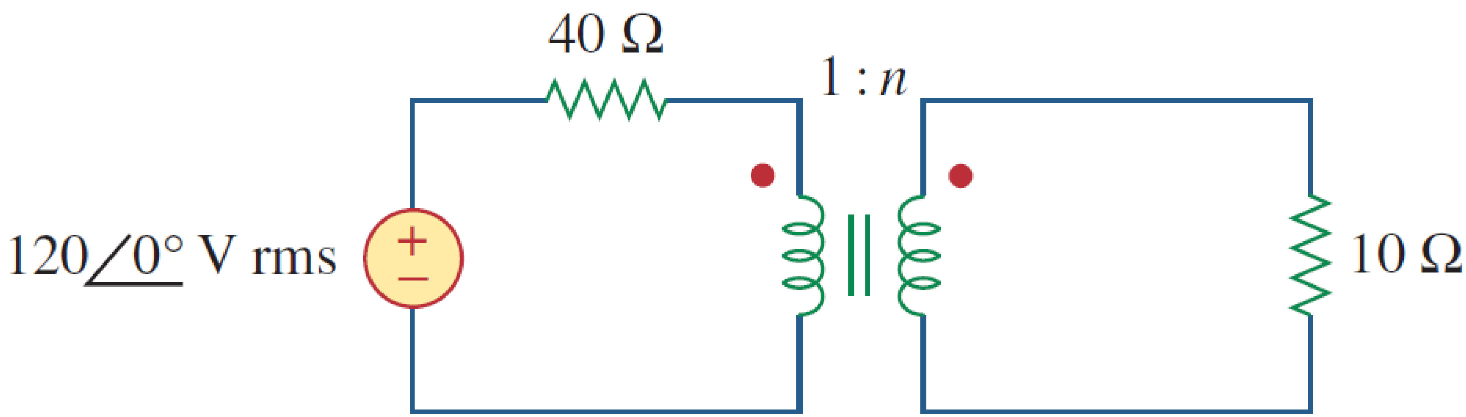
$\therefore \quad I_1 = 5\angle 0° \text{ A}$

$I_2 = 2.5\angle 0° \text{ A}$

$V_1 = \sqrt{5}\angle 63° \text{ V}$

$V_2 = 2\sqrt{5}\angle 63° \text{ V}$

For the circuit shown , determine the turns ratio n that will cause maximum average power transfer to the load . Calculate that maximum average power



For maximum average power transfer to the load : $Zth = 10\Omega$

$$n^2.40 = 10$$

$$\therefore n = 0.5$$

$$I = \frac{60\angle 0}{10+10} = 3\angle 0 Arms$$

$$Vth = 60\angle 0 Vrms$$

$$P = I^2.R = 90W$$