Birzeit University
Faculty of Engineering
Department of Electrical Engineering
Engineering Probability and Statistics ENEE 331
Problem Set (5)
Estimation Theory

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1) A manufacturer of semiconductor devices takes a random sample of size n of chips and tests them, classifying each chip as defective or non-defective. Let $X_i = 0$ if the chip is non-defective and $X_i = 1$ if the chip is defective.

a. Find the mean and variance of the sample average defined as $\overline{X} = \frac{1}{n}\sum_{i=1}^{n} X_i$.
b. Compare the sample variance for the case when n = 50 and n = 100. Comment on the effect of sample size on the variance of the sampling distribution
c. If p is the probability of a defective chip, find an unbiased estimator of p.

2) Consider a random sample of size n taken from a discrete distribution, the pmf of which is given by: $f(x) = \theta^x (1-\theta)^{1-x}$, x = 0, 1. Two estimators for θ are proposed

$$\hat{\Theta}_1 = \overline{X} = \frac{1}{n}\sum_{i=1}^{n} X_i$$

$$\hat{\Theta}_2 = \frac{n\overline{X} + 1}{n+2}$$

a. Which one of these two estimators is an unbiased estimator of the parameter θ?
b. Which one has a smaller variance?

3) In a random sample of 500 persons in the city of Ramallah, it was found that 372 voted for Abu Mazen in the 2005 presidential elections for the Palestinian Authority. Determine a 95% confidence interval for p, the actual proportion of Ramallah residents supporting Abu Mazen.

4) The compressive strength of concrete is being tested by a civil engineer. He tests 12 specimens and obtains the following data (in psi)

2216	2237	2249	2204	2225	2301	2281	2283
2318	2255	2275	2295				

a. Find point estimates for the mean and variance of the strength
b. Construct a 95% confidence interval on the mean strength
c. Construct a 95% confidence interval on the variance of the strength.

5) A random sample of n = 36 observations has been drawn from a normal distribution with mean 50 and standard deviation 12. Find the probability that the sample mean is in the interval 47 to 53.

6) Given the following pair of measurements, which are suspected to be linearly related. Do a regression analysis to find the linear relationship $y = \alpha x + \beta$

X_i	0.77	4.39	4.11	2.91	0.56	0.89	4.09	2.38	0.78	2.52
Y_i	14.62	22.21	20.12	19.42	14.69	15.23	24.48	16.88	8.56	16.24

7) A machine produces metal rods used in an automobile suspension system. A random sample of 9 rods is selected and the diameter is measured. The resulting data (in mm) are:

8.24 8.23 8.20 8.21 8.22 8.28 8.17 8.26 8.19

If the sampling comes from a normal population with a mean rod diameter μ and a variance σ^2, find

a. point estimates for the mean and the variance
b. a 95% confidence interval on the mean
c. a 95% confidence interval on the variance

8) A random sample of n = 10 structural elements is tested for compressive strength. We know that the true mean compressive strength is $\mu = 5000$ psi and the standard deviation is $\sigma = 100$ psi. Find the probability that the sample mean compressive strength exceeds 4985 psi.

9) Let X_1 and X_2 be a sample of size two drawn from a population with mean μ and variance σ^2. Two estimators for μ are proposed:

$$\hat{\mu}_1 = \frac{X_1 + X_2}{2}$$

$$\hat{\mu}_2 = \frac{X_1 + 2X_2}{3}$$

Which is the better estimator and in what sense?

10) Suppose that X has the following discrete distribution

$$P(X = x) = \begin{cases} 1/3 & x = 1,2,3 \\ 0 & otherwise \end{cases}$$

A random sample of n = 200 is selected from this population. Approximate the probability that the sample mean is greater than 2.1 but less than 2.5.

11) The amount of waiting time that a customer spends waiting at a bank is a random variable with mean 8.2 minutes and standard deviation 1.5 minutes. Suppose that a random sample of n = 50 customers is observed. Find the probability that the average waiting time for these customers is less than 8 minutes.

12) A computer, in adding numbers, round each number to the nearest integer. Suppose that all rounding errors are independent and uniformly distributed over (-0.5, 0.5). If 1500 numbers are added, what is the probability that the magnitude of the total error exceeds 15?

13) Suppose that X has a normal distribution with mean μ and variance σ^2, where μ and σ^2 are unknown. A sample of size 15 yielded the values $\sum_{i=1}^{15} X_i = 8.7$ and $\sum_{i=1}^{15} X_i^2 = 27.3$. Obtain a 95% confidence interval on the variance.