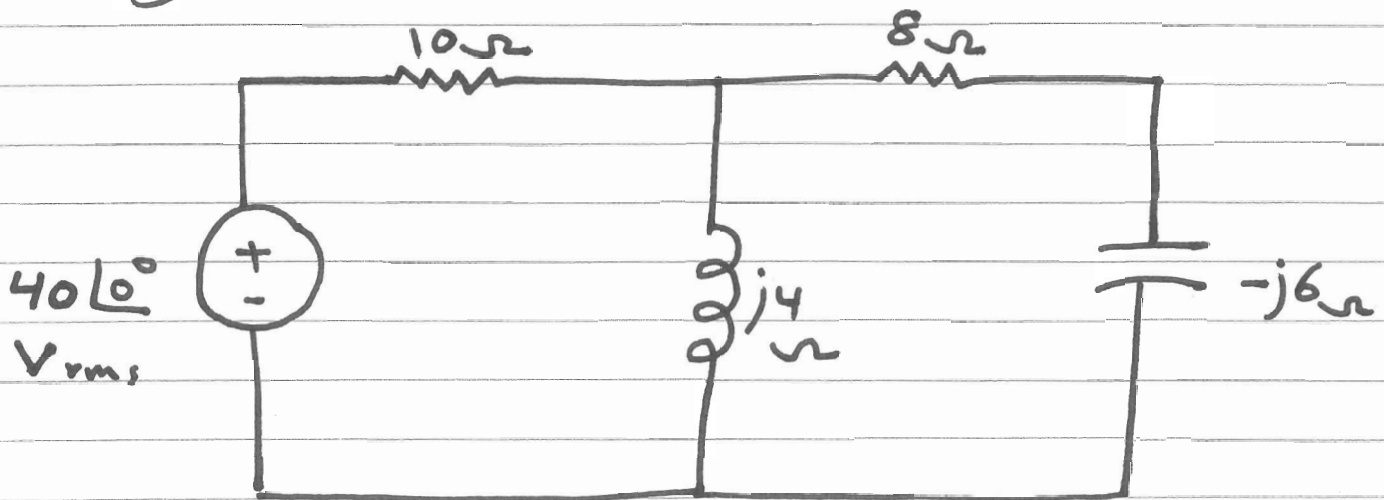# Example

Calculate the power factor seen by the source and the average power supplied by the source.

$$Z = 10 + j4 \| (8 - j6)$$

$$Z = 12.69 \angle 20.6\overset{\circ}{2} \quad \Omega$$

$$\vec{I}_s = \frac{40\angle 0^\circ}{Z} = 3.152 \angle -20.6\overset{\circ}{2} \ A_{rms}$$

$$\Theta_s = 0^\circ \quad , \quad \Phi_i = -20.6\overset{\circ}{2}$$

$$PF = \cos(\Theta_s - \Phi_i)$$

$$PF = \cos(20.62^\circ)$$

$$PF = 0.936 \ Lagging$$

The average power supply by the source is equal to the average power absorbed by the Circuit

----

$$P_{av} = V_{rms} \, I_{rms} \, \cos(\Theta_v - \Phi_i)$$

$V_{rms} = 40 \, V \, rms$

$I_{rms} = 3.152 \, A_{rms}$

$\Theta_v = 0°$
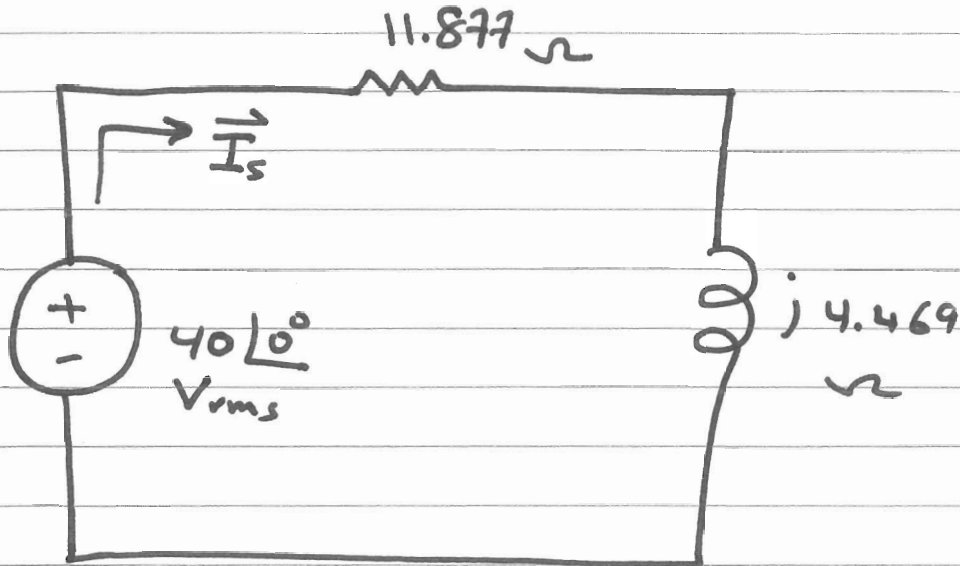
$\Phi_i = -20.62°$

$\therefore P_{av} = (40)(3.152) \cos(0 - (-20.62°))$

$\therefore P_{av} = 118 \, Watt$

$$Z = 12.69 \,\underline{/20.62°} \; \Omega$$

$$Z = 11.877 + j\,4.469 \; \Omega$$

$$\therefore \; P_{av} = I_{vms}^2 \, R$$

$$P_{av} = (3.152)^2 (11.877)$$

$$P_{av} = 118 \, W$$

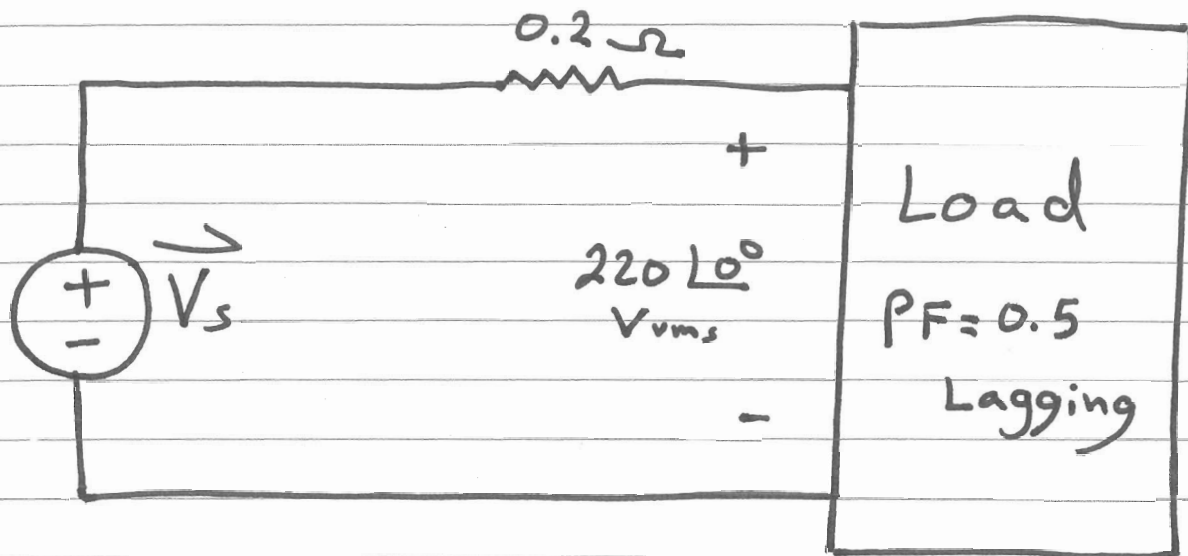also $P_{av} = P_{av}_{10\Omega} + P_{av}_{8\Omega} + P_{av}_{-j6} + P_{av}_{j4}$

$$P_{av} = P_{av}_{10\Omega} + P_{av}_{8\Omega}$$

Example

An industrial Load Consumer 11KW at 0.5 PF Lagging from a 220V rms Line. The transmission Line resistance from the power Company to the plant is 0.2 Ω.

1) Determine the average power that must be Supplied by the power Company

2) Repeat ① if the Power factor is Changed to unity.

The circuit shows a source V_s (marked $+/-$) connected through a $0.2\ \Omega$ resistor to a Load with $PF = 0.5$ Lagging. The load voltage is $220\angle 0^\circ$ V_{rms}.

$$P_{av_L} = V_{rms} \cdot I_{rms} \cdot PF$$
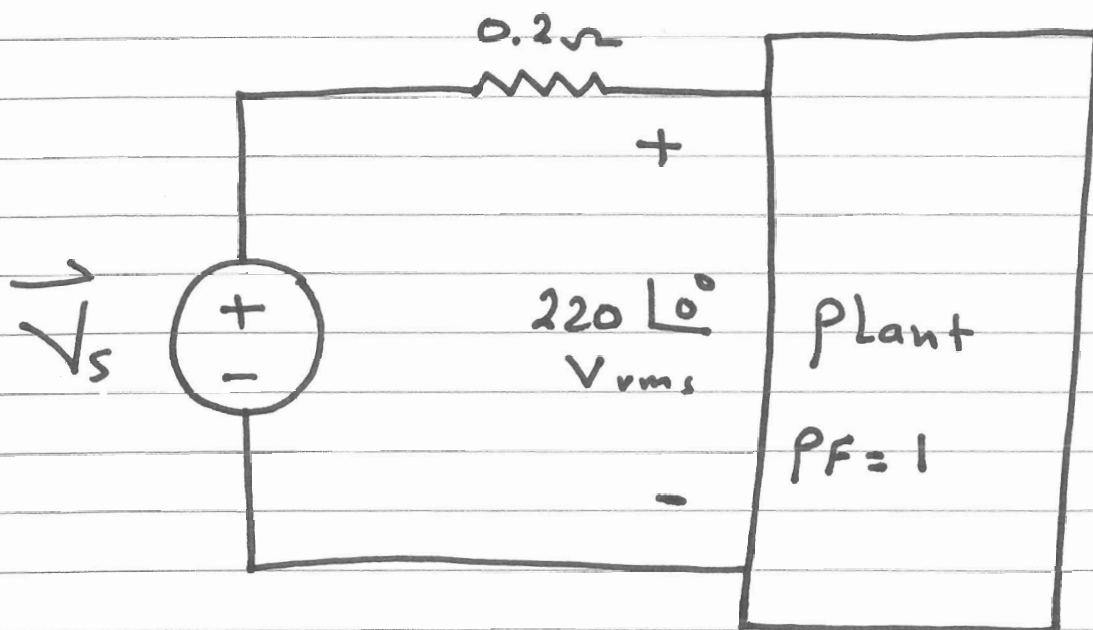
$$\therefore I_{rms} = \frac{P_{av_L}}{V_{rms} \cdot PF}$$

$$I_{rms} = \frac{11\,KW}{(220)(0.5)} = 100\ A_{rms}$$

$$P_{Loss} = (I_{rms}^2) \cdot (0.2) = 2\,KW$$

$$\therefore P_{av_{sup}} = P_{av_L} + P_{av_{Loss}}$$

$$P_{av_{sup}} = 13\,KW$$

$$P_{av}_{Load} = V_{vms} \cdot I_{vms} \cdot PF$$

$$\therefore I_{vms} = \frac{P_{av}}{V_{vms} \cdot PF} = 50 \, A \, vms$$

$$P_{Loss} = I_{vms}^2 \cdot R = (50)^2 \cdot (0.2) = 0.5 \, Kw$$

$$\therefore P_{av}_{sup} = 0.5 \, Kw + 11 \, Kw$$

$$P_{av}_{sup} = 11.5 \, Kw$$

Example

Find the power factor of the two loads
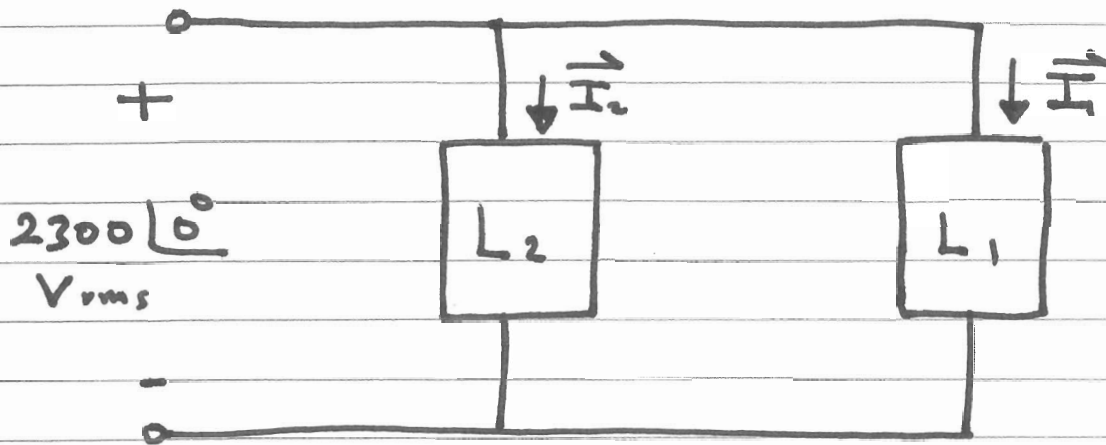
Load 1 :   10 KW , 0.9 Lagging PF

Load 2 :   5 KW , 0.95 Leading PF

$$\vec{I_1} = \frac{10,000}{(2300)(0.9)} \underline{/- Cos^{-1} 0.9}$$

$$\vec{I_1} = 4.83 \underline{/- 25.84°} \quad A\,rms$$

$$\vec{I_2} = \frac{5000}{(2300)(0.95)} \underline{/+ Cos^{-1} 0.95}$$

$$\vec{I_2} = 2.288 \underline{/18.195°} \quad A\,rms$$

$$\vec{I_s} = \vec{I_1} + \vec{I_2} = 6.78 \underline{/-12°} \quad A\,rms$$

$$PF = Cos(\theta_v - \theta_i) = Cos(0 - (-12°))$$

$$PF = 0.978 \ Lagging$$

-30-
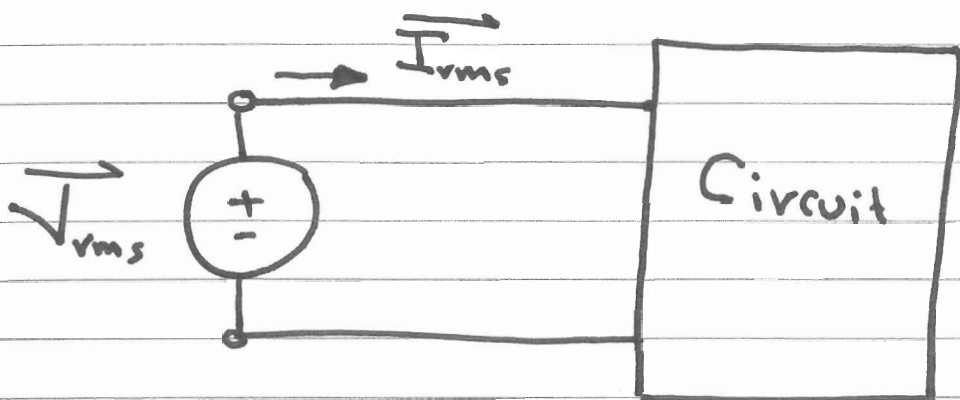
Complex Power

$$\vec{V}_{rms} = V_{rms} \; \underline{/\Theta_v}$$

$$\vec{I}_{rms} = I_{rms} \; \underline{/\Phi_i}$$

$$\vec{S} \equiv \text{Complex Power}$$

$$\vec{S} = \vec{V}_{rms} \cdot \vec{I}_{rms}^{\;*}$$

$$\vec{S} = V_{rms} \, I_{rms} \; \underline{/\Theta_v - \Phi_i}$$

$$\vec{S} = V_{rms} \, I_{rms} \, \cos(\Theta_v - \Phi_i)$$
$$+ j \, V_{rms} \, I_{rms} \, \sin(\Theta_v - \Phi_i)$$

$$\vec{S} = P_{av} + j \, Q$$

$$\vec{S} = P_{av} + j\,Q$$

$P_{av} \equiv$ Average power in Watt

$Q \equiv$ Reactive power in VAR

$\therefore P_{av} = Real\left[\vec{S}\right]$

$Q = Imajinary\left[\vec{S}\right]$

$Q = V_{rms}\,I_{rms}\,sin\,(\Theta_v - \phi_i)$

1) For pure resistance

$\Theta_v - \phi_i = 0$

$\therefore Q_R = 0$

2) For pure inductance

$\Theta_v - \phi_i = +90°$

$\therefore Q_L = V_{rms}\,I_{rms}$

$V_{rms} = WL\,I_{rms}$

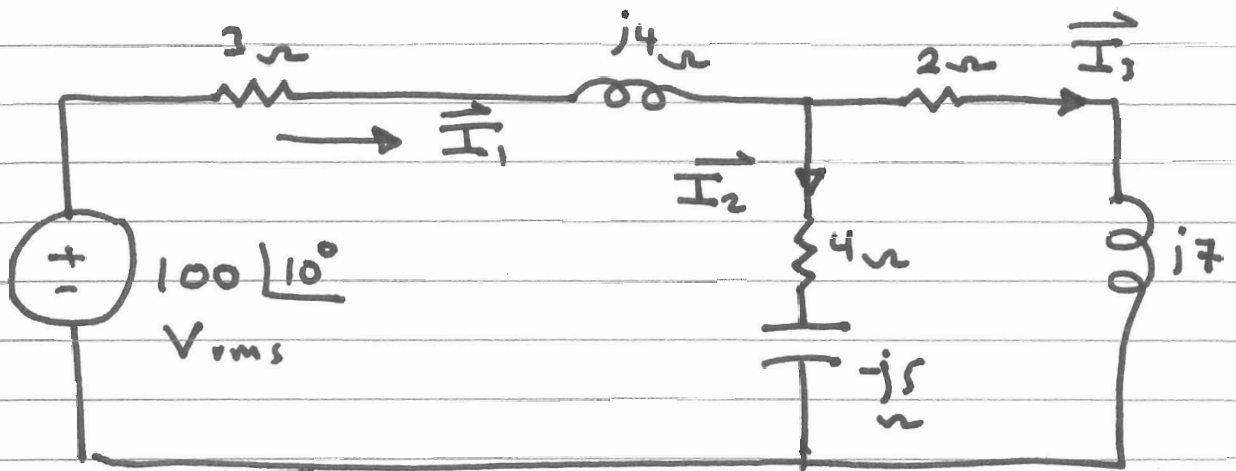$\therefore Q_L = WL\,I_{rms}^2 = \dfrac{V_{rms}^2}{WL}$

3) For pure Capacitance

$$\theta_v - \theta_i = -90°$$

$$Q_c = - V_{rms} I_{rms}$$

$$I_{rms} = \omega C V_{rms}$$

$$\therefore Q_c = - \frac{I_{rms}^2}{\omega C} = - \omega C V_{rms}^2$$

What are the VARS Consumed by the Circuit

$$Q = V_{rms} \, I_{rms} \, Sin \left(\theta_v - \phi_i \right)$$

$$\vec{I_1} = \frac{\vec{J_s}}{Z}$$

$$Z = (2+j7) \| (4-j5) + 3 + j4$$

$$Z = 10.35 + j4.55 = 11.3 \, \underline{/27.7°} \quad \sqrt{2}$$

$$\therefore \vec{I_1} = \frac{100 \, \underline{/10°}}{11.3 \, \underline{/27.7°}} = 8.84 \, \underline{/-13.7°} \, Arms$$

$$\therefore Q = (100)(8.84) \, Sin \left(10 - (-13.7°) \right)$$

$$\therefore Q = 355 \, VARS$$

$$I_2 = 10.2 \, Arms$$

$$I_3 = 8.95 \, Arms$$

$$P_{av} = V_{rms} \, I_{rms} \, Cos \, (\Theta_v - \Phi_i)$$

$$Q = V_{rms} \, I_{rms} \, Sin \, (\Theta_v - \Phi_i)$$

$$\frac{Q}{P_{av}} = tan \, (\Theta_v - \Phi_i)$$

$$\therefore \quad Q = P_{av} \, tan \, (\Theta_v - \Phi_i)$$

$$Q = P_{av} \, tan \left[Cos^{-1}(PF) \right]$$

$$\vec{S} = P_{av} + j \, Q$$

$$\vec{S} = \sqrt{P_{av}^2 + Q^2} \; \underline{\big|\tan^{-1} \frac{Q}{P_{av}}}$$

$$\vec{S} = V_{rms} \, I_{rms} \; \underline{\big|\Theta_v - \Phi_i}$$

$$\therefore \quad P_a = |S| = \sqrt{P_{av}^2 + Q^2}$$

$$\Theta_v - \Phi_i = tan^{-1} \frac{Q}{P_{av}}$$

$$\theta_v - \phi_i = \tan^{-1} \frac{Q}{P_{av}}$$

To increase P.F, we need to decrease Q

∴ For inductive Circuit, we
add a Capacitor in parallel
to increase the power factor
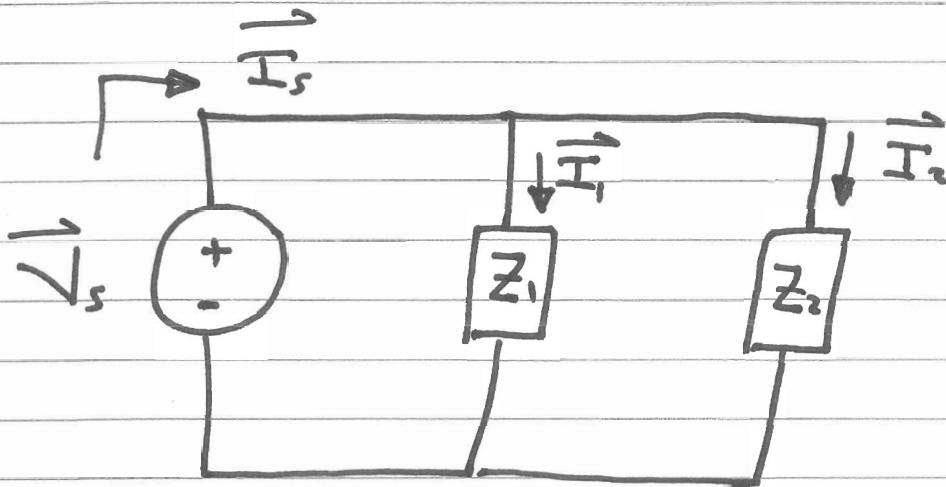
$$P_{av_T} = P_{av_1} + P_{av_2} + --- P_{av_n}$$

$$Q_T = Q_1 + Q_2 + -- Q_n$$

$$\vec{S}_T = P_{av_T} + j\, Q_T$$

$$\vec{S}_T = \vec{S}_1 + \vec{S}_2 + ----- \vec{S}_n$$

Conservation of Ac Power

The Complex, real, and reactive
powers of the Sources equal
the respective Sum of the complex
, real, and reactive powers of
the individual Loads.
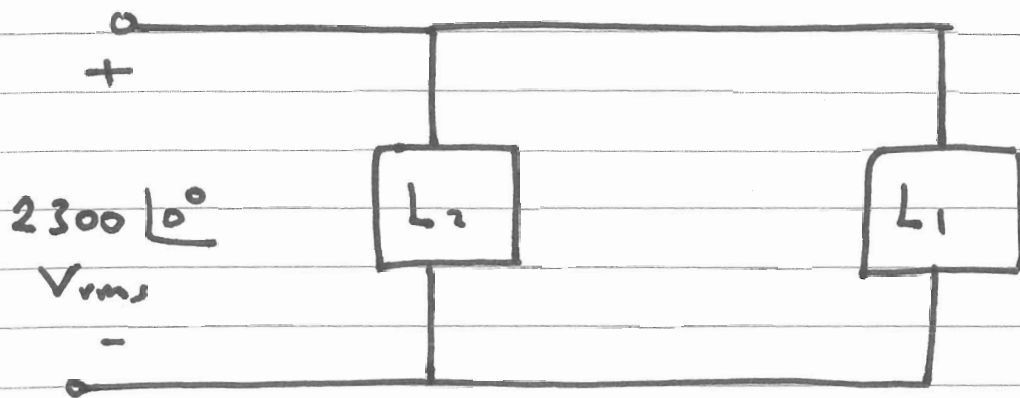

$$\vec{S}_{Source} = \vec{V}_s \cdot \vec{I}_s^*$$

$$\vec{S}_{Source} = \vec{V}_s \cdot \left(\vec{I}_1^* + \vec{I}_2^* \right)$$

$$\vec{S}_{Source} = \vec{V}_s \cdot \vec{I}_1^* + \vec{V}_s \cdot \vec{I}_2^*$$

$$\vec{S}_{Source} = \vec{S}_1 + \vec{S}_2$$

The Same results can be obtained for a series
Connection.

Find the power factor of the two Loads

Load 1 :   10 KW , 0.9 Lagging P.F

Load 2 :   5 KW   , 0.95 Leading P.F

$$\vec{S_1} = P_{av_1} + j\,Q_1$$

$$Q_1 = P_{av_1} \tan\left[\cos^{-1}(P.F_1)\right] = 4843 \text{ VARS}$$

$$\therefore \vec{S_1} = 10000 + j\,4843$$

$$\vec{S_2} = P_{av_2} + j\,Q_2$$

$$Q_2 = - P_{av_2} \tan\left[\cos^{-1}(P.F_2)\right]$$

$$Q_2 = - 1643 \text{ VARS}$$

$$\therefore \vec{S_2} = 5000 - j\,1643$$

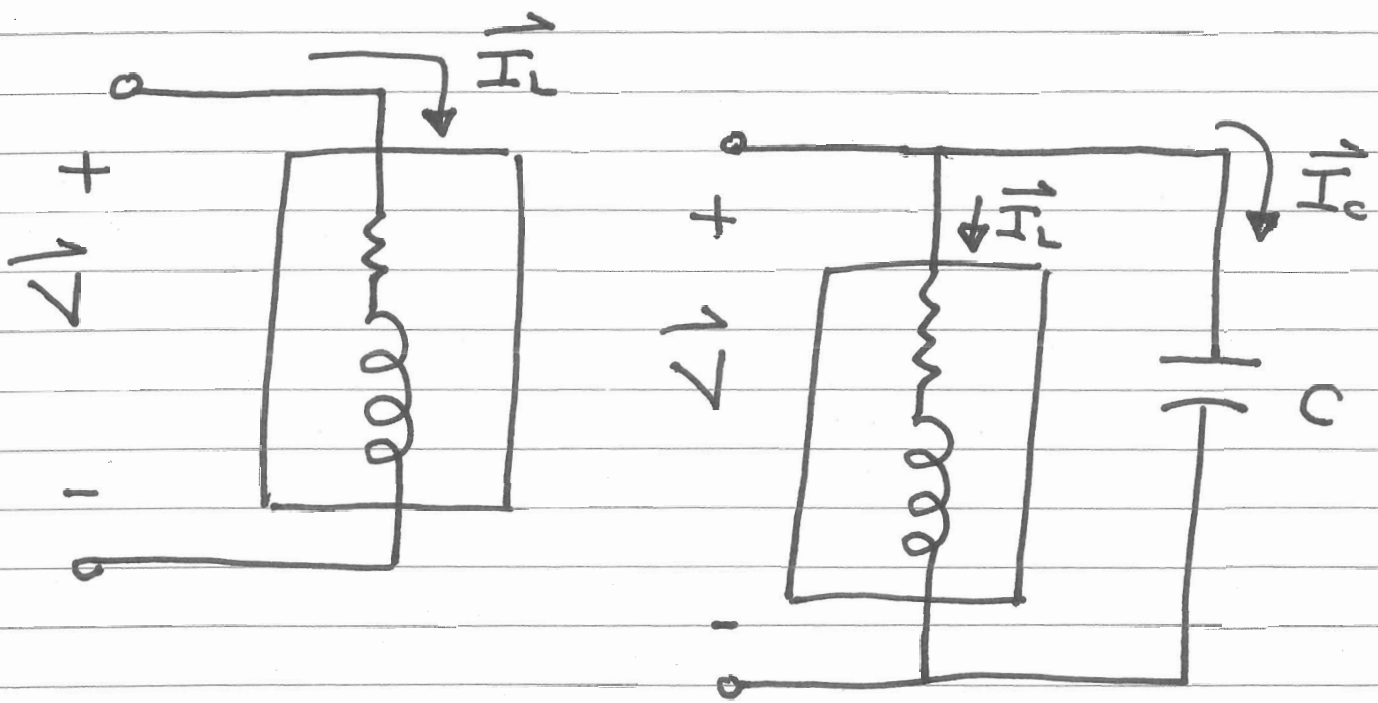$$\vec{S_T} = \vec{S_1} + \vec{S_2}$$

$$\vec{S_T} = 15000 + j\ 3200$$

$$\vec{S_T} = 15337.5 \ \underline{/12.02^\circ} \qquad VA$$

$$P.F = Cos(12.02)$$

$$P.F = 0.978 \ Lagging$$

Power Factor Correction

Power factor Correction is the process
of increasing the power factor
without altering the voltage or
Current to the original load.

Power factor Correction is necessary
for Economic Reason.

$$PF = \cos(\theta_v - \phi_i)$$

For R

$$P.F = 1$$
$$Q_R = 0$$

∴ To improve the power factor
we must decrease the Reactive
power

∴ For inductive Circuit, we add
a Capacitor in parallel to the
load.

$$Q_c = Q_{Final} - Q_{init}$$

$$C = -\frac{Q_c}{\omega V_{rms}^2}$$

Example

A certain industrial plant Consumes
1 MW at 0.7 Lagging Power factor
and a 2300 V rms.

What is the minimum Capacitor required
to improve the power factor to
0.9 Lagging. $\omega = 377$ v/s

$Q_{ini} = P_{av} \tan \left[\cos^{-1} PF_1 \right]$

$Q_{ini} = 1M \tan \left[\cos^{-1} 0.7 \right]$

$Q_{ini} = 1.02 \text{ MVARS}$

$Q_{Fin} = P_{av} \tan \left[\cos^{-1} PF_2 \right]$

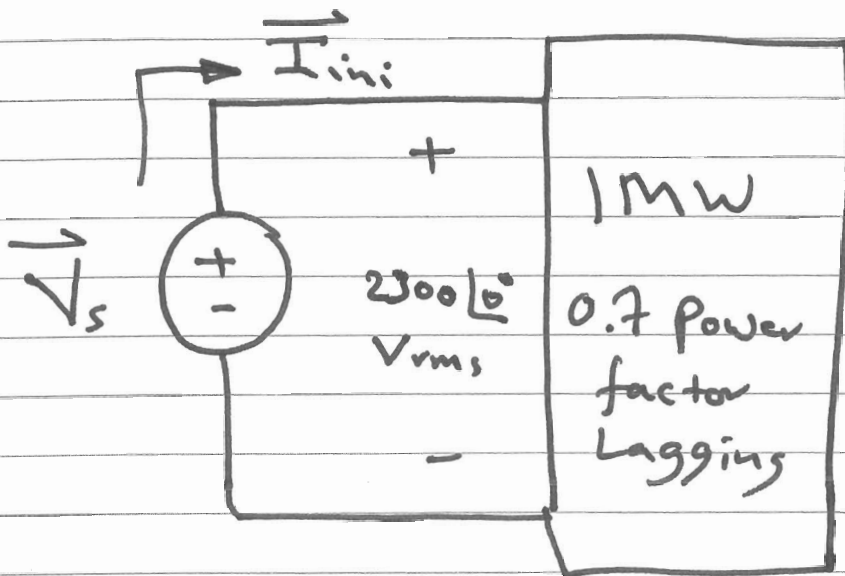$Q_{Fin} = P_{av} \tan \left[\cos^{-1} 0.9 \right]$

$Q_{Fin} = 0.484 \text{ MVARS}$
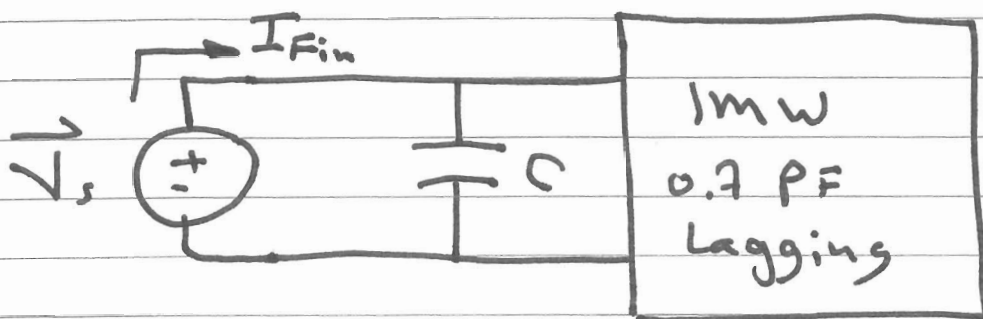
$Q_c = Q_{Fin} - Q_{ini}$

$Q_c = -0.536 \text{ MVARS}$

$$Q_c = -\frac{V_{rms}^2}{X_c}$$

$$Q_c = -WC\, V_{rms}^2$$

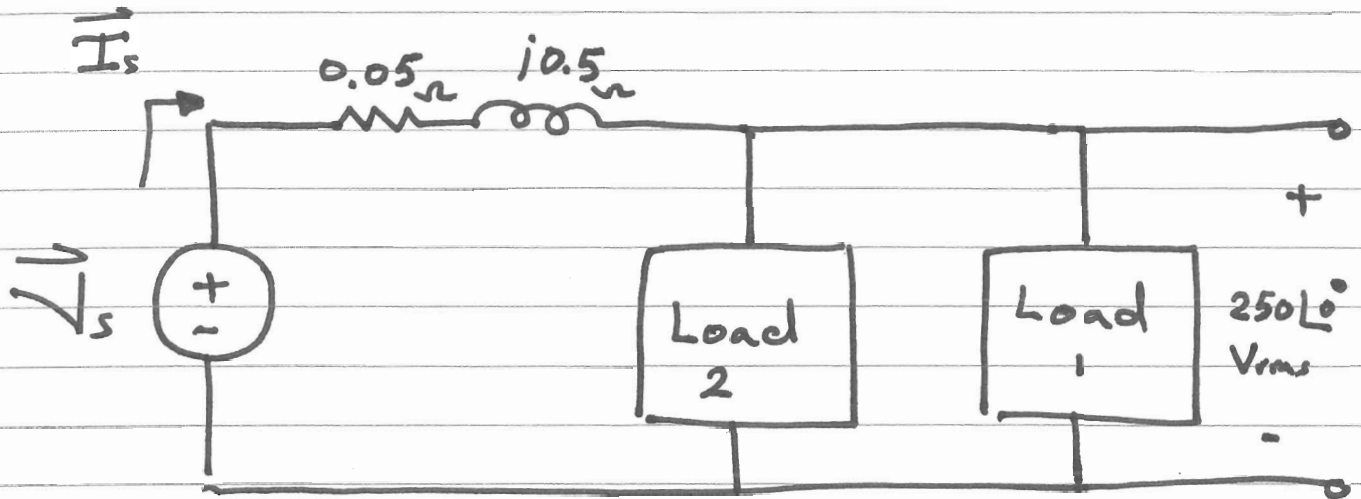$$\therefore\; C = -\frac{Q_c}{W V_{rms}^2} = 269\, MF$$

$$I_{ini} = \frac{P_{av}}{(V_{rms})(PF_1)} = 621\, A_{rms}$$

$$I_{Fin} = \frac{P_{av}}{(V_{rms})(PF_2)} = 483\, A_{rms}$$

Example

Load 1 :   8 KW ; 0.8 PF Leading

Load 2 :   20 KVA ;   0.6 PF Lagging

1) Determine the power factor of the two
   Loads in parallel

2) Determine the apparent power required
   to supply the loads; the magnitude of
   the Current I_s ; the average power loss in
   the transmission Line

3) Compute the value of the Capacitor that
   would Correct the power factor to 1 if

placed in parallel with the two loads

$$\omega = 377 \text{ r/s}$$

4) Repeat step 2

Load 1 : 8 KW ; 0.8 PF_1 leading

Load 2 : 20 KVA ; 0.6 PF_2 Lagging

P_{av_1} = 8000 W

$\therefore Q_1 = - P_{av_1} \tan\left[\cos^{-1}(PF_1)\right] = -6000$ VARS

$\therefore \vec{S_1} = P_{av_1} + j Q_1$

$\vec{S_1} = 8000 - j 6000$ VA

P_{a_2} = 20000 VA ; PF_2 = 0.6 Lagging

$\therefore P_{av_2} = P_a \cdot PF_2 = 12000$ W

$Q_2 = P_{av_2} \tan\left[\cos^{-1}(PF_2)\right] = +16000$ VARS

$\therefore \vec{S_2} = P_{av_2} + j Q_2$

$\vec{S_2} = 12000 + j 16000$ VA

$\vec{S_{T_L}} = \vec{S_1} + \vec{S_2}$

$\vec{S_{T_L}} = 20000 + j 10000$ VA

$\vec{S_{T_L}} = 22360 \underline{\big| 26.565°}$ VA

$\therefore PF_{Loads} = \cos(26.565) = 0.8944$ Lagging

$$\vec{S}_{L_T} = \vec{V}_{rms} \vec{I}_s^*$$

$$\therefore \vec{I}_s^* = \frac{\vec{S}_{L_T}}{\vec{V}_{rms}} = \frac{22360 \, \underline{|26.565°}}{250}$$

$$\therefore \vec{I}_s = 89.44 \, \underline{|-26.565°} \quad A \, rms$$

Since $\vec{S}_{L_T} = 22360 \, \underline{|26.565°}$

$$\therefore P_a = |\vec{S}_{L_T}| = 22360 \quad \vee A$$

$$P_{av \atop Loss} = |\vec{I}_s|^2 \cdot (0.05)$$

$$= 400 \, W$$

3) Since $\vec{S}_{L_T} = 20000 + j \, 10000$

$$\therefore Q_{ini} = 10000 \quad VARS$$

$$Q_{Fin} = 0$$

$$\therefore Q_c = Q_{Fin} - Q_{ini} = -10000 \, VARS$$

$$\therefore C = \frac{-Q_c}{W V_{rms}^2} = 424.4 \, \mu F$$

4) Since $Q_{Fin} = 0$

$\therefore \vec{S_F} = P_{av} = 20000 \; VA$

$\therefore P_a = P_{av} = 20000 \; VA$

$\therefore \vec{S_F} = 20000 \lfloor 0° \; VA$

$\vec{S_F} = \vec{V_{rms}} \cdot \vec{I_s^*}$

$\therefore \vec{I_s^*} = \dfrac{20000 \lfloor 0°}{250 \lfloor 0°} = 80 \lfloor 0° \; A_{rms}$
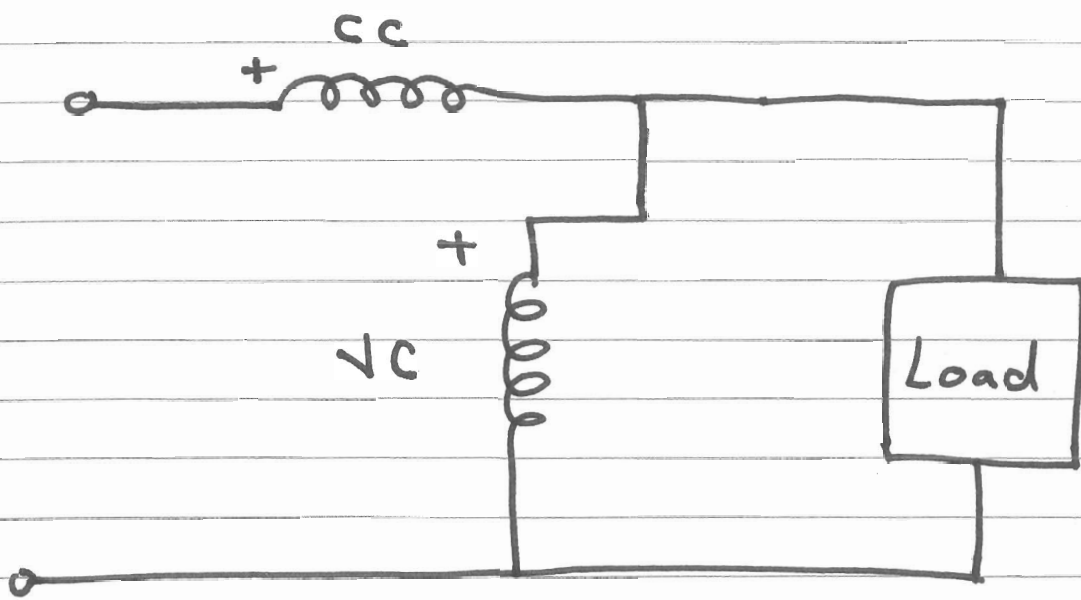
$\therefore \vec{I_s} = 80 \lfloor 0° \; A_{rms}$

$P_{Loss} = |I_s|^2 \cdot (0.05)$

$P_{Loss} = 320 \; W$

Power Measurement

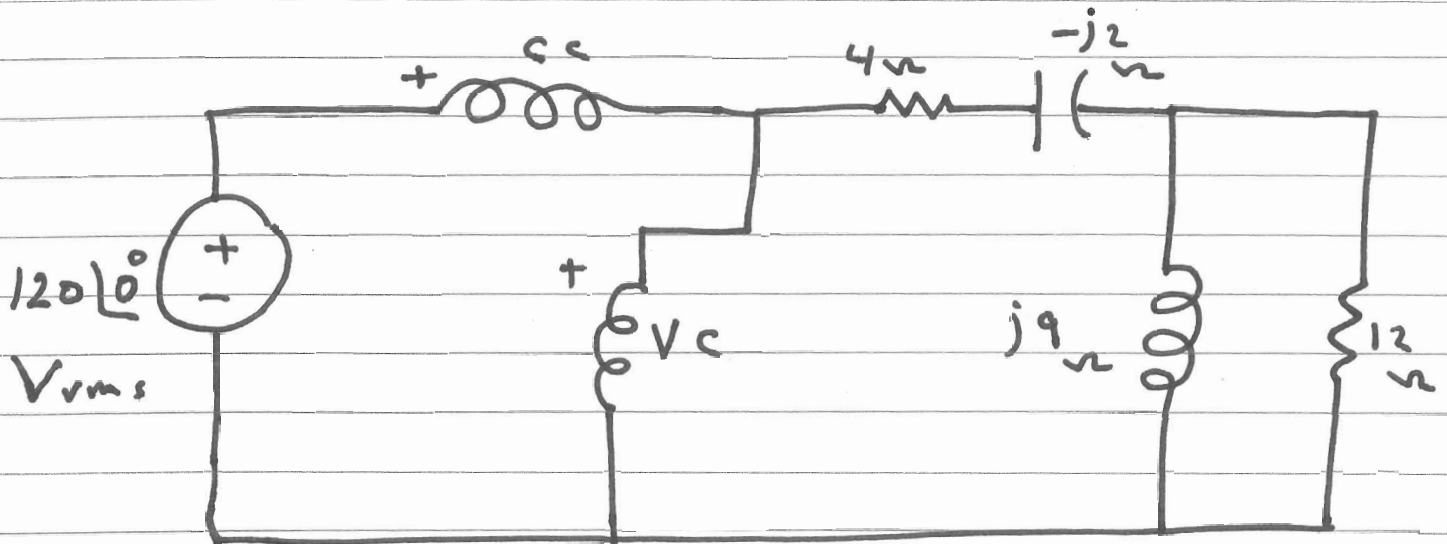Wattmeter is the instrument for measuring the average power

Two Coils are used, the high impedance Voltage Coil and the Low impedance Current Coil.

$$P = V_{rms} \, I_{rms} \, \cos(\theta_v - \phi_i)$$

Example

Find the Wattmeter reading

$$Z = 4 - j2 + (j9 \| 12)$$

$$Z = 9.13 \angle 24.32°$$

$$\vec{I} = \frac{120 \angle 0°}{9.13 \angle 24.32°} = 13.14 \angle -24.32° \; A_{rms}$$

$$P = (120)(13.14) \cos(0 + 24.32°)$$

$$P = 1436.9 \; w$$