Birzeit University Faculty of Engineering Dep. of Electrical Engineering

Probability ENEE 331 Problem Set (2) Single Random Variables

P1) Let X be a random variable with mean $E\{X\}=11/12$ , and probability density function *Where a,b constants*

a) What is the value of a,b?
b) What is the cumulative distribution function of X?

$$f(x)=\begin{cases} ax^2+b & for\, 0\le x\le 1 \\ 0 & otherwise \end{cases}$$

$$1=\int_0^1 f(x)dx=\int_0^1 (ax^2+b)dx=\frac{ax^3}{3}+bx\Big]_0^1=\frac{a}{3}+b-0=\frac{a}{3}+b$$

$$a=3-3b$$

$$\frac{11}{12}=\int_0^1 xf(x)dx=\int_0^1 x(ax^2+b)dx=\frac{ax^4}{4}+\frac{bx^2}{2}\Big]_0^1=\frac{a}{4}+\frac{b}{2}$$

$$a=-2b+\frac{11}{3}$$

$$b=\frac{-2}{3},a=5$$

$$F_x(X\le x)=\int_0^x\left(5x^2-\frac{2}{3}\right)dx=\frac{5x^3}{3}-\frac{x}{3}\Big]_0^x=\frac{5x^3-2x}{3}$$

P2) If you reach bus stop at 11 o'clock, knowing that the bus will arrive at some time uniformly distributed between 11 and 11:30.

a) What is the probability that you will have to wait longer than 5 minutes?
b) If at 11:15 the bus has not yet arrived, what is the probability that you will have to what at least an additional 10 minutes?

Uniformly distributed a)

$$P(X\le x)=\frac{x}{b-a}$$

$$P(X>5)=1-P(X\le 5)=1-\frac{5}{30-0}=\frac{5}{6}$$

b) If at 11:15 the bus has not yet arrived, what is the probability that you will have to what at least an additional 10 minutes

$$P(X\ge(15+10)|X\ge 15)=\frac{P(X\ge 25\cap X\ge 15)}{P(X\ge 15)}=\frac{P(X\ge 25)}{P(X\ge 15)}=\frac{1-\frac{25}{30}}{1-\frac{15}{30}}=\frac{1}{3}$$

P3) Let X be a continuous random variable that has the following probability density function

$$f_X(x)=\begin{cases} 2x & 0<\text{x}<1 \\ 0 & \text{Elsewhere} \end{cases}$$

a) Find the mean and variance of X.
b) Find and plot the cumulative distribution function of X.
c) What is P(0.3 < X < 0.6)?

d) Let Y = 1/X compute E(1/X).

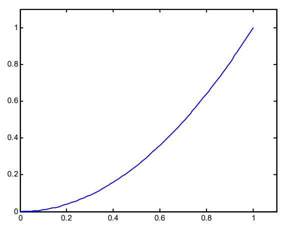
a) Find the mean and variance of X.

$$\mu_x = E(x) = \int_{-\infty}^{\infty} x f_x(x) dx = \int_0^1 2x^2 dx = \frac{2}{3}$$

$$\sigma_x^2 = \int_{-\infty}^{\infty} (x-\mu_x)^2 f_x(x) dx = \int_0^1 2x\left(x-\frac{2}{3}\right)^2 dx = \frac{2x^4}{4} - \frac{8}{9}x^3 + \frac{4}{9}x^2 \Big|_0^1 = \frac{1}{18}$$

b) Find and plot the cumulative distribution function of X.

$$F_x(X \le x) = \int_0^x 2x dx = x^2 \Big]_0^x = x^2$$




c) What is P(0.3 < X < 0.6)?

$$P(0.3 < \mathrm{X} < 0.6) = F_x(0.6) - F_x(0.3) = 0.6^2 - 0.3^2 = 0.27$$

$$or \int_{0.3}^{0.6} 2x dx = x^2 \Big]_{0.3}^{0.6}$$

d) Let Y = 1/X compute E(1/X).

$$E(\frac{1}{x}) = \int_{-\infty}^{\infty} \frac{1}{x} f_x(x) dx = \int_0^1 \frac{1}{x}(2x) dx = 2x \Big|_0^1 = 2$$

P4) Given a random variable X having a normal distribution with $\mu_x = 50$ and $\sigma_x = 10$. A new Random Variable $Y = X^2$

a) find the probability that X is less than 50

$$P(\mathrm{X} < 50) = \Phi(\frac{50-50}{10}) = 0.5$$

b) find the probability that X is between 45 and 62

$$P(45 < \mathrm{X} < 62) = \Phi(\frac{62-50}{10}) - \Phi(\frac{45-50}{10}) = 0.7580 - 0.3085 = 0.4495$$

c) find the mean of Y

$$\sigma_x^2 = 100 = E(x^2) - \mu_x^2 = E(x^2) - 50^2$$

$$E(x^2) = 2600$$

d) find the probaility density function of Y.

$$f_y(y) = 2\frac{f_x(x)}{|dx/dy|} = \frac{2}{2x}\frac{e^{\frac{-x^2}{2}}}{\sqrt{2\pi}} = \frac{e^{\frac{-y}{2}}}{\sqrt{2\pi y}}; y \ge 0$$

P5) Bits are sent over a communications channel in packets of 12. If the probability of a bit being corrupted over this channel is 0.1 and such errors are independent, what is the probability that no more than 2 bits in a packet are corrupted?

a) If 6 packets are sent over the channel, what is the probability that at least one packet will contain 3 or more corrupted bits?

$$P(X \le 2) = \sum_{0}^{2} \binom{n}{x_i} P^x (1-P)^{n-x} = 0.282 + 0.377 + 0.23 = 0.889$$

b) Let X denote the number of packets containing 3 or more corrupted bits. What is the probability that X will exceed its mean by more than 2 standard deviations?

$$P(X > 3) = 1 - P(X \le 2) = 1 - 0.889 = 0.111$$

$$\mu_x = np = 6 * 0.111 = 0.666$$

$$\sigma_x^2 = np(1-p) = 0.77^2$$

$$P(X - \mu_x > 2\sigma_x) = P(X > 2.2) = P(X \ge 3) = 1 - P(X \le 2) = \sum_{0}^{2} \binom{n}{x_i} P^x (1-P)^{n-x}$$

$$= 1 - \sum_{0}^{2} \binom{12}{x_i} 0.111^x (0.889)^{12-x} = 0.009$$

P6) Suppose that the lifetime X of a tower, measured in years, is described by an exponential distribution with mean equals to 25 years

$$f_x(x) = \begin{cases} \frac{1}{25} e^{-x/25} & x \ge 0 \\ 0 & otherwise \end{cases}$$

If three towers, operated independently, were erected at the same time, what is the probability that at least two will still stand after 35 years.

Solution

Exponential distribution $\lambda e^{-\lambda x}$

$$P(X \le x) = 1 - e^{-\lambda x} = 1 - e^{-\frac{x}{25}}$$

$$\mu_x = E\{X\} = \frac{1}{\lambda} = 25$$

The probability that a given tower still stands after 35 years is

$$P(X > 35) = 1 - P(X \le 35) = 1 - 1 + e^{-\frac{35}{25}} = e^{-1.4}$$

Finally, we use the binomial distribution with n=3 trials, success probability p=e^(-1.4), k>=2 successes. The probability that at least 2 will still stand after 35 years is

$$P(X \ge 2) = \sum_{2}^{3} \binom{n}{x_i} P^x (1-P)^{n-x} = \sum_{2}^{3} \binom{3}{x_i} e^{-1.4x} (1 - e^{-1.4})^{3-x}$$

$$= 3e^{-2*1.4} (1 - e^{-1.4})^1 + e^{-3*1.4} = 0.1524$$

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