BIRZEIT UNIVERSITY

Faculty of Engineering and Technology
Department of Electrical and Computer Engineering
Engineering Probability and Statistics ENEE 2307

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Final Exam

Date: Wednesday 25/1/2017 Time: 120 minutes
Name: Student #:

Opening Remarks:

- This is a 120-minute exam. Calculators are allowed. Mobile phones, books, notes, formula sheets, and other aids are not allowed.
- You are required to show all your work and provide the necessary explanations everywhere to get full credit.
- There are two extra problems for those who were absent in the midterm exam.

Problem 1(16 points) **(ABET criterion a)**

The voltage V across a 1-Ω resistor R is a uniform random variable over the interval (0, 1). The instantaneous power is $P = V^2/R$.

a. Find the expected value of the power P.
b. Find the pdf of the instantaneous power P.

Problem 2 (18 points)**(ABET criterion a)**

Let X and Y denote the lengths of life, in years, of two components in an electronic system. If the joint density function of these variables is

$$f_{X,Y}(x,y) = \begin{cases} ke^{-(x+y)} & 0 < x, 0 < y \\ 0 & otherwise \end{cases}$$

a. Find k so that $f_{X,Y}(x,y)$ is a valid joint probability density function.
b. Are X and Y statistically independent? Explain.
c. Are X and Y correlated? Explain.
d. Let W=2X+3Y, determine the standard deviation of W.

Problem 3(18 points)**(ABET criterion e)**

The time, in hours, it takes for computer programmer A to complete his program is a uniform random variable X, which is uniformly distributed over the interval (0, 2). The time it takes for programmer B is also a random variable Y (independent of X), which is uniformly distributed over the interval (0, 2).

a. Write down the marginal probability density functions $f_X(x)$ and $f_Y(y)$.
b. Find the joint probability density functions $f_{X,Y}(x,y)$ and the region over which it is defined.
c. Find the probability that B needs at least twice the time needed by A to complete his program.

Problem 4(18 points) **(ABET criterion e)**

The weights of cement bags are normally distributed with a mean of (50) kg and a standard deviation of 2 kg.

a. What is the probability that one randomly selected cement bag will weigh more than 52 kg?
b. What is the probability that 5 randomly selected cement bags will have a mean weight of more than 52 kg
c. Find n, such that the probability that the mean weight of n randomly selected cement bags be larger than 51 kg is less than 0.01.

Problem 5 (14 points) **(ABET criterion a)**

Given a random sample $X_1, X_2, \ldots, X_n$ of size n drawn from a distribution with pdf

$f(x) = \theta e^{-\theta x}$, $x > 0$.

Find a maximum likelihood estimator for the unknown parameter θ in terms of the observations.

Problem 6 (16 points) **(ABET criterion e)**

An electrical firm manufactures light bulbs that have a length of life that is approximately normally distributed with a standard deviation of 40 hours.

a. If a sample of 30 bulbs has an average life of 780 hours, find a 96% confidence interval for the population mean of all bulbs produced by this firm.
b. How large a sample is needed if we wish to be 96% confident that our sample mean will be within 10 hours of the true mean?

End of Exam

Good Luck

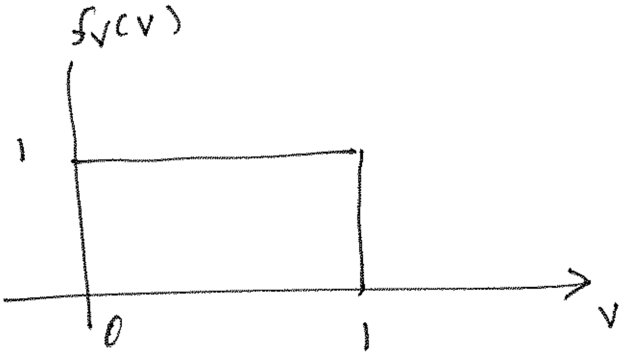
Final Exam Solutions

January 25, 2017

Problem 1 :

$P = V^2/R \quad ; \quad R = 1\ \Omega \quad \Rightarrow$

$P = V^2$



a. $E(P) = E(V^2)$

$$= \int_0^1 v^2 f_V(v)\,dv$$

$$= \int_0^1 v^2 (1)\,dv = \frac{v^3}{3}\Big|_0^1 = \frac{1}{3}$$

b. $f_P(p) = \dfrac{f_V(v)}{|dP/dv|} = \dfrac{(1)}{2V}$ "one-to-one mapping)

$$= \frac{(1)}{2\sqrt{P}}$$

when $v = 0 \Rightarrow P = 0$

$v = 1 \Rightarrow P = 1$

$$\Rightarrow \quad f_P(p) = \begin{cases} \dfrac{1}{2\sqrt{p}} & 0 < P \leq 1 \\ 0 & \text{o.w} \end{cases}$$

Problem 2: $f_{X,Y}(x,y) = \begin{cases} k\,e^{-(x+y)} & x>0\ ,\ y>0 \quad (2) \\ 0 & o.w \end{cases}$

a. $\int_0^{\infty}\int_0^{\infty} k\,e^{-(x+y)}\,dx\,dy = 1 \quad \Rightarrow \quad k = 1$

b. $f_X(x) = \int_0^{\infty} k\,e^{-(x+y)}\,dy = e^{-x} \qquad x>0$

$f_Y(y) = \int_0^{\infty} k\,e^{-(x+y)}\,dx = e^{-y} \qquad y>0$

since $f_{X,Y}(x,y) = f_X(x)\cdot f_Y(y) \Rightarrow X$ & Y are indep.

c. Independence $\Rightarrow$ uncorrelated $\Rightarrow \rho = 0$

d. $W = 2X + 3Y$

$\sigma_W^2 = 4\sigma_X^2 + 9\sigma_Y^2$

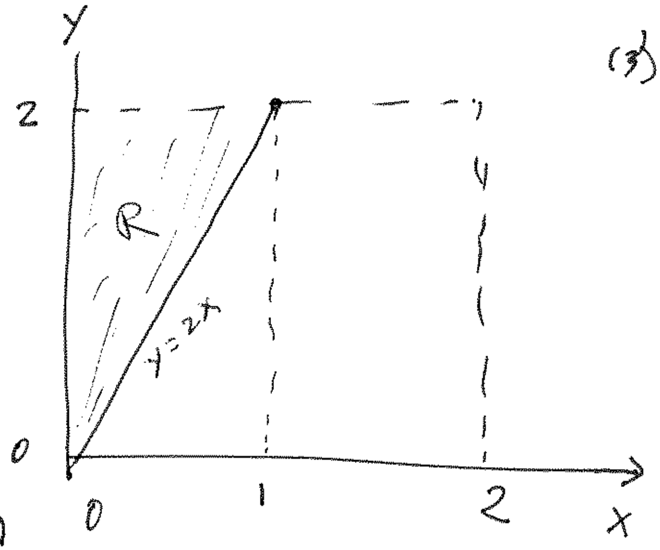
$f_X(x) = e^{-x}\,u(x) \quad \Rightarrow \sigma_X^2 = \frac{1}{(\lambda^2)} = \frac{1}{(1)} = 1$

$\sigma_W^2 = 4(1) + 9(1)$

$= 13$

$\sigma_W = \sqrt{13}$

(3)



a. $f_X(x) = \begin{cases} \frac{1}{2} & 0 < x \le 2 \\ 0 & \text{o.w} \end{cases}$

$f_Y(y) = \begin{cases} \frac{1}{2} & 0 < y \le 2 \\ 0 & \text{o.w} \end{cases}$

b. $f_{X,Y}(x,y) = \begin{cases} \frac{1}{4} & 0 < x \le 2 \cap 0 < y \le 2 \\ 0 & \text{o.w} \end{cases}$

c. $P(Y \geq 2X) = \int_0^1 \int_{2x}^2 \left(\frac{1}{4}\right) dy\, dx$

$= \frac{1}{4} \int_0^1 (2 - 2x)\, dx$

$= \frac{1}{4} \left[2x - x^2 \right|_0^1 = \frac{1}{4}[2 - 1] = \frac{1}{4}$

Problem 4 (4)

$\mu_x = 50, \quad \sigma_x = 2$

a. $P(X > 52) = 1 - \Phi\left(\frac{52 - \mu_x}{\sigma_x}\right) = 1 - \Phi\left(\frac{52-50}{2}\right)$

$= 1 - \Phi(1) = 1 - 0.8413 = 0.1587$ From Φ tables

$= \boxed{Q(1) = 0.15866}$ from Q functions

b. $Y_1 = \frac{\sum x_i}{5} \Rightarrow \mu_y = \mu_x = 50$

$\sigma_y^2 = \frac{\sigma_x^2}{n} = \frac{4}{5} \Rightarrow \sigma_y = \frac{2}{\sqrt{n}} = \sqrt{0.8}$

$P(Y > 52) = 1 - \Phi\left(\frac{52 - \mu_y}{\sigma_y}\right) = 1 - \Phi\left(\frac{52-50}{\sqrt{0.8}}\right)$

$= 1 - \Phi(2.236) = 1 - 0.9871 = 0.0129$

$= Q(2.236) \simeq \frac{Q(2.0) + Q(2.25)}{2} \simeq \boxed{0.013}$

c. $Y_2 = \frac{\sum x_i}{n} \Rightarrow E(Y_2) = \mu_x = 50$

$\sigma_{Y_2} = \frac{\sigma_x}{\sqrt{n}} = \frac{2}{\sqrt{n}}$

$P(Y_2 > 51) < 0.01$

$1 - \Phi\left(\frac{51-50}{2/\sqrt{n}}\right) \le 0.01 \Rightarrow \Phi\left(\frac{51-50}{2/\sqrt{n}}\right) = 0.99$

$\Phi\left(\frac{\sqrt{n}}{2}\right) = 0.99 \Rightarrow \frac{\sqrt{n}}{2} = 2.33$

$\Rightarrow n = (2 \times 2.33)^2 = 21.7156$

$\Rightarrow \boxed{n = 22}$

$Q\left(\frac{\sqrt{n}}{2}\right) = 0.01 \Rightarrow \frac{\sqrt{n}}{2} \simeq 2.3 \Rightarrow n = 21.16$

$\Rightarrow n = 22$

Problem 5

$$f_x(x;\theta) = \theta\, e^{-\theta x}$$

$$L(x_1, x_2, \dots, \theta) = \theta e^{-\theta x_1} \cdot \theta e^{-\theta x_2} \cdots \theta e^{-\theta x_n}$$

$$L = \theta^n e^{-\theta \sum x_i}$$

$$\ln L(\theta) = n \ln \theta - \theta \sum x_i$$

$$\frac{\partial \ln L(\theta)}{\partial \theta} = \frac{n}{\theta} - \sum x_i = 0$$

$$\frac{n}{\hat{\theta}} = \sum x_i \Rightarrow \boxed{\hat{\theta} = \frac{n}{\sum x_i}}$$

Problem 6

a. $\sigma_x = 40$

$n = 30$

$$P\left(\hat{\mu} - z_{\alpha/2}\frac{\sigma}{\sqrt{n}} \leq \mu_x \leq \hat{\mu} + z_{\alpha/2}\frac{\sigma}{\sqrt{n}}\right) \geq 1-\alpha$$

$\alpha = 0.04$

$\frac{\alpha}{2} = 0.02$

$z_{\alpha/2} \simeq 2.05$

$$\Rightarrow P\left(780 - 2.05\frac{40}{\sqrt{30}} \leq \mu_x \leq 780 + 2.05\frac{40}{\sqrt{30}}\right) \geq 0.96$$

$$P(780 - 14.971 \leq \mu_x \leq 780 + 14.971) \geq 0.96$$

$$\boxed{P(765.02 \leq \mu_x \leq 794.97) \geq 0.96}$$

b. Need $z_{\alpha/2}\frac{\sigma}{\sqrt{n}} = 10$

$$2.05\frac{40}{\sqrt{n}} = 10$$

$$\Rightarrow \sqrt{n} = \frac{2.05 * 40}{10} \Rightarrow 8.2$$

$$\Rightarrow n = 67.24$$

$$\Rightarrow \boxed{n \geq 68}$$