BIRZEIT UNIVERSITY

# Faculty of Engineering

## Electrical Engineering Department

***Probability and Statistical Engineering, ENEE331***

Dr. W. Hashlamoun, Mr. N. Nofal, Dr. M. Jubran

*Final exam*

Date: Tuesday 18/5/2010 | Time: 150 minutes

Name: | Student #:

---

**Problem 1** (16pts)**:**

A sample space S consists of three events, *A*, *B*, and *C* such that $P(A^c) = 0.5$, $P(A \cap B) = 0.25$, and $P(B \cup C) = 0.75$. The pair of events (*A* and *B*), (*B* and *C*) are independent. Events *A* and *C* are mutually exclusive. Find the followings:

a. $P(A)$, $P(B)$, and $P(C)$.
b. $P(B / A)$

**Problem 2** (16pts)**:**

Let X be a continuous random variable that has the following cumulative distribution function

$$F_X(x) = \begin{cases} 0 & x \le 0 \\ Kx^2 & 0 < x \le 10 \\ 100K & x > 10 \end{cases}$$

a. Find K so that $F_X(x)$ is a valid cumulative distribution function.
b. Find $P(X \le 5)$.
c. Find the probability density function.
d. Find the mean value of X.

**Problem 3** (16pts)**:**

The internet connection speed at any time from your home can depend on the amount of overall internet traffic at that time. Let the random variable X denote the speed of connection in megabits per second (MBPS). Assuming X has a normal probability distribution function with mean μ= 1 MBPS and standard deviation σ= 0.1 MBPS, answer the following questions:

a. What is the probability that the connection speed will be less than 0.837 MBPS at any given time?
b. What is the probability that the connection speed will be between 0.837 MBPS and 1 MBPS at any given time?
c. Find a value d such that the connection speed is between $1 - d$ MBPS and $1 + d$ MBPS with probability 0.8664

**Problem 4** (18pts):

a. Let the joint pdf of two random variables X and Y be given as

$$f_{X,Y}(x,y) = \begin{cases} 2(x+y) & 0 \le x \le 1, 0 \le y \le 1 \\ 0 & otherwise \end{cases}$$

1. Find the marginal probability density functions $f_X(x)$ and $f_Y(y)$.
2. Are X and Y independent?

b. If X and Y are independent, normal random variables with E(X) = 10, Var(X) = 4, E(Y) =0, and Var(Y) = 9. Let T = X – 2Y, find the mean and variance of T

**Problem 5** (18pts):

a. Given a random sample $X_1, X_2, \ldots, X_n$ of size n drawn from a distribution with pdf $f(x) = \theta e^{-\theta x}$, x > 0. Use the maximum likelihood method to find a point estimator for the unknown parameter θ in terms of the observations.

b. Suppose that a random sample of size 25 is taken from a a normal distribution with mean $\mu_X = 9$ and $\sigma_X{}^2 = 4$. Write down the probability density function of the sample average defined as $\overline{X} = \frac{\sum_{i=1}^{25} X_i}{25}$

**Problem 6** (16pts):

The annual rainfall in a region is normally distributed with unknown mean value $\mu_x$ and unknown variance $\sigma_x^2$. Annual rainfall measurements, $X_1, X_2, \ldots, X_{10}$, were collected over a period of 10 years and it was found that the sample mean is 112.4 cm and the sample standard deviation is 37.6 cm

a. Find a 95% confidence interval on the population mean $\mu_x$.
b. Find a 95% confidence interval on the population variance $\sigma_x^2$.

GOOD LUCK ☺