Chapter I

$0 \le p(x) \le 1$

$\sum p(x) = 1$

III. Counting Rules

1. *mn* Rule; extended *mn* Rule

If an experiment is performed in two stages, with ***m*** ways to accomplish the first stage and ***n*** ways to accomplish the second stage, then there are ***mn*** ways to accomplish the experiment. For ***k*** stages, with the number of ways equal to $n_1 n_2 n_3 \ldots n_k$

2. Permutations:

The number of ways you can arrange ***n*** distinct objects, taking them ***r*** at a time $P_r^n = \frac{n!}{(n-r)!}$

3. Combinations:

The number of distinct combinations of ***n*** distinct objects that can be formed, taking them ***r*** at a time is $C_r^n = \frac{n!}{r!(n-r)!}$

IV. Event Relations

1. Events

a. Disjoint or mutually exclusive: $P(A \cap B) = 0$ and $P(A \cup B) = P(A) + P(B)$.

b. Complementary: $P(A) = 1 - P(A^C)$

c. Independent events $P(A \cap B) = P(A)P(B)$ and $P(A|B) = P(A)$, $P(B|A) = P(B)$

2. Conditional probability: $P(A \mid B) = \frac{P(A \cap B)}{P(B)}$

3. Additive Rule of Probability: $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

4. Multiplicative Rule of Probability: $P(A \cap B) = P(A)P(B \mid A)$

5. Law of Total Probability $P(A) = P(A \cap X_1) + P(A \cap X_2) + \ldots + P(A \cap X_k) = P(X_1)P(A|X_1) + P(X_2)P(A|X_2) + \ldots + P(X_k)P(A|X_k)$;
where X_1 , X_2 , X_3 ,..., X_k be mutually exclusive

6. Bayes' Rule $P(X_i \mid A) = \frac{P(X_i)P(A \mid X_i)}{\sum P(X_i)P(A \mid X_i)}$ for $i = 1, 2, \ldots k$ where X_1 , X_2 , X_3 ,..., X_k be mutually exclusive

ChapterII

	$f_x(x) = P(X=x)$	$F_x(x) = P(X \le x)$		$\mu_x = E[X]$ $\sigma_x^2 = E[(x-\mu_x)^2]$	Examples
General Discrete	$P(X=x)$	$\sum_0^x P(X=x_i)$		$\mu_x = \sum_0^x x_i f(x_i)dx; and \sigma_x^2 == \sum_0^x (x_i-\mu_x)^2 f(x_i)$	
General Continuous	$P(X=x)$	$\int_{-\infty}^x f(x)dx$		$\mu_x = \int_{-\infty}^x xf(x)dx;\ and\ \sigma_x^2 == \int_{-\infty}^x (x-\mu_x)^2 f(x)dx$	
Discrete Uniform $a \le x \le b$	$\frac{1}{b-a}$	$\frac{\text{int}(x)-a}{b-a}$	any integer value between *a* and *b* inclusive, each equally likely	$\mu_x = \frac{a+b}{2}$ $\sigma_x = \frac{b-a}{\sqrt{12}}$	-Rolling a single die -Follows a uniform distribution over the interval -all events within that class are equally likely to occur
Binomial *x*=0,1,…,*n* *n*,*p*	$\binom{n}{x_i} P^x (1-P)^{n-x}$	$\sum_0^x \binom{n}{x_i} P^x (1-P)^{n-x}$	1. The experiment consists of n repeated trials; 2. Each trial results in an outcome that may be classified as a success or a failure (hence the name, binomial); 3. The probability of a success, denoted by p, remains constant from trial to trial and repeated trials are independent.	$\mu_x = E\{X\} = np$ $\sigma_x = \sqrt{np(1-p)}$	-Experiments Success or a failure -Coin toss -Male and Female -The patient dies, or does not

Geometric *x*=1,…,*n* *n*,*p*	$P(1-P)^{x-1}$	$P\sum_{1}^{x}(1-P)^{x-1}$	The number of Bernoulli trials needed until the first Success occurs (P(*S*)=*p*)	$\mu_x = E\{X\} = \frac{1}{p}$ $\sigma_x = \sqrt{\frac{(1-p)}{p^2}}$	-Number of tosses to first head - Number of inspections to obtain first defec- tive -Number of bits transmitted until the first error
Poisson *x*=1,…,*n* λ _ $occurance$ T _ $\text{int}\ erval$ $b = \lambda T$	$\frac{e^{-b}b^x}{x!}$	$\sum_{0}^{x}\frac{e^{-b}b^x}{x!}$	Distribution often used to model the number of incidences of some characteristic in time or space	$\mu_x = E\{X\} = b$ $\sigma = \sqrt{b} = \sqrt{\lambda T}$	-Arrivals of customers in a queue -Numbers of flaws in a roll of fabric -Number of typos per page of text.
Hypergeometric $x =$ $0,1,\ldots,\min(n,k)$ $p = \left(\frac{k}{N}\right)$	$\frac{\binom{k}{x}\binom{N-k}{n-x}}{\binom{N}{n}}$	$\sum_{0}^{x}\frac{\binom{k}{x}\binom{N-k}{n-x}}{\binom{N}{n}}$	Finite population generalization of Binomial Distribution Population: N Elements k Successes	$E\{X\} = np$ $\sigma_x =$ $\sqrt{np(1-p)\frac{N-n}{N-1}}$	- A sample of n elements are selected at random without replacement. - A batch of 100 piston rings is known to contain 10 defective rings. If two piston rings are drawn from the batch
Exponential $x \geq 0$	$\lambda e^{-\lambda x}$	$1-e^{-\lambda x}$	a process in which events occur continuously and independently at a constant average rate.	$E\{X\} = \frac{1}{\lambda}$ $\sigma_x = \sqrt{\frac{1}{\lambda^2}}$	-The number of calls that arrive each day over a period of a year -Records show that job submissions have a Poisson distribution with an average of 4 per minute -Cracks in specific length

Rayleigh	$\frac{2x}{b}e^{-x^2/b}$	$1-e^{-x^2/b}$		$E\{X\}=\sqrt{\frac{\pi b}{4}}$ $\sigma_x=\sqrt{\frac{b(4-\pi)}{4}}$	-Model multiple paths of dense scattered signals reaching a receiver. -Wind speed, wave heights and sound/light radiation. -The lifetime of an object, where the lifetime depends on the object's age.
Cauchy	$\frac{\alpha\pi}{x^2+\alpha^2}$	$\frac{1}{2}+\frac{1}{\pi}\tan^{-1}\left(\frac{x}{\alpha}\right)$		$E\{X\}=\infty$ $\sigma_x=\infty$	
Gaussian (Normal) $-\infty\le x\le\infty$ Normal approximation of the Binomial and Poisson If n>>k	$\frac{1}{\sqrt{2\pi\sigma_x^2}}e^{-\frac{(x-\mu_x)^2}{s\sigma_x^2}}$	$\Phi\left(\frac{x-\mu_x}{\sigma_x}\right)$	represent real-valued random variables whose distributions are not known	$E[X]=\mu_x$ $\sigma_x^2=\sigma_x^2$	When we repeat an experiment numerous times and average our results, the random variable representing the average or mean tends to have a normal distribution as the number of experiments becomes large. -Errors in measurement or production processes can often be approximated by a normal distribution

Linear Functions : $g(Y)=aY+b \quad (a,b\equiv\text{constants})$

$E[aY+b]=a\mu+b$

$Var[aY+b]=a^2\sigma^2$

$\sigma_{aY+b}=|a|\sigma$

Let Y = g(X) be a monotonically increasing or decreasing function of (x). $f_y(y)=\frac{f_x(x)}{|dy/dx|}$