

ENEE 2307
Midterm Exam

July 28, 2018

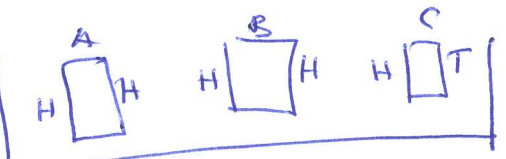
Problem 1: $P(A) = 0.6$, $P(B) = 0.5$, $P(C) = 0.4$

$$\begin{aligned}
 \text{a. } P(\text{at least one event}) &= P(A \cup B \cup C) \\
 &= P(A) + P(B) + P(C) \\
 &\quad - P(A)P(B) - P(A)P(C) - P(B)P(C) \quad \left. \begin{array}{l} \\ \\ \end{array} \right\} \text{ due to independence} \\
 &\quad + P(A)P(B)P(C) \\
 &= 1.5 - 0.3 - 0.24 - 0.2 + 0.12 = 0.88 \quad 8
 \end{aligned}$$

$$\text{b. } P(A \cap B \cap C) = (0.6)(0.5)(0.4) = 0.12 \quad 6$$

$$\text{c. } P(A \cap B) = (0.6)(0.5) = 0.3 \quad 6$$

Problem 2

$$P(H) = P(A)P(H|A) + P(B)P(H|B) + P(C)P(H|C)$$


$$\text{a. } = \frac{1}{3}(1) + \frac{1}{3}(1) + \frac{1}{3}\left(\frac{1}{2}\right)$$

$$P(H) = \frac{5}{6} \quad (12)$$

$$\text{b. } P(C|H) = \frac{P(C \cap H)}{P(H)} = \frac{P(C)P(H|C)}{P(H)}$$

$$= \frac{\frac{1}{3} \cdot \frac{1}{2}}{\frac{5}{6}} = \frac{1}{5} \quad (8)$$

Problem 3 : $F_X(x) = \begin{cases} 0 & x < 0 \\ 1 - e^{-2x} & x \geq 0 \end{cases}$

a. $f_X(x) = \frac{dF_X(x)}{dx} = \begin{cases} 0 & x < 0 \\ 2e^{-2x} & x \geq 0 \end{cases}$ 7

b. $P(X \leq 2) = \int_0^2 2e^{-2x} dx = -e^{-2x} \Big|_0^2$

$P(X \leq 2) = 1 - e^{-4}$ 7

c. $E(X) = \int_0^{\infty} 2x e^{-2x} dx = \int_0^{\infty} x f_X(x) dx = \frac{1}{\lambda} = \frac{1}{2}$ 6

Problem 4 :

$n = 20$
 $p = 0.05$

$P(X=x) = \binom{20}{x} (0.05)^x (0.95)^{20-x}$ 6

a. $P(X \geq 2) = 1 - [P(X=0) + P(X=1)]$ 3
 $= 1 - \left[\binom{20}{0} (0.05)^0 (0.95)^{20} + \binom{20}{1} (0.05)^1 (0.95)^{19} \right]$
 $= 1 - [0.3584 + 0.3773]$ 3
 $= 0.2642$ 2

b. $E(X) = np = (20)(0.05) = 1$ 6
 $\frac{6}{20}$

Problem 5

$$p = 0.005$$

$$n = 6000$$

Find $P(X \leq 32)$

$$\mu_x = np = 6000 * 0.005 = 30 \quad \text{\$}$$

$$\sigma_x^2 = np(1-p) = 6000 * 0.005 * (1 - 0.005) = 29.85$$

$$\sigma_x = 5.463 \quad \text{\$}$$

$$P(X \leq 32) = \Phi\left(\frac{32 - \mu_x}{\sigma_x}\right) \quad \text{\$}$$

$$= \Phi\left(\frac{32 - 30}{5.463}\right) \quad \text{2}$$

$$= \Phi(0.366)$$

$$= 0.6406 \quad \text{2} \quad (0.6443)$$

