Faculty of Engineering and Technology
Probability and Statistics
Quiz

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Question#1

Let X and Y be two jointly continuous random variables with joint PDF

$$f_{XY}(x,y) = \begin{cases} cxy & 0 \le x \le 1, \sqrt{x} \le y \le 1 \\ 0 & otherwise \end{cases}$$

Find:

a) Determine c
b) Marginal pdfs $f_X(x)$, and $f_Y(y)$
c) Are X and Y statically independent?
d) Find the conditional PDF of X given Y=y, $f_{X|Y}(x|y)$.
e) Find E[X|Y=y], for $0 \le y \le 1$.

Question#2

The number of accidents in a certain city is modeled by a Poisson random variable with an average rate of 10 accidents per day. Suppose that the number of accidents on different days are independent. Find the probability that there will be more than 4000 accidents in a certain year. Assume that there are 365days in a year.

Good Luck

a) $1 = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f_{X,Y}(x,y)\, dy\, dx$

$1 = \int_{0}^{1} \int_{\sqrt{x}}^{1} cxy \; dy\, dx \rightarrow \int_{0}^{1} \left(\frac{c}{2} x y^2\right)\Big|_{\sqrt{x}}^{1} dx$

$= \frac{c}{2} \int_{0}^{1} x \left(1 - (\sqrt{x})^2\right) dx = \frac{c}{2} \int_{0}^{1} (x - x^2)\, dx$

$= \frac{c}{2} \left(\frac{x^2}{2} - \frac{x^3}{3}\right)\Big|_{0}^{1} = \frac{c}{2}\left(\frac{1}{2} - \frac{1}{3}\right) = 1$

$1 = 0.08\bar{3}\, c \rightarrow \boxed{c = 12}$.

b) $f_X(x) = \int_{-\infty}^{\infty} 12xy\; dy = \int_{\sqrt{x}}^{1} 12xy\, dy$

$= 12x\left(\frac{y^2}{2}\right)\Big|_{\sqrt{x}}^{1} = 12x\left(\frac{1}{2} - \frac{x}{2}\right) = 6x - 6x^2,$

if $0 \leq x \leq 1$, and if $x > 1$ or $x < 0$

$f_X(x) = \int_{-\infty}^{\infty} 0\, dy = 0.$

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$\hookrightarrow \therefore f_X(x) = \begin{cases} 6x - 6x^2 & 0 \leq x \leq 1 \\ 0 & \text{O.W} \end{cases}$

* if $y > 1$ or $y < \sqrt{x}$:

$f_Y(y) = \int_{-\infty}^{\infty} 0 \, dx = 0.$

* if $\sqrt{x} \leq y \leq 1$:

$f_Y(y) = \int_0^1 12xy \, dx = \left(\frac{\overset{6}{\cancel{12}} x^2 y}{\cancel{2}} \right) \Big|_0^1$

$= 6y - 0 = 6y$

$\hookrightarrow \therefore f_Y(y) = \begin{cases} 6y & \sqrt{x} \leq y \leq 1 \\ 0 & \text{O.W} \end{cases}$

(c) X & Y Statically independent, if:

$$f_X(x)\, f_Y(y) = f_{X,Y}(x,y)$$

$$(6x - 6x^2)\, 6y \overset{?}{=} 12xy$$

$$36xy - 36x^2y \neq 12xy.$$

$3xy - 3x^2y \neq xy \rightarrow \therefore$ X & Y are <u>not</u> S.I

(d) $$f_{X|Y}(x|y) = \left. \frac{f_{X,Y}(x,y)}{f_Y(y)} \right|_{Y=y}$$

$$= \begin{cases} \dfrac{\overset{2}{\cancel{12}}\, x \cancel{y}}{\cancel{6y}} = 2x & 0 \leq x \leq 1 \\ 0 & \text{o.w} \end{cases}$$

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(e) $E\{2x\} = \int_0^1\int_0^1 2x * 12xy \, dy \, dx$

$= \int_0^1\int_0^1 24x^2y \, dy \, dx = \int_0^1 \frac{24x^2}{2}(y^2)\Big|_0^1 dx$

$= \int_0^1 12x^2 \cancel{(1-0)} \, dx$

$= \frac{\overset{4}{\cancel{12}}x^3}{\cancel{3}}\Big|_0^1 = 4\cancel{(1-0)} = 4.$