Probability and Statistical Engineering, ENEE2307

Quiz #4

Name: Sec #1 12 Dec 2017

Std. No.:

Suppose continuous r.v.s $(X,Y)\in R^2$ have joint pdf

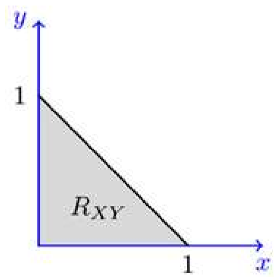
$$f_{x,y}(x,y)=\begin{cases} cxy & ;\ x\geq 0; and\ 0\leq x+y\leq 1 \\ 0 & ;\ otherwise \end{cases}$$

a. (3 points) Find the c?

b. (3 points) Determine the marginal pdfs for X and Y?

c. (2 points) Are X and Y independent?

d. (2 points) Find $P(X+Y\leq 0.5)$.



$$\int_0^1\int_0^{1-x} f_{x,y}(x,y)dydx=\int_0^1\int_0^{1-x} cxydydx=1$$

$$\int_0^1 cx\frac{(1-x)^2}{2}dx=1$$

$$\frac{c}{2}\left[\frac{x^2}{2}-\frac{2x^3}{3}+\frac{x^4}{4}\right|_0^1=1$$

$$c=24$$

$$b)\ \ f_x(x)=24\int_0^{1-x} xydy=12(x-x^2)$$

$$f_y(y)=24\int_0^{1-y} xydy=12(y-y^2)$$

$$f_{x,y}(x,y)\stackrel{?}{=}f_x(x)f_x(y)$$

$$24xy\neq 12(x-x^2)*12(y-y^2)$$

X and Y are not statistically independent

$$d)\ \ \mathrm{P(X+Y<0.5)=P(Y<0.5-X)}=24\int_0^{0.5}\int_0^{0.5-x} xydydx=$$

$$24\int_0^{0.5} x\frac{(0.5-x)^2}{2}dx=\frac{24}{2}\left[\frac{x^2}{2}-\frac{x^3}{3}+\frac{x^4}{4}\right|_0^{0.5}=\frac{3}{16}$$

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Quiz #5

Name: Sec #4 2-Jan-1

Std. No.:

An insurance company has 25000 automobile policy holders. If the yearly claim of a policy holder is a random variable with mean 320 and standard deviation 540, approximate the probability that the total yearly claim exceeds 8.3 million.

$$Y = X_1 + X_2 + \cdots + X_{25000}$$

$$\mu_y = n\mu_x = 25000 * 320$$

$$\sigma_y^2 = n\sigma_x^2 = 25000 * 540^2$$

$$P\{z > 8.3 \times 10^6\} = 1 - \phi\left(\frac{8.3 \times 10^6 - 25000 * 320}{540 * \sqrt{25000}}\right)$$

$$= 1 - \phi(3.51) = 0.00023$$

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Quiz #MakeUp

Name: Sec # 1/13/2018

Std. No.:

Assume the r.v. X has a Bernoulli distribution, that is,

$$f\ \ (x,p)=\begin{cases} p^{x}(1-p)^{1-x} & ;\ x=0,1 \\ 0 & ;\ otherwise \end{cases}$$

Where $0 < p \equiv \theta < 1$

Use the ML technique for n samples to find an estimator for p ?

$$L(\theta)=\prod_{1}^{n} p^{x_i}(1-p)^{1-x_i}=$$

$$=p^{\sum_1^n x_i}(1-p)^{1-\sum_1^n x_i}$$

$$\ln L(\theta)=\ln\left(p^{\sum_1^n x_i}(1-p)^{1-\sum_1^n x_i}\right)=$$

$$\ln L(\theta)=\left(\sum_{1}^{n} x_i\right)\ln p+\left(1-\sum_{1}^{n} x_i\right)\ln(1-p)$$

$$\frac{d}{dp}\ln L(p)=\frac{d}{dp}\left[\left(\sum_{1}^{n} x_i\right)\ln p+\left(1-\sum_{1}^{n} x_i\right)\ln(1-p)\right]=0$$

$$\left(\sum_{1}^{n} x_i\right)\frac{1}{p}+\left(1-\sum_{1}^{n} x_i\right)\frac{1}{1-p}=0$$

$$\left(\sum_{1}^{n} x_i\right)\frac{1}{p}=\left(1-\sum_{1}^{n} x_i\right)\frac{1}{p-1}$$

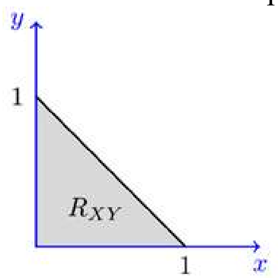
$$\frac{p-1}{p}=\frac{1-\sum_1^n x_i}{\sum_1^n x_i}$$

$$\frac{1}{p}=1-\frac{1-\sum_1^n x_i}{\sum_1^n x_i}=1-\frac{1}{\sum_1^n x_i}+1$$

$$p=\frac{1}{2-\frac{1}{\sum_1^n x_i}}=\frac{\sum_1^n x_i}{2\sum_1^n x_i-1}$$

Let X and Y be random variables with a joint pdf $f_{X,Y}(x,y) = C$ for $0 \le X + Y \le 1$, $0 \le X \le 1$, $0 \le Y \le 1$

a. Find C so that this is a valid joint pdf
b. Find the marginal density functions of X and Y.
c. Are X and Y independent?
d. Find the conditional pdf of Y given X = 0.5



$$\int_0^\infty \int_0^\infty f(x,y)dydx = 1 = \int_0^1 \int_0^{1-x} cdydx = c\int_0^1 (1-x)dx = c\left(x - \frac{x^2}{2}\right)_0^1 = \frac{c}{2}$$

$c = 2$

the marginal density functions of X and Y

$$f(x) = \int_0^{1-x} 2dy = 2 - 2x$$

$$f(y) = \int_0^{1-y} 2dy = 2 - 2y$$

Are X and Y independent? No $f(x)f(y) \neq f(x,y)$

$(2-2x)(2-2y) \neq 2$

Find the conditional pdf of Y given X = 0.5

$$f(y \mid x = 0.5) = \frac{f(x,y)}{f(x=0.5)} = \frac{2}{2-0.5*2} = 2$$