Birzeit University
Faculty of Engineering
Department of Electrical Engineering
Engineering Probability and Statistics ENEE 331
Problem Set (1)
Fundamental Concepts of Probability

1) Three people, A, B, and C, are running for the same office, and we assume that one and only one of them wins. Suppose that A and B have the same chance of winning, but that C has only 1/2 the chance of A or B. Let E be the event that either A or C wins. Find P(E)?
   Solution $P[A] = P[B] = 2P[C]$
   $P[A] + P[B] + P[C] = 1$
   $p + p + 0.5p = 1$
   $p = \frac{2}{5} = 0.4$
   $P[E] = P[A] + P[C] = 0.4 + 0.2 = 0.6$
2) The student body is re-electing the class President, Vice-President, Treasurer, and Secretary from a group of 28 students consisting of 10 seniors, 8 juniors, 6 sophomores, and 4 freshmen. This time each class must be represented. What is the probability of electing one officer from each class?

Solution: the four elected members President, Vice-President, Treasurer, and Secretary can be
senior, junior, sophomore, and freshman respectively
Or junior, senior, sophomore, and freshman respectively
Or …

We have four position from for groups so $N_G = \frac{4!}{(4-4)!} = 24 ways$

Where we have
10 ways to select one from 10 seniors,
8 ways to select one from juniors,
6 ways to select one from sophomores,
and 4 ways to select one from freshmen.
So the number of total ways that we can select in is
$N_A = 24 * 10 * 8 * 6 * 4 = 46080 ways$
Where we have N different ways four the four elected members regardless of the number from each

$$N = \binom{28}{4} = \frac{28!}{(28-4)!} = 491400 ways$$

The probability of electing one officer from each class is

$$p = \frac{46080}{491400} = 0.0938$$

3) Suppose you are taking a multiple-choice test with c choices for each question. In answering a question on a test, the probability that you know the answer is p. If

you don't know the answer, you choose one at random. What is the probability that you knew the answer to a question, given that you answered it correctly?

Solution:

p: knew the answer to a question

A: correct answer.

$$P(A) = p + \frac{1}{c}(1-p)$$

The probability that you knew the answer to a question, given that you answered it correctly

$$P(p/A) = \frac{P(p \cap A)}{P(A)} = \frac{p}{p + \frac{1}{c}(1-p)}$$

4) A sample space consists of three events, A, B and C. If $P(A^c) = 0.5$, and $P(A \cap B) = 0.25$, $P(B \cup C) = 0.75$. The pair of events (A and B), (B and C) are independent. Events A and C are mutually exclusive. Find the followings:

a. Probability that exactly one event will occur.

b. P(B/A)

Solution:

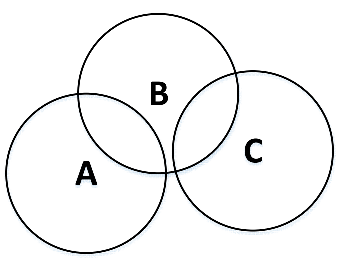
$$P(A) = 1 - 0.5 = 0.5$$

$$P(B) = \frac{P(A \cap B)}{P(A)} = \frac{0.25}{0.5} = 0.5$$

$$P(C) = P(B \cup C) - P(B) + P(B) * P(C)$$

$$P(C) = 0.75 - 0.5 + 0.5 * P(C)$$

$$P(C) = \frac{0.25}{0.5} = 0.5$$



a) Probability that exactly one event will occur= $\{A\overline{BC},\ \bar{A}B\bar{C}, \overline{AB}C\}$

Events A and C are mutually exclusive $A \cap \bar{C} = A \cap (1-C) = A, and\ \bar{A} \cap C = C$

$= P(A \cap \bar{B} \cap \bar{C}) + P(\bar{A} \cap B \cap \bar{C}) + P(\bar{A} \cap \bar{B} \cap C)$

$= P(A \cap \bar{B}) + P(\bar{A} \cap B \cap \bar{C}) + P(\bar{B} \cap C)$

$= P\big(A \cap (1-B)\big) + P\big((1-A) \cap B \cap (1-C)\big) + P\big((1-B) \cap C\big)$

$= P(A) - P(A \cap B) + P(B) - P(A \cap B) - P(B \cap C) + P(A \cap B \cap C) + P(C) - P(B \cap C)$

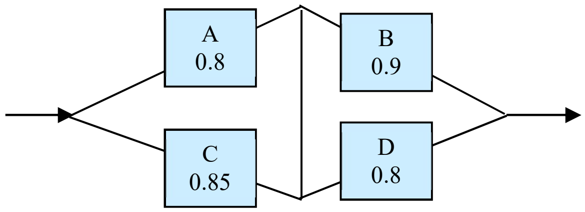
$= 0.5 - 0.25 + 0.5 - 0.25 - 0.25 + 0 + 0.5 - 0.25 = 0.5$

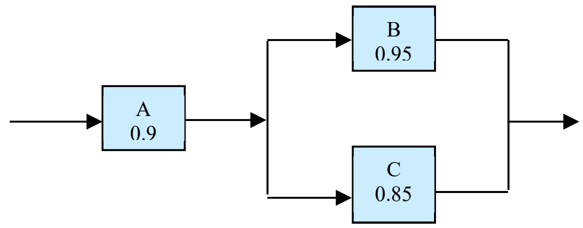
a.

5) Consider the following systems made up of independent components. The probability that each component functions is indicated in the figures.

a. Find the probability that the systems work properly (system reliability).

b. Is it possible to increase the reliability up to 99.5% for the system in (b) by adding more components in the parallel connection?





a) $=P((A_a \cup C_a) \cap (B_a \cup D_a) \cap A_b \cap (B_b \cup C_b)$

$$= P(A_a \cup C_a)P(B_a \cup D_a)P(A_b)P(B_b \cup C_b)$$

$=[P(A_a) + P(C_a) - P(A_a)P(C_a)][P(B_a) + P(D_a) - P(B_a)P(D_a)]P(A_b)[P(B_b) + P(C_b) - P(B_b)P(C_b)]$

$=[0.8 + 0.85 - 0.8 * 0.85][0.9 + 0.8 - 0.9 * 0.8]0.9[0.95 + 0.85 - 0.95 * 0.85]$

=0.97*0.98*0.9*0.9925=0.85

b) No it is impossible because system P(system Ab)=0.9 and it is series, so the output will be less or equal 0.9.

6) In a factory of Plastic tow machines produce 1000 pieces, the first produce 70% of these pieces, its produce 350 cups, and 250 plates, the second machine produce 150 cups and 120 bowls and the rest is plates. If one of these tow machines product is selected at random, find the probability that

Let M1 :First Machine; M2 :Second Machine; C :Cups, L :Plates; B:Bowls.

a. The product is plate?
b. The product is either cups, plates, either bowls?
c. The product is produced by the first machine if it was cups?

Let M1 :First Machine; M2 :Second Machine; C :Cups, L :Plates; B:Bowls.

Solution

$$P(M1) = 0.7; P(M2) = 0.3; P(C/M1) = \frac{0.35}{0.7} = 0.5\ ;\ P(C/M2) = \frac{0.12}{0.3} = 0.4;$$

$$P(L/M1) = \frac{0.25}{0.7} = 0.357\ ;\ P(L/M2) = \frac{0.03}{0.3} = 0.1; P(B) = 0.12\ ;$$

a- The product is plate is $P(L) = 0.25 + 0.03 = 0.28$;

Total probabilty theorm is the same.

$= P(M1) * P\left(\frac{L}{M1}\right) + P(M2) * P\left(\frac{L}{M2}\right) = 0.7 * 0.357 + 0.3 * 0.1$=0.28

d. The product is either cups, plates, either bowls

$$P(C \cup L \cup B) = (0.35 + 0.25 + 0.15 + 0.12 + 0.03) = 0.9;$$

The rest is produced by machine one 100 gives the same. The product is produced by the first machine if it was cups.

$$P(M1/C) = \frac{P(M1 \cap C)}{P(C)} = \frac{0.35}{0.35 + 0.15} = 0.7$$

7) A manufacturing firm employs three analytical plans for the design and development of a particular product. For cost reasons, all three are used at

varying times. In fact, plans 1, 2, and 3 are used for 30%, 20%, and 50% of the products, respectively. The defect rate is different for the three procedures as follows: P(D | P1) = 0.01, P(D | P2) = 0.03, P(D | P3) = 0.02, where P(D | Pj) is the probability of a defective product, given plan j. If a random product was observed and found to be defective, which plan was most likely used and thus responsible?

Solution:
$$P(P1/D) = P(P1)P(D/P1) = 0.3 * 0.01 = 0.003$$
$$P(P2/D) = P(P2)P(D/P2) = 0.2 * 0.03 = 0.006$$
$$P(P3/D) = P(P3)P(D/P3) = 0.5 * 0.02 = 0.01$$

So the third plan was most likely used and thus responsible since
$$P(P3/D) > Max(P(P1/D), P(P2/D))$$

8) A box contains colored balls: 5 Red balls and 7 Green balls. Suppose that three balls are drawn from this box.
   a. If the three balls are drawn one after another without replacement, what is the probability that the colors observed will be Red, Green in this order?
   b. If the three balls are drawn one after another with replacement, what is the probability that all three of the selected balls will be of the same color?
   c. If the three balls are drawn simultaneously from this urn (thus without replacement), what is the probability that the selected balls will all be of the same colors?

Solution: The probability that the three balls color will be in order Red, Green without replacement,
$$p = P(RGG \cup RRG) = \frac{5}{12} * \frac{7}{11} * \frac{6}{10} + \frac{5}{12} * \frac{4}{11} * \frac{7}{10} = 0.265$$
The probability that the three balls have the same color with replacement,
$$p = P(RRR \cup GGG) = \frac{5}{12} * \frac{5}{12} * \frac{5}{12} + \frac{7}{12} * \frac{7}{12} * \frac{7}{12} = \frac{13}{48}$$
The probability that the three balls have the same color are drawn simultaneously,
$$p = P(RRR \cup GGG) = \frac{5}{12} * \frac{4}{11} * \frac{3}{10} + \frac{7}{12} * \frac{6}{11} * \frac{5}{10} = \frac{9}{44}$$

9) For a storm-sewer system, estimates of annual maximum flow rates (AMFR) and their likelihood of occurrence [assuming that a maximum of 12 cubic feet per second, cfs, is possible] are given as:

   Event A: (5 ≤ AMFR ≤ 10 cfs) P(A) = 0.6
   Event B: (8 ≤ AMFR ≤ 12 cfs) P(B) = 0.6
   Event C=A∪B P(C) = 0.7

   Determine P (8 ≤ AMFR ≤ 10 cfs), the probability that AMFR is between 8 and 10 cfs.

Solution P (8 ≤ AMFR ≤ 10 cfs)=
$$P(A \cap B) = P(A) + P(B) - P(A \cup B) = 0.6 + 0.6 - 0.7 = 0.5$$