Birzeit University
Faculty of Engineering
Department of Electrical Engineering
Engineering Probability and Statistics ENEE 331
Problem Set (3)
Single Random Variables

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1)The distribution of resistance for resistors of a certain type is known to be normal. 10% of all resistors have a resistance exceeding 10.256 ohms, and 5% have resistance smaller than 9.671 ohms. What are the mean value and standard deviation of the resistance distribution?

2) The tensile strength of paper is modeled by a normal distribution with a mean of 35 pounds per square inch and a standard deviation of 12 pounds per square inch. The specifications require that the tensile strength exceed 30 pounds per square inch. What is the probability that a randomly selected piece of paper will exceed the specifications?

3) The specifications for a component require that its length be between 2.53 cm and 2.57 cm. The current manufacturing process produces parts whose length is a normally distributed random variable, with a mean of 2.55 and a standard deviation of 0.015. What percentage of the parts will be within specifications?

4) A complex chemical manufacturing process is difficult to control. Only 70% of all batches of chemicals that are produced are within specifications. What is the probability that at least 270 of the most recent 400 batches are within specifications?

5) A biased coin comes up heads 30% of the time. The coin is tossed 400 times. Let X be the number of heads in the 400 tossing.
   a. Use Chebyshev's inequality to bound the probability that X is between 100 and 140.
   b. Use Gaussian approximation to compute the probability that X is between 100 and 140.

6) The lifetime of a structure **T** is a Gaussian random variable which is dependent on the strength of used concrete. B250 has $\mu = 35$ years, $\sigma = 10$ years, whereas B300 has $\mu = 50$ years, $\sigma = 5$ years. If a structure with a design period of 40 years is be designed, which concrete is better to be used?

7) The rainfall over Ramallah district follows the normal distribution with a mean of 600 mm and a standard deviation of 80 mm. The rainfall is distributed over a 500 km^2 area. Find:
   a- The probability of obtaining a rainwater volume less than 206 MCM (MCM = Million Cubic Meter)
   b- Find the mean and the standard deviation of the volume of rainfall in MCM.
   c- Flooding condition will be considered if the rainfall is higher than 900 mm.

Find the probability of flooding for any given year.

8) An exponential random variable X has a pdf

$$f_X(x) = \begin{cases} e^{-x} & x \geq 0 \\ 0 & x \leq 0 \end{cases}$$

Find the pdf of a new random variable $Y = X^2$

9) If a projectile is fired at an angle θ with an initial velocity v, the distance R (called the horizontal range) that it travels can be expressed as $R = (v^2/g)\sin 2\theta$, where g is the gravitational constant. If θ is uniformly distributed on (0, π/2),
  a. find the mean distance the projectile travels
  b. find the pdf of R.

10) The stored energy in a spring of stiffness constant k N/m displaced a distance x meters from an equilibrium position is $E = \dfrac{1}{2}kx^2$ J. If the displacement X is known to be a random variable uniformly distributed between (-Δ, Δ), find the mean value of the stored energy.

11) A complex system is made up of 100 components functioning independently. The probability that any one component will fail during the period of operation is 0.1. In order for the entire system to function, at least 85 of the components must be working. Evaluate the probability of this.

12) The temperature, X, achieved in a certain chemical reaction varies from experiment to experiment but appears to be described well by a pdf of the form $f(x) = xe^{-x^2/2}$, x > 0, where x is measured in Fahrenheit. The conversion formula for going from degrees Fahrenheit (X) to degrees Celsius (Y) is $Y = \dfrac{5}{9}(X - 32)$. Describe the distribution of temperature in terms of degrees Celsius. That is, find f(y).