Statistics

Chapter 3: Single Random Variables and Probability Distributions

Random Variables

Definition:

- A real-valued function whose domain is the sample space is called a *random variable* (r.v).
- - The random variable is given an uppercase letter X, Y, Z, … while the values assumed by this random variable are given lowercase letters x, y, z, …
- - The whole idea behind the r.v is a one to one *mapping* from the sample space on the real line via the mapping function X(s).

Random Variables

- **Definition:** Associated with each discrete r.v (X) is a *Probability Mass Function P(X = x)*. This density function is the sum of all probabilities associated with the outcomes in the sample space that get mapped into (x) by the mapping function (random variable X).
- -Associated with each continuous r.v (X) is a *Probability Density Function (pdf)* $f_X(x)$.This $f_X(x)$ is not the probability that the random variable (X) takes on the value (x), rather $f_X(x)$ is a continuous curve having the property that:

$$P(a \leq x \leq b) = \int_a^b f_x(x)dx$$

Two Types of Random Variables

- A **random variable** is a variable hat assumes numerical values associated with the random outcome of an experiment, where one (and only one) numerical value is assigned to each sample point.

Two Types of Random Variables

- A **discrete random variable** can assume a countable number of values.
- Discrete random variables
  - Success fail experiments' outcomes
  - Number of sales
  - Number of Trials
  - People in line
  - Mistakes per page
  - Success fail experiments' outcomes

- A **continuous random variable** can assume any value along a given interval of a number line
- **Continuous random variables**
  - Length
  - Depth
  - Volume
  - Time
  - Weight

4.2: Probability Distributions for Discrete Random Variables

- The **probability distribution** of a discrete random variable is a graph, table or formula that specifies the probability associated with each possible outcome the random variable can assume.
  - $p(x) \geq 0$ for all values of x
  - $\Sigma p(x) = 1$

Expected Values of Random Variables

- The **mean**, or **expected value**, of a **discrete random variable** is

$$\mu_x = E[X] = \sum_{i=0}^{\infty} x_i P(x_i)$$

- The **mean**, or **expected value**, of a **Continuous random variable** is

$$\mu_x = E[X] = \int_{-\infty}^{\infty} x f_x(x) d$$

Expected Values of Discrete Random Variables

- The **variance** of a **discrete random variable** *x* is

$$\sigma_x^2 = E[(x-\mu_x)^2] = \sum_{i=0}^{\infty} (x_i - \mu_x)^2 P(x_i)$$

- The **standard deviation** of a **discrete random variable** *x* is

$$\sigma_x = \sqrt{\sigma_x^2} = \sqrt{E[(x-\mu_x)^2]} = \sqrt{\sum_{i=0}^{\infty} (x_i - \mu_x)^2 P(x_i)}$$

Expected Values of continuous Random Variables

- The **variance** of a **d continuous random variable** *x* is
- $\sigma_x^2 = E[(x-\mu_x)^2] = \int_{-\infty}^{\infty}(x_i-\mu_x)^2 f_x(x)dx$

- The **standard deviation** of a **continuous random variable** *x* is

$$\sqrt{\sigma_x^2} = \sqrt{E[(x-\mu_x)^2]} = \sqrt{\int_{-\infty}^{\infty}(x_i-\mu_x)^2 f_x(x)dx}$$

The Binomial Distribution

- A Binomial Random Variable
  - *n* identical trials
  - Two outcomes: **S**uccess or **F**ailure
  - P(**S**) = *p*; P(**F**) = *q* = 1 – *p*
  - Trials are independent
  - *x* is the number of **S**uccesses in *n* trials

The Binomial Distribution

- A Binomial Random Variable
  - *n* identical trials → Flip a coin 3 times
  - Two outcomes: **S**uccess or **F**ailure → Outcomes are Heads or Tails
  - P(**S**) = *p*; P(**F**) = *q* = 1 – *p* → P(H) = .5; P(F) = 1-.5 = .5
  - Trials are independent → A head on flip *i* doesn't change P(H) of flip *i* + 1
  - *x* is the number of **S**'s in *n* trials

The Binomial Distribution

Results of 3 flips	Probability	Combined	Summary
HHH	$(p)(p)(p)$	p^3	$(1)p^3q^0$
HHT	$(p)(p)(q)$	p^2q	
HTH	$(p)(q)(p)$	p^2q	$(3)p^2q^1$
THH	$(q)(p)(p)$	p^2q	
HTT	$(p)(q)(q)$	pq^2	
THT	$(q)(p)(q)$	pq^2	$(3)p^1q^2$
TTH	$(q)(q)(p)$	pq^2	
TTT	$(q)(q)(q)$	q^3	$(1)p^0q^3$

The Binomial Distribution

- The Binomial Probability Distribution
  - p = P(**S**) on a single trial
  - n = number of trials
  - x = number of successes

$$P(x) = \binom{n}{x} p^x (1-p)^{n-x}$$

4.4: The Binomial Distribution

- Say 40% of the class is female.
- What is the probability that 6 of the first 10 students walking in will be female?

$$P(x) = \binom{n}{x} p^x q^{n-x}$$

$$= \binom{10}{6} (.4^6)(.6^{10-6})$$

$$= 210(.004096)(.1296)$$

$$= .1115$$

The Binomial Distribution

- A Binomial Random Variable has

Mean $\mu_x = E[X] = np$

Variance $\sigma_x^2 = E[(x-\mu_x)^2]$ =np(1-p)

II. The Geometric Distribution

- Let the outcome of an experiment be either a success with probability (p) or a failure with probability (1 – p). Let (X) be the number of times the experiment is performed to the first occurrence of a success. Then (X) is a discrete random variable with integer values ranging from one to infinity. The probability mass function of (X) is:

$$P(X = x) = P(FFF \dots FS)$$
$$P(X = x) = (1-p)^{x-1}p; x = 1,2,3, \dots..$$

II. The Geometric Distribution

- ***Theorem:***

The mean and the variance of (X) are:

$$\mu_x = E[X] = \frac{1}{p}$$

$$\sigma_x^2 = E[(x - \mu_x)^2] = \frac{1-p}{p^2}$$

II. The Geometric Distribution

- **EXAMPLE (3-17):**

Let the probability of occurrence of a flood of magnitude greater than a critical magnitude in a given year be 0.02. Assuming that floods occur independently, determine the "return period" defined as the average number of years between floods.

SOLUTION:

(X) has a geometric distribution with p = 0.02

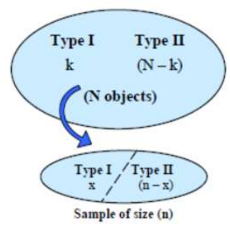
$\mu_x = E[X] = \frac{1}{p} = \frac{1}{0.02} = 50 \text{ year}$

Hyper-geometric Distribution

Consider the sampling without replacement of a lot of (N) items, (k) of which are of one type and (N – k) of a second type. The probability of obtaining (x) items in a selection of (n) items without replacement obeys the hyper-geometric distribution:

$$P(X=x)=\frac{\binom{k}{x}\binom{N-k}{n-x}}{\binom{N}{n}}$$

$x=1,2,3,\ldots,\min(k,n)$



Hyper-geometric Distribution

- *NOTE:* is the ratio of items of type (I) to the total population $P=\frac{k}{N}$
- - ***Theorem:***

The mean and the variance of the hyper-geometric random variable are:

$$\mu_x=E[X]=n\frac{k}{N}=np$$

$$\sigma_x^2=Var(X)=\frac{nk(N-k)(N-n)}{N^2(N-1)}=np(1-p)\left(\frac{N-n}{N-1}\right)$$

Hyper-geometric Distribution

EXAMPLE (3−19):

Fifty small electric motors are to be shipped. But before such a shipment is accepted, an inspector chooses 5 of the motors randomly and inspects them. If none of these tested motors are defective, the lot is accepted. If one or more are found to be defective, the entire shipment is inspected. Suppose that there are, in fact, three defective motors in the lot. What is the probability that the entire shipment is inspected?

SOLUTION:

Let (X) be the number of defective motors found, then (X) assumes the values (0 , 1 , 2 , 3).

$P(entire\ shipment\ is\ inspected) = P(X \geq 1) = 1 - P(X = 0)$

$$1 - P(X = 0) = 1 - \frac{\binom{3}{0}\binom{50-3}{5-0}}{\binom{50}{5}} = 1 - 0.72 = 0.28$$

Hyper-geometric Distribution

EXAMPLE (3-20):

A committee of seven members is to be formed at random from a class with 25 students of whom 15 are girls. Find the probability that:

a- No girls are among the committee

SOLUTION:

Let (X) represents the number of girls in the committee.

$$P(X = 0) = \frac{\binom{15}{0}\binom{25-15}{7-0}}{\binom{25}{7}} = 2.4964 * 10^{-4}$$

f(x,N,k,n)= hygepdf(0,25,15,7)

b- All committee members are girls

c- The majority of the members are girls

Hyper-geometric Distribution

EXAMPLE (3-20):

b- All committee members are girls

$$P(X=7)=\frac{\binom{15}{7}\binom{25-15}{7-7}}{\binom{25}{7}}=0.0134$$

c- The majority of the members are girls

$$P(X\geq 4)=\sum_{x=4}^{7}\frac{\binom{15}{x}\binom{25-15}{7-x}}{\binom{25}{7}}=0.7394$$

Hyper-geometric Distribution

- ***Theorem:***

For large (N), one can use the approximation:

$$P(X=x)\cong\binom{n}{x}p^x(1-p)^{n-x}; where \;\; p=\frac{k}{N}$$

This approximation gives very good results if $\frac{n}{N}\leq 0.1$

For the ex3.19 :$P(X=0)\cong\binom{5}{0}\left(\frac{3}{50}\right)^0\left(\frac{47}{50}\right)^5$ =0.733

The Hypergeometric Distribution

- In the binomial situation, each trial was independent.
  - Drawing cards from a deck and replacing the drawn card each time
- If the card is *not* replaced, each trial depends on the previous trial(s).
  - The hypergeometric distribution can be used in this case.

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The Hypergeometric Distribution

- Suppose a customer at a pet store wants to buy two hamsters for his daughter, but he wants two males or two females (i.e., he wants only two hamsters in a few months)
- If there are ten hamsters, five male and five female, what is the probability of drawing two of the same sex? (With hamsters, it's virtually a random selection.)

$$P(M=2)=P(F=2)=\frac{\binom{5}{2}\binom{10-5}{2-2}}{\binom{10}{2}}=\frac{(10)(1)}{45}=.22$$

$$P(M=2\text{ or }F=2)=P(M=2)+P(F=2)=2\times .22=.44$$

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The Poisson Distribution

- Evaluates the probability of a (usually small) number of occurrences out of many opportunities in a …
  - Period of time
  - Area
  - Volume
  - Weight
  - Distance
  - Other units of measurement

The Poisson Distribution

Definition:
A discrete random variable (X) is said to have a *Poisson distribution* if it has the following probability mass function:

$$P(x) = \frac{\lambda^x e^{-\lambda}}{x!} = \frac{b^x e^{-b}}{x!}$$

- b= λ = mean number of occurrences in the given unit of time, area, volume, etc.
- *e* = 2.71828….
- $\mu = \lambda$
- $\sigma^2 = \lambda$

To verify that this is, indeed, a valid probability mass function we need to show that:

The Poisson Distribution

- Say in a given stream there are an average of 3 striped trout per 100 yards. What is the probability of seeing 5 striped trout in the next 100 yards, assuming a Poisson distribution?

$$P(x=5)=\frac{\lambda^x e^{-\lambda}}{x!}=\frac{3^5 e^{-3}}{5!}=.1008$$

The Poisson Distribution

- ***Poisson Process:*** Consider a counting process in which events occur at a rate of λ occurrence per unit time. Let X(t) be the number of occurrences recorded in the interval (0 ,t), we define the *Poisson process* by the following assumptions:

1- X(0) = 0 , i.e., we begin the counting at time t = 0.

2- For non-overlapping time intervals $(0, t_1)$, (t_2, t_3), the number of occurrences $\{X(t_1) - X(0)\}$ and $\{X(t_3) - X(t_2)\}$ are independent.

3- The probability distribution of the number of occurrences in any time interval depends only on the length of that interval.

4- The probability of an occurrence in a small time interval (Δt) is approximately (λ Δt).

Using the above assumptions, one can show that the probability of exactly (x) occurrences in any time interval of length (T) follows the Poisson distribution and,

$$P(x=5)=\frac{(\lambda T)^x e^{-(\lambda T)}}{x!};where\_x=0,1,2,3,...$$

The Poisson Distribution

- ***Theorem:***

Let (λ) be a fixed number and (n) any arbitrary positive integer. For each nonnegative integer (x):

$$\lim_{n\to\infty}\binom{n}{x}p^x(1-p)^{n-x}=e^{-\lambda}\frac{\lambda^x}{x!};\qquad \textit{where } p=\frac{\lambda}{n};$$

The Poisson Distribution

From previous Example of towers

- How about in the next 50 yards, assuming a Poisson distribution?
  - Since the distance is only half as long, λ is only half as large.

$$P(x=5)=\frac{\lambda^x e^{-\lambda}}{x!}=\frac{1.5^5 e^{-1.5}}{5!}=.0141$$

The Poisson Distribution

- **EXAMPLE (3-21):**

Messages arrive to a computer server according to a Poisson distribution with a mean rate of 10 messages/hour.

a- What is the probability that 3 messages will arrive in one hour.

b- What is the probability that 6 messages will arrive in 30 minutes.

SOLUTION:

a- λ =10 messages/hour → T = 1 hour

$$P(X = x) = e^{-1\ *1}\frac{(10*1)^x}{x!}; x=0,1,2,3,......$$

$$P(X = 3) = e^{-1\ *1}\frac{(10*1)^3}{3!}=0.0076$$

b- λ =10 messages/hour → T = 0.5 hour

$$P(X = x) = e^{-10*0.5}\frac{(10*0.5)^x}{x!}; x=0,1,2,3,......$$

$$P(X = 6) = e^{-10*0.5}\frac{(10*0.5)^6}{6!}=0.1462$$

The Poisson Distribution

- **EXAMPLE (3-22):**

The number of cracks in a section of a highway that are significant enough to require repair is assumed to follow a Poisson distribution with a mean of two cracks per mile.

a- What is the probability that there are no cracks in 5 miles of highway?
b- What is the probability that at least one crack requires repair in ½ miles of highway?

c- What is the probability that at least one crack in 5 miles of highway?
SOLUTION:

a- λ =2 Cracks/mile → T = 5 Miles $P(X = 0) = e^{-2*5}\frac{(2*5)^0}{0!}=e^{-10}$

b- λ =2 Cracks/mile → T = 5 Miles

$$P(X \geq 1) = \sum_{x=1}^{\infty} e^{-2*0.5}\frac{(2*0.5)^x}{x!}=1-e^{-2*0.5}\frac{(2*0.5)^0}{0!}=1-e^{-1}$$

c- λ =2 Cracks/mile → T = 5 Miles

$$P(X \geq 1) = \sum_{x=1}^{\infty} e^{-2*5}\frac{(2*5)^x}{x!}=1-e^{-2*5}\frac{(2*5)^0}{0!}$$

The Poisson Distribution

- **EXAMPLE (3-23):**

Given 1000 transmitted bits, find the probability that exactly 10 will be in error. Assume that the bit error probability is 1/365.

SOLUTION:
X: random variable representing number of bits in error.
Exact solution:

$P(bit\ error) = \frac{1}{365}$; Number of trials (n) = 1000

Required number of bits in error (k) = 10

$P(X = k) = \binom{n}{k} p^k (1-p)^{n-k} = \binom{1000}{10} \frac{1}{365}^{10} (1 - \frac{1}{365})^{990}$

Approximate solution: $e^{-\lambda} \frac{\lambda^x}{x!} = e^{-\frac{1000}{365}} \frac{(\frac{1000}{365})^x}{x!}$;

$P(X = 10) = e^{-\frac{1000}{365}} \frac{(\frac{1000}{365})^{10}}{10!}$

Common Continuous Random Variables:

I. Exponential Distribution:

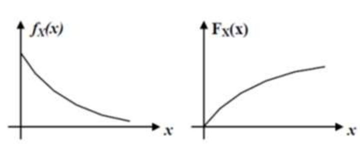
- ***Definition:***

It is said that a random variable (X) has an exponential distribution with a parameter λ (λ > 0) if (X) has a continuous distribution for which the pdf $f_X(x)$ is given as:

$f_X(x) = \lambda e^{-\lambda x};\ x \geq 0$

The cumulative distribution function is:

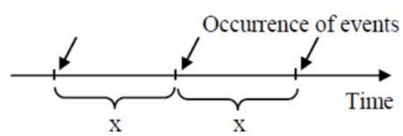
$F_X(x) = 1 - e^{-\lambda x};\ x \geq 0$



Exponential Distribution:

The exponential distribution is often used in a practical problem to represent the distribution of the time that elapses before the occurrence of some event. It has been used to represent the such periods of time as the period for which a machine or an electronic component will operate without breaking down, the period required to take care of a customer at some service facility, and the period between the arrivals of two successive customers at a facility.

If the event being considered occurs in accordance with a Poisson process, then both the waiting time until an event will occur and the period of time between any two successive events will have exponential distribution.



Exponential Distribution:

- ***Theorem:***

If the random variable (X) has an exponential distribution with parameter (λ), then:

$$\mu_x = E[X] = \int_0^{\infty} x\lambda e^{-\lambda x} dx = \lambda\left(\frac{x}{-\lambda} - \frac{1}{\lambda^2}\right) e^{-\lambda x}\Big|_0^{\infty} = \frac{1}{\lambda}$$

$$\sigma_x^2 = E[(x-\mu_x)^2] = E[X^2] - \mu_x^2$$

$$= \int_0^{\infty} x^2 \lambda e^{-\lambda x} dx - \mu_x^2 =$$

$$\lambda\left(\frac{x^2}{-\lambda} - \frac{2x}{\lambda^2} + \frac{2}{-\lambda^3}\right) e^{-\lambda x}\Big|_0^{\infty} - \mu_x^2 = \frac{2}{\lambda^2} - \frac{1}{\lambda^2} = \frac{1}{\lambda^2}$$

Distribution:

- **Exercise**

The number of telephone calls that arrive at a certain office is modeled by a Poisson random variable. Assume that on the average there are five calls per hour.

a. What is the average (mean) time between phone calls?

b. What is the probability that at least 30 minutes will pass without receiving any phone call?

c. What is the probability that there are exactly three calls in an observation interval of two consecutive hours?

d. What is the probability that there is exactly one call in the first hour and exactly two calls in the second hour of a two-hour observation interval?

Exponential Distribution:

- **EXAMPLE (3-24):**

Suppose that the depth of water, measured in meters, behind a dam is described by an exponential random variable with pdf:

$$f_X(x) = \begin{cases} \lambda e^{-\lambda x} & ; \ x \geq 0 \\ 0 & Otherwise \end{cases}$$

There is an emergency overflow at the top of the dam that prevents the depth from exceeding 40.6 m. There is a pipe placed 32.0 m below the overflow that feeds water to a hydroelectric generator (turbine).

a- What is the probability that water is wasted though emergency overflow?

b- What is the probability that water will be too low to produce power?

c- Given that water is not wasted in overflow, what is the probability that the generator will have water to derive it?

Exponential Distribution:

- **EXAMPLE (3-24):**

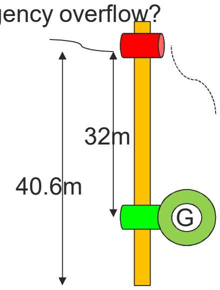
Solution:

$$f_X(x) = \begin{cases} \frac{1}{13.5} e^{-\frac{1}{13.5}x} & ;\ x \geq 0 \\ 0 & Otherwise \end{cases}$$

a-The probability that water is wasted though emergency overflow?

$$P(X \geq 40.6) = \int_{40.6}^{\infty} \frac{1}{13.5} e^{-\frac{1}{13.5}x} dx$$

$$= \frac{-13.5}{13.5} e^{-\frac{1}{13.5}x} |_{40.6}^{\infty} = e^{-3}$$



Exponential Distribution:

- **EXAMPLE (3-24):**

b- What is the probability that water will be too low to produce power?

$$P(X < (40.6 - 32)) = \int_{0}^{8.6} \frac{1}{13.5} e^{-\frac{1}{13.5}x} dx$$

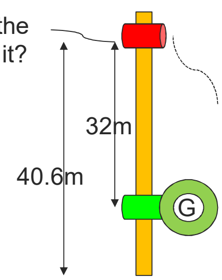
$$= \frac{-13.5}{13.5} e^{-\frac{1}{13.5}x} |_{0}^{8.6} = 1 - e^{-0.637} = 0.47$$

c- Given that water is not wasted in overflow, what is the probability that the generator will have water to derive it?

$$P(X > 8.6 / X < 40.6) = \frac{P(X > 8.6 \cap X < 40.6)}{P(X < 40.6)}$$

$$= \frac{P(8.6 < X < 40.6)}{P(X < 40.6)} = \frac{\int_{8.6}^{40.6} \frac{1}{13.5} e^{-\frac{1}{13.5}x} dx}{\int_{0}^{40.6} \frac{1}{13.5} e^{-\frac{1}{13.5}x} dx}$$

$$= 0.504$$



Common Continuous Random Variables:

- **II. Rayleigh Distribution:**

The Rayleigh density and distribution functions are:

$$f_X(x) = \frac{2}{\lambda} x e^{-\frac{x^2}{\lambda}}; \; x \geq 0$$

The cumulative distribution function is:

$$F_X(x) = 1 - e^{-\frac{x^2}{\lambda}}; \; x \geq 0$$

The Rayleigh pdf describes the envelope of white noise when passed through a band pass filter. It is used in the analysis of errors in various measurement systems.

\- ***Theorem:*** $\mu_x = E[X] = \sqrt{\frac{\pi\lambda}{4}}; \sigma_x^2 = E[(x - \mu_x)^2] = \frac{\lambda(4 - \pi)}{4}$

Cauchy Random Variable:

This random variable has:

$$f_X(x) = \frac{\alpha/\pi}{x^2 + \alpha^2}$$

The cumulative distribution function is:

$$F_X(x) = \frac{1}{2} + \frac{1}{\pi} tan^{-1}\left(\frac{x}{\alpha}\right)$$

Gaussian (Normal) Distribution:

- ***Definition:***

A random variable (X) with pdf:

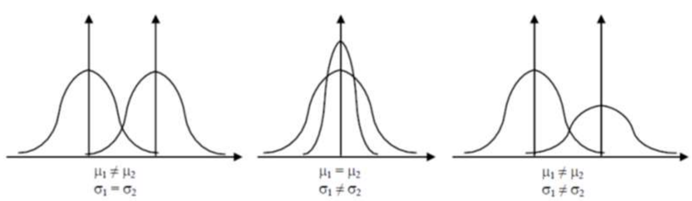
$$f_X(x) = \frac{1}{\sqrt{2\pi\sigma_x^2}} e^{-\frac{(x-\mu_x)^2}{2\sigma_x^2}} ; -\infty < x < \infty$$

has a normal distribution with parameters μ_x and σ_x^2 where $-\infty < x < \infty$ and $\sigma_x^2 \le 0,$ Furthermore:

$$E[X] = \mu_x; \;\; Var(x) = E[(x-\mu_x)^2] = \sigma_x^2$$

Gaussian (Normal) Distribution:

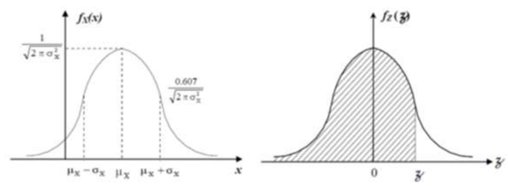
Infinite number of normal distributions can be formed by different combination of parameters.



Gaussian (Normal) Distribution:

- ***Definition:***

A normal random variable with mean zero and variance one is called a standard normal random variable. A standard normal random variable is denoted as Z.



Definition:

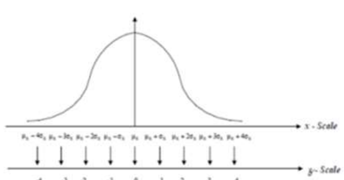
The function $\Phi(\mathfrak{z}) = P\{Z \leq \mathfrak{z}\}$ is used to denote the cumulative distribution function of a standard normal random variable:

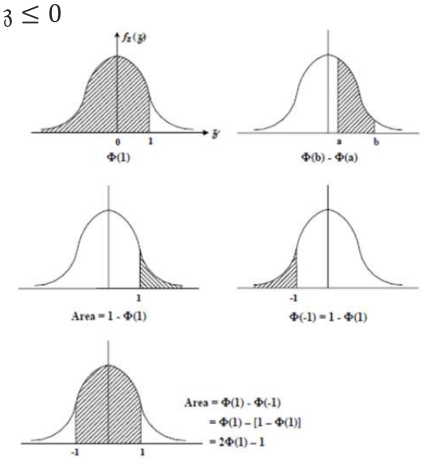
$$\Phi(\mathfrak{z}) = \int_{-\infty}^{Z} \frac{1}{\sqrt{2\pi}} e^{-\frac{u^2}{2}} du$$

Gaussian (Normal) Distribution:

- This function is tabulated for $\mathfrak{z} \leq 0$

For $\mathfrak{z} < 0; \Phi(\mathfrak{z}) = 1 - \Phi(-\mathfrak{z})$





Gaussian (Normal) Distribution:

- ***Cumulative Distribution Function:***

$$P(X \le x) = F_X(x) = \int_{-\infty}^{x} \frac{2}{\sqrt{2\pi\sigma_x^2}} e^{-\frac{(x-\mu_x)^2}{2\sigma_x^2}} dx;$$

Let $u = \left(\frac{x-\mu_x}{\sigma_x}\right) \rightarrow du = \frac{dx}{\sigma_x} \rightarrow dx = \sigma_x du$

$$\Phi(\mathfrak{z}) = \int_{-\infty}^{\frac{x-\mu_x}{\sigma_x}} \frac{2}{\sqrt{2\pi\sigma_x^2}} e^{-\frac{u^2}{2}} \sigma_x du; \qquad \Phi(\mathfrak{z}) = \int_{-\infty}^{\mathfrak{z}} \frac{2}{\sqrt{2\pi}} e^{-\frac{u^2}{2}} du;$$

$$\Phi(\mathfrak{z}) = \left(\frac{Z-\mu_x}{\sigma_x}\right)$$

Therefore, we conclude that: $P(X \le x_o) = \left(\frac{x_o-\mu_x}{\sigma_x}\right)$

$$P(x_o \le X \le x_1) = \left(\frac{x_1-\mu_x}{\sigma_x}\right) - \left(\frac{x_o-\mu_x}{\sigma_x}\right)$$

Gaussian (Normal) Distribution:

- **EXAMPLE (3-25):**

Suppose the current measurements in a strip of wire are assumed to follow a normal distribution with a mean of 10 mA and variance 4 $(mA)^2$. What is the probability that a measurement will exceed 13 mA?

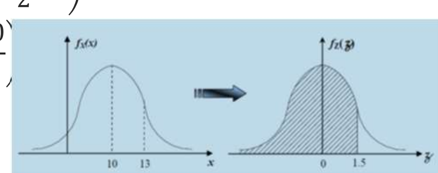
- **SOLUTION:**

X = current in mA

$$P(X \le x_o) = \Phi\left(\frac{x_o-\mu_x}{\sigma_x}\right) = \Phi\left(\frac{x_o-10}{2}\right)$$

$$P(X > 13) = 1 - \Phi\left(\frac{13-10}{2}\right)$$

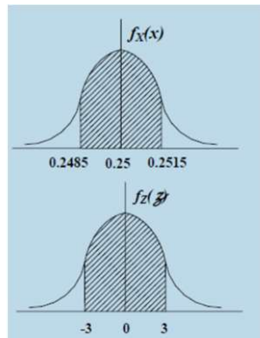
= 1 – 0.93319 = 0.06681



Gaussian (Normal) Distribution:

- **EXAMPLE (3-26):**

The diameter of a shaft in an optical storage drive is normally distributed with mean 0.25 inch and standard deviation of 0.0005 inch. The specifications on the shaft are 0.25 ± 0.0015 inch. What proportion of shafts conforms to specifications?



- **SOLUTION:**

X = diameter of a shaft in inch

$$P(0.25 - 0.0015 \le X \le 0.25 + 0.0015)$$

$$= \Phi\left(\frac{0.2515 - 0.25}{0.0005}\right) - \Phi\left(\frac{0.2485 - 0.25}{0.0005}\right)$$

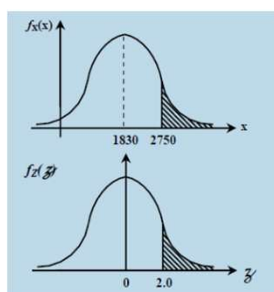
$$\Phi(3) - \Phi(-3) = 2\Phi(3) - 1$$

= (2 x 0.99865) – 1 = 0.9973

Gaussian (Normal) Distribution:

- **EXAMPLE (3-27):**

Assume that the height of clouds above the ground at some location is a Gaussian random variable (X) with mean 1830 m and standard deviation 460 m. find the probability that clouds will be higher than 2750 m.



- **SOLUTION:**

X = diameter of a shaft in inch

$$P(X > 2750) = 1 - P(X \le 2750)$$

$$= 1 - \Phi\left(\frac{2750 - 1830}{460}\right)$$

$$= 1 - \Phi(2)$$

= 1 – 0.9772 = 0.0228