



Faculty of Engineering and Technology
 Department of Electrical and Computer Engineering
 Engineering Probability and Statistics ENEE 2307

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Final Exam

Time: 120 minutes
 Student #:

Date: Wednesday 25/1/2017

Name:

Opening Remarks:

- This is a 120-minute exam. Calculators are allowed. Mobile phones, books, notes, formula sheets, and other aids are not allowed.
- You are required to show all your work and provide the necessary explanations everywhere to get full credit.
- There are two extra problems for those who were absent in the midterm exam.

Problem 1 (16 points) (ABFT criterion a)

The voltage V across a $1\text{-}\Omega$ resistor R is a uniform random variable over the interval $(0, 1)$. The instantaneous power is $P = V^2/R$.

- Find the expected value of the power P .
- Find the pdf of the instantaneous power P .

Problem 2 (18 points) (ABFT criterion a)

Let X and Y denote the lengths of life, in years, of two components in an electronic system. If the joint density function of these variables is

$$f_{X,Y}(x,y) = \begin{cases} k e^{-(x+y)} & 0 < x, 0 < y \\ 0 & \text{otherwise} \end{cases}$$

- Find k so that $f_{X,Y}(x,y)$ is a valid joint probability density function.
- Are X and Y statistically independent? Explain.
- Are X and Y correlated? Explain.
- Let $W = 2X + 3Y$, determine the standard deviation of W .

Problem 3 (18 points) (ABFT criterion e)

The time, in hours, it takes for computer programmer A to complete his program is a uniform random variable X , which is uniformly distributed over the interval $(0, 2)$. The time it takes for programmer B is also a random variable Y (independent of X), which is uniformly distributed over the interval $(0, 2)$.

- Write down the marginal probability density functions $f_X(x)$ and $f_Y(y)$.
- Find the joint probability density functions $f_{X,Y}(x,y)$ and the region over which it is defined.
- Find the probability that B needs at least twice the time needed by A to complete his program.

- Problem 4 (18 points) (ABET criterion e)
- The weights of cement bags are normally distributed with a mean of (50) kg and a standard deviation of 2 kg.
- What is the probability that one randomly selected cement bag will weigh more than 52 kg?
 - What is the probability that 5 randomly selected cement bags will have a mean weight of more than 52 kg?
 - Find n , such that the probability that the mean weight of n randomly selected cement bags be larger than 51 kg is less than 0.01.

Problem 5 (14 points) (ABET criterion a)

Given a random sample X_1, X_2, \dots, X_n of size n drawn from a distribution with pdf

$$f(x) = \theta e^{-\theta x}, x > 0.$$

Find a maximum likelihood estimator for the unknown parameter θ in terms of the observations.

Problem 6 (16 points) (ABET criterion e)

An electrical firm manufactures light bulbs that have a length of life that is approximately normally distributed with a standard deviation of 40 hours.

- If a sample of 30 bulbs has an average life of 780 hours, find a 96% confidence interval for the population mean of all bulbs produced by this firm.
- How large a sample is needed if we wish to be 96% confident that our sample mean will be within 10 hours of the true mean?

End of Exam
Good Luck

Problem 1:

$$P = \mathbb{R}^2 / \mathbb{R}$$

$$P = \mathbb{R}^2$$

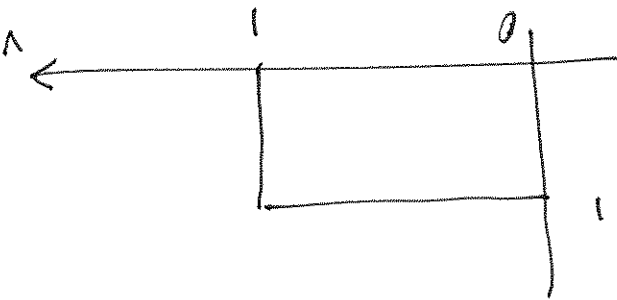
$$a \cdot E(P) = E(C \vee 2)$$

$$= \int_1^0 \int_1^0 f_{(C \vee 2)} dV$$

$$= \int_1^0 \int_1^0 \sqrt{2} dV = \frac{1}{3}$$

$$P = \mathbb{R}^2 / \mathbb{R} \Rightarrow$$

$f_{(C \vee 2)}$



"one-to-one mapping"

$$b. f_P(p) = \frac{|f_{(C \vee 2)}|}{|f_{(P \vee 2)}|} = \frac{2\sqrt{2}}{(1)}$$

$$= \frac{2\sqrt{2}}{(1)}$$

$$\text{when } v=0 \Rightarrow 0=0$$

$$v=1 \Rightarrow 1=1$$

$$1 \leq p \leq 1 \quad \left. \begin{matrix} 0 \\ 2\sqrt{2} \\ 1 \end{matrix} \right\} = (p) d_f \Rightarrow$$

3.0

$$\sigma^2 = \sqrt{13}$$

$$= 13$$

$$(1) b + (1) + q(1) = \sigma^2 = 2$$

$$1 = \frac{(1)}{1} = \frac{(2)}{1} = \sigma^2 \Rightarrow (x) \sigma^2 = (x) f(x)$$

$$\sigma^2 = 4 + 2 + q = 2$$

$$w = 2x + 3y \quad \cdot p$$

C. Independence \Rightarrow uncorrelated $\Rightarrow \rho = 0$

Since $f_{X|Y}(x|y) = f_X(x) \cdot f_Y(y) \Rightarrow X$ & Y are indep.

$$y > 0 \quad f_Y(y) = \int_{-\infty}^0 k e^{-(x+y)} dx = (k) f_Y$$

$$x > 0 \quad f_X(x) = \int_{-\infty}^0 k e^{-(x+y)} dy = (k) f_X \quad \cdot q$$

$$1 = k \int_{-\infty}^0 \int_{-\infty}^0 k e^{-(x+y)} dx dy = 1 \quad \cdot b$$

Problem 2: $f_{X|Y}(x|y) = \begin{cases} k e^{-(x+y)} & 0 < x < \infty, 0 < y < \infty \\ 0 & \text{elsewhere} \end{cases}$

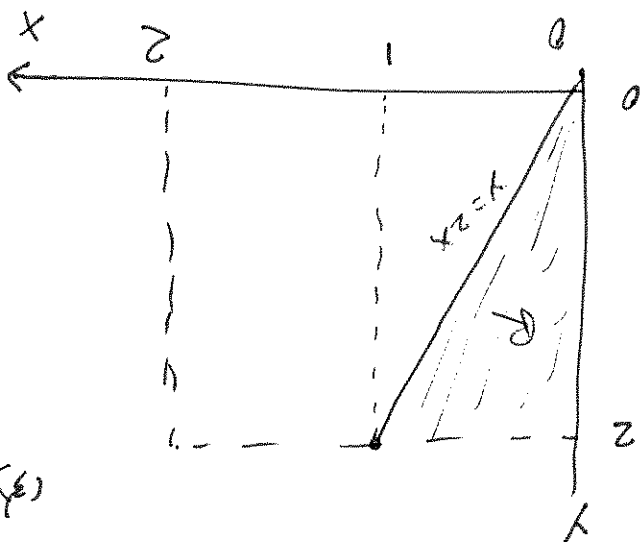
$$\frac{1}{4} = [1 - z] \frac{1}{1} = \int_1^0 (2x - x^2) \frac{1}{1} dx = \int_1^0 (2 - x) \frac{1}{1} dx =$$

$$x_p R_p \left(\frac{1}{1} \right) \int_1^0 \int_2^0 = (x \geq 1) \cdot 0$$

$$\left. \begin{array}{l} m \cdot 0 \\ z \geq x > 0 \\ \cup z \geq x > 0 \end{array} \right\} = (R(x))^{1/4} \cdot a$$

$$\left. \begin{array}{l} m \cdot 0 \\ z \geq x > 0 \end{array} \right\} = (R) \cdot \frac{1}{2} \cdot a$$

$$\left. \begin{array}{l} m \cdot 0 \\ z \geq x > 0 \end{array} \right\} = (x) \cdot \frac{1}{2} \cdot a$$



(2)

(3)

$$\boxed{22 = n} \Rightarrow n = 21.16 \quad \Phi\left(\frac{z}{\sqrt{n}}\right) = 0.01 \Rightarrow \sqrt{\frac{z}{n}} \approx 2.3$$

$$\Rightarrow \boxed{22 = n} \Rightarrow n = (2 \times 2.33)^2 = 21.7156$$

$$33.2 = \frac{z}{\sqrt{n}} \Rightarrow \Phi\left(\frac{z}{\sqrt{n}}\right) = 0.99$$

$$b.b.0 = \Phi\left(\frac{z/\sqrt{n}}{51-50}\right) \Rightarrow 0.01 \leq \Phi\left(\frac{z/\sqrt{n}}{51-50}\right) < 1 - \Phi\left(\frac{z/\sqrt{n}}{51-50}\right)$$

$$P(Y_2 > 51) < 0.01$$

$$\frac{Y_2}{\sqrt{2}} = \frac{Y_2}{\sqrt{2}} = \frac{Y_2}{\sqrt{2}} = \frac{Y_2}{\sqrt{2}} \Rightarrow E(Y_2) = \mu = 50$$

$$\boxed{0.013} \approx \frac{z}{\sqrt{n}} \approx \Phi(2.25) + \Phi(2.25) = 0.013$$

$$\boxed{0.0129} = 1 - 0.9871 = 1 - \Phi(2.236)$$

$$P(Y > 52) = 1 - \Phi\left(\frac{Y - \mu}{\sigma/\sqrt{n}}\right) = 1 - \Phi\left(\frac{2}{\sqrt{25}}\right) = 1 - \Phi(0.8)$$

$$b. \quad Y_1 = \frac{5}{\sqrt{25}} = \frac{5}{5} = 1 \Rightarrow \frac{Y_1}{\sqrt{2}} = \frac{1}{\sqrt{2}} = \frac{1}{1.414} = 0.707 = \sqrt{0.8}$$

$$\boxed{\Phi(1) = 0.15866}$$

from Φ functions

$$1 - \Phi(1) = 1 - 0.15866 = 0.8413$$

$$a. \quad P(X > 52) = 1 - \Phi\left(\frac{52-50}{\sqrt{25}}\right) = 1 - \Phi\left(\frac{2}{5}\right) = 1 - \Phi(0.4)$$

$$\mu = 50, \sigma = 2$$

Problem 4

$$\boxed{\frac{\sum x_i}{n} = \theta} \Leftrightarrow \sum x_i = \frac{\theta}{n}$$

$$0 = \sum x_i - \frac{\theta}{n} = \frac{\partial \ell(\theta)}{\partial \theta}$$

$$\ln L(\theta) = n \ln \theta - \theta \sum x_i$$

$$\sum x_i \theta^{-2} = \frac{1}{n}$$

$$[x_1, x_2, \dots, x_n] = \theta^{-1} = \theta^{-2} x_1 + \theta^{-2} x_2 + \dots + \theta^{-2} x_n$$

$$f(x; \theta) = \theta^{-x}$$

Problem 5

$$\boxed{99 \leq n} \Rightarrow$$

$$\Rightarrow n = 67.24$$

$$\Rightarrow \sqrt{n} = \frac{10}{2.05 + 40} \Rightarrow 8.2$$

$$2.05 = \frac{40}{\sqrt{n}} = 10$$

$$10 = \frac{\sqrt{n}}{2} \quad z_{\alpha/2} \quad p = 2\alpha$$

$$\boxed{96.0 \leq (\pm 6.4) \leq \hat{m} < 20.59 \pm 0.96}$$

$$96.0 \leq (\pm 6.4) \leq \hat{m} < 20.59 \pm 0.96$$

$$96.0 \leq \left(\frac{\sqrt{30}}{40} \right) \leq \hat{m} < 20.59 \pm 0.96$$

$$z_{\alpha/2} \approx 2.05$$

$$\frac{2}{z} = 0.02$$

$$\alpha = 0.04$$

$$P(\hat{m} - z_{\alpha/2} \frac{\sqrt{n}}{9} < \hat{m} < \hat{m} + z_{\alpha/2} \frac{\sqrt{n}}{9}) \geq 1 - \alpha$$

$$n = 30$$

$$9 = 40$$

problem 6