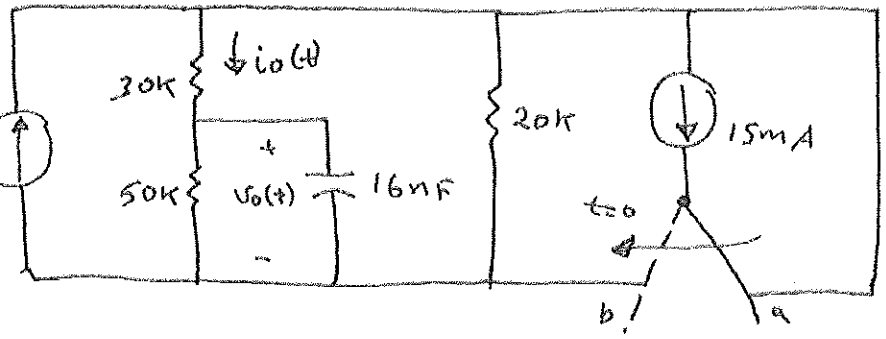
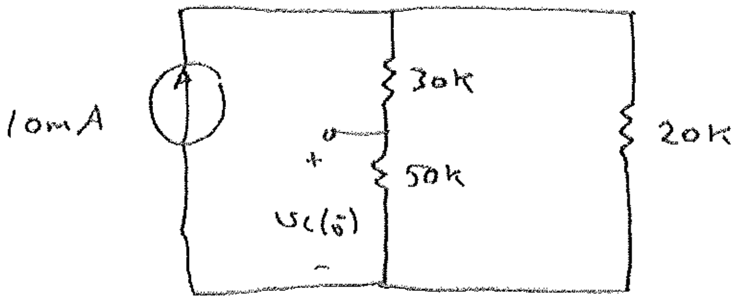
7.54 Q2

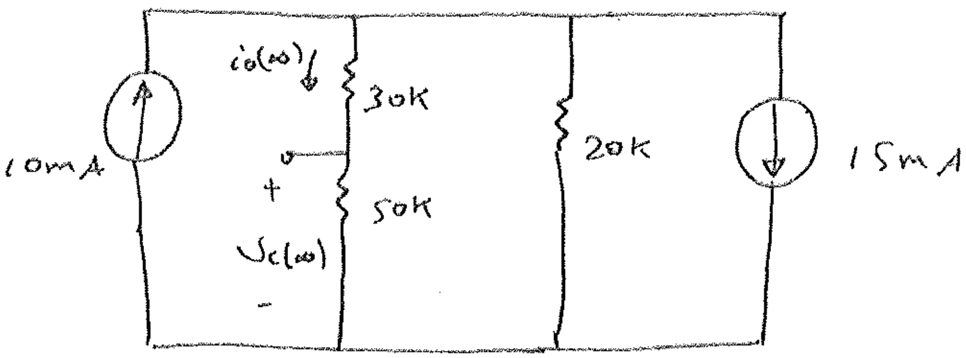



for t < 0




$$v_c(0^-) = (50k)\left(\frac{20k}{20k+80k}\right)(10mA) = 100\,V$$

at t = ∞



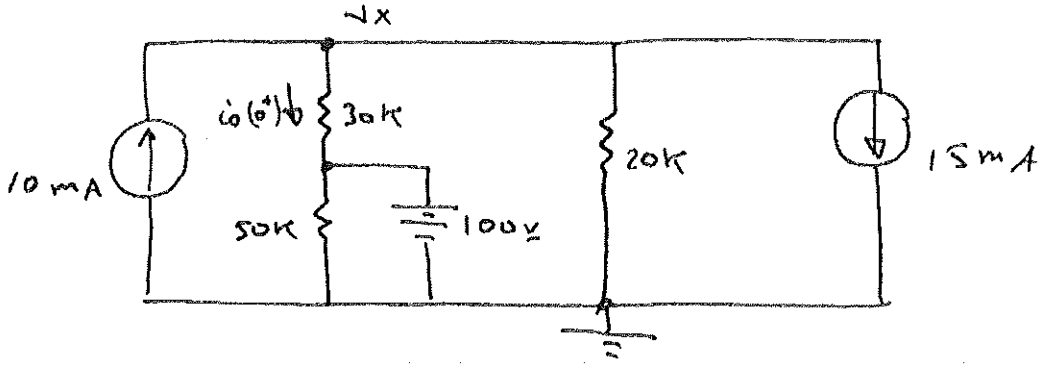

$$v_c(\infty) = (10mA - 15mA)\left(\frac{20k}{20k+80k}\right)(50k) = -50\,V$$

Q'2



$$i_o(\infty) = (10mA - 15mA)\left(\frac{20k}{20k + 80k}\right) = -1mA$$

at $t = 0^+$




$$v_o(0^+) = 100\,v$$

$$10mA = \frac{v_x - 100}{30k} + 15mA + \frac{v_x}{20k}$$

$$v_x = -20\,v$$

$$i_o(0^+) = \frac{v_x - v_c(0^+)}{30k} = -4mA$$

$$\tau = R_{TH} C$$

$$R_{TH} = (30k + 20k) \| 50k = 25k$$

$$\tau = 0.4ms$$

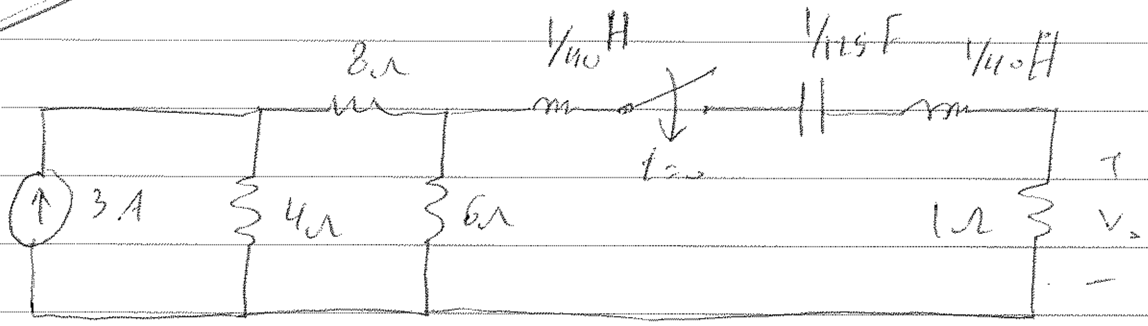
$$i_o(t) = i_o(\infty) + \left(i_o(0^+) - i_o(\infty)\right) e^{-t/\tau} \quad t > 0$$

$$i_o(t) = -\left(1 + 3e^{-2500t}\right) mA \quad \text{for } t > 0$$

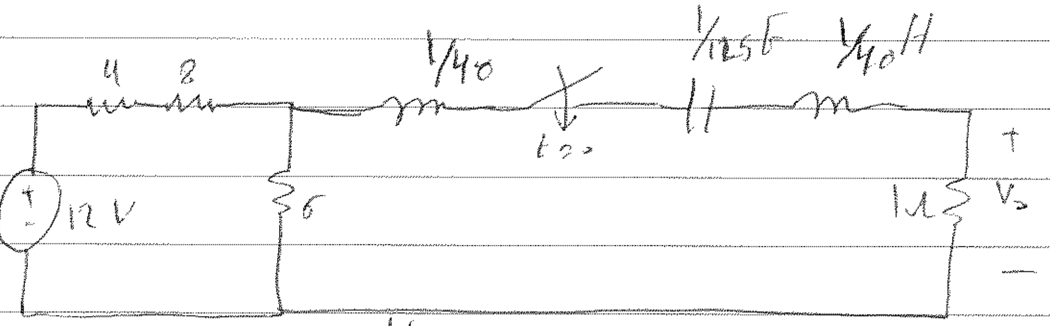
$$v_o(t) = v_o(\infty) + \left[v_o(0^+) - v_o(\infty)\right] e^{-t/\tau} \quad \text{for } t > 0$$

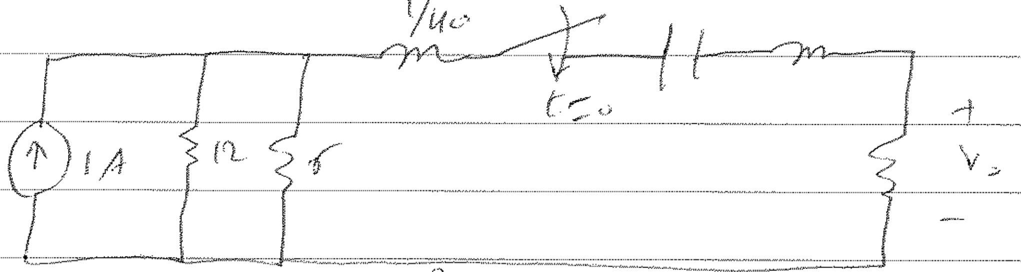
$$v_o(t) = \left(-50 + 150e^{-2500t}\right) v \quad \text{for } t > 0$$

# Q3

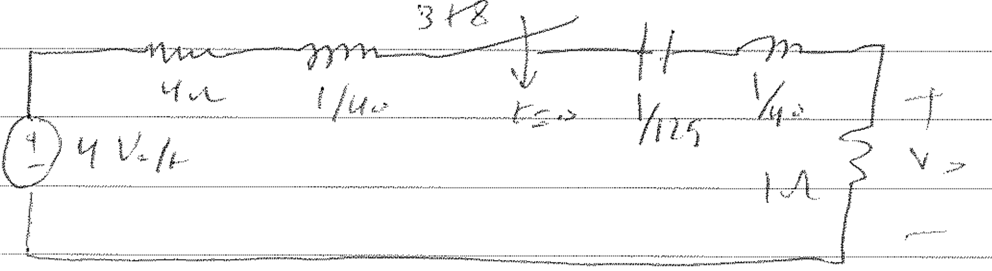



find $V_o$ for $t > 0$



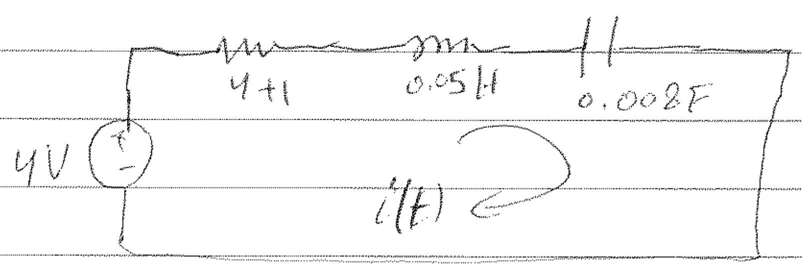

$12 \,//\, 6 = \frac{12 \times 6}{3+8} = 4\,\Omega$




for $t < 0$ or $t = 0^-$ :

$V_c(0^-) = 0$ Volt $\qquad i_L(0^-) = 0$ A

for $t > 0$



Q3

find $i(t)$ for $t>0$

$$-V_s + Ri(t) + L\frac{di(t)}{dt} + \frac{1}{C}\int_{0^-}^{t} i(t)\,dt + V_c(0^-) = 0 \quad (1)$$

by differentiation:

$$R\frac{di(t)}{dt} + L\frac{d^2 i(t)}{dt^2} + \frac{1}{C}i(t) = 0 \quad (2)$$

$$\alpha = \frac{R}{2L} = \frac{5}{2(0.05)} = 50$$

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{(0.05)(0.008)}} = 50$$

$\alpha = \omega_0$ (critical damped response)

$$i(t) = i_n(t) + i_f(t)$$

$i(t)$ Second order homogeneous differential equation:

i.e. $i(t) = i_n(t)$ $\quad i_f(t) = 0$

$$i(t) = A_1 t e^{-\alpha t} + A_2 e^{-\alpha t} \quad \text{for } t>0$$

$$i(t) = A_1 t e^{-50t} + A_2 e^{-50t}$$

To find $A_1$ & $A_2$ we need $i(0^+)$ & $\frac{di(0^+)}{dt}$

$i(0^+) = i_L(0^-) = i_L(0^+) = 0$

Using eq (1) at $t=0^+$

$$-V_s + Ri(0^+) + L\frac{di(0^+)}{dt} + \frac{1}{C}\int_{0^-}^{0^+} i(t)\,dt + 0 = 0$$

$$-4 + R(0) + 0.05 \frac{di(0^+)}{dt} + 0 = 0$$

$$\frac{di(0^+)}{dt} = \frac{4}{0.05} = 80$$

$$i(t) = A_1 t e^{-50t} + A_2 e^{-50t}$$

at $i(0^+)$ :

$$i(0^+) = A_1(0) + A_2(1) = 0$$

$$\Rightarrow A_2 = 0$$

$$\Rightarrow i(t) = A_1 t e^{-50t}$$

$$\frac{di(t)}{dt} = A_1 \left[ t(-50) e^{-50t} + e^{-50t}(1) \right]$$

at $t = 0^+$ :

$$\frac{di(0^+)}{dt} = 80 = \left[ A_1 \left( (0) + 1 \right) \right] \Rightarrow A_1 = 80$$

$$\Rightarrow i(t) = 80 t e^{-50t}$$

$$V_o(t) = \underset{1\Omega}{R}\, i(t) = 80 t e^{-50t} \quad \text{for } t > 0$$