

Birzeit University
 Faculty of Engineering
 Department of Electrical Engineering
 Engineering Probability and Statistics ENEE 2309
 Problem Set (1)
 Fundamental Concepts of Probability

Instructor: Dr. Wael Hashlamoun

1) Let A and B denote two events defined over a sample space S . Suppose it is given that $P(A \cup B) = 0.76$

- a. If A and B are mutually exclusive events with $P[\bar{A}] = 0.45$, then what is $P[B]$?
- b. Suppose, instead, that A and B are independent events with $P[B/A] = 0.12$. What is $P[A]$?
- c. If it is given that $P[A] = 0.30$ and that $P[B/A] = 0.60$, what is $P[B]$?

2) In a certain lot of personal computers, it is known that 1 % have some minor defect as they come off the production line. They are put through a test procedure, which detects any defect 98 % of the time if a defect is really present, and indicates a defect 1 % of the time even though there is none present. What is the probability that

- a. a computer will be classified defective as a result of the test procedure?
- b. a computer is in fact defective if the test indicates that it is defective?

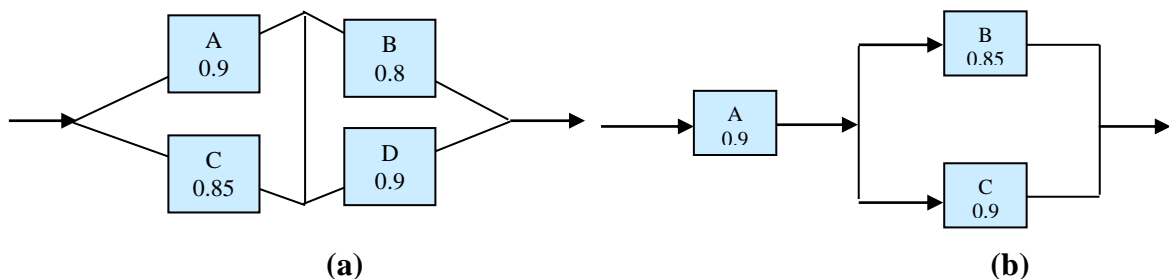
3) A sample space consists of three events, A , B and C . If $P(A^c) = 0.5$, and $P(A \cap B) = 0.25$, $P(B \cup C) = 0.75$. The pair of events (A and B), (B and C) are independent.

Events A and C are mutually exclusive. Find the followings:

- a. Probability that exactly one event will occur.
- b. $P(B/A)$

4) Consider the following systems made up of independent components. The probability that each component functions is indicated in the figures.

- a. Find the probability that the systems work properly (system reliability).
- b. Is it possible to increase the reliability up to 99.5% for the system in (b) by adding more components in the parallel connection?

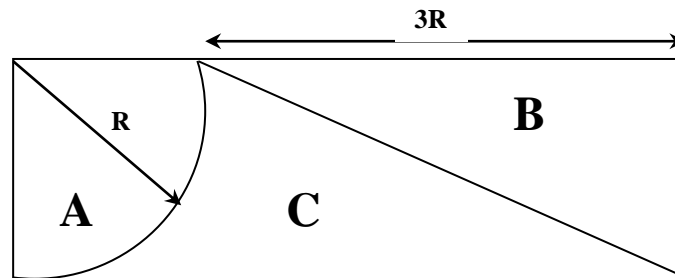


5) Items in a production line have to pass two successive quality control tests. If the probability of producing items of high quality is 0.95, the probability of misclassifying the items through the first and second tests is 0.05 and 0.02 respectively.

- a. Find the percentage of items classified as high quality.

- b. If an item is classified as high quality, what is the probability that it came out of the production line as high quality?

- 6) An irrigation well is to be drilled in the area shown in the figure. The probability of obtaining water for each sub-area A, B and C is 0.8, 0.1 and 0.3 respectively. The probability of selecting any of the three sub-areas is proportional to the area
- Find the probability of obtaining water.
 - If no water was obtained, what is the probability that the well was drilled in area C?



- 7) An urn contains colored balls: 4 Red balls and 6 Green balls. Suppose that three balls are drawn from this urn.
- If the three balls are drawn one after another without replacement, what is the probability that the colors observed will be Red, Green in this order?
 - If the three balls are drawn one after another with replacement, what is the probability that all three of the selected balls will be of the same color?
 - If the three balls are drawn simultaneously from this urn (thus without replacement), what is the probability that the selected balls will all be of the same colors?

8) Prove that a set with n elements has 2^n subsets.

9) For a storm-sewer system, estimates of annual maximum flow rates (AMFR) and their likelihood of occurrence [assuming that a maximum of 12 cubic feet per second, cfs, is possible] are given as:

Event A: ($5 \leq \text{AMFR} \leq 10$ cfs)	$P(A) = 0.6$
Event B: ($8 \leq \text{AMFR} \leq 12$ cfs)	$P(B) = 0.6$
Event C= $A \cup B$	$P(C) = 0.7$

Determine $P(8 \leq \text{AMFR} \leq 10 \text{ cfs})$, the probability that AMFR is between 8 and 10 cfs.

- 10) Given that $P(A) = P(B) = P(C) = \frac{1}{4}$, $P(A \cap B) = P(C \cap B) = 0$, $P(A \cap C) = \frac{1}{8}$, find
- the probability that at least one of the events A, B, or C occurs
 - $P(A \cup B \cup C)$
 - the probability that exactly one even occurs

11) A box contains three coins. One coin is two-headed, a second is fair, and the third is biased with p being the probability of getting a head. A coin is chosen at random from the box and flipped once

- What is the probability that the flip results in a head?
- Suppose that the flip yields a head, what is the probability that the chosen coin is the biased one?

12) An experiment is independently repeated five times. The probability of a success in every trial is 0.5, find the probability of obtaining three consecutive successes.

13) An experiment is independently repeated until a success is obtained for the first time. What is the probability that it will take five trials for that to happen assuming that the probability of a success in each trial is p .

14) A motor drives an electric generator. During a 30-day period, the motor needs repair with probability 8% and the generator, independently of the motor, needs repair with probability 4%. What is the probability that during the given period, the entire apparatus will need repair?

15) Assume that the reaction time of a driver over the age of 70 to a certain visual stimulus is described by a continuous probability function of the form $f(x) = xe^{-x}$, $x > 0$, where x is measured in seconds. Let A be the event "Driver requires longer than 1.5 seconds to react". Find $P(A)$.

16) One model to describe the mortality is $f(t) = kt^2(100 - t)^2$, $0 \leq t \leq 100$, where t describes the age at which a person dies.

- a. Find k
- b. Let A be the event "Person lives over 60". Find $P(A)$
- c. What is the probability that a person will die between the age of 80 and 85 given that the person has lived to be at least 70?