Birzeit University
Faculty of Engineering
Department of Electrical Engineering
Engineering Probability and Statistics ENEE 331
Problem Set (2)
Single Random Variables

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1) The time X (in minutes) for a lab assistant to prepare the equipment for a certain experiment is believed to have a uniform distribution over (25, 35).

a. Find the values of E (X) and Var (X).
b. What is the probability that the preparation time is within 2 minutes of the mean time?
c. Plot the graph of the cumulative distribution function of X.

2) The time X (in second) it takes a librarian to locate an entry in a file of records on checked out books has an exponential distribution with expected time of 20 seconds. Calculate the following probabilities:

a. $P(20 < X < 30)$.
b. $P(X > 15 / X < 30)$.

3) The number of failures of a testing instrument from contamination particles on the product is a Poisson random variable with a mean of 0.04 failures per hour. What is the probability that the instrument does not fail in an 8-hour shift?

4) Because of design problem in a system, there is a 40% chance that the system will fail before the warranty period is up and be brought in for repair. In a group of 8 customers, what is the probability that at least 3 customers will bring the system in for repair before warranty period is over.

5) Let X be a continuous random variable that has the following probability density function

$$f_X(x) = \begin{cases} 2x & 0 < x < 1 \\ 0 & \text{Elsewhere} \end{cases}$$

a. Find the mean and variance of X.
b. find and plot the cumulative distribution function of X.
c. What is $P(0.3 < X < 0.6)$?
d. Let $Y = 1/X$ compute $E(1/X)$.

6) Suppose that the lifetime X of a power transmission tower, measured in years, is described by an exponential distribution with mean equals to 25 years

$$f_x(x) = \begin{cases} \frac{1}{25} e^{-x/25} & x \geq 0 \\ 0 & otherwise \end{cases}$$

If three towers, operated independently, were erected at the same time, what is the probability that at least two will still stand after 35 years.

7) Marketing estimates that a new instrument for the analysis of soil samples will be very successful, moderately successful, or unsuccessful, with probabilities 0.3, 0.6, and 0.1 respectively. The yearly revenue associated with a very successful, moderately successful, or unsuccessful instrument is JD 10,000, JD 7,000, and JD 2,000 respectively. Let X denote the yearly revenue of the instrument

a. Determine the probability mass function of X.
b. Determine the expected value of the yearly revenue.

8) The probability density function of the length X of a metal bar is f (x) = 2 for 2.3 < x < 2.8 meters. If the specifications require the length of the metal bars to be from 2.25 to 2.75 meters, what proportion of the bars fails to meet the specifications?

9) The radial probability density function for the ground state of the hydrogen atom (the pdf of the electron position from the atom) is given by

$f(r) = \frac{4}{a^3} r^2 e^{-2r/a}$ for r > 0

where a is the Bohr radius (a = 52.9 pm).

a. What is the distance from the center of the atom that the electron is most likely to be found? This value of r is called the mode of the random variable.
b. Find the average value of r?, (the mean distance of the electron from the center of the atom).
c. What is the probability that the electron will be found within a sphere of radius a centered at the origin?

10) The number of telephone calls that arrive at a certain office is modeled by a Poisson random variable. Assume that on the average there are five calls per hour.

a. What is the average (mean) time between phone calls?
b. What is the probability that at least 30 minutes will pass without receiving any phone call?
c. What is the probability that there are exactly three calls in an observation interval of two consecutive hours?
d. What is the probability that there is exactly one call in the first hour and exactly two calls in the second hour of a two-hour observation interval?

11) The lifetime of a system, expressed in weeks, is a Rayleigh random variable X for which $f(x) = \frac{x}{200} e^{-x^2/400}$ for x ≥ 0.

a. What is the probability that the system will not last a full week?
b. What is the probability that the system lifetime will exceed one year?
c. Find a value t for which P{X ≤ t} = P{X ≥ t}. This value of X is called the median of the random variable

12) An intercom system master station provides power to four offices. The probability that any one office will be switched on and draw power at any time is 0.4. When on, an office draws 0.5 Watts

a. The master station is overloaded when 2 Watts is demanded. Find the probability of overload.
b. Find the mean value of the power delivered by the master station.

13) The number of knots in a certain type of wood used in the making of quality kitchen cabinets has been observed to follow a Poisson distribution with a mean of 1.5 knots per cubic meter of wood. What is the probability that

a. there is at most one knot in a one m^3 sample of this wood?
b. there is at least one knot in a one m^3 sample of this wood?
c. there are exactly two knots in a two m^3 sample of this wood?

14) Two shooters A and B point their guns towards a disk. For shooter A, the probability that any bullet hits the disk is 0.8, while for shooter this probability is 0.85. If shooter A fires 5 bullets, and independently of A, shooter B fires 6 bullets.

a. Find the probability that the disk is free from bullets
b. Find the probability that two bullets from shooter A and three bullets from shooter B reside on the disk.

15) The current I passing through an electronic device is related to the voltage V at its terminals by the relation $I = I_0 e^{kV}$ where I_0 and k are constants and V is a random variable uniformly distributed between (0, 0.25). Find the expected value of the current passing through the device.

16) The skewness of a random variable X can be measured in terms of its third moment about the mean. If a pdf is symmetric, $E\{(X-\mu_X)^3\}$ will obviously be 0; for pdf's not symmetric, $E\{(X-\mu_X)^3\}$ will not be zero. In practice, the symmetry (or lack of symmetry) of a pdf is often measured by the coefficient of skewness defined as: $\gamma_1 = \frac{1}{\sigma_X{}^3} E\{(X-\mu_X)^3\}$.

a. Find the skewness of a random variable X uniformly distributed over (-a, a)
b. Find the skewness of the distribution $f_x(x) = \begin{cases} 2(1-x) & 0 < x < 1 \\ 0 & otherwise \end{cases}$

17) Let X be the number of independent trials until a success occurs for the first time and let p be the probability of a success in a single trial.

a. Show that X has the discrete probability mass function $P(X = X) = (1-p)^{x-1} p$, x =1, 2, 3, …
b. Find the mean value of X.

18) Let X be a binomial random variable with parameters n and p.

a. Show that the mean value of X is np
b. Show that the variance of X is np(1-p)

19) Let X be a Poisson random variable with parameter b. Show that the probability that X is even is $\frac{1}{2}(1+e^{-2b})$

20) A company makes castings for steel stress-monitoring gauges. The annual profit Q, in hundreds of thousands of dollars, can be expressed as a function of product demand, x: $Q = Q(x) = 2(1-e^{-2x})$. Suppose that the demand (in thousands) for their castings follows an exponential pdf $f(x) = 6e^{-6x}$, x > 0. Find the company's expected profit.